國立交通大學

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碩 士 論 文

多變量 JS 管制圖

Multivariate JS Control Chart

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摘　　要

對於多個變數的管制圖，這篇論文探討如何改進階段一參數估計來建立更精確的階段二管制界限。利用文獻上已證明，當多維度常態分配的平均數未知時，在平方損失的情況下，若變數維度高於 2，一般常使用的樣本平均數估計量具有不容許性。許多的收縮估計量已經被證明比傳統估計量有更好的表現，例如 James-Stein 估計量。在低損壞或高良率的製程，我們可使用 James-Stein 估計量來改進階段一的參數估計。這篇論文提出利用改進估計量來構造多維度管制界限。利用電腦模擬和數值計算的結果顯示調整過後的管制界限比現有的管制界限有明顯的改進效果。

關鍵字：Hotelling$-T^2$ 管制圖、多變量累積和、多變量指數加權移動平均、James-Stein 估計量、平均連串長度。

Multivariate JS Control Chart

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Abstract

In this study, we focus on improving Phase I study to construct more accurate Phase II control limits for multivariate variables. For a multivariate normal distribution with unknown mean, the usual mean estimator is known to be inadmissible under the squared error loss when the dimension of variables is greater than 2. Shrinkage estimators, such as the James-Stein etc., are shown to have better performance than the conventional estimator in the literature. When considering a low defect or high yield process, we utilize the James-Stein estimator to improve the Phase I parameter estimation. Multivariate control limits based on the improved estimator are proposed in this study. The adjusted control limits are shown to have substantial improvements than the existing control limits.

Key words: Hotelling$-T^2$ chart, MCUSUM, MEWMA, James-Stein estimator, average run length.

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兩年的碩士生涯很快地過了，還有許多來不及遺留在統計所上。

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Contents

List of Tables

List of Figures

1 Introduction

Multivariate control charts are a useful tool in detecting a shift in a manufacturing process when the interesting characteristics are multivariate. Early research goes back to the T^2 control (Hotelling 1947), which detects a shift based on only the recent observation, resulting in insensitive detection of small mean shift.

There are many other multivariate control charts proposed in the literature. Crosier (1988) proposed multivariate CUSUM control charts by either reducing each multivariate observation to a scalar or formed a CUSUM vector from the observations. Pignatiello and Runger (1990) proposed a CUSUM control chart, MC1, and showed it has better performance than several other CUSUM control charts. In addition, the MEWMA control chart established by Lowry (1992) uses all the data information from the early to the last observation to construct a chart which has the advantage of smaller average run length for detecting small shifts in the process mean. Reynolds (2006, 2008) proposed combining different multivariate control charts for monitor. More established charts refer to Seber (1984), Tracy et al. (1992), Jolayemi (1995), Aparisi (1996), Sullivan and Woodall (1996), Chou et al. (1999), Kano et al. (2001), Mason et al. (2001, 2003), Kim et al. (2003), Mahmoud and Woodall (2004), He and Grigoryan (2005), Reynolds and Kim (2005), Koutras et al. (2006).

Most of the control limits studies focus on providing improved methods for Phase II monitoring, the investigation of the Phase I study is not as depth as the Phase II study in the literature. Before monitoring the process, one must use Phase I studies to estimate the in-control process parameters to determine the location of the control limits. Therefore the accuracy of Phase I estimation is more important than Phase II study.

For monitoring a univariate characteristic which is assumed to follow a normal distribution, the number of unknown parameters is usually less than or equal to 2. For example there are only two parameters, mean and variance, of a univariate normal distribution. In this case, the sample mean and sample variance are optimal estimators under the squared error loss. On the contrary, when the number of unknown parameters is greater than 2 such as the case of a multivariate normal distribution, it is well known that the conventional estimators are not optimal under the squared error loss . For example, the sample mean is not admissible for estimating the multivariate normal mean under the squared error loss when the dimension is greater than 2 (Hwang and Casella 1982, DasGupta, Ghosh and Zen 1995). Due to the inadmissible property of the sample mean, the parameter estimation in Phase I for monitoring multivariate process can be improved.

In a high yield or low defective manufacturing process, although the true value of the parameter is unknown, we expect that the values should be within a region because the shift of this high yield process should be small. Thus, in this study, we propose using shrinkage estimators by shrinking the estimators to a center point of this region instead of the usual estimator in Phase I estimation. One of the well-known shrinkage estimators is the James-Stein estimator.

In this study, we propose adjusted multivariate control charts based on the James-Stein estimator and show substantial improvements of the adjusted charts compared with the conventional charts in a high-yield manufacturing process.

The comparison is to compare average run lengths (ARLs) for several well-known charts based on the conventional estimator and James-Stein estimator provided in this study to show the substantial improvement of the new approach.

This paper is organized as follows. The shrinkage approach is discussed in Section 2. The existing control charts when covariance matrix is known and the new approach for estimating the mean in phase I and the new charts are proposed in Section 3. In order to evaluate the performance of the proposed procedures in terms of in and out-of-control average run lengths we perform some simulation studies. The comparisons between the conventional charts and our improved charts and the ARL performances of the improved charts are shown in Section 4. An example from chemical process data is given in Section 5. Our conclusions and discussions are mentioned in Section 6.

2 Shrinkage approach

Suppose a p-dimensional sample $X_1, ..., X_n$ following a multivariate normal distribution $N(\mu, \Sigma)$, with unknown μ and known Σ where μ is a p-dimensional vector and Σ is a $p \times p$ matrix. Without loss of generality, Σ is assumed to be a $p \times p$ diagonal matrix $\sigma^2 I_{p\times p}$. The conventional estimators for μ is the sample mean $\bar{X} = \sum_{i=1}^{n} X_i/n$. Stein (1956) has proved that $\bar{X}$ is inadmissible for $p \geq 3$ for estimating μ under the squared error loss. An improved estimator, called James-Stein estimator, is proposed by James and Stein (1961) with a smaller mean squared error (MSE) than $\bar{X}$. The approach mainly shrinks the conventional estimator to a point. Since then, many studies for established shrinkage estimators are developed (Strawderman and Cohen 1971, Efron and Morris 1972, Draper and Van Nostrand 1979, Casella 1980).

In this study, we mainly used the James-Stein estimator to construct improved control charts. The standard form of the James-Stein estimator is

$$\bar{X}^{JS} = (1 - \frac{(p-2)\sigma^2}{\|\bar{X} - \nu\|^2})^+ \cdot (\bar{X} - \nu) + \nu, \tag{1}$$

where ν is a fixed vector we want to shrink, and the notation $(x)^+$ denotes

$$(x)^+ = \begin{cases} x & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

A $1-\alpha$ confidence region for μ based on the sample mean is

$$C = \{\mu : ||\bar{X} - \mu||^2 <= c\}$$

where c is the $1-\alpha$ cutoff point of a chi-square distribution with p degrees of freedom. The exact coverage probability of the confidence region C is $1-\alpha$.

The coverage probability of

$$C^{JS} = \{\mu : ||\bar{X}^{JS} - \mu||^2 <= c\}$$

is shown to have a higher coverage probability than $1 - \alpha$ analytically and numerically (Brown 1966, Joshi 1967, Morris 1977, Hwang and Casella 1982, DasGupta, Ghosh and Zen 1995).

3 Improved Control Charts

In this section, we review several well-known charts and provide adjusted forms of these charts. For simplifying the notations, we consider the case of subgroup size 1 in Phase I and Phase II processes. The notation $\bar{X}$ here denotes the sample mean from the Phase I process.

1. The T^2 control chart.

   This first control chart is the Hotelling$-T^2$ chart (Hotelling 1947). When covariance matrix is known, the test statistic for sample i is

   $$T_i = (X_i - \bar{X})'\Sigma^{-1}(X_i - \bar{X}).$$

   It gives an out-of-control signal if the statistic

   $$T_i > c_1, \tag{2}$$

   (Tracy, Young, and Mason 1992). where c_1 is an upper control limit.

2. MC1 chart.

   Pignatiello and Runger (1990) proposed MC1 control chart and showed that it can improve Crosier (1988) multivariate CUSUM control charts.

   The statistic for MC1 chart is

   $$MC1_i = max\{\|C_i\| - kn_i, 0\}, k > 0,$$

   where

   $$C_i = \sum_{k=i-n_i+1}^{i} (X_k - \bar{X}), \tag{3}$$

$$n_i = \begin{cases} 1 & \text{if } MC1_{i-1} \le 0, \\ n_{i-1}+1 & \text{if } MC1_{i-1} > 0. \end{cases}$$

and

$$\|C_i\| = \sqrt{C_i'\Sigma^{-1}C_i}.$$

MC1 chart gives an out of control signal when

$$MC1_i > c_2, \tag{4}$$

where c_2 is an upper control limit. They choose the referential parameter k to be the half of the distance of μ and μ_0, where μ_0 represents the on-target state and μ represents a specified, unacceptable off-target state, and c_2 such that the on-target ARL_0 of this chart achieves the desired value.

3. MEWMA control chart.

MEWMA chart is first developed by Lowry (1992) with the statistic

$$E_i = Z_i'\Sigma_{Z_i}^{-1}Z_i,$$

where

$$Z_i = \lambda(X_i - \bar{X}) + (1-\lambda)Z_{i-1}, \tag{5}$$

$$\Sigma_{Z_i} = \frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]\Sigma.$$

$Z_0 = 0$ and λ is a constant between 0 and 1. The parameter λ determines the rate at which 'older' data enter into the calculation of the statistic. Thus, a large value of λ gives more weight to recent data and less weight to older data. When choosing the value of λ used for weighting, it is recommended to use small values (such as 0.2) to detect small shifts, and larger values (between 0.2 and 0.4) for larger shifts.

It gives an out-of-control signal if

$$E_i > c_3, \tag{6}$$

where c_3 is a specified constant. The choice of c_3 is chosen to achieve a specified in-control ARL_0.

4. SZ & MZ combined control chart.

   Reynolds (2006, 2008) proposed combining different multivariate control charts for monitoring. One of them is the combination of SZ and MZ control chart, where SZ denotes the Hotelling$-T^2$ chart and MZ denotes the MEWMA chart.

   The combined chart signals if

$$T_i > c_4 \tag{7}$$

   in SZ chart or

$$c_\infty^{-1} G_i > c_5 \tag{8}$$

   in MZ chart, where $G_i = Z_i' \Sigma^{-1} Z_i$, and $c_\infty = \lambda/(2-\lambda)$.

   More details are given in Appendix.

In this study, we propose charts modified by James-Stein estimator for these four existing charts. The forms of the modified charts are as follows.

1. JS-T^2 control chart.

   The modified T^2 control chart is based on the statistic

$$T_i^{JS} = (X_i - \bar{X}^{JS})'\Sigma^{-1}(X_i - \bar{X}^{JS}).$$

It gives an out-of-control signal if the statistic

$$T_i^{JS} > c_6. \tag{9}$$

2. JS-MC1 chart.

The statistic for JS-MC1 chart is

$$MC1_i^{JS} = max\{\|C_i\| - kn_i, 0\}, k > 0,$$

where

$$C_i = \sum_{k=i-n_i+1}^{i} (X_k - \bar{X}^{JS}),$$

$$n_i = \begin{cases} 1 & \text{if } MC1_{i-1}^{JS} \le 0, \\ n_{i-1} + 1 & \text{if } MC1_{i-1}^{JS} > 0. \end{cases}$$

and

$$\|C_i\| = \sqrt{C_i'\Sigma^{-1}C_i}.$$

JS-MC1 chart gives an out of control signal when

$$MC1_i^{JS} > c_7, \tag{10}$$

where c_7 is an upper control limit.

