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# Scattering mechanisms of the Hall effect and transverse magnetoresistance in non-degenerate piezo-electric semiconductors

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**Abstract.** The Hall effect and transverse magnetoresistance in the intrinsic non-degenerate semiconductors InSb and GaAs have been investigated according to the scattering processes of carriers in solids. These scattering processes include the acoustic phonon scattering, piezo-electric scattering and ionised-impurity scattering. The energy band structure of carriers is assumed to be non-parabolic. Results show that the Hall angle, the Hall coefficient, and the transverse magnetoresistance depend strongly on the DC magnetic field due to the energy-dependent relaxation time. The holes in semiconductors play a major role in the electrical transport phenomenon for the piezo-electric scattering and ionised-impurity scattering, while for the acoustic phonon scattering the conduction electrons in semiconductors play the major role in the electrical transport phenomenon. However, the numerical results for the piezo-electric scattering and ionised-impurity scattering are shown to be rather smaller than those for the acoustic phonon scattering in the intrinsic semiconductors. Therefore, the conduction electrons play a dominant role in the carrier transport phenomenon in the intrinsic semiconductors. A comparison between experimental work and theory for the Hall effect and transverse magnetoresistance in InSb has been made. The effect of non-parabolicity in InSb has also been discussed in comparison with the numerical results for the parabolic band structure.

## 1. Introduction

The Hall effect has been used for many years as an aid to understand electric properties of solids (Putley 1960, Beer 1963, Cohen and Jortner 1973, Abakumov *et al* 1975, Golovkina *et al* 1976, Blood and Orton 1978, Chien and Westgate 1980, Klitzing *et al* 1980, Beaulac *et al* 1981, Aoki and Ando 1981, Thouless 1981, Středa 1982, Look 1982, Brandt *et al* 1983, Yoshioka *et al* 1983, Aers and MacDonald 1984, Bumaĭ *et al* 1984, Dolgov *et al* 1984, Wu and Lin 1984, 1985, Heinonen and Taylor 1985, Neustroev and Osipov 1986, Yoshihiro *et al* 1986, Furneaux and Reinecke 1986, Chui 1986). Most piezo-electric semiconductors exhibit non-linear phenomena owing to the non-parabolicity of the energy–momentum relation as the energy band structure for these materials (Nag 1972, 1980). Because of the energy-dependence of the carrier relaxation time, the Hall coefficient and transverse magnetoresistance could be affected by the DC magnetic field and temperature at low temperatures. The sign of the Hall coefficient $R_H$ tells whether we have an n-type or a p-type semiconductor. Thus a study of the non-linear transport phenomenon in the Hall effect provides an interesting method for detecting the energy band properties of semiconductors. The effect of non-parabolicity of the energy band structure in semiconductors can lead to a magnetic field dependence of the non-linear transport phenomenona with a uniform DC magnetic field (Rustagi *et al* 1968, Wu and Tsai 1980). Hansen (1981) proposed a correct form of the velocity operator from the Hamiltonian operator to show that the Hall effect is not influenced by non-parabolicity in the limit of vanishing scattering. However, the different

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scattering processes due to the interaction of carriers and imperfections in semiconductors produce different electrical transport effects (Nag 1972, 1980, Bumaĭ *et al* 1984, Wu and Tsai 1986).

These scattering processes can be analysed by the method based on the relaxation time approximation. The energy-dependent relaxation time can give rise to a strong magnetic field dependence of the electron–phonon interaction in solids. Consequently, the scattering of carriers in semiconductors plays an important role in the Hall effect and transverse magnetoresistance in semiconductors. The energy-dependent relaxation time for carriers with type $i$ non-parabolic band structure for several fundamental scattering processes can be expressed in the forms (Smith 1969, Bumaĭ 1984, Wu and Tsai 1986):

(i) for acoustic-phonon scattering mechanism

$$\tau_i = \tau_{i0}(E_g/2E_{kn}^{(i)})^{1/2}\theta_{kn}^{(i)3/2} \tag{1}$$

(ii) for piezo-electric scattering mechanism

$$\tau_i = \tau_{i0}(2E_{kn}^{(i)}/E_g)^{1/2}\theta_{kn}^{(i)1/2} \tag{2}$$

(iii) for ionised-impurity scattering mechanism

$$\tau_i = \tau_{i0}(2E_{kn}^{(i)}/E_g)^{3/2}\theta_{kn}^{(i)-1/2}. \tag{3}$$

Here, $\tau_{i0}$ is the energy-independent relaxation time of carriers with the type $i$ and can be considered as a constant, $E_{kn}^{(i)}$ is the energy level carriers of type $i$, $E_g$ is the energy gap between the conduction and valence bands, and the parameter

$$\theta_{kn}^{(i)} = (1 + 2E_{kn}^{(i)}/E_g)^{-1} \tag{4}$$

comes from the correction for the effective mass of carriers in semiconductors.

In this paper we shall study the Hall effect and transverse magnetoresistance in non-degenerate intrinsic semiconductors of III–V compounds such as InSb and GaAs in the presence of a uniform DC magnetic field by taking into account the effect of carrier relaxation time from scattering processes in solids as shown in equations (1)–(3). In intrinsic or near-intrinsic semiconductors both electrons and holes are present and we have to deal with sets of electrons or sets of holes with different effective mass tensors. We also make the following assumptions:

(i) for non-degenerate semiconductors, the distribution function of carriers can be represented by the Maxwell–Boltzmann distribution;

(ii) the relaxation time of carriers is assumed to be dominated by the type of scattering given by equations (1)–(3);

(iii) the energy band structure of semiconductors is assumed to be non-parabolic.

In §2, we derive the Hall angle and Hall coefficient in the presence of DC magnetic field $\boldsymbol{B}$ along the $z$ axis. The electric field is taken to be in the $x$–$y$ plane and the current density is restricted to be along the $x$ direction. We also calculate the transverse magnetoresistance for both types of carriers in intrinsic semiconductors. In §3, we calculate the average value of the parameters given in §2. These parameters contain the energy-dependent relaxation time of carriers according to the scattering processes in solids. In §4, numerical results for the Hall effect and transverse magnetoresistance in InSb and GaAs are presented. Finally, a brief discussion about our numerical analysis is given and a comparison between the parabolic and non-parabolic band for InSb has been made.

