

然而我們要找出穩定性就需要令：

$$\left[\begin{array}{l} 2H_1 + 2H_2 + 2H_3 \\ +\alpha(8H_1^3 + 16H_1^2H_2 + 16H_1H_2^2 + 8H_2^3 + 16H_1^2H_3 + 24H_1H_2H_3 + 16H_2^2H_3 + 16H_1H_3^2 + 16H_2H_3^2 + 8H_3^3) \\ +\beta(6H_1^3 + 4H_1^2H_2 + 4H_1H_2^2 + 6H_2^3 + 4H_1^2H_3 + 4H_2^2H_3 + 4H_1H_3^2 + 4H_2H_3^2 + 6H_3^3) \\ +v[1 + \alpha(4H_1^2 + 4H_1H_2 + 4H_2^2 + 4H_1H_3 + 4H_2H_3 + 4H_3^2) + \beta(3H_1^2 - 4H_1H_2 + 3H_2^2 - 4H_1H_3 - 4H_2H_3 + 3H_3^2)] \\ -2v^2\beta(H_1 + H_2 + H_3) - v^3\beta \end{array} \right] = 0 \quad (C.50)$$

因此我們由(C.50)可以找到三組解

$$v = -(H_1 + H_2 + H_3)$$

$$v = -\frac{1}{2} \left(H_1 + H_2 + H_3 \pm \sqrt{9H_1^2 + 9H_2^2 + 9H_3^2 - 6H_1H_2 - 6H_2H_3 - 6H_1H_3} \right) \quad (C.51)$$

我要找出 $v > 0$ 才能為不穩定性

$$v = -(H_1 + H_2 + H_3) \text{ 無法確定大於 } 0$$

我們來看

$$v = -\frac{1}{2} \left(H_1 + H_2 + H_3 \pm \sqrt{9H_1^2 + 9H_2^2 + 9H_3^2 - 6H_1H_2 - 6H_2H_3 - 6H_1H_3} \right)$$

需要

$$\sqrt{9H_1^2 + 9H_2^2 + 9H_3^2 - 6H_1H_2 - 6H_2H_3 - 6H_1H_3} > H_1 + H_2 + H_3 \quad (C.52)$$

才會有負號出現，因平方相減

$$\begin{aligned} & (9H_1^2 + 9H_2^2 + 9H_3^2 - 6H_1H_2 - 6H_2H_3 - 6H_1H_3) - (H_1 + H_2 + H_3)^2 \\ & = 4(H_1 - H_2)^2 + 4(H_2 - H_3)^2 + 4(H_1 - H_3)^2 \geq 0 \end{aligned} \quad (C.53)$$

而我們不考慮 $H_1 = H_2 = H_3$ 這組，則確實有一組微擾 $v > 0$ ，故得證。

Appendix D Calculate Eigenvalues

在計算 eigenvalues 處，會用矩陣的方式表示，在分別顯示每一列矩陣的數值以方便計算，然再做乘法計算處因為過於複雜，所以會用 mathematica 計算，由於計算複雜所以用電腦計算，以下是計算 eigenvalues 的過程：

我們已知平衡點為

$$B = 2(\chi - 4) - 4\Sigma^2(1 + 2\chi), \Omega_\Lambda = 1 + \frac{1}{2}\Sigma^2, N = 0, Q = 0, \Sigma_+ = \Sigma_+, \Sigma_- = \Sigma_-, \Sigma_{+1} = 0$$

$$\Sigma_{-1} = 0, \Sigma_{+2} = 0, \Sigma_{-2} = 0, Q_2 = 0 \quad (D.1)$$

使用線性化系統

$$\frac{dx_i}{dt} = a_{i1}x_1 + \dots + a_{in+1}x_{n+1}$$

$$(a_{ij} = \left. \frac{\partial A_i}{\partial x_j} \right|_{x_j=0}; i, j = 1, 2, \dots, n+1) \quad (D.2)$$

