國 立 交 通 大 學

財務金融研究所

碩 士 論 文

隨機波動性和雙重指數跳躍模型之
美式選擇權近似解

An Efficient Approximation for Pricing
American Options under
Stochastic Volatility and Double Exponential Jumps

研 究 生：邱允鼎

指導教授：郭家豪 博士

中 華 民 國 九 十 九 年 六 月

隨機波動性和雙重指數跳躍模型之

美式選擇權近似解

An Efficient Approximation for Pricing

American Options under

Stochastic Volatility and Double Exponential Jumps

研 究 生：邱允鼎　　Student：Yun-Ting Chiu

指導教授：郭家豪 博士　　Advisor：Dr. Jia-Hau Guo

國 立 交 通 大 學

財務金融研究所

碩 士 論 文

A Thesis

Submitted to Graduate Institute of Finance

College of Management

National Chiao Tung University

for the Degree of Master in Finance

June 2010

Hsinchu, Taiwan, Republic of China

中 華 民 國 九 十 九 年 六 月

隨機波動性和雙重指數跳躍模型之
美式選擇權近似解

研究生：邱允鼎　　　　　　　　　　　　　指導教授：郭家豪 博士

國立交通大學財務金融研究所碩士班

摘要

這篇論文的目標是提供一個在隨機波動性和雙重指數跳躍模型之下讓評價美式選擇權快速而且有效率的近似解方程式。我們的數值結果說明了不對稱跳躍與提早履約溢酬的關係：在美式賣權的時候，提早履約溢酬會隨著往上跳的機率增加而增加。

關鍵字：美式選擇權；隨機波動性；雙重指數跳躍；提早履約溢酬。

An Efficient Approximation for Pricing American Options under Stochastic Volatility and Double Exponential Jumps

Student: Yun-Ting Chiu Advisor: Dr. Jia-Hau Guo

Institute of Finance
National Chiao Tung University

Abstract

The goal of the paper is to provide a useful and efficient analytic formula for pricing American options applied by quadratic approximation method that allows for stochastic volatility and double exponential jump. Our results also show that asymmetric jumps play an important role on the early-exercise premium. The early-exercise premium increases as the probabilities of upward jumps increase of put options.

Keywords: American options; stochastic volatility; double exponential jump; early-exercise premium.

誌謝

這篇論文可以走到完成的這一步，我要非常感謝我的指導教授郭家豪博士，老師在我的研究之路上不斷的悉心指導及不厭其煩的教導駑鈍的我。我也要感謝口試委員謝文良博士、王之彥博士、張龍福博士，感謝口委們在口試中惠予我許多寶貴的建議，讓本論文可以更加精彩完整。感謝在研究期間一起咪聽的好夥伴宏琦、偉倫、曉茹、任萱，一起在老師辦公室等待的時光，一起在課堂上討論的場景，尤其快接近口試的階段我們見面得越頻繁，每每想到都是研究路上深刻的回憶。

感謝交大財金所的師長們兩年來諄諄教誨，讓我能在知識學習上更加扎實精進；感謝交大財金所97級的同學們，特別是室友群軒、家誠、京翰在生活上的相互幫忙；此外，怡錞、詠晴、冠文、玫竹、文盈、紀純、怡樺、宛余、思蓉、嘉恩、淑雯、偲維、亮志、昭毅、譽獻、政宏、敏誠、民發、汶傑、才逸、育倫、為德、修豪、偉良…等等一同參與我研究生涯的同學們，真的感謝你們，能夠認識你們很令人開心。

在新竹的兩千多個日子，遇過了多少的颱風、艷陽、豪雨，參與了多少的迎新、送舊、離別，突然要離開了，滿是不捨這裡的點點滴滴回憶。然而往前邁進的動力就是一路上扶持我的好家人、好同學、好朋友，我很感謝也感覺很幸運，上天讓我活在這裡、體驗這裡，也讓我在這過得很好、很安穩。

新竹，我會懷念這裡的風，這裡的街道，這裡的建築物，還有這裡遇見的你。

邱允鼎 謹誌

國立交通大學財務金融研究所

中華民國九十九年六月

Content

List of Tables

I. Introduction

Because American option could be exercised at any time before expiry, the valuation and optimal exercise boundary is one of the most difficult problems in derivative securities. Past two decades, pricing American-style-options has been investigated widely in numerical methods and analytical approximation. In this paper, we present an efficient approximation for pricing American call and put options that allows for stochastic volatility and double exponential jump.

Ramezani and Zeng (1999) used empirical tests to illustrate that the double exponential jump-diffusion model to fit stock data better than the normal jump-diffusion model. Kou and Wang (2004) extended the analytical tractability of Black-Scholes analysis for the classical geometric Brownian motion to the alternative model with double exponential jump diffusion. One may regard the jump part of the model as the market affected by the outside impacts. Good or bad impacts occur in terms of a Possion process, and the asset price changes in response in terms of the jump size distribution. Owing to the double exponential distribution has high peak and heavy tails, it can justify the overreaction and underreaction to outside impacts.