3. JS-MEWMA control chart.

$$E_i^{JS} = (Z_i^{JS})'\Sigma_{Z_i}^{-1}(Z_i^{JS}),$$

where

$$Z_i^{JS} = \lambda(X_i - \bar{X}^{JS}) + (1-\lambda)Z_{i-1}^{JS},$$

$$\Sigma_{Z_i} = \frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]\Sigma.$$

$Z_0^{JS} = 0$. It gives an out-of-control signal if

$$E_i^{JS} > c_8, \tag{11}$$

where c_8 is a specified constant. The choice of c_8 is chosen to achieve a specified on-target ARL_0.

4. JS-SZ & JS-MZ combined control chart.

   The combined chart signals if

$$T_i^{JS} > c_9 \tag{12}$$

in JS-SZ chart or

$$c_\infty^{-1} G_i^{JS} > c_{10} \tag{13}$$

in JS-MZ chart, where $G_i^{JS} = (Z_i^{JS})'\Sigma^{-1}(Z_i^{JS})$.

These proposed charts are modified by replacing the sample mean with the James-Stein estimate in appropriate places. We call these modified charts as JS-type charts.

4 Simulation Comparisons

In this section, we conduct a simulation study to compare the JS-type control charts and the original control charts. Let $D = ||\mu_1 - \mu_2||$, where μ_1 is the target value and μ_2 is the process mean.

The process is stable when $\mu_1 = \mu_2$, and it is out of control when $||\mu_1 - \mu_2||$ is greater than a tolerance error. We compare the average run lengths (ARLs) of these charts. To have an objective comparison, we set the control limits of each control chart with ARL about 200 when it is in control. After that, the ARLs for different out-of-control cases are calculated. Since each chart is set to have the same ARL_0, to compare their performance, we only need to compare their ARLs for out-of-control cases. A chart with a longer ARL in a out-of-control case is not preferable than a chart with shorter ARL in a out-of-control case because the chart with longer ARL cannot detect the out-of-control signal earlier.

In Tables 1-3, we consider that the process mean is estimated from 100 samples in phase I. In practice, large sample size does not occur frequently, Tables 6-27 give control limits for small different sample size. Tables 1-3 show the ARLs for the JS-type charts and original charts for different D values. The last lines of Tables 1-3 give the c_i values used in the control limits such that they have in-control ARL_0 around 200.

To compare with the result of Reynolds (2008), we adopt the same situation of $\text{ARL}_0 = 800$ in Table 4, which shows ARLs for the JS-type SZ & MZ combined chart and SZ & MZ combined chart for different D values. Those c_i values listed in the tables are obtained by simulation using 6000 runs.

4.1 The T^2 chart comparison

Table 1 shows that the JS-T^2 chart has smaller ARL than the T^2 chart when D is greater than zero. Although the improvement is not as significant as those shown in the comparisons for the other charts discussed below, it reveals that the JS-T^2 can detect an out of control signal earlier than the T^2 charts on the average. Tables 6-9 give c_6 values of JS-T^2 charts for different ARL_0.

Table 1: ARL performances of the T^2 chart and JS-T^2 chart for $m = 100$.

	$p=3$		$p=5$		$p=10$	
D	T^2	JS-T^2	T^2	JS-T^2	T^2	JS-T^2
0.0	197.5816	197.6206	198.0983	198.2043	197.5816	197.7731
0.5	127.9476	127.6503	142.1606	141.9809	159.2788	158.5314
1.0	52.0097	51.8524	68.0545	67.7801	92.4855	91.9608
1.5	20.7174	20.5585	28.7142	28.4834	44.9762	44.2102
2.0	8.8885	8.8173	12.5098	12.4091	21.0724	20.5803
2.5	4.4215	4.4000	6.0596	5.9939	10.0698	9.8768
3.0	2.5796	2.5626	3.3609	3.3203	5.2693	5.1915
$c_1 \backslash c_6$	12.9507	12.8908	16.8934	16.7755	25.4090	25.1860

4.2 The MC1 chart comparison

The comparison of ARLs for MC1 and JS-MC1 charts are presented in Table 2. It is common that k is selected as 0.5. It shows the substantial improvement of JS-MC1 chart compared with the original chart.

The improvement is more significant when p increases. It reveals that even when the dimension of the characteristics is large, the JS-MC1 chart still can detect the shift quickly

Table 2: ARL performances of MC1 and JS-MC1 chart for $k = 0.5$ and $m = 100$.

	$p=3$		$p=5$		$p=10$	
D	MC1	JS-MC1	MC1	JS-MC1	MC1	JS-MC1
0.0	200.1542	200.7395	200.0733	200.3680	200.0495	200.0503
0.5	67.0385	45.9877	76.8583	45.7562	98.8505	48.2117
1.0	14.6455	11.9562	18.8227	12.5557	35.5640	13.8275
1.5	7.770333	6.6028	10.1753	7.0617	21.0493	8.4037
2.0	5.429167	4.6765	7.1412	5.0610	15.0847	6.1437
2.5	4.188333	3.6370	5.5480	3.9772	11.8073	4.9152
3.0	3.459333	3.0042	4.5838	3.3292	9.6903	4.1425
$c_2 \backslash c_7$	7.51	6.39	10.72	7.53	24.71	10.33

and the adjustment enhances its detective ability in high dimensional process. It improves the performance by adjusting c_7. Tables 10-11 give c_7 values for $\text{ARL}_0 = 200, 500$, and 1000.

4.3 The MEWMA chart comparison

Table 3 displays the ARL performances with $\lambda = 0.2$ and $\text{ARL}_0 = 200$ for MEWMA and JS-MEWMA charts. The results show that the JS-MEWMA chart always has a shorter ARL than the MEWMA chart when a shift occurs. It is similar as the MC1 chart case that when p increases, the improvement of JS-type chart is more significant. Note that the absolute value of difference of the two ARLs for two charts is more than 1 when $D = 0.5$. Moreover, adjusted chart signals much faster than the conventional chart when p goes larger if a shift occurs. The c_8 values for different ARL_0 are displayed in Tables 12-27.

Table 3: ARL performances of MEWMA chart and JS-MEWMA chart for $\lambda = 0.2$ and $m = 100$.

	$p = 3$		$p = 5$		$p = 10$	
D	MEWMA	JS-type	MEWMA	JS-type	MEWMA	JS-type
0.0	200.1557	200.6612	200.5333	200.5392	200.0513	200.2657
0.5	46.7785	45.5408	56.0000	52.3313	77.7200	72.4402
1.0	10.4612	10.2300	12.7418	12.0912	17.3615	16.3577
1.5	4.3692	4.2978	5.181	4.9882	6.7430	6.4725
2.0	2.2812	2.2405	2.7675	2.6743	3.6265	3.4783
2.5	1.2995	1.2812	1.6338	1.581	2.2032	2.1057
3.0	0.7863	0.7735	0.9977	0.9558	1.3935	1.3250
$c_3 \backslash c_8$	12.62	12.48	16.64	16.25	25.32	24.70

4.4 The SZ & MZ combined chart comparison

To compare the SZ & MZ combined chart and JS-SZ & JS-MZ combined chart, we adopt the same situation by setting $\text{ARL}_0 = 800$. In this case, since the ARL_0 is set to be larger than the other three charts, it is more time-consuming to obtain c_4, c_5, c_9 and c_{10} values. Thus, we consider to compare the cases of smaller m values for $p = 4$ case. Note that different from the other three charts whose performance only depends on one c_i value, the performance of SZ & MZ combined chart depends on two c_i values. To set a fix ARL_0 of SZ & MZ combined chart, there are many combinations of (c_4, c_5) which can achieve the same ARL_0 value. In this study, we use (c_4, c_5) and (c_9, c_{10}) such that the SZ & MZ combined chart and JS-SZ & JS-MZ combined chart have better performance than the other c_i values to make the comparison.

For convenience, we refer the conventional combined chart to RS2008. Table 4 compares the ARL performances of two combined control procedures with the control limits at the bottom line.

In the $D = 0.5$ case of Table 4, when the sample size m of phase I process is 10, SZ & MZ combined chart detects a small shift after 392 data collected on the average, however, JS-type chart signals just 76 data on the average. The improvement obtained by JS-SZ & MZ combined chart compared with JS-SZ & MZ combined chart is more substantial than the other charts comparison. The comparison for the other m values are shown in Table 28.

Combined the above results, the JS-type charts are shown to be competitors of the conventional charts. From Tables 1-3, when $p = 3$, JS-MEWMA chart has the shortest ARL_1 among the three JS-type charts no matter what D value is. The JS-T^2 chart has longest ARL_1 for $D \leq 2.5$, however, the JS-MC1 chart signals latest in the $D = 3$ case. For the case of $p = 10$, the performance of the JS-T^2 chart is worse than the other charts no matter how far the process mean shifts. The JS-MC1 chart has better ARL_1 performance than the JS-MEWMA chart if $D \leq 1$, whereas JS-MEWMA outperforms JS-MC1 when $D \geq 1.5$.

Table 4: ARL performances for combined chart with $p = 4$ and $\lambda = 0.026$.

	$m = 10$		20		30	
D	RS2008	JS-type	RS2008	JS-type	RS2008	JS-type
0.0	800.8250	800.9403	799.6271	800.7813	801.1713	802.5317
0.5	392.9000	76.3125	248.0854	73.2663	175.0188	73.3125
1.0	85.3417	27.1350	44.4704	25.3242	34.4675	24.7525
1.5	29.0950	15.3629	21.7117	15.0525	19.5133	14.6383
2.0	17.7017	10.7117	14.1500	10.3688	13.0075	10.0933
2.5	11.4178	7.4204	9.5758	7.3575	8.8004	7.1092
3.0	7.0156	5.1179	6.0725	4.9758	5.9317	4.9188
4.0	2.6428	2.2213	2.3229	2.2017	2.2821	2.1071
5.0	1.3539	1.2654	1.2842	1.2467	1.2729	1.2367
$c_5 \backslash c_{10}$	52.1250	21.3708	37.0550	20.1323	30.8600	19.4980
$c_4 \backslash c_9$	21.2350	20.2868	20.6075	20.1227	20.5225	19.8993

	$m = 40$		50	
D	RS2008	JS-type	RS2008	JS-type
0.0	800.8084	802.1742	801.4341	802.0572
0.5	142.5094	67.4667	115.4050	62.4083
1.0	32.2967	23.9006	30.2922	24.0072
1.5	18.4289	14.7022	17.5456	14.4250
2.0	12.3044	9.9661	11.8361	9.7272
2.5	8.2222	7.0789	8.2239	7.0050
3.0	5.5378	4.8778	5.4817	4.7278
4.0	2.3183	2.0906	2.2244	2.0839
5.0	1.2828	1.2578	1.2556	1.2317
$c_5 \backslash c_{10}$	27.7025	18.9847	25.6675	18.5525
$c_4 \backslash c_9$	20.4925	19.8100	20.4525	19.7275

5 Example

In this section, we use a data example on page 516 of Montgomery (2005) to illustrate the proposed method and show the substantial improvement of the JS-type charts. The data include 4 process variables from a chemical process, which are shown in Table 5. By a similar way as in Montgomery (2005), the first 20 are used for Phase I examination, and the other 10 are used for Phase II monitoring. There is a mean shift at about the 24th sample to the 25th sample. The monitoring process for the 10 points of original charts and JS-type charts are presented in Figures 1-4. Figure 1 shows that the T^2 chart detects an out-of-control signal at the 10th point and the JS-T^2 chart can accurately signal at the 4th point. Figure 2 shows that MC1 chart detects an out-of-control signal at the 10th point and the JS-MC1 chart can detect an out-of-control signal at the 8th point. Figure 3 shows that MEWMA chart detects an out-of-control signal at the 8th point and the JS-MEWMA chart can detect an out-of-control signal at the 6th point. Figure 4 shows the outcomes of two combined charts. The lower line and the upper line on the left panel of Figure 4 are the control limits for SZ and MZ, respectively. The control limits for JS-SZ and JS-MZ on the right panel overlap in this case since they are very close. In this example, both charts do not signal. Combining the above results, we conclude that the JS-type charts can detect an out-of-control signal earlier than the existing charts when a shift occurs.