表示成矩陣為：

$$\frac{d}{dt} \begin{pmatrix} B \\ \Omega_\Lambda \\ N \\ Q \\ \Sigma_+ \\ \Sigma_- \\ \Sigma_{+1} \\ \Sigma_{-1} \\ \Sigma_{+2} \\ \Sigma_{-2} \\ Q_2 \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial A_1}{\partial B} \right)_0 & \left(\frac{\partial A_1}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_1}{\partial N} \right)_0 & \left(\frac{\partial A_1}{\partial Q} \right)_0 & \left(\frac{\partial A_1}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_1}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_1}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_1}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_1}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_1}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_1}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_2}{\partial B} \right)_0 & \left(\frac{\partial A_2}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_2}{\partial N} \right)_0 & \left(\frac{\partial A_2}{\partial Q} \right)_0 & \left(\frac{\partial A_2}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_2}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_2}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_2}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_2}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_2}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_2}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_3}{\partial B} \right)_0 & \left(\frac{\partial A_3}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_3}{\partial N} \right)_0 & \left(\frac{\partial A_3}{\partial Q} \right)_0 & \left(\frac{\partial A_3}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_3}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_3}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_3}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_3}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_3}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_3}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_4}{\partial B} \right)_0 & \left(\frac{\partial A_4}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_4}{\partial N} \right)_0 & \left(\frac{\partial A_4}{\partial Q} \right)_0 & \left(\frac{\partial A_4}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_4}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_4}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_4}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_4}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_4}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_4}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_5}{\partial B} \right)_0 & \left(\frac{\partial A_5}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_5}{\partial N} \right)_0 & \left(\frac{\partial A_5}{\partial Q} \right)_0 & \left(\frac{\partial A_5}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_5}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_5}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_5}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_5}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_5}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_5}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_6}{\partial B} \right)_0 & \left(\frac{\partial A_6}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_6}{\partial N} \right)_0 & \left(\frac{\partial A_6}{\partial Q} \right)_0 & \left(\frac{\partial A_6}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_6}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_6}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_6}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_6}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_6}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_6}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_7}{\partial B} \right)_0 & \left(\frac{\partial A_7}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_7}{\partial N} \right)_0 & \left(\frac{\partial A_7}{\partial Q} \right)_0 & \left(\frac{\partial A_7}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_7}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_7}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_7}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_7}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_7}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_7}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_8}{\partial B} \right)_0 & \left(\frac{\partial A_8}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_8}{\partial N} \right)_0 & \left(\frac{\partial A_8}{\partial Q} \right)_0 & \left(\frac{\partial A_8}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_8}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_8}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_8}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_8}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_8}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_8}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_9}{\partial B} \right)_0 & \left(\frac{\partial A_9}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_9}{\partial N} \right)_0 & \left(\frac{\partial A_9}{\partial Q} \right)_0 & \left(\frac{\partial A_9}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_9}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_9}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_9}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_9}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_9}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_9}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_{10}}{\partial B} \right)_0 & \left(\frac{\partial A_{10}}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_{10}}{\partial N} \right)_0 & \left(\frac{\partial A_{10}}{\partial Q} \right)_0 & \left(\frac{\partial A_{10}}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_{10}}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_{10}}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_{10}}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_{10}}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_{10}}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_{10}}{\partial Q_2} \right)_0 \\ \left(\frac{\partial A_{11}}{\partial B} \right)_0 & \left(\frac{\partial A_{11}}{\partial \Omega_\Lambda} \right)_0 & \left(\frac{\partial A_{11}}{\partial N} \right)_0 & \left(\frac{\partial A_{11}}{\partial Q} \right)_0 & \left(\frac{\partial A_{11}}{\partial \Sigma_+} \right)_0 & \left(\frac{\partial A_{11}}{\partial \Sigma_-} \right)_0 & \left(\frac{\partial A_{11}}{\partial \Sigma_{+1}} \right)_0 & \left(\frac{\partial A_{11}}{\partial \Sigma_{-1}} \right)_0 & \left(\frac{\partial A_{11}}{\partial \Sigma_{+2}} \right)_0 & \left(\frac{\partial A_{11}}{\partial \Sigma_{-2}} \right)_0 & \left(\frac{\partial A_{11}}{\partial Q_2} \right)_0 \end{pmatrix} \begin{pmatrix} B \\ \Omega_\Lambda \\ N \\ Q \\ \Sigma_+ \\ \Sigma_- \\ \Sigma_{+1} \\ \Sigma_{-1} \\ \Sigma_{+2} \\ \Sigma_{-2} \\ Q_2 \end{pmatrix} \quad (D.3)$$