Compared to Black and Scholes (1973) proposed European options with closed form solutions, American options with an early exercise feature complicated their valuation.

MacMillan (1986) developed the quadratic approximation method to solve the American put option on a non-dividend-paying stock pricing problem. Barone-Adesi and Whaley (1987) applied this method to evaluate American options by using the separable technique (but may generate some pricing errors in some cases). Bates (1991) first adopted jumps to the process of the underlying asset return. Chang, Kang, Kim, and Kim (2007) applied this scheme to American barrier (knock-out) and floating-strike lookback options. Guo, Hung and So (2009) extended the method for stochastic volatility and the normal return jump model. These articles illustrate that quadratic approximation method plays an important role on pricing American options. We used stochastic volatility and double exponential jumps as examples to compare with the least-squares simulation approach proposed by Longstaff and Schwartz (2001). The results illustrate that the quadratic approximation scheme is efficient in pricing American options based on these diffusion processes. And for put options, the estimates of approximation and the values of early-exercise premium are decrease by reducing the jump size on the probabilities of downward jumps.

The remainder of the paper is organized in third sections: Section 2 describes the stochastic volatility model with double exponential jumps. Section 3 is the numerical results and Conclusions summarize in Section 4.

II. Model

The following presents the stochastic volatility model with double exponential jump diffusion. Under the risk-neutral probability measure P^*, the dynamics of the underlying asset price, $S(t)$ with the conditional variance, $V(t)$ that allows for mean-reverting process :

$$\frac{dS(t)}{S(t)} = (r - d - \lambda^* E^*(Y^* - 1))dt + \sqrt{V^*(t)}dW_s^*(t) + d(\sum_{i=1}^{N^*(t)}(Y_i^* - 1)) \tag{1}$$

$$dV^*(t) = (\overline{V} - \kappa_V V^*(t))dt + \sigma_V \sqrt{V^*(t)}dW_V{}^*(t) \tag{2}$$

where r is the risk-free interest rate and d is the dividend yield. $W(t)$ represents a standard Brownian motion, $N(t)$ is a Poisson process with rate λ . $\{Y_i\}$ is a sequence of independent identically distributed (i.i.d.) nonnegative random variables such that $G = \log(Y)$ has an asymmetric double exponential distribution with the density :

$$f_G(g) = p\eta_1 e^{-\eta_1 g} 1_{\{g \geq 0\}} + q\eta_2 e^{\eta_2 g} 1_{\{g<0\}} \text{ , } \eta_1 > 1 \text{ , } \eta_2 > 0$$

where $p, q \geq 0$, p+q=1,represent the probabilities of upward and downward jumps. $N(t), W(t)$ *and* Gs , are assumed to be independent . ρ is the instantaneous correlation coefficient between the stock price return process and its variance process. In order to hold the Martingale property,

$$E^*(Y^* - 1) = \frac{p^* \eta_1^*}{\eta_1^* - 1} + \frac{q^* \eta_2^*}{\eta_2^* + 1} - 1$$

is subtracted from the stock price process, it remains the growth rate of the stock return rate as $r - d$.

The Partial Integro-differential equation for the price P is therefore

$$
\begin{aligned}
0 = {} & \frac{1}{2} P_{SS} S^2 V + \frac{1}{2} P_{VV} \sigma_V^2 V + P_{SV} \rho \sigma_V S V \\
& + P_S [(r-d) - \lambda^* E^* (Y^* - 1)] S + P_V (\overline{V} - \kappa_V V) - P_T - rP \\
& + \lambda^* \int_{-\infty}^{\infty} [P(Sy, V) - P(S,V)] \Phi(y) dy
\end{aligned}
\tag{3}
$$

American options with an early exercise feature complicated their valuation. The price of a basic American call option, $C^A(S,V,T;K)$ with a strike price, K, and a maturity date, T, can be described as

$$C^A(S,V,T;K) = C^E(S,V,T;K) + \xi(S,V,T;K) \tag{4}$$

where $C^E(S,V,T;K)$ represents the price of European call option and $\xi(S,V,T;K)$ is the value of the corresponding early-exercise premium. The early-exercise premium must satisfy Equation (3) because American option values, as well as European option values, satisfy the above-mentioned partial differential equation in the nonstopping region under the risk-neutral measure.

The analytic European option solution with the stochastic volatility model has existed (see Appendix), the unsolved part in pricing formula of an American option is a good approximation for the early-exercise premium. Since options are homogeneous in *S* and *K*, the premium is also homogeneous in *S* and *K*.