## 2. The Hall effect and transverse magnetoresistance in a uniform DC magnetic field

In the non-parabolic model, the relation between the energy $E$ and the momentum $\boldsymbol{p}(p_x, p_y, p_z)$ of carriers can be represented by (Nag 1972, 1980)

$$E(1 + E/E_g) = \boldsymbol{p}^2/2m_i^* \tag{5}$$

where $m_i^*$ is the effective mass of carriers with $i = \mathrm{e}$ for electrons and $i = \mathrm{h}$ for holes. For carriers in a uniform DC magnetic field $\boldsymbol{B}$ directed along the $z$ axis, the energy of carriers can be obtained by using the vector potential of the Landau gauge $\boldsymbol{A}_0 = (0, Bx, 0)$ as (Wu and Tsai 1980)

$$E_{kn}^{(i)} = -\tfrac{1}{2}E_g[\![1 + \hat{\imath}\{1 + (4/E_g)[(n + \tfrac{1}{2})\hbar\omega_{ci} + \hbar^2k_z^2/2m_i^*]\}^{1/2}]\!] \tag{6}$$

where $\hat{\imath} = 1$ for holes, $\hat{\imath} = -1$ for electrons, $k_z$ is the $z$ component of the carrier wavevector $\boldsymbol{k}$, and $\omega_{ci} = |e|B/m_i^*c$ is the cyclotron frequency of carriers with the type $i$.

If the DC magnetic field $\boldsymbol{B}$ is applied at right angles to an applied electric field, then the interaction between the DC magnetic field and the moving carriers produces a potential difference in the third direction mutually orthogonal to the electric field $\bar{\mathscr{E}} = (\mathscr{E}_x, \mathscr{E}_y, 0)$ which lies in the $x$–$y$ plane. Thus the average current densitities in the $x$–$y$ plane due to the contributions from electrons and holes can be expressed by (Smith 1969)

$$J_x = \left(\frac{n_c e^2}{m^*_c}\right)\left[\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle \mathscr{E}_x - \left(\frac{eB}{m^*_c c}\right)\left\langle\frac{\tau^2_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle \mathscr{E}_y\right]$$
$$+\left(\frac{n_h e^2}{m^*_h}\right)\left[\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle \mathscr{E}_x + \left(\frac{eB}{m^*_h c}\right)\left\langle\frac{\tau^2_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle \mathscr{E}_y\right] \quad (7)$$

and

$$J_y = \left(\frac{n_c e^2}{m^*_c}\right)\left[\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle \mathscr{E}_y + \left(\frac{eB}{m^*_c c}\right)\left\langle\frac{\tau^2_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle \mathscr{E}_x\right]$$
$$+\left(\frac{n_h e^2}{m^*_h}\right)\left[\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle \mathscr{E}_y - \left(\frac{eB}{m^*_h c}\right)\right.$$
$$\left.\times\left\langle\frac{\tau^2_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle \mathscr{E}_x\right] \quad (8)$$

where $n_i$ is the carrier concentration with the type $i$, and $\langle\ldots\rangle$ denotes the average value taken over all energies in the system. If the current density is taken in the $x$ direction only, i.e. $J_y = 0$, then one can obtain from equation (8)

$$\mathscr{E}_y = -\left(\frac{eB}{c}\right)\left[\frac{n_c}{(m^*_c)^2}\left\langle\frac{\tau^2_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle - \frac{n_h}{(m^*_h)^2}\left\langle\frac{\tau^2_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]$$
$$\times\left[\left(\frac{n_c}{m^*_c}\right)\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle + \left(\frac{n_h}{m^*_h}\right)\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^{-1}\mathscr{E}_x. \quad (9)$$

The electric field initially is $\mathscr{E}_x$ when the DC magnetic field is absent, whereas the electric field can be represented by $\bar{\mathscr{E}} = (\mathscr{E}_x, \mathscr{E}_y, 0)$ in the presence of a DC magnetic field. The effect of a DC magnetic field causes a rotation of the electric field through a specific angle $\theta_H$, called the Hall angle. Thus, from equation (9), the Hall angle $\theta_H$ is given by the relation

$$\tan\theta_H = \frac{\mathscr{E}_y}{\mathscr{E}_x}$$
$$= -\left(\frac{c}{eB}\right)\left\{(n_c - n_h) - \left[n_c\left\langle\frac{1}{1+\omega^2_{cc}\tau^2_c}\right\rangle\right.\right.$$
$$\left.\left.- n_h\left\langle\frac{1}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]\right\}\left[\left(\frac{n_c}{m^*_c}\right)\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle\right.$$
$$\left.+\left(\frac{n_h}{m^*_h}\right)\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^{-1}. \quad (10)$$

From equations (7) and (9), one can also obtain

$$J_x = -\left(\frac{ce}{B}\right)\left\{\left[\left(\frac{n_c}{m^*_c}\right)\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle + \left(\frac{n_h}{m^*_h}\right)\right.\right.$$
$$\left.\times\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^2 + \left(\frac{eB}{c}\right)^2\left[\left(\frac{n_c}{m^*_c}\right)\left\langle\frac{\tau^2_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle\right.$$
$$\left.\left.-\left(\frac{n_h}{m^{*2}_h}\right)\left\langle\frac{\tau^2_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^2\right\}\left[\left(\frac{n_c}{m^{*2}_c}\right)\left\langle\frac{\tau^2_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle\right.$$
$$\left.-\left(\frac{n_h}{m^{*2}_h}\right)\left\langle\frac{\tau^2_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^{-1}\mathscr{E}_y. \quad (11)$$

Hence the Hall coefficient $R_H$ can be obtained as

$$R_H = \frac{\mathscr{E}_y}{J_x B}$$
$$= -\left(\frac{c}{e^3B^2}\right)\left[(n_c - n_h) - \left(n_c\left\langle\frac{1}{1+\omega^2_{cc}\tau^2_c}\right\rangle\right.\right.$$
$$\left.\left.- n_h\left\langle\frac{1}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right)\right]\left\{\left[\left(\frac{n_c}{m^*_c}\right)\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle\right.\right.$$
$$\left.+\left(\frac{n_h}{m^*_h}\right)\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^2 + \left(\frac{c}{eB}\right)^2\left[(n_c - n_h)\right.$$
$$\left.\left.- n_c\left\langle\frac{1}{1+\omega^2_{cc}\tau^2_c}\right\rangle + n_h\left\langle\frac{1}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^2\right\}^{-1}. \quad (12)$$

Similarly, from equations (7) and (9), one can obtain

$$J_x = e^2\left\{\left(\frac{n_c}{m^*_c}\right)\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle + \left(\frac{n_h}{m^*_h}\right)\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right.$$
$$+\left(\frac{eB}{e}\right)^2\left[\left(\frac{n_c}{m^{*2}_c}\right)\left\langle\frac{\tau^2_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle - \left(\frac{n_h}{m^{*2}_h}\right)\left\langle\frac{\tau^2_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^2$$
$$\times\left[\left(\frac{n_c}{m^*_c}\right)\left\langle\frac{\tau_c}{1+\omega^2_{cc}\tau^2_c}\right\rangle + \left(\frac{n_h}{m^*_h}\right)\right.$$
$$\left.\left.\times\left\langle\frac{\tau_h}{1+\omega^2_{ch}\tau^2_h}\right\rangle\right]^{-1}\right\}\mathscr{E}_x. \quad (13)$$