For comparing the existing charts, both T^2 chart and MC1 chart do not detect an out-of-control signal until the last sample. In this case, the MEMMA chart can detect

Table 5: Chemical process data.

i	x_1	x_2	x_3	x_4	i	x_1	x_2	x_3	x_4
1	10	20.7	13.6	15.5	16	9.7	20.1	10	16.6
2	10.5	19.9	18.1	14.8	17	8.3	18.4	12.5	14.2
3	9.7	20	16.1	16.5	18	11.9	21.8	14.1	16.2
4	9.8	20.2	19.1	17.1	19	10.3	20.5	15.6	15.1
5	11.7	21.5	19.8	18.3	20	8.9	19	8.5	14.7
6	11	20.9	10.3	13.8	21	9.9	20	15.4	15.9
7	8.7	18.8	16.9	16.8	22	8.7	19	9.9	16.8
8	9.5	19.3	15.3	12.2	23	11.5	21.8	19.3	12.1
9	10.1	19.4	16.2	15.8	24	15.9	24.6	14.7	15.3
10	9.5	19.6	13.6	14.5	25	12.6	23.9	17.1	14.2
11	10.5	20.3	17	16.5	26	14.9	25	16.3	16.6
12	9.2	19	11.5	16.3	27	9.9	23.7	11.9	18.1
13	11.3	21.6	14	18.7	28	12.8	26.3	13.5	13.7
14	10	19.8	14	15.9	29	13.1	26.1	10.9	16.8
15	8.5	19.2	17.4	15.8	30	9.8	25.8	14.8	15

an out-of-control signal better than the two existing charts. For comparing the JS-type charts, the JS-T^2 chart can accurately detect an out-of-control signal at the 4th points, which is better than the three adjusted charts.

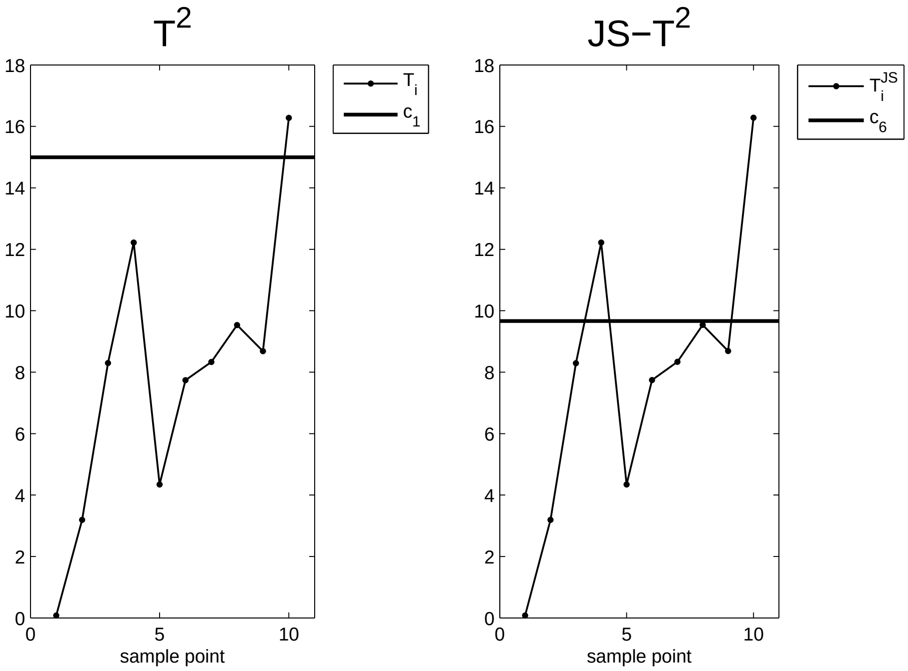

Figure 1: T^2 chart and JS-T^2 chart for monitoring the last ten points in Table 5

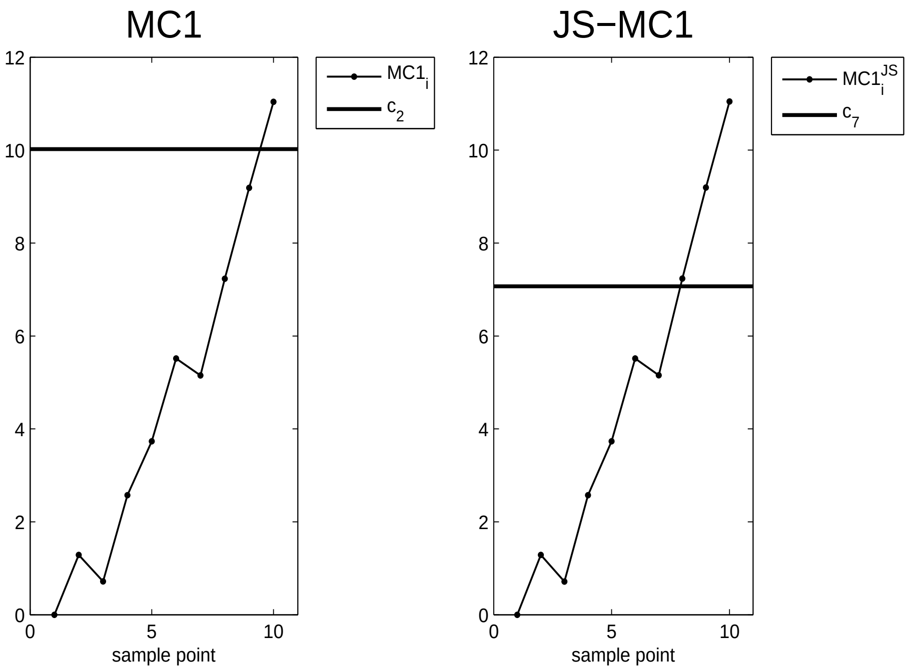

Figure 2: MC1 chart and JS-MC1 chart for monitoring the last ten points in Table 5 with $k = 0.5$

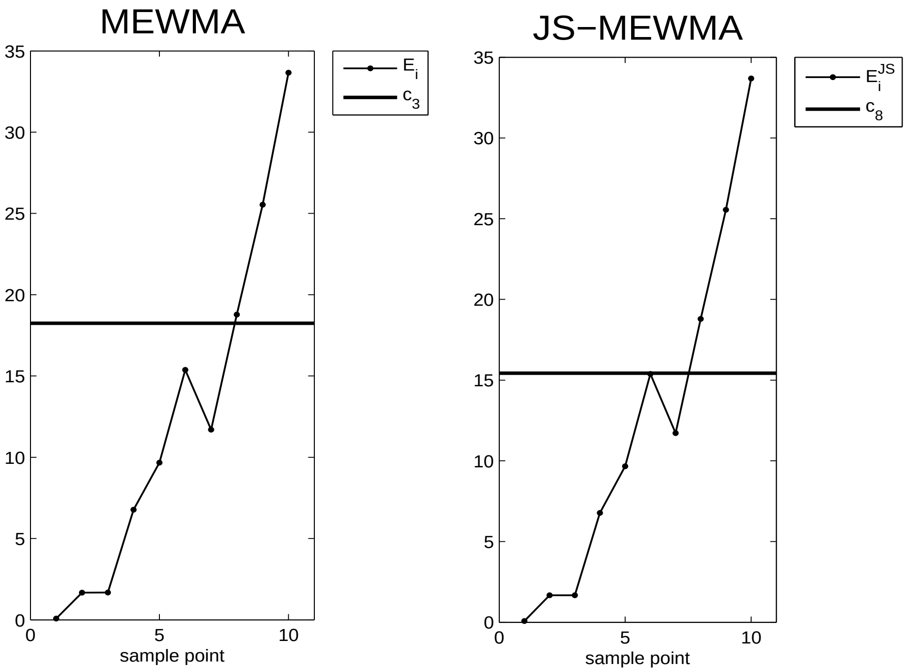

Figure 3: MEWMA chart and JS-MEWMA for monitoring the last ten points in Table 5 with $\lambda = 0.2$

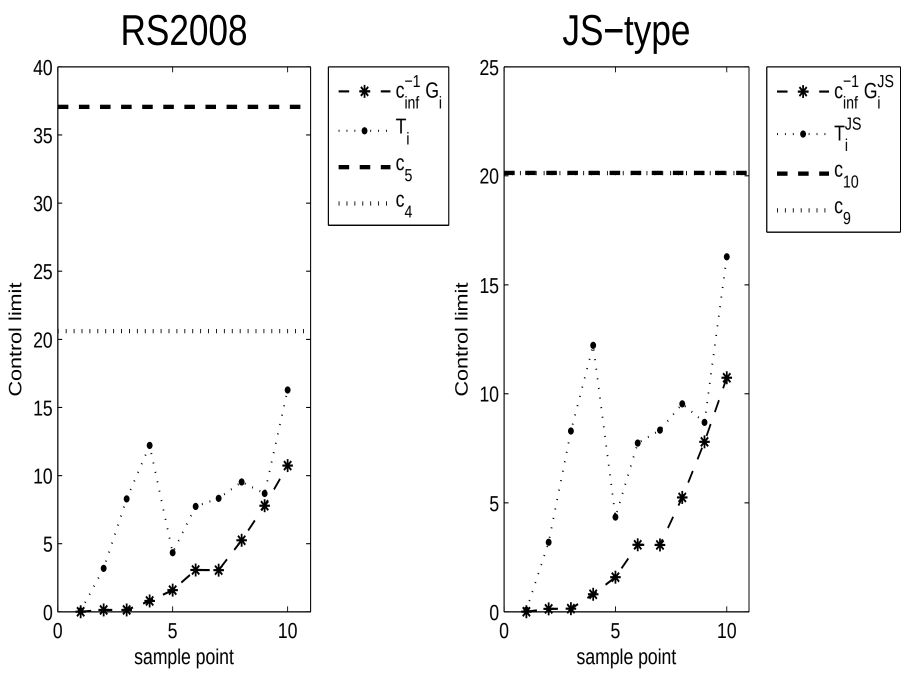

Figure 4: SZ & MZ combined chart and JS-type chart for monitoring the last ten points in Table 5 with $\lambda = 0.026$

6 Conclusion

In this study, adjusted charts based on James-Stein estimator for monitoring a high yield process are proposed. Since in a low defect or high yield process, the true parameter values of the distributions of interested variables are within a region, shrinkage estimators centered at a central point of the region can be expected to outperform the usual estimators. Thus, we adopt the well-known James-Stein estimators in constructing improved charts. Nowadays, processes usually come with a high yield in order to earn a great benefit. Off-line examination is more important than before. In this paper, we compare the ARL performances of T^2 chart, MC1 chart, MEWMA chart, and the combined SZ & MZ chart. Adjusted control charts have better performances than conventional ones, and JS-SZ & JS-MZ combined chart has the most significant improvement than the other adjusted charts. The control limits c_i of the proposed JS-type charts are obtained by simulation and tabulated in this study.

