Bidirectional Optical Fiber Transmission Systems Using Raman Amplification

SIEN CHI AND MING-SENG KAO

Abstract—**Bidirectional optical fiber transmission systems using Raman amplification are discussed. The analytical expressions of the signals amplified by both forward and backward Raman scattering are presented. We have found that there exists an optimal pump power for the maximum unrepeatered transmission length in a bidirectional system, and the maximum length is about 450 km, which is about the same as that of the unidirectional system.**

I. Introduction

NONLINEAR optical effects in fibers such as stimulated Raman scattering, stimulated Brillouin scattering, self-phase modulation, and parameteric wave mixing can lead to undesirable signal loss and signal distortion in optical fiber transmission systems [1]. In particular, stimulated Raman scattering can cause crosstalks in wavelength-division-multiplexed optical fiber systems [2]–[5].

On the other hand, stimulated Raman scattering can be used as direct optical amplification to extend the repeater spacing in optical fiber transmission systems [6], [7]. Stimulated Raman amplification has been analyzed by Smith [8] analytically on the assumption that pump depletion is neglected. Recently Mochizuki [7] has carried out theoretical investigations of signals amplified by stimulated Raman scattering taking account of pump depletion. Based on the results, a unidirectional fiber transmission system of unrepeated spacing at about 400 km is proposed.

In this paper, bidirectional optical fiber transmission systems using stimulated Raman amplification are analyzed by taking account of pump depletion and analytical results are presented. From these results, the bidirectional and the unidirectional transmission systems are compared.

II. Analysis

In this section we analyze signal propagation in bidirectional fiber transmission systems using both forward and backward Raman amplification. We consider the case where the signal is pulse modulated while the pump is a continuous wave. We further assume that the initial signal power is much larger than the spontaneous emission power.

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Here we deal with single-mode fiber transmission systems with distance L. Pump power $P_1(0)$ and signal power $S_1(0)$ are injected at $z = 0$ and travel in $+z$ direction; pump power $P_2(L)$ and signal power $S_2(L)$ are injected at $z = L$ and travel in the $-z$ direction. The core radius of the fiber is r and the effective cross section A is assumed to be πr^2. α and g are loss coefficient and Raman gain constant of the fiber, respectively. In our examples, $r = 4.5\ \mu m$, $\alpha = 0.2$ dB/km and $g = 0.8 \times 10^{-11}$ cm/W.

A. Forward Raman Amplification

We analyze forward stimulated Raman amplification taking account of the pump depletion. The differential equations for the pump power $P_1(z)$ and the signal power $S_1(z)$ propagating in $+z$ direction are given by

$$\frac{dS_1(z)}{dz} = \left[\frac{g}{A} P_1(z) - \alpha_s\right] S_1(z) \tag{1}$$

$$\frac{dP_1(z)}{dz} = -\left[\frac{g \cdot \nu_p}{A \cdot \nu_s} S_1(z) + \alpha_p\right] P_1(z) \tag{2}$$

where α_s, α_p are loss coefficients of the fiber for the signal and the pump, respectively, and ν_s, ν_p are the frequencies of the signal and the pump, respectively. Here we have neglected the Raman amplification term due to spontaneous emission. Similarly, the differential equations for the pump $P_2(z)$ and the signal $S_2(z)$ propagating in $-z$ direction are given by

$$\frac{dS_2(z)}{dz} = -\left[\frac{g}{A} P_2(z) - \alpha_s\right] S_2(z) \tag{3}$$

$$\frac{dP_2(z)}{dz} = \left[\frac{g \cdot \nu_p}{A \cdot \nu_s} S_2(z) + \alpha_p\right] P_2(z). \tag{4}$$

Equations (1)–(4) can be solved analytically under the assumption

$$\alpha_s = \alpha_p = \alpha.$$

The assumption is valid around the 1.5-μm wavelength region in extremely low-loss fibers.

