國 立 交 通 大 學

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碩 士 論 文

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A Note on the Default Probability with Price Limits

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摘要

本篇論文提出一個在市場有漲跌幅限制下的破產機率算法。除了使用歷史股價報酬率來求得破產機率外，亦可使用跌停頻率的方法。因為在市場有漲跌幅限制之下，使用歷史股價報酬率會低估波動度，所以破產機率亦會被低估。然而使用跌停頻率的方法可以獲得較準確且一致的破產機率。

關鍵字：漲跌幅限制；破產機率；波動度。

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ABSTRACT

A new approach to derive the default probability with price limits is proposed in this paper. In addition to using historical return data, the default probability can be derived from the frequency of limit down. Because of the underestimated volatility, the default probability may be extremely underestimated with historical return data in price-limit markets. However, the price volatility estimated from the hitting frequency is always consistent with the true model and results in a better estimated default probability.

Keywords: price limits; default probability; volatility.

Content

List of Tables

List of Figures

I. Introduction

Estimating the asset value and volatility is always an important issue in default probability models, such as the Merton's (KMV) model (1974) and the first passage model (Jarrow and Turnbull, 1995). Both models define a firm's default as the situation that its asset value falls below the Debt level. To derive the asset value and volatility in the Merton's model, the method of solving two equations for two unknown variables (TETU) can be used (Vassalou and Xing, 2004). In 2009, Miyake and Inoue proposed an estimated method, which uses the moments of assets, for assets value and the volatility under the Merton's model. The first passage model also needs to estimate the asset value and volatility. In these models, the estimations in volatility of assets are always relative to the volatility of the equity. A more exact method of volatility estimation for the stock price in the price-limit market is proposed; moreover, a comparison with traditional method shows its importance of deriving the default probability.

Government agencies often impose daily price movement limits, on the stock market with to avoid violent changes of stock prices. According to the Brady Report (1998), after the October 1987 crash, price limits might be useful in preventing excess market volatility; thus, adoption of price limit mechanisms became apparent. For example, China and Mexico only allow stock prices to fluctuate within a range of 10%; France, Korea, Spain and Peru all allow fluctuating within a range of 15%; 7% fluctuating range is allowed in Taiwan. In Japan and South Africa, the fluctuating range differs from the stock price level, even for only 2%. In addition to equity markets, such stabilizing measures are also widely used in commodity markets.

In the price-limit market, a unique feature that the frequencies of limit down, which is strictly increasing in volatility, is used to estimate the volatility and default probability in this paper. With price limits, the frequencies of limit down and limit up can be associated with transition density (Guo and Huang, 2010) of the stock price at the end of a trading day. Because of the limited movement of the stock price, the volatility of the stock price estimated from common-used historical returns can be undervalued, while the hitting-frequency can be directly observed from markets without bias. Having the exact volatility of stock price, the credit risk models which take Merton's model for illustration can be applied to estimate the default probability of a company.

In this paper, a new approach to estimate the default probability with the transition density of the stock price is proposed. The fact that the probability of hitting-boundary is monotone increasing in volatility sheds light on how to estimate the volatility from hitting-boundary frequency. Having the volatility of stock price, the Merton's model can be applied to estimate the asset level and the default probability of the company.

The remainder of this paper is organized as follows. In section II, the price process with price limits is briefly described, and a new volatility estimation method with the transition density of price is provided. Section III derives the default probability in price-limit markets. Section IV concludes this paper.

II. The Model

The intraday model of the stock pricing process with price limits was developed by Ban, Choi, and Ku (BCK, 2000). Adjusted the Black-Scholes model to price-limit markets, the price process is a function of geometric Brownian motion until the price limits are reached. Its boundary behavior should encompass the slowly reflecting case as well as the absorbing or instantaneously reflecting case. The complicated, natural stochastic differential equation is given by following:

$$\begin{cases} dS_t = \theta I_{(a,b)}(S_t)dt + \sigma I_{(a,b)}(S_t)dW_t + \delta_1 d\phi_t - \delta_2 d\varphi_t \\ I_{\{a\}}dt = \rho_1 d\phi_t \\ I_{\{b\}}dt = \rho_2 d\varphi_t \end{cases} \tag{1}$$

ϕ and φ are local times at a (the lower bound, limit down) and b (the upper bound, limit up), respectively. ρ is the viscosity of the boundary with $\rho > 0$. Larger values of ρ inhibit the change of the stock price on boundary. δ (>0) denotes the elasticity of the boundary. As δ increases, the stock price rebounds more violently.

