國 立 交 通 大 學

財務金融研究所

碩 士 論 文

不完全資訊下的債務協商

How to Renegotiation with Imperfect Information?

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摘要

本研究假設債權人無法得知真實價格，只能以觀察價格來做為債務協商的依據。 從假設推導出股東及債權價值的封閉解後，重新探討在可以債權協商的情形下，是 否有債務不效率的情形。本文指出，當股東比較有協商的權力時，債權人會要求股東支付資訊溢酬來補償資訊不對稱的損失。不過，當債權人比較有 協商的權力時，債權人不會主動的提出債務協商的請求。

關鍵字： 債務協商、不完全資訊、資訊不對稱

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ABSTRACT

When firms experience financial distress, equityholders may act strategically, forcing concessions from debthodlers and paying less than the originally-contracted interest payment. This article incorporates strategic debt service under imperfect information, which debthodlers catch the observation price instead of real price, and develops simple closed-form expression for debt and equity values. We analyze the efficient implication of renegotiation, showing that debthodlers will ask for information premium when equityhodlers can make take-it-or-leave-it offers and debtholders will never renegotiate actively when debthodlers can make take-it-or-leave-it offers.

key word: renegotiation, imperfect formation, asymmetry information

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1 Introduction

The valuation of risky debt is central to theoretical and empirical work in corporate finance. There are many studies, such Mella-Barral and Parraudin (1997) and Anderson and Sundaresan (1996), which focus on perfect model on the firm's value and claim that costless debt renegotiation never obtains inefficient liquidations. Both the creditor and the firm may experience a Pareto-improvement in their positions by renegotiating the loan. By renegotiating the terms of the debt, the financially-distressed firm can pay less than the originally-contracted interest payment and avoid the stigmatization of bankruptcy, and the creditor can avoid the costs of taking the firm. Hence, debt renegotiation can eliminate inefficient liquidations. However, inefficient liquidations often occur in many markets even after renegotiation.

Hackbarth, Hennessy, and Leland (2007) first used the trade-off theory between tax-shield and bankruptcy cost to explain these inefficient liquidations for weak firms after introducing market debts. They also show that banks always accept strong firms' renegotiation offers and never liquidate these firms, no matter how the information on the debt contract conditions evolves over time. Their results are consistent with the findings of Blackwell and Kidwell (1988), who suggest that small firms issue privately-placed debt almost exclusively, and larger firms are more likely to issue market debt. Nevertheless, Bourgeon and Dionne (2007) argued that this scenario does not necessarily corresponding to the reality. They introduced asymmetric information on the LGD (loss given default) value at the renegotiation date to explain why banks do not renegotiate with strong firms under certain circumstances. They found that the presence of asymmetric information between banks and firms indicates that banks will not always renegotiate with strong firms with a high LGD or a low liquidation value. Their model helps to explain the empirical findings of Carey and Gordy (2008).

Nevertheless, much recent research has focused on perfect information models on the firm's value for creditors. For examples of notable studies, see Mella-Barral and Parraudin (1997), Bourgeon and Dionne (2007), and Hackbarth, Hennessy, and Leland (2007). However, indeed, there is asymmetric information between the firm and the creditor because it is typically difficult for the creditor to observe the firm's value directly. Hence, the creditor must instead draw inferences about the state variable from publicly-available information. As claimed in Duffie and Lando (2001), the creditor's imperfect information on the firm's value makes default intensity strictly positive at zero maturity because the creditor is uncertain about the nearness of the current state variable to the trigger level at which the firm would declare default. The existence of the default intensity makes it reasonable that observed bond prices often drop abruptly at or around the time of default. Bond prices with perfect information instead converge continuously to its default-contingent value as default approaches. Moreover, yield spreads for risky firms' debts with complete information climbs rapidly with maturity, but bond-market participants' imperfect information on the firm causes a more moderate variation in yield spreads with maturity. Lots of empirical studies, such as Fons (1994), Helwege and Turner (1999), and Sarig and Warga (1989), show that severe variation in the shape of the term structure of yield spreads is seldom observed in bond markets.

Our research focuses on the implication of strategic debt service with imperfect market. With some informational assumptions, we set up an incomplete accounting information model and derive the creditor's conditional distribution of the firm's value. We then considered the firm value with renegotiation and without renegotiation. In addition, we review the debt efficiency problem.

To compare the differences between perfect market and imperfect market easily, we set the unbiased observation price and changed the variance of the noisy account to interpret the discrepancy. In the result, we found three effects under imperfect market. First, even under the unbiased observation price, which is close to the real liquidation price, p_c, as the firm does not disclose the full information, debtholders will overestimate the firm value, debt value, and equity value if the observation price is low enough. The reason is that the firm still operates well if there is no negative news, such as that of a financial crisis, when the observed price is low. Second, if debtholders and equityholders can renegotiate coupon payments, and equityholders can make take-it-or-leave-it offers to debtholders, then leverage will still cause some debt inefficiency because of the information asymmetry. We found that leverage still reduces the ex ante value of the firm when equityholders can make take-it-or-leave-it offers to debtholders, but renegotiation still increases the firm value. Under the imperfect market, equityholders can make take-it-or-leave-it offers to debtholders to reduce the inefficient of bankruptcy, but it cannot reduce the information asymmetry. Thus when equityholders want to renegotiate with debtholders, they have to sacrifice some benefit, like a part value of equity, to convince debtholders of the coupon reduction. Third, if debtholders can make take-it-or-leave-it offers to equityholders, then the renegotiation will not occur. When debtholders have the power to renegotiate with equityholders, it is hard for debtholders to decide the best timing to exercise their right. Because the decision only depends on the observation price, and debtholders have no idea whether the observation price is overestimated or underestimated, they have to take more risk if they follow the price to renegotiate. Under this situation, debtholders will not renegotiate actively, and the firm with leverage will cause debt inefficiency.

This article is structured as follows. Section II presents the basic setup and assumptions under imperfect market, and then calculates the firm, equity, and debt values without renegotiation. Section III reconsiders the firm, equity, and debt values with renegotiation and readdresses the debt efficiencies. Section IV concludes the paper.

2 Debt and Equity Value under Imperfect Market

2.1 Assumptions and basic setup

For this study, we have assumed that capital markets are frictionless and can borrow and lend freely at a constant, sure interest rate, r. Consider a firm that produces a unit of item for consumption whose price is denoted by p_t. Let p_t follow the Geometric Brownian Motion (GBM),

$$dp_t = \mu p_t dt + \sigma p_t dB_t \tag{2.1}$$

where μ and σ are constants and dB_t is a standard Brownian motion. If we set $p_t = e^x$, then

$$dX_t = (\mu - \frac{\sigma^2}{2})dt + \sigma dB_t \tag{2.2}$$

While in production, the firm incurs costs per period of $w > 0$, so its net earnings flow is

$$p_t - w. \tag{2.3}$$

Let us assume that bankruptcy impairs the firm's efficiency in that, after bankruptcy, the new owners of the firm can only generate earnings of

$$\xi_1 p_t - \xi_0 w \tag{2.4}$$

where $\xi_1 \leq 1$ and $\xi_0 \geq 1$. Since bond or outer investors are not kept fully informed of the status of the firm, it is not so easy for them to capture the firm value accurately. If they want to calculate the firm value, all they can do is to record the observation price of the outputs, estimate the size of the noisy accounting, and try to predict the real firm value. We assume that there is a noisy accounting report of assets, given by $\hat{p}_t = p_t e^{U_t} = e^{X_t}$, where U_t is normally distributed with mean $\bar{u} = E[U_t]$ and variance $a^2 = var[U_t]$.