Thus the conductivity due to the contribution from both types of carriers (electrons and holes) can be expressed by

$$\sigma(B)=e^2\left\{\left(\frac{n_\mathrm{c}}{m_\mathrm{c}^*}\right)\left\langle\frac{\tau_\mathrm{c}}{1+\omega_\mathrm{cc}^2\tau_\mathrm{c}^2}\right\rangle+\left(\frac{n_\mathrm{h}}{m_\mathrm{h}^*}\right)\left\langle\frac{\tau_\mathrm{h}}{1+\omega_\mathrm{ch}^2\tau_\mathrm{h}^2}\right\rangle\right.$$

$$+\left(\frac{c}{eB}\right)^2\left[n_\mathrm{c}-n_\mathrm{h}-n_\mathrm{c}\left\langle\frac{1}{1+\omega_\mathrm{cc}^2\tau_\mathrm{c}^2}\right\rangle+n_\mathrm{h}\left\langle\frac{1}{1+\omega_\mathrm{ch}^2\tau_\mathrm{h}^2}\right\rangle\right]^2$$

$$\left.\times\left[\left(\frac{n_\mathrm{c}}{m_\mathrm{c}^*}\right)\left\langle\frac{\tau_\mathrm{c}}{1+\omega_\mathrm{cc}^2\tau_\mathrm{c}^2}\right\rangle+\left(\frac{n_\mathrm{h}}{m_\mathrm{h}^*}\right)\left\langle\frac{\tau_\mathrm{h}}{1+\omega_\mathrm{ch}^2\tau_\mathrm{h}^2}\right\rangle\right]^{-1}\right\}. \qquad (14)$$

If $\rho_0$ is the resistivity at zero magnetic field and $\rho(B)=1/\sigma(B)$, then we have an expression as follows:

$$\frac{\Delta\rho}{\rho_0}=1-\frac{\sigma(B)}{\sigma(0)} \qquad (15)$$

where $\Delta\rho=\rho(B)-\rho_0$ is the change in resistivity from the zero-field value $\rho_0$, and $\sigma(0)$ is given by

$$\sigma(0)=e^2\,(n_\mathrm{c}\tau_\mathrm{c0}/m_\mathrm{c}^*+n_\mathrm{h}\tau_\mathrm{h0}/m_\mathrm{h}^*) \qquad (16)$$

where $\tau_\mathrm{c0}$ and $\tau_\mathrm{h0}$ are energy-independent relaxation times for electrons and holes, respectively, and can be considered as constants in our expression.

## 3. Calculations of $\langle 1/(1+\omega_{\mathrm{c}i}^2\tau_i^2)\rangle$and $\langle \tau_i/(1+\omega_{\mathrm{c}i}^2\tau_i^2)\rangle$

Since $(\hbar k_{z\,\mathrm{max}})^2/2m_i^*\simeq k_\mathrm{B}T\ll E_\mathrm{g}$ at the low temperatures ($T<100\,\mathrm{K}$) in which we are interested, equation (6) can be expressed by

$$E_{kn}^{(i)}\simeq-\tfrac{1}{2}E_\mathrm{g}-\hat{\imath}[\tfrac{1}{2}E_\mathrm{g}a_n^{(i)}+\hbar^2k_z^2/2m_i^*a_n^{(i)}] \qquad (17)$$

with

$$a_n^{(i)}=[1+(4\hbar\omega_{\mathrm{c}i}/E_\mathrm{g})(n+\tfrac{1}{2})]^{1/2}. \qquad (18)$$

The factor $a_n^{(i)}$ defined by equation (18) plays an important role of the non-parabolic band structure in solids at low temperatures and high magnetic fields. From the processes of different scattering mechanisms of carriers in semiconductors as shown in equation (1)–(3), one can obtain the expressions for $\langle 1/(1+\omega_{\mathrm{c}i}^2\tau_i^2)\rangle$ and $\langle \tau_i/(1+\omega_{\mathrm{c}i}^2\tau_i^2)\rangle$ as follows.

(i) For the acoustic phonon scattering mechanism

$$\left\langle\frac{1}{1+\omega_{\mathrm{c}i}^2\tau_i^2}\right\rangle=(2E_\mathrm{g}/k_\mathrm{B}T)^{1/2}\left[\sum_{n=0}^{\infty}(a_n^{(i)})^{1/2}\exp\left(\frac{E_\mathrm{g}a_n^{(i)}}{2k_\mathrm{B}T}\right)\right]^{-1}$$

$$\times\left[\sum_{n=0}^{\infty}F(\alpha_n)\exp\left(-\frac{E_\mathrm{g}a_n^{(i)}}{2k_\mathrm{B}T}\right)\left(\frac{\omega_{\mathrm{c}i}\tau_{i0}(4\mathrm{a}_n^{(i)}-3)^{-1/2}}{(a_n^{(i)})^{5/2}(a_n^{(i)}-1)}\right)^{-1}\right.$$

$$\left.\times\left(1+\frac{\omega_{\mathrm{c}i}^2\tau_{io}^2}{(a_n^{(i)})^3(a_n^{(i)}-1)}\right)^{-1/2}\right. \qquad (19)$$

where

$$\alpha_n=\frac{(E_\mathrm{g}/2k_\mathrm{B}T)^{1/2}(a_n^{(i)})^2(a_n^{(i)}-1)}{\omega_{\mathrm{c}i}\tau_{i0}(4a_n^{(i)}-3)^{1/2}}$$

$$\times\left[1+\frac{\omega_{\mathrm{c}i}^2\tau_{i0}^2}{(a_n^{(i)})^3(a_n^{(i)}-1)}\right]^{1/2} \qquad (20)$$

and

$$F(x)=\exp(-x^2)\int_0^x\exp(t^2)\,\mathrm{d}t \qquad (21)$$

is the Dawson integral (Abramowitz and Stegun 1964).

