國立交通大學

財務金融研究所

碩 士 論 文

以 copula-based GARCH 模型探討原油價格與匯率

共移性的經濟價值

The economic value of co-movement between oil price and exchange rate using

copula-based GARCH models

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摘要

由於美元為國際原油交易的主要貨幣。近幾年來，美元的大幅貶值導致了原油價格的飆升。本研究採用關連結構 GARCH 模型試圖更有彈性的去探討原油與匯率之間的依賴結構。而實證結果也表示，對稱的關連結構 GARCH 模型具有較好的解釋能力。此外我們使用動態資產配置策略去評估模型的經濟價值及其實際的效率性。在樣本外的預測中，使用 Frank 關連結構 GARCH 模型要比其他靜態及動態模型具有較高的經濟價值。而較為保守的投資者也願意付出較高的費用將靜態的投資策略轉為關連結構GARCH模型的動態策略。

關鍵字：原油，匯率，共移性，關連結構，經濟價值。

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Abstract

The US dollar is used as the major currency of international crude oil trading, and thus the substantial depreciation of US dollar results in the soar of crude oil prices in recent years. In addition, the oil and exchange rate returns have been shown to be skew and leptokurtic and exhibit asymmetric or tail dependence structure. Therefore, this study uses the dynamic copula-based GARCH models to flexibly explore the dependence structure between the oil and US dollar exchange rate, and the empirical results demonstrate that the GARCH model with symmetric copulas has better explanatory ability. Furthermore, an asset allocation strategy is implemented to evaluate economic value and confirm the efficiency of the copula-based GARCH models. In terms of out-of-sample forecasting performance, a dynamic strategy based on the GARCH model with Frank copula exhibits larger economic benefits than static and other dynamic strategies. An investor with a higher risk aversion attitude also generates higher fee for switching from a static strategy to a dynamic strategy based on copula-based GARCH models.

Key words：Oil, Exchange rate, Co-movement, Copula, Economic value

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1. Introduction

The energy commodities differ from other trading products in their uniqueness as well as non-renewable nature. Most countries must rely on energy imports owing to the rareness of oil-producing countries. This also leads to that the prices of energy commodities are easily affected by many factors, such as government policy, politics, season, demand, supply, etc. In particular, the US dollar is commonly used as a major currency in the international energy commodity market, and hence the change in US dollar exchange rate will induce the commodity price fluctuation and then affect the economic actions of energy commodity importing and exporting countries.

Over the last few years, energy commodity prices have been experiencing an unprecedented high fluctuation. For example, the crude oil price has risen steadily from $20 per barrel in January 2002 to the tiptop $145 per barrel in July 2008 while fallen sharply and returned to $40 in December 2008 (see, Figure 1). In the meantime, the US dollar index (USDX[1]) after 2002 exhibits a greatly different tendency to that before 2002 and shows a significantly opposite direction to the crude oil price, that is to say, crude oil prices have been soaring while the US dollar has depreciated to a historical low price meantime and vice versa. This negative relationship will enable the crude oil commodity and the US dollar currency tool to have diversification and hedging benefits. As a result, modeling and forecasting the volatility and dependence structures of oil and exchange rate returns accurately are of considerable interest to investors and financial institutions.

In recent years, there have been a number of methods proposed to explore the relationship between oil price and US dollar exchange rate. For example, Yousefi and Wirjanto (2004) investigated the impact of US dollar exchange rate fluctuation on the

[1] The US Dollar Index (USDX®) is an average of six major world exchange rates: Euro (57.6 %), Japanese Yen (13.6 %), UK Pound (11.9 %), Canadian Dollar (9.1 %), Swedish Krona (4.2%) and Swiss Franc (3.6 %).

formation of OPEC[2] by using Hansen's GMM model and verified that the correlation of oil and US dollar exchange rate is negative. Akram (2004) presented evidence of a non-linear negative relationship between oil prices and the Norwegian exchange rate, and pointed out that the nature of the relationship varies with the level and trend in oil prices. Cifarelli and Paladino (2010) used a multivariate CCC GARCH-M model to discover oil price dynamics are associated with exchange rate behavior and found strong evidence that oil price shifts are negatively related to exchange rate changes.

Furthermore, there are other studies focus on the discussion of lead-lag relationship between oil and exchange rate and their interactive influence. Although those studies different from our studies aim but they also support the negative relationship between oil and exchange rate. For Example, Krichene (2005) used the vector error correction model (VECM) and demonstrated the negative impact that falling nominal effective exchange rate could lead to a surge in oil prices, and inversely either long-term effect or short-term effect. Sari et al. (2009) employed the generalized forecast error variance decompositions and generalized impulse response functions to find evidence of weak long-run equilibrium relationship but strong feedback in the short run. Lizardo (2009) used the vector autoregressive (VAR) model and revealed that oil prices significantly explain movements in the value of the US dollar against major currencies. The currencies of oil importers depreciate relative to the USD when the real oil price goes up.

According to the majority of literatures, they point out the negative relationship between oil price and US dollar exchange rate. A number of possible explanations for this negative relationship between the US dollar and crude oil price are summarized as follows. First, oil-exporting countries want to stabilize the purchasing power of their export revenues (US dollar) in terms of their imports (non-US dollar), so they might adopt currencies pegged to the

[2] The Organization of the Petroleum Exporting Countries is a cartel of twelve countries. The principal goals are safeguarding the cartel's interests and securing a steady income to the producing countries.

US dollar in order to avoid causing loss. Second, the depreciation of US dollar makes oil cheaper for consumers in non-US dollar regions, thus changing their crude oil demand, which eventually causes adjustments in the oil price, denominated in US dollars. Third, a falling US dollar reduces the returns on US dollar denominated financial assets, increasing the attractiveness of oil and other commodities to foreign investors. Commodity assets are also regarded as a hedge against inflation, since the US dollar's depreciation raises the risk of inflationary pressures in the United States. Based on above reasons, we must consider the change of exchange rate and oil price at the same time.

The analysis of financial market movements and co-movements are important for effective diversification in portfolio management. Previous researches commonly use multivariate GARCH models to provide one way to estimate time-varying dependence structure, but it is often based on severe restrictions to guarantee a well-defined covariance matrix. The VAR model and multivariate GARCH models assume that the asset returns follows a multivariate normal or student-t distribution with linear dependence. This assumption is at odds with numerous empirical researches, in which it has been shown that crude oil and exchange rate returns are skewed, leptokurtic and fat-tail. And the dependence relationship between oil and exchange rate is non-linear or asymmetrical. To improve the drawbacks, we use the copula-based GARCH models to capture the volatility and dependence structures of crude oil and exchange rate returns. The copula-based GARCH models allow for better flexibility in joint distributions than bivariate normal or student-t distribution. In addition, three types of marginal models are employed to capture a variety of characteristics of oil and exchange rate returns including of volatility clustering, leverage effect, or the long-run effect. Five types of copula functions are also used to provide a more general dependence structures rather than treat it as simple linear correlation.

Furthermore, model performs better statistically does not equivalently imply that the model performs well in practice, and hence we follow Fleming et al. (2001) to evaluate the out-of-sample covariance forecast performance based on the copula-based GARCH models by the use of a strategic asset allocation problem. We also take the transaction cost problem into consideration and compute the break-even transaction cost, discussed in Han (2006). Based on the break-even cost, an investor would decide to trade or not if the real transaction cost is much higher than the estimated break-even cost.

Our contribution to the literature is twofold. First, we propose the copula-based GARCH models to elastically describe the volatility and dependence structure of oil and US dollar exchange rate returns. The copula-based GARCH model can be used to capture the probable skewness and leptokurtosis in the oil and exchange rate returns as well as the possibly asymmetric and tail dependence between the oil and exchange rate returns. We find that the symmetric copulas seem superior to the asymmetric copulas in the description of dependence structure between the oil and exchange rate returns, The GARCH model with Student-t copula exhibits better explanatory ability of the oil and USDX futures returns. We also observe that the dependence structure between oil and US dollar exchange rate returns is not very significant before 2003 while becomes negative and descends continuously after 2003. Second, rather than statistical criteria, we examine whether the copula-based GARCH models can benefit an investor by implementing an asset allocation strategy. In terms of out-of-sample results, we find that the dynamic strategies based on the copula-based GARCH models outperform the static strategy and other dynamic strategies based on the CCC GARCH and DCC GARCH models, which demonstrates that skewness and leptokurtosis of crude oil and USDX futures returns are economically significant. Furthermore, a more risk-averse investor would be willing to pay higher fees to switch his strategy from the static strategy to the dynamic strategies based on copula-based GARCH models.

The remainder of this paper is organized as follows. In the next section, we introduce the copula-based GARCH models in detail. Section 3 presents the empirical estimation results. Section 4 introduces an economic evaluation methodology and investigates the out-of-sample forecasts of the copula-based GARCH models. Finally, Section 5 concludes.

2. Econometric Model

2.1 Time-varying copula

According to the Sklar's theorem, a joint distribution function can be separated into the marginal distributions and dependence structure. For any bivariate cumulative distribution function, $F(x_1, x_2) = P(X_1 \le x_1, X_2 \le x_2)$, which has continuous marginal cumulative functions, $F_i = P(X_i \le x_i)$ for $1 \le i \le 2$, there exist a unique copula function $C(u, v)$ such as $F(x_1, x_2) = C(F_1(x_1), F_2(x_2))$. Thus, different copula functions can be used to depict a flexible dependence structure between two random variables.

