國立交通大學

財務金融研究所

碩 士 論 文

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摘要

估計違約風險是定價公司債、交換合約與信用衍生性金融商品的一個關鍵因素。2007 年全球金融風暴過後，企業的違約風險更加受到學術界與實務界的重視，特別像是有些公司原本營運狀況良好，卻受到金融風暴影響而突然發生破產危機，因此，如何能夠準確的預測企業的違約風險比以往更受到重視。

本篇研究提出一種新的架構來評價公司證券，以結構性信用風險模型為基礎，加入了巴黎選擇權的架構，並且以跳躍擴散模型 (Kou 2002) 來當作評價公司市場價值的模型，此模型較之前文獻中的模型具有較彈性的參數設定，更符合實證上之需求。此外，我們並改善數值模擬方法以加快數值計算的速度，以此方法來估計公司債價值。

關鍵字：巴黎選擇權、跳躍擴散模型、蒙地卡羅模擬法、結構性信用風險模型

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Abstract

After the worldwide financial crisis in 2007, credit risk of the company is getting vast attention not only from academic but also from people in practice. Specifically, many firms had good rating but suddenly default during the financial crisis. Hence, how to accurately model the default risk of the firm is a much more important issue nowadays.

In this paper, we develop a more efficient numerical simulation method to value the corporate risky bond. Our model employs the structural approach for valuing corporate bonds under the double exponential jump diffusion process (Kou 2002). This approach has more flexibility in matching the empirical data than previous models. In addition, to make our model more realistic, we adopt the caution time setting, which is parallel to the Parisian option in option pricing, to model the bond safety covenant.

Keywords: Parisian option, double exponential jump diffusion process, Monte Carlo simulation, structural credit risk model

Contents

List of Exhibit

List of Figures

1. Introduction

There are two fundamental approaches to model the default risk of corporations. One approach is the structural form which models the firm value pioneer by Black and Scholes (1973) and Merton (1974), and extended by Black and Cox (1976), Longstaff and Schwartz (1995), Leland (1998), Zhou (2001), and others. The other is the reduced form, brought up by Jarrow et al.(1997), Madan and Unal (1998), Duffie and Singleton (1999), and others, using the Possion distribution to model the default rate. The reduced form does not consider the relation between corporation's capital structural and default risk.

The primary purpose of this paper is to build a model extending the structural form under the jump-diffusion process that combines the concept of safety covenants and features of empirical data that return has features of heavier tails and left-skewed. Under the above condition, we provide a new model composed of the Parisian option framework and double exponential jump diffusion process. The new model should better fit in with the empirical data of both short-term and long-term default rates and yield spreads.

The main reason that we use the Parisian option framework under double exponential jump diffusion to simulate asset value is that this model is more reasonable and flexible for pricing bond values than barrier option under Merton jump-diffusion. The assumption of barrier option is that if firm value drops below the pre-specific level, the firm will shut down immediately. This first-passage time model helps us to model the safety covenants, but such safety covenants are often too strict to firms. Fujita and Ishizaka (2002) use the Parisian option framework to present the concept of "caution time" into original first-time passage model. In their model, if firm value hits the barrier, the firm is into caution time. The firm defaults if the time that firm value is below the barrier exceeds caution time. Their model gives more flexibility to the structural form model and safety covenant. In this paper, we follow the concept of caution time to pricing bond value plus double exponential jump diffusion process. The double exponential jump diffusion model has many nice features for structural model, including:

A. According to research, bond price often drops surprisingly around the time of default (Beneish and Press, 1995; Duffie and Lando, 2001). The jump-diffusion model, including the double exponential jump diffusion mode, is consistent with this evidence. Many situations may cause the jump of bond price, such as a nature disaster, lawsuits and sudden financial turmoil.

B. In practice, the empirical data of return distribution are skewed to left, and have higher peak and heavier tails than normal distribution. The double exponential jump diffusion model is more flexible in parameter setting than Merton jump-diffusion. This model can adjust the probability of up-side jump and down-side jump. In addition, it also can set the up-side jump and down-side jump amplitude separately. These features of the double exponential jump diffusion let the return fit in with the empirical data.

C. The double exponential jump diffusion has more flexibility to match empirical credit spreads. Credit spreads styled facts are: (1) Credit spreads do not converge to zero even for very short maturity bonds. (2) Credit spreads have downward, humped, and upward shapes. These shapes present firms' financial distresses.

All of the above points are motives for pricing bonds under the double exponential jump diffusion.

The remaining sections of this paper are as follows: Section 2 reviews the literature related to this paper. Section 3 presents the structural model which is based on Parisian option framework under the double exponential jump diffusion. Section 4 proposes a fast numerical method to simulate bond value and presents the results of simulations. Section 5 is the conclusions.

2. Literature Review

2.1 Option Pricing Model Reviews

Black and Scholes (1973) offer an explicit model for option pricing. They derive a close-form expression from Brownian motion for pricing the European option. They provide a new vision of pricing option.

Following the Black and Scholes model, Merton (1976) extends the Black and Scholes diffusion process model to the jump-diffusion process model. He is the first to derive the close-form expression of jump diffusion model. This model has an advantage to match the real world in that asset return sometimes has a discontinuous jump due to incomplete information. However, the assumption of this model is that rate of return follows log-normal distribution. It is not consistent with the empirical research that return distribution has left skewed and heavier tail than normal distribution.

Kou (2002) provides an option pricing approach under double the exponential jump diffusion process. This process has many good features, including the probability and tendency that up-side and down-side jumps could be given separately. Because of the nice features of the double exponential jump diffusion, the log-normal return assumption of Merton model could be corrected. In addition, double exponential jump diffusion process is easy to use for option pricing.

2.2 Structural Form Model

Early theorization of structural form model can be traced back to Merton (1974). Merton provides an approach that can use corporate capital structure to price corporate debt and default risk. He points out that equity value could be considered as a call option which is priced by the Black-Scholes model (1973). There are some disadvantages of using Black-Sholes model when pricing equity value, such as ignoring that low liquidity makes corporate bond default and bond default happen only at maturity, hence the following literatures modified the original structural model of Merton.

Black and Cox (1976) extend the Black-Sholes model and solve the problem that bond default only occurs at maturity. It allows for corporate bond default anytime before maturity only if the bond value hits a pre-specific level. Once the bond value reaches the pre-specific level, the corporation goes into default or is liquidated immediately. Although Black and Cox relax the assumption of default time of Black-Scholes and Merton framework, this model still shares some assumptions with the Merton model. One of the drawbacks of this approach is that interest rate is assumed to be constant.

After Black and Cox, Longstaff and Schwartz (1995) develop a new approach to pricing risky bonds. This model incorporates the Black and Cox model with interest rate risk. This approach has an important advantage in that close-form expression for both risky fixed-rate and floating-rate bonds could be derived. It relaxes the assumption of a constant interest rate.