In considering the signals amplified by stimulated Raman scattering, we assume

$$S_1(0),\ S_2(L) \ll P_1(0),\ P_2(L).$$

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The solutions of (1)–(4) are given by [7]:

$$S_1(z) = \frac{S_1(0)\exp\left[K'(1 - e^{-\alpha z}) - \alpha z\right]}{1 + B'\exp\left[K'(1 - e^{-\alpha z})\right]} \tag{5}$$

$$P_1(z) = \frac{P_1(0)e^{-\alpha z}}{1 + B'\exp\left[K'(1 - e^{-\alpha z})\right]} \tag{6}$$

$$S_2(z) = \frac{S_2(L)\exp\left\{K\left[1 - e^{-\alpha(L-z)}\right] - \alpha(L - z)\right\}}{1 + B\exp\left\{K\left[1 - e^{-\alpha(L-z)}\right]\right\}} \tag{7}$$

$$P_2(z) = \frac{P_2(L)e^{-\alpha(L-z)}}{1 + B\exp\left\{K\left[1 - e^{-\alpha(L-z)}\right]\right\}} \tag{8}$$

where

$$B = \frac{\nu_p S_2(L)}{\nu_s P_2(L)}$$

$$B' = \frac{\nu_p S_1(0)}{\nu_s P_1(0)}$$

$$K = \frac{gP_2(L)}{\alpha A}$$

$$K' = \frac{gP_1(0)}{\alpha A}.$$

B. Backward Raman Amplification

In considering the backward Raman amplification of $S_1(z)$ near $z = L$, where $S_1(z) << P_2(z)$, the depletion of $P_2(z)$ due to $S_1(z)$ is neglected. The differential equation for $S_1(z)$ is given by

$$\frac{dS_1(z)}{dz} = \left[\frac{g}{A}P_2(z) - \alpha\right]S_1(z) \tag{9}$$

where $P_2(z)$ is given by (8). Here again we have neglected the Raman amplification term due to spontaneous emission. Similarly, in considering the backward Raman amplification of $S_2(z)$ near $z = 0$, where $S_2(z) << P_1(z)$, the depletion of $P_1(z)$ due to $S_2(z)$ is neglected. The differential equation for $S_2(z)$ is given by

$$\frac{dS_2(z)}{dz} = -\left[\frac{g}{A}P_1(z) - \alpha\right]S_2(z) \tag{10}$$

where $P_1(z)$ is given by (6).

Equation (9) can be solved analytically and $S_1(z)$ is given by (see Appendix A):

$$S_1(z) = S_1(0)\frac{\exp\left[Ke^{\alpha(z-L)}\right] + Be^K}{\exp\left[Ke^{-\alpha L}\right] + Be^K}e^{-\alpha z}. \tag{11}$$

Similarly, $S_2(z)$ is given by

$$S_2(z) = S_2(L)\frac{\exp\left[K'e^{-\alpha z}\right] + B'e^{K'}}{\exp\left[K'e^{-\alpha L}\right] + B'e^{K'}}e^{-\alpha(L-z)}. \tag{12}$$

Here, we define the gain G in the $+z$ direction due to stimulated Raman amplification by

$$G = 10\log\frac{S_1(L)}{S_1(0)e^{-\alpha L}}. \tag{13}$$

Similarly, we define the gain G' in the $-z$ direction due to stimulated Raman amplification by

$$G' = 10\log\frac{S_2(0)}{S_2(L)e^{-\alpha L}}. \tag{14}$$

Considering the gain due to backward Raman amplification, we can obtain G from (13) and (11):