Duffie and Lando (2001) show that

$$\Psi(x_0, x, \sigma\sqrt{t} = k) = P(\min_{0 \le s \le t} x_s > 0 | X_0 = x_0 > 0, X_t = x > 0) = 1 - e^{-\frac{2x_0 x}{k^2}} \tag{2.5}$$

and the density of X_t, killed at $\tau = \inf\{t : X_t \ge \underline{v}_j\}$, conditional on $Y_t = X_t + U_t$ is

$$b(x|Y_t, x_0, t) = \frac{\Psi(x_0, x, \sigma\sqrt{t} = k)\phi_U(Y_t - x)\phi_X(x)}{\phi_Y(Y_t)} \tag{2.6}$$

where $X_t \sim N(mt + x_0, \sigma^2 t)$, $U_t \sim N(\bar{u}, a^2)$, $Y_t \sim N(\bar{u} + mt + x_0, a^2 + \sigma^2 t)$, and $\underline{v}_j = \log(p_j)$ is the logarithm of the product price as bankruptcy. The density of X_t, conditional on $\tau > t$, and Y_t is defined by

$$g_{p_j}(x|y, x_0, t) \equiv \frac{b(x|Y_t, x_0, t)}{\int_{\underline{v}_j}^{\infty} b(x|Y_t, x_0, t)dx} \tag{2.7}$$

which it can be derived as:

$$g_{p_j}(x|y, x_0, t) = \frac{\sqrt{\frac{\beta_0}{\pi}}(1 - e^{-\frac{2\tilde{x}_0\tilde{x}}{k^2}})\exp[-J(\tilde{y}, \tilde{x}, \tilde{x}_0)]}{\exp(\frac{\beta_1^2}{4\beta_0} - \beta_3)\Phi(\frac{\beta_1}{\sqrt{2\beta_0}}) + \exp(\frac{\beta_2^2}{4\beta_0} - \beta_3)\Phi(-\frac{\beta_2}{\sqrt{2\beta_0}})} \tag{2.8}$$

where $\beta_0 = \frac{a^2+\sigma^2 t}{2a^2\sigma^2 t}$, $\beta_1 = \frac{\tilde{y}}{a^2} + \frac{\tilde{x}_0 + mt}{\sigma^2 t}$, $\beta_2 = -\beta_1 + \frac{2\tilde{x}_0}{k^2}$, $\beta_3 = \frac{1}{2}\left[\frac{\tilde{y}^2}{a^2} + \frac{(\tilde{x}_0+mt)^2}{\sigma^2 t}\right]$, and j represents the different trigger price of bankruptcy.

2.2 The unlevered firm

Let $W(p_t)$ denote the total value of the pure equity firm in the hands of its initial equityholders, and $X(p_t)$ denote the total value of the pure equity firm in the hands of the other owners after bankruptcy; then, Pierre and William (1997) show that

$$W(p) = \begin{cases} \frac{p}{r-\mu} - \frac{w}{r} + \left[\gamma - \frac{p_c}{r-\mu} + \frac{w}{r}\right]\left(\frac{p}{p_c}\right)^{\lambda} & \text{for } p \le p_c \\ \gamma & \text{for } p < p_c \end{cases} \tag{2.9}$$

$$X(p) = \begin{cases} \frac{\xi_1 p}{r-\mu} - \frac{\xi_0 w}{r} + \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right]\left(\frac{p}{p_x}\right)^{\lambda} & \text{for } p \le p_x \\ \gamma & \text{for } p < p_x \end{cases} \tag{2.10}$$

where $p_c = -\frac{\lambda}{1-\lambda}\frac{w+r\gamma}{r}(r-\mu)$, $p_x = -\frac{\lambda}{1-\lambda}\frac{\xi_0 w+r\gamma}{\xi_1 r}(r-\mu)$.

After issuance, the outer investors are not kept the fully informed of the status of the firm. If, conditional on X_t, we start at some given level x_0, the noisy observation Y_t and $\tau > t$, then the outer investors will obtain the density function of the X_t and the expectation of the firm's value.

Proposition 1. *Under imperfect market, we assume the noisy accounting report of assets is given by $\hat{p}_t = p_t e^{U_t} = e^{Y_t}$, conditional on $\tau > t$ and the starting level x_0. The total value of the pure equity firm in the hands (i) of its initial equityholders , $W_{ulnr}(\hat{p})$, and (ii) of other owners after bankruptcy, $X_{ulnr}(\hat{p})$, under imperfect market are equal to*

$$W_{ulnr}(\hat{p}) = \frac{p_c}{r-\mu}A_{1c} - \frac{w}{r} + \left[\gamma - \frac{p_c}{r-\mu} + \frac{w}{r}\right]A_{\lambda c} \tag{2.11}$$

$$X_{ulnr}(\hat{p}) = \gamma(1-B_{0c}) + \frac{\xi_1 p_c}{r-\mu}B_{1cx} - \frac{\xi_0 w}{r}B_{0cx} + \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right]\left(\frac{p_c}{p_x}\right)^{\lambda} B_{\lambda cx} \tag{2.12}$$

where $p_c = -\frac{\lambda}{1-\lambda}\frac{w+r\gamma}{r}(r-\mu)$, $A_{ij}(\hat{p}_t)$ and $B_{ijk}(\hat{p}_t)$ refer to the Appendix, and λ is the negative root of the quadratic equation $\lambda(\lambda-1)\sigma^2/2 + \lambda\mu = r$.

2.3 The levered firm

Suppose that the firm has issued perpetual debt with principal b/r and a contractual coupon flow b per period of time. We assume that equityholders are free to cover the firm's operating losses by injecting capital and that, so long as they do this, bankruptcy cannot occur. The stationary nature of the payoffs involved implies that bankruptcy occurs when the output price, p_t, first hits some constant level p_b. In the absence of arbitrage, and assuming that strict seniority of claims is respected, $\hat{L}$ and $\hat{V}$ must satisfy

$$\hat{L}(P_b) = \min\left\{X(p_b), \tfrac{b}{r}\right\} \quad \text{and} \quad \hat{V}(P_b) = \max\left\{0, W(p_b) - \tfrac{b}{r}\right\} \tag{2.13}$$

where $\hat{L}$ and $\hat{V}$ denote the levered firm's debt and equity value. Suppose that the terms of the debt cannot be renegotiated; in this instance, Pierre and William (1997) show that

$$\hat{V}(p) = \begin{cases} \dfrac{p}{r-\mu} - \dfrac{w+b}{r} + \left[\dfrac{p_b}{r-\mu} + \dfrac{w+b}{r}\right]\left(\dfrac{p}{p_b}\right)^{\lambda} & \text{for } \gamma < \frac{b}{r} \\ \hat{W}(p) - \frac{b}{r} & \text{for } \gamma \geq \frac{b}{r} \end{cases} \tag{2.14}$$

$$\hat{L}(p) = \begin{cases} \dfrac{b}{r}\left[X(p_b) - \dfrac{b}{r}\right]\left(\dfrac{p}{p_b}\right)^{\lambda} & \text{for } \gamma < \frac{b}{r} \\ \frac{b}{r} & \text{for } \gamma \geq \frac{b}{r} \end{cases} \tag{2.15}$$

where $p_b = -\frac{\lambda}{1-\lambda}\frac{w+b}{r}(r-\mu)$. The interest thing is that $\hat{V}(p) + \hat{L}(p)$ does not equal $W(p)$ and is even slightly lower than $W(p)$. The inefficiencies arise because of the presence of debt. In order to understand the inefficiencies that result from the presence of debt, consider how the total value of the levered firm, $\hat{W}(p) \equiv \hat{V}(p) + \hat{L}(p)$, depends on the contracted coupon flow, b. When b becomes larger, then p_b will becomes larger, and the timing of bankruptcy will be different. As $r\gamma < b < \frac{\xi_0 w + r\gamma}{\xi_1}$, p_b gets slightly smaller than p_x, and $X(p_b) = \gamma$. Therefore, when the real price first hits the lower boundary p_b, debtholders who take over at bankruptcy will prefer to liquidate the firm instantly. If b exceeds $(\xi_0 w + r\gamma)/\xi_1$, then $X(p_b)$ is larger than γ, which means debtholders will take over at bankruptcy until the real price first hits p_x.