$$\left\langle\frac{\tau_i}{1+\omega_{\mathrm{c}i}^2\tau_i^2}\right\rangle=(E_\mathrm{g}/k_\mathrm{B}T)^{1/2}\left[2\surd 2\omega_{\mathrm{c}i}\sum_{n=0}^{\infty}(a_n^{(i)})^{1/2}\right.$$

$$\left.\times\exp\left(-\frac{E_\mathrm{g}a_n^{(i)}}{2k_\mathrm{B}T}\right)\right]^{-1}\sum_{n=0}^{\infty}\left\{a_n^{(i)}(a_n^{(i)}-1)\exp\left(-\frac{E_\mathrm{g}a_n^{(i)}}{2k_\mathrm{B}T}\right)\right.$$

$$\left.\times\left[(4a_n^{(i)}-3)^{1/2}\left(1+\frac{\omega_{\mathrm{c}i}^2\tau_{i0}^2}{(a_n^{(i)})^3(a_n^{(i)}-1)}\right)^{1/2}\right]^{-1}\right\}\left(\frac{2k_\mathrm{B}T}{E_\mathrm{g}}\right)^{1/2}$$

$$\times\frac{a_n^{(i)}(4a_n^{(i)}-3)^{1/2}}{\omega_{\mathrm{c}i}\tau_{i0}}\left(1+\frac{\omega_{\mathrm{c}i}^2\tau_{i0}^2}{(a_n^{(i)})^3(a_n^{(i)}-1)}\right)^{1/2}$$

$$\left.+2\left(1-\frac{(a_n^{(i)})^3(a_n^{(i)}-1)}{\omega_{\mathrm{c}i}^2\tau_{i0}^2}\right)F(\alpha_n)\right]. \qquad (22)$$

(*ii*) For the piezo-electric scattering mechanism

$$\left\langle\frac{1}{1+\omega_{\mathrm{c}i}^2\tau_i^2}\right\rangle=\surd(\pi/2)(E_\mathrm{g}/k_\mathrm{B}T)^{1/2}\left[\omega_{\mathrm{c}i}\tau_i\sum_{n=0}^{\infty}(a_n^{(i)})^{1/2}\right.$$

$$\times\exp\left(-\frac{E_\mathrm{g}a_n^{(i)}}{2k_\mathrm{B}T}\right)\right]^{-1}\sum_{n=0}^{\infty}\left[(a_n^{(i)})^{3/2}\exp\left(-\frac{E_\mathrm{g}a_n^{(i)}}{2k_\mathrm{B}T}\right)W(\mathrm{i}\beta_n)\right.$$

$$\left.\times\left(1+\frac{\omega_{\mathrm{c}i}^2\tau_{i0}^2[a_n^{(i)}-1]}{(a_n^{(i)})}\right)^{1/2}\right] \qquad (23)$$

where

$$\beta_n=\frac{(E_\mathrm{g}/2k_\mathrm{B}T)^{1/2}a_n^{(i)}}{\omega_{\mathrm{c}i}\tau_{i0}}\left(1+\frac{\omega_{\mathrm{c}i}^2\tau_{i0}^2(a_n^{(i)}-1)}{a_n^{(i)}}\right)^{1/2}. \qquad (24)$$

The function $W(\mathrm{i}x)$ is given by (Abramowitz and Stegun 1964)

$$W(\mathrm{i}x) = \exp(x^2)\ \mathrm{erfc}(x)$$

$$= \exp(x^2)\left(1 - \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)\,\mathrm{d}t\right). \quad (25)$$

$$\left\langle \frac{\tau_i}{1+\omega_{ci}^2\tau_i^2} \right\rangle = \left[ 2\omega_{ci}^2\tau_{i0} \sum_{n=0}^{\infty} (a_n^{(i)})^{1/2} \exp\left( -\frac{E_g a_n^{(i)}}{2k_B T} \right) \right]^{-1}$$

$$\times \sum_{n=0}^{\infty} \frac{a_n^{(i)}\ \exp[-E_g a_n^{(i)}/2k_B T]}{[a_n^{(i)}-1]^{1/2}} \left[ 1 - \sqrt{\left(\frac{\pi}{2}\right)\left(\frac{E_g}{k_B T}\right)}^{1/2} \right.$$

$$\times W(\mathrm{i}\beta_n)(\omega_{ci}\tau_{i0})^{-1}(a_n^{(i)}-1) - (\omega_{ci}\tau_{i0})^{-3}a_n^{(i)}$$

$$\left. \times \left( 1 + \frac{\omega_{ci}^2\tau_{i0}^2(a_n^{(i)}-1)}{a_n^{(i)}} \right)^{1/2} \right]. \quad (26)$$

(iii) For the ionised-impurity scattering mechanism

$$\left\langle \frac{1}{1+\omega_{ci}^2\tau_i^2} \right\rangle = \sqrt{(\pi/2)}(E_g/k_B T)^{1/2}$$

$$\times \left[ \omega_{ci}\tau_{i0} \sum_{n=0}^{\infty} (a_n^{(i)})^{1/2} \exp\left( -\frac{E_g a_n^{(i)}}{2k_B T} \right) \right]^{-1}$$

$$\times \sum_{n=0}^{\infty} \frac{(a_n^{(i)})^{1/2} \exp(-E_g a_n^{(i)}/2k_B T) W(\mathrm{i}\gamma_n)}{(a_n^{(i)}-1)(4a_n^{(i)}-1)^{1/2}[1+\omega_{ci}^2\tau_{i0}^2 a_n^{(i)}(a_n^{(i)}-1)^3]^{1/2}} \quad (27)$$

where

$$\gamma_n = \frac{(E_g/2k_B T)^{1/2}[1+\omega_{ci}^2\tau_{i0}^2 a_n^{(i)}(a_n^{(i)}-1)^3]^{1/2}}{\omega_{ci}\tau_{i0}(a_n^{(i)}-1)(4a_n^{(i)}-1)^{1/2}}. \quad (28)$$

$$\left\langle \frac{\tau_i}{1+\omega_{ci}^2\tau_{i0}^2} \right\rangle = \left[ 2\omega_{ci}^2\tau_{i0} \sum_{n=0}^{\infty} (a_n^{(i)})^{1/2} \exp\left( -\frac{E_g a_n^{(i)}}{2k_B T} \right) \right]^{-1}$$

$$\times \sum_{n=0}^{\infty} (a_n^{(i)}-1)^{-3/2} \exp\left( -\frac{E_g a_n^{(i)}}{2k_B T} \right)$$

$$\times \left( 1 - \frac{\sqrt{(\pi/2)}(E_g/k_B T)^{1/2}[\omega_{ci}^2\tau_{i0}^2 a_n^{(i)}(a_n^{(i)}-1)^3 - 1]W(\mathrm{i}\gamma_n)}{\omega_{ci}\tau_{i0}(a_n^{(i)}-1)(4a_n^{(i)}-1)^{1/2}[1+\omega_{ci}^2\tau_{i0}^2 a_n^{(i)}(a_n^{(i)}-1)^3]^{1/2}} \right). \quad (29)$$

These results have been approximated by the expansion of $(1+\omega_{ci}^2\tau_i^2)^{-1}$ with the assumption that at low temperatures $(\hbar^2 k_z^2/2m_i^*)_{\max} \simeq k_B \mathrm{T} \ll E_g$. This turns out to be $\hbar^2 k_z^2/m_i^* a_n^{(i)2} E_g \ll 1$ for $a_n^{(i)} > 1$ in high magnetic fields.