$\xi(S,V,T;K) = K\xi(S/K,V,T;1)$. The Barone-Adesi and Whaley (1987) defined the premium as

$$\xi(S,V,T;K) = KH(T)F(S/K,V,H) \quad KH(T)F(\tag{5}$$

where $z = S/K$ and $H(T)$ is an arbitrary function of time-to-maturity, T.

The partial derivatives of ξ are $\xi_S = HF_z, \xi_{SS} = HF_{zz}/K, \xi_V = KHF_V, \xi_{VV} = KHF_{VV}, \xi_{SV} = HF_{zV},$ and $\xi_T = KFH_T = KHF_H H_T$. Substitute Equation (5) into Equation (3):

$$\begin{aligned}0 &= \frac{1}{2}KHF_{zz}z^2V + \frac{1}{2}KHF_{VV}\sigma_V^2 V + KHF_{zV}\rho\sigma_V zV \\ &+ KHF_z[(r-d) - \lambda^* E^*(Y^* - 1)]z + KHF_V(\overline{V} - \kappa_V V) - H_T(KF + KHF_H) - rKHF \\ &+ \lambda^* KH \int_{-\infty}^{\infty}[F(zy,V) - F(z,V)]\Phi(y)dy\end{aligned} \tag{6}$$

Barone-Adesi and Whaley (1987) choose $H(T)$ *as* 1 - exp(- rT) for simplicity. Chang et al. (2007) further adjusted $H(T)$ to equal 1 - exp(- αrT) for controlling α to reduce barrier option pricing errors of the quadratic approximation. After substituting $H(T)$ =1 - exp(- αrT) into Equation (6), Equation (7) is:

$$\begin{aligned}0 &= \frac{1}{2}KHF_{zz}z^2V + \frac{1}{2}KHF_{VV}\sigma_V^2 V + KHF_{zV}\rho\sigma_V zV \\ &+ KHF_z[(r-d) - \lambda^* E^*(Y^* - 1)]z + KHF_V(\overline{V} - \kappa_V V) - \alpha r(1-H)KHF_H - \alpha r(1-H)KF \\ &+ \lambda^* KH \int_{-\infty}^{\infty}[F(zy,V) - F(z,V)]\Phi(y)dy\end{aligned} \tag{7}$$

As described in Barone-Adesi and Whaley (1987), Bates (1991), and Chang et al. (2007), $\alpha r(1-H)KHF_H$ is negligible. Substituting $F(z,V) = a_1\exp[B_1V]z^{\phi_1} + a_2\exp[B_2V]z^{\phi_2}$ into equation (7) and separating variables a_1 and a_2, generating yields

$$\begin{aligned}0 &= \frac{1}{2}\phi(\phi-1)V + \frac{1}{2}B^2\sigma_V^2 V + \phi B\rho\sigma_V V \\ &+ \phi[(r-d) - \lambda(\frac{p\ \eta_1}{\eta_1 - 1} + \frac{q\ \eta_2}{\eta_2 + 1} - 1)] + B(\overline{V} - \kappa_V V) - \alpha r(1/H - 1) - r \\ &+ \lambda\ [\frac{-p\phi}{\phi - \eta_1} + \frac{-q\phi}{\phi + \eta_2}]\end{aligned} \tag{8}$$

Further separating Equation (8) into two equations for *V*-terms and non-*V*-terms, respectively, Equations (9) and (10) are resulted:

$$0=\frac{1}{2}\phi^2+\phi(B\rho\sigma_V-\frac{1}{2})+\frac{1}{2}B^2\sigma_V^2-\kappa_V B \tag{9}$$

and

$$\begin{aligned}0=&B\overline{V}-(\alpha r(1/H-1)+r)+\lambda\,[\frac{-p\phi}{\phi-\eta_1}+\frac{-q\phi}{\phi+\eta_2}]\\&+\phi[(r-d)-\lambda(\frac{p\,\eta_1}{\eta_1-1}+\frac{q\,\eta_2}{\eta_2+1}-1)]\end{aligned} \tag{10}$$

After given the value of the parameters $r,\ d,\ \overline{V},\ \kappa_V,\ \rho,\ \sigma_V,\ \lambda, \alpha,\ p, q,\ \eta_1,\ \eta_2$ *and* T the values of ϕ_1, ϕ_2, B_1 and B_2 can be rapidly deduced from Equations (9) and (10) by using Newton's method. If $\eta_1, \eta_2 >> \phi$, we replace the Equation (10) term of

$\lambda\,[\frac{-p\phi}{\phi-\eta_1}+\frac{-q\phi}{\phi+\eta_2}]\cong\lambda[\frac{-p\phi}{-\eta_1}+\frac{-q\phi}{\eta_2}]$ to obtain the value *B*.