Another assumption of the Black and Cox model is that the remaining value of the firm at default has to be equal to the default boundary. Zhou (2001) provided a new model for solving this assumption. He combines Merton jump-diffusion process with the Black and Cox structural model; hence this model is able to endogenously produce random variation in recovery rate. Besides this, the jump-diffusion model solves another problem that the default rate reaches to zero when time maturity is in a very short-term.

Because of the features of a down-and-out Parisian option that expires if the underlying asset price goes down, hits a specific barrier level and stays below this level for a period window, Fujita and Ishizaka (2002) propose a new concept, "caution time," for relaxing safety covenant. Their model states if firm value drops below the barrier, the bondholders will have observations on operation of the firm; this is what is meant by “caution time”. If the time in which firm value stays below the barrier exceeds “caution time”, bondholders think the firm defaults. Also, if the firm value is below the barrier at maturity, bondholders believe the firm to be in default.

Francois and Morellec (2004) use the down-and-out Parisian option for modeling risky bonds under Chapter 11 of the U.S. Bankruptcy Code. They point out that Parisian option’s special feature of period window could fundamentally represent that a corporation renegotiate in financial distress under Chapter 11 of the U.S. Bankruptcy Code. This model lets bondholders and shareholders have an unambiguous effect on default incentives and credit spread.

Chen and Kou (2009) extend the model under the double exponential jump diffusion model of the barrier option framework for credit risk. This model presents that jump risk and endogenous default can have significant effect on credit spread. This model has more flexible shapes of jump to explain the empirical data than jump-diffusion model.

2.3 Parisian Option Reviews

Chesney, Jeanblanc and Yor (1997) define a new option called Parisian option which is extended from the barrier option framework. A down-and-out (up-and-out) Parisian option is an option that expires if the underlying asset price goes down (up), hits a specific barrier level and stays below (above) for a period window. Conversely, A down-and-in (up-and-in) Parisian option is an option that comes into existence if the underlying asset price goes down (up), hits a specific barrier level and stays below (above) the period window. They derive a formula based on the Brownian motion theory for pricing Parisian option.

According to the definition of Parisian option, Avellaneda and Wu (1999) formulate a partial differential equation (PDE) for Parisian option. The PDE solves Parisian option pricing numerically on a trinomial lattice. They also characterize the value function of Parisian option in the continuous limit.

Bernard, Le Courtois and Quittard (2005) develop a new inverse Laplace which transforms the method used to price Parisian option. They provide a quick and simple numerical method to compute the price and Greeks of Parisian option.

3. Model

3.1 Asset Model

In this section, we describe the model of pricing a firm's assets using the double exponential jump diffusion process model of Kou (2002). Under the double exponential jump diffusion process model, the firm's asset value has two parts. One is a continuously pure diffusion process worked by geometric Brownian motion. The other is a jump part. Jump sizes follow the double exponential distribution and the jump times are driven by the event times of a Possion distribution.

To price the asset under the double exponential jump diffusion process, following the research of Lucas (1978) with a HARA type of utility function for the representative agent, we could consider that equity and debt are contingent claims of an asset. The rational equilibrium price of an asset is given by the expectation of discounted asset payoff, where the expectation is estimated under the risk-neutral probability measure P. More precisely, we build the following equation used for modeling value of firm's assets $V(t)$ following a double exponential jump diffusion process under risk-neutral measure P :

$$\frac{dV(t)}{V(t-)} = \left(r - \lambda \xi\right) dt + \sigma dW(t) + d\left(\sum_{i=1}^{N(t)} (Z_i - 1)\right) \tag{3.1}$$

The solution of the equation is given by

$$V(t) = V(0)\exp\left\{\left(r - \frac{1}{2}\sigma^2 - \lambda\xi\right)t + \sigma W(t)\right\}\prod_{i=1}^{N(t)} Z_i \tag{3.2}$$

where r is the risk-free interest rate (we assume that interest rate is constant), σ is the volatility of the asset, and ξ is the mean of percentage jump size:

$$\xi \equiv E\left[Z-1\right] = E\left[e^Y - 1\right] = \frac{p\eta_u}{\eta_u - 1} + \frac{q\eta_d}{\eta_d + 1} - 1 \tag{3.3}$$

$W(t)$ is a standard Brownian motion under risk-neutral measure P, $N(t)$ is a homogenous Possion process with mean λ, and Z_i is a series of independent identically distribution nonnegative random variables so that $Y = \ln(Z)$ has a density of the double exponential distribution:

$$f_y(y) = p \cdot \eta_u e^{-\eta_u y} \cdot I_{\{y \ge 0\}} + q \cdot \eta_d e^{\eta_d y} \cdot I_{\{y<0\}},\ \eta_u > 1,\ \eta_d > 0 \tag{3.4}$$

where $p,\ q \ge 0,\ p+q=1$, $I_{\{y \ge 0\}}$, $I_{\{y<0\}}$ are indicator functions. The condition $\eta_u > 1$ is to confirm that expectation of $V(t)$ is finite. p and q are the probability of up-side jump and down-side jump. The means of two exponential distributions are $1/\eta_u$ and $1/\eta_u$. The mean of Y is $p/\eta_u - q/\eta_d$. In this model, $W(t)$, $N(t)$, and Y are assumed to be independent. The return process $X(t) \equiv \ln\left(V(t)/V(0)\right)$ is the following equation:

$$X(t) = \left(r - \frac{1}{2}\sigma^2 - \lambda\xi\right)t + \sigma W(t) + \sum_{i=1}^{N(t)} Y_i \tag{3.5}$$

where $X(0) = 0$, and the equation is still under risk-neutral probability measure P. If Y is a normal distribution, the model is the same as the Merton jump-diffusion model.

3.2 Pricing corporate debts

The next step is to build the bond value model. We follow the assumptions of the asset pricing model described in section 3.1 and Parisian option framework. We assume that the bond defaults if a firm's asset value is under a level, H, which is a exponential barrier, and the time of asset value below the barrier $\hat{t}$ is over a window period