In this case, debtholders are still not kept the fully informed of the status of the firm. If, conditional on X_t, we start at some given level x_0, the noisy observation Y_t, and $\tau > t$, then debtholders have to check the real bankruptcy point when the debtholders want to take the expectation of the equity value and the debt value. When $p_x > p_b$, the firm will be liquidated at p_b, so $\underline{v}$ is set to $\log(p_b)$. For $p_x < p_b$, the firm will be taken over by debtholders until the real price first hits p_x; then, $\underline{v}$ is set to $\log(p_x)$, and

$$\hat{V}(p) = 0 \quad \text{for } p_x < p < p_b \tag{2.16}$$

$$\hat{L}(p) = X(p) \quad \text{for } p_x < p < p_b \tag{2.17}$$

Proposition 2. *Under imperfect market, we assume the noisy accounting report of assets is given by $\hat{p}_t = p_t e^{U_t} = e^{X_t + U_t} = e^{Y_t}$, conditional on $\tau > t$ and the starting level x_0. If $L_{wlnr}(\hat{p})$ and $V_{wlnr}(\hat{p})$ denote the total values of the firm's equity and debt under these assumptions, then, if $\gamma \le b/r$, the debt is riskless, and*

$$L_{wlnr}(\hat{p}) = \frac{b}{r}\ , \quad V_{wlnr}(\hat{p}) = W_{ulnr}(\hat{p}) - \frac{b}{r} \tag{2.18}$$

If $\gamma < b/r$, then the expected value of the $\hat{V}(p_t)$ and $\hat{L}(p_t)$ is

$$V_{wlnr}(\hat{p}) = \begin{cases} \dfrac{p_b}{r-\mu}A_{1b} - \dfrac{w+b}{r} + \left[\dfrac{p_b}{r-\mu} + \dfrac{w+b}{r}\right] A_{\lambda b} & \text{for } p_b < p_x \\ \dfrac{p_x}{r-\mu}B_{1xb} - \dfrac{w+b}{r}B_{0xb} + \left[\dfrac{p_b}{r-\mu} + \dfrac{w+b}{r}\right]\left(\dfrac{p_x}{p_b}\right)^{\lambda} B_{\lambda xb} & \text{for } p_b > p_x \end{cases} \tag{2.19}$$

$$L_{wlnr}(\hat{p}) = \begin{cases} \frac{b}{r} + \left[X(p_b) - \frac{b}{r}\right] A_{\lambda b} & \text{for } p_b < p_x \\ \\ \dfrac{\xi_1 p_x}{r-\mu}(A_{1x} - B_{1xb}) - \dfrac{\xi_0 w + b}{r}(1 - B_{0xb}) & \\ \qquad + \left[\gamma - \dfrac{\xi_1 p_x}{r-\mu} + \dfrac{\xi_0 w}{r}\right](A_{\lambda x} - B_{\lambda xb}) & \\ \qquad + \frac{b}{r}B_{0xb}\left[X(p_b) - \dfrac{b}{r}\right]\left(\dfrac{p_x}{p_b}\right)^{\lambda} B_{\lambda xb} & \text{for } p_b > p_x \end{cases} \tag{2.20}$$

where $p_x = -\frac{\lambda}{1-\lambda}\frac{\xi_0 w + r\gamma}{\xi_1 r}(r-\mu)$, $p_b = -\frac{\lambda}{1-\lambda}\frac{w+b}{r}(r-\mu)$, $A_{ij}(\hat{p}_t)$ and $B_{ijk}(\hat{p}_t)$ refer to the Appendix, and λ is the negative root of the quadratic equation $\lambda(\lambda-1)\sigma^2/2 + \lambda\mu = r$.

The prediction value, $W_{ulnr}(\hat{p}_t)$, $V_{ulwd}(\hat{p}_t)$, and $L_{ulwd}(\hat{p}_t)$ from Proposition 1 and 2 are illustrated in Figure 1. Even if outer investors know that the observed price is unbiased, they still feel frightened when the observed price decreases to p_c. As long as the firm does not declare bankruptcy, outer investors will overestimate the firm value because debtholders will be optimistic about the firm under this situation. It is familiar to the expected value of the debt and the equity value. Because of the inferior information about the price, the drawback will reflect on the firm value, equity value, and debt value. Therefore, debtholders will overestimate the firm and equity values and underestimate the debt value as the observation price increases.

2.4 Firm value and leverage

Generally speaking, the firm value is equal to the equity value pluses the bond value, no matter what firm has leverage or unleverage under perfect market. When the debt principal, b/r, is greater than the scrapping value γ, bondholders are the residual claimants, and the debt is risky. Therefore, the value of the firm, $W_{ulwd}(\hat{p}_t)$, which is defined by $V_{ulwd}(\hat{p}_t) + L_{ulwd}(\hat{p}_t)$, will decrease slightly. Leverage generates losses from an ex ante point of view because of the direct bankruptcy costs it entails under perfect market. From Figure 2, it is easy to see the difference between perfect market and imperfect market. When b is small, such that b/r is smaller than γ, the debt is riskless, and the payment of the coupon does not affect debtholders' estimation of the firm value under the unbiased observation price. When b get larger and b/r is slightly larger than γ, leverage becomes costly because it may results in liquidation at p_b. The firm may goes to "liquidation bankruptcy," which means debtholders will prefer to liquidate the firm than take over at bankruptcy. When b was large enough, we found that there was some trouble with

$E[\hat{L}(p)|\hat{p}_t, x_0, t] > W_{ulnr}(\hat{p})$ as $b/r > \gamma$. As a result, debtholders need to modify the debt, firm, and equity value; this is the difference between a levered firm without renegotiation under perfect market and under imperfect market. There is some difference in estimating the value of the debt as $p_b > p_x$, because the bankruptcy occurs at p_x. When $p_b > p_x$, we assert that the firm is liable to "an operating concern bankruptcy," since the real price first hits p_b; when this happens, bonderholders will take over the firm. Before bonderholders take over the firm, they are unable to obtain all information on the firm, but when they evaluate the value of the debt, they are still concerned with the value between p_x and p_b. As b gets large, such that $p_b > p_t$, then the value of the firm will converge to X_{ulnr}, which is still larger than the real firm value, $X(p)$, because of the information asymmetry.

3 Debt and Equity Value with Renegotiation under Imperfect market

3.1 Service Flows with Equityholder Offers

We want to consider how the value of the firm's security is affected if debtholders and equityholders can renegotiate coupon payments. When equityholders can make take-it-or-leave-it offers to bondholders, since the asymmetric information, they will take advantage of bondholders. We shall assume that possible strategies for equityholders consist of piecewise right-continuous service flow functions of $\hat{p}_t$, the observation price. First, we notice $X(p)$ satisfy the following PDE:

$$rX(p) = s(p) + \mu p X'(p) + \frac{\sigma^2}{2} p^2 X''(p) \quad \text{for } p < p_s \tag{3.1}$$

$X_{ulnr}(\hat{p}_t)$ is also assumed to satisfy the same PDE with different μ and σ^2, say $\hat{\mu}$ and $\hat{\sigma}^2$. Since the parameter μ and σ^2 is from the mean and variance of the logarithm of the price under perfect market, we can easily to get the mean and variance of the logarithm of the price under imperfect market and set them be the $\hat{\mu}$ and $\hat{\sigma}^2$. Second, Pierre and William (1997) show that $s(p)$ is the optimal debt service flow function under perfect market, so we assume there is another optimal debt service flow function, $\tilde{s}(\hat{p})$, under imperfect market. The intuitive explanation of the service flow function is that debthodlers require a service flow to dissuade them from equityholders need provide debtholders with an income flow whose capitalized value is sufficient to dissuade them from. Therefore, equityholders must to provide enough income flow to capture $X_{ulnr}(\hat{p}_t)$, the value of the firm in the hands of the new owners under imperfect market, when $p < \hat{p}_s$. From above, $X_{ulnr}(\hat{p}_t)$ satisfy the following PDE,

$$rX_{ulnr}(\hat{p}_t) = \tilde{s}(\hat{p}_t) + \hat{\mu}\hat{p}_t X'_{ulnr}(\hat{p}_t) + \frac{\hat{\sigma}^2}{2}\hat{p}_t^2 X''_{ulnr}(\hat{p}_t) \quad \text{for } \hat{p}_t < \hat{p}_s \tag{3.2}$$

