國立交通大學

財務金融研究所

碩 士 論 文

考量破產風險後分紅保單在保險公司的負債評價

Fair Valuation of Life Insurance Liabilities in Participating Contracts with Insolvency Risk

研 究 生：葉家誠

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摘要

本篇論文分析了一個很受歡迎，內含許多鑲嵌選擇權的保險商品 – 分紅保單。我們根據保戶的存款準備金用蒙地卡羅法去模擬未來可能的現金流去應用在一個分紅的機制上面。合約可以被拆解為合約本身、分紅選擇權、解約選擇權和違約選擇權。我們的目標是希望可以讓這些內在鑲嵌的選擇權可以被公平的評價在保險公司負債裡面。值得注意的是我們內加了一個違約選擇權於保單公平價值評價內，然而該違約選擇權其實侵蝕保戶的保單價值，應該要受到監管機關的限制。此外，本篇論文引用較貼切實際的隨機資產過程去合乎真實世界的情形。

關鍵字：分紅保單, 破產風險, 最小平方法蒙地卡羅, 公平價值。

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In this research, we analyze the fair value of popular insurance product – participating contract (or with-profit contracts) which is embedded with some options. We use a credit mechanism by means of Monte Carlo Simulation to generate the possible cash flow of policyholder base on benefit reserve. The contract can be decomposed to policy claim, bonus option, surrender option and default option. The purpose of this paper is to make them fair presented in the liabilities category. It is noticeable that we add additional default option to the contract valuation framework. However, the default option we added to the contract erodes the contract value which should be restricted by regulatory authorities. Moreover, we use a more practical stochastic asset process to fit the real world situation.

Key words：Participating contract, Insolvency Risk, LSMC, Fair Valuation

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1. Introduction

How to evaluate the Fair value of insurance contract has been a popular research subjects since the roaring eighties which caused several insurance companies go bankruptcy (see Briys and de Varenne (1994) for detailed description). Since then, the insolvency risk of insurance company has become a non-ignorable issue. However, the history repeated itself, the Financial Tsunami happened in 2008, not only the insurance companies but also financial service institution. It was mainly because of the low-rate government policy and over credit expansion which is caused by mortgage derivative and high leverage operation. Both fair valuation of complicated derivative and insurance contract are not an easy job. Until now, accounting tends to make the assets and liabilities fairly presented to its fair value in each account. IFRS 4 is an insurance contract accounting standard which is put into practice by most of European Countries like England, German and France. The soul in the accounting standard promote insurance contracts should be presented in fair value on balance sheet; moreover, any embedded options should be take into consideration, and the future cash flow should be discounted under current interest rate. To deal with the tremendous works, there are two phase to go. In phase1, the life insurance companies require a test for the adequacy of recognized insurance liabilities and an impairment test for reinsurance assets; in phase 2, each contract should be discounted under current interest rate and any options embedded should be considered. The first phase is ongoing during 2005 by EU member countries and there are more and more countries follow. The second phase is started off by seldom countries like England and Dutch. To see the effect of IFRS 4 made, we can glimpse into Taiwan's insurance market. There have been four European insurance companies sold their business to Taiwan local company since 2007[1]; by year, they are ING Group (2007),

[1] Taiwan insurance industry hasn't follow IFRS 4 standard second phase yet, but will put into practice at 2011.

TransGlobe Life Insurance Inc. (2009), American International Group (2009), and Prudential Assurance Company Limited (2009). Not including American International Group into consideration since the main reason it sell its business to Taiwan local company might be caused by Financial Tsunami. All other insurance companies come from Europe – Dutch and England. They cannot afford high interest spread loss of insurance contracts in Taiwan business since the high technical rate guarantee in early year. Under IFRS4, they are recognized as loss in liabilities.

1.1 Literature Reviews and Motivations

Participating contracts is a popular product when current interest rate is low and expect it will surge up in the future since it participate insurance company's profit. Thus, even though the technical rate is restricted in low level but there is possible future benefits for policyholder to share. We call this basic option in participating contract as bonus option. Besides bonus option, there are another embedded options covered in the participating contracts, such as surrender option, default option, etc. Surrender option can be seeing as American option which gives the policyholder the right to early exercise the contracts before maturity. In practice, the surrender option involved punishment when policyholder surrender the contract, i.e., it won't return all the premium you had paid. Bonus option has the main feature of participating contract which mentioned before. When the profit is greater than the interest rate guarantee, policyholder has the right to participate insurance company's profit. There are several ways to receive the bonus, such as, paid-up insurance premium, save-in agreed interest rate and paid-up additions. In this analysis, we use the paid-up additions form to calculate the future bonus since our framework is mainly focus on single premium insurance. However, in spite of it is an attractive feature to attract policyholder buying the contract, insurance company started cutting their bonuses in order to ensure its survival. At last, default option is the right for policyholder to protect himself from insurance company's financial distress.

When insurance company suffers insolvency (i.e. company's assets are not enough to cover its liabilities), policyholder can liquidate residual asset of insurance company. These three options are always considered by most of papers (Bacinello et al., 2003a, 2003b, 2009; Grosen et al., 2000, 2001, 2002; Chen and Suchanecki, 2007, etc.). Some of these papers consider mortality risk in the framework (Bacinello , Biffis and Millossovich (2009), Bacinello (2003a, 2003b); Grosen, Jensen and Jorgensen (2001)), and some consider default risk (Chen and Suchanecki (2007), Grosen, Jensen and Jorgensen (2002)). , but none of them considers both mortality risk and default risk. In this analysis, we combine both of them and use Parisian option mechanism to construct default option. We allow policyholder liquidate insurance company's assets if insurance company suffer insolvency for a continue period or cumulative period. This idea is come up by the American bankruptcy law – Chapter 11. Chapter 11 allows insurance company a grace period to reorganize the company before it is liquidated. A company survives if it walks through financial distress or else it goes bankrupt. Such a bankruptcy procedure with a given "grace" period does not only exist in the United States, but also in Japan and in France. Table 1 provides detailed information on the bankruptcy procedure and the number of days spent in default for some exemplary bankruptcies of life insurance companies in the United States.

Table 1[2]

Some defaulted insurance companies in the United States

American defaulted companies	Year	Bankruptcy code	Days spent in default
Executive Life Insurance Co.	1991	Ch.11	462
First Capital Life Insurance Co.	1991	Ch.11	1669
Monarch Life Insurance Co.	1994	Ch.11	392
ARM Financial Group	1999	Ch.11	245
Penn Corp. Financial Group	2000	Ch.11	119
Conseco Inc.	2002	Ch.11	266

[2] The data come from http://www.bankruptcydata.com/ and http://www.chapter11blog.com/.