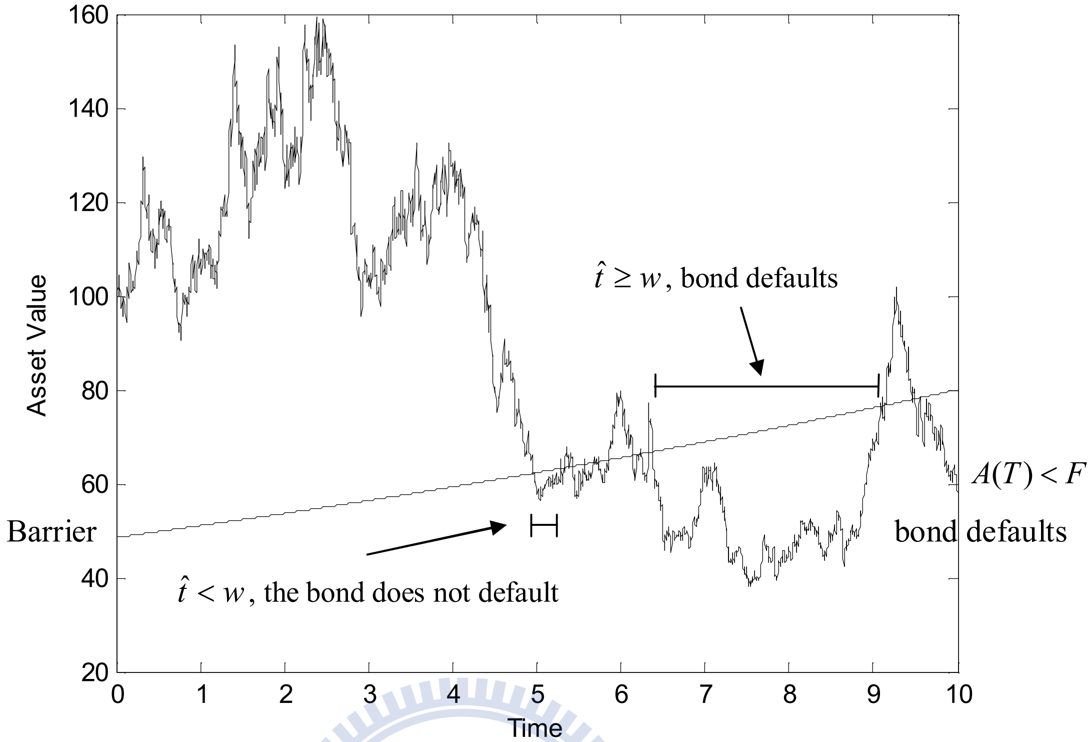

Figure 1. The “caution time” and the situation defaults.

(caution time) w[1]. The first time that bond default is defined as time τ.

Mathematically,

$$H(t) = e^{-\phi(T-t)} F, \ \phi \geq r, \ T > 0, \ 0 \leq t \leq T \tag{3.6}$$

$$\tau \equiv \inf \{ t \geq 0 \mid \hat{t} \geq w \} \tag{3.7}$$

$$\hat{t}(V(t), t) = \begin{cases} 0 & \text{if V(t)} > H(t) \\ t - g_t & \text{if V}(t) \leq H(t) \end{cases} \tag{3.8}$$

$$g_t = \sup \{ s \leq t \mid V(s) = H(s) \} \tag{3.9}$$

where ϕ is barrier discount rate, F is face value of bond, g_t is the last time before t that asset value hit the barrier. If $\tau \geq T$ it means that the bond does not default before maturity. If $\tau < T$ means that the bond defaults before maturity, the bondholders only receive the asset value of the firm minus the write-down value at

[1] Barrier option framework is a special case of Parisian option framework, if caution time equal to zero.

default time. If the asset value is under the barrier with bond life-expired, bondholder consider the bond as default. Figure 1 shows the “caution time" framework. In general, write-down value is a non-increasing function of asset value. We assume the equation of write-down value is a linear form:

$$R(V) = R_1 \cdot V \tag{3.10}$$

where R_1 is a non-negative constant. Because we follow the concept of caution time, we consider the firm in default if the firm’s asset value is below the barrier at maturity. In this model, we also assume that coupon rate does not affect the result of our research such that we focus on zero-coupon bond for our research. We can derive the price of a risky bond by using a fundamental bond pricing approach that discounted cash flow. The bond price $B(V,\ T)$:

$$B(V,\ T) = E^P\left[\left\{\exp(-rT)\cdot\left(F\cdot I_{\{V(T)\geq H(T)\}} + \left(V(T) - R(V(T))\right)\cdot I_{\{V(T)<H(T)\}}\right)\right\} I_{\{\tau\geq T\}} + \exp(-r\tau)\left(V(\tau) - R(V(\tau))\right) I_{\{\tau<T\}}\right] \tag{3.11}$$

where P is risk-neutral probability measure, I is indicator function, T is time of maturity, and τ is time of default. This equation is combined with two parts. The first part of equation is the present value of cash flow which bondholder could receive at maturity. The second part is the present value of cash flow which bondholder could receive if bond default before maturity. For a tractable approach in Monte Carlo method, we provided the following equation:

$$B(V,\ T) = \lim_{N\to\infty}\sum_{i=1}^{N}\frac{1}{N}\left[\left\{\exp(-rT)\cdot\left(F\cdot I_{\{V_i(T)\geq H(T)\}} + \left(V_i(T) - R(V_i(T))\right)\cdot I_{\{V_i(T)<H(T)\}}\right)\right\} I_{\{\tau\geq T\}} + \exp(-r\tau)\left(V_i(\tau) - R(V_i(\tau))\right) I_{\{\tau<T\}}\right] \tag{3.12}$$

This equation for pricing bond value could be easily programmed by the Monte Carlo Method.

4. Result

4.1. Numerical Method

Parisian option is a path-dependent option which takes considerable time to simulate. In order to reduce the computing time of the Monte Carlo method, Metwally and Atiya (2002) provide an approach called Uniform Sampling for speeding up the simulation time of calculation. This approach is based on the Brownian bridge concept which is proposed by Karatzas and Shreve (1991), and Revuz and Yor (1994). The Brownian bridge concept is that, if you have a Wiener process defined by a series of time-indexed random variables $\{W(t_1), W(t_2), \ldots\}$. You could use the Brownian bridge method to insert a random variable $W(t_k)$, where $t_i < t_k < t_{i+1}$, into the series in such a manner that the result of series remains unchanged. Given $W(t)$ and $W(t+\Delta t_1+\Delta t_2)$, we want to get $W(t+\Delta t_1)$. We use Brownian Bridge method to assume that we could get $W(t+\Delta t_1)$ by a weighted average of $W(t)$ and $W(t+\Delta t_1+\Delta t_2)$ plus an independent normal random variable:

$$W(t+\Delta t_1) = \alpha W(t) + \beta W(t+\Delta t_1+\Delta t_2) + \gamma Z \tag{4.1}$$

$$\alpha = \frac{\Delta t_2}{\Delta t_1 + \Delta t_2}$$

$$\beta = 1-\alpha$$

$$\gamma = \sqrt{\Delta t_1 \alpha}$$

where α, β and γ are constants to be determined, and Z is a standard normal random variable.