$$G = 10\log\frac{1 + B}{\exp\left[K(e^{-\alpha L} - 1)\right] + B}. \tag{15}$$

When $B = 0$, i.e., $S_2(L) = 0$, (15) reduces to

$$G = 4.34\left[K(1 - e^{-\alpha L})\right] \tag{16}$$

which is the same result obtained by [7] for the unidirectional transmission. The gain given by (15) as a function of $P_2(L)$ is shown in Fig. 1 for several values of $S_2(L)$. When $P_2(L)$ is small, the gain increases linearly with $P_2(L)$ and is independent of $S_2(L)$. When $P_2(L)$ is large, the gain for $S_2(L) = 0$ still increases with $P_2(L)$, but the gain for $S_2(L) > 0$ increases in a much slower rate, and for a fixed value of $P_2(L)$, the gain decreases as $S_2(L)$ increases since a larger $S_2(L)$ causes a larger depletion of $P_2(z)$ and a smaller gain for the backward Raman amplification. Therefore, the unidirectional transmission system has a larger gain than the bidirectional transmission system with the same pump. The gain must be small enough (<80 dB) to assure the assumption of nondepleted pump due to the backward Raman amplification.

The calculated examples for (11) are shown in Fig. 2 and Fig. 3. There is a point where minimum signal power $(S_1)_{\min}$ occurs, backward amplification is made mainly in the range from this point to $z = L$. The point ($Z_{\min}$) of minimum signal power can be obtained by differentiating (11) with respect to z and is given by

$$\exp\left[Ke^{\alpha(Z_{\min}-L)}\right] + Be^K = \exp\left[Ke^{\alpha(Z_{\min}-L)}\right]Ke^{\alpha(Z_{\min}-L)} \tag{17}$$

when $B = 0$, i.e., $S_2(L) = 0$, $Z_{\min}$ is given by

$$Z_{\min} = \frac{1}{\alpha}\ln\frac{A\alpha}{gP_2(L)} + L \tag{18}$$

which is the same result obtained by [7] for the unidirectional transmission. The relation between $L - Z_{\min}$ and $P_2(L)$ given by (17) is shown in Fig. 4 for several values of $S_2(L)$. When $P_2(L)$ is small, $L - Z_{\min}$ increases with $P_2(L)$ and is independent of $S_2(L)$. When $P_2(L)$ is large, $L - Z_{\min}$ for $S_2(L) = 0$ still increases with $P_2(L)$; but for $S_2(L) > 0$, there exists a pump power $P_{2m}(L)$ such that $L - Z_{\min}$ reaches its maximum value, and when $P_2(L)$ is larger than $P_{2m}(L)$, $L - Z_{\min}$ decreases with $P_2(L)$. This means as $P_2(L)$ increases, $Z_{\min}$ shifts toward $Z = L$ for a

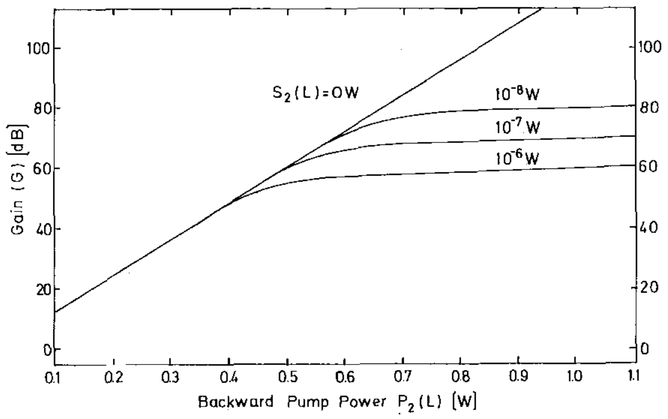

Fig. 1. Backward amplification gain of S_1 as a function of backward pump power $P_2(L)$ for several values of $S_2(L)$.

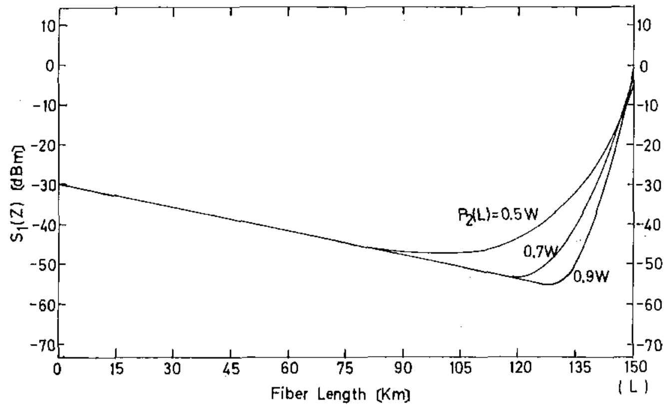

Fig. 2. Signal power variation of $S_1(z)$ as a function of fiber length in backward amplification for several values of $P_2(L)$. $S_2(L) = 10^{-6}$ W.