Metropolitan Mortgage & Securities	2004	Ch.11	n/a
U.S. Insurance Group, LLC	2009	Ch.11	n/a
All American Title Agency, LLC	2009	Ch.11	n/a

So far, there are three kinds of method to calculate the initial value of insurance contract by generate future scenario. They are binomial tree (Bacinello, 2003), finite difference (Grosen and Jorgensen, 2001) and Monte Carlo method (Grosen and Jorgensen, 2000; Bacinello , Biffis and Millossovich, 2009). Each method has each pros and cons. We use Monte Carlo Simulation in this analysis since it is easier to construct a complicated stochastic asset value than finite difference method. Moreover; it's faster than binomial tree when we are dealing a long-term path-dependent contract. In order to generate a model to fit the real world in a more practical and efficient way, we follow the stochastic process in Bakshi, Cao and Chen (1997) to generate Monte Carlo simulation. Besides, we use an algorithm similar to Bacinello , Biffis and Millossovich (2009) algorithm 1 to calculate the contract value recursively by LSMC.

1.2 Contribution

Our contribution in this study is three-fold; first, we extend the default option with Parisian option framework to deal with the fact that insurance companies may go bankruptcy. And policyholder can liquidate insurance company's residual value after insurance company can't go over the grace period of bankruptcy procedure. Second, we use traditional actuarial method to calculate the actual premium for simulation in a more practical view. Third, in fact, it's hard to use binomial model or finite difference method to deal with the three dimension stochastic valuation. It's either wasting of time or too complicated to solve the stochastic differential equation. We use LSMC to simplify the timing and complex math equation problem. In comparison to Bacinello , Biffis and Millossovich (2009), we consider about insurance financial face in our cash flow simulation.

This paper is organized as follows. Section 2 describes the model we use to analyze the contacts and presents the basic modeling framework. We also show the state variable and mortality law we use in this part. In section 3 we demonstrate how contract values can be decomposed into their basic elements. In section 4 we construct the detailed algorithm of each contract. In section 5 we have numerical result for parameter implication. In section 6 we come to a conclusion and future prospects. In section 7 we have regression formula details for section 4.

2. The Model

In this section we provide a more detailed description of the participating contract, the Monte Carlo simulation in financial risk, mortality law and the LSMC idea. Furthermore, we take a closer look at our extension on Grosen and Jorgensen's (2000) framework. Besides, we extend Grosen and Jorgensen's (2000) framework with mortality risk by uncertain future life time.

We first describe our assumption in our model. The contracts is operated in a continuous time frictionless economy with a perfect financial market to ease the complexity of the contracts itself, so there are no tax effects, transaction costs, divisibility, liquidity, and short-sales constraints and other imperfections can be ignored. We also ignore expense charge and fluctuation of mortality; instead of Lee-Carter mortality law (1992), we use Makeham's mortality law (1860) to simulate future life time. The existence of mortality risk implies the uncertainty as to the expiration of the individual contracts. Two assumptions for mortality risk are needed. First, we assumed financial risk and mortality risk are uncorrelated. It's a reasonable assumption in general case. Second, we follow standard actuarial practice by assuming that mortality risk for insurance company can be easily diversified by a sufficient large number of contracts. The implication of this assumption is that insurance company won't go bankruptcy if the asset value of single policyholder scenario is less than the benefit payment.

Furthermore, the contracts can be priced under a probability weighted average of values of pure financial contracts spanning all feasible expiration dates with the weighting probabilities derived from the relevant mortality tables. However, we must notice the previous assumptions we just made are paradoxical. In practice, insurance product is neither in a frictionless economy nor in a perfect financial market. The contract must include the cost and expense inside the premium. Besides, the contract is hardly to be composed by other financial products. So, frictionless economy and perfect market assumptions fail in real world. However, in order to deal with the analysis and see the parameter implication, we made these assumptions to simplify the framework. In this paper, we use the standard actuarial symbol that IAA (International Actuarial Association) uses; otherwise, we'll describe the detailed description.

We deal with the endowment insurance with maturity T in this analysis. At time zero, policyholder makes a single premium P_0 with the insurance company. The policyholder then acquires an insurance contract to ensue future life contingency. If policyholder dies before maturity, his beneficiary gets the claim payment by insurance company. Either or, if policyholder lives to maturity, he'll have the benefit claim. Furthermore, claims at each time till maturity might greater than the initial claim that policyholder insure since we are dealing with the participating contract which bonus will be paid by the additional paid-up claim payment.

At the inception of the contract, the insurance company invests the trusted funds in the financial market and commits to crediting interest on the policy's account balance. The initial asset value is composed by policyholder's premium and stockholder's contribution. The pay-out scheme is linked to this and previous years' market return. We will describe more detail in the interest rate crediting mechanism later. We merely note that the interest credit in year t is determined by $r_p(t)$ and $r_p(t-1)$ which are the credit interest in year t and $t-1$ respectively.

In this analysis, we follow Grosen and Jorgensen's (2000) interest rate crediting mechanism, i.e. the policy for the determination of each year's $r_p(\cdot)$. We now turn to model out the main issue – the participating contracts.

2.1 Model basics

To model the contracts, we use the following simplified time t balance sheet as its departure.

Figure 1

Assets	L/E
$A(t)$	$\text{P}(t-1)$ $B(t)=\eta A(t)-P(t-1)$ $E(t)=(1-\eta)A(t)-B(t)$
$A(t)$	$A(t)$

First, notice the left-hand side $A(t)$, the market value of the assets backing the contract. Second, The top of the right-hand side $P(t)$, the benefit reserve; η, the wealth distribution ratio (or liability ratio), it is the ratio that asset belongs to policyholder initially. To joint this parameter is for the purpose that we are trying to calculate the default option. A realistic value of wealth distribution ratio would be 85~95%. Third, $B(t)$, the bonus reserve, or just called the buffer. The last but not the least, $E(t)$, equity value. To be clear, $P(t)$, $B(t)$ and $E(t)$ are not represented as market value but book value, and figure 1 is just individual policy and a snap-shot of the balance sheet situation at a certain point in time.

2.2 The Financial Risk in Asset Side

The insurance company is assumed to keep the asset base invested in a well-diversified portfolio at all times. Instead of well-known Geometric

Brownian motion process, we use more realistic model that Bakshi, Cao and Chen (1997) propose, which include stochastic risk-free rate r_t, stochastic volatility K_t, unexpected jump J_t^Y and stochastic asset value S_t. Under risk-neutral measure, the well-diversified portfolio value and its component is according to the following stochastic equations:

$$\begin{aligned} dr_t &= \varsigma_r(\delta_r - r_t)dt + \sigma_r\sqrt{r_t}dZ_t^r \\ dK_t &= \varsigma_K(\delta_K - K_t)dt + \sigma_K\sqrt{K_t}dZ_t^K \\ dY_t &= (r - \frac{1}{2}K_t - \lambda_Y\mu_Y)dt + \sqrt{K_t}\left(\rho_{SK}dZ_t^K + \rho_{Sr}dZ_t^r \right. \\ &\quad \left. + \sqrt{1-\rho_{SK}^2-\rho_{Sr}^2}dZ_t^S\right) + dJ_t^Y \\ S_t &= e^{Y_t} \end{aligned} \tag{2.2}$$

Where the process Z^r, Z^S, Z^K are mutually independent Brownian motion, J^Y is a compound Poisson process with jump arrival rate $\lambda_Y > 0$ and i.i.d. lognormal jumps Δ_Y. Specifically, we assume that $\log(1+\Delta_Y)$ is Normal with mean μ_Y and standard deviation $\sigma_Y > 0$. J^Y is assumed to be independent of the vector $\left(Z^r, Z^S, Z^K\right)$.We define the state variable as $X_t \equiv (r_t, K_t, Y_t)$. Details on the estimation of model are provided in Bakshi, Cao and Chen (1997).