## 4. Numerical analysis and discussion

We consider the intrinsic semiconductors of III–V compounds such as InSb and GaAs. The relevant values of physical parameters for these materials are given in table 1 (Sze 1981, Wu and Tsai 1986). We plot the Hall angle $\theta_H$ as a function of the DC magnetic field B for intrinsic semiconductors as shown in figure 1. Figure 1(*a*) for the acoustic phonon scattering shows that the Hall angle $\theta_H$ decreases monotonically with the magnetic field. It can be seen that the behaviour Hall angle with temperature for InSb becomes quite complicated in certain regions $B < 8$ kG, $B = 55$–$60$ kG, and $B >$ 125 kG. In these regions the Hall angle shows small oscillations with the magnetic field. However, for a large-gap semiconductor such as GaAs, this abnormal region occurs only in a lower magnetic field region B < 25 kG.

Since the relaxation time for carriers is energy-dependent as shown in equations (1)–(3) and the energy band structure of III–V semiconductors is assumed to be non-parabolic, the Hall angle exhibits a small oscillation in a non-degenerate semiconductor. These oscillations come from the factor $a_n^{(i)}$ in equation (18) owing to the magnetophonon effect. This magnetophonon effect requires no degeneracy of carriers and these oscillations occur if the phonon energy is an integer multiple of the Landau level spacing $\hbar\omega_{ci}$ (Gurevich and Firsov 1961, Firsov *et al* 1964). For large-gap semiconductor GaAs, however, this magnetophonon effect becomes insignificant due to a small change of $a_n^{(i)}$ with the DC magnetic field in lower fields, and the Hall angle $\theta_H$ becomes a monotonic function of the magnetic field. In figure 1(*b*) for the piezo-electric scattering and figure 1(*c*) for the ionised-impurity scattering, it is shown that the Hall angle $\theta_H$ increases rapidly with the magnetic field in the low-field region and then increases moderately with the magnetic field in the high-field region. But the numerical results for the piezo-electric scattering are larger than those for the ionised-impurity scattering. For the numerical values of GaAs, the Hall angle becomes quite small even in the high-field region. In figure 2, we plot the Hall coefficient as a function of the DC magnetic field in intrinsic semiconductors. It shows that the numerical values for InSb are much larger than those for GaAs. For the acoustic phonon scattering as shown in figure 2(*a*), $-R_H$ increases with magnetic field and then decreases with the field in InSb. The maximum point appears in the range of magnetic

**Table 1.** Physical parameters for InSb and GaAs:$n_i$ is the carrier density for type *i*, $m_i^*$ is the effective mass of carriers of type *i*, $m_0$ is the mass of free electrons, $\tau_{i0}$ is the energy-independent relaxation time of carriers of type *i*, and $E_g$ is the energy gap of the semiconductors.

| Samples | $n_e = n_h$ (cm$^{-3}$) | $m_e^*/m_0$ | $m_h^*/m_0$ | $\tau_{e0}$ (s) | $\tau_{h0}$ (s) | $E_g$ (eV) |
|---|---|---|---|---|---|---|
| InSb | $1.75 \times 10^{14}$ | 0.0145 | 0.4 | $6.5 \times 10^{-13}$ | $2.8 \times 10^{-13}$ | 0.2 |
| GaAs | $1.73 \times 10^{15}$ | 0.067 | 0.082 | $3.24 \times 10^{-13}$ | $1.86 \times 10^{-14}$ | 1.51 |