The result of the approximation of Equation (10) is

$$B=\frac{1}{\overline{V}}(\psi_0-\psi_1\phi) \tag{11}$$

where $\psi_0=\alpha r(1/H-1)+r$ and $\psi_1=(r-d)+\lambda(\frac{p}{\eta_1}-\frac{p\,\eta_1}{\eta_1-1}-\frac{q}{\eta_2}-\frac{q\,\eta_2}{\eta_2+1}+1)$

Substituting Equation (11) into Equation (9) generates

$$\begin{aligned}0=&(\frac{1}{2}\overline{V}^2-\rho\sigma_V\psi_1\overline{V}-\frac{1}{2}\sigma_V^2\psi_1^2)\phi^2\\&+(\rho\sigma_V\psi_0\overline{V}+\sigma_V^2\psi_0\psi_1+\kappa_V\psi_1\overline{V}-\frac{1}{2}\overline{V}^2)\phi\\&-(\kappa_V\psi_0\overline{V}+\frac{1}{2}\sigma_V^2\psi_0^2)\end{aligned} \tag{12}$$

Parameters that satisfy the relationship $\psi_0>2\frac{\kappa_V\overline{V}}{\sigma_V^2}$ ensure that one root (ϕ_1) is negative for puts, and the other (ϕ_2) is positive for calls. Because the relationships $\phi_1 < 0$ and $a_1 \neq 0$ imply that the $\lim_{S \to 0} C^A(S,V,T;K) = \infty$, therefore that $a_1 = 0$.

Once we have the values for a_2 and B_2, a_2 and $\bar{S}$ (the early-exercise price for calls) can be determined from the value-match condition and the high contact condition:

$$\xi(\bar{S},V,T;K) = \bar{S} - K - C^E(\bar{S},V,T;K) \tag{13}$$

and

$$\xi_S(\bar{S},V,T;K) = 1 - C_S^E(\bar{S},V,T;K) \tag{14}$$

where $\xi_S(\bar{S},V,T;K)$ *and* $C_S^E(\bar{S},V,T;K)$ are the differential-form for early-exercise premium and European option on *S*.

Equations (13) and (14) infers that:

$$\bar{S} = \frac{\phi_2 [\bar{S} - K - C^E(\bar{S},V,T,K;\ )]}{1 - C_z^E(\bar{S},V,T;K)} \tag{15}$$

and a_2 can be determined by

$$a_2 = \frac{\bar{S} - K - C^E(\bar{S},V,T;K)}{K(1 - \exp(-\alpha r T))\exp[B_2 V](\frac{\bar{S}}{K})^{\phi_2}} \tag{16}$$

The resulting formula for a basic American call option is

$$C^A(S,V,T;K) = C^E(S,V,T;K) + [\bar{S} - K - C^E(\bar{S},V,T;K)]\,(S/\bar{S})^{\phi_2} \tag{17}$$

for $S<\bar{S}$ or S-K for $S \geq \bar{S}$.

For the American puts, the boundary conditions are different to those for calls. But it must satisfy the same Partial Integro-differential equation. The boundary conditions for puts are

$$\mathrm{P}^{A}(\underline{S},V,T;K)=\mathrm{K}-\underline{S} \tag{18}$$

and

$$\mathrm{P}_{S}^{A}(\underline{S},V,T;K)=-1 \tag{19}$$

where $\underline{S}$ is the early-exercise price. The positive root (ϕ_2) is excluded for the puts because it implies that the $\lim_{S\to\infty} P^{A}(S,V,T;K)=\infty$. Resulting $a_2=0$ and therefore,

$$\begin{aligned} P^{A}(S,V,T;K) &= P^{E}(S,V,T;K) \\ &\quad + [K-\underline{S}-P^{E}(\underline{S},V,T;K)]\,(\mathrm{S}/\underline{S})^{\phi_1} \end{aligned} \tag{20}$$

for $\mathrm{S}>\underline{S}$ or K-S for $\mathrm{S}\le\underline{S}$.

where

$$\underline{S}=\frac{\phi_1[K-\underline{S}-P^{E}(\underline{S},V,T,K;\ )]}{-1-P_{s}^{E}(\underline{S},V,T;K)} \tag{21}$$

III. Numerical Results

The least-squares method (LSM) proposed by Longstaff and Schwartz (2001) is a well-known simulation method for pricing American options .Table 1 illustrates a comparison of the LSM and the quadratic approximation proposed in this research for the stochastic volatility model with double exponential jumps. The simulation estimates are based on 50,000 paths for the stock-price process and the option is exercisable 90 times before maturity. Table 1 shows that our approximation is consistent with the LSM. In addition, it is more efficient than the LSM because its computing time is much less than that of the LSM. Simulated American represents the results of the LSM. Approximation means the quadratic approximation and Diff is the difference between Simulated American and Approximation.