The probability space here is given by $\left(\Omega, F, \mathbb{F}, \mathbb{Q}\right)$, where $\mathbb{Q}$ is a probability measure that equivalent to real world probability measure and the gain from holding the financial product is a $\mathbb{Q}$-*martingale* after deflation by the money market account. And $\mathbb{F} = \left(F_t\right)_{t\geq 0}$ is a filtration satisfying the usual conditions of right continuity and $\mathbb{Q}$-*completenes* and such that $F_0 = \{0, \Omega\}$.

2.3 The Mortality Risk in Liability Side

For the force of mortality, we use the Makeham's mortality law (1860)

$$\mu_x(t) = A + Bc^{x+t} \tag{2.3.1}$$

, and we can transfer the force of mortality (2.3.1) into the probability density function of future life time of (x)[3], i.e. $T(x)$

$$f_{T(x)}(t) = \exp\left(-\int_0^t A + Bc^{x+s} ds\right) \times \left(A + Bc^{x+t}\right) \tag{2.3.2}$$

At last, the cumulative probability function is given by

$$\begin{aligned} F_{T(x)}(t) &= P(T(x) < t) \\ &= \int_0^t \exp\left(-\int_0^u A + Bc^{x+s} ds\right) \times \left(A + Bc^{x+t}\right) du \end{aligned} \tag{2.3.3}$$

Then, we can use the Inverse Probability integral Transform to generate the future life time of (x).

$$\begin{aligned} &U \sim unif(0,1) \\ &\tau_d = F_{T(x)}^{-1}(U) \end{aligned} \tag{2.3.4}$$

Where U is a uniform random variable between 0 to 1.

Details on parameters estimation are provided by Melnikov and Romaniuk (2006).

Here, we modify our filtration that combines the financial risk and mortality risk[4]. We consider an individual aged x at a reference time 0. We denote the filtration $\mathbb{H}$ generated by the process $N_t = 1_{T(x) \le t}$ which equals zero as long as the individual is alive and jumps to one at death. We enlarge our filtration we set before as $\mathbb{G} \doteq \mathbb{F} \vee \mathbb{H}$. Then we work with the enlargement probability space $(\Omega, G, \mathbb{G}, \mathbb{Q})$ instead of $(\Omega, F, \mathbb{F}, \mathbb{Q})$.

2.4 Interest Rate Credit Mechanism and Bonus Policy

Before entering the subject of credit mechanism, we first distinguish the difference between technical rate and risk-free rate. Technical rate is the rate insurance company expects to earn by using the policy premium, and it is the rate that insurance company use to discount future policy claim payment to

[3] In IAA, the symbol (x) denotes the life-age-x.

calculate policy premium. In other views, we can see technical rate as minimum interest rate guarantee. The risk-free rate is the interest rate that it is assumed can be obtained by investing in financial instrument without default risk.

To introduce the credit mechanism, we first introduce two parameters: target buffer ratio γ and distribution ratio α. Target buffer ratio is the ratio that insurance company's bonus reserve mechanism to protect its solvency. The realistic value would be in the order of 10~15%. The distribution ratio is the ratio that insurance company distributes its profit to policyholder base on, i.e., the percentage can be distributed to policyholder. A realistic value is in the area 20~30%. Before we proceed, let us briefly recapitulate the most frequently applied notation:

T	: maturity time of contract
$r_p(t)$	: policy interest rate in year t
b_t	: claim payment
i	: technical rate (minmum interest rate guarantee)
$r_B(t)$	: bonus interest rate in year t
$A(t)$	: market value of insurance company's asset at time t
$P(t)$	: policy reserve at time t
$B(t)$	: bonus reserve at time t
${}_tV$	: benefit reserve
${}_tV^{adj}$	: adjuted benefit reserve with additional paid-up claim
γ	: target buffer ratio
α	: distribution ratio
(x)	: life-age x
τ	: death time
$A_{x:\overline{T}\rceil}$	: Endowment Insurance for age x, T year maturity

The discussion above is now be formulized as following analytical scheme for the interest rate credited to policyholder's accounts in year t.

[4] See Bacinello , Biffis and Millossovich (2009) for more detailed description.

$$B(t) = \eta A(t) - P(t-1)$$
$$r_p(t) = \max\left(i, (\alpha \frac{B(t)}{P(t-1)} - \gamma) \right) \quad (2.4.1)$$

This implies a bonus interest rate as stated below,

$$r_p(t) = \max\left(0, (\alpha \frac{B(t)}{P(t-1)} - \gamma) - i \right) \quad (2.4.2)$$

Then, we can generate the policy reserve at year t as below[5],

$$b_t = b_{t-1} + \left(\frac{{}_{t-1}V + {}_tV}{2} \right) r_p(t) / \mathrm{A}_{x+t:\overline{T-t}|};$$
$${}_tV_{adj} = b_t \mathrm{A}_{x+t:\overline{T-t}|} \ ;$$
$$\text{where } \mathrm{A}_{x+t:\overline{T-t}|} = \sum_{i=1}^{T-t} \frac{1}{(1+r_G)^i} {}_{i-1}p_{x+t} q_{x+t+i} + \frac{1}{(1+r_G)^{T-t}} {}_{T-t}p_{x+t} \quad (2.4.3)$$
$$P(t) = \begin{cases} {}_tV_{adj} & \text{if (x) is alive} \\ b_t & \text{o.w.} \end{cases}$$

${}_tp_x \equiv$ the probability that someone age x lives t years

$q_x \equiv$ the probability taht someone age x dies within one year

Our framework is different from the research done by Grosen and Jorgensen (2000). We extend the framework with mortality risk and fit mortality risk to the realistic life insurance contracts – endowment insurance contracts. This extension with mortality risk also works for the whole life insurance contracts. All procedures are the same, instead of the fixed maturity date, replacing it to the maximum future life time that we simulate.

According to the Bacinello, Biffis and Millossovich (2009) algorithm 1 that we are going to explore in the next chapter, it is an efficient way to calculate the fair value of insurance liabilities.