$$\begin{aligned}
& \left(\left(\frac{\partial A_9}{\partial B} \right)_0, \left(\frac{\partial A_9}{\partial \Omega_\Lambda} \right)_0, \left(\frac{\partial A_9}{\partial N} \right)_0, \left(\frac{\partial A_9}{\partial Q} \right)_0, \left(\frac{\partial A_9}{\partial \Sigma_+} \right)_0, \left(\frac{\partial A_9}{\partial \Sigma_-} \right)_0, \left(\frac{\partial A_9}{\partial \Sigma_{+1}} \right)_0, \left(\frac{\partial A_9}{\partial \Sigma_{-1}} \right)_0, \left(\frac{\partial A_9}{\partial \Sigma_{+2}} \right)_0, \left(\frac{\partial A_9}{\partial \Sigma_{-2}} \right)_0, \left(\frac{\partial A_9}{\partial Q_2} \right)_0 \right) \\
& = \left(-\frac{6\Sigma_- (4\Sigma^2 - 1)}{4\Sigma^2 + B + 8}, 0, 0, -\frac{7\Sigma_- (36\Sigma^2 + B)}{4\Sigma^2 + B + 8}, \frac{48\Sigma_+ (B + 9)}{4\Sigma^2 + B + 8}, \frac{48\Sigma_+ \Sigma_- (B + 9)}{4\Sigma^2 + B + 8}, \right. \\
& \quad \left. \frac{16B\Sigma_+^2 - 9(B + 8) - 36(\Sigma^2 - 4\Sigma_+^2)}{4\Sigma^2 + B + 8}, \frac{16\Sigma_+ \Sigma_- (B + 9)}{4\Sigma^2 + B + 8}, -6, 0, -\frac{\Sigma_+ (36\Sigma^2 + B)}{4\Sigma^2 + B + 8} \right)
\end{aligned} \tag{D.12}$$

$$\begin{aligned}
& \left(\left(\frac{\partial A_{10}}{\partial B} \right)_0, \left(\frac{\partial A_{10}}{\partial \Omega_\Lambda} \right)_0, \left(\frac{\partial A_{10}}{\partial N} \right)_0, \left(\frac{\partial A_{10}}{\partial Q} \right)_0, \left(\frac{\partial A_{10}}{\partial \Sigma_+} \right)_0, \left(\frac{\partial A_{10}}{\partial \Sigma_-} \right)_0, \left(\frac{\partial A_{10}}{\partial \Sigma_{+1}} \right)_0, \left(\frac{\partial A_{10}}{\partial \Sigma_{-1}} \right)_0, \left(\frac{\partial A_{10}}{\partial \Sigma_{+2}} \right)_0, \left(\frac{\partial A_{10}}{\partial \Sigma_{-2}} \right)_0, \left(\frac{\partial A_{10}}{\partial Q_2} \right)_0 \right) \\
& = \left(-\frac{6\Sigma_- (4\Sigma^2 - 1)}{4\Sigma^2 + B + 8}, 0, 0, -\frac{7\Sigma_- (36\Sigma^2 + B)}{4\Sigma^2 + B + 8}, \frac{48\Sigma_+ \Sigma_- (B + 9)}{4\Sigma^2 + B + 8}, \frac{48\Sigma_- (B + 9)}{4\Sigma^2 + B + 8}, \right. \\
& \quad \left. \frac{16\Sigma_+ \Sigma_- (B + 9)}{4\Sigma^2 + B + 8}, \frac{16B\Sigma_-^2 - 9(B + 8) - 36(\Sigma^2 - 4\Sigma_+^2)}{4\Sigma^2 + B + 8}, 0, -6, -\frac{\Sigma_- (36\Sigma^2 + B)}{4\Sigma^2 + B + 8} \right)
\end{aligned} \tag{D.13}$$

$$\begin{aligned}
& \left(\left(\frac{\partial A_{11}}{\partial B} \right)_0, \left(\frac{\partial A_{11}}{\partial \Omega_\Lambda} \right)_0, \left(\frac{\partial A_{11}}{\partial N} \right)_0, \left(\frac{\partial A_{11}}{\partial Q} \right)_0, \left(\frac{\partial A_{11}}{\partial \Sigma_+} \right)_0, \left(\frac{\partial A_{11}}{\partial \Sigma_-} \right)_0, \left(\frac{\partial A_{11}}{\partial \Sigma_{+1}} \right)_0, \left(\frac{\partial A_{11}}{\partial \Sigma_{-1}} \right)_0, \left(\frac{\partial A_{11}}{\partial \Sigma_{+2}} \right)_0, \left(\frac{\partial A_{11}}{\partial \Sigma_{-2}} \right)_0, \left(\frac{\partial A_{11}}{\partial Q_2} \right)_0 \right) \\
& = \left(-\frac{3}{8}\Sigma^2, \frac{3}{4}B, 0, -\frac{1}{4}\frac{9\Sigma^2(16+B+4\Sigma^2)}{4\Sigma^2-1}, \frac{3}{4}\frac{\Sigma_+(36\Sigma^2+B)}{4\Sigma^2-1}, \frac{3}{4}\frac{\Sigma_-(36\Sigma^2+B)}{4\Sigma^2-1}, \right. \\
& \quad \left. \frac{1}{2}\frac{\Sigma_+(36\Sigma^2+B)}{4\Sigma^2-1}, \frac{1}{2}\frac{\Sigma_+(36\Sigma^2+B)}{4\Sigma^2-1}, \frac{1}{4}\frac{\Sigma_+(36\Sigma^2+B)}{4\Sigma^2-1}, \frac{1}{4}\frac{\Sigma_+(36\Sigma^2+B)}{4\Sigma^2-1}, -6 \right)
\end{aligned} \tag{D.14}$$