Because previous studies had indicated that the comprehensive economic factors will induce dependence structure to change over time, thus Patton (2006) extended the Sklar's theorem and introduced the conditional copula function to model the time-varying conditional dependence. Let $r_{o,t}$ and $r_{e,t}$ be random variables that denote oil and exchange rate returns at period t, respectively, with marginal conditional cumulative distribution functions $u_{o,t} = G_{o,t}(r_{o,t} \mid \Psi_{t-1})$ and $u_{e,t} = G_{e,t}(r_{e,t} \mid \Psi_{t-1})$, where Ψ_{t-1} denotes the past information. Then, the conditional copula function $C_t(u_{o,t}, u_{e,t} \mid \Psi_{t-1})$ can be written by the two time-varying cumulative distribution functions. Extending Sklar's theorem, the bivariate conditional cumulative distribution functions of random variables $r_{o,t}$ and $r_{e,t}$ can be written as

$$F(r_{o,t}, r_{e,t} \mid \Psi_{t-1}) = C_t\left(u_{o,t}, u_{e,t} \mid \Psi_{t-1}\right) \tag{2.1}$$

Assume the cumulative distribution function is differentiable, and the conditional joint density can be expressed as

$$\begin{aligned} f(r_{o,t}, r_{e,t} \mid \Psi_{t-1}) &= \frac{\partial^2 F(r_{o,t}, r_{e,t} \mid \Psi_{t-1})}{\partial r_{o,t} \partial r_{e,t}} \\ &= c_t\left(u_{o,t}, u_{e,t} \mid \Psi_{t-1}\right) \times g_{o,t}\left(r_{o,t} \mid \Psi_{t-1}\right) \times g_{e,t}\left(r_{e,t} \mid \Psi_{t-1}\right) \end{aligned} \tag{2.2}$$

where $c_t(u_t, v_t \mid \Psi_{t-1}) = \partial^2 C_t(u_t, v_t \mid \Psi_{t-1}) / \partial u_t \partial v_t$ is the conditional copula density function and $g_i(\cdot)$ is the density function corresponding to $G_i(\cdot)$.

2.2 Marginal density

Many financial time series have been shown to have a number of important features, including leptokurtosis, volatility clustering, long memory, volatility smile, leverage effect and so on. That is, assumption of constant variance is not appropriate and in such instances it is preferable to examine patterns that allow the variance to depend upon its history. Therefore, we consider three types of GARCH-type models to describe the time-varying volatility structures of oil and exchange rate returns. Except to the traditional GARCH model, we further use GJR-GARCH and component GARCH models to construct marginal distributions. On the description of volatility structures, one takes asymmetry effect into consideration, and the other distinguishes the difference of duration.

2.2.1 GARCH model

First, we utilize the GARCH model to specify the conditional marginal densities of oil and exchange rate returns, defined by

$$\begin{aligned} & r_{i,t} = \alpha_i + \varepsilon_{i,t}, \varepsilon_{i.t} \mid \Psi_{t-1} = h_{i.t} z_{i.t}, z_{i.t} \sim skewed - t(z_i \mid \eta_i, \lambda_i) \\ & h_{i,t}^2 = c_i + a_i \varepsilon_{i,t-1}^2 + b_i h_{i,t-1}^2 \ , i = o, e \end{aligned} \tag{2.3}$$

The parameters restrictions in variance equation are $c_i > 0$, $a_i, b_i \geq 0$, and $a_i + b_i < 1$. The error term $\varepsilon_{i,t}$ is assumed to be skewed-t distribution which can be used to describe the possibly asymmetric and heavy tail characteristics of oil and exchange rate returns. Following Hansen (1994), the density function is

$$skewed - t(z \mid \eta, \lambda) = \begin{cases} bc\left(1 + \dfrac{1}{\eta - 2}\left(\dfrac{bz + a}{1 - \lambda}\right)^2\right)^{-(\eta+1)/2}, z < -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{\eta - 2}\left(\dfrac{bz + a}{1 + \lambda}\right)^2\right)^{-(\eta+1)/2}, z \geq -\dfrac{a}{b} \end{cases} \tag{2.4}$$

The value of a, b, c are defined as

$$a \equiv 4\lambda c \frac{\eta - 2}{\eta - 1}, \quad b^2 \equiv 1 + 2\lambda^2 - a^2 \quad \text{and} \quad c \equiv \frac{\Gamma(\eta + 1/2)}{\sqrt{\pi(\eta - 2)}\Gamma(\eta / 2)}$$

where λ and η are the asymmetry and kurtosis parameters, respectively. These are restricted to be $-1 < \lambda < 1$ and $2 < \eta < \infty$. When $\lambda = 0$, it will turn to the Student-t distribution. If $\lambda = 0$ and η diverge to infinite, it will be the normal distribution.

2.2.2 GJR-GARCH model

Another style feature of financial time series is the leverage effect whereby there is an asymmetric reaction of volatility changes in response to positive and negative shocks of the same magnitude. To this effect, we employ the GJR-GARCH model, proposed by Glosten et al. (1993), to take into account the asymmetric effect in the volatility structure, which is given by

$$\begin{aligned} r_{i,t} &= \alpha_i + \varepsilon_{i,t}, \varepsilon_{i.t} \mid \Psi_{t-1} = h_{i.t} z_{i.t}, z_{i.t} \sim skewed - t(z_i \mid \eta_i, \lambda_i) \\ h_{i,t}^2 &= c_i + a_i \varepsilon_{i,t-1}^2 + b_i h_{i,t-1}^2 + d_i k_{i.t-1} \varepsilon_{i.t-1}^2 \end{aligned} \tag{2.5}$$

where $k_{i.t-1} = 1$ if $\varepsilon_{i,t-1}$ is negative, otherwise $k_{i,t-1} = 0$, and the parameter d_i regards as an asymmetric impact to volatility from the lagged one residual. If there is leverage effect on oil or exchange rate markets, the parameter d_i will be expected to be positive.

2.2.3 Component GARCH model

The component GARCH (CGARCH) model can be used to decompose the conditional volatility into a long-run trend component and a short-run transitory component. Contrary to the traditional GARCH model, the component GARCH model allows the conditional volatility reverts to the time-varying long-run volatility level rather than the constant long-run volatility level. Engle and Lee (1999) replaced the constant unconditional variance with a time-varying permanent component, which represents the long-run volatility, to ensure that the volatility is not constant in the long-run and proposed the following component GARCH model:

$$\begin{aligned}
&\varepsilon_{i,t} \mid \Psi_{t-1} = h_{i,t} z_{i,t}, z_{i,t} \sim skewed - t(z_i \mid \eta_i, \lambda_i) \\
&h_{i,t}^2 = q_{i,t} + a_i(\varepsilon_{i,t-1}^2 - q_{i,t-1}) + b_i(h_{i,t-1}^2 - q_{i,t-1}) \\
&q_{i,t} = \varpi_i + \phi_i q_{i,t-1} + \zeta_i(\varepsilon_{i,t-1}^2 - h_{i,t-1}^2)
\end{aligned} \tag{2.6}$$

where $\phi_i < 1$ and $a_i + b_i < 1$. The parameter ϕ_i measures the persistence in the permanent component, and the forecast error $\left(\varepsilon_{i,t-1}^2 - h_{i,t-1}^2\right)$ serves as the driving factor for the time-dependent movement of the permanent component. The parameters ζ_i and a_i regard as the short-run shock effect of the permanent component and the transitory component, respectively.

2.3 Copula function

In the past, the correlation is usually employed to describe co-movement. But it is only a simple measure of dependence structure so that cannot express the relationship completely. Generally, studies use multivariate normal distribution to measure the relationship of assets movement. However, empirical evidence has shown that the distributions of financial asset returns are usually skewed and fat-tail differs from normality. Hence, we adopt copula functions which provide a flexible method to construct multivariate distributions given the marginal distributions and the dependence structures separately.

Here we use two families of copula function to describe the dependence structure between oil and exchange rate returns, in order to fit various phenomenons. Two elliptical and three Archimedean's copula functions are employed, so as to capture different dependence structures. The advantage of elliptical copula is that one can specify different levels of correlation between the marginals but they are restricted to have radial symmetry. On the other hand, Archimedean copulas exhibit greater explanation for asymmetric and tail dependence.

Figure 2 demonstrates the different copula contour plots under standard normal and skewed-t marginal distributions. Under the skewed-t marginal distribution, the axis of symmetry becomes a concave curve and the distribution becomes more centralize. These plots indicate that even use the same copula, the difference of marginal still causes enormous dissimilarity. Hence, under false marginal distribution hypothesize will induce fault to estimate copula function.

2.3.1 Elliptical copulas

Gaussian copula

$$C_t^{Gaussian}(u_{o,t},u_{e,t} \mid \rho_t)=\int_{-\infty}^{\Phi^{-1}(u_{o,t})}\int_{-\infty}^{\Phi^{-1}(u_{o,t})}\frac{1}{2\pi\sqrt{1-\rho_t^2}}\exp\left\{-\frac{x^2-2\rho xy+y^2}{2(1-\rho_t^2)}\right\}dxdy \qquad (2.7)$$

Gaussian copula density function

$$c_t^{Gaussian}(u_{o,t},u_{e,t} \mid \rho_t)=\frac{1}{\sqrt{1-\rho_t^2}}\exp\left\{-\frac{\rho_t^2\left(\left(\Phi^{-1}(u_{o,t})\right)^2+\left(\Phi^{-1}(u_{e,t})\right)^2\right)-2\rho_t\Phi^{-1}(u_{o,t})\Phi^{-1}(u_{e,t})}{2(1-\rho_t^2)}\right\} \qquad (2.8)$$

where $\Phi^{-1}(\cdot)$ is the inverse of standard normal cumulative density function.

It is the well knows bivariate normal distribution under the normal distribution marginal. The property of Gaussian copula is symmetric and implies zero dependence in the extreme tails.

Student-t copula

$$C_t^{Student-t}(u_{o,t},u_{e,t} \mid \rho_t)=\int_{-\infty}^{t_\upsilon^{-1}(u_{o,t})}\int_{-\infty}^{t_\upsilon^{-1}(u_{o,t})}\frac{1}{2\pi\sqrt{1-\rho_t^2}}\exp\left\{1+\frac{x^2-2\rho xy+y^2}{\upsilon(1-\rho_t^2)}\right\}dxdy \qquad (2.9)$$

where $t_\upsilon^{-1}(\cdot)$ is the inverse of Student-t cumulative density function, $\upsilon > 2$.