Metwally and Atiya follow the Brownian bridge concept to calculate the probability of no crossing barrier, if we know the two-end point value. Let the jump times be $T_1, T_2, \ldots, T_M$, these are the first variables that should be generated. We assume $x(T_i^-)$ is the instant process value before the *i-th* jump and $x(T_i^+)$ is the instant process value after the i-th jump. Between any two jumps, $x\left(T_i^+\right)$ and $x\left(T_{i+1}^-\right)$, the process is under the pure Brownian motion. From Metwally and Atiya (2002), whose model is jump diffusion model and they assume barrier is flat. Let B_s be a Brownian bridge in the interval $\left[T_i, T_{i+1}\right]$ and $\tau = \left(T_{i+1} - T_i\right)$. The probability of no barrier crossing in the interval $\left[T_i, T_{i+1}\right]$:

$$P_i = P\left(\inf_{T_i \le s \le T_{i+1}} \left\{B_s > \ln H\right\} \mid B_{T_i^+} = x\left(T_i^+\right), B_{T_{i+1}^-} = x\left(T_{i+1}^-\right)\right)$$

$$= \begin{cases} 1 - \exp\left(-\dfrac{2\left[\ln H - x\left(T_i^+\right)\right]\left[\ln H - x\left(T_{i+1}^-\right)\right]}{\tau\sigma^2}\right) & \text{if } \ x\left(T_{i+1}^-\right) > \ln H \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

In our model, we assume that barrier is an exponential function of time so that we have to modify the drift-term of the double exponential jump diffusion process to let the barrier be constant. In this case, our new process of return and barrier become:

$$X'(t) = \left(r - \phi - \frac{1}{2}\sigma^2 - \lambda\xi\right)t + \sigma W(t) + \sum_{i=1}^{N(t)} Y_i \tag{4.3}$$

$$H' = \mathrm{e}^{-\phi T} F \tag{4.4}$$

We let c equal the drift-term of $X'(t)$. The following approach is the Monte Carlo method for a bond pricing under Parisian option structural and modified double exponential jump diffusion process:

Step 1. For n=1 to N do Monte Carlo simulations as follow *Steps 2~5*.

Step 2. Generate the jump-timing t_i from a given density function (In this paper, we use exponential distribution.). We repeat *Step 2* until $\sum t_i > T$.

Step 3. For $i = 1$ to $M + 1$ (M is the number of jumps that happen during the whole life of asset). Generate the return of asset for all jump point.

(a). We let $x(t) = \ln V(t)$, the initial value $x(t_0) = x(0) = \ln V(0)$ and generate the return of asset before jump $x(t_i -)$ from Gaussian distribution under mean $x(t_{i-1}) + c(t_i - t_{i-1})$ and standard deviation $\sigma\sqrt{t_i - t_{i-1}}$.

(b). Generate a random variable from Uniform [0, 1]. Put the random variable in CDF of the double exponential distribution[2] and generate the jump size J_i[3].

(c). Generate return of asset after jump $x(t_i+) = x(t_i-) + J_i$.

Step 4. For intervals $i = 0$ to M, set $default = 0$, $check\text{-}time = 0$, $i = 0$ at first, let $x(t_0+) = x(0)$, while $(default = 0)$ or $(i < M+1)$, we continue the loop.

(a). If $x(t_i+) > \ln(H')$, set $check\text{-}time = 0$

1. if $x(t_{i+1}-) > \ln H'$, compute the probability of no barrier crossing P_i based on equation (4.2)

2. Let $b = (T_{i+1} - T_i)/(1 - P_i)$

3. Generate s from a uniform distribution in the interval $[T_i, T_i + b]$

4. If $s \in [T_i, T_{i+1}]$, then the asset value crosses the barrier for the first time at time s in interval $[T_i, T_{i+1}]$. Since we know the asset value at time s and T_{i+1} is $\ln H'$ and $x(t_{i+1}-)$, we assume that 1 year could divide by K days and use the Brownian bridge method (4.1) to simulate the asset process from s to T_{i+1}. We check the process of each point whether crossing above the barrier before the time T_{i+1}.

For intervals $j = 1$ to $[T_{i+1} - s] \cdot K$

(1). If $x(s + j/K) < \ln H'$, then $check\text{-}time = check\text{-}time + 1/K$

(2). If $x(s + j/K) \geq \ln H'$, then we reset $check\text{-}time = 0$

i. If $check\text{-}time \geq w$, then $default = 1$

$$DiscBond_n = \exp[(\phi - r)\tau] \cdot [(1-R)(\exp(x(\tau)))]$$

Exit loop, compute another Monte Carlo cycle (*Step 2~5*).

ii. else, $j = j + 1$

(3). when $j = [T_{i+1} - s] \cdot K$, and $check\text{-}time < w$, let $i = i + 1$, repeat *Step 4*.

---

[2] The proof of the double exponential jump diffusion CDF is in appendix A.

[3] Appendix B. shows the method to generate jump size under the double exponential jump diffusion.

5. If $s \notin [T_i, T_{i+1}]$, then asset does not cross the barrier,

$i = i+1$, repeat *Step 4*.

(b). If $x(t_i+) \le \ln(H')$, since we know the asset value at time T_i and T_{i+1} is $x(t_i+)$ and $x(t_{i+1}-)$, we directly use the Brownian Bridge method (4.1) to simulate the asset process in interval $[T_i, T_{i+1}]$ such as *Step 4.(a).4.*. We check the process of each point whether crossing above the barrier before the time T_{i+1}. For intervals $j=1$ to $[T_{i+1}-T_i]\cdot K$

(1). If $x(t_i + j/K) < \ln H'$,

then $check\text{-}time = check\text{-}time + 1/K$

(2). If $x(t_i + j/K) \ge \ln H'$, then we reset $check\text{-}time = 0$.

i. If $check\text{-}time \ge w$, then $default = 1$

$$DiscBond_n = \exp[(\phi - r)\tau]\cdot\left[(1-R)\left(\exp\left(x(\tau)\right)\right)\right]$$

Exit loop, compute another Monte Carlo cycle (*Step 2~5*).

ii. else, $j = j+1$

(3). when $j = [T_{i+1} - T_i]\cdot K$, and $check\text{-}time < w$, let $i = i+1$, repeat *Step 4*.

(c). When $i = M+1$, check $x(T)$

(1). If $x(T) \ge \ln(H')$ and $default = 0$

$$DiscBond_n = \exp(-rT)\cdot F$$

(2). Else, let $default = 1$

$$DiscBond_n = \exp[(\phi - r)T]\cdot\left[(1-R)\left(\exp\left(x(T)\right)\right)\right]$$

Exit loop, compute another Monte Carlo cycle (*Step 2~5*).

Step 5. If $n = N$, we finish the Monte Carlo simulation. We could calculate the estimate for the risky bond price:

$$DiscBond = \frac{1}{N}\sum_{n=1}^{N} DiscBond_n$$

We run a MATLAB program on an Intel T4400 2.20 GHz CPU 1 million Monte

Exhibit 1. The comparisons between different simulation methods. $F = 80$.