The standard errors of the simulation estimates are given in parentheses. As shown, the differences (Diff) between the LSM algorithms and the quadratic approximation are typically small. Table 2 gives the comparison of Approximation (approx) and European value for put options with T=0.25 year. *Prem* represents the value of early-exercise premium. Intuitively, the approximation values increase after incorporating into the double exponential jump diffusion (arrival rate λ from 0 to 7).Table 3 shows that early exercise premium of in-the-money options decrease after incorporating into the double exponential jump diffusion. The results are consistent with Amin (1993). However, it may not be the case for at-the-money or out-of-the-money options. We found that early-exercise premium has an upward trend as reducing the probabilities of upward-jumps (*p* from 0.7 to 0.3) of put options. Table 4 also presents the relations between the early-exercise premium (*Prem*) and the difference probability of upward jumps (*p* from 0 to 1). For call options, the early-exercise premium increases as the probabilities of upward jumps increase, and vice versa. Besides, by reducing the jump size of the downward jump results in the decrease of early-exercise premium of put options and the increase of call options.

IV. Conclusion

In this paper, we provide a useful and efficient analytic formula for pricing American options applied by quadratic approximation method that allows for stochastic volatility and double exponential jump.Comparison with the least-squares method algorithms presents that the quadratic approximation technique is accurate and efficient.

As Amin (1993) incorporating the double exponential jump diffusion into the pricing model results in the decrease of early-exercise premium for in-the money options. Our results also show that asymmetric jumps play an important role on the early-exercise premium. The early-exercise premium increases as the probabilities of upward jumps increase of put options.

References

[1]. Amin, K. I. (1993). “Jump Diffusion Option Valuation in Discrete Time,” Journal of Finance, 48, 1833-1863.

[2]. Bakshi, G., & Cao, C. (2003). “Risk-neutral Kurtosis, Jumps, and Option Pricing: Evidence from 100 Most Actively Traded Firms on the CBOE,” EFA 2003 Annual Conference Paper No. 953.

[3]. Barone-Adesi, G., & Whaley, R. (1987). “Efficient Analytic Approximation of American Option Values,” Journal of Finance, 42, 301–320.

[4]. Bates, D. (1991). “The Crash of ‘87: Was It Expected? The Evidence from Options Markets,” Journal of Finance, 46, 69–107.

[5]. Black, F., & Scholes, M. (1973). “The Pricing of Options and Corporate Liabilities,” Journal of Political Economy, 81, 637-654.

[6]. Chang, G., Kang, J., Kim, H., & Kim, I. (2007). “An Efficient Approximation Method for American Exotic Options,” Journal of Futures Markets, 27, 1, 2007.

[7]. Duffie, D., Pan, J., & Singleton, K. (2000). “Transform Analysis and Asset Pricing for Affine Jump-duffusions,” Econometrica, 68, 6, 1343–1376.

[8]. Guo, J. H., Hung, M. W., & So, L. C. (2009). “A Generalization of the Barone-Adesi and Whaley Approach for the Analytic Approximation of American Options,” Journal of

Futures Markets, 29, 478-493.

[9]. Horng, T. L., & Tien, C. Y. (2006). “American Options Valuation: A Parsimoniously Numerical Approach”.

[10]. Hull, J. (2000).Options, Futures, and Other Derivative Securities, 4th ed. Prentice Hall, NJ.

[11]. Kou, S.G. (2002). “A Jump Diffusion Model for Option Pricing,” Management Science, 48, 1086-1101.

[12]. Kou, S. G. & Wang, H. (2004). “Option Pricing under a Double Exponential Jump Diffusion Model,” Management Science, 50, 1178-1192.

[13]. Longstaff, F., & Schwartz, E. (2001). “Valuing American Options by Simulation: A Simple Least-squares Approach,” Review of Financial Studies, 14, 113–147.

[14]. MacMillan, L. (1986). “Analytic Approximation for the American Put Option,” Futures and Options Research, 1A, 119–140.

[15]. Ramezani, C. A., & Zeng, Y. (1999). “Maximum Likelihood Estimation of Asymmetric Jump-diffusion Process: Application to Security Prices,” Working paper, Department of Statistics, University of Wisconsin, Madison, WI.

Appendix

Stochastic Volatility Model with Double Exponential Jumps

The present value of a basic European call option can be formulated as

$C^E(S,V,T;K)=E^*[e^{-rT}\max\{S(T)-K,0\}]$ and is given by

$$C^E(S,V,T;K)=\frac{1}{2}J(t,T;-i)-\frac{1}{\pi}\int_0^\infty\frac{\operatorname{Im}[e^{iv\log(K)}J(t,T;-i-v)]}{v}dv$$
$$\text{-K}(\frac{1}{2}J(t,T;0)-\frac{1}{\pi}\int_0^\infty\frac{\operatorname{Im}[e^{iv\log(K)}J(t,T;-v)]}{v}dv)$$

where $J(t,T;\phi)$ is the characteristic function of the state density. $\operatorname{Im}(c)$ denotes the imaginary part of $c\in C$. The characteristic function is given by