We suppose the following:

Hypothesis 1. *If equityholders can make take-it-or-leave-it offers to bondholders regarding debt service than there exists trigger levels, p_s and p_c such that*

(1) *bankruptcy occurs when p_t first hits p_c,*

(2) *for all $\hat{p} < \hat{p}_s$, $\tilde{s}(\hat{p}) < b$ and $L_{wlwd}(\hat{p}) = X_{ulnr}(\hat{p})$ (i.e., when debt service is less than the contracted coupon, the value of debt equals that of debtholder's observation outside option $X_{ulnr}(\hat{p})$),*

(3) *for all $\hat{p} \geq \hat{p}_s$, $\tilde{s}(\hat{p}) = b$(i.e., the contracted coupon is paid).*

Now, we can get the service flow function $\tilde{s}(\hat{p})$:

$$\tilde{s}(\hat{p}) = \begin{cases} \text{satisfy the equation (3.2)} & \text{for } \hat{p} < \hat{p_s} \\ b & \text{for } \hat{p} \geq \hat{p_s} \end{cases} \tag{3.3}$$

From this service flow function, $L_{wlwre}(\hat{p})$satisfy the following PDE,

$$rL_{wlwre}(\hat{p}_t) = \tilde{s}(\hat{p}_t) + \hat{\mu}\hat{p}_t L'_{wlwre}(\hat{p}_t) + \frac{\hat{\sigma}^2}{2}\hat{p}_t^2 L''_{wlwre}(\hat{p}_t) \tag{3.4}$$

The absence of arbitrage implies that $L_{wlwre}(\hat{p}) = X_{ulnr}(\hat{p})$ for all $\hat{p}_t < \hat{p}_s$. No bubble condition includes $\lim_{\hat{p}\to\infty} L_{wlwre}(\hat{p}) = \frac{b}{r}$. Under this situation, equityholders know the real price of the output, so the value of equityholders is the real firm value minus the estimated value of debtholders, i.e. $V_{wlwre}(p,\hat{p}) = W(p) - L_{wlwre}(\hat{p})$.

Proposition 3. *Under imperfect market, we assume the noisy accounting report of asset is given by $\hat{p}_t = p_t e^{U_t} = e^{X_t+U_t} = e^{Y_t}$, and Hypothesis 1, equityholders adopt the service flow function, $\tilde{s}(\hat{p})$. The values of equity, $V_{wlwre}(p,\hat{p})$, and debt, $L_{wlwre}(\hat{p})$, are as follows:*

$$V_{wlwre}(p,\hat{p}) = W(p) - L_{wlwre}(\hat{p}) \tag{3.5}$$

where, if $\gamma \leq \frac{b}{r}$, then debt is riskless and $L_{wlwre}(\hat{p}) = \frac{b}{r}$. If $\gamma < \frac{b}{r}$, then the debt is risky and

$$L_{wlwre}(\hat{p}) = \begin{cases} \frac{b}{r} + \left[X_{ulnr}(\hat{p}_s) - \frac{b}{r}\right]\left(\frac{\hat{p}}{\hat{p}_s}\right)^{\hat{\lambda}} & \text{for } \hat{p} > \hat{p}_s \\ X_{ulnr}(\hat{p}) & \text{for } \hat{p} \leq \hat{p}_s \end{cases} \tag{3.6}$$

where $X_{ulnr}(\hat{p})$ is define by equation 2.12, $\hat{p}_s$ is solved by $L'_{wlwd}(\hat{p}_s) = X'_{ulnr}(\hat{p}_s)$, and $\hat{\lambda}$ is the negative root of the quadratic equation $\hat{\lambda}(\hat{\lambda}-1)\hat{\sigma}^2/2 + \hat{\lambda}\hat{\mu} = r$.

Equity and debt value with equityholder offers is shown in Figure 3. It is clear that renegotiation, even under the imperfect market, still generates the fully efficient outcome. The main difference between the perfect market and the imperfect market is the point of the trigger price p_s and $\hat{p}_s$. If we set unbiased observation price under imperfect market, debtholders would not agree this contact when equityholders want to renegotiate at the real trigger price, p_s. The main reason is debtholders believe that equityholders have some information that debtholders do not know or there is some advantage for equityholders. Therefore, when the observation price hits the real trigger price they still reject the renegotiation. As they reject the renegotiation, even the real price hits the real trigger price, debtholders will not get huge loss since they can be the new owner of the firm. If equityholders still want to own the firm, then they have to renegotiate later and it means debtholders will ask some *information premium* to make up for information asymmetry.

Now we try to change the variable a, which is the volatility of the noisy accounting, the outcome is shown in Figure 4. It is easy to find that trigger price under the imperfect market becomes smaller when a becomes larger. Basic on the intuition, debtholders will ask more information premium when the market is more imperfect or the information is more asymmetry. If the time, t, between we observe the price of the firm, $\hat{p}_t = e^{Y_t}$, and $p_0 = e^{X_0}$ is shorter, and the firm does not operate so well that the one might goes to bankruptcy, then it is much difficult to convince debtholders to agree the deal when equityholders want to renegotiate the coupon payments.In order to capture this situation, if we fix the other variable and change the different t, then from penal.1, we can see when t gets smaller then $\hat{p}_s$ gets smaller. Therefore, debtholders might ask more information premium because of inferior information. Image that at initial time the firm still operate, what's the difference between the equityholders want to renegotiate because of financial distress next day and next year? When equityhodlers want to renegotiate next day, debthodlers can keep more full information than one year later, so equityholders need to release more information to persuade debtholders accept the agreement which will reflect on the value of firm.

3.2 Service Flows with Debtholder Offers

If the bargaining power is witched to debtholders, then the situation will be totally different. When equityholders make the decision to renegotiate, they may use their information advantage and release some information to decide the best timing of renegotiation. Nevertheless, debtholders do not have this kind of advantage to decide the best timing of renegotiation. We still assume that possible strategies for equityholders consist of piecewise right-continuous service flow functions of $\hat{p}_t$, the observation price. Similarly, Pierre and William (1997) show that $q(p)$ is the optimal debt service flow function under perfect market, so we assume there is another optimal debt service flow function, $\tilde{q}(\hat{p})$, under imperfect market. The explanation of the service flow function is that debtholders like a residual claimants, maximizing firm value subject to the constraint placed upon them by the "outside option" of equityholders. Since the outside option of equityholders is supplied by limit liability, they will abandon the firm and precipitate bankruptcy when the equity value is negative. Under this strategic debt service, $L_{wlwrd}(\hat{p})$ and $V_{wlwrd}(\hat{p})$ satisfy the following PDE:

$$rL_{wlwrd}(\hat{p}_t) = \tilde{q}(\hat{p}_t) + \hat{\mu}\hat{p}_t L'_{wlwrd}(\hat{p}_t) + \frac{\hat{\sigma}^2}{2}\hat{p}_t^2 L''_{wlwrd}(\hat{p}_t) \tag{3.7}$$

$$rV_{wlwrd}(\hat{p}_t) = \hat{p} - w - \tilde{q}(\hat{p}_t) + \hat{\mu}\hat{p}_t V'_{wlwrd}(\hat{p}_t) + \frac{\hat{\sigma}^2}{2}\hat{p}_t^2 V''_{wlwrd}(\hat{p}_t) \tag{3.8}$$

and $\tilde{q}(\hat{p}_t)$ is defined as following:

$$\tilde{q}(\hat{p}) = \begin{cases} \text{satisfy the equation (3.7) and (3.8)} & \text{for } \hat{p} < \hat{p}_b \\ b & \text{for } \hat{p} \geq \hat{p}_b \end{cases} \tag{3.9}$$

The absence of arbitrage implies that $V_{wlwrd}(\hat{p}_t) = 0$ and no bubble condition includes $\lim_{\hat{p}\to\infty} L_{wlwrd}(\hat{p}_t) = \frac{b}{r}$.