Student-t copula density function

$$c_t^{Student-t}\left(u_{o,t},u_{e,t}\middle|\rho_t\right)=\frac{\Gamma\left(\frac{\upsilon+2}{2}\right)\Gamma\left(\frac{\upsilon}{2}\right)}{\sqrt{1-\rho_t^2}\left(\Gamma\left(\frac{\upsilon+1}{2}\right)\right)^2}\left(1+\frac{\left(t_\upsilon^{-1}\left(u_{o,t}\right)\right)^2}{\upsilon}\right)^{\frac{\upsilon+1}{2}}\left(1+\frac{\left(t_\upsilon^{-1}\left(u_{e,t}\right)\right)^2}{\upsilon}\right)^{\frac{\upsilon+1}{2}}$$
$$\times\left(1+\frac{\left(t_\upsilon^{-1}\left(u_{o,t}\right)\right)^2-2\rho_t t_\upsilon^{-1}\left(u_{o,t}\right)t_\upsilon^{-1}\left(u_{e,t}\right)+\left(t_\upsilon^{-1}\left(u_{e,t}\right)\right)^2}{\upsilon\left(1-\rho_t^2\right)}\right)^{-\frac{\upsilon+1}{2}} \tag{2.10}$$

When the number of degrees of freedom becomes larger, the copula converges to the Gaussian one. Student-t copula also symmetrical, specially, this copula has the nature of tail dependence.

2.3.2 Archimedean copulas

The families of Archimedean copulas have been named by Ling (1965) and were realized by Schweizer and Sklar (1961). Differ from Elliptical copula, Archimedean copulas are characterized by some generator function which have many useful properties. They have upper tail dependence, lower tail dependence or both, so that, they could describe better the reality of the behavior of the financial markets. Here three types of Archimedean copulas are used to combine the marginal distributions into the joint distributions. In general, people use three Archimedean copulas commonly: the Clayton, Frank and Gumbel. Unfortunately, the Gumbel copula is limited to the description of positive dependence structure. Hence, we turn to use the survival Clayton copula which possesses the similar property to the Gumbel copula but does not have positive dependence restriction. First we should define the survival function:

$$\overline{C}_t(u_{o,t},u_{e,t}\mid\alpha_t)=u_{o,t}+u_{e,t}-1+C_t(1-u_{o,t},1-u_{e,t}\mid\alpha_t)$$

The density of survival function is

$$\overline{c}_t(u_{o,t},u_{e,t}\mid\alpha_t)=c_t(1-u_{o,t},1-u_{e,t}\mid\alpha_t)$$

Here use Kendall's τ to measure the co-movements between different markets in the presence of non-linear relationships.

Clayton copula

$$C_t^{Clayton}(u_{o,t},u_{e,t}\mid\alpha_t)=\max\left[\left(u_{o,t}{}^{-\alpha_t}+u_{e,t}{}^{-\alpha_t}-1\right)^{-1/\alpha_t},0\right] \tag{2.11}$$

Clayton copula density function

$$c_t^{Clayton}(u_{o,t},u_{e,t}\mid\alpha_t)=\frac{(1+\alpha_t)\left(u_{o,t}{}^{-\alpha_t}+u_{e,t}{}^{-\alpha_t}-1\right)^{-2-\alpha_t^{-1}}}{\left(u_{o,t}u_{e,t}\right)^{\alpha_t+1}} \tag{2.12}$$

where $\alpha_t=2\tau_t/(1-\tau_t)$, $\alpha_t\in[-1,\infty)\setminus\{0\}$

The Clayton copula was first proposed by Clayton (1978). It is an asymmetric Archimedean copula, exhibiting greater dependence in the negative tail than in the positive. Because of this lower tail dependence property, Clayton copula is commonly used to captures the markets collapse.

Survival Clayton copula

$$C_t^{SClayton}(u_{o,t},u_{e,t}\mid\alpha_t)=u_{o,t}+u_{e,t}-1+\left(\left(1-u_{o,t}\right)^{-\alpha_t}+\left(1-u_{e,t}\right)^{-\alpha_t}-1\right)^{-1/\alpha_t} \tag{2.13}$$

Survival Clayton copula density function

$$c_t^{SClayton}(u_{o,t},u_{e,t}\mid\alpha_t)=\frac{(1+\alpha_t)\left(\left(1-u_{o,t}\right)^{-\alpha_t}+\left(1-u_{e,t}\right)^{-\alpha_t}-1\right)^{-2-\alpha_t^{-1}}}{\left(\left(1-u_{o,t}\right)\left(1-u_{e,t}\right)\right)^{\alpha_t+1}} \tag{2.14}$$

The Survival Clayton copula is an asymmetric Archimedean copula, exhibiting greater dependence in the positive tail than in the negative one.

Frank copula

$$C_t^{Frank}(u_{o,t},u_{e,t}\mid\alpha_t)=-\frac{1}{\alpha_t}\ln\left(1+\frac{(e^{-\alpha_t u_{o,t}}-1)(e^{-\alpha_t u_{e,t}}-1)}{e^{-\alpha_t}-1}\right) \tag{2.15}$$

Frank copula density function

$$c_t^{Frank}(u_{o,t},u_{e,t}\mid\alpha_t)=\frac{\alpha_t\left(1-e^{-\alpha_t}\right)e^{-\alpha_t\left(u_{o,t}+u_{e,t}\right)}}{\left[1-e^{\alpha_t}-\left(1-e^{-\alpha_t u_{o,t}}\right)\left(1-e^{-\alpha_t u_{e,t}}\right)\right]^2} \tag{2.16}$$

where $\alpha_t \in (-\infty,\infty)\backslash\{0\}$. The association parameter $\tau_t = 1 + 4[D(\alpha_t) - 1]/\alpha_t$, so that $\tau_t \in (-1,1)$. $D(\cdot)$ is called Debye functions, defined as

$$D(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} \frac{t}{\exp(t) - 1} dt$$

The Frank copula firstly appeared in Frank (1979). It is a symmetric Archimedean copula. There is neither lower nor upper tail dependence in Frank copula. Form Figure 2 we can see the distribution in the similar way with the Gaussian copula.

2.3.3 Dynamic dependence structure

In the description of dependence structure, Person's correlation coefficient is commonly used in Gaussian copula and Student-t copula. On the other hand, we use the Kendall's tau in Archimedean copulas. In addition, we follow the concept of Patton (2006) and Bartram et al. (2007), to assume that the dependence parameters rely on the past dependence and historical information, $(u_{o,t-1} - 0.5)(u_{e,t-1} - 0.5)$. If both $u_{o,t-1}$ and $u_{e,t-1}$ are either bigger or smaller than 0.5, then we infer that the dependence is higher than last. Let ρ_t^* and τ_t^* be appropriate logistic transformation[3] of dependence parameters ρ_t and τ_t, respectively, and the time-varying parameters ρ_t^* and τ_t^* can be expressed as:

$$\begin{aligned} \rho_t^* &= \alpha_c + \beta_c \rho_{t-1}^* + \gamma_c \left(u_{o,t-1} - 0.5\right)\left(u_{e,t-1} - 0.5\right) \\ \tau_t^* &= \alpha_c + \beta_c \tau_{t-1}^* + \gamma_c \left(u_{o,t-1} - 0.5\right)\left(u_{e,t-1} - 0.5\right) \end{aligned} \tag{2.17}$$

where $0 \le \beta_c < 1$.

2.4 Parameter estimation method

When the maximum likelihood method is implemented over a high dimension case will confront the huge computation and the accuracy of parameters estimation, the optimization

[3] The appropriate logistic transformation is used to ensure the dependence parameters to be in the interval (-1,1), which can be written as $\rho_t^* = \left(1 - e^{\rho_t}\right)/\left(1 + e^{\rho_t}\right)$, $\tau_t^* = \left(1 - e^{\tau_t}\right)/\left(1 + e^{\tau_t}\right)$

problem becomes more difficult. Consequently, we use the two-stage estimation method, or called inference functions for margins (IFM), to estimate parameters of our copula-based GARCH models. In addition, Joe (1997) showed that this estimator is close to the maximum likelihood estimator and asymptotic efficiency to it. Hence, the two-stage estimation method will compute the estimator efficiently without losing the realist information.

The log-likelihood function of the observation t can be derived by taking the logarithm of (2.2):

$$\log f_t = \log g_{o,t} + \log g_{e,t} + \log c_t \tag{2.18}$$

Let Θ_o and Θ_e be the parameters of marginal distributions of oil and exchange rate returns, respectively, and Θ_c be the parameters be the parameters in the copula function, c_t. The likelihood function of $\Theta = (\Theta_o, \Theta_e, \Theta_c)$ can be expressed as:

$$L_{o,e}(\Theta) = L_o(\Theta_o) + L_e(\Theta_e) + L_c(\Theta_c) \tag{2.19}$$

In the first stage, we estimate the parameters of marginal distributions by the use of maximum likelihood method, respectively,

$$\begin{aligned} \hat{\Theta}_o &\equiv \arg\max \sum_{t=1}^{T} \log g_{o,t}(r_{o,t} \mid \Psi_{t-1}; \Theta_o) \\ \hat{\Theta}_e &\equiv \arg\max \sum_{t=1}^{T} \log g_{e,t}(r_{e,t} \mid \Psi_{t-1}; \Theta_e) \end{aligned} \tag{2.20}$$

In the second stage, given the marginal estimates obtained above, the dependence parameters are estimated by

$$\hat{\Theta}_c \equiv \arg\max \sum_{t=1}^{T} \log c_t(u_{o,t}, u_{e,t}, \hat{\Theta}_o, \hat{\Theta}_e; \Theta_c) \tag{2.21}$$

where $u_{o,t} = G_{o,t}(r_{o,t} \mid \Psi_{t-1}, \hat{\Theta}_o)$ and $u_{e,t} = G_{e,t}(r_{e,t} \mid \Psi_{t-1}, \hat{\Theta}_e)$.