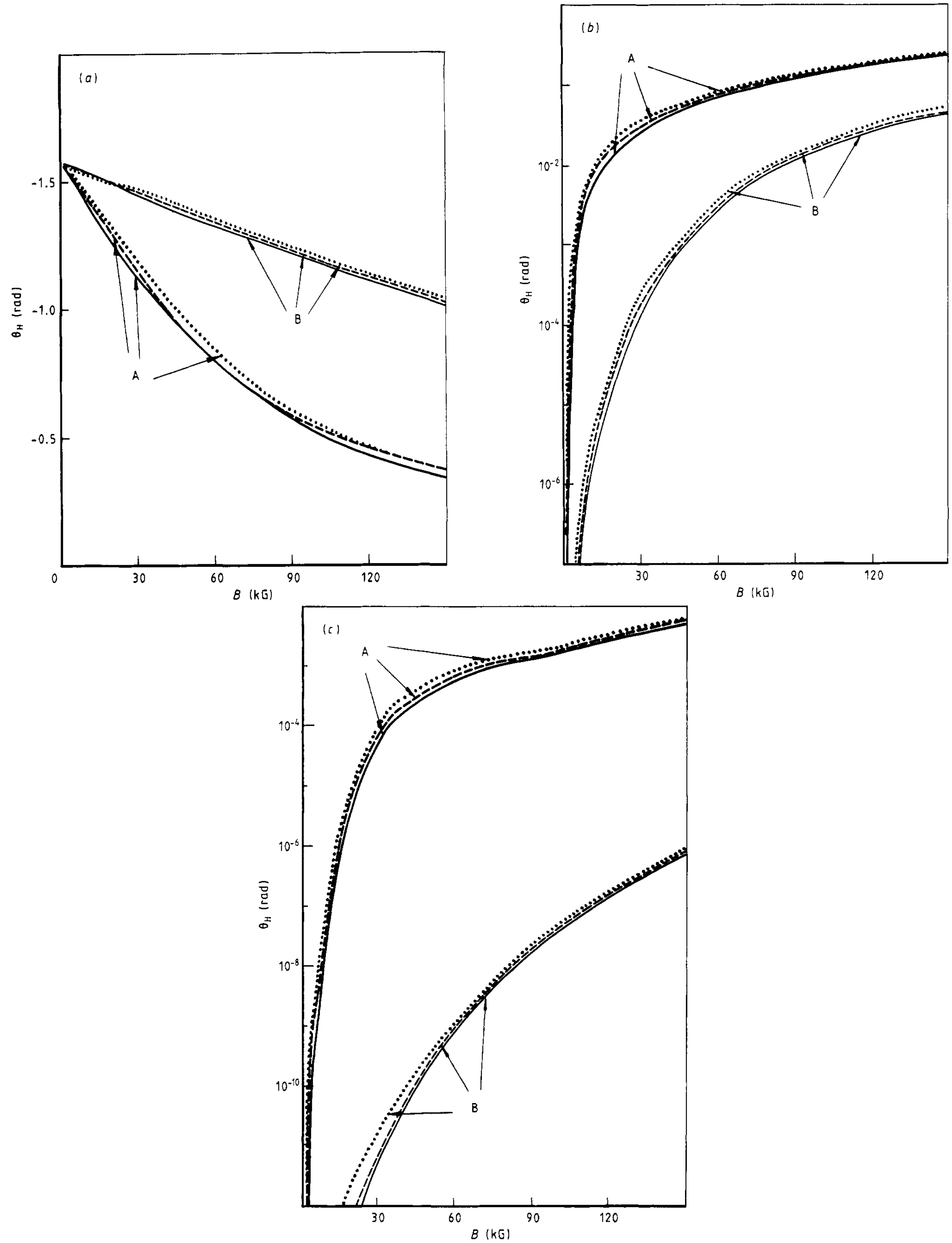

**Figure 1.** The Hall angle $\theta_H$ as a function of DC magnetic field $B$ in intrinsic semiconductors for: (*a*) acoustic phonon scattering; (*b*) piezo-electric scattering; (*c*) ionised-impurity scattering. A, InSB; B, GaAs. Dotted curves, 19.7 K; broken curves, 10 K; full curves, 4.2 K.

fields around $B = 50$–90 kG at low temperatures. This maximum point will be shifted to the higher DC magnetic field region with increasing temperature. However, for the case of GaAs, $-R_H$ increases monotonically with the magnetic field. For the piezo-electric scattering as shown in figure 2(*b*), the Hall coefficient $R_H$ of InSb

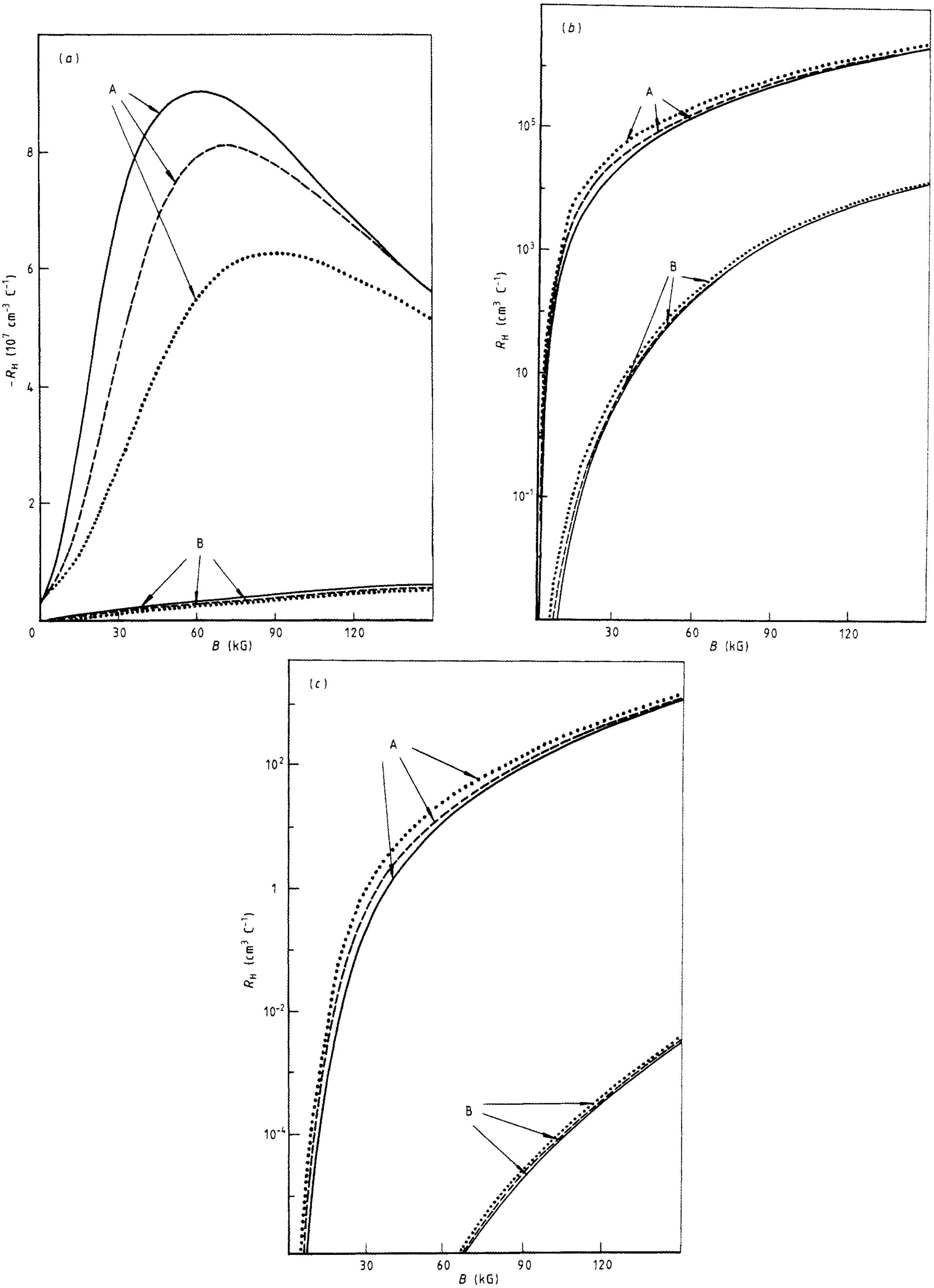

**Figure 2.** The Hall coefficient $R_H$ as a function of DC magnetic field $B$ in intrinsic semiconductors for (*a*) the acoustic phonon scattering; (*b*) the piezo-electric scattering; (*c*) the ionised-impurity scattering. A, InSb; B, GaAs. Dotted curves, 19.7 K; broken curves, 10 K; full curves, 4.2 K