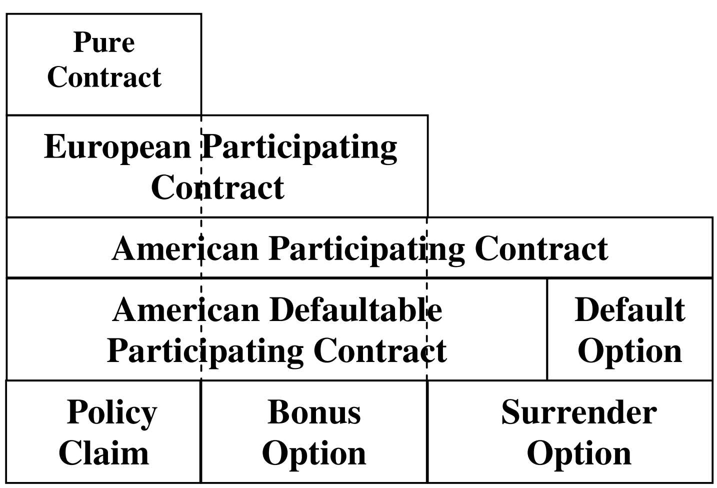
[5] The adjustment framework refers to the policy distribution rule is refer to Cathay Life Insurance.

3. Valuation of Contract

In this section, we deal with four kinds of contracts as in Figure 2. First, we consider a pure insurance contract without any options embedded but policy claim. Second, European participating contract gives policyholder the right to share additional bonus besides benefit claim in each year. However, there is no early exercise right for European participating contract before contracts naturally terminate (Time to maturity or death). Third, American participating polices give policyholder the right to share additional bonus and surrender the contract before contracts naturally terminate. Policyholder can terminate the contract if policyholder thinks there are no more bonuses in the future. The last but not the least, American defaultable participating polices give policyholder another right to liquidate insurance company when insurance company's assets cannot afford basic benefit reserve which is calculated by expected loss in actuarial practice (Due to different policy premium).

Since in different technical rate, we'll have different initial asset value. We can see each option value by simply minus from downstairs to the upstairs in Figure 2. To see the parameter implication, we'll interchange the parameter value we mention before to see different impact, like distribution ratio, buffer ratio, technical rate, etc.

Figure 2



3.1 Pure Contract and European contract

As described before, the pure endowment polices and European participating contract pays the claim payment only when policyholder dies before maturity or lives to maturity. The difference between pure contract and European contract is merely on its benefit claim. Pure contract pays the same amount of benefits whenever contract naturally terminate. European contract pay the initial benefit plus additional paid-up insurance benefit by bonus in each year.

$$
\begin{aligned}
&V_s^P \equiv \text{Pure policies} \\
&V_s^E \equiv \text{European participating policies} \\
&V_s^E = E^Q\left\{e^{-r(\tau_d - s)} b_t \middle| G_s\right\} \\
&\text{, where } \tau_d = \begin{cases} T(x) & \text{if } T(x) < T \\ T & \text{if } T(x) \geq T \end{cases} \\
&V_s^P = V_s^E \text{ except } b_t = b_0 \ \forall t \\
&(b_0 \text{ is initial claim})
\end{aligned} \tag{3.1.1}
$$

3.2 American participating contract

The difference between American and European participating contract is that, besides policy claim and bonus option, American participating contract have additional surrender option. In fact, in practical contracts, there is punishment if policyholder surrender before contract naturally terminate, which is called Market Value Adjustment. We ignore the feature instead to see the actual parameter implication.

We use Least Square Monte Carlo algorithm which is initiated by Longstaff and Schwartz (2001) to calculate the American option. Bacinello , Biffis and Millossovich (2009) change the regression to more complicated form since the stochastic asset value they use is more complicated than Geometric Brownian Motion that Longstaff and Schwartz (2001) initialize in the Least Square algorithm. Since we have three dimensional stochastic value and unexpected jump as Bacinello , Biffis and Millossovich (2009), we use polynomial basis function of order 3 to run the regression continuation value. We'll describe more details in next chapter.

$$V_s^A \equiv \text{Americna participating policies}$$
$$V_s^A = \sup_{\tau_s \in \Im_{s,\tau_d}} E^Q\left\{ e^{-r(\tau_s - s)} P(t) \middle| F_s \right\} \tag{3.2.1}$$

, where $\Im_{s,\tau_d}$ denotes the class of $G_s - \textit{stopping times}$ taking values in $[s,T]$

3.3 American defaultable participating contract

The whole option base contract – American defaultable participating contract possesses all option mentioned above and another default option. We include insolvency risk into account to value the insurance polices. There is chance for the insurance company to default when their asset are less than benefit reserve. Instead of immediate bankruptcy, we design a mechanism similar to Parisian option. Insurance company will actually go bankrupt when insurance company's asset is lower than benefit reserve for continue period or cumulative period d. If the stopping time is earlier comparing with surrender and future life time, then insurance company goes bankrupt and policyholder only have the residual value of asset. Before constructing the default option, we construct the default barrier which is given by

$$Ba_t = b_0 \mathrm{A}_{x+t:\overline{T-t}|} \tag{3.3.1}$$

The barrier set as Equation (3.3.1) is because the insurance company cannot ensure anything when insurance company does not even have the ability to meet initial benefit claim reserve.[6] Then we have the contract value

$V_s^{ADO} \equiv$ American defaultable participating policy with continue solvent period

$V_s^{ADC} \equiv$ American defaultable participating policy with cumulative solvent period

---

[6] There is a tougher barrier we can use; that is, the adjusted benefit reserve which we can just revise b_0 in (3.3.1) into b_t.

$$
\begin{aligned}
V_s^{ADO} &= \sup_{\tau_s \in \Im_{s,\min(\tau_d,\tau_{do})}} E^Q\left\{e^{-r(\tau_s - s)} P(t) \middle| F_s\right\} \\
V_s^{ADC} &= \sup_{\tau_s \in \Im_{s,\min(\tau_d,\tau_{dc})}} E^Q\left\{e^{-r(\tau_s - s)} P(t) \middle| F_s\right\}
\end{aligned} \qquad (3.3.2)
$$

, where $\tau_{do} \equiv$ contine solvent period stopping time and

$\tau_{dc} \equiv$ cumulative solvent period stopping time[7]

4 Computational Aspects

4.1 Monte Carlo Simulation

Stochastic process we use is mentioned as before, and the participating contract can be valued by standard Monte Carlo techniques. We need run the state valuable and calculate the policies in a Bermudan claim framework instead of American claim framework. With the continuous asset value, we just calculate the contract value discretely. We call the Backward Discretization Step (BDS) the length in years of each time interval arising from this discretization. To simulate the state variable, we need a finer grid which we called Forward Discretization Step (FDS).