$$J(t,T;\phi)=\exp(A(T;\phi)+B(T;\phi)V)S^{i\phi}$$

Where $A(T;\phi)$ and $B(T;\phi)$ are

$$A(T;\phi)\equiv(i\phi)[r-d-\lambda(\frac{p\eta_1}{\eta_1-1}+\frac{q\eta_2}{\eta_2+1}-1)]T-rT$$
$$+\lambda T(\frac{-p\eta_1}{-\eta_1+i\phi}+\frac{q\eta_2}{\eta_2+i\phi}-p-q)-\lambda T$$
$$-\frac{\overline{V}}{\sigma_V^2}[(\varepsilon+i\phi\sigma_V\rho-\kappa_V)T+2\log(1-\frac{(\varepsilon+i\phi\sigma_V\rho-\kappa_V)(1-\exp(-\varepsilon T))}{2\varepsilon})]$$

$$B(T;\phi)\equiv\frac{i\phi(i\phi-1)(1-\exp(-\varepsilon T))}{2\varepsilon-(\varepsilon+i\phi\sigma_V\rho-\kappa_V)(1-\exp(-\varepsilon T))}$$

$$\varepsilon\equiv\sqrt{(i\phi\sigma_V\rho-\kappa_V)^2-i\phi(i\phi-1)\sigma_V^2}$$

The proof is published in Duffie [2000].Once we have the solution for European calls, the formula for puts, $P^E(S,V,T;K)$, can be obtained by the put-to-call conversion equation

Grabbe [1983] :

$$P^E(S,V,T;K) = C^E(S,V,T;K) - S\exp(-dT) + K\exp(-rT)$$

Table 1 Comparison of American Options: Stochastic Volatility with Double Exponential Jumps

	S	European option	Simulated American (a)	CPU time	(s.e.)	approximation (b)	CPU time	Diff(b)-(a)
call	140	39.424	39.844	47.099	0.034	40.000	1.837	-0.156
	130	29.643	29.886	47.856	0.025	30.001	1.758	-0.115
	120	20.090	20.238	45.536	0.029	20.258	1.781	-0.021
	110	11.344	11.436	46.931	0.015	11.419	1.810	0.018
	100	4.648	4.690	47.763	0.038	4.678	1.798	0.012
	90	1.134	1.142	46.972	0.020	1.146	1.876	-0.004
put	60	39.405	39.928	45.554	0.031	40.000	1.852	-0.072
	70	29.563	29.948	45.947	0.033	30.000	1.912	-0.052
	80	19.842	20.085	46.319	0.024	20.048	1.839	0.037
	90	10.985	11.048	46.555	0.032	11.068	1.747	-0.020
	100	4.648	4.712	46.489	0.033	4.684	1.844	0.028
	110	1.493	1.514	44.542	0.016	1.510	1.813	0.003

Note: The CPU times are in seconds.

$K = 100,\ r = 0.06,\ d = 0.06,\ V_0 = 0.01,\ \overline{V} = 0.49,\ \rho = -0.25,\ \lambda = 3,\ \sigma_V = 0.2,\ \kappa_V = 5.08,\ \alpha = 1.64,$
$p = 0.6,\ q = 0.4,\ \eta_1 = 25,\ \eta_2 = 25,\ T = 0.25$

Table 2 Comparison of Approximation (approx) and European value for Put Option with p=0.5