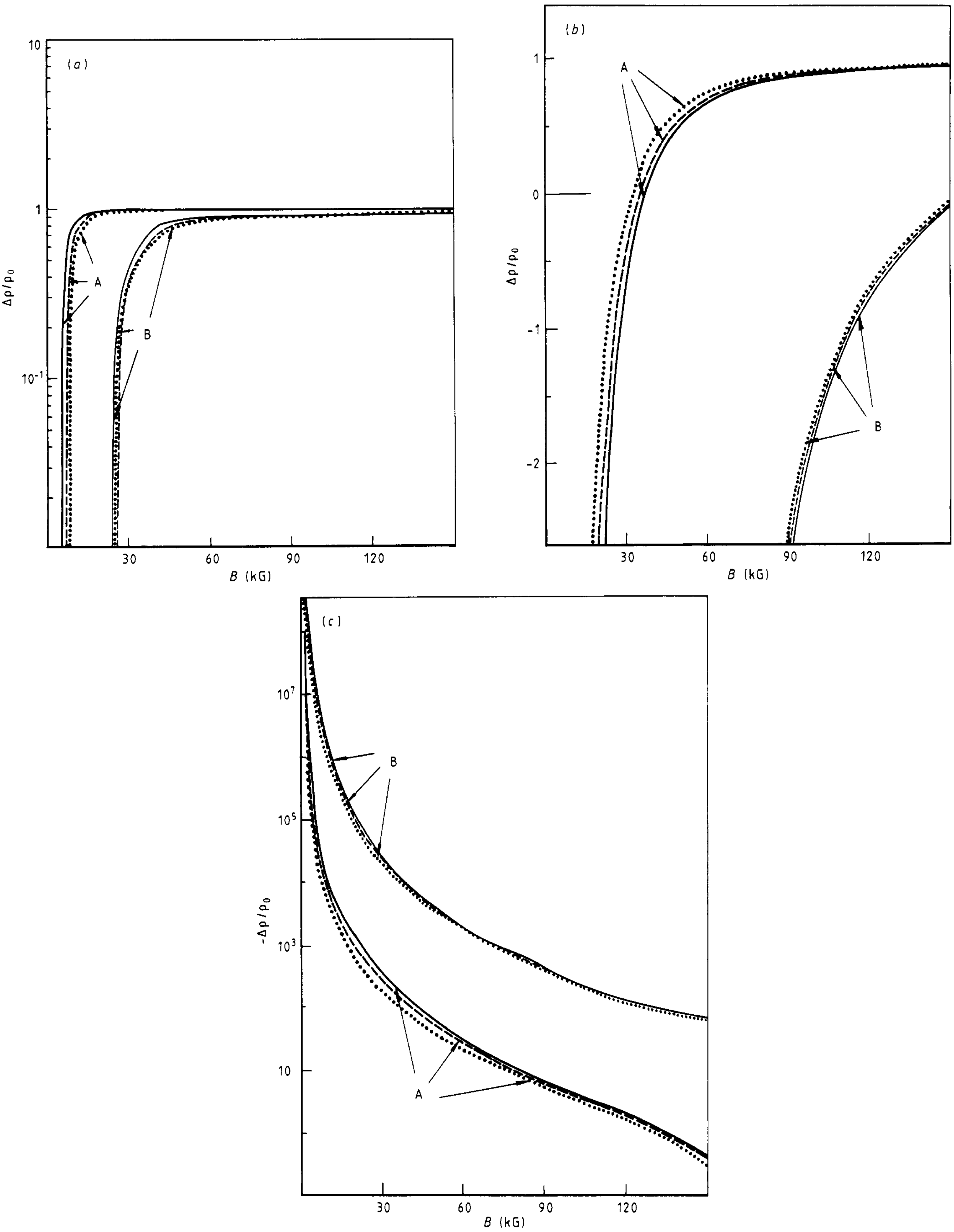

**Figure 3.** The transverse magnetoresistance $\Delta\rho/\rho_0$ as a function of DC magnetic field $B$ in intrinsic semiconductors for (*a*) the acoustic phonon scattering; (*b*) the piezo-electric scattering; (*c*) the ionised-impurity scattering. A, InSb; B, GaAs. Dotted curves, 19.7 K; broken curves, 10 K; full curves, 4.2 K.

increases rapidly with the magnetic field in the lower-field region and increases moderately with the field for $B > 15$ kG, while for the case of GaAs, $R_H$ increases monotonically with the magnetic field. It can also be seen that the numerical values for InSb are much larger than those for GaAs. For the ionised-impurity scattering in InSb, $R_H$ increases rapidly with the magnetic field and then increases moderately with the field for $B <$ 25 kG. For the case of GaAs, $R_H$ increases monotonically with the magnetic field and its numerical value is much smaller than that for InSb. For the acoustic phonon scattering, the values of the Hall angle and Hall coefficient are negative, thus the conduction electrons of intrinsic semiconductors play a major role in the electrical conduction phenomenon based on our numerical calculations. While the values of the Hall angle and Hall coefficient for the piezo-electric scattering and ionised-impurity scattering are positive, and the holes of intrinsic semiconductors play a major role in the electrical conduction phenomenon due to the piezo-electric scattering and the ionised-impurity scattering. However, the absolute values for the acoustic phonon scattering are much larger than those for the piezo-electric and ionised-impurity scatterings, hence the total contributions of the Hall angle and Hall coefficient for the scattering in intrinsic semiconductors are negative. Consequently, the conduction electrons in intrinsic semiconductors play a major role in the electrical conduction phenomenon based on our present numerical results. In figure 3 we plot the transverse magnetoresistance of the intrinsic semiconductors as a function of the DC magnetic field. For the acoustic phonon scattering as shown in figure 3(*a*), it can be seen that the transverse magnetoresistance depends strongly on the magnetic field in the lower-field region. The transverse resistivity increases rapidly with the magnetic field in the range of magnetic fields $B = 6$–$20$ kG for InSb and $B = 24$–$50$ kG for GaAs. After passing this range, the change of the transverse magnetoresistance becomes quite small. It is shown that at quite high magnetic fields $\Delta\rho \simeq \rho_0$ for InSb and $\Delta\rho \simeq 0.95\,\rho_0$ for GaAs. For the piezo-electric scattering as shown in figure 3(*b*), the change in resistivity increases monotonically with the magnetic field. For the case of InSb, the change in resistivity will be $\Delta\rho \simeq 0.98\,\rho_0$ as the DC magnetic field increases up to 150 kG, but for the case of GaAs $\Delta\rho$ is still negative at $B = 150$ kG. For the ionised-impurity scattering as shown in figure 3(*c*), the change in resistivity $\Delta\rho$ does not easily reach $\rho_0$ as the magnetic field increases up to very high values for both InSb and GaAs. Therefore, our numerical results show that the acoustic phonon scattering still plays the predominant scattering role in the effect of DC magnetic field on the magnetoresistance in intrinsic semiconductors.

From the numerical analysis presented here, it can be seen that the Hall angle, Hall coefficient and transverse magnetoresistance can be influenced by the DC magnetic field due to the non-parabolicity of energy bands for carriers in intrinsic semiconductors. This effect of non-parabolicity of energy band structure comes from the introduction of the energy-dependent relaxation time. Although the holes play a major role in the electrical transport phenomenon for the piezo-electric scattering and ionised-impurity scattering in intrinsic semiconductors, the absolute values of the Hall angle and Hall coefficient, and the values of the transverse magnetoresistance for these scattering processes, appear much smaller than those for the acoustic phonon scattering of carriers in intrinsic semiconductors. Consequently, the conduction electrons in intrinsic semiconductors play a major role for the carrier transport phenomenon.