The actual steps involved in simulating a single path are the following:

STEP 0: (Initialization)

$N_2 = 1/FDS; N_1 = 1/BDS;$

$X_{11}^{m,l}{}_{11}, \ldots X_{1N_2}^{m,l}, \tau_d$ for $m = 1, \ldots, M; l = 1, \ldots, L$

$P^{m,l}(0) = A_{x:\overline{T}|}$; $A^{m,l}(0) = P^{m,l}(0)/\eta$

STEP 1: (Forward Iteration) For $j = 1, \ldots, N_1$

Set $D_j^{m,l} = I(j-1 < \tau_d^{m,l} \le j)$ and $L_j^{m,l} = I(\tau_d^{m,l} > j)$ to represent the man die this year or the man still alive.

$B^{m,l}(j) = L_j^{m,l}(\eta A^{m,l}(j) - P^{m,l}(j-1))$

---

7 τ_{do} is always less than τ_{dc} since $\Im_{s,\tau_{do}} \subseteq \Im_{s,\tau_{dc}}$.

$$r_p^{m,l}(j) = L_j^{m,l} \max\left\{ i, \alpha\left(\frac{B^{m,l}(j)}{P^{m,l}(j-1)} - \gamma \right) \right\}$$

$$b_j^{m,l} = L_j^{m,l}\left(b_{j-1}^{m,l} + \left(\frac{{}_{j-1}V + {}_jV}{2} \right)(r_p^{m,l}(j) - i) / {}_jV \right)$$

$${}_jV_{adj}^{m,l} = L_j^{m,l} b_j^{m,l} A_{x-1:\overline{T-1}|}$$

$$P^{m,l}(j) = L_j^{m,l}\, {}_jV + D_j^{m,l} b_j^{m,l}$$

STEP 2: (Initial Value)

$$\hat{V}_0^{E,l} = \frac{1}{M}\sum_{m=1}^{M}\left(\left(\prod_{jj=1}^{\min\left(\left\lceil \tau_d^{m,l} \right\rceil, T\right)} \left(\frac{1}{1+r_{jj}^{m,l}}\right) \right) \times b_{\left\lceil \tau_d^{m,l} \right\rceil}^{m,l} \right) \tag{4.1.1}$$

$$\hat{V}_0^{E} = \frac{1}{L}\sum_{l=1}^{L} \hat{V}_0^{E,l}$$

These steps are repeated M times and the Monte Carlo estimate of the initial contract value $V_0^{E,l\ 7}$, is founded by averaging the $P^{m,l}(\tau_d)$ with different discount rate that we've simulated in each state each time to time zero. In order to keep the contract value more accurate, we repeat different seeds L times, and average them for initial contract value. Initial value of pure insurance contract can be calculated simply by replacing $b_{\tau_d^{m,l}}^{m,l}$ in (4.1.1) into b_0. The superscript ^{m,l} represent m^{th} simulation in l^{th} seed. The superscript is also applied in the following section.

4.2 The recursive method for path-dependent contracts

In order to calculate the American (Bermudan) and Parisian type contract, we use LSMC algorithm which is similar to Bacinello , Biffis and Millossovich (2009) to compute the continuation value of American type contract. This method is first presented by Longstaff and Schwartz (2001).

The valuation algorithm requires the execution of the steps 0 and 1 mentioned above and the following additional steps:

STEP 2: (Initialization) Set $\tau_s^{m,l} = \min\left(\left\lceil \tau_d^{m,l} \right\rceil, T\right)$ and

$P_{\tau_s^{m,l}} = P(\tau_s^{m,l})$ for m=1…M.

STEP 3: (Backward Iteration) For $j = N_1 - 1, N_1 - 2 \ldots, 1$:

(1) (Continuation values) Set $I_j = \left\{1 \le m \le M : \tau_d^{m,l} > j\right\}$ and, for $m \in I_j$,

set $C_j^{m,l} = \sum_{h=j+1}^{\tau_s^{m,l}} P_h^{m,l} \times \left(\prod_{jj=j}^{h} \frac{1}{1 + r_{jj}^{m,l}} \right)$.

(2) (Regression) Regress the continuation values $\left(C_j^{m,l}\right)_{m \in I_j}$ against $\left(e(X_j^{m,l}, \tau_d^{m,l})\right)_{m \in I_j}$ to obtain the $\hat{C}_j^{m,l} = \hat{\beta}_j \cdot e\left(X_j^{m,l}, \tau_d^{m,l}\right)$ for $m \in I_j$.

If $C_j^{m,l} > \hat{C}_j^{m,l}$ then set $\tau_s^{m,l} = j$ and $P_j^{m,l} = P^{m,l}(j)$.[8]

STEPS 4: (Initial value) Compute the single premium of the contract

$$\hat{V}_0^{A,l} = \frac{1}{M} \sum_{m=1}^{M} \left(\left(\prod_{jj=1}^{\tau_s^{m,l}} \left(\frac{1}{1 + r_{jj}^{m,l}} \right) \right) \times P^{m,l}(\tau_s^{m,l}) \right)$$

$$\hat{V}_0^{A} = \frac{1}{L} \sum_{l=1}^{L} \hat{V}_0^{A,l} \qquad (4.2.1)$$

In order to calculate the whole option base contract, we examine the asset value forwardly. The valuation algorithm follows steps 0~4 mentioned before and the algorithm below,

STEP 5: (Initialize) $ct = 0, (ct_2 = 0)$; $\tau_{do}^{m,l} = T, (\tau_{dc}^{m,l} = T)$

STEP 6: (Forward Iteration) For $j = 1, \ldots N_1$

(1)(Continue Insolvent time)

---

[8] See Appendix for details.

If $\left(A^{m,l}(j) < Ba^{m,l}(j)\right)$, $ct = ct + 1$,

If $\left(ct \geq d\right)$, $\tau_{do}^{m,l}$=*j*, break;

else, ct=0

(2)(Cumulative Insolvent time)

If $\left(A^{m,l}(j) < Ba^{m,l}(j)\right)$, $ct_2 = ct_2 + 1$,

If $\left(ct_2 \geq d\right)$ $\tau_{dc}^{m,l}$=j, break;

STEP 7: (Initial Value)

$$\tau_{sdo}^{m,l} = \min(\tau_s^{m,l}, \tau_{do}^{m,l})$$

$$\hat{V}_0^{ADO,l} = \frac{1}{M}\sum_{m=1}^{M}\left(\left(\prod_{jj=1}^{\tau_{sd}^{m,l}}(\frac{1}{1+r_{jj}^{m,l}})\right)\times P^{m,l}(\tau_{sd}^{m,l})\right) \tag{4.2.2}$$

$$\hat{V}_0^{ADO} = \frac{1}{L}\sum_{l=1}^{L}\hat{V}_0^{ADO,l}$$

Then we have American defaultable participating contract. In next chapter, we run Monte Carlo simulation to see the parameter implication in our model. Our focus is on (4.1.1), (4.2.1) and (4.2.2); we'll also see the option value by minus each contract value.