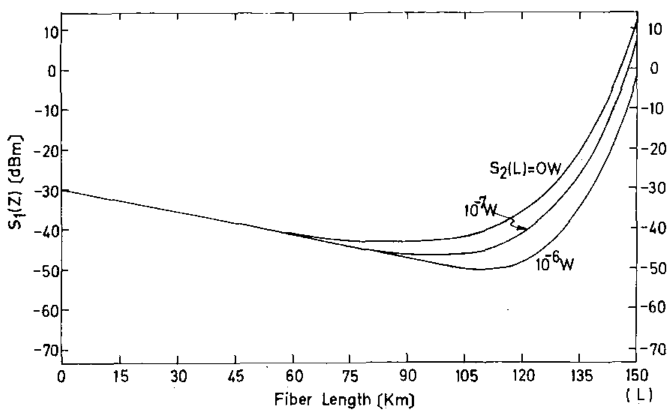

Fig. 3. Signal power variation of $S_1(z)$ as a function of fiber length in backward amplification for several values of $S_2(L)$. $P_2(L) = 0.6$ W.

bidirectional transmission system, but shifts away from $Z = L$ for a unidirectional transmission system.

From (11) and (17), the minimum signal power $(S_1)_{\min}$ as a function of $P_2(L)$ is shown in Fig. 5 for several values of $S_2(L)$. Fig. 5 is quite similar to Fig. 4. When $P_2(L)$ is small, $(S_1)_{\min}$ increases with $P_2(L)$ and is independent of $S_2(L)$. When $P_2(L)$ is large, $(S_1)_{\min}$ for $S_2(L) = 0$ still increases with $P_2(L)$, but for $S_2(L) > 0$, there exists a pump power $P_{2m}(L)$ such that $(S_1)_{\min}$ reaches its maximum value, and when $P_2(L)$ is larger than $P_{2m}(L)$, $(S_1)_{\min}$ decreases with $P_2(L)$. From Fig. 5 we can also see that when $P_2(L) > P_{2m}(L)$, the unidirectional trans-

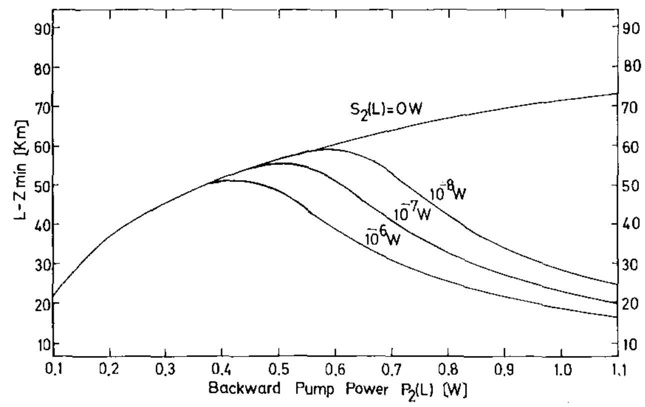

Fig. 4. L-$Z_{\min}$ as a function of backward pump power $P_2(L)$ for several values of $S_2(L)$.