Lemma 1. *Under imperfect market, we assume the noisy accounting report of asset is given by $\hat{p}_t = p_t e^{U_t} = e^{X_t+U_t} = e^{Y_t}$. Then the values of equity, $V_{wlwrd}(p, \hat{p}_t)$, and debt, $L_{wlwrd}(\hat{p}_t)$, are as follows:*

$$V_{wlwrd}(p, \hat{p}_t) = W(p) - L_{wlwrd}(\hat{p}_t) \tag{3.10}$$

where,if $\gamma \leq \frac{b}{r}$, then debt is riskless and $L_{wlwrd}(\hat{p}) = \frac{b}{r}$. If $\gamma < \frac{b}{r}$, then the debt is risky and

$$L_{wlwrd}(\hat{p}) = \begin{cases} \frac{b}{r} + \left[W_{ulnr}(\hat{p}_b) - \frac{b}{r}\right]\left(\frac{\hat{p}}{\hat{p}_b}\right)^{\hat{\lambda}} & \text{for } \hat{p} > \hat{p}_b \\ W_{ulnr}(\hat{p}) & \text{for } \hat{p} \leq \hat{p}_b \end{cases} \tag{3.11}$$

where $W_{ulnr}(\hat{p})$ is define by equation 2.11, p_b is solved by $L'_{wlwrd}(\hat{p}_b) = W'_{ulnr}(\hat{p}_b)$, and $\hat{\lambda}$ is the negative root of the quadratic equation $\hat{\lambda}(\hat{\lambda}-1)\hat{\sigma}^2/2 + \hat{\lambda}\hat{\mu} = r$.

From the Lemma, we can get equity and debt value when debtholders can make take-if-or-leave-it offers and both of them is shown in Figure 5. If we set unbiased observation price under imperfect market, debtholders would not renegotiate at the real trigger price, p_b. In this case, they need some information premium, because there is some asymmetry information such that debtholders do not know the relationship between the observation price and the real price even the observation price is unbiased. If the observation price is higher than the real price, then the debtholders need to renegotiate when the real price first hits p_b. Otherwise, they will lose the debt value as $p < p_b$. If the observation price is lower than the real price, then when the observation price is higher than p_b, the debthodlers will not want to renegotiate with equityholders because the real price still does not hit the trigger price, p_b. Because the information asymmetry, they will renegotiate as $\hat{p}_b < p_b$. In this case, the equity value is follow that the real firm value takes of the observation debt value. If debthodlers renegotiate as above, then real equity value will less then 0 and equityhodlers will not accept the accord. Because of the failure of the agreement, equityhodlers will declare bankruptcy and liquidate when the real price first hits p_b. It is also means that renegotiation does not generate efficient outcome even the observation price is unbiased.

Proposition 4. *Under imperfect market, we assume the noisy accounting report of asset is given by $\hat{p}_t = p_t e^{U_t} = e^{X_t+U_t} = e^{Y_t}$. If debtholders can make take-it-or-leave-it offers, then renegotiation will not occur and the firm will bankruptcy at p_b, i.e. the issuance of debt can not generate efficient outcome when the observation price is unbiased.*

4 Conclusion

Our study shows that, if equityholders can make take-it-or-leave-it offers, then equityholders have to give up some equity value in order to convince the debtholders to lower the bond coupon, and debt values will approximate the firm's taken-over value when the firm is in financial distress. Clearly, when the information on the product price is more transparent, there is less information asymmetry, and debtholders will require a lower information premium when equityholders want to renegotiate the debt service.

When debtholders can make take-it-or-leave-it offers, no matter how low the observation price is under the unbiased assumption, they will never renegotiate actively with the unbiased observation price. The observation price is the only source for debtholders to decide the renegotiation timing. Hence, they really care about the price being underestimated or overestimated, and these two situations will lead to opposite decisions. In order to avoid taking more risk, they are more passive,which results in inefficient bankruptcy.

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t(year)	$\hat{p}_s$
10	4.008668
5	3.979913
2	3.889488
1	3.792926
0.1	3.445957
0.01	3.339659

Table 1: $\hat{p}_s$ for different times with $a = 0.1$

$b = 4$, $w = 1$, $\sigma = 0.1$, $r = 0.05$, $\mu = 0$, $\gamma = 60$, $\xi_1 = 0.9$, $\xi_0 = 1.1$

	α				
a	0	0.25	0.5	0.75	1
0.1	3.3928	3.4733	3.5701	3.6776	3.7929
	(92.97%)	(92.34%)	(92.02%)	(91.84%)	(91.71%)
0.2	3.2023	3.2655	3.3443	3.4391	3.5466
	(87.75%)	(86.81%)	(86.20%)	(85.88%)	(85.75%)
0.5	2.9873	3.0093	3.0441	3.1047	3.2097
	(81.86%)	(80.02%)	(78.46%)	(77.54%)	(77.61%)
p_α	3.6492	3.7615	3.8797	4.0043	4.1358

Table 2: Table 2: $\hat{p}_\alpha$ for different α with different a

$b = 4$, $w = 1$, $\sigma = 0.1$, $r = 0.05$, $\mu = 0$, $\gamma = 60$, $\xi_1 = 0.9$, $\xi_0 = 1.1$, $t = 1$

(%) is the percentage of $\hat{p}_\alpha$

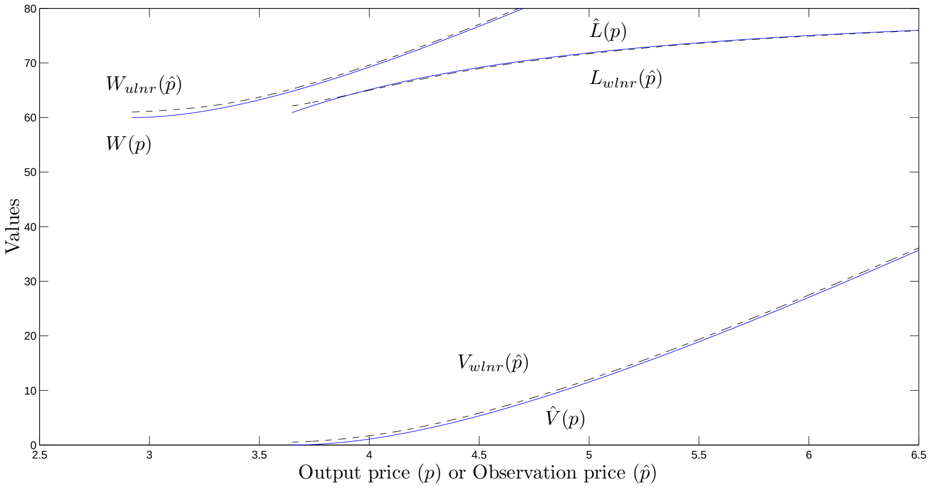

Figure 1: Security valuation with no renegotiation

$b = 4$, $w = 1$, $\sigma = 0.1$, $r = 0.05$, $\mu = 0$, $\gamma = 60$, $\xi_1 = 0.9$, $\xi_0 = 1.1$, $a = 0.1$, $t = 1$,