Method	Std Error	CPU Time (per million iterations)	Std Error × CPU Time
Standard Monte Carlo $\Delta = 1/12$	0.0227	1096	24.8792
Standard Monte Carlo $\Delta = 1/52$	0.0217	4695	101.8815
Standard Monte Carlo $\Delta = 1/252$	0.0201	21007	422.2407
Uniform Sampling $K = 252$	0.0123	66	0.8118

The CPU time is seconds per million iterations.

Exhibit 2. The comparisons between different simulation methods. $F = 90$.

Method	Std Error	CPU Time (per million iterations)	Std Error × CPU Time
Standard Monte Carlo $\Delta = 1/12$	0.0342	1123	38.4066
Standard Monte Carlo $\Delta = 1/52$	0.0334	5236	174.8824
Standard Monte Carlo $\Delta = 1/252$	0.0325	20629	670.4425
Uniform Sampling $K = 252$	0.0184	83	1.5272

The CPU time is seconds per million iterations.

Carlo iterations for each method to value the bond price. We compare the Uniform sampling method with standard Monte Carlo method. In exhibit 1, we use parameter settings as follows: $V(0)=100$, $F=80$, $r=0.05$, $\phi=0.05$, $\lambda=0.2$, $R_1=0.4$, $p=0.5$, $q=0.5$, $\eta_1=2.79667154579233$, $\eta_2=2.12168612641381$, $T=1$, $w=1/12$, $\sigma^2=0.02$. In exhibit 1, the Uniform Sampling method greatly reduces time of simulation. From the result of Std Error × CPU time, we know the Uniform Sampling

Exhibit 3. The comparisons between different simulation methods. $F = 95$.

Method	Std Error	CPU Time (per million iterations)	Std Error × CPU Time
Standard Monte Carlo $\Delta = 1/12$	0.0382	905	34.5710
Standard Monte Carlo $\Delta = 1/52$	0.0381	4148	158.0388
Standard Monte Carlo $\Delta = 1/252$	0.0379	19437	736.6623
Uniform Sampling $K = 252$	0.0184	93	1.7112

The CPU time is seconds per million iterations.

method is more efficient than standard Monte Carlo. The standard error of the Uniform Sampling method is also smaller than standard Monte Carlo. It presents that Uniform Sampling has a more accurate Monte Carlo simulation result with the same Δt. In addition to more efficiency, the Uniform Sampling method has lower bias than standard Monte Carlo method. The reason is that the Uniform Sampling method uses uniform distribution to generate the time of hitting barrier. This action make the Uniform Sampling has less discontinuous simulation. In exhibit 2 and 3, we change the face value and other parameters remain the same. We increase the face value from 80 to 90 and 95 in exhibit 2 and exhibit 3. The standard error significantly increases with higher face value in standard Monte Carlo methods. The probability of using Brownian bridge method is increased with higher face value in the Uniform Sampling method, but the simulation time is almost unchanged.

4.2. Numerical Result

In this paper, we propose a difference between a Parisian option framework and a barrier option framework. Also, we want to present a difference between the double exponential jump diffusion model and the Merton jump-diffusion model. We follow the concept of Zhou's approach (2001). We control the overall mean and volatility of the firm's value to be constants as we change the parameter values which domain the random component of asset value. Therefore we know that the variations of bond values are truly caused by different combinations of parameter values rather than by the changes in overall mean and volatility of the firm's value. To do this, we have to know under the risk-neutral measure P, the mean and volatility of return in these models. From Ramezani and Zeng (2006), we know the moment of return under physical measure Q in these models, thus we can use this result to easily get the moment of return under risk-neutral measure P. We let X be the return of asset value. We control EX and $Var(X)$ by these moments[4] of return X in different models to observe the effect caused by changing the parameter. In the following figures, each point is simulated 1 million times for precise value.

First, we want to present the difference of structure model between a barrier option and a Parisian option framework. In this case, we simulate the asset value under the double exponential jump diffusion. We control the parameter settings that total variance $=0.09$, total mean $=0.005$, $V(0)=100$, $F=80$, $r=0.05$, $\phi=0.05$, $\lambda=0.05$, $R_1=0.4$, $p=0.5$, $q=0.5$, and jump variance $=0.35$ thus we find one set that $\eta_u=2.79667154579233$, $\eta_d=2.12168612641381$, and variance of pure diffusion $\sigma^2=0.0725$. We change the caution time by no caution time, 5 days, 10 days, 15 days, 1 months, 6 months, and 1 year to observe the effects caused by these changes. Because there are no apparent differences after 15 days caution time, figure 2 and 3 only presents the result of no caution time to 15 days caution time. Figure 2. presents the relationship between cumulative default probability and maturity under different caution time. It shows that longer caution time has less cumulative default probability. Figure 3 shows

[4] Appendix C presents the moments in Merton jump-diffusion model and the double exponential jump diffusion model.

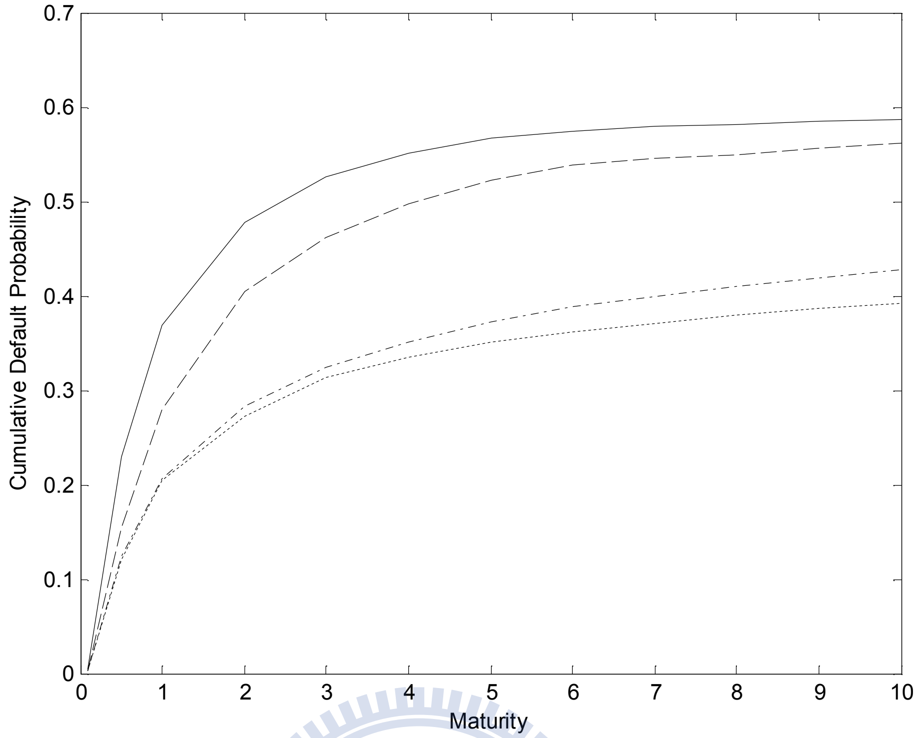

Figure 2. The relationship between cumulative default probability and maturity in different caution time: (—) : no caution time; (--): 5 days; (—·) : 10 days; (···) : 15 days.