The proposed charts can signal more accurately when the process is out of control and have the less probability of signal when the process is in control. They certainly can provide useful alternatives in monitoring process.

Appendix

Details for SZ & MZ combined chart:

It will be convenient to let σ represent the vector of standard deviations of the p variables (i.e, the square roots of the diagonal elements of Σ). Let μ_0, Σ_0, and σ_0 represent the in-control values for μ, Σ, and σ, respectively. In practice, the in-control parameter values would be estimated in Phase I. We use $\bar{X}$ to estimate the process mean μ_0 and assume the covariance matrix Σ_0 is known. Let X_{ij} represent the sample point i $(i = 1, 2, \ldots, m)$ for variable j $(j = 1, 2, \ldots, p)$, and let

$$Z_{ij} = (X_{ij} - \bar{X}_j)/\sigma_{0j}.$$

where $\bar{X}_j$ is the jth component of $\bar{X}$ and σ_{0j} is the jth component of σ_0. The SZ chart is based on the statistic

$$(Z_{i1}, Z_{i2}, \ldots, Z_{ip})\Sigma_{Z0}^{-1}(Z_{i1}, Z_{i2}, \ldots, Z_{ip})'$$

with an upper control limit UCL_1.

The MZ control chart is based on the following values. Let

$$E_{ij}^Z = \lambda Z_{ij} + (1-\lambda)E_{i-1,j}^Z, j = 1, 2, \ldots, p,$$

where $E_{0j}^Z = 0$. And let constant $c_\infty = \lambda/(2-\lambda)$. The control chart uses the statistic

$$M_i^Z = c_\infty^{-1}(E_{i1}^Z, E_{i2}^Z, \ldots, E_{ip}^Z)\Sigma_{Z0}^{-1}(E_{i1}^Z, E_{i2}^Z, \ldots, E_{ip}^Z)'$$

where Σ_{Z0} is the in-control covariance matrix of the standardized observations. It signals if M_i^Z exceeds an upper control limit UCL_2, where UCL_2 is a specified constant. The choice of UCL_2 is chosen to achieve a specified on-target ARL_0.

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Table 6: Values of c_6 with $\mathrm{ARL}_0 = 10$.

m	p = 3	p = 4	p = 5	p = 6	p = 7	p = 8	p = 9	p = 10
5	6.89	8.32	9.72	11.10	12.47	13.79	15.11	16.37
6	6.79	8.25	9.64	11.04	12.40	13.71	15.02	16.31
7	6.71	8.18	9.61	10.98	12.33	13.66	14.97	16.27
8	6.66	8.12	9.56	10.93	12.29	13.63	14.95	16.24
9	6.61	8.10	9.51	10.90	12.28	13.59	14.91	16.21
10	6.57	8.05	9.50	10.88	12.24	13.58	14.88	16.19
11	6.55	8.03	9.47	10.86	12.22	13.55	14.87	16.16
12	6.52	8.02	9.45	10.84	12.20	13.54	14.85	16.15
13	6.50	7.99	9.43	10.83	12.19	13.52	14.84	16.14
14	6.49	7.97	9.42	10.81	12.17	13.51	14.83	16.13
15	6.47	7.97	9.41	10.80	12.17	13.50	14.82	16.12
16	6.45	7.95	9.39	10.79	12.16	13.50	14.80	16.11
17	6.44	7.94	9.39	10.79	12.15	13.49	14.80	16.10
18	6.43	7.94	9.38	10.78	12.14	13.48	14.80	16.10
19	6.42	7.93	9.37	10.77	12.13	13.48	14.79	16.10
20	6.41	7.92	9.36	10.77	12.13	13.47	14.79	16.09
25	6.38	7.89	9.34	10.74	12.10	13.45	14.77	16.07
30	6.36	7.87	9.32	10.72	12.09	13.43	14.75	16.05

Table 7: Values of c_6 with $\text{ARL}_0 = 20$.

m	p = 3	p = 4	p = 5	p = 6	p = 7	p = 8	p = 9	p = 10
5	8.59	10.14	11.65	13.12	14.58	15.98	17.37	18.76
6	8.46	10.04	11.57	13.04	14.49	15.90	17.30	18.69
7	7.95	9.60	11.17	12.68	14.15	15.59	16.99	18.38
8	8.30	9.90	11.44	12.93	14.40	15.80	17.21	18.58
9	8.26	9.86	11.39	12.91	14.35	15.78	17.18	18.56
10	8.21	9.83	11.37	12.86	14.31	15.75	17.14	18.52
11	8.18	9.79	11.33	12.85	14.29	15.73	17.13	18.51
12	8.14	9.76	11.31	12.81	14.28	15.70	17.12	18.49
13	8.12	9.75	11.31	12.79	14.26	15.70	17.10	18.48
14	8.10	9.73	11.28	12.78	14.25	15.67	17.09	18.47
15	8.08	9.71	11.28	12.77	14.24	15.67	17.08	18.46
16	8.06	9.70	11.26	12.76	14.23	15.66	17.07	18.45
17	8.05	9.69	11.25	12.75	14.22	15.65	17.05	18.44
18	8.04	9.67	11.23	12.74	14.21	15.64	17.05	18.43
19	8.03	9.67	11.23	12.74	14.20	15.63	17.04	18.42
20	8.02	9.66	11.23	12.73	14.19	15.63	17.04	18.42
25	7.98	9.62	11.19	12.70	14.16	15.60	17.01	18.39
30	7.95	9.60	11.17	12.68	14.15	15.59	16.99	18.38

Table 8: Values of c_6 with $\mathrm{ARL}_0 = 100$.

m	p = 3	p = 4	p = 5	p = 6	p = 7	p = 8	p = 9	p = 10
5	12.39	14.12	15.85	17.51	19.11	20.69	22.26	23.75
6	12.24	14.03	15.74	17.36	19.02	20.61	22.15	23.66
7	12.11	13.90	15.63	17.30	18.93	20.52	22.08	23.59
8	12.03	13.83	15.57	17.25	18.89	20.47	22.01	23.54
9	11.96	13.77	15.49	17.21	18.84	20.43	21.98	23.51
10	11.90	13.73	15.47	17.16	18.79	20.38	21.96	23.48
11	11.84	13.68	15.43	17.12	18.78	20.37	21.92	23.47
12	11.80	13.66	15.40	17.10	18.75	20.34	21.91	23.44
13	11.77	13.64	15.39	17.08	18.72	20.32	21.88	23.42
14	11.74	13.61	15.37	17.06	18.71	20.31	21.87	23.40
15	11.72	13.58	15.36	17.05	18.69	20.28	21.85	23.39
16	11.69	13.55	15.32	17.03	18.68	20.28	21.84	23.38
17	11.69	13.53	15.32	17.01	18.67	20.26	21.83	23.37
18	11.66	13.53	15.30	17.00	18.65	20.25	21.82	23.35
19	11.63	13.52	15.29	16.99	18.65	20.25	21.81	23.34
20	11.63	13.50	15.28	16.99	18.63	20.24	21.80	23.34
25	11.57	13.47	15.24	16.95	18.60	20.20	21.77	23.31
30	11.53	13.42	15.21	16.92	18.58	20.19	21.75	23.29

Table 9: Values of c_6 with $\mathrm{ARL}_0 = 200$.

m	p = 3	p = 4	p = 5	p = 6	p = 7	p = 8	p = 9	p = 10
5	13.99	15.81	17.58	19.31	20.95	22.61	24.18	25.76
6	13.84	15.65	17.46	19.19	20.84	22.48	24.11	25.66
7	13.67	15.56	17.33	19.09	20.77	22.42	24.03	25.59
8	13.59	15.46	17.26	19.02	20.70	22.35	23.96	25.55
9	13.51	15.39	17.22	18.98	20.65	22.31	23.92	25.50
10	13.44	15.33	17.16	18.91	20.63	22.29	23.90	25.46
11	13.39	15.30	17.15	18.89	20.60	22.25	23.86	25.44
12	13.34	15.25	17.10	18.86	20.56	22.21	23.83	25.42
13	13.32	15.23	17.07	18.83	20.54	22.19	23.80	25.40
14	13.26	15.19	17.05	18.81	20.51	22.17	23.79	25.38
15	13.24	15.18	17.03	18.79	20.50	22.18	23.78	25.36
16	13.21	15.16	17.00	18.78	20.48	22.14	23.78	25.34
17	13.19	15.14	17.00	18.77	20.47	22.13	23.76	25.34
18	13.17	15.13	16.98	18.74	20.47	22.11	23.75	25.34
19	13.17	15.10	16.97	18.73	20.45	22.11	23.73	25.31
20	13.14	15.10	16.95	18.73	20.43	22.11	23.73	25.31
25	13.08	15.05	16.90	18.69	20.40	22.07	23.70	25.28
30	13.04	15.01	16.88	18.66	20.37	22.05	23.67	25.25