$p_c = 2.9194$, $p_b = 3.6492$

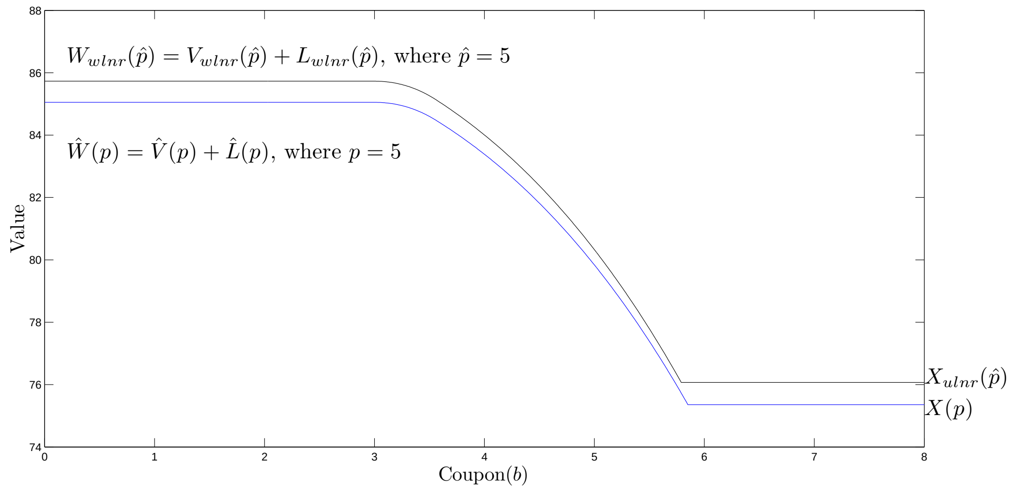

Figure 2: Total firm value and leverage

$w = 1,\ \sigma = 0.1,\ r = 0.05,\ \mu = 0,\ \gamma = 60,\ \xi_1 = 0.9,\ \xi_0 = 1.1,\ a = 0.1,\ t = 1$

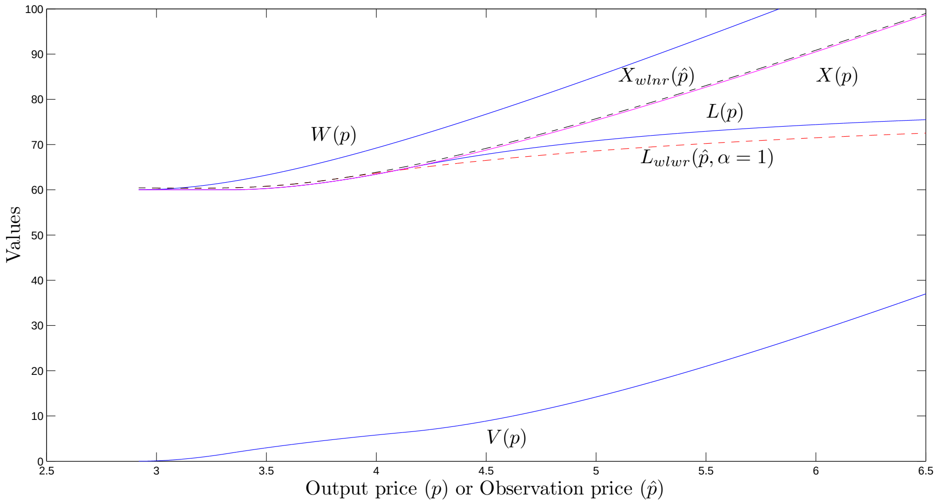

Figure 3: Security valuation with equityholder offer

$b = 4$, $w = 1$, $\sigma = 0.1$, $r = 0.05$, $\mu = 0$, $\gamma = 60$, $\xi_1 = 0.9$, $\xi_0 = 1.1$, $a = 0.1$, $t = 1$,

$p_c = 2.9194$, $p_s = 4.1358$, $\hat{p}_{\alpha=1} = 3.7929(91.71\%)$

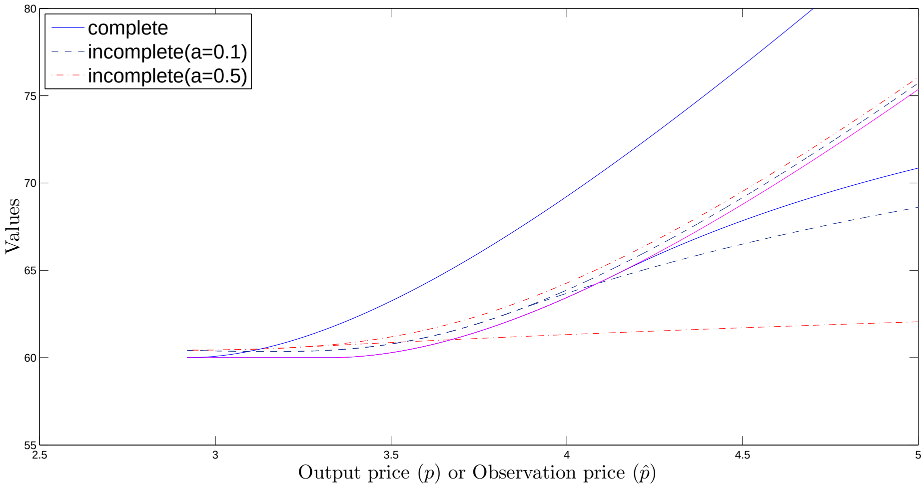

Figure 4: Security valuation with equityholder offers and different a

$b = 4$, $w = 1$, $\sigma = 0.1$, $r = 0.05$, $\mu = 0$, $\gamma = 60$, $\xi_1 = 0.9$, $\xi_0 = 1.1$, $t = 1$, $p_c = 2.9194$, $p_s = 4.1358$, $\hat{p}_{\alpha=1}(a = 0.1) = 3.7929(91.71\%)$, $\hat{p}_{\alpha=1}(a = 0.5) = 3.2097(77.61\%)$

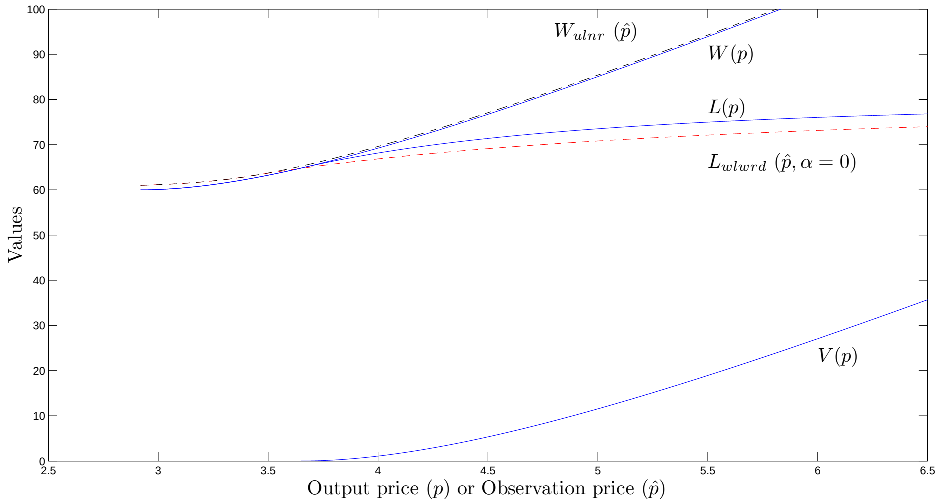

Figure 5: Security valuation with debtholder offers

$b = 4$, $w = 1$, $\sigma = 0.1$, $r = 0.05$, $\mu = 0$, $\gamma = 60$, $\xi_1 = 0.9$, $\xi_0 = 1.1$, $a = 0.1$, $t = 1$,