parameter values							
K	ρ	λ	η_1	η_2	approximation (a)	European option (b)	Diff (a)-(b)
100	0	0	25	25	10.8400	10.7550	0.0850
100	0	0	25	50	10.8400	10.7550	0.0850
100	0	0	50	25	10.8400	10.7550	0.0850
100	0	0	50	50	10.8400	10.7550	0.0850
100	-0.25	0	25	25	10.8057	10.7124	0.0933
100	-0.25	0	25	50	10.8057	10.7124	0.0933
100	-0.25	0	50	25	10.8057	10.7124	0.0933
100	-0.25	0	50	50	10.8057	10.7124	0.0933
100	-0.75	0	25	25	10.7342	10.6232	0.1109
100	-0.75	0	25	50	10.7342	10.6232	0.1109
100	-0.75	0	50	25	10.7342	10.6232	0.1109
100	-0.75	0	50	50	10.7342	10.6232	0.1109
100	0	7	25	25	11.4069	11.3349	0.0720
100	0	7	25	50	11.2653	11.1963	0.0690
100	0	7	50	25	11.1310	11.0468	0.0843
100	0	7	50	50	10.9857	10.9039	0.0818
100	-0.25	7	25	25	11.3811	11.3040	0.0771
100	-0.25	7	25	50	11.2376	11.1630	0.0747
100	-0.25	7	50	25	11.1027	11.0118	0.0909
100	-0.25	7	50	50	10.9549	10.8657	0.0892
100	-0.75	7	25	25	11.3275	11.2398	0.0877
100	-0.75	7	25	50	11.1801	11.0938	0.0863
100	-0.75	7	50	25	11.0442	10.9391	0.1050
100	-0.75	7	50	50	10.8904	10.7858	0.1045
90	0	0	25	25	3.8757	3.8446	0.0311
90	0	0	25	50	3.8757	3.8446	0.0311
90	0	0	50	25	3.8757	3.8446	0.0311
90	0	0	50	50	3.8757	3.8446	0.0311
90	-0.25	0	25	25	3.8765	3.8410	0.0356
90	-0.25	0	25	50	3.8765	3.8410	0.0356
90	-0.25	0	50	25	3.8765	3.8410	0.0356
90	-0.25	0	50	50	3.8765	3.8410	0.0356
90	-0.75	0	25	25	3.8784	3.8328	0.0456
90	-0.75	0	25	50	3.8784	3.8328	0.0456
90	-0.75	0	50	25	3.8784	3.8328	0.0456
90	-0.75	0	50	50	3.8784	3.8328	0.0456
90	0	7	25	25	4.6399	4.6109	0.0290
90	0	7	25	50	4.3897	4.3631	0.0266
90	0	7	50	25	4.3586	4.3254	0.0332
90	0	7	50	50	4.0913	4.0606	0.0307
90	-0.25	7	25	25	4.6391	4.6072	0.0319
90	-0.25	7	25	50	4.3863	4.3566	0.0297
90	-0.25	7	50	25	4.3616	4.3247	0.0369
90	-0.25	7	50	50	4.0917	4.0569	0.0348
90	-0.75	7	25	25	4.6365	4.5983	0.0382
90	-0.75	7	25	50	4.3784	4.3418	0.0366
90	-0.75	7	50	25	4.3671	4.3220	0.0451
90	-0.75	7	50	50	4.0922	4.0486	0.0436

	parameter values						
K	ρ	λ	η_1	η_2	approximation (a)	European option (b)	Diff (a)-(b)
80	0	0	25	25	0.6610	0.6508	0.0101
80	0	0	25	50	0.6610	0.6508	0.0101
80	0	0	50	25	0.6610	0.6508	0.0101
80	0	0	50	50	0.6610	0.6508	0.0101
80	-0.25	0	25	25	0.6960	0.6839	0.0121
80	-0.25	0	25	50	0.6960	0.6839	0.0121
80	-0.25	0	50	25	0.6960	0.6839	0.0121
80	-0.25	0	50	50	0.6960	0.6839	0.0121
80	-0.75	0	25	25	0.7632	0.7463	0.0169
80	-0.75	0	25	50	0.7632	0.7463	0.0169
80	-0.75	0	50	25	0.7632	0.7463	0.0169
80	-0.75	0	50	50	0.7632	0.7463	0.0169
80	0	7	25	25	1.1213	1.1108	0.0105
80	0	7	25	50	0.9065	0.8973	0.0092
80	0	7	50	25	0.9959	0.9842	0.0117
80	0	7	50	50	0.7796	0.7693	0.0103
80	-0.25	7	25	25	1.1466	1.1347	0.0119
80	-0.25	7	25	50	0.9344	0.9238	0.0106
80	-0.25	7	50	25	1.0235	1.0101	0.0135
80	-0.25	7	50	50	0.8111	0.7990	0.0121
80	-0.75	7	25	25	1.1953	1.1802	0.0151
80	-0.75	7	25	50	0.9903	0.9763	0.0140
80	-0.75	7	50	25	1.0770	1.0594	0.0175
80	-0.75	7	50	50	0.8721	0.8557	0.0164

Note: $S_0 = 90,\ r = 0.06,\ d = 0.06,\ \mathrm{V}_0 = 0.01,\ \overline{V} = 0.49,\ \sigma_V = 0.2,\ \kappa_V = 5.06,\ \alpha = 1.64,\ \mathrm{T} = 0.25$

Table 3 The early-exercised premium (Prem) with different probability of upward jumps (p)