3. Data and Empirical Results

3.1 Data and descriptive statistics

This study uses West Texas Intermediate (WTI) crude oil futures and US dollar index (USDX) futures data to stand for oil and exchange rate markets. WTI crude oil, also known as light sweet oil, is the futures contract traded on New York Mercantile Exchange (NYMEX). The USDX represents the trade-weighted value of the US dollar in terms of a basket of six major foreign currencies. There exist a futures contract and an option contract traded on the New York Board of Trade (NYBOT). Both WTI crude oil and USDX futures prices data[4] with the nearest to maturity for the period from January 1, 1990 to December 31, 2009 are obtained from DATASTREAM and 5,008 daily return observations are generated for each asset. In addition, we use the three-month Treasury bill as the risk-free rate, which are obtained from the Federal Reserve Board. The daily close prices, daily excess returns, and annual trading volumes of WTI crude oil and USDX futures over the sample period are graphed in Figure 1.

The descriptive statistics of crude oil and exchange rate excess returns are reported in Table 1. Both crude oil and exchange rate returns exhibit the left-skew and leptokurtic phenomenon. So that, we use the Jarque-Bera statistic to test the normality of distribution then we get the conclusion of both oil and exchange rate all reject the null hypothesis of normality. Consequently, we adapt the Hansen's skewed-t distribution to dovetail with the feature in our study.

3.2 Estimation results

3.2.1 Marginal distribution

[4] The futures price data are continuous series, as defined by DATASTREAM.

Table 2 presents the estimate results of three classes of copula-based GARCH models. Panel A reports the parameter estimates of marginal distributions with the GARCH, GJR-GARCH and component GARCH models. We use the Akaike information criteria (AIC) and Bayesian information criteria (BIC) to examine which model has better goodness of fit. The two methods resolve this problem by adding a corrected term to avoid the overfitting problems caused by different number of parameters in each models. According to these two information criteria, we can find the GARCH model has smaller value than the GJR-GARCH and component GARCH models based on each copula function, which imply that the GARCH model exhibits the best goodness of fit.

As can be seen, the asymmetry parameters, λ_i, are significant and negative for crude oil returns while insignificant for USDX returns, exhibiting that crude oil returns are skewed to left. In addition, in the GARCH model, the parameters a_i and b_i are significant to explain the crude oil and exchange rate returns have volatility clustering. And the sum of a_i and b_i are very close to 1 implies that there is high volatility persistence in both crude oil and exchange rate markets. Further, the asymmetric parameters d_i in the GJR-GARCH model are insignificant and exhibit no asymmetric effect in the volatility structures of crude oil and exchange rate markets, which is consistent to Manera et al. (2004) and Wang and Yang (2006). The result may be evidence that the asymmetric reaction to equities markets do not bring into the crude oil and USDX futures market. Next, we adapt component GARCH model to distinguish the return volatilities into permanent and transitory components. The result demonstrates the parameters ϕ_i of both crude oil and exchange rate markets are very close to 1 and shows there is high persistence in the permanent component. The result also reveals that it significantly diminishes the value of estimate $a_i + b_i$ of crude oil from the GARCH model to the component GARCH model. Such that $a_i + b_i$ much less than ϕ_i, which signifies the transitory component persistence will decline faster than the permanent component. The parameters ζ_i and a_i in the component GARCH model are regarded as the reaction of shocks to the permanent and transitory components, respectively. From Table 2, we can also see the impact on the permanent component is significantly greater than that on the transitory component. And the parameter a_i turns to insignificant which explain sudden information will not cause volatility impulsion.

3.2.2 Copula function

The parameter estimates for different copula functions are reported in Panels B-F of Table 2. In terms of AIC and BIC, the Student-t dependence structure exhibits better explanatory ability than other dependence structures no matter what marginal models are employed, while Clayton and survival Clayton copulas have worse explanatory ability. The results imply the tail dependence between oil and exchange rate returns may be significant while not asymmetric. In addition, the GARCH model with Student-t copula perform superior to any other selected model. Moreover, we can see the autoregressive parameter β_c is closely to 1 in each copula function, indicating the dependence structure between oil and exchange rate returns is high persistent. And the latent parameter γ_c is also significant in every copula function which displays that latest return information is a meaningful measure. Specially, γ_c in Clayton copula is much larger than others and mean it had more short-run response than others copula functions.

3.2.3 Volatility estimates plot

Figure 3 plots the volatility estimates of crude oil and USDX returns based on GARCH, GJR-GARCH and CGARH models. The crude oil had undergone two periods of larger unrest in our sample period. First period began in August 1990 which commonly known as “The 3rd energy crisis” due to the Persian Gulf War. Because the oil demand of most countries must rely on imports, the wars of oil-producing countries cause supply diminish so that price soaring. Another period began in September 2008, which stemmed from the American subprime mortgage crisis and then the OPEC claim of oil output reduction. By comparison, the volatility of USDX is much stable. The gravest period is from 2008 to 2009. That reason might come from the purposely control by US government in order to rescue the American economic decline after the financial tsunami. In addition, the volatility estimates based on three different marginal models are similar consistent with the results of goodness of fit. We also can find the circumstance that the crude oil and USDX volatilities usually rise at the same time, implying there exist some connections between crude oil and USDX.

3.2.4 Correlation estimates plot

The correlation parameter estimates between oil and exchange rate returns over the sample period generated from different copula models are plotted in Figure 4[5]. During the period 1990 to 2003, the dependence structure between crude oil and USDX returns keeps a lower level or zero correlation. But since 2003, the correlation started descending continuously to this day due to the crude oil prices have steadily increased caused the international oil price reaches a historical break-through. On the other hand, because the US government wanted to pull the export effectively and reducing the international trade deficit, causing US dollar tremendously decreased in value relative to most other countries' currencies. Moreover, the depreciation of US dollar against other currencies has helped to drive up the oil price over the past few years. The most major reason is that the US dollar is the main invoicing currency of crude oil futures trading. Thus, the falling of US dollar motivated speculators to buy an abundance of crude oil futures contracts to get greater profits, and then promote raise oil price uncommonly.

[5] We transform all dependence structures into the correlation by numerical integral, in order to more clear compare the estimate results.

In addition, in Figure 4, the two paths from Gaussian and Student-t copulas are very close consistent with the results in Panel C of Table 2, which present the degree of freedom of Student-t copula is considerable. The Clayton and survival Clayton copulas exhibit similar dependence trend for each other while display low level dependence relative to the symmetric copulas. Moreover, the main difference in correlation estimates between Clayton and survival Clayton copulas is that the Clayton copula exhibits larger ripples. Finally, the correlation trend based on the Frank copula falls in between and close to Gaussian and Student-t copulas.

4. An Economic Evaluation Methodology

In previous section, we have checked the explanatory ability of each selected model. However, the estimation results perform well does not equivalently imply economically useful consequences. Thus, in this section, we follow Fleming et al. (2001) to evaluate the economic value of volatility timing by a dynamic asset allocation strategy. We use crude oil futures, USDX futures and three-month Treasury bill to construct a portfolio. First, the optimal portfolio weights of selected assets are constructed under the mean-variance framework. Second, the quadratic utility function is employed to assess the performance of dynamic strategies based on different models and to quantify how personal opinion affects the performance. Finally, this framework establishes a concise approach to assess the significance and robustness of results.

4.1 Evaluation model

First we consider an investor who wants to minimize portfolio variance subject to achieving a particular expected return. Let r_t be $N \times 1$ vector of returns on the risky assets, the investor solves the following optimization at each period t,

$$\begin{aligned} &\min_{\mathbf{w_t}} \mathbf{w_t}' \Sigma_t \mathbf{w_t} \\ &\text{s.t. } \mathbf{w_t}'\mu + (1 - \mathbf{w_t}'\mathbf{1}) r_f = \mu_p \end{aligned} \tag{4.1}$$

where $\mathbf{w_t}$ is an $N \times 1$ vector of portfolio weights on risky assets, μ_t and Σ_t are the vector of conditional expected returns and conditional covariance matrix of risk assets, respectively, r_f is return on the riskless asset, μ_p is the target conditional expected return of portfolio. The solution of the optimization problem is

$$\mathbf{w_t} = \frac{\left(\mu_p - r_f\right) \Sigma_t^{-1} \left(\mu - r_f \mathbf{1}\right)}{\left(\mu - r_f \mathbf{1}\right)' \Sigma_t^{-1} \left(\mu - r_f \mathbf{1}\right)} \tag{4.2}$$

which is the optimal weights on risky assets. The weight on the riskless asset is $1-\mathbf{w_t}'\mathbf{1}$. In order to focus on the evaluation of volatility and dependence structures based on different models, we assume the conditional expected returns of selected risky assets at time t equal their unconditional means, i.e., $\mu_t = \mu = E\left[r_{t+1}\right]$, and the optimal time-varying weights only rely on the one-step-ahead covariance matrix forecasts of selected risky assets, $\Sigma_t = E_t\left[(r_{t+1}-\mu)(r_{t+1}-\mu)'\right]$

In order to measure the value of our models, we compare the performance of the dynamic strategies based on copula-based GARCH models to that of the static strategy based on sample covariance matrix. By the Taylor series, we can obtain the quadratic utility as a second-order approximation to the investor true utility function. Under this specification, the investor's realized utility in period $t+1$ can be written as

$$U\left(W_{t+1}\right) = W_t r_{p,t+1} - \frac{aW_t^2}{2} r_{p,t+1}^2 \tag{4.3}$$

where W_{t+1} is the investor's wealth at $t+1$, a is his absolute risk aversion (ARA), $r_{p,t+1} = r_f + \mathbf{w}_t^{'} r_{t+1}$ is the portfolio return at period $t+1$. Under the assumption of constant relative risk aversion, the average realized utility can be used to estimate the expected utility generated by a given level of initial wealth W_0, which is as follows

$$\bar{U}(\cdot) = W_0\left(\sum_{t=0}^{T-1} r_{p,t+1}^d - \frac{\gamma}{2(1+\gamma)}\left(r_{p,t+1}^d\right)^2\right) \tag{4.4}$$

For the purposes of comparison between the static strategy and dynamic strategies based on selected models, we estimate the switching fees by equating the two average utility equations as follows:

$$\sum_{t=0}^{T-1}\left(r_{p,t+1}^d - \Delta\right) - \frac{\gamma}{2(1+\gamma)}\left(r_{p,t+1}^d - \Delta\right)^2 = \sum_{t=0}^{T-1} r_{p,t+1}^s - \frac{\gamma}{2(1+\gamma)}\left(r_{p,t+1}^s\right)^2 \tag{4.5}$$

where $r_{p,t+1}^s$ and $r_{p,t+1}^d$ denote the portfolio returns based on the static and dynamic strategies, respectively, and Δ is explained as the maximum fee that an investor would be willing to pay to switch from the static strategy to the dynamic strategy.