Table 10: Values of c_7 with $k = 0.5$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	8.09	12.11	15.65	7.76	11.12	13.67	7.58	10.52	12.17
p = 4	8.14	11.15	13.27	7.91	10.56	12.4	7.68	10.16	11.75
p = 5	8.57	11.26	12.91	8.44	10.84	12.32	8.35	10.46	11.97
p = 6	8.95	11.57	12.74	8.92	11.13	12.62	8.83	11.12	12.28
p = 7	9.58	12.07	13.34	9.45	11.73	13.02	9.3	11.56	12.86
p = 8	10.03	12.51	13.95	9.98	12.28	13.51	9.91	12.18	13.27
p = 9	10.62	13.03	14.49	10.53	12.72	14.19	10.33	12.66	13.88
p = 10	11.11	13.73	14.96	11.01	13.47	14.7	10.94	13.18	14.63
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	7.43	10.07	12.04	7.23	9.79	11.27	7.11	9.36	10.9
p = 4	7.68	9.94	11.5	7.67	9.9	11.2	7.48	9.67	10.83
p = 5	8.24	10.39	11.7	8.11	10.2	11.48	8.08	10.16	11.27
p = 6	8.71	10.77	12.11	8.67	10.67	11.89	8.57	10.56	11.78
p = 7	9.26	11.48	12.64	9.17	11.3	12.57	9.17	11.23	12.38
p = 8	9.84	11.95	13.21	9.71	11.89	13.04	9.61	11.86	12.89
p = 9	10.23	12.47	13.79	10.21	12.35	13.65	10.16	12.39	13.54
p = 10	10.85	13.13	14.35	10.71	12.98	14.19	10.7	12.92	14.13
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	6.98	9.27	10.66	6.97	8.97	10.28	6.83	8.92	10.24
p = 4	7.43	9.49	10.64	7.38	9.33	10.51	7.34	9.19	10.31
p = 5	7.97	10	11.16	7.91	9.82	10.89	7.79	9.76	10.99
p = 6	8.49	10.51	11.63	8.47	10.38	11.6	8.42	10.4	11.53
p = 7	9.06	11.09	12.23	9.02	10.95	12.14	8.98	10.94	12.1
p = 8	9.6	11.68	12.92	9.57	11.61	12.71	9.52	11.53	12.82
p = 9	10.11	12.26	13.49	10.1	12.19	13.39	10.05	12.09	13.3
p = 10	10.74	12.87	14.02	10.58	12.72	13.99	10.62	12.74	13.85

Table 11: Values of c_7 with $k = 0.5$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	6.79	8.7	9.92	6.76	8.71	12.04	6.67	8.62	9.8
p = 4	7.35	9.17	10.35	7.28	9.08	11.78	7.2	8.98	10.16
p = 5	7.81	9.67	10.81	7.81	9.66	12.31	7.8	9.53	10.71
p = 6	8.37	10.25	11.33	8.34	10.26	12.8	8.36	10.07	11.3
p = 7	8.9	10.95	11.94	8.89	10.81	13.3	8.87	10.65	11.94
p = 8	9.48	11.4	12.6	9.43	11.35	14.02	9.36	11.36	12.49
p = 9	9.99	12.02	13.29	9.97	11.94	14.49	9.97	11.92	13.08
p = 10	10.65	12.63	13.92	10.51	12.61	15.14	10.47	12.44	13.72
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	6.64	8.63	9.57	6.6	8.39	9.59	6.59	8.29	9.45
p = 4	7.19	8.92	10.11	7.1	8.95	9.98	7.13	8.89	9.95
p = 5	7.73	9.57	10.59	7.7	9.47	10.53	7.68	9.42	10.54
p = 6	8.28	10.12	11.24	8.27	10.09	11.16	8.24	10.02	11.13
p = 7	8.89	10.73	11.88	8.85	10.71	11.76	8.78	10.59	11.73
p = 8	9.37	11.31	12.4	9.32	11.24	12.39	9.32	11.23	12.31
p = 9	9.93	11.86	13.11	9.95	11.79	13.02	9.86	11.81	12.97
p = 10	10.47	12.45	13.72	10.48	12.4	13.63	10.41	12.33	13.61
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	6.57	8.28	9.45	6.42	8.04	9.13	6.3	7.86	8.87
p = 4	7.07	8.79	9.87	6.96	8.58	9.75	6.86	8.46	9.54
p = 5	7.66	9.41	10.46	7.55	9.26	10.25	7.48	9.08	10.15
p = 6	8.25	9.99	11.05	8.15	9.87	10.92	8.07	9.7	10.74
p = 7	8.73	10.57	11.68	8.67	10.49	11.53	8.57	10.31	11.41
p = 8	9.34	11.13	12.33	9.2	10.98	12.22	9.21	10.95	12.01
p = 9	9.83	11.8	13.01	9.78	11.6	12.82	9.67	11.47	12.66
p = 10	10.41	12.38	13.51	10.32	12.26	13.39	10.24	12.08	13.3

Table 12: Values of c_8 with $\lambda = 0.1$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	15.96	22.08	24.04	15.86	20.81	23.14	15.26	19.93	22.63
p = 4	17.13	22.01	24.2	16.82	21.39	23.39	16.37	20.83	22.54
p = 5	18.42	22.9	25.7	18.08	22.45	24.4	17.89	21.8	24.08
p = 6	19.98	24.25	26.71	19.52	23.69	25.81	19.35	23.51	25.35
p = 7	21.49	25.79	27.98	21.2	25.34	27.59	20.97	24.98	27.28
p = 8	23.02	27.4	29.26	22.71	26.8	29.37	22.5	26.57	28.8
p = 9	24.46	28.74	30.99	24.25	28.36	30.72	24.12	28.13	30.79
p = 10	26.06	30.51	32.83	25.75	30.09	32.13	25.64	29.76	31.89
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	15.17	19.45	21.63	14.58	19.12	21.39	14.56	18.55	20.67
p = 4	16.3	20.35	22.89	15.87	19.94	21.96	15.87	19.64	21.83
p = 5	17.74	21.65	23.6	17.66	21.22	23.44	17.33	21.37	23.34
p = 6	19.24	23.2	25.26	19.11	23.05	25.13	19.1	22.78	24.82
p = 7	20.98	24.59	26.7	20.67	24.63	26.74	20.69	24.29	26.53
p = 8	22.47	26.28	28.54	22.18	26.06	28.34	22.18	26.01	28.14
p = 9	24	28.04	30.04	23.79	27.65	30.06	23.74	27.5	29.83
p = 10	25.42	29.44	31.83	25.36	29.29	31.54	25.07	29.16	31.48
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.21	18.17	20.31	14.13	18.01	20.1	14.08	18.1	19.93
p = 4	15.7	19.41	21.54	15.55	19.19	21.3	15.5	18.99	21.22
p = 5	17.25	20.9	22.96	17.34	20.84	23	17.09	20.67	22.68
p = 6	18.93	22.58	24.59	18.77	22.35	24.48	18.74	22.31	24.37
p = 7	20.54	24.16	26.35	20.31	24.1	26.24	20.28	24.05	26.16
p = 8	22.12	25.82	28.09	21.99	25.71	27.85	21.9	25.68	27.81
p = 9	23.71	27.29	29.65	23.58	27.27	29.38	23.46	27.17	29.48
p = 10	25.13	29.1	31.18	25.17	28.76	31.22	25.01	28.73	31.1

Table 13: Values of c_8 with $\lambda = 0.1$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.7	17.53	19.59	13.67	17.62	19.46	13.72	17.33	19.21
p = 4	15.35	18.97	20.85	15.29	18.92	20.82	15.07	18.7	20.57
p = 5	16.99	20.63	22.47	16.97	20.49	22.44	16.9	20.41	22.25
p = 6	18.5	22.28	24.23	18.5	22.05	24.11	18.57	21.91	24.23
p = 7	20.22	23.85	25.88	20.13	23.9	25.88	20.05	23.59	25.89
p = 8	21.83	25.66	27.49	21.8	25.29	27.45	21.62	25.36	27.38
p = 9	23.4	27.05	29.33	23.29	27.1	29.32	23.29	26.98	29.15
p = 10	24.9	28.65	30.81	24.74	28.42	31	24.93	28.44	30.79
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.66	17.08	19.04	13.46	16.97	19.04	13.39	17.06	18.65
p = 4	15.15	18.65	20.6	15.09	18.42	20.65	14.98	18.42	20.3
p = 5	16.85	20.17	22.22	16.71	20.23	22.12	16.72	20.04	22.13
p = 6	18.37	21.92	23.91	18.37	21.89	23.89	18.4	21.76	23.77
p = 7	20.08	23.63	25.58	19.99	23.34	25.67	19.88	23.48	25.48
p = 8	21.69	25.39	27.41	21.69	25.22	27.3	21.49	24.99	27.17
p = 9	23.22	26.71	29.04	23.09	26.78	28.93	23.17	26.69	28.86
p = 10	24.7	28.43	30.75	24.67	28.49	30.57	24.61	28.29	30.59
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.31	16.78	18.9	13.09	16.39	18.14	12.93	15.94	17.92
p = 4	14.96	18.22	20.23	14.71	17.96	19.87	14.57	17.69	19.48
p = 5	16.6	19.96	22.08	16.47	19.71	21.56	16.31	19.46	21.49
p = 6	18.3	21.85	23.82	18.11	21.45	23.34	17.98	21.21	23.16
p = 7	19.95	23.4	25.49	19.73	23.2	25.24	19.63	22.88	24.88
p = 8	21.54	25.06	27.11	21.3	24.77	26.94	21.17	24.66	26.69
p = 9	23.09	26.76	28.76	22.87	26.2	28.56	22.75	26.15	28.51
p = 10	24.64	28.29	30.51	24.39	28.02	30.23	24.29	27.79	30.05

Table 14: Values of c_8 with $\lambda = 0.2$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	15.78	19.7	21.71	15.3	19.18	20.93	15.07	18.76	20.69
p = 4	17.07	20.81	22.72	16.74	20.24	22.24	16.53	19.95	21.86
p = 5	18.73	22.12	24.16	18.42	21.77	23.95	18.05	21.65	23.36
p = 6	20.07	23.7	25.85	20.15	23.38	25.45	19.76	23.32	25.04
p = 7	21.92	25.47	27.51	21.71	25.04	27.22	21.47	24.84	26.9
p = 8	23.45	26.92	29.05	23.19	26.75	28.91	23.11	26.54	28.44
p = 9	24.93	28.66	30.66	24.76	28.42	30.56	24.6	28.09	30.18
p = 10	26.62	30.26	32.46	26.4	30	32.12	26.17	29.79	31.89
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.85	18.32	20.16	14.69	17.98	19.8	14.53	17.87	19.74
p = 4	16.38	19.72	21.62	16.32	19.57	21.38	16.02	19.25	21.3
p = 5	17.98	21.26	23.28	18	21.29	22.98	17.74	21.03	22.9
p = 6	19.76	23.12	24.94	19.63	22.94	24.74	19.46	22.8	24.66
p = 7	21.42	24.7	26.67	21.28	24.45	26.6	21.17	24.48	26.38
p = 8	23.05	26.52	28.4	22.82	26.13	28.25	22.72	26.13	28.15
p = 9	24.58	28.02	29.96	24.58	27.95	29.88	24.44	27.71	29.72
p = 10	26.01	29.64	31.74	26.04	29.53	31.42	25.95	29.2	31.46
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.52	17.35	19.43	14.3	17.26	19.17	14.24	17.3	19.12
p = 4	16.01	19.11	21.11	15.87	19.02	20.72	15.84	18.86	20.78
p = 5	17.75	20.81	22.78	17.61	20.74	22.64	17.57	20.66	22.58
p = 6	19.41	22.54	24.54	19.31	22.45	24.42	19.31	22.38	24.35
p = 7	21	24.19	26.24	20.98	24.23	26.07	20.95	24.08	26.02
p = 8	22.72	25.98	27.97	22.65	25.8	27.77	22.58	25.71	27.83
p = 9	24.21	27.75	29.75	24.08	27.56	29.54	24.11	27.39	29.41
p = 10	25.83	29.21	31.42	25.89	29.1	31.2	25.67	29.03	31.13