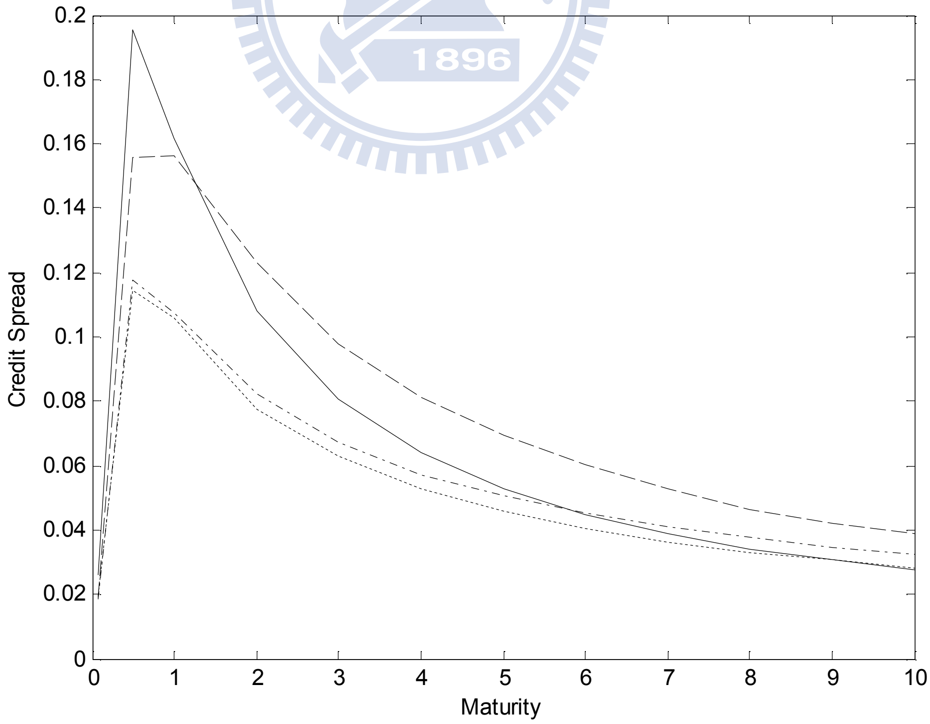

Figure 3. The relationship between credit spreads and maturity in different caution time: (—) : no caution time; (--): 5 days; (—·) : 10 days; (···) : 15 days.

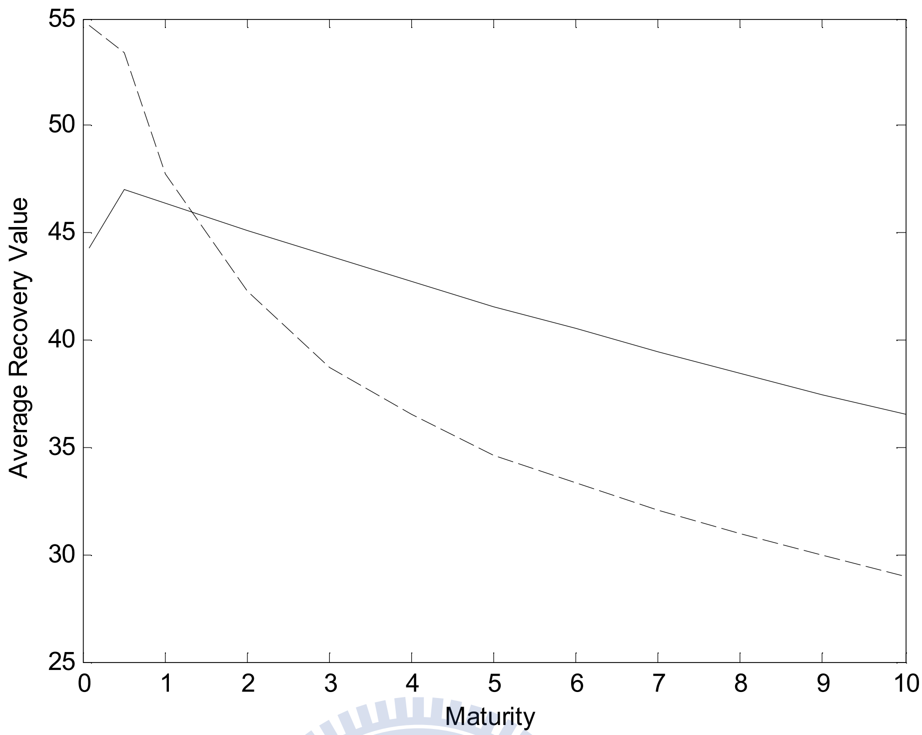

Figure 4. The relationship between average recovery value and maturity in different caution time: $(-)$: no caution time; (--): 5 days.

that credit spread after 2 years maturity under barrier option framework is lower than Parisian option framework with 5 days caution time. The credit spreads decrease under Parisian framework with caution time increasing. To examine the result, we check the average recovery value of no caution time and 5 days caution time. We save the recovery value of no caution time and 5 days caution time in the same iteration. The result of average recovery value in figure 4. is consistent with figure 3. that no caution time has more average recovery value than 5 days caution time after 2 years maturity. The high credit spread under 5 days caution is due to the low recovery value. This combination of parameter settings leads this result that the process of asset value usually goes down in 5 days after first hit time.

Second, we want to present the flexibility of the double exponential jump diffusion model. We compare the double exponential jump diffusion model with the Merton