On the other hand, transaction cost is important consideration for any dynamic strategy. It is an important fact to impact our profitability of trading strategies. But making an accurate determination of the size of transaction costs is difficult because it involves many factors. According to Han (2006), be assumed that transaction costs equal a fixed proportion tc of the value traded in each asset.

$$\text{cost} = tc\left| w_t - w_{t-1}\frac{1+r_t}{1+r_{d,t}} \right| \tag{4.6}$$

In the lack of reliable estimates of suitable transaction costs, we consider the break-even transaction cost, which is the maximum transaction cost. In comparing the dynamic strategy with the static strategy, an investor will prefer the dynamic strategy when the break-even transaction cost is high enough. Furthermore, the fact that the break-even transaction cost is much higher will make it easier to implement the dynamic strategy.

4.2 Out-of-sample evaluation result

In this section, we consider that a constant relative risk aversion investor can allocate wealth between the risk-free asset, crude oil futures and USDX futures based on different models. We involve rolling the five years sample data to compute the one-period-ahead forecasted in order to determine the series of optimal portfolio weights. The out-of-sample period for the dates covers five years ranging from January 2005 to December 2009. Then we measure the economic value of the short-term covariance forecasts between crude oil and exchange rate futures returns by a strategic asset allocation problem. We compare the out-of-sample performance of the dynamic strategies based on selected models with the static strategy based on a constant covariance matrix. In this part, our research focuses on the performance fees Δ, which an investor is willing to pay for switching from the static strategy to the dynamic strategy. The fees display the economic value of each selected models relative to the static strategy with target return 5%, 10% and 15%. We present the fees with the relative risk aversion level of $\gamma = 1$ and $\gamma = 5$.

Table 3 presents the out-of-sample performance fees and break-even transaction costs for the dynamic strategies based on selected models versus the static strategy under different target returns and risk aversion level with the minimum variance strategy. The most of dynamic strategy models have positive performance fee which demonstrate that the dynamic strategy is superior to the static strategy. For instance, when using the copula-based GARCH models, the investor is willing to pay form 50 to 407 annualized basis points (bps) for using that dynamic strategy instead of the static strategy. Next we compare the different dynamic models to verify their merits. We can find that $GARCH^{Gaussian}$ is better than DCC everywhere. The discrepancy of the two models is produced by its residual distributions. Because crude oil and exchange rate returns are not normality, the skewed-t distribution has better ability to describe the characterization and then leads to higher economic value.

Furthermore, comparing with three different marginal distributions, we find the GARCH model performs better than the others based on each copula function. This phenomenon is also concordant to the previous estimate result. We conclude the GARCH model is the best volatility model to explain the variation of crude oil and exchange rate. For example, using the copula-based GARCH dynamic strategy instead of the static strategy, the performance fee is between 40 and 104 basis points. Among them, $GARCH^{Frank}$ has excellent achievement. In fact, Frank copula has better achievement on economic value among all selected copula functions no matter what marginal distributions. Finally, the survival Clayton has the poorest performance even worse than static strategy on some place.

The impact of transaction costs is an important consideration in constructing the profitability of trading strategies. Here we compute the break-even transaction costs tc^{be} as the minimum proportional cost. Because if the transaction costs are sufficiently high, the period-by-period changes in the dynamic weights of an optimal strategy will cause the strategy too costly to implement relative to the static model. Comparing the dynamic strategy with the static strategy, an investor prefers the dynamic strategy when he pays a transaction costs lower than break-even transaction costs. The break-even transaction costs values are expressed in basis points per trade and are reported only when performance fee Δ is positive. Besides, we assume the transaction costs of crude oil and USDX futures are at the same level.

Under different relative risk aversion levels, the high level commonly accompanies a high break-even transaction costs. Results demonstrates that the tc^{be} value of $GARCH^{Frank}$ are positive and high; they tend to be higher almost 50 bps and can be as high as 59 bps. In a word, as the tc^{be} values are generally positive and reasonably high, we conclude that the performance fees we have reported is robust to reasonably high transaction costs for the dynamic strategy. After examining the forecast performance of all models by performance fee and break-even transaction cost, we can find that the GRACH marginal has excellent accomplishment in all respects. Among them, Frank copula is the most prominent.

5. Conclusions

In recent years, both oil commodity and US dollar currency have been experiencing an unprecedented high fluctuation while exhibit the significantly opposite trends. This negative relationship will enable the oil commodity and the US dollar currency to be useful tools for strategic asset allocation and risk management. For these reasons, the forecast of the volatility and co-movement structures of oil and exchange rate returns have attracted much attention among academics and institutional investors.

However, it has been demonstrated that oil and exchange rate returns are skew and leptokurtic and perhaps follow extremely dissimilar marginal distributions as well as different degrees of freedom parameters. The relationship structure between the oil and exchange rate returns may also exhibit asymmetric or tail dependence structure. Therefore, in order to improve the drawbacks of conventional multivariate GARCH model, this paper proposes three classes of copula-based GARCH models to elastically describe the volatility and dependence structure of oil and US dollar exchange rate returns. We find that the GARCH model with Student-t copula possesses better explanatory ability of crude oil and USDX futures returns, suggesting that there is symmetric tail dependence structure between crude oil and USDX futures returns. In addition, the leverage effects are demonstrated to be insignificant for both crude oil and USDX futures. Based on the marginal distribution with the component GARCH model, we can find that the persistence of short-run volatility is apparently smaller than that if long-run volatility for the crude oil futures, while it is not significant for the USDX futures. We also observe that the dependence structure between crude oil and US dollar exchange rate returns becomes negative and descends continuously after 2003 unlike the pattern before.

In addition, in order to examine whether the copula-based GARCH models can benefit an investor, we evaluate the economic value of our models by implementing a strategic asset allocation problem. In terms of out-of-sample results, we find that the dynamic strategies based on the copula-based GARCH models outperform the static strategy and other dynamic strategies based on the CCC GARCH and DCC GARCH models, which demonstrates that skewness and leptokurtosis of crude oil and USDX futures returns are economically significant. Furthermore, the GARCH model with Frank copula yields highest performance fees and break-even transaction costs to attract investors to switch their trading strategy and performs the most prominent among all selected models. More risk-averse investors are also willing to pay higher fees to switch their strategies from the static strategy to the dynamic strategies based on copula-based GARCH models.

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Table 1. Summary Statistics for Crude oil and USDX rate Excess Returns

	Crude oil Futures	USDX Futures
Mean(%)	0.0108	-0.0186
SD(%)	2.5404	0.5655
Skewness	-0.8964	-0.028
Kurtosis	19.6644	4.6938
Max(%)	16.4097	2.8167
Min(%)	-40.072	-2.7401
JB	58617.8208***	599.3234***

Note: This table reports the descriptive statistics for daily crude oil and USDX futures excess returns for the sample period from January 2, 1990 to December 31, 2009. JB is the Jarque-Bera statistic, which is used to test for normality. The symbols *, **, and *** represent statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 2. Estimation Results of Copula-Based GARCH Models

	GARCH		GJR-GARCH		Component GARCH	
	Crude oil	USDX	Crude oil	USDX	Crude oil	USDX
Panel A: Estimation of Marginals						
α_i	0.03033	-0.00753	0.02846	-0.00793	0.01660	-0.02430***
	(0.02761)	(0.00722)	(0.02718)	(0.00725)	(0.02696)	(0.00718)
c_i	0.04376***	0.00106**	0.04405***	0.00100**		
	(0.01186)	(0.00047)	(0.01189)	(0.00047)		
a_i	0.05639***	0.02967***	0.05383***	0.02763***	0.01257	0.00019
	(0.00650)	(0.00399)	(0.00819)	(0.00506)	(0.01486)	(0.00065)
b_i	0.93700***	0.96772***	0.93679***	0.96778***	0.70329***	0.99295***
	(0.00673)	(0.00435)	(0.00671)	(0.00432)	(0.17045)	(0.01921)
η_i	6.74588***	7.36002***	6.75076***	7.35605***	6.80645***	7.39712***
	(0.11656)	(0.25135)	(0.12199)	(0.24559)	(0.18823)	(0.53335)
λ_i	-0.04734**	-0.02554	-0.04806**	-0.02584	-0.04662**	-0.01543
	(0.01931)	(0.01893)	(0.01946)	(0.01902)	(0.01942)	(0.01872)
d_i			0.00533	0.00419		
			(0.01098)	(0.00682)		
ϖ_i					0.04244***	0.00118**
					(0.01144)	(0.00053)
ϕ_i					0.99349***	0.99679***
					(0.00298)	(0.00206)
ζ_i					0.05465***	0.03183***
					(0.00662)	(0.00413)
Panel B: Estimation of Gaussian Dependence Structure						
α_c	0.00004		0.00005		0.00005	
	(0.00005)		(0.00005)		(0.00005)	
β_c	0.99901***		0.99900***		0.99902***	
	(0.00074)		(0.00075)		(0.00074)	
γ_c	0.05573***		0.05649***		0.05563***	
	(0.01509)		(0.01539)		(0.01510)	
AIC	29622.112		29626.776		29629.609	
BIC	29719.894		29737.595		29753.466	
Panel C: Estimation of Student-t Dependence Structure						
α_c	0.00004		0.00004		0.00004	
	(0.00005)		(0.00005)		(0.00005)	
β_c	0.99908***		0.99908***		0.99909***	
	(0.00078)		(0.00079)		(0.00078)	
γ_c	0.05468***		0.05526***		0.05463***	
	(0.01604)		(0.01623)		(0.01606)	
υ	14.75600***		15.10001***		14.78200***	
	(0.35900)		(0.19900)		(0.30349)	
AIC	29598.641		29603.089		29605.945	
BIC	29702.941		29720.427		29736.321	