Table 15: Values of c_8 with $\lambda = 0.2$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.06	17.16	18.91	14.02	17.04	18.68	14	16.96	18.76
p = 4	15.84	18.81	20.52	15.77	18.72	20.45	15.67	18.62	20.39
p = 5	17.58	20.55	22.26	17.53	20.41	22.43	17.42	20.39	22.21
p = 6	19.16	22.35	24.17	19.18	22.19	24.05	19.06	22.14	24.11
p = 7	20.86	23.97	26.03	20.8	24	25.96	20.8	23.91	25.83
p = 8	22.51	25.74	27.77	22.38	25.6	27.57	22.38	25.57	27.51
p = 9	24.07	27.35	29.4	24.04	27.28	29.25	23.96	27.15	29.23
p = 10	25.63	28.96	31.03	25.55	28.94	30.95	25.49	28.9	30.84
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.72	16.82	18.6	13.75	16.57	18.41	13.68	16.63	18.29
p = 4	15.59	18.62	20.28	15.51	18.51	20.23	15.51	18.43	20.2
p = 5	17.34	20.26	22.14	17.34	20.28	22.11	17.3	20.22	22.04
p = 6	19.02	22.14	24	19.06	22.01	23.95	18.93	21.92	23.89
p = 7	20.73	23.78	25.79	20.61	23.77	25.59	20.62	23.68	25.65
p = 8	22.33	25.49	27.59	22.34	25.58	27.43	22.28	25.42	27.39
p = 9	23.95	27.2	29.2	23.8	27.15	29.08	23.82	27.07	29.16
p = 10	25.43	28.8	30.96	25.46	28.76	30.87	25.4	28.67	30.76
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.65	16.58	18.3	13.48	16.25	17.94	13.38	16	17.73
p = 4	15.43	18.38	20.07	15.32	17.98	19.85	15.12	17.92	19.59
p = 5	17.23	20.14	21.97	17	20.1	21.73	16.98	19.73	21.73
p = 6	18.88	21.98	23.84	18.74	21.82	23.69	18.71	21.62	23.46
p = 7	20.54	23.7	25.58	20.48	23.54	25.38	20.34	23.18	25.16
p = 8	22.23	25.3	27.32	22.06	25.23	27.07	21.94	25.12	26.98
p = 9	23.76	27.06	29.2	23.62	26.91	28.83	23.61	26.72	28.74
p = 10	25.33	28.66	30.76	25.34	28.49	30.45	25.08	28.41	30.51

Table 16: Values of c_8 with $\lambda = 0.3$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	15.41	18.68	20.66	15.07	18.34	20.21	14.85	17.91	19.8
p = 4	16.89	20.02	21.97	16.69	19.92	21.46	16.51	19.6	21.26
p = 5	18.63	21.8	23.6	18.41	21.49	23.44	18.22	21.38	22.98
p = 6	20.26	23.44	25.33	20.06	23.4	25.11	19.84	23.12	24.89
p = 7	21.8	25.13	27.01	21.66	24.88	26.73	21.67	24.75	26.63
p = 8	23.36	26.86	28.67	23.34	26.53	28.44	23.28	26.24	28.41
p = 9	25.04	28.51	30.4	24.95	28.24	30.21	24.76	28.08	29.96
p = 10	26.58	29.98	32.1	26.48	29.78	31.8	26.24	29.55	31.75
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.74	17.93	19.58	14.47	17.51	19.32	14.39	17.31	19.11
p = 4	16.44	19.41	21.16	16.28	19.26	20.92	16.09	19.21	20.85
p = 5	18.09	21.1	22.88	18.02	20.93	22.83	17.86	20.8	22.56
p = 6	19.81	22.84	24.69	19.71	22.7	24.61	19.5	22.47	24.34
p = 7	21.53	24.53	26.48	21.39	24.46	26.26	21.25	24.25	26.28
p = 8	23.11	26.27	28.16	22.97	26.14	28.06	22.91	25.93	27.91
p = 9	24.66	27.93	29.93	24.67	27.75	29.75	24.5	27.7	29.64
p = 10	26.29	29.57	31.48	26.16	29.35	31.34	26.05	29.16	31.34
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.33	17.13	19.02	14.27	17.02	18.75	14.15	16.9	18.69
p = 4	16.01	18.89	20.64	15.94	18.77	20.55	15.91	18.74	20.54
p = 5	17.81	20.76	22.5	17.69	20.57	22.33	17.62	20.59	22.38
p = 6	19.53	22.58	24.32	19.51	22.26	24.22	19.44	22.29	24.2
p = 7	21.11	24.3	26.09	21.15	24.15	26	21.01	24.05	25.94
p = 8	22.81	25.95	27.78	22.86	25.91	27.81	22.79	25.81	27.76
p = 9	24.46	27.53	29.63	24.43	27.48	29.46	24.22	27.39	29.39
p = 10	26.12	29.18	31.24	25.94	29.16	31.22	25.9	28.97	30.99

Table 17: Values of c_8 with $\lambda = 0.3$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.16	16.77	18.59	13.97	16.7	18.41	14.01	16.61	18.36
p = 4	15.8	18.57	20.27	15.85	18.58	20.19	15.71	18.41	20.26
p = 5	17.56	20.46	22.26	17.52	20.46	22.14	17.61	20.39	22.1
p = 6	19.38	22.25	24.15	19.2	22.25	23.94	19.28	22.05	23.96
p = 7	21.02	23.99	25.84	20.94	23.97	25.82	20.9	23.82	25.69
p = 8	22.68	25.7	27.57	22.66	25.68	27.56	22.53	25.62	27.56
p = 9	24.34	27.42	29.42	24.2	27.35	29.19	24.26	27.31	29.24
p = 10	25.71	29.03	30.98	25.81	29.06	30.91	25.75	28.86	30.93
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.83	16.6	18.17	13.85	16.42	18.17	13.77	16.54	18.12
p = 4	15.66	18.49	20.16	15.66	18.32	20.04	15.61	18.3	19.91
p = 5	17.47	20.33	22.07	17.42	20.23	21.93	17.5	20.14	21.88
p = 6	19.28	22.06	23.81	19.19	22.02	23.99	19.17	22.01	23.77
p = 7	20.92	23.88	25.77	20.91	23.72	25.71	20.85	23.86	25.54
p = 8	22.58	25.59	27.5	22.55	25.45	27.43	22.44	25.49	27.4
p = 9	24.19	27.24	29.14	24.11	27.24	29.06	24.1	27.12	29.12
p = 10	25.72	28.85	30.86	25.74	28.9	30.86	25.73	28.87	30.77
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.68	16.37	18.07	13.57	16.22	17.74	13.42	16.07	17.67
p = 4	15.6	18.27	19.98	15.5	18.15	19.81	15.38	17.9	19.62
p = 5	17.45	20.24	21.84	17.33	19.95	21.75	17.18	19.91	21.61
p = 6	19.19	21.96	23.72	19	21.77	23.57	18.89	21.7	23.42
p = 7	20.85	23.83	25.6	20.71	23.52	25.49	20.59	23.5	25.3
p = 8	22.43	25.39	27.33	22.36	25.3	27.21	22.3	25.28	27.05
p = 9	24.08	27.05	29.11	23.96	26.95	28.91	23.88	26.97	28.87
p = 10	25.59	28.82	30.76	25.59	28.56	30.65	25.47	28.51	30.52

Table 18: Values of c_8 with $\lambda = 0.4$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	15.09	18.24	19.98	14.88	17.74	19.45	14.68	17.59	19.35
p = 4	16.82	19.63	21.44	16.58	19.59	21.24	16.5	19.26	20.89
p = 5	18.43	21.57	23.34	18.29	21.15	22.97	18.22	20.99	22.85
p = 6	20.15	23.22	25.13	19.99	23.02	24.76	19.94	22.87	24.62
p = 7	21.92	24.96	26.8	21.66	24.69	26.44	21.55	24.61	26.35
p = 8	23.42	26.5	28.54	23.19	26.28	28.34	23.28	26.18	28.03
p = 9	25.06	28.21	30.18	24.93	28.03	29.9	24.75	27.96	29.79
p = 10	26.59	29.76	31.76	26.5	29.57	31.74	26.36	29.41	31.49
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.54	17.37	19.09	14.45	17.2	18.78	14.31	16.99	18.59
p = 4	16.32	19.11	20.94	16.2	18.88	20.66	16.1	18.79	20.51
p = 5	18.05	20.9	22.6	17.92	20.79	22.51	17.85	20.66	22.42
p = 6	19.72	22.64	24.43	19.58	22.46	24.36	19.69	22.46	24.1
p = 7	21.48	24.38	26.12	21.42	24.29	26.12	21.28	24.23	26.08
p = 8	23.07	26.18	27.93	22.87	25.95	27.87	22.9	25.85	27.74
p = 9	24.66	27.77	29.68	24.57	27.62	29.52	24.48	27.58	29.52
p = 10	26.26	29.39	31.33	26.09	29.3	31.22	26.01	29.23	31.14
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.26	16.95	18.52	14.15	16.76	18.44	14.13	16.72	18.33
p = 4	16	18.74	20.4	15.94	18.58	20.44	16.01	18.46	20.18
p = 5	17.81	20.67	22.34	17.72	20.51	22.22	17.75	20.41	22.09
p = 6	19.47	22.27	24.2	19.45	22.25	24.09	19.46	22.23	23.94
p = 7	21.28	24.09	25.93	21.2	24.04	25.83	21.06	24.02	25.73
p = 8	22.83	25.85	27.65	22.83	25.71	27.64	22.67	25.71	27.6
p = 9	24.54	27.5	29.39	24.32	27.35	29.26	24.38	27.33	29.23
p = 10	26.05	29.08	31	25.97	29.11	31.05	25.95	28.9	30.97

Table 19: Values of c_8 with $\lambda = 0.4$ (continued).

Table A.9 (continued) Values of c_8 with $\lambda = 0.4$.