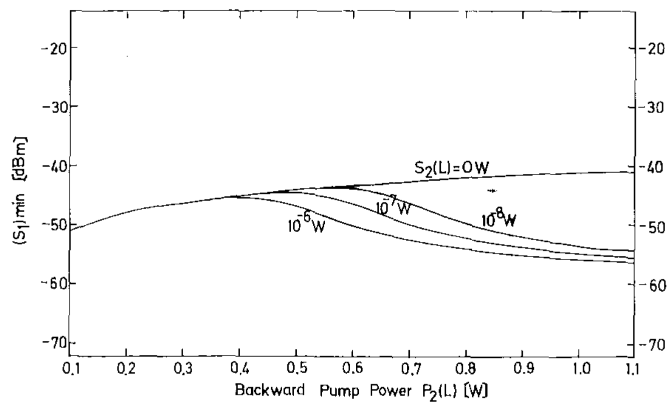

Fig. 5. Relation of minimum signal power of $S_1(z)$ and backward pump power $P_2(L)$ in backward amplification with fiber length 200 km, under the condition $S_1(0) = 10^{-5}$ W.

mission system has a larger $(S_1)_{\min}$ than the bidirectional transmission system.

III. Discussion

From (5)–(8), (11) and (12), the signals and the pumps for a bidirectional symmetric transmission system using both forward and backward Raman amplification is shown in Fig. 6.

The maximum transmission lengths without a repeater for a bidirectional symmetric and a unidirectional transmission systems are given respectively by (see Appendix B):

$$L_b = \frac{1}{\alpha} \ln \frac{S_1(0)}{(S_1)_{\min}} \frac{Ke^K \exp\left[Ke^{\alpha(Z_{\min}-L)}\right]}{(1 + Be^K)^2} \qquad (19)$$

where $L - Z_{\min}$ can be found from (17), and

$$L_u = -\frac{1}{\alpha} \ln \left[\frac{(S_1)_{\min}}{S_1(0)} \left(e^{-K'} + B'\right)\right] + \frac{1}{\alpha} \ln K + \frac{1}{\alpha} \qquad (20)$$

which has an additional term $1/\alpha$, compared to the same formula in [7]. This is caused by different mathematical assumptions in calculating maximum transmission length L. The result of [7] is obtained by considering forward Raman pump only in the range from $z = 0$ to $z = z_{\min}$ and backward pump only from $z = z_{\min}$ to $z = L$. However, the former assumption is not good since in the range $z = z_{\min}$ there almost no forward pump but backward pump starts to affect S_1. To get a more accurate result, we consider forward pump only from $z = 0$ to $z = L/2$ and backward pump only from $z = L/2$ to $z = L$. Under this assumption, the result we get has an additional term $1/\alpha$, and it is a more precise result.

Equations (19) and (20) as functions of $P_2(L)$ are shown in Fig. 7 for both the unidirectional and the bidirectional transmission systems. When $P_2(L)$ is small, $L_b = L_u$ and increases with $P_2(L)$. When $P_2(L)$ is large, L_u increases with $P_2(L)$ in a much slower rate; L_b reaches a maximum value at certain $P_2(L)$, and when $P_2(L)$ is further increased, L_b decreases. Therefore, a bidirectional transmission system has an optimal pump power for maximum transmission lengths without a repeater. As an example, we choose 1.49 and 1.6 μm as the wavelengths of the

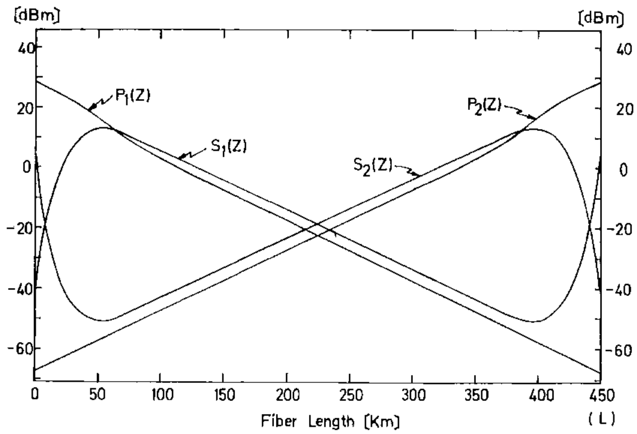

Fig. 6. Signal and pump power variation as a function of fiber length for a symmetrical bidirectional system. $L = 450$ km.