$p_c = 2.9194$, $p_b = 3.6492$, $\hat{p}_{\alpha=0}(a = 0.1) = 3.3928(92.97\%)$

Appendix

Proof of Proposition 1. *First, we claim:*

$$A_{ij}(\hat{p}_t) \equiv \int_{\underline{v}=\log(p_j)}^{\infty} p^i g_{p_j}(x|\log(\hat{p}_t), x_0, t)dx$$

$$= \int_{\underline{v}=\log(p_j)}^{\infty} e^{i\log(\hat{p}_t)} g_{p_j}(x|\log(\hat{p}_t), x_0, t)dx$$

$$= \frac{\exp\left(\frac{(\beta_1+i)^2}{4\beta_0}\right)\Phi\left(\frac{\beta_1+i}{\sqrt{2\beta_0}}\right) + \exp\left(\frac{(\beta_2-i)^2}{4\beta_0}\right)\Phi\left(-\frac{\beta_2-i}{\sqrt{2\beta_0}}\right)}{\exp\left(\frac{\beta_1^2}{4\beta_0}\right)\Phi\left(\frac{\beta_1}{\sqrt{2\beta_0}}\right) + \exp\left(\frac{\beta_2^2}{4\beta_0}\right)\Phi\left(-\frac{\beta_2}{\sqrt{2\beta_0}}\right)}$$

for $i = 1, \lambda$; $j = c, x, b$

$$B_{ijk}(\hat{p}_t) \equiv \int_{\underline{v}'=\log(p_k)-\underline{v}=\log(p_j)}^{\infty} p^i g_{p_j}(x|\log(\hat{p}_t), x_0, t)dx$$

$$= \int_{\underline{v}'=\log(p_k)-\underline{v}=\log(p_j)}^{\infty} e^{i\log(\hat{p}_t)} g_{p_j}(x|\log(\hat{p}_t), x_0, t)dx$$

$$= \frac{\exp\left(\frac{(\beta_1+i)^2}{4\beta_0}\right)\Phi\left[\frac{\beta_1+i}{\sqrt{2\beta_0}} - \sqrt{2}(\underline{v}'-\underline{v})\right] + \exp\left(\frac{(\beta_2-i)^2}{4\beta_0}\right)\Phi\left[-\frac{\beta_2-i}{\sqrt{2\beta_0}} - \sqrt{2}(\underline{v}'-\underline{v})\right]}{\exp\left(\frac{\beta_1^2}{4\beta_0}\right)\Phi\left(\frac{\beta_1}{\sqrt{2\beta_0}}\right) + \exp\left(\frac{\beta_2^2}{4\beta_0}\right)\Phi\left(-\frac{\beta_2}{\sqrt{2\beta_0}}\right)}$$

for $i = 0, 1, \lambda$; $j = c, x, b$; $k = x, b$

From definition,

$$A_{ij}(\hat{p}_t)$$

$$= \int_{\underline{v}=\log(p_j)}^{\infty} e^{i\log(\hat{p}_t)} g_{p_j}(x|\log(\hat{p}_t), x_0, t)dx$$

$$= \int_0^{\infty} e^{i\tilde{x}} \frac{(1-e^{-\frac{2\tilde{x_0}\tilde{x}}{k^2}})\exp\left[-J(\tilde{y},\tilde{x},\tilde{x_0})\right]}{\int_0^{\infty}(1-e^{-\frac{2\tilde{x_0}\tilde{x}}{k^2}})\exp\left[-J(\tilde{y},\tilde{x},\tilde{x_0})\right]d\tilde{x}} d\tilde{x}$$

$$= \frac{\int_0^{\infty}\exp\left[\frac{(\beta_1+i)^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}-\frac{\beta_1+i}{2\beta_0^{1/2}}\right)^2\right]d\bar{x} - \int_0^{\infty}\exp\left[\frac{(\beta_2-i)^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}+\frac{\beta_2-i}{2\beta_0^{1/2}}\right)^2\right]d\bar{x}}{\int_0^{\infty}\exp\left[\frac{\beta_1^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}-\frac{\beta_1}{2\beta_0^{1/2}}\right)^2\right]d\bar{x} - \int_0^{\infty}\exp\left[\frac{\beta_2^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}+\frac{\beta_2}{2\beta_0^{1/2}}\right)^2\right]d\bar{x}}$$

$$= \frac{\exp\left[\frac{(\beta_1+i)^2}{4\beta_0}\right]\Phi\left[\frac{\beta_1+i}{\sqrt{2\beta_0}}\right]-\exp\left[\frac{(\beta_2+i)^2}{4\beta_0}\right]\exp\left[-\frac{\beta_2-i}{\sqrt{2\beta_0}}\right]}{\exp\left[\frac{\beta_1^2}{4\beta_0}\right]\Phi\left[\frac{\beta_1}{\sqrt{2\beta_0}}\right]-\exp\left[\frac{\beta_2^2}{4\beta_0}\right]\exp\left[-\frac{\beta_2}{\sqrt{2\beta_0}}\right]}$$

$B_{ijk}(\hat{p}_t)$

$$= \int_{\underline{v}'=\log(p_k)-\underline{v}=\log(p_j)}^{\infty} e^{i\log(\hat{p}_t)} g_{p_j}(x|\log(\hat{p}_t), x_0, t)dx$$

$$= \int_{\underline{v}'-\underline{v}}^{\infty} e^{i\tilde{x}} \frac{(1-e^{-\frac{2\tilde{x_0}\tilde{x}}{k^2}})\exp\left[-J(\tilde{y},\tilde{x},\tilde{x_0})\right]}{\int_0^{\infty}(1-e^{-\frac{2\tilde{x_0}\tilde{x}}{k^2}})\exp\left[-J(\tilde{y},\tilde{x},\tilde{x_0})\right]d\tilde{x}} d\tilde{x}$$

$$= \frac{\int_{\underline{v}'-\underline{v}}^{\infty}\exp\left[\frac{(\beta_1+i)^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}-\frac{\beta_1+i}{2\beta_0^{1/2}}\right)^2\right]d\bar{x}-\int_{\underline{v}'-\underline{v}}^{\infty}\exp\left[\frac{(\beta_2-i)^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}+\frac{\beta_2-i}{2\beta_0^{1/2}}\right)^2\right]d\bar{x}}{\int_{\underline{v}'-\underline{v}}^{\infty}\exp\left[\frac{\beta_1^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}-\frac{\beta_1}{2\beta_0^{1/2}}\right)^2\right]d\bar{x}-\int_{\underline{v}'-\underline{v}}^{\infty}\exp\left[\frac{\beta_2^2}{4\beta_0}\right]\exp\left[-\left(\bar{x}+\frac{\beta_2}{2\beta_0^{1/2}}\right)^2\right]d\bar{x}}$$

$$= \frac{\exp\left[\frac{(\beta_1+i)^2}{4\beta_0}\right]\Phi\left[\frac{\beta_1+i}{\sqrt{2\beta_0}}-\sqrt{2}(\underline{v}'-\underline{v})\right]-\exp\left[\frac{(\beta_2+i)^2}{4\beta_0}\right]\exp\left[-\frac{\beta_2-i}{\sqrt{2\beta_0}}-\sqrt{2}(\underline{v}'-\underline{v})\right]}{\exp\left[\frac{\beta_1^2}{4\beta_0}\right]\Phi\left[\frac{\beta_1}{\sqrt{2\beta_0}}\right]-\exp\left[\frac{\beta_2^2}{4\beta_0}\right]\exp\left[-\frac{\beta_2}{\sqrt{2\beta_0}}\right]}$$

Now, liquidation will occur the first time that p_t hits some constant level p_c; then, by definition, we know that $p_c = e^{\underline{v}}$.

$$W_{ulnr}(\hat{p}) = E\left[W(p)|\hat{p}_t, x_0, t\right]$$

$$= E\left[W(e^x)|e^{y_t}, x_0, t\right]$$

$$= \int_{\underline{v}}^{\infty}\left\{\frac{p}{r-\mu}-\frac{w}{r}+\left[\gamma-\frac{p_c}{r-\mu}+\frac{w}{r}\right]\left(\frac{p}{p_c}\right)^{\lambda}\right\} g_{p_c}(x|y, x_0, t)dx$$

$$= \int_0^\infty \left\{ \frac{e^{\tilde{x}+\underline{v}}}{r-\mu} - \frac{w}{r} + \left[\gamma - \frac{p_c}{r-\mu} + \frac{w}{r}\right] \left(\frac{e^{\tilde{x}+\underline{v}}}{p_c}\right)^\lambda \right\} g_{p_c}(x|y, x_0, t) d\tilde{x}$$

$$= \frac{p_c}{r-\mu} A_{1c} - \frac{w}{r} + \left[\gamma - \frac{p_c}{r-\mu} + \frac{w}{r}\right] A_{\lambda c}$$