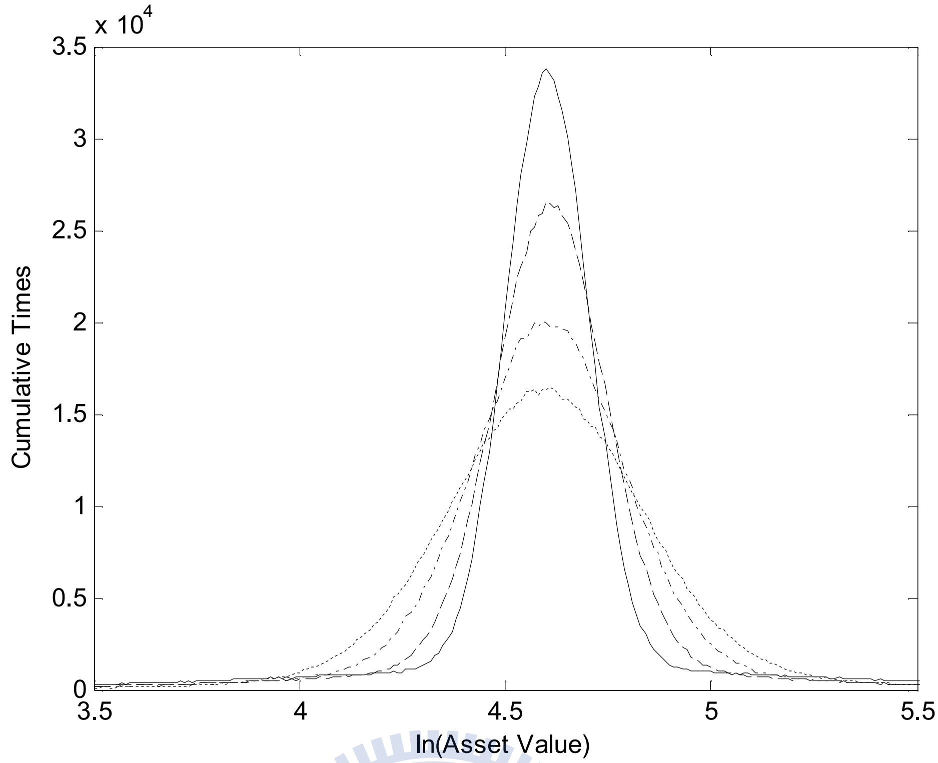

Figure 5. The relationship between cumulative times and asset value in different model and skewness. $(-)$: Merton jump-diffusion model with $skewness = 0$, $\lambda = 0.2$, $\sigma^2 = 0.01$, $\alpha = 0.016250257$, $\beta = 0.399738782$; the double exponential jump diffusion model $\lambda = 0.2$, $p = 0.5$ and $q = 0.5$: (--): $skewness = -0.5$, $\eta_u = 2.470527501$, $\eta_d = 2.241299129$ and $\sigma^2 = 0.017418446$; (-·): $skewness = 0$, $\eta_u = 2.616159165$, $\eta_d = 2.616159142$ and $\sigma^2 = 0.03155712$; (···): $skewness = 0.5$, $\eta_u = 2.85627674$, $\eta_d = 3.658956675$ and $\sigma^2 = 0.050546344$

jump-diffusion model with the same EX, $Var(X)$ and λ. We control the total mean $EX = 0$, total variance $Var(X) = 0.09$, $r = 0.05$ and $\lambda = 0.2$ that we adjust the parameter settings to generate different skewness. In Merton jump-diffusion model, the flexibility of skewness is limited. In this case, we let $skewness = 0$ and variance of pure diffusion $\sigma^2 = 0.01$ generate a setting combination so that mean of jump size $\alpha = 0.016250257$ and variance of jump size $\beta = 0.399738782$ under the Merton jump-diffusion model. Also, we generate three setting combinations with different skewness under the double exponential jump diffusion model. First, we control $skewness = -0.5$, $p = 0.5$ and $q = 0.5$ to generate a setting combination so that

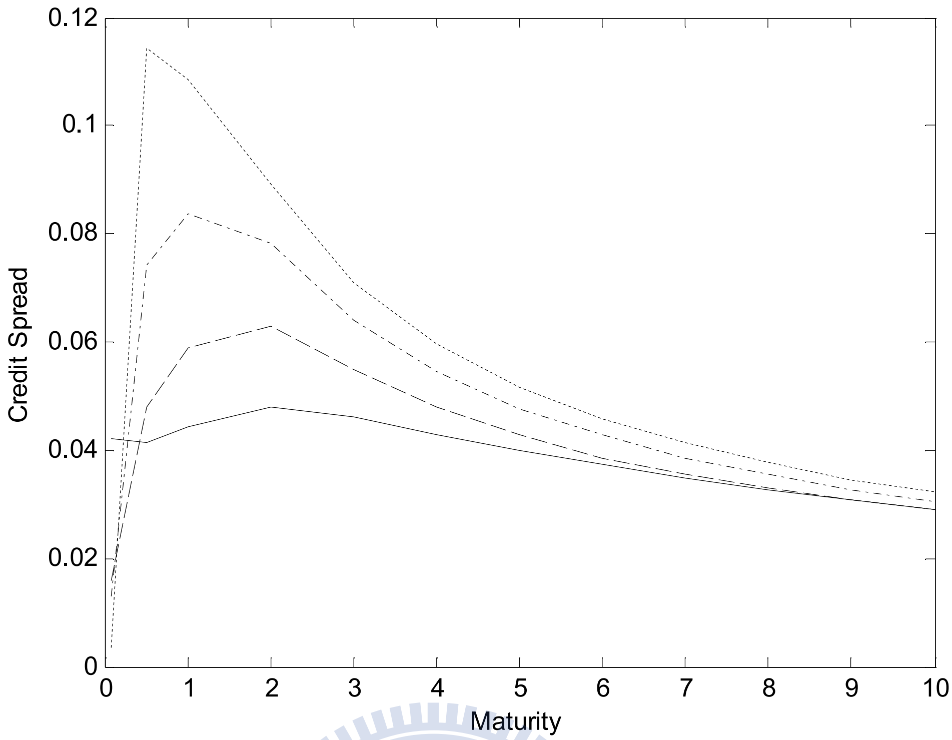

Figure 6. the relationship between credit spreads and maturity in different model and skewness. (−) : Merton jump-diffusion model with $skewness = 0$, $\lambda = 0.2$, $\sigma^2 = 0.01$, $\alpha = 0.016250257$, $\beta = 0.399738782$; the double exponential jump diffusion model $\lambda = 0.2$, $p = 0.5$ and $q = 0.5$: (--): $skewness = -0.5$, $\eta_u = 2.470527501$, $\eta_d = 2.241299129$ and $\sigma^2 = 0.017418446$; (-·) : $skewness = 0$, $\eta_u = 2.616159165$, $\eta_d = 2.616159142$ and $\sigma^2 = 0.03155712$; (···) : $skewness = 0.5$, $\eta_u = 2.85627674$, $\eta_d = 3.658956675$ and $\sigma^2 = 0.050546344$.

