General analysis of frequency-modulation reticles

Zu Wen Chao
National Chiao Tung University
Institute of Electronics
45 Po-Ai Street
Hsin-Chu 30039
Taiwan

Jih Liang Chu
Chung San Institute of Science and Technology
P.O. Box 1-3
Lung-Tang, Tao-Yuan 32526
Taiwan

Abstract. A general method for the analysis of frequency-modulation reticles is presented. The formulation starts by introducing the spoke function $f(\rho)$ as an offsetting factor to the phase angle θ. Through the process of coordinate transformation, the output signal after demodulation is obtained. The result contains two main parts: the first part represents the contribution from the basic configuration of a straight-edge spoke; the second part, containing the term $df/d\rho$, represents the modifying effect of changing the spoke edge to an arbitrary curve. By means of four examples, the method is shown to be valid in verifying the dependence of the output signal on the spoke shape. Any frequency-modulation reticle can be analyzed by this general method so long as its corresponding spoke function is known.

Subject terms: optical tracking; frequency-modulation reticles; spoke shape.

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CONTENTS

1. INTRODUCTION

Reticles[1,2] used in optical trackers or seekers as an element to modulate the light from a remote target can produce signals representing the target's deviation from the optical axis of the tracker. The performance of a reticle depends mainly on the type of modulation and the geometrical shape of its spoke. In this paper we present a general method to analyze the signal produced by frequency-modulation (FM) reticles with spokes of arbitrary shape.

2. MATHEMATICAL FORMULATION

Consider an FM reticle with 2m equally spaced spokes of alternating transmittance 1 and 0. Figure 1(a) shows the configuration of such a reticle in operation. The spoke shape is described by the function $f(\rho)$, which introduces offsetting of the phase angle θ at various radial distances ρ, as shown in Fig. 1(b).

The light from the target is focused on the image plane as a spot P, rotating with a constant angular velocity Ω around O_1. The reticle is fixed on this plane. The center of the reticle, which is at O, in general does not coincide with O_1.

Referring to the O-X-Y coordinate system first, we assume that the intensity transmittance function of the reticle is $r(\mathbf{x})$ and the intensity distribution function of the image is $p(\mathbf{x},t)$. The intensity behind the reticle is

$$u(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(\mathbf{x})\,p(\mathbf{x},t)\,d^2\mathbf{x} \ . \qquad (1)$$

For a tiny spot it is reasonable to assume that $p(\mathbf{x},t)$ is a delta function of $\mathbf{x}$; i.e.,

$$p(\mathbf{x},t) = \delta(x - x')\,\delta(y - y') \ , \qquad (2)$$

where $x'(t)$ and $y'(t)$ are the instantaneous positional coordinates of the image spot.

For the function $r(\mathbf{x})$, notice in Fig. 1(a) that $[\theta - f(\rho)]/2\pi = b/m$. Here b stands for a noninteger parameter that takes different values in different spoke zones. For example, in the zone OAB, we have $0 \le b < 1/2$; in OBC, $1/2 \le b < 1$; in ODC, $1 \le b < 3/2$; in ODE, $3/2 \le b < 2, \ldots$, and in the last zone, $m - 1/2 \le b < m$. Thus, we have

$$r(\mathbf{x}) = \mathrm{STP}(\sin\{m[\theta - f(\rho)]\}) \ , \qquad (3)$$

where

$$\mathrm{STP}(w) = \begin{cases} 1 \ , & w \ge 0 \ , \\ 0 \ , & w < 0 \ . \end{cases} \qquad (4)$$

Substituting Eq. (3) into Eq. (1), we have

$$\begin{aligned} u(t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{STP}(\sin\{m[\theta - f(\rho)]\})\,\delta(x - x')\,\delta(y - y')\,dx\,dy \\ &= \mathrm{STP}(\sin\{m[\theta - f(\rho)]\}) \\ &= \mathrm{STP}[\sin(\text{argument})] \ . \end{aligned} \qquad (5)$$

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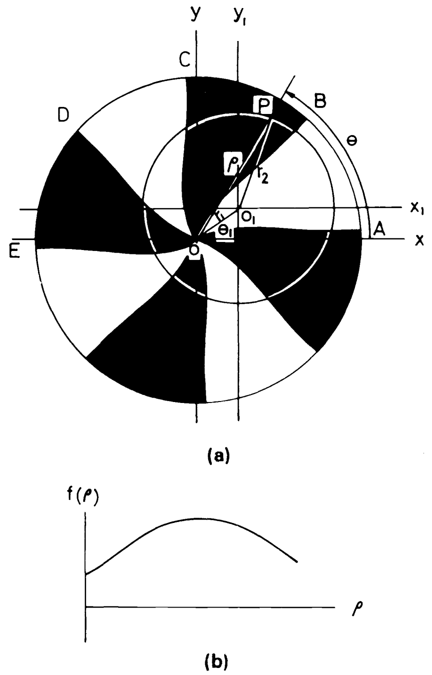

Fig. 1. (a) Reticle system. (b) Arbitrary spoke function.