Table 2. (Continued)

Panel D: Estimation of Clayton Dependence Structure			
α_c	-0.00463**	-0.00443**	-0.00446**
	(0.00199)	(0.00219)	(0.00180)
β_c	0.94601***	0.94728***	0.94741***
	(0.01897)	(0.02169)	(0.01670)
γ_c	0.22416***	0.22639***	0.22268***
	(0.06326)	(0.07430)	(0.05571)
AIC	29715.952	29719.963	29723.991
BIC	29813.734	29830.782	29847.848
Panel E: Estimation of Survival Clayton Dependence Structure			
α_c	-0.00051**	-0.00052**	-0.00050**
	(0.00025)	(0.00026)	(0.00024)
β_c	0.99367***	0.99349***	0.99375***
	(0.00229)	(0.00240)	(0.00226)
γ_c	0.03840***	0.03720***	0.03678***
	(0.00916)	(0.00900)	(0.00872)
AIC	29715.007	29721.328	29722.405
BIC	29812.789	29832.147	29846.262
Panel F: Estimation of Frank Dependence Structure			
α_c	0.000004	0.000006	0.000005
	(0.00003)	(0.00005)	(0.00004)
β_c	0.99814***	0.99817***	0.99817***
	(0.00008)	(0.00007)	(0.00010)
γ_c	0.04656***	0.04685***	0.04639***
	(0.00021)	(0.00018)	(0.00022)
AIC	29627.313	29630.515	29633.982
BIC	29725.094	29741.334	29757.839

Note: The table presents the maximum likelihood estimates of three classes of copula-based GARCH models, which are based on the daily crude oil and USDX futures excess returns for the sample period from January 2, 1990 to December 31, 2009. Three types of marginal distributions (GARCH, GJR-GARCH and component GARCH models) are used, and they are expressed as follows:

(A) GARCH model:

$$r_{i,t}=\alpha_i+\varepsilon_{i,t},\quad \varepsilon_{i,t}\left|\Psi_{t-1}=h_{i,t}z_{i,t},\quad z_{i,t}\sim skewed-t(z_i\left|\eta_i,\lambda_i\right.),\quad h_{i,t}^2=c_i+a_i\varepsilon_{i,t-1}^2+b_ih_{i,t-1}^2,\quad ,i=o,e.\right.$$

(B) GJR-GARCH model:

$$h_{i,t}^2=c_i+b_ih_{i,t-1}^2+a_i\varepsilon_{i,t-1}^2+d_ik_{i,t-1}\varepsilon_{i,t-1}^2,\quad k_{i,t-1}=1\ \ if\ \ \varepsilon_{i,t-1}<0$$

(C) Component GARCH model:

$$h_{i,t}^2=q_{i,t}+a_i\left(\varepsilon_{i,t-1}^2-h_{i,t-1}^2\right)+b_i\left(h_{i,t-1}^2-q_{i,t-1}\right),\quad q_{i,t}=\varpi_i+\phi_iq_{i,t-1}+\zeta_i\left(\varepsilon_{t-1}^2-h_{i,t-1}^2\right).$$

In addition, five types of copula functions are utilized to describe the dependence structure, and their densities are expressed as follows:

(A) Gaussian Copula:

$$c_t^{Gaussian}\left(u_{o,t},u_{e,t}\left|\rho_t\right.\right)=\frac{1}{\sqrt{1-\rho_t^2}}\exp\left\{-\frac{\rho_t^2\left(\left(\Phi^{-1}\left(u_{o,t}\right)\right)^2+\left(\Phi^{-1}\left(u_{e,t}\right)\right)^2\right)-2\rho_t^2\Phi^{-1}\left(u_{o,t}\right)\Phi^{-1}\left(u_{e,t}\right)}{2\left(1-\rho_t^2\right)}\right\}.$$

(B) Student-t Copula :

$$c_t^{Student-t}\left(u_{o,t},u_{e,t}\left|\rho_t\right.\right)=\frac{\Gamma\left(\frac{\upsilon+2}{2}\right)\Gamma\left(\frac{\upsilon}{2}\right)}{\sqrt{1-\rho_t^2}\left(\Gamma\left(\frac{\upsilon+1}{2}\right)\right)^2}\left(1+\frac{\left(t_\upsilon^{-1}\left(u_{o,t}\right)\right)^2}{\upsilon}\right)^{\frac{\upsilon+1}{2}}\left(1+\frac{\left(t_\upsilon^{-1}\left(u_{e,t}\right)\right)^2}{\upsilon}\right)^{\frac{\upsilon+1}{2}}$$

$$\times\left(1+\frac{\left(t_{\upsilon}^{-1}\left(u_{o,t}\right)\right)^{2}-2\rho_{t}t_{\upsilon}^{-1}\left(u_{o,t}\right)t_{\upsilon}^{-1}\left(u_{e,t}\right)+\left(t_{\upsilon}^{-1}\left(u_{e,t}\right)\right)^{2}}{\upsilon\left(1-\rho_{t}^{2}\right)}\right)^{-\frac{\upsilon+1}{2}}.$$

(C) Clayton Copula : $c_{t}^{Clayton}(u_{o,t},u_{e,t}\mid\alpha_{t})=\dfrac{\left(1+\alpha_{t}\right)\left(u_{o,t}{}^{-\alpha_{t}}+u_{e,t}{}^{-\alpha_{t}}-1\right)^{-2-\alpha_{t}^{-1}}}{\left(u_{o,t}u_{e,t}\right)^{\alpha_{t}+1}}$.

(D) Survival Clayton Copula: $c_{t}^{SClayton}\left(u_{o,t},u_{e,t}\mid\alpha_{t}\right)=c_{t}^{Clayton}\left(1-u_{o,t},1-u_{e,t}\mid\alpha_{t}\right)\cdot$

(E) Frank Copula: $c_{t}^{Frank}(u_{o,t},u_{e,t}\mid\alpha_{t})=\alpha_{t}\left(1-e^{-\alpha_{t}}\right)e^{-\alpha_{t}\left(u_{o,t}+u_{e,t}\right)}\Big/\left[1-e^{\alpha_{t}}-\left(1-e^{-\alpha_{t}u_{o,t}}\right)\left(1-e^{-\alpha_{t}u_{e,t}}\right)\right]^{2}\cdot$

The proper logistic transformation of dependence parameters, ρ_t and τ_t, obey the following process $\rho_{t}^{*}=\alpha_{c}+\beta_{c}\rho_{t-1}^{*}+\gamma_{c}\left(u_{o,t-1}-0.5\right)\left(u_{e,t-1}-0.5\right)$ and $\tau_{t}^{*}=\alpha_{c}+\beta_{c}\tau_{t-1}^{*}+\gamma_{c}\left(u_{o,t-1}-0.5\right)\left(u_{e,t-1}-0.5\right)$, respectively, where $\rho_{t}^{*}=\left(1-e^{\rho_{t}}\right)/\left(1+e^{\rho_{t}}\right)$ and $\tau_{t}^{*}=\left(1-e^{\tau_{t}}\right)/\left(1+e^{\tau_{t}}\right)$

The Akaike information criteria (AIC) and Bayesian information criteria (BIC) are used to evaluate the goodness of fit of the selected models. The numbers in parentheses are standard deviations. The superscripts *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 3. Out-of-sample Economic Value for Dynamic Strategy Based on selected Models versus Static Strategy with the Minimum Variance Strategy

	CCC				DCC			
μ_p^*	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}
5%	37	8	41	9	51	10	46	9
10%	82	8	103	10	79	7	59	6
15%	146	8	195	12	95	5	52	3

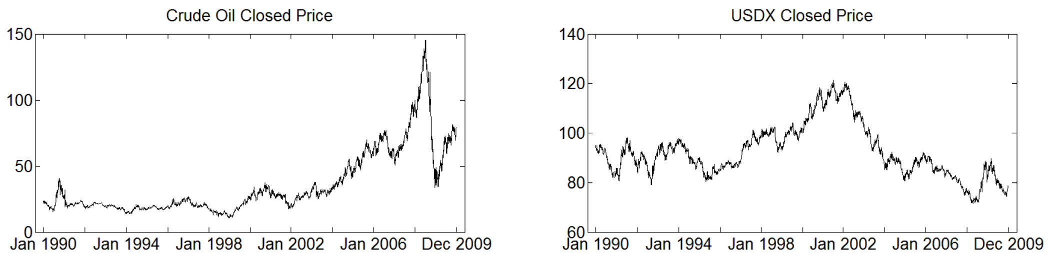
	$GARCH^{Gaussian}$				$GARCH^{Student-t}$				$GARCH^{Clayton}$				$GARCH^{SClayton}$				$GARCH^{Frank}$			
μ_p^*	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}
5%	77	40	80	43	75	39	78	42	75	40	85	46	50	27	49	27	85	44	92	49
10%	161	38	176	43	157	37	173	42	175	41	217	53	90	22	87	22	190	44	222	54
15%	260	37	299	45	257	37	298	45	311	44	407	60	130	19	124	19	324	46	403	59