5. Numerical Implication

In this section we present the result from the numerical analysis of the model. We execute the algorithm in chapter 4 by Monte Carlo method. The reference parameter is showed in table 2. If there is no more refer to the parameter value, the parameter value is according to the value in table 2. Some details about first column we need to clarify. $P(0)$ is the initial premium calculated by traditional actuarial method by $r_0 = 0.05$, and $E(0)$ is calculated by a certain ratio such that the following equation holds.

$$\begin{aligned} P_0 &= \eta A_0 \\ E_0 &= (1-\eta) A_0 \end{aligned} \tag{5.1}$$

The wealth distribution ratio η is set as 0.9 because of realistic situation that a life insurance company often finances its asset by 0.1 to 0.2 to cover its liability. The mortality law's parameter is set by using the empirical test result of Melnikov and Romaniuk (2006). Participating ratio is decided by the lowest regulation requirement in Europe.

We ran 100,000 simulations with 25 different seeds in table 4 to table 7 but table 6, i.e., $M = 100{,}000$ ，$L = 25$; 50,000 simulations with 10 different seeds for table 6, $M = 50{,}000$ ，$L = 5$. We ran this number of simulations because it converges well. The standard deviation is around 0.05 to 0.01.

Table 2
Some parameter details

	r	K	S	μ
$BDS = 1$	$r_0 = 0.05$	$K_0 = 0.04$	$A_0 = P(0) + E(0)$	$A = 9.5666 \times 10^{-4}$
$FDS = 0.01$	$\zeta_r = 0.6$	$\zeta_K = 1.5$	$\rho_{SK} = -0.7$	$B = 5.162 \times 10^{-5}$
$\eta = 0.9$	$\delta_r = 0.05$	$\delta_K = 0.04$	$\rho_{Sr} = 0$	c=1.09369
$\alpha = 0.6$	$\sigma_r = 0.03$	$\sigma_K = 0.4$	$\lambda_Y = 0.50$	
$\gamma = 0.15$			$\mu_Y = 0$	
$P(0) = 55.865$			$\sigma_Y = 0.07$	
$E(0) = 6.207$				
$T = 15$				
$d = 2$				
$b_0 = 100$				

We explore the inside meaning of each options in the very beginning. The basic part of participating contracts is bonus option. The bonus option means the future possible profit for policyholder. The greater profit policyholder can share the greater liabilities insurance company should recognize because of future payments. Another option is the surrender options which represent the right for policyholder when he doesn't want to hold the policy anymore. Moreover, he can get away from the contracts with his contracts value plus the additional bonus he has earned. The greater surrender option value means the greater right for policyholder when policyholder decide whenever to surrender from the contract. In other words, the greater the surrender option value represents the greater incentive for policyholder to surrender the contracts before maturity. The last option we are exploring is the default option which is meaningful since the Financial Tsunami in 2008. "Too big to fail" seems to be nonsense since then. In our view, the greater the default option value means the greater value that policyholder loss in the event of insurance company's default.

Table 3 interprets the abbreviation in Table 4, 5, 6, 7.

Table 3
Some detailed description for the table below

Name	Parenthesis	Name Represent
Pure	(1)	Pure Insurance contract
Euro	(2)	European participating contract
Bonus	(3)=(2)-(1)	Bonus Option
Am	(4)	American Participating contract
Surrender	(5)=(4)-(2)	Surrender Option
Paris1	(6)	American defaultable participating contract with continue insolvent period
Def1	(7)=(4)-(6)	Default Option in continue insolvent period
Pairs2	(8)	American defaultable participating contract with cumulative insolvent period
Def2	(9)=(4)-(8)	Default Option in cumulative insolvent period

In Table 4, we have similar results with Grosen and Jorgensen (2000), the greater the distribution ratio and less target buffer ratio, the greater bonus value and less surrender value. Both bonus option and surrender option are decreasing against technical rate. It's comprehensible that high distribution ratio enriches bonus value for policyholder, and technical rate guarantee minimum interest deteriorates bonus option but enrich pure contract. Moreover, it is no need to surrender the contract if there is high future profit guarantee, so less surrender value if distribution ratio and technical rate are high. However, let's see the impact at default option. The lower the profit guarantee and higher buffer, the lower default option. It's also a comprehensible result that default value is high when profit guarantee is too much or buffer is not enough to protect policyholder's interest. Moreover, we can see default value is almost zero when technical rate and distribution rate are low in spite of target buffer ratio is high or low. The policyholder benefits much from a higher regulation parameter γ because higher values of γ provide the policyholder a better protection against losses.

As the same setting with Grosen and Jorgensen (2000), we set up some competitive contracts to see the option value. A large α obviously implies a more favorable bonus option, ceteris paribus, so contract values are rising in α. Conversely, an increase in the target buffer ratio, γ, means less favorable terms for the option elements, so contract values decrease as γ increases. Hence, bonus policies with relatively low αs and high γs can be classified as *conservative* whereas contracts with the opposite characteristics can be labeled *aggressive*. In this connection, we can see $\gamma = 0$ implies insurance company only use it's equity as buffer and without any reserves. In table 5, we can see an aggressive policy can attract policyholder, and it can keep policyholder by its attractive bonus might have in the future, but it suffer greater default value. Still, in this scenario we also propose that supervisory authority should limit its maximum distribution ratio and lowest buffer ratio to protect policyholder's interest.

In table 6, we change policyholder's age and contract maturity. It's obvious that the surrender value is decreasing against policyholder's age. Surprisingly, surrender value is increasing against contract maturity if policyholder is young. It is increasing at first and decreasing when contracts maturity is long if policyholder is in middle age. It is decreasing if policyholder is in elder age. The condition changes as we are in different technical rate. We suppose that surrender option is neither strictly increasing nor strictly decreasing against contract maturity. Moreover, we can see default value is increasing either against policyholder's age or contract maturity.

In table 7, we focus on default option value. As we can see, I examine different asset structure toward different insolvency period allowed by bankruptcy law. η is wealth distribution ratio which is also called debt-to asset ratio. As larger as η, as less as equity. We separate the asset structure we examine as follow, sufficient equity capital $(\eta = 0.85)$, suitable equity capital $(\eta = 0.90)$ and inadequate equity capital $(\eta = 0.95)$. The default value goes up when equity buffer $(1-\eta)$ are low. Comparing with different γ, we can see that default value is higher if target buffer ratio is lower. In considering the bankruptcy law, default value is lower if grace period is longer.