After FM demodulation, the electric output voltage is

$$v(t) = \frac{d}{dt}(\text{argument})$$

$$= m\left[\frac{d\theta}{dt} - f'(\rho)\frac{d\rho}{dt}\right] . \qquad (6)$$

This signal is then used for control to perform a tracking mission in the sense that v(t) approaches a constant.

To work out Eq. (6), we transfer to the O_1-X_1-Y_1 coordinate system. Using the relationships

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) , \qquad (7)$$

$$\rho = \sqrt{x^2 + y^2} , \qquad (8)$$

$$\begin{aligned} x &= r_2\cos\Omega t - r_1\cos\theta_1 , \\ y &= r_2\sin\Omega t - r_1\sin\theta_1 , \end{aligned} \qquad (9)$$

and introducing them into Eq. (6), we have

$$v(t) = m\Omega\left\{1 + \sum_{n=1}^{\infty}\left(\frac{r_1}{r_2}\right)^n \cos[n(\Omega t - \theta_1)] - \frac{df}{d\rho}\frac{1}{\rho} r_1 r_2 \sin(\Omega t - \theta_1)\right\} . \qquad (10)$$

3. EXAMPLES

Reticles of different spoke shapes can be examined by applying different values of f in Eq. (10). Each yields a corresponding value for v(t).

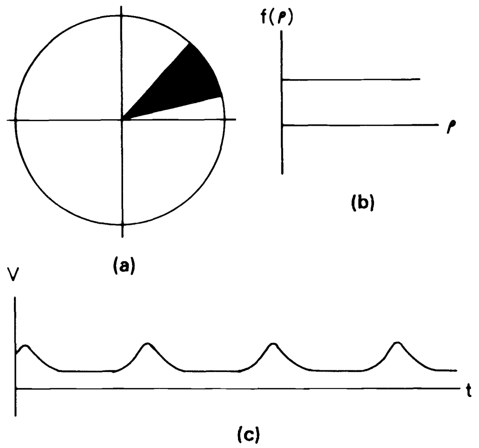

Fig. 2. (a) Reticle with straight-edge spoke; only one spoke is shown. (b) Spoke function. (c) Output signal.

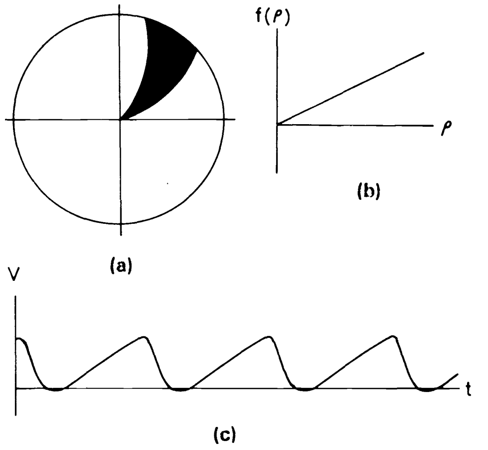

Fig. 3. (a) Typical reticle with spiral-edge spoke. (b) Spoke function. (c) Output signal.

3.1. Straight-edge spoke

For a straight-edge spoke, the simplest geometry, there is no angular variation with respect to ρ, so we assign $f(\rho) = \epsilon_1 = \text{const}$, obtaining

$$v(t) = m\Omega\left\{1 + \sum_{n=1}^{\infty}\left(\frac{r_1}{r_2}\right)^n \cos[n(\Omega t - \theta_1)]\right\} . \qquad (11)$$

Readers interested in this case may refer to Suzuki[3] and Anderson and Callary.[4] Figure 2 shows our result for such a reticle, its spoke function, and its typical output signal v(t).