Guo and Huang (2010) showed that the logarithm price at the end of a day ($Z_t = \ln(S_t)$) follows the backward equation:

$$\begin{cases} \dfrac{\partial p}{\partial t} = \dfrac{1}{2}\sigma^2 \dfrac{\partial^2 p}{\partial x^2} + \mu \dfrac{\partial p}{\partial x}, & t > 0, L < x < U, L < y < U \\ \lim\limits_{x \downarrow L} p(t; x, y) = 0, & t > 0, L < y < U \\ \lim\limits_{x \uparrow U} p(t; x, y) = 0, & t > 0, L < y < U \\ \lim\limits_{t \downarrow 0} p(t; x, y) = \delta(y - x), & L < x < U, L < y < U \end{cases} \tag{2}$$

where *x* denotes the initial position, Z_0; *y* denotes the position of price at time *t*, Z_t; *L* and *U* denote the limit-down price and limit-up price with limit rate γ ($L = \ln(Z_0 \cdot (1-\gamma))$ and $U = \ln(Z_0 \cdot (1+\gamma))$), respectively. Solving the Eq. 2, the transition density is given as following[1]:

$$p(t;x,y) = \begin{cases} \dfrac{2}{U-L}\exp\left\{\dfrac{\mu(y-x)}{\sigma^2} - \dfrac{\mu^2 t}{2\sigma^2}\right\}\displaystyle\sum_{m=1}^{\infty}\exp\left\{-\dfrac{m^2\pi^2\sigma^2 t}{2(U-L)^2}\right\} & \\ \times \sin\left(\dfrac{m\pi(x-L)}{U-L}\right)\sin\left(\dfrac{m\pi(y-L)}{U-L}\right) & \text{, if } L < \mathrm{y} < U \quad (3) \\ \mathbb{P}_x(\tau_L < \tau_U) - \int_L^U \mathbb{P}_y(\tau_L < \tau_U)p(t;x,y)dy & \text{, if } \mathrm{y} = L \quad (4) \\ \mathbb{P}_x(\tau_U < \tau_L) - \int_L^U \mathbb{P}_y(\tau_U < \tau_L)p(t;x,y)dy & \text{, if } \mathrm{y} = U \quad (5) \end{cases}$$

where $\mu = r - \frac{1}{2}\sigma^2$, r denotes the risk-free rate; τ_L and τ_U denote the stopping time at L and U, respectively. Given the initial position, x, the expression $\mathbb{P}_x(\tau_L < \tau_U)$ and $\mathbb{P}_x(\tau_U < \tau_L)$ can be derived[2]:

$$\begin{cases} \mathbb{P}_x(\tau_L < \tau_U) = \dfrac{1 - exp(2\mu(U-x)/\sigma^2)}{1 - exp(2\mu(U-L)/\sigma^2)} \\ \mathbb{P}_x(\tau_U < \tau_L) = \dfrac{1 - exp(-2\mu(x-L)/\sigma^2)}{1 - exp(-2\mu(U-L)/\sigma^2)} \end{cases} \qquad (6)$$

From Table 1, it can be shown that using the transition density to compute the hitting-boundary probability is consistent with the simulations.

The hitting-boundary probabilities (Eq. 4 and Eq. 5), which are monotonic increasing in volatility (see figure 1), can be computed from the derived transition density, and then the volatility of stock price can be implied from the hitting-boundary probabilities. For example, during the past N days, if the limit down were n_d days, the limit down frequency was $\frac{n_d}{N}$, which can be an estimator for left side of Eq. 4 and imply the volatility from bisection for its

[1] Please refer to Guo, J. H., and W. L. Huang (2010), for the details.
[2] Please refer to Bhattacharya, R. N., and E. C. Waymire (1990), for the details.

monotonic property. For a high default-risk firm, it may be a high frequency of limit down in its stock price, so the frequency of limit down is used in this paper for derived the default probability.

Credit risk models that treat the equity volatility as an input have an unbiased estimator of the default probability with the proposed method. As shown in Table 2, in price-limit markets, the estimated equity volatility of the traditional method with historical equity return data can be extremely underestimated over than 15%. In contrast, the implied equity volatility with hitting frequency is more consistent with the true volatility.

III. Default Probability

In this section, the Merton's Model (1974) is introduced to illustrate how to derive the default probability with the implied equity volatility. Merton's model states that a company's equity is a call option on assets of the company if the company has a zero-coupon bond that matures at time *T*. Let V_t denote the value of a company's assets at time *t* with volatility σ_V, E_t denote the value of a company's equity at time *t* with volatility σ_E, and D denote the debt repayment due at maturity *T*. Under the risk-neutral measure, the assets value follows a stochastic process: $\frac{dV_t}{V_t} = rdt + \sigma_V dW_t$. Hence, the Black-Scholes formula gives the current value of equity as

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2), \tag{7}$$

where $d_1 = \frac{\ln V_0/D + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}}$ and $d_2 = d_1 - \sigma_V \sqrt{T}$.