	$GJR-GARCH^{Gaussian}$				$GJR-GARCH^{Student-t}$				$GJR-GARCH^{Clayton}$				$GJR-GARCH^{SClayton}$				$GJR-GARCH^{Frank}$			
μ_p^*	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}
5%	53	25	55	27	50	24	53	26	53	26	62	30	22	11	20	10	63	30	69	33
10%	100	21	110	24	98	21	112	25	118	25	156	34	24	5	18	4	131	28	154	34
15%	160	21	289	25	162	21	199	27	217	28	305	40	24	3	11	2	221	28	279	37

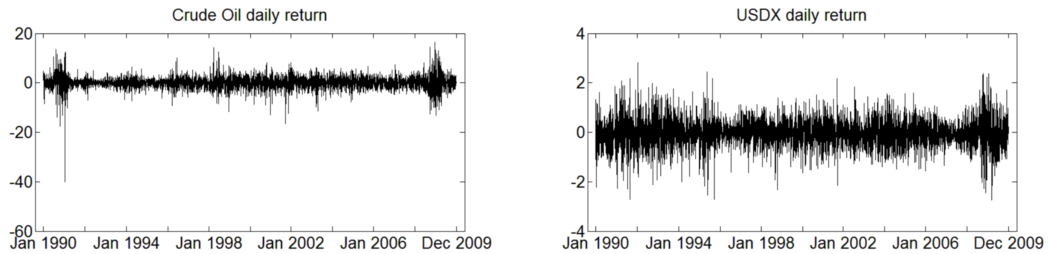
	$CGARCH^{Gaussian}$				$CGARCH^{Student-t}$				$CGARCH^{Clayton}$				$CGARCH^{SClayton}$				$CGARCH^{Frank}$			
μ_p^*	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}	Δ_1	tc_1^{be}	Δ_5	tc_5^{be}
5%	50	25	45	24	46	24	43	22	49	26	53	28	27	14	22	12	54	28	56	29
10%	78	18	59	14	74	17	60	14	100	23	114	27	27	6	3	1	106	24	112	26
15%	93	13	50	7	93	13	61	9	162	23	193	28	7	1	-49	--	163	23	178	26

Note: The table presents the out-of-sample performance fee (Δ) and break-even transaction costs (tc^{be}) for a dynamic strategy based on selected models versus the static strategy for three target returns (5%, 10% and 15%) with a minimum variance strategy. Each minimum variance strategy builds an efficient portfolio by investing in the daily returns of the crude oil futures, USDX futures, and a risk-free asset. The fees are denoted as the amount which an investor is willing to pay for switching from the static strategy to another dynamic strategy with the relative risk aversion level γ =1 and 5. The performance fee (Δ) is expressed in annualized basis points. The break-even transaction cost (tc^{be}) is defined as the minimum proportional cost per trade for which the dynamic strategies would have the same utility as the static strategy. In addition, (tc^{be}) values are reported only when Δ is positive. The out-of-sample period runs from January 2, 2005 to December 31, 2009.

Panel A: Close Prices



Panel B: Excess Returns



Panel C: Trading Volumes

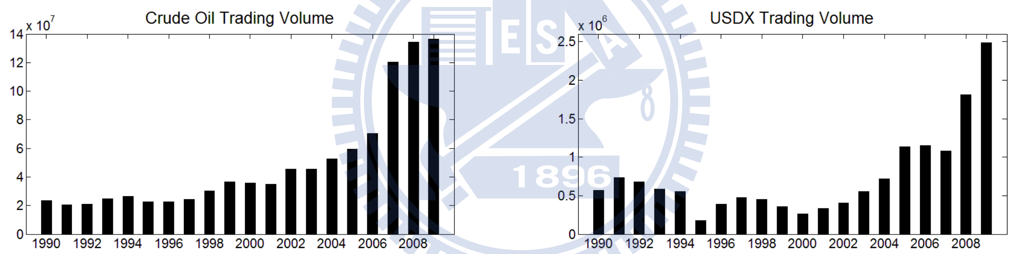
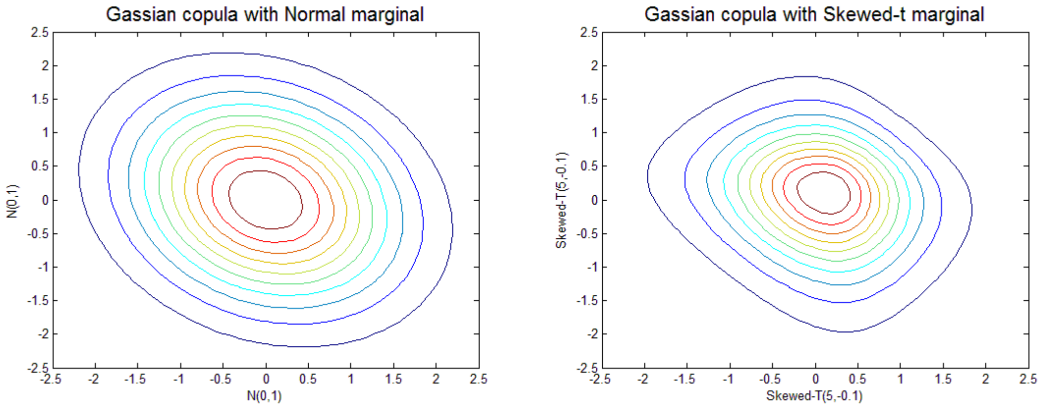
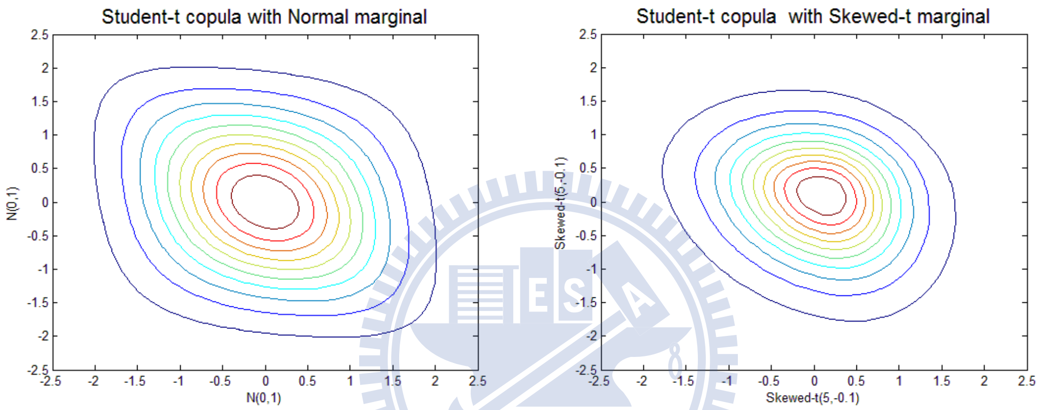


Figure 1. Daily close prices, daily excess returns and annual trading volumes of crude oil and USDX futures for the sample period from January 2, 1990 to December 31, 2009.

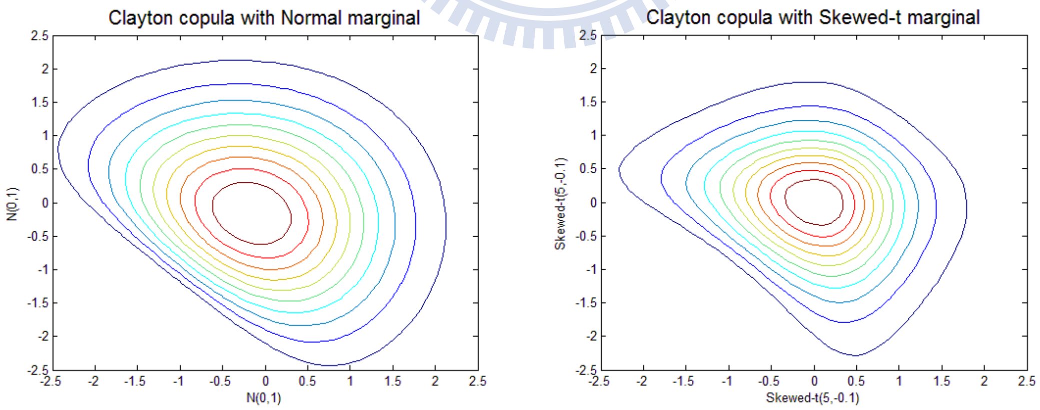
Panel A: Gaussian copula with Normal and Skewed-t marginal distributions



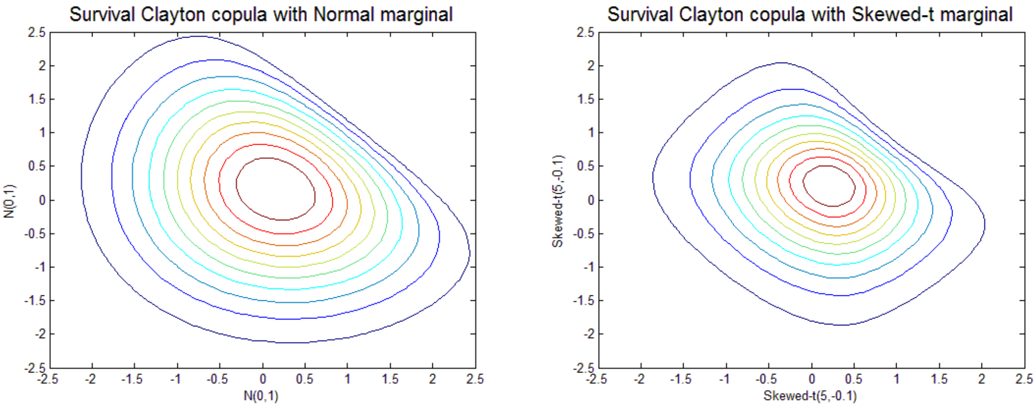
Panel B: Student-t copula with Normal and Skewed-t marginal distributions