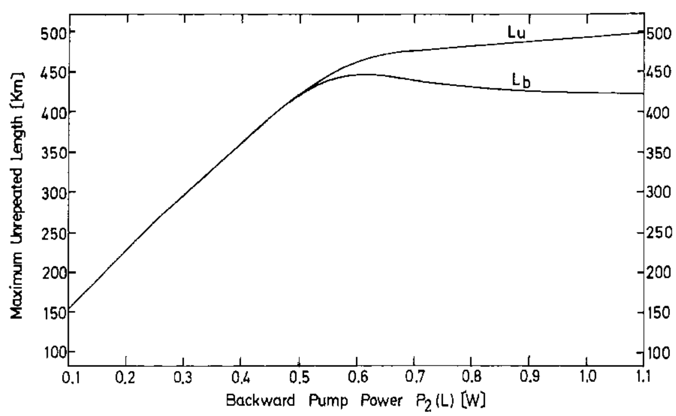

Fig. 7. Maximum unrepeated length of unidirectional (L_u) and bidirectional (L_b) systems under the condition $S_1(0) = 10^{-7}$ W, $(S_1)_{\min} = 10^{-8}$ W.

pump and the signal respectively, $P_1(0) = P_2(L) = 0.6$ W, the lowest signal power is 10^{-8} W, the fiber loss is 0.2 dB/km, and the core diameter is 9 μm. The maximum transmission length is then about 470 km for the unidirectional transmission system and 450 km for the bidirectional transmission system.

IV. Conclusion

We have derived the analytical expressions of the signals amplified by both forward and backward Raman scattering in a bidirectional transmission system. We have also derived the analytic expression of the maximum transmission length without a repeater for the unidirectional transmission system. We have found an optimal pump power for the maximum unrepeatered transmission length in a bidirectional system. The maximum unrepeatered transmission length in the bidirectional system can be easily found numerically and is about 450 km which is about the same as that of the unidirectional transmission system.

The nonlinear Kerr effect may affect the pulse shape as the pulse propagates in the fiber. This effect will be considered in the future studies.

Appendix A

We rewrite (19) into

$$\frac{dS_1(z)}{S_1(z)} = \left[\frac{g}{A} P_2(z) - \alpha\right] dz \tag{A1}$$

where $P_2(z)$ is given in (8) and is reproduced here:

$$P_2(z) = \frac{P_2(L) e^{\alpha(z-L)}}{1 + B \exp\left\{K[1 - e^{\alpha(z-L)}]\right\}}. \tag{A2}$$

Integrating (A1), from $z = 0$ to z, we have

$$\ln \frac{S_1(z)}{S_1(0)} = \frac{g}{A} \int_0^z P_2(z)\, dz - \alpha z. \tag{A3}$$

The integration $\int_0^z P_2(z)\, dz$ in (A3) can be calculated as follows.

Letting

$$y = e^{\alpha(z-L)} \tag{A4}$$

we have

$$dy = \alpha e^{\alpha(z-L)}\, dz. \tag{A5}$$

Then

$$\int_0^z P_2(z)\,dz = \int_0^z \frac{P_2(L)e^{\alpha(z-L)}}{1 + B\exp\left\{K[1 - e^{\alpha(z-L)}]\right\}}\,dz$$
$$= \frac{P_2(L)}{\alpha}\int_{e^{-\alpha L}}^{y} \frac{dy}{1 + Be^{K}e^{-Ky}}$$
$$= \frac{P_2(L)}{\alpha K}\left[\ln\left(\frac{e^{Ky}}{Be^{K}} + 1\right)\right]_{e^{-\alpha L}}^{y}$$
$$= \frac{A}{g}\ln\left\{\frac{\exp\left[Ke^{\alpha(z-L)}\right] + Be^{K}}{\exp\left[Ke^{-\alpha L}\right] + Be^{K}}\right\}. \quad (A6)$$