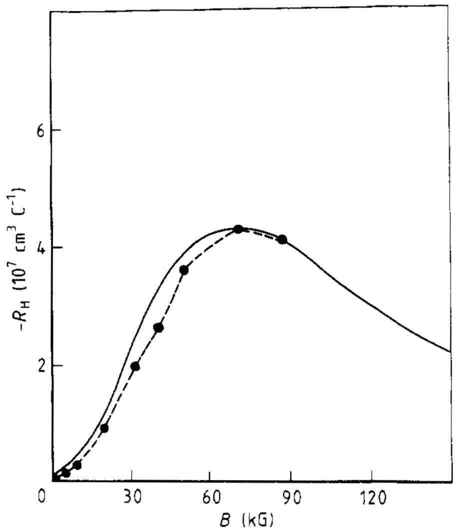

**Figure 4.** The Hall coefficient $R_H$ as a function of DC magnetic field in InSb with a carrier density of $4 \times 10^{14}$ cm$^{-3}$ at $T = 4.2$ K. The full curve shows the theoretical and the broken curve the experimental results.

To compare our theory with experimental data for a carrier density of $4 \times 10^{14}$ cm$^{-3}$ at $T = 4.2$ K in InSb (Keyes and Sladek 1956), we perform the numerical calculation for the Hall coefficient versus DC magnetic field with $n_e = n_h = 4 \times 10^{14}$ cm$^{-3}$ as shown in figure 4. It can be seen that the theory is in good agreement with experimental data.

A small numerical difference appears in the lower magnetic field region. Since in the lower magnetic field region, the assumption of the Maxwell–Boltzmann distribution for carriers is not a good approximation for our numerical calculations at low temperatures. Moreover, the parameter $a_n^{(i)}$ due to the non-parabolic band structure of InSb in equation (18) causes the summation over $n$ in equations (19), (22), (23), (26), (27) and (29) to converge slowly. Consequently, a small numerical difference appears in our calculations in the lower magnetic field region.

Finally, in order to demonstrate the effect of non-parabolicity of the energy band structure in semiconductors, we plot the numerical results of the parabolic band structure and non-parabolic band structure as shown in figure 5. In figure 5(*a*), the Hall angle for the parabolic band structure is smaller than that for the non-parabolic band structure at $T = 4.2$ K for InSb. In figure 5(*b*), the Hall coefficient for the parabolic band structure is shown to be insignificant compared with that for the non-parabolic band structure. In figure 5(*c*) for the transverse magnetoresistance, it is also shown that the effect of the parabolic band structure in InSb appears to be insignificant. Therefore, the energy band

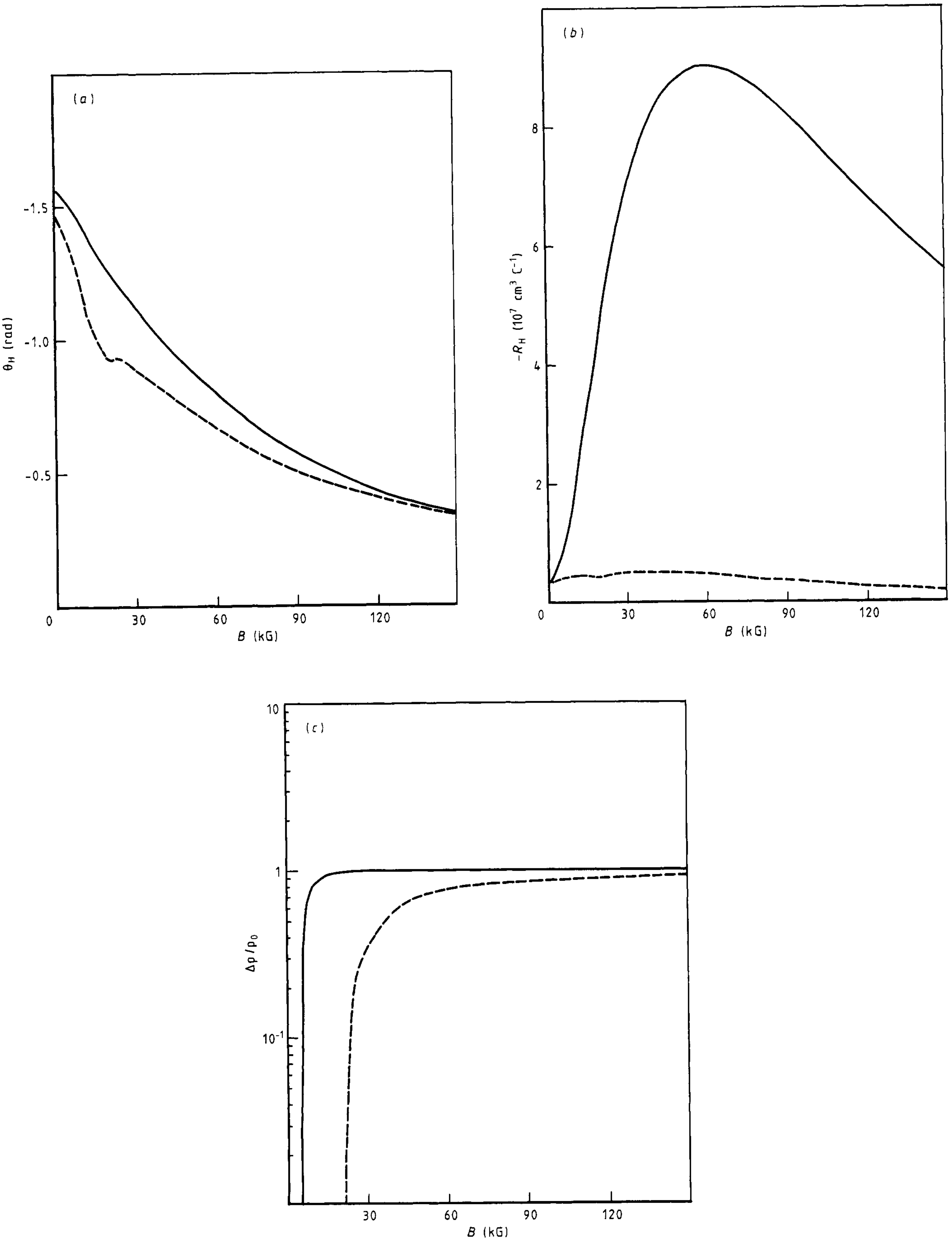

**Figure 5.** (*a*) The Hall angle $\theta_H$, (*b*) the Hall coefficient $R_H$ and (*c*) the transverse magnetoresistance $\Delta\rho/\rho_0$, as a function of DC magnetic field *B* for InSb at $T = 4.2$ K: Full curves, non-parabolic band structure; broken curves, parabolic band structure.

structure of III–V compounds such as InSb can be assumed to be non-parabolic.

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