parameter values							*Prem*				
κ	ρ	η_1	η_2	λ	*Prem*	λ	*p=0.7*	*p=0.6*	*p=0.5*	*p=0.4*	*p=0.3*
100	0	25	25	0	0.0850	7	0.0670	0.0694	0.0720	0.0747	0.0777
100	0	25	50	0	0.0850	7	0.0651	0.0669	0.0690	0.0713	0.0739
100	0	50	25	0	0.0850	7	0.0824	0.0833	0.0843	0.0851	0.0860
100	0	50	50	0	0.0850	7	0.0808	0.0813	0.0818	0.0823	0.0828
100	-0.25	25	25	0	0.0933	7	0.0695	0.0744	0.0771	0.0800	0.0831
100	-0.25	25	50	0	0.0933	7	0.0704	0.0724	0.0747	0.0772	0.0802
100	-0.25	50	25	0	0.0933	7	0.0892	0.0901	0.0909	0.0917	0.0925
100	-0.25	50	50	0	0.0933	7	0.0881	0.0886	0.0892	0.0898	0.0904
100	-0.75	25	25	0	0.1109	7	0.0823	0.0849	0.0877	0.0908	0.0943
100	-0.75	25	50	0	0.1109	7	0.0813	0.0837	0.0863	0.0893	0.0929
100	-0.75	50	25	0	0.1109	7	0.1036	0.0987	0.1050	0.1057	0.1065
100	-0.75	50	50	0	0.1109	7	0.1034	0.1040	0.1045	0.1053	0.1062
90	0	0	25	25	0.0311	7	**0.0268**	0.0279	0.0290	0.0303	0.0316
90	0	0	25	50	0.0311	7	0.0253	0.0259	0.0266	0.0274	0.0282
90	0	0	50	25	0.0311	7	0.0319	0.0326	0.0332	0.0339	0.0345
90	0	0	50	50	0.0311	7	0.0303	0.0305	0.0307	0.0309	0.0312
90	-0.25	0	25	25	0.0356	7	0.0285	0.0307	0.0319	0.0332	0.0346
90	-0.25	0	25	50	0.0356	7	0.0282	0.0290	0.0297	0.0306	0.0317
90	-0.25	0	50	25	0.0356	7	0.0356	0.0363	0.0369	0.0375	0.0381
90	-0.25	0	50	50	0.0356	7	0.0343	0.0345	0.0348	0.0350	0.0352
90	-0.75	0	25	25	0.0456	7	0.0357	0.0369	0.0382	0.0397	0.0413
90	-0.75	0	25	50	0.0456	7	0.0439	0.0356	0.0366	0.0378	0.0392
90	-0.75	0	50	25	0.0456	7	0.0439	0.0421	0.0451	0.0457	0.0462
90	-0.75	0	50	50	0.0456	7	0.0431	0.0434	0.0436	0.0439	0.0443
80	0	0	25	25	0.0101	7	0.0096	0.0100	0.0105	0.0110	0.0116
80	0	0	25	50	0.0101	7	0.0088	0.0090	0.0092	0.0094	0.0096
80	0	0	50	25	0.0101	7	0.0110	0.0114	0.0117	0.0121	0.0125
80	0	0	50	50	0.0101	7	0.0101	0.0102	0.0103	0.0104	0.0104
80	-0.25	0	25	25	0.0121	7	0.0106	0.0114	0.0119	0.0124	0.0130
80	-0.25	0	25	50	0.0121	7	0.0102	0.0104	0.0106	0.0109	0.0112
80	-0.25	0	50	25	0.0121	7	0.0128	0.0131	0.0135	0.0138	0.0142
80	-0.25	0	50	50	0.0121	7	0.0119	0.0120	0.0121	0.0122	0.0123
80	-0.75	0	25	25	0.0169	7	0.0140	0.0145	0.0151	0.0157	0.0164
80	-0.75	0	25	50	0.0169	7	0.0134	0.0137	0.0140	0.0144	0.0149
80	-0.75	0	50	25	0.0169	7	0.0169	0.0163	0.0175	0.0179	0.0182
80	-0.75	0	50	50	0.0169	7	0.0162	0.0163	0.0164	0.0165	0.0167

Note: $S_0 = 90,\ r = 0.06,\ d = 0.06,\ \mathrm{V}_0 = 0.01,\ \overline{V} = 0.49,\ \sigma_V = 0.2,\ \kappa_V = 5.06,\ \alpha = 1.64,\ \mathrm{T} = 0.25$

Prem: the value of early-exercise premium (approximates – european option)

Table 4 Comparison of the early-exercised premium (Prem) with different probability of upward jumps (P)

	parameter values					Prem (approx-euro)				
	K	ρ	λ	η_1	η_2	P				
						0	0.25	0.5	0.75	1
put	100	-0.75	7	25	50	0.1086	0.0949	0.0863	0.0802	0.0755
	100	-0.75	7	50	50	0.1086	0.1066	0.1045	0.1030	0.1014
	100	-0.75	7	50	25	0.1092	0.1068	0.1050	0.1032	0.1014
call	80	-0.75	7	25	50	0.0596	0.0635	0.0668	0.0696	0.0721
	80	-0.75	7	50	50	0.0596	0.0605	0.0615	0.0626	0.0636
	80	-0.75	7	50	25	0.0482	0.0509	0.0542	0.0583	0.0636

Note:

$S_0 = 90,\ r = 0.06,\ d = 0.06,\ V_0 = 0.01,\ \overline{V} = 0.49,\ \rho = -0.75,\ \sigma_V = 0.2,\ \kappa_V = 5.06,\ \alpha = 1.64,\ T = 0.25$