Table 4 – distribution ratio vs. technical rate vs. buffer ratio

$\gamma = 0.15$										
i	α	**Pure (1)**	**Euro (2)**	**Bonus (3)**	**Am (4)**	**Surrender (5)**	**Paris1 (6)**	**Def1 (7)**	**Pairs2 (8)**	**Def2 (9)**
0 $P_0 = 100$	**0.2**	49.350	78.754	29.403	97.564	18.810	97.562	0.002	97.562	0.002
	0.4	49.348	88.713	39.366	103.919	15.206	103.875	0.044	103.884	0.035
	0.6	49.347	94.526	45.179	108.782	14.256	108.556	0.226	108.622	0.160
0.02 $P_0 = 74.510$	**0.2**	49.352	69.369	20.016	77.238	7.870	77.179	0.059	77.193	0.045
	0.4	49.347	77.430	28.083	84.087	6.657	83.234	0.853	83.418	0.669
	0.6	49.350	81.896	32.545	88.235	6.339	86.549	1.685	86.820	1.415
0.04 $P_0 = 55.865$	**0.2**	49.352	61.582	12.230	63.066	1.484	61.159	1.907	61.425	1.641
	0.4	49.348	67.903	18.555	69.063	1.160	65.090	3.973	65.462	3.602
	0.6	49.348	71.475	22.127	72.511	1.036	67.425	5.085	67.829	4.682

$\gamma = 0.10$										
i	α	**Pure (1)**	**Euro (2)**	**Bonus (3)**	**Am (4)**	**Surrender (5)**	**Paris1 (6)**	**Def1 (7)**	**Pairs2 (8)**	**Def2 (9)**
0 $P_0 = 100$	**0.2**	49.347	80.396	31.048	98.430	18.034	98.427	0.002	98.428	0.002
	0.4	49.350	91.032	41.682	105.828	14.796	105.734	0.095	105.761	0.068
	0.6	49.343	97.229	47.886	111.270	14.040	110.845	0.425	110.955	0.315
0.02 $P_0 = 74.510$	**0.2**	49.353	70.747	21.394	78.227	7.480	78.115	0.113	78.149	0.079
	0.4	49.346	79.381	30.035	85.791	6.410	84.549	1.242	84.785	1.006
	0.6	49.345	84.332	34.987	90.520	6.188	88.320	2.200	88.634	1.886
0.04 $P_0 = 55.865$	**0.2**	49.343	62.604	13.261	63.989	1.385	61.646	2.343	61.943	2.046
	0.4	49.349	69.510	20.161	70.583	1.073	66.063	4.520	66.448	4.135
	0.6	49.353	73.419	24.066	74.388	0.969	68.790	5.598	69.219	5.169
$\gamma = 0.05$										
i	α	**Pure (1)**	**Euro (2)**	**Bonus (3)**	**Am (4)**	**Surrender (5)**	**Paris1 (6)**	**Def1 (7)**	**Pairs2 (8)**	**Def2 (9)**
0 $P_0 = 100$	**0.2**	49.350	82.176	32.825	99.502	17.326	99.497	0.005	99.497	0.004
	0.4	49.348	93.439	44.091	107.971	14.532	107.782	0.189	107.838	0.133
	0.6	49.347	100.055	50.708	114.000	13.945	113.304	0.695	113.464	0.536
0.02 $P_0 = 74.510$	**0.2**	49.352	72.224	22.872	79.380	7.156	79.162	0.219	79.229	0.151
	0.4	49.347	81.565	32.218	87.821	6.256	86.167	1.654	86.441	1.380
	0.6	49.350	86.766	37.416	92.855	6.088	90.175	2.679	90.529	2.326
0.04 $P_0 = 55.865$	**0.2**	49.352	63.730	14.379	65.018	1.288	62.263	2.755	62.578	2.440
	0.4	49.348	71.283	21.935	72.276	0.993	67.296	4.980	67.710	4.565
	0.6	49.348	75.552	26.204	76.437	0.885	70.337	6.100	70.814	5.623

Table 5

Different contract type

	Pure	**Euro**	**Bonus**	**Am**	**Surrender**	**Paris1**	**Def1**	**Pairs2**	**Deft2**
Conservative[9]	49.358	61.577	24.410	63.270	1.693	61.092	2.177	69.098	1.923
Neutral[10]	49.363	69.525	25.897	70.729	1.204	66.014	4.715	59.446	4.318
Aggressive[11]	49.353	77.850	25.954	78.760	0.910	71.981	6.780	51.134	6.226

[9] Conservative scenario with $\alpha = 0.2, \gamma = 0.15$

[10] Neutral scenario with $\alpha = 0.4, \gamma = 0.10$

[11] Aggressive scenario with $\alpha = 0.6, \gamma = 0.00$.

Table 6 – policyholder's age vs. policy maturity vs. technical rate

i=0.02

x	T	Pure (1)	Euro (2)	Bonus (3)	Am (4)	Surrender (5)	Paris1 (6)	Def1 (7)	Pairs2 (8)	Deft2 (9)
20	**15**	48.599	81.728	33.129	88.314	6.586	86.598	1.716	86.875	1.439
	20	38.492	75.557	37.065	83.652	8.094	80.839	2.812	81.222	2.429
	25	30.664	69.601	38.936	78.834	9.233	75.157	3.676	75.582	3.252
40	**15**	49.348	81.896	32.548	88.262	6.366	86.620	1.643	86.882	1.380
	20	39.840	76.053	36.212	83.651	7.598	80.842	2.809	81.209	2.442
	25	32.818	70.504	37.686	78.878	8.374	75.035	3.844	75.467	3.411
60	**15**	53.476	82.875	29.399	87.940	5.066	86.367	1.573	86.607	1.333
	20	46.837	78.533	31.696	83.728	5.196	80.972	2.757	81.307	2.421
	25	43.157	75.593	32.436	80.174	4.581	75.593	4.581	75.982	4.192

i=0.04

x	T	Pure (1)	Euro (2)	Bonus (3)	Am (4)	Surrender (5)	Paris1 (6)	Def1 (7)	Pairs2 (8)	Deft2 (9)
20	**15**	48.610	71.072	22.462	72.322	1.250	67.358	4.964	66.550	4.564
	20	38.485	62.548	24.064	63.975	1.426	57.668	6.307	69.163	5.837
	25	30.667	55.047	24.381	56.522	1.475	49.445	7.077	70.556	6.576
40	**15**	49.344	71.472	22.128	72.638	1.166	67.430	5.208	64.694	4.805
	20	39.847	63.500	23.653	64.745	1.245	57.919	6.825	67.878	6.363
	25	32.813	56.670	23.857	57.856	1.186	50.252	7.604	69.610	7.103
60	**15**	53.449	73.656	20.207	74.391	0.736	67.855	6.536	62.627	6.167
	20	46.883	68.272	21.389	68.747	0.474	59.999	8.747	66.244	8.302
	25	43.128	64.751	21.622	64.754	0.003	54.158	10.596	68.481	10.067