Panel C: Clayton copula with Normal and Skewed-t marginal distributions



Panel D: Survival Clayton copula with Normal and Skewed-t marginal distributions



Panel E: Frank copula with Normal and Skewed-t marginal distributions

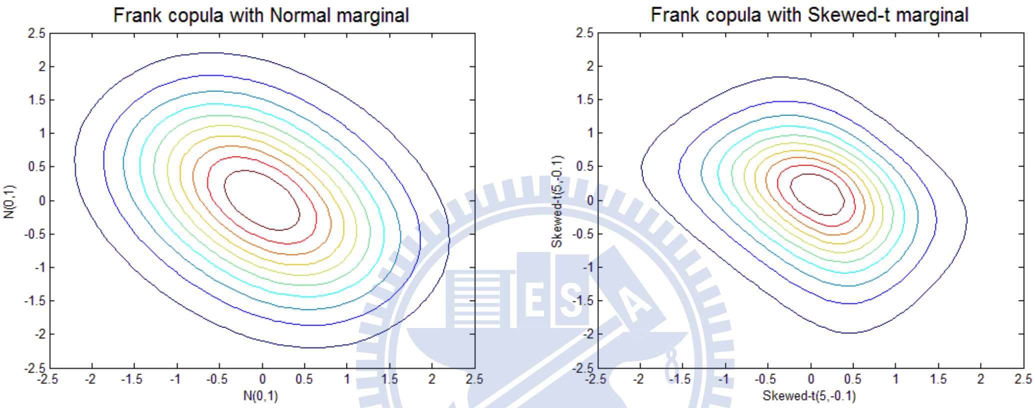
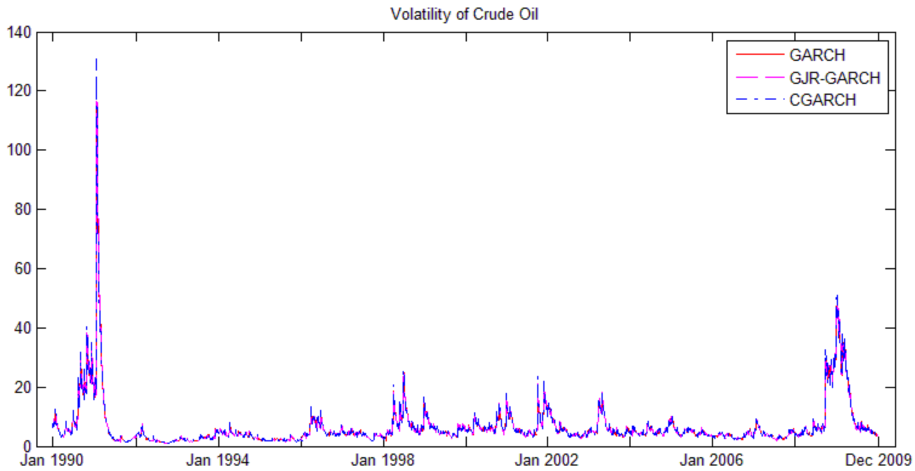


Figure 2. Contour plot based on two types of marginal distributions (Normal(0,1) and Skewed-t (5,-0.1)) and five types of copula functions (Gaussian, Student-t, Clayton, survival Clayton and Frank) under the specific dependence parameter, $\tau = -0.2$.

Panel A: Crude Oil



Panel B: USDX

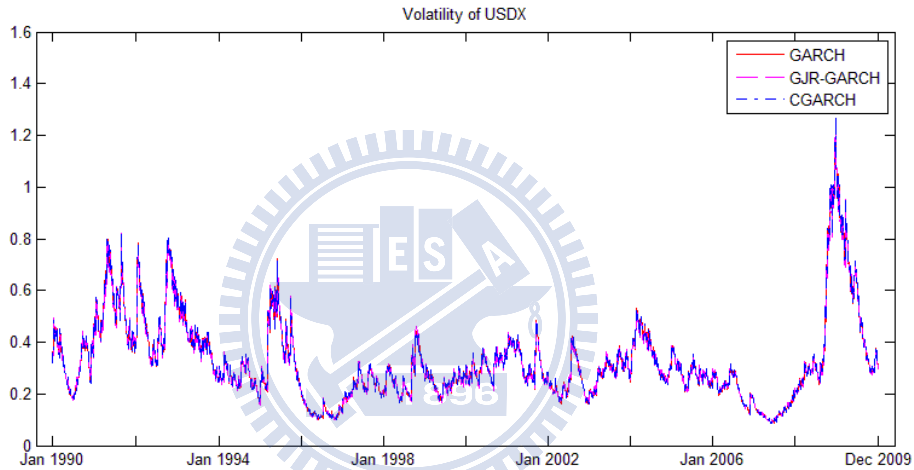
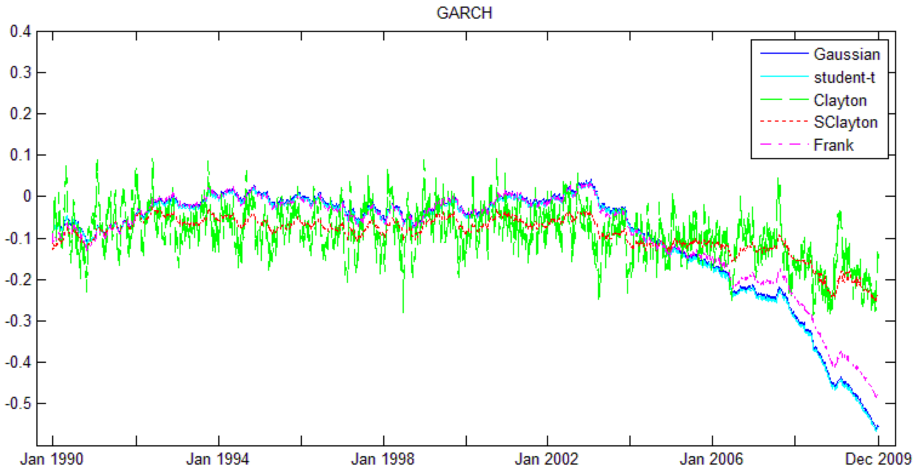
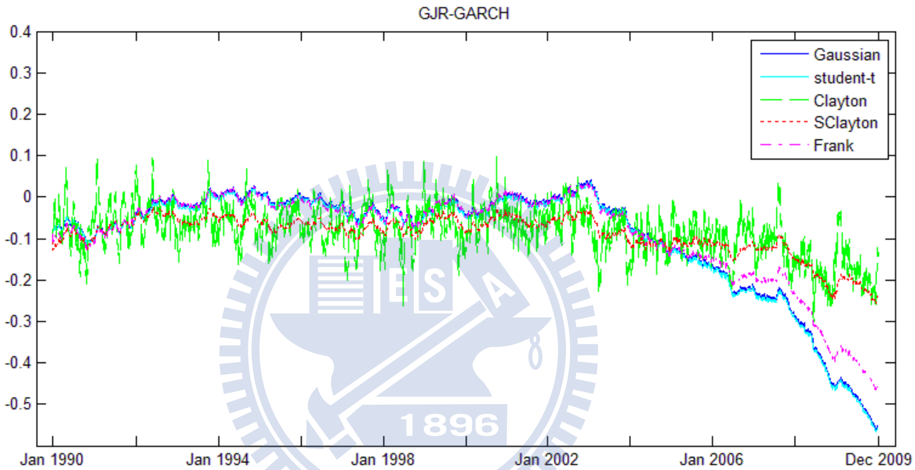


Figure 3. Volatility estimates of crude oil and USDX futures excess returns based on the GARCH, GJR-GARCH, and Component GARCH models for the sample period from January 2, 1990 to December 31, 2009.

Panel A: GARCH Model



Panel B: GJR-GARCH Model



Panel C: Component GARCH Model

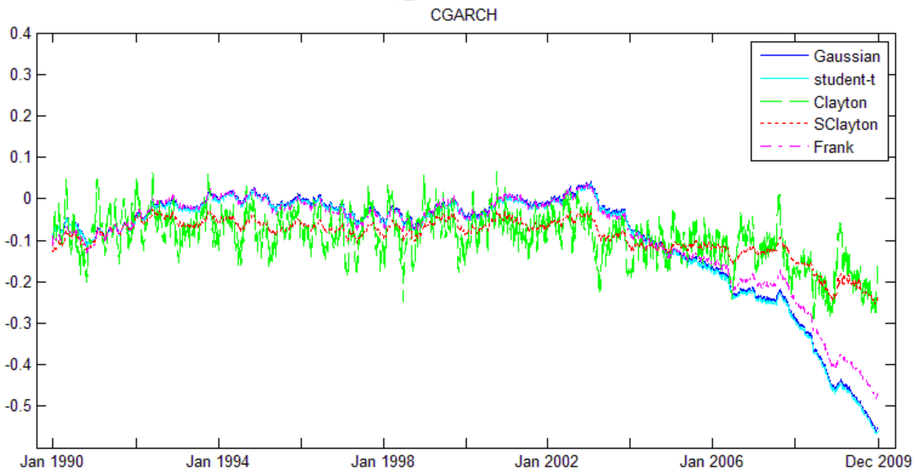


Figure 4. Correlation estimates between crude oil and USDX futures excess returns based on marginal distributions of the GARCH, GJR-GARCH, and component GARCH models and dependence structures of the Gaussian, Student-t, Clayton, survival Clayton and Frank copulas. The sample period is from January 2, 1990 to December 31, 2009.