Thus, $N(-d_2)$ is the risk-neutral probability that the company will default on the debt. However, V_0 and σ_V can not be directly observed. If the equity of the company is publicly traded, E_0 and σ_E can be observed or obtained from markets. Itô's lemma also shows that

$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0$, or

$$\sigma_E E_0 = N(d_1)\sigma_V V_0 \tag{8}$$

Having two equations, (7) and (8), V_0 and σ_V can be solved and the default probability can also be derived (Vassalou and Xing, 2004).

Since E_0 and σ_E are obtained from the stock price and the hitting-frequency method, respectively, V_0 and σ_V can be computed to derive the default probability. It can be shown from Table 3 that the estimation error of the asset's volatility in the traditional method with historical return data can be 16.48%. For the default probability estimation, the error in the traditional method even can be 52.92%. In contrast, the hitting-frequency method gives a more consistent result to true values; for example, the errors are always less than 4%. From Table 4, it can be shown that the traditional method can be replaced by hitting frequency method when price limits is high or the volatility of stock price is high; in other words, higher frequency of limit down decreases the accuracy of using historical return to estimate default probabilities, but increases the efficiency of using hitting frequency.

IV. Conclusion

Under the price process in price-limit markets, a new price volatility estimation method obtained with transition density is proposed. The transition density shows that the larger volatility leads to the higher hitting frequencies. Using the hitting frequency to estimate the volatility of stock price is always consistent, while the traditional method may undervalue over than 15%. The default probability estimated with hitting frequency always consistent with the true default probability, whereas it is undervalued about 50% in the traditional method if the Merton's model is applied.

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Table 1: Probabilities on Boundaries Comparing with the Monte Carlo Simulation

σ (%)	Hitting U before L		Hitting L before U	
	Transition Probability	Simulation	Transition Probability	Simulation
30	3.34×10^{-4}	3.00×10^{-4} (5.12×10^{-5})	1.27×10^{-4}	1.17×10^{-4} (2.79×10^{-5})
50	0.0307	0.0293 (4.10×10^{-4})	0.0219	0.0215 (3.63×10^{-4})
70	0.1033	0.1000 (0.0012)	0.1209	0.1163 (6.78×10^{-4})

Model parameter specifications: $S_0 = 100$, $\gamma = 7\%$, $r = 1\%$. Each simulation has 100,000 paths and 1,000 time steps in one day. The numbers inside the brackets stand for the standard deviations of simulations.

Table 2: Volatility estimation from Hitting Frequency and Traditional Method (%)

True Volatility	50		70	
Method	Estimation	Error (%)	Estimation	Error (%)
Hitting Frequency	49.53 (2.86)	-0.94	68.03 (4.97)	-2.81
Historical Return	46.36 (1.72)	-7.28	59.83 (2.00)	-15.51

Model parameter specifications: $S_0 = 100$, $\alpha = 3.5\%$, $\beta = 7\%$, $r = 1\%$, $T = 1\,(year)$. Each simulation has 100 paths and 1,000 time steps in one day. The numbers inside the brackets stand for the standard deviations of simulations. True volatility denotes the setting volatility of stock price in simulations. The historical return method calculates the standard deviation from the last one-year return. The hitting frequency method counts the frequency of limit down, and the volatility is implied from the Newton's Method.

Table 3a: Default Probabilities – Case 1: $\sigma = 50\%$

	Assets		Volatility of Assets (%)		Default Probability (%)	
True Model	297.9049		16.89		0.98	
Hitting Frequency	Estimation	Error (%)	Estimation	Error (%)	Estimation	Error (%)
	297.9068 (0.0491)	0.00	16.70 (0.0092)	-1.12	0.95 (0.0037)	-3.06
Historical Return	297.9520 (0.0176)	0.02	15.73 (0.0052)	-6.87	0.60 (0.0017)	-38.78

Table 3b: Default Probabilities (%) – Case 2: $\sigma = 70\%$

	Assets		Volatility of Assets (%)		Default Probability (%)	
True Model	296.7959		24.52		6.33	
Hitting Frequency	Estimation	Error (%)	Estimation	Error (%)	Estimation	Error (%)
	296.8199 (0.4517)	0.01	24.22 (0.0176)	-1.22	6.12 (0.0163)	-3.32
Historical Return	297.5709 (0.0956)	0.26	20.48 (0.0069)	-16.48	2.98 (0.0049)	52.92

Model parameter specifications: $S_0 = 100$, $\alpha = 3.5\%$, $\beta = 7\%$, $r = 1\%$, $T = 1\,(year)$