我們要確實有真解所以矩陣依照(2.50)線性化 DE 方程式為 $\dot{x} = Ax = \lambda x$ ，因此矩陣的 determinant 要為 0:

$$\det \begin{bmatrix} a_{11} - \lambda, \dots, a_{1n+1} \\ \dots \\ a_{n+1}, \dots, a_{n+1n+1} - \lambda \end{bmatrix} = 0 \tag{D.14}$$

接著我們可以算出一元多次方程式，是利用 mathematica 計算程式來做計算，計算顯示在下頁可以看出很難用手算出，所以拜託林益弘學長教我用電腦算，我們用電腦可算出，在此我們可以用因式分解的方式找出 eigenvalues

$$\begin{aligned}
& -\lambda^{11} + (-4\Sigma_+ - 19)\lambda^{10} + \left(18\Sigma_+^2 + 18\Sigma_-^2 - 72\Sigma_+ - 153 - \frac{B}{2}\right)\lambda^9 + (72\Sigma_+^3 + 72\Sigma_+\Sigma_-^2 + 288\Sigma_+^2 + 288\Sigma_-^2 - 2B\Sigma_+ - 540\Sigma_+ - 8B - 675)\lambda^8 \\
& + \left(1080\Sigma_+^3 + 1080\Sigma_+\Sigma_-^2 + 1890\Sigma_+^2 + 9B\Sigma_+^2 + 1890\Sigma_-^2 + 9B\Sigma_-^2 - 30B\Sigma_+ - 2160\Sigma_+ - \frac{105}{2}B - 1755\right)\lambda^7 \\
& + (6480\Sigma_+^3 + 36B\Sigma_+^3 + 6480\Sigma_+\Sigma_-^2 + 36B\Sigma_+\Sigma_-^2 + 6480\Sigma_+^2 + 117B\Sigma_+^2 + 6480\Sigma_-^2 + 117B\Sigma_-^2 - 180B\Sigma_+ - 4860\Sigma_+ - 180B - 2673)\lambda^6 \\
& + \left(19440\Sigma_+^3 + 432B\Sigma_+^3 + 19440\Sigma_+\Sigma_-^2 + 432B\Sigma_+\Sigma_-^2 + 12150\Sigma_+^2 + 594B\Sigma_+^2 + 12150\Sigma_-^2 + 594B\Sigma_-^2 - 540B\Sigma_+ - 5832\Sigma_+ - \frac{675}{2}B - 2187\right)\lambda^5 \\
& + (29160\Sigma_+^3 + 1944B\Sigma_+^3 + 29160\Sigma_+\Sigma_-^2 + 1944B\Sigma_+\Sigma_-^2 + 11664\Sigma_+^2 + 1458B\Sigma_+^2 + 11664\Sigma_-^2 + 1458B\Sigma_-^2 - 810B\Sigma_+ - 2916\Sigma_+ - 324B - 729)\lambda^4 \\
& + \left(17496\Sigma_+^3 + 3888B\Sigma_+^3 + 17496\Sigma_+\Sigma_-^2 + 3888B\Sigma_+\Sigma_-^2 + 4374\Sigma_+^2 + 1701B\Sigma_+^2 + 4374\Sigma_-^2 + 1701B\Sigma_-^2 - 486B\Sigma_+ - \frac{243}{2}B\right)\lambda^3 \\
& + (2916B\Sigma_+^3 + 2916B\Sigma_+\Sigma_-^2 + 729B\Sigma_+^2 + 729B\Sigma_-^2)\lambda^2 \\
& = -\frac{1}{2}(\lambda+3)^4(4\Sigma_+ + \lambda + 1)(-18\Sigma^2 + \lambda^2 + 3\lambda)(2\lambda^2 + 6\lambda + B)\lambda^2 = 0
\end{aligned} \tag{D.15}$$

我們的等號為零，所以可以得出下列 eigenvalues:

$$\lambda = 0, 0, -3, -3, -3, -3, -\frac{3}{2}\left(1 \pm \sqrt{1+8\Sigma^2}\right), -\frac{3}{2} \pm \frac{1}{2}\sqrt{9-2B}, -1-4\Sigma_+ \tag{D.16}$$



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