Substituting (A6) into (A3), we obtain

$$\ln\frac{S_1(z)}{S_1(0)} = \ln\left\{\frac{\exp\left[Ke^{\alpha(z-L)}\right] + Be^{K}}{\exp\left[Ke^{-\alpha L}\right] + Be^{K}}\right\} - \alpha z. \quad (A7)$$

Equation (A7) reduces to

$$S_1(z) = S_1(0)\left\{\frac{\exp\left[Ke^{\alpha(z-L)}\right] + Be^{K}}{\exp\left[Ke^{-\alpha L}\right] + Be^{K}}\right\}e^{-\alpha z} \quad (A8)$$

which is (11).

Appendix B

For a given minimum $(S_1)_{\min}$ at $z = z_{\min}$, the maximum transmission length L without a repeater can be found in the following way. For $z \le L/2$, we apply (5), and for $z \ge L/2$ we apply (11); then $(S_1)_{\min}$ can be expressed as

$$(S_1)_{\min} = \frac{S_1(0)\exp\left[K'(1 - e^{-\alpha(L/2)}) - \alpha(L/2)\right]}{1 + B'\exp\left[K'(1 - e^{-\alpha(L/2)})\right]} \cdot \frac{\exp\left[Ke^{-\alpha(L-z_{\min})}\right] + Be^{K}}{\exp\left[Ke^{-\alpha(L/2)}\right] + Be^{K}}\,e^{-\alpha(z_{\min}-(L/2))} \quad (B1)$$

where $L - z_{\min}$ satisfies (17) which is reproduced here:

$$\exp\left[Ke^{-\alpha(L-z_{\min})}\right] + Be^{K} = \exp\left[Ke^{-\alpha(L-z_{\min})}\right]Ke^{-\alpha(L-z_{\min})} \quad (B2)$$

By using (B2) and the condition $\alpha(L/2) >> 1$, (B1) can be reduced to

$$(S_1)_{\min} = \frac{S_1(0)e_{K'}}{1 + B'e^{K'}} \cdot \frac{\exp\left[Ke^{-\alpha(L-z_{\min})}\right]Ke^{-\alpha L}}{1 + Be^{K}}. \quad (B3)$$

For a symmetric bidirectional transmission system, (B3) reduces to

$$(S_1)_{\min} = \frac{S_1(0)e^{K}\exp\left[Ke^{-\alpha(L-z_{\min})}\right]Ke^{-\alpha L}}{(1 + Be^{K})^2}. \quad (B4)$$

Then the maximum unrepeatered transmission length L_b for the symmetric bidirectional system is

$$L_b = \frac{1}{\alpha}\ln\left\{\frac{S_1(0)}{(S_1)_{\min}}\,\frac{e^{K}\exp\left[Ke^{-\alpha(L-z_{\min})}\right]K}{(1 + Be^{K})^2}\right\} \quad (B5)$$

which is (19), where $L - z_{\min}$ can be found from (B2). For a unidirectional transmission system, (B3) reduces to

$$(S_1)_{\min} = \frac{S_1(0)e^{K'}}{1 + B'e^{K'}} \cdot K \cdot e^{1-\alpha L}. \quad (B6)$$

Then the maximum unrepeatered transmission length L_u for the unidirectional system is

$$L_u = -\frac{1}{\alpha}\ln\left[\frac{(S_1)_{\min}}{S_1(0)}(e^{-K'} + B')\right] + \frac{1}{\alpha}\ln K + \frac{1}{\alpha} \quad (B7)$$

which is (20).

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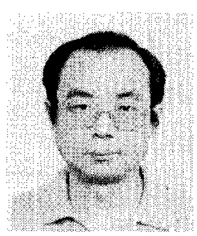

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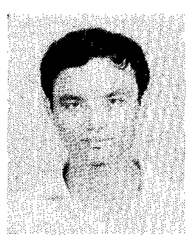

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