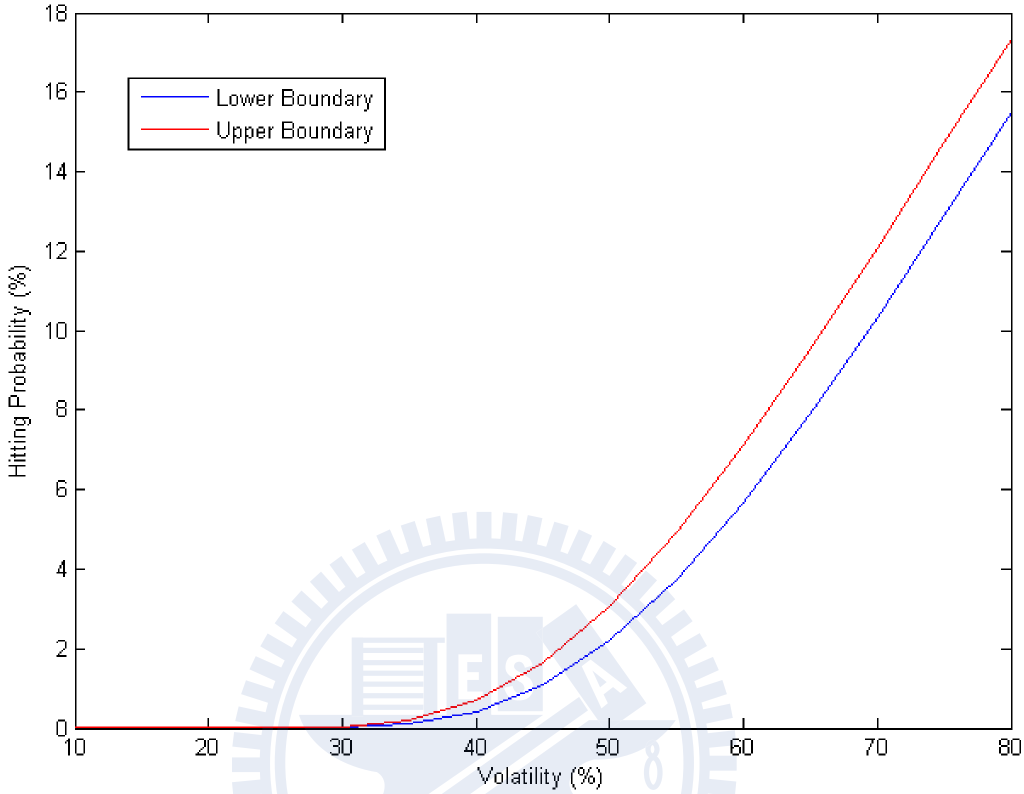
Assume total debt D of the firm is 200 with maturity 1 year, and equity value is equal to the current stock price. The numbers inside the brackets stand for the standard deviations in 100 simulations. Volatilities of stock are estimated by simulating 1 year data. True model represents data that calculated from Merton's model by the setting volatility of price for simulations; that is, data in the true-model row is unbiased. The historical return method is that calculates the standard deviation from the simulated one-year return. The hitting frequency method counts the frequency of limit down, and the volatility is implied from the Newton's Method.

Table 4: Sensitive Analysis for Default Probability in Price Volatility and Price Limits

			Volatility of Stock Price		
			50%	70%	90%
Default Probabilities in True Model (%)			0.98	6.33	16.09
Hitting Frequency (%)	Price Limits	10%	0.50 (0.0111)	5.91 (0.0111)	15.70 (0.0290)
		7%	0.99 (0.0046)	6.33 (0.0146)	15.77 (0.0312)
		3.5%	0.98 (0.0040)	6.33 (0.0233)	16.36 (0.0688)
Historical Return (%)	Price Limits	10%	0.98 (0.0028)	6.04 (0.0114)	14.72 (0.0194)
		7%	0.95 (0.0025)	5.32 (0.0083)	11.36 (0.0102)
		3.5%	0.32 (0.0007)	1.12 (0.0010)	1.68 (0.0007)

Model parameter specifications: $S_0 = 100$, $r = 1\%$, $T = 1\,(year)$. Assume total debt D of the firm is 200 with maturity 1 year, and equity value is equal to the current stock price. The numbers inside the brackets stand for the standard deviations in 100 simulations. Volatilities of stock are estimated by simulating 1 year data. True model represents data that calculated from Merton's model by the setting volatility of price for simulations; that is, data in the true-model row is unbiased. The historical return method is that calculates the standard deviation from the simulated one-year return. The hitting frequency method counts the frequency of limit down, and the volatility is implied from the Newton's Method.

Figure 1: Hitting Probability



Model parameter specifications: $r = 1\%$, $\alpha = \beta = 7\%$.