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.99	16.6	18.26	13.96	16.46	18.18	13.93	16.47	18.04
p = 4	15.93	18.41	20.22	15.73	18.38	20.11	15.83	18.39	20.02
p = 5	17.66	20.32	22.13	17.59	20.27	22.05	17.57	20.25	21.97
p = 6	19.33	22.22	23.91	19.27	22.11	23.91	19.33	22.08	23.83
p = 7	21.12	23.92	25.73	21	23.94	25.77	21.06	23.92	25.71
p = 8	22.71	25.6	27.56	22.72	25.52	27.45	22.66	25.53	27.42
p = 9	24.39	27.29	29.2	24.3	27.34	29.22	24.26	27.22	29.14
p = 10	25.91	28.96	30.9	25.88	28.9	30.87	25.86	28.84	30.79
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.86	16.34	18.03	13.84	16.38	18.15	13.8	16.3	17.89
p = 4	15.75	18.36	20.03	15.66	18.33	19.87	15.66	18.2	19.84
p = 5	17.52	20.18	21.92	17.47	20.15	21.84	17.49	20.14	21.76
p = 6	19.25	22.02	23.76	19.29	21.99	23.72	19.2	21.93	23.78
p = 7	20.94	23.69	25.64	20.91	23.75	25.51	20.87	23.73	25.62
p = 8	22.56	25.49	27.33	22.61	25.56	27.37	22.61	25.4	27.3
p = 9	24.25	27.19	29.13	24.24	27.19	29.04	24.15	27.15	29.04
p = 10	25.81	28.88	30.76	25.84	28.81	30.68	25.7	28.85	30.72
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.84	16.25	17.85	13.64	16.09	17.61	13.51	15.9	17.52
p = 4	15.63	18.21	19.85	15.49	18.01	19.7	15.47	17.94	19.56
p = 5	17.43	20.11	21.8	17.36	19.92	21.67	17.22	19.84	21.54
p = 6	19.11	21.95	23.69	19.14	21.75	23.55	19.03	21.75	23.49
p = 7	20.88	23.76	25.5	20.8	23.65	25.31	20.67	23.37	25.27
p = 8	22.57	25.47	27.29	22.52	25.36	27.23	22.36	25.23	27.12
p = 9	24.13	27.21	29.03	24.03	26.98	28.86	24	26.97	28.85
p = 10	25.74	28.8	30.68	25.71	28.63	30.56	25.6	28.54	30.54

Table 20: Values of c_8 with $\lambda = 0.5$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.9	17.77	19.53	14.76	17.49	19.09	14.45	17.26	19.02
p = 4	16.62	19.38	21.22	16.35	19.14	20.89	16.36	19.05	20.66
p = 5	18.46	21.18	23.03	18.24	20.97	22.77	18.13	20.73	22.65
p = 6	20.06	22.89	24.76	19.87	22.73	24.61	19.8	22.74	24.38
p = 7	21.65	24.65	26.51	21.64	24.42	26.26	21.45	24.34	26.18
p = 8	23.4	26.46	28.19	23.2	26.17	28.03	23.02	26.06	27.87
p = 9	25.03	28.02	29.97	24.87	27.88	29.8	24.69	27.71	29.5
p = 10	26.46	29.65	31.72	26.34	29.58	31.39	26.31	29.35	31.27
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.36	17.1	18.65	14.35	16.95	18.5	14.53	17.87	19.74
p = 4	16.26	18.93	20.56	16.1	18.85	20.44	16.02	19.25	21.3
p = 5	17.97	20.71	22.36	17.87	20.54	22.32	17.74	21.03	22.9
p = 6	19.65	22.42	24.25	19.69	22.4	24.22	19.46	22.8	24.66
p = 7	21.37	24.22	26.01	21.39	24.14	25.96	21.17	24.48	26.38
p = 8	22.97	26	27.87	22.99	25.92	27.62	22.72	26.13	28.15
p = 9	24.63	27.59	29.55	24.55	27.55	29.47	24.44	27.71	29.72
p = 10	26.11	29.27	31.2	26.08	29.29	31.02	25.95	29.2	31.46
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.11	16.66	18.25	14.15	16.51	18.23	13.97	16.47	18.13
p = 4	15.95	18.55	20.16	15.96	18.55	20.07	15.92	18.38	20.07
p = 5	17.73	20.52	22.12	17.69	20.37	22.15	17.59	20.32	21.99
p = 6	19.6	22.24	24.02	19.49	22.2	23.86	19.38	22.12	23.82
p = 7	21.15	24.11	25.84	21.14	24	25.7	21.1	23.89	25.68
p = 8	22.82	25.67	27.54	22.77	25.71	27.51	22.77	25.63	27.44
p = 9	24.54	27.34	29.29	24.41	27.35	29.26	24.41	27.24	29.17
p = 10	26.03	29.08	30.92	25.93	28.95	30.94	25.92	28.95	30.84

Table 21: Values of c_8 with $\lambda = 0.5$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.94	16.42	17.99	13.89	16.34	17.94	13.85	16.32	17.9
p = 4	15.76	18.39	19.94	15.74	18.32	19.91	15.72	18.26	19.91
p = 5	17.59	20.28	21.88	17.55	20.18	21.9	17.54	20.2	21.85
p = 6	19.39	22.06	23.77	19.3	22.05	23.79	19.29	22.01	23.75
p = 7	21.13	23.8	25.59	20.95	23.8	25.61	20.99	23.75	25.55
p = 8	22.69	25.52	27.44	22.69	25.56	27.36	22.68	25.44	27.32
p = 9	24.33	27.3	29.08	24.31	27.25	29.12	24.24	27.2	29.06
p = 10	25.91	28.88	30.84	25.93	28.93	30.78	25.87	28.77	30.79
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.77	16.28	17.84	13.78	16.23	17.71	13.7	16.16	17.72
p = 4	15.68	18.22	19.88	15.63	18.17	19.86	15.64	18.15	19.79
p = 5	17.46	20.13	21.72	17.4	20.03	21.77	17.47	20.07	21.76
p = 6	19.25	21.98	23.73	19.28	21.96	23.67	19.22	21.9	23.62
p = 7	20.97	23.75	25.47	21	23.78	25.44	20.94	23.65	25.46
p = 8	22.57	25.46	27.26	22.59	25.35	27.23	22.59	25.41	27.24
p = 9	24.24	27.13	29.03	24.14	27.15	29.05	24.16	27.17	28.97
p = 10	25.76	28.84	30.69	25.85	28.78	30.62	25.86	28.74	30.65
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.82	16.06	17.72	13.58	15.95	17.57	13.4	15.85	17.42
p = 4	15.59	18.13	19.75	15.54	18.02	19.64	15.44	17.95	19.52
p = 5	17.34	20.01	21.73	17.32	19.96	21.57	17.31	19.79	21.47
p = 6	19.15	21.83	23.59	19.07	21.79	23.45	19.04	21.69	23.39
p = 7	20.91	23.64	25.38	20.76	23.55	25.31	20.72	23.57	25.27
p = 8	22.56	25.36	27.2	22.47	25.33	27.06	22.43	25.15	27.08
p = 9	24.2	27.07	28.89	24.12	26.99	28.85	24.14	26.91	28.71
p = 10	25.81	28.78	30.66	25.74	28.64	30.59	25.67	28.58	30.55

Table 22: Values of c_8 with $\lambda = 0.6$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.74	17.38	19.04	14.57	17.22	18.68	14.32	16.94	18.56
p = 4	16.47	19.13	20.81	16.29	18.9	20.73	16.2	18.74	20.45
p = 5	18.23	20.99	22.65	18.09	20.82	22.53	17.9	20.59	22.34
p = 6	19.87	22.78	24.48	19.85	22.57	24.34	19.72	22.37	24.2
p = 7	21.61	24.45	26.37	21.43	24.38	26.09	21.38	24.19	26
p = 8	23.25	26.14	28.07	23.11	26.01	27.91	23.06	25.83	27.76
p = 9	24.82	27.89	29.71	24.71	27.71	29.6	24.59	27.65	29.42
p = 10	26.49	29.53	31.51	26.36	29.33	31.18	26.24	29.19	31.17
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.36	16.8	18.46	14.12	16.7	18.24	14.1	16.43	18.07
p = 4	16.17	18.68	20.37	16.01	18.58	20.27	15.84	18.4	20.12
p = 5	17.88	20.55	22.24	17.77	20.41	22.08	17.74	20.25	22.02
p = 6	19.64	22.41	24.15	19.54	22.26	24.01	19.48	22.27	23.89
p = 7	21.27	24.09	25.95	21.3	23.99	25.82	21.2	23.98	25.76
p = 8	22.89	25.81	27.65	22.9	25.75	27.58	22.82	25.64	27.5
p = 9	24.59	27.4	29.3	24.49	27.38	29.27	24.45	27.38	29.19
p = 10	26.14	29.12	31.09	26.07	29.11	30.91	26.09	29.01	30.83
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.94	16.44	17.99	13.92	16.38	17.97	13.8	16.34	17.87
p = 4	15.87	18.36	20.01	15.78	18.36	19.95	15.71	18.29	19.9
p = 5	17.7	20.27	21.97	17.64	20.19	21.87	17.64	20.18	21.83
p = 6	19.41	22.14	23.82	19.42	22.06	23.77	19.32	22.05	23.73
p = 7	21.11	23.86	25.58	21.01	23.86	25.56	21.08	23.78	25.56
p = 8	22.83	25.62	27.49	22.81	25.49	27.34	22.62	25.49	27.3
p = 9	24.33	27.28	29.15	24.34	27.24	29.11	24.34	27.2	29.05
p = 10	26.06	28.92	30.89	25.91	28.84	30.76	25.93	28.96	30.8

Table 23: Values of c_8 with $\lambda = 0.6$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.81	16.19	17.93	13.77	16.22	17.71	13.75	16.21	17.75
p = 4	15.7	18.22	19.88	15.69	18.2	19.76	15.7	18.16	19.66
p = 5	17.57	20.11	21.75	17.54	20.09	21.8	17.5	20.1	21.72
p = 6	19.36	21.96	23.66	19.32	21.93	23.61	19.29	21.91	23.63
p = 7	21	23.75	25.48	21.01	23.67	25.46	20.95	23.62	25.52
p = 8	22.72	25.43	27.28	22.64	25.45	27.28	22.58	25.44	27.2
p = 9	24.32	27.18	29.02	24.34	27.18	29.03	24.25	27.15	28.93
p = 10	25.88	28.78	30.68	25.8	28.77	30.65	25.82	28.75	30.68
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.74	16.1	17.61	13.67	16.08	17.6	13.61	16.07	17.63
p = 4	15.66	18.1	19.75	15.58	18.07	19.76	15.54	18.02	19.67
p = 5	17.44	20.04	21.71	17.42	19.96	21.66	17.44	20.02	21.67
p = 6	19.3	21.82	23.57	19.14	21.84	23.46	19.13	21.77	23.51
p = 7	20.88	23.63	25.47	20.91	23.67	25.36	20.92	23.58	25.36
p = 8	22.62	25.34	27.13	22.62	25.33	27.22	22.58	25.33	27.22
p = 9	24.24	27.15	28.98	24.22	27.06	28.94	24.21	27	28.91
p = 10	25.75	28.73	30.58	25.77	28.65	30.56	25.82	28.7	30.59
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.66	16.02	17.51	13.57	15.9	17.4	13.54	15.76	17.3
p = 4	15.59	18.11	19.61	15.53	17.92	19.5	15.35	17.82	19.45
p = 5	17.48	19.91	21.54	17.32	19.78	21.52	17.33	19.77	21.4
p = 6	19.13	21.8	23.47	19.14	21.73	23.39	19.05	21.69	23.28
p = 7	20.96	23.6	25.29	20.84	23.45	25.28	20.75	23.48	25.21
p = 8	22.51	25.34	27.11	22.55	25.24	27.07	22.48	25.23	26.99
p = 9	24.14	27.03	28.88	24.11	26.9	28.73	24.04	26.88	28.76
p = 10	25.77	28.7	30.55	25.68	28.61	30.51	25.69	28.58	30.5

Table 24: Values of c_8 with $\lambda = 0.7$.