3.2. Spiral-edge spoke[5]

If we assign $f(\rho) = \epsilon_2\rho$ for the spoke shape, we have a reticle with spiral-edge spokes, as shown in Fig. 3. The electric output is

$$v(t) = m\Omega\left\{1 + \sum_{n=1}^{\infty}\left(\frac{r_1}{r_2}\right)^n \cos[n(\Omega t - \theta_1)] - \frac{\epsilon_2 r_1 r_2 \sin(\Omega t - \theta_1)}{\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\Omega t - \theta_1)}}\right\} . \qquad (12)$$

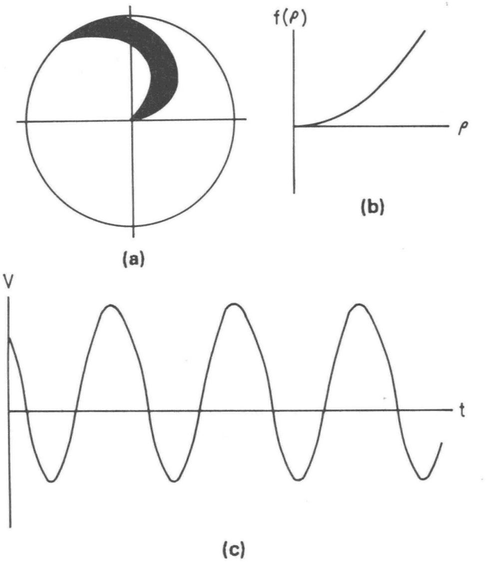

Fig. 4. (a) Reticle with parabolic spiral-edge spoke. (b) Spoke function. (c) Output signal.

3.3. Parabolic spiral-edge spoke

If we assign $f(\rho) = \epsilon_3 \rho^2$ for the spoke shape, we have a reticle with parabolic spiral-edge spokes, as shown in Fig. 4. The output is

$$v(t) = m\Omega \left\{ 1 + \sum_{n=1}^{\infty} \left(\frac{r_1}{r_2} \right)^n \cos[n(\Omega t - \theta_1)] - 2\epsilon_3 r_1 r_2 \sin(\Omega t - \theta_1) \right\} . \tag{13}$$

3.4. Complicated spiral-edge spoke

If we assign $f(\rho) = \epsilon_4 \sin \rho$ for the spoke shape, we have a reticle with complicated spiral-edge spokes, as shown in Fig. 5. The output is

$$v(t) = m\Omega \left\{ 1 + \sum_{n=1}^{\infty} \left(\frac{r_1}{r_2} \right)^n \cos[n(\Omega t - \theta_1)] - \frac{\cos[r_1^2 + r_2^2 - 2r_1 r_2 \cos(\Omega t - \theta_1)]^{1/2}}{[r_1^2 + r_2^2 - 2r_1 r_2 \cos(\Omega t - \theta_1)]^{1/2}} \times \epsilon_4 r_1 r_2 \sin(\Omega t - \theta_1)] \right\} . \tag{14}$$

4. CONCLUSION

A general method is introduced for analyzing the signal produced, after demodulation, by an FM reticle. Since the spoke shape varies from reticle to reticle, the dependence of the output signal on spoke geometry is evident. For the reduced case, i.e., the straight-edge spoke, the same result is obtained as that previously shown by Suzuki[3] and Anderson and Callary.[4]

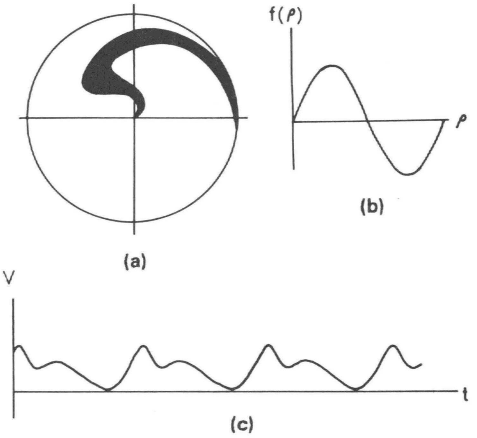

Fig. 5. (a) Reticle with complicated spiral-edge spoke. (b) Spoke function. (c) Output signal.

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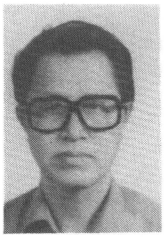
Zu Wen Chao was born in 1947. He received his BS degree from National Taiwan Normal University in 1969 and his MS degree from National Tsing Hua University, Hsin Chu, Taiwan, in 1974, both in physics. From 1974 to 1984 he was with the National Science Council, Taiwan. From 1981 to 1982 he was with the U.S. National Bureau of Standards, Maryland, as a visiting guest worker. He is now working toward his Ph.D. degree at the National Chiao Tung University. His interests include optical tracking systems, optical data processing, and spectrophotometry. He is a member of the Optical Engineering Society of Taiwan.



Jih Liang Chu received the BS degree in electrical engineering from National Cheng Kung University, Taiwan, in 1962. He completed his graduate work at National Chiao Tung University in Taiwan, where he earned the M.Sc. (1967) and Ph.D. (1972) degrees. Since joining Chung San Institute of Science and Technology in 1967, he has been engaged in developing electro-optic systems for various applications. He has been an invited professor at the Electrical Physics Department of National Chiao Tung University since 1979.