It is easy to know $p_c \leq p_x$*, so we set* $p_x = e^{\underline{v}'}$*; then,*

$$X_{ulnr}(\hat{p}) = E\left[X(p)|\hat{p}_t, x_0, t\right]$$

$$= E\left[X(e^x)|e^{y_t}, x_0, t\right]$$

$$= \int_{\underline{v}}^\infty X\left(e^x\right) g_c(x|y, x_0, t) dx$$

$$= \int_{\underline{v}}^{\underline{v}'} \gamma g_c(x|y, x_0, t) d\tilde{x} +$$

$$\int_{\underline{v}'}^\infty \left\{ \frac{\xi_1 p}{r-\mu} - \frac{\xi_0 w}{r} + \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right] \left(\frac{p}{p_x}\right)^\lambda \right\} g_{p_c}(x|y, x_0, t) dx$$

$$= \int_{\underline{v}}^\infty \gamma g_{p_c}(x|y, x_0, t) dx - \int_{\underline{v}'}^\infty \gamma g_{p_c}(x|y, x_0, t) dx +$$

$$\int_{\underline{v}'-\underline{v}}^\infty \left\{ \frac{\xi_1 \tilde{x}+\underline{v}}{r-\mu} - \frac{\xi_0 w}{r} + \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right] \left(\frac{\tilde{x}+\underline{v}}{p_x}\right)^\lambda \right\} g_{p_c}(x|y, x_0, t) d\tilde{x}$$

$$= \int_0^\infty \gamma g_{p_c}(x|y, x_0, t) d\tilde{x} - \int_{\underline{v}'-\underline{v}}^\infty \gamma g_{p_c}(x|y, x_0, t) d\tilde{x} +$$

$$\int_{\underline{v}'-\underline{v}}^\infty \left\{ \frac{\xi_1 \tilde{x}+\underline{v}}{r-\mu} - \frac{\xi_0 w}{r} + \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right] \left(\frac{\tilde{x}+\underline{v}}{p_x}\right)^\lambda \right\} g_{p_c}(x|y, x_0, t) d\tilde{x}$$

$$= \gamma(1 - B_{0c}) + \frac{\xi_1 p_c}{r-\mu} B_{1cx} - \frac{\xi_0 w}{r} B_{0cx} + \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right] \left(\frac{p_c}{p_x}\right)^{\lambda} B_{\lambda cx}$$

Q.D.E

Proof of Proposition 2. *For $\gamma > b/r$, debt is riskless, so $L_{wlnr}(\hat{p}) = \frac{b}{r}$ and $V_{wlnr}(\hat{p}) = W_{nlnr}(\hat{p}) - \frac{b}{r}$. As $\gamma < b/r$ and $p_b < p_x$, liquidation will occur the first time that p_t hits some constant level p_b. Then, by definition, we set $p_b = e^{\underline{v}}$.*

$$V_{wlnr}(\hat{p}) = E\left[\hat{V}(p)|\hat{p}_t, x_0, t\right]$$

$$= \int_{\underline{v}}^{\infty} \left\{ \frac{e^x}{r-\mu} - \frac{w+b}{r} - \left[\frac{p_b}{r-\mu} + \frac{w+b}{r}\right]\left(\frac{e^x}{p_b}\right)^{\lambda} \right\} g_{p_b}(x|y, x_0, t)dx$$

$$= \int_{0}^{\infty} \left\{ \frac{e^{\tilde{x}+\underline{v}}}{r-\mu} - \frac{w+b}{r} - \left[\frac{p_b}{r-\mu} + \frac{w+b}{r}\right]\left(\frac{e^{\tilde{x}+\underline{v}}}{p_b}\right)^{\lambda} \right\} g_{p_b}(x|y, x_0, t)d\tilde{x}$$

$$= \frac{p_b}{r-\mu} A_{1b} - \frac{w+b}{r} - \left[\frac{p_b}{r-\mu} + \frac{w+b}{r}\right] A_{\lambda b}$$

$$L_{wlnr}(\hat{p}) = E\left[\hat{L}(p)|\hat{p}_t, x_0, t\right]$$

$$= \int_{\underline{v}}^{\infty} \left\{ \frac{b}{r} + \left[X(p_b) - \frac{b}{r}\right]\left(\frac{e^x}{p_b}\right)^{\lambda} \right\} g_{p_b}(x|y, x_0, t)dx$$

$$= \frac{b}{r} + \left[X(p_b) - \frac{b}{r}\right] A_{\lambda b}$$

If $\gamma < b/r$ and $p_b > p_x$, then the firm will be taken over as the real price first hits p_b, and liquidation will occur the first time that p_t hits some constant level p_b. By definition, we set $p_x = e^{\underline{v}}$ and $p_b = e^{\underline{v}'}$.

$$V_{wlnr}(\hat{p}) = E\left[\hat{V}(p)|\hat{p}_t, x_0, t\right]$$

$$= \int_{\underline{v}}^{\underline{v}'} 0 g_x(x|y, x_0, t)dx + \int_{\underline{v}'}^{\infty} \left\{ \frac{e^x}{r-\mu} - \frac{w+b}{r} - \left[\frac{p_b}{r-\mu} + \frac{w+b}{r}\right]\left(\frac{e^x}{p_b}\right)^{\lambda} \right\} g_{p_x}(x|y, x_0, t)dx$$

$$= \frac{p_x}{r-\mu} B_{1xb} - \frac{w+b}{r} B_{0xb} - \left[\frac{p_b}{r-\mu} + \frac{w+b}{r}\right]\left(\frac{p_x}{p_b}\right)^{\lambda} B_{\lambda xb}$$

$$L_{wlnr}(\hat{p}) = E\left[\hat{L}(p)|\hat{p}_t, x_0, t\right]$$

$$= \int_{\underline{v}}^{\underline{v}'} X(p) g_{p_x}(x|y, x_0, t)dx + \int_{\underline{v}'}^{\infty} \hat{L}(p) g_{p_x}(x|y, x_0, t)dx$$

$$= \frac{\xi_1 p_x}{r-\mu}(A_{1x} - B_{1xb}) - \frac{\xi_0 w + b}{r}(1 - B_{0xb})$$

$$+ \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right](A_{\lambda x} - B_{\lambda}) + \frac{b}{r} B_{0xb}\left[X(p_b) - \frac{b}{r}\right]\left(\frac{p_x}{p_b}\right)^{\lambda} B_{\lambda xb}$$

Q.D.E

Proof of Proposition 3. *There are two cases. If $\gamma \leq b/r$, debt is riskless and the firm is liquidated efficiently by equityholders when p_t first hits p_c since equityholders know the real price. If $\gamma < b/r$, suppose that Hypothesis 1 applies and that $\hat{p}_c$ is less than $\hat{p}_s$,* *where $\hat{p}_s$ is the trigger for renegotiation. For $\hat{p} < \hat{p}_s$, $L_{wlwre}(\hat{p}) = X_{ulnr}(\hat{p})$. For $\hat{p} > \hat{p}_s$, $L_{wlwre}(\hat{p})$ satisfies the PDE with $\tilde{s}(\hat{p}) = b$. Similarly, the general solution of $L_{wlwre}(\hat{p})$ is $L_{wlwre}(\hat{p}) = \frac{b}{r} + B_1\hat{p}^{\lambda_1} + B_2\hat{p}^{\lambda_2}$. Again, from the asymptotic condition, it implies that B_2 is zero. B_1 and $\hat{p}_s$ are determined by the no arbitrage condition $L_{wlwre}(\hat{p}_s) = X_{ulnr}(\hat{p}_s)$ and $L'_{wlwde}(\hat{p}_s) = X'_{ulnr}(\hat{p}_s)$. Solving these equation yields the expression in proposition 5.*

Q.D.E

Proof of Lemma 1. *Similar to the proof of Proposition 5, it is easy to solve the $L_{wlwrd}(\hat{p})$, which satisfies (3.7). For $\hat{p} \geq \hat{p}_b$, $L_{wlwrd}(\hat{p}_b) = W_{ulnr}(\hat{p}_b)$. For $\hat{p} > \hat{p}_b$, $L_{wlwrd}(\hat{p}_b)$ satisfies the PDE with $\tilde{q}(\hat{p})$ set equal to b. Then we can get the conclusion of the lemma easily.*

Q.D.E