	$m=5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.56	17.08	18.73	14.38	16.92	18.61	14.2	16.73	18.34
p = 4	16.37	18.88	20.51	16.17	18.83	20.36	16.03	18.54	20.25
p = 5	18.18	20.78	22.43	17.92	20.76	22.23	17.95	20.47	22.09
p = 6	19.81	22.54	24.26	19.68	22.4	24.15	19.62	22.38	24.02
p = 7	21.44	24.4	26.15	21.37	24.24	25.89	21.26	24.04	25.79
p = 8	23.02	26.04	27.82	23.03	25.93	27.71	22.91	25.78	27.52
p = 9	24.76	27.69	29.55	24.61	27.52	29.46	24.62	27.49	29.32
p = 10	26.34	29.38	31.28	26.24	29.18	31.11	26.16	29.15	31.07
	$m=8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.15	16.66	18.17	14.09	16.46	18	13.94	16.38	17.94
p = 4	16.04	18.5	20.06	15.97	18.46	20	15.85	18.36	19.96
p = 5	17.75	20.33	22	17.76	20.22	22	17.66	20.23	21.9
p = 6	19.5	22.1	23.91	19.42	22.12	23.86	19.41	22.07	23.83
p = 7	21.29	23.87	25.74	21.19	23.88	25.67	21.07	23.81	25.61
p = 8	22.87	25.62	27.47	22.76	25.56	27.44	22.72	25.63	27.36
p = 9	24.48	27.35	29.26	24.42	27.3	29.07	24.39	27.29	29.11
p = 10	26.07	29.02	30.96	26.03	28.96	30.79	25.98	28.9	30.84
	$m=11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.84	16.28	17.86	13.87	16.3	17.79	13.81	16.2	17.72
p = 4	15.82	18.23	19.82	15.77	18.19	19.86	15.67	18.09	19.78
p = 5	17.56	20.16	21.83	17.54	20.09	21.81	17.58	20.09	21.77
p = 6	19.36	22.04	23.77	19.34	21.88	23.7	19.25	21.85	23.67
p = 7	21.08	23.81	25.49	20.95	23.81	25.55	20.99	23.72	25.47
p = 8	22.69	25.5	27.23	22.74	25.49	27.22	22.68	25.44	27.26
p = 9	24.37	27.25	29.07	24.31	27.16	28.97	24.27	27.08	28.98
p = 10	25.98	28.79	30.77	25.92	28.8	30.74	25.88	28.8	30.66

Table 25: Values of c_8 with $\lambda = 0.7$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.74	16.19	17.66	13.75	16.06	17.64	13.69	16.03	17.69
p = 4	15.67	18.18	19.64	15.66	18.07	19.66	15.57	18	19.61
p = 5	17.47	20	21.75	17.45	19.9	21.66	17.46	19.97	21.65
p = 6	19.26	21.85	23.55	19.27	21.85	23.58	19.28	21.79	23.51
p = 7	20.97	23.66	25.4	20.89	23.61	25.38	20.94	23.61	25.38
p = 8	22.55	25.46	27.13	22.62	25.4	27.13	22.55	25.31	27.16
p = 9	24.19	27.07	28.93	24.22	27.09	28.89	24.28	27	28.92
p = 10	25.78	28.82	30.68	25.84	28.72	30.69	25.85	28.67	30.58
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.68	15.99	17.56	13.62	15.95	17.47	13.62	15.95	17.45
p = 4	15.6	18.05	19.6	15.5	17.98	19.54	15.48	17.92	19.52
p = 5	17.42	19.87	21.63	17.45	19.91	21.52	17.37	19.89	21.49
p = 6	19.23	21.82	23.57	19.17	21.77	23.54	19.19	21.69	23.43
p = 7	20.94	23.57	25.3	20.86	23.56	25.32	20.91	23.51	25.29
p = 8	22.53	25.34	27.13	22.52	25.38	27.06	22.57	25.28	27.08
p = 9	24.15	27.02	28.91	24.14	26.95	28.85	24.16	26.95	28.84
p = 10	25.79	28.73	30.59	25.8	28.68	30.57	25.79	28.58	30.56
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.53	15.85	17.39	13.5	15.75	17.28	13.44	15.71	17.26
p = 4	15.48	17.87	19.51	15.45	17.85	19.4	15.41	17.77	19.34
p = 5	17.33	19.81	21.49	17.31	19.8	21.33	17.3	19.71	21.43
p = 6	19.11	21.69	23.39	19.07	21.65	23.37	19.07	21.59	23.24
p = 7	20.83	23.5	25.39	20.81	23.5	25.2	20.73	23.42	25.11
p = 8	22.51	25.36	26.99	22.53	25.15	27	22.44	25.18	26.9
p = 9	24.19	26.94	28.81	24.11	26.89	28.76	24.15	26.86	28.71
p = 10	25.68	28.59	30.54	25.68	28.59	30.48	25.6	28.6	30.45

Table 26: Values of c_8 with $\lambda = 0.8$.

	$m = 5$			6			7		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	14.29	16.86	18.47	14.2	16.73	18.27	14.13	16.56	18.17
p = 4	16.15	18.71	20.32	16.03	18.51	20.22	15.96	18.46	20.08
p = 5	17.92	20.54	22.33	17.88	20.39	22.16	17.72	20.34	21.95
p = 6	19.64	22.45	24.13	19.58	22.23	23.96	19.5	22.17	23.88
p = 7	21.38	24.19	25.9	21.24	24.05	25.78	21.17	23.96	25.71
p = 8	22.99	25.83	27.66	22.91	25.74	27.62	22.86	25.67	27.44
p = 9	24.67	27.52	29.49	24.57	27.44	29.25	24.42	27.32	29.12
p = 10	26.23	29.18	30.98	26.13	28.97	30.94	26.02	28.98	30.82
	$m = 8$			9			10		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.99	16.44	17.99	13.84	16.41	17.79	13.87	16.21	17.75
p = 4	15.8	18.38	19.96	15.78	18.29	19.93	15.73	18.28	19.78
p = 5	17.67	20.29	21.89	17.56	20.2	21.7	17.54	20.17	21.69
p = 6	19.46	22.09	23.76	19.43	21.93	23.64	19.35	21.96	23.6
p = 7	21.18	23.81	25.56	21.08	23.84	25.47	21.01	23.72	25.4
p = 8	22.84	25.56	27.39	22.78	25.44	27.28	22.66	25.51	27.22
p = 9	24.29	27.27	29.13	24.39	27.22	29.03	24.34	27.11	29
p = 10	26.01	28.87	30.73	25.94	28.87	30.72	25.95	28.83	30.71
	$m = 11$			12			13		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.82	16.1	17.75	13.67	16.13	17.63	13.68	16.04	17.57
p = 4	15.81	18.14	19.73	15.61	18.07	19.69	15.63	18.02	19.55
p = 5	17.56	20.04	21.72	17.56	19.93	21.66	17.46	20.03	21.64
p = 6	19.35	21.96	23.62	19.31	21.83	23.51	19.26	21.9	23.56
p = 7	21.01	23.72	25.42	20.98	23.61	25.43	20.93	23.62	25.38
p = 8	22.7	25.42	27.22	22.7	25.45	27.19	22.56	25.37	27.13
p = 9	24.27	27.08	28.97	24.28	27.08	28.92	24.21	27.1	28.97
p = 10	25.84	28.84	30.66	25.88	28.76	30.65	25.81	28.67	30.57

Table 27: Values of c_8 with $\lambda = 0.8$ (continued).

	$m = 14$			15			16		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.62	15.97	17.52	13.57	15.95	17.51	13.59	15.94	17.45
p = 4	15.59	18	19.57	15.57	17.97	19.56	15.5	17.94	19.52
p = 5	17.45	19.96	21.58	17.33	19.91	21.57	17.38	19.89	21.51
p = 6	19.18	21.79	23.51	19.23	21.76	23.45	19.16	21.74	23.43
p = 7	20.91	23.53	25.38	20.94	23.6	25.31	20.91	23.58	25.29
p = 8	22.56	25.35	27.14	22.5	25.31	27.07	22.55	25.31	27.02
p = 9	24.21	27	28.87	24.21	27.02	28.86	24.2	27.03	28.76
p = 10	25.77	28.66	30.61	25.8	28.65	30.54	25.78	28.74	30.57
	$m = 17$			18			19		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.64	15.87	17.38	13.55	15.84	17.35	13.51	15.83	17.36
p = 4	15.56	17.86	19.54	15.53	17.95	19.52	15.5	17.88	19.49
p = 5	17.43	19.87	21.55	17.3	19.83	21.43	17.37	19.76	21.51
p = 6	19.21	21.66	23.41	19.18	21.75	23.41	19.07	21.67	23.38
p = 7	20.88	23.55	25.26	20.79	23.42	25.22	20.86	23.47	25.25
p = 8	22.55	25.27	27.03	22.61	25.27	27	22.48	25.26	27.04
p = 9	24.22	26.98	28.72	24.14	26.99	28.72	24.07	26.93	28.72
p = 10	25.71	28.59	30.48	25.75	28.62	30.52	25.73	28.53	30.48
	$m = 20$			25			30		
ARL_0	200	500	1000	200	500	1000	200	500	1000
p = 3	13.55	15.8	17.36	13.46	15.71	17.26	13.37	15.62	17.13
p = 4	15.42	17.87	19.39	15.41	17.82	19.33	15.38	17.79	19.29
p = 5	17.27	19.81	21.45	17.28	19.7	21.33	17.21	19.72	21.32
p = 6	19.13	21.7	23.33	19.05	21.67	23.31	19.02	21.55	23.27
p = 7	20.78	23.53	25.17	20.75	23.44	25.15	20.75	23.32	25.14
p = 8	22.51	25.16	26.97	22.41	25.11	26.98	22.41	25.09	26.88
p = 9	24.12	26.84	28.76	24.08	26.95	28.69	24.03	26.82	28.67
p = 10	25.74	28.58	30.42	25.67	28.53	30.45	25.66	28.48	30.37

Table 28: ARL performances for combined chart with $p = 4$ and $\lambda = 0.026$ (continued).

	$m = 15$		25		35	
D	RS2008	JS-type	RS2008	JS-type	RS2008	JS-type
0.0	801.2405	801.2757	800.5775	800.6816	801.3025	801.6725
0.5	322.9917	71.6533	190.4038	72.0625	154.6356	70.8133
1.0	53.9800	25.8408	36.9725	24.7446	33.0678	24.4850
1.5	23.9044	15.1779	19.9108	14.7725	18.6639	14.6725
2.0	15.2789	10.4396	13.1954	10.2467	12.6533	10.1288
2.5	9.9344	7.4567	9.0908	7.1500	8.6789	7.0138
3.0	6.3922	5.0379	5.7279	4.9188	5.8783	4.9383
4.0	2.4350	2.2004	2.3479	2.1875	2.3244	2.1358
5.0	1.2961	1.2879	1.2804	1.2433	1.2844	1.2446
$c_5 \backslash c_{10}$	42.3900	20.3687	32.3225	19.7433	28.8400	19.3333
$c_4 \backslash c_9$	20.7200	20.2118	20.5675	20.0300	20.5150	19.8475

	$m = 45$	
D	RS2008	JS-type
0.0	801.0425	801.2415
0.5	130.1133	65.7361
1.0	31.1561	24.0456
1.5	17.6911	14.4661
2.0	12.0189	9.8267
2.5	8.3144	6.9422
3.0	5.6772	4.8044
4.0	2.2667	2.0917
5.0	1.3067	1.2328
$c_5 \backslash c_{10}$	26.4875	18.7855
$c_4 \backslash c_9$	20.4700	19.7650