Table 7 – wealth distribution ratio vs. insolvency period vs. buffer ratio

$\gamma = 0.15$										
η	d	**Pure (1)**	**Euro (2)**	**Bonus (3)**	**Am (4)**	**Surrender (5)**	**Paris1 (6)**	**Def1 (7)**	**Pairs2 (8)**	**Def2 (9)**
0.85	**1**	49.352	71.455	22.103	72.495	1.040	66.550	5.945	66.550	5.945
	2	49.345	71.480	22.136	72.519	1.039	68.852	3.667	69.163	3.356
	3	49.347	71.439	22.092	72.490	1.051	70.170	2.320	70.556	1.934
0.9	**1**	49.354	71.434	22.080	72.476	1.042	64.694	7.782	64.694	7.782
	2	49.347	71.500	22.153	72.533	1.034	67.481	5.052	67.878	4.656
	3	49.348	71.429	22.082	72.465	1.035	69.116	3.349	69.610	2.855
0.95	**1**	49.343	71.470	22.127	72.509	1.039	62.627	9.882	62.627	9.882
	2	49.344	71.453	22.108	72.486	1.033	65.732	6.753	66.244	6.241
	3	49.359	71.504	22.145	72.539	1.035	67.840	4.699	68.481	4.059
$\gamma = 0.05$										
η	d	**Pure (1)**	**Euro (2)**	**Bonus (3)**	**Am (4)**	**Surrender (5)**	**Paris1 (6)**	**Def1 (7)**	**Pairs2 (8)**	**Def2 (9)**
0.85	**1**	49.347	75.530	26.183	76.410	0.880	69.327	7.083	69.327	7.083
	2	49.350	75.524	26.175	76.394	0.869	71.874	4.519	72.221	4.173
	3	49.345	75.546	26.201	76.445	0.899	73.320	3.125	73.743	2.702
0.9	**1**	49.345	75.546	26.201	76.419	0.873	67.339	9.080	67.339	9.080
	2	49.339	75.609	26.270	76.490	0.880	70.419	6.071	70.896	5.594
	3	49.351	75.592	26.241	76.489	0.897	72.200	4.289	72.751	3.738
0.95	**1**	49.346	75.548	26.201	76.436	0.889	64.917	11.519	64.917	11.519
	2	49.340	75.579	26.240	76.456	0.876	68.503	7.953	69.134	7.322
	3	49.342	75.553	26.211	76.442	0.889	70.643	5.799	71.384	5.059

6 Conclusions

In this paper we have presented a general framework for fair valuation of participating contract combining with insolvency risk and mortality risk. A contract is constructed by pure insurance, bonus option, surrender option and default option. We use the Least Squares Monte Carlo method to calculate the surrender option value. As a practical example, we use endowment insurance to observe the parameter implication. Moreover, we use actual insurance premium to calculate bonus in each years.

Life insurance companies have traditionally not given much attention to the proper valuation of the various option elements with which their policies have been issued, and this has undoubtedly contributed to the problems now experienced in the life insurance. In considering the asset structure and default option, insurance companies must either have sufficient equity capital or larger target buffer ratio to buffer the unexpected shortfall in the future. With greater distribution ratio of profit or interest rate guarantee in the policy, insurance company need more equity to avoid the insolvency incident happened. Alternatively, to reduce the default value, the insurance company could consider more conservative bonus policies to the extent that this is permitted by law and the contractual terms. In regulation views, if an insurance company is going to sell a competitive contract, supervisory authority must inspect if its capital is sufficient as a buffer or not. In fact, the analysis showed that contract values are highly dependent on the assumed bonus policy and the spread between the market interest rate and the guaranteed rate of interest built into the contract.

Some other future research can be considered in the model. First, participating contract is more expensive than pure contract in the practice. However, we only use pure contract value to consider the asset value in our framework. The future research might use the real premium annually to construct the framework. Second, we ignore the fluctuation of mortality in our framework. In practice, insurance company use more conservative interest rate and mortality table to calculate the insurance premium. The surplus in interest rate we've already discus in our study. The surplus in mortality spread earning is also distributed in participating contract which is also an extension in the future research. Lastly, our model can be also used in scenario analysis. We can construct an insurance contract portfolio composed by different age. We can use the scenario to arrange a proper asset and liability management to ensure insurance company's solvency.

7 Appendix

In this section, we show the regression we use to calculate the continuation value in this analysis. First we note that the backward iteration in step j – $\left(e(X_j^{m,l},\tau_d^{m,l})\right)_{m\in I_j}$ is the combination of 3 order of state variable and future life time and $(X_j^{m,l},\tau_d^{m,l})=\left(r_j^{m,l},K_j^{m,l},Y_j^{m,l},\tau_d^{m,l}\right)$ with $\tau_d^{m,l}>j$. We've mentioned the reason why we use the polynomial basis function of order 3 in section 3.2. However, since $\tau_d^{m,l}$ does not concern with asset value, we only use order 1 in future life time $\tau_d^{m,l}$. In regression terms speaking, we use 19 explanation variable from the combination of $(X_j^{m,l},\tau_d^{m,l})$ and interception to explain the response variable $C_j^{m,l}$. The following equation is the regression function we use in calculating the continuation value of the contracts,

$$
\begin{aligned}
C_j^{m,l} &= b_0+b_1\left(Y_j^{m,l}\right)+b_2\left(Y_j^{m,l}\right)^2+b_3\left(Y_j^{m,l}\right)^3+b_4\left(r_j^{m,l}\right)+b_5\left(r_j^{m,l}\right)^2+b_6\left(r_j^{m,l}\right)^3\\
&+b_7\left(K_j^{m,l}\right)+b_8\left(K_j^{m,l}\right)^2+b_9\left(K_j^{m,l}\right)^3+b_{10}\left(r_j^{m,l}\times Y_j^{m,l}\right)+b_{11}\left(r_j^{m,l}\times K_j^{m,l}\right)\\
&+b_{11}\left(K_j^{m,l}\times Y_j^{m,l}\right)+b_{12}\left(\left(r_j^{m,l}\right)^2\times Y_j^{m,l}\right)+b_{13}\left(\left(r_j^{m,l}\right)^2\times K_j^{m,l}\right)+b_{14}\left(\left(S_j^{m,l}\right)^2\times r_j^{m,l}\right)\\
&+b_{15}\left(\left(S_j^{m,l}\right)^2\times K_j^{m,l}\right)+b_{16}\left(\left(K_j^{m,l}\right)^2\times r_j^{m,l}\right)+b_{17}\left(\left(K_j^{m,l}\right)^2\times S_j^{m,l}\right)\\
&+b_{18}\left(r_j^{m,l}\times K_j^{m,l}\times S_j^{m,l}\right)+b_{19}\tau_d^{m,l}
\end{aligned}
$$

All we need in regression constructing is to estimate the 20 parameter above, and then you can have the estimated continuation value of each step. We use this as a signal whether we surrender the contracts or not. Here $b_i, i=0,\ldots 19$ is parameter in regression not the benefit claim. Sorry for the misleading symbol.

In fact, there is a bug we need to deal with at $j=N_1-1$, that is $\tau_d^{m,l}$ will be the same for all $m\in I_j$. We just cut b_{19} at $j=N_1-1$ instead. At $j=N_1-2,\ldots,1$, the regression function is ditto. In a much more general form, if $\tau_d^{m,l}$ is the same for all $m \in I_j$, we'll cut b_{19} to run the regression instead in any backward step j.

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