$\eta_u = 2.470527501$, $\eta_d = 2.241299129$ and $\sigma^2 = 0.017418446$. Second, we control $skewness = 0$, $p = 0.5$ and $q = 0.5$ to generate a setting combination so that $\eta_u = 2.616159165$, $\eta_d = 2.616159142$ and $\sigma^2 = 0.03155712$. Third, we control $skewness = 0.5$, $p = 0.5$ and $q = 0.5$ to generate a setting combination so that $\eta_u = 2.85627674$, $\eta_d = 3.658956675$ and $\sigma^2 = 0.050546344$. Figure 5 is the asset value distribution under these parameter settings with 1 year maturity. It shows the skewness with different parameter settings. Figure 6 shows the relationship between credit spreads and maturity with different skewness under the Merton model and the double exponential jump diffusion model. The result of figure 6, that lower skewness

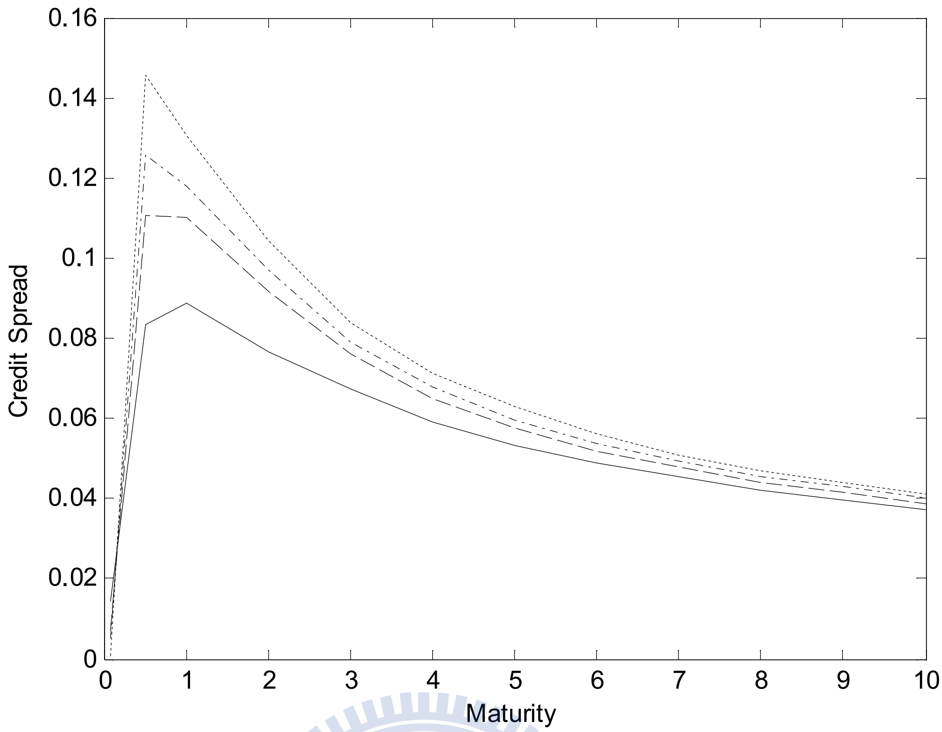

Figure 7. The relationship between credit spreads and maturity in different model and skewness. $(-)$: Merton jump-diffusion model with $skewness = 0$, λ =0.05, $\alpha = 0.03403074$, $\beta = 0.825697084$ and $\sigma^2 = 0.48714694$; the double exponential jump diffusion model λ =0.05, $p = 0.5$ and $q = 0.5$: (--): $skewness = -0.5$, $\eta_u = 1.83694595044413$, $\eta_d = 1.58459906319395$ and $\sigma^2 = 0.0552696595816093$; $(-\cdot)$: $skewness = 0$, $\eta_d = 1.92390672759791$ and $\sigma^2 = 0.0629838660328502$ with $skewness = 0$; $(\cdots)$: $skewness = 0.5$, $\eta_u = 2.07131120647897$, $\eta_d = 3.54067693331008$ and $\sigma^2 = 0.0743575057803415$.

has lower credit spread, is not consist with the comment sense. The lower skewness has larger lose in a short maturity. It should have higher credit spread due to more probability to makes bonds default. In this case, We observe that the lower skewness has higher credit spread in a very short maturity. Therefore, we infer that the weight of variance between pure diffusion and jump size generate this result instead of skewness. To check our inference, we let λ =0.05 so that we can change the weight of variance between pure diffusion and jump size with the same skewness, p and q. We generate a setting combination that $\alpha = 0.03403074$, $\beta = 0.825697084$ and $\sigma^2 = 0.48714694$ under Merton jump-diffusion model. In addition, we generate the three parameter settings under double exponential jump diffusion, 1. $\eta_u = 1.83694595044413$, $\eta_d = 1.58459906319395$ and $\sigma^2 = 0.0552696595816093$ with $skewness = -0.5$, 2. $\eta_u = 1.92390672760078$, $\eta_d = 1.92390672759791$ and $\sigma^2 = 0.0629838660328502$ with $skewness = 0$, 3. $\eta_u = 2.07131120647897$, $\eta_d = 3.54067693331008$ and $\sigma^2 = 0.0743575057803415$ with $skewness = 0.5$. We increase the weight of variance in pure diffusion part in each setting. Figure 7. shows that the credit spread increases after 0.5 year maturity and decrease in a very short maturity. In the long term, the credit spreads under the same EX and $Var(X)$ are very close. This result is consistent with our inference. Although the skewness is not a main reason affecting the shape of credit spread, skewness still limits the varieties of parameter combinations. However, the double exponential jump diffusion model has more flexibility of parameter setting if we control the moments of models. We can use this model generate more shapes of credit spreads.

5. Conclusion

This paper provides a Parisian option framework for corporation risky bond valuation and default risk estimation under the double exponential jump diffusion process. This framework has more flexibility of parameter settings than a barrier option framework under the Merton jump-diffusion model. We demonstrate the shapes of credit spreads in different caution time. Caution time leads to a variety of shapes for credit spreads, default probability and recovery value in every maturity. Besides, the two-side jumps also make bond valuation have more shapes in different maturity. This paper also presents an approach that has more efficient and accurate method to compute the Monte Carlo method under a Parisian option framework. This approach significantly reduces time of computation and bias compared to a standard approach. This is significantly beneficial when we need a quick calculation in a short time.

There are some directions for future research. First, the variance reduction could be used in the Monte Carlo simulation for improving the estimate. Second, it will be interest to study the corporate bond which is composed of senior bond and junior bond.

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Appendix A.

The proof of the double exponential distribution CDF

$$f_y(y) = p \cdot \eta_u e^{-\eta_u y} \cdot I_{\{y \geq 0\}} + q \cdot \eta_d e^{\eta_d y} \cdot I_{\{y<0\}},\ \eta_u > 1,\ \eta_d > 0$$

$$F_Y(y) = \int_{-\infty}^{y} f(x)dx$$

1. If $y < 0$, Let $x' = -x$

$$\begin{aligned} F(y) &= \int_{-\infty}^{y} q\eta_d e^{\eta_d x} dx \\ &= \int_{\infty}^{-y} q\eta_d e^{-\eta_d x'} dx'(-1) \\ &= \int_{-y}^{\infty} q\eta_d e^{-\eta_d x'} \mathrm{dx}' \\ &= -qe^{-\eta_d x'} \Big|_{-y}^{\infty} \\ &= qe^{\eta_2 y} \end{aligned}$$

2. If $\mathrm{y} \geq 0$

$$\begin{aligned} F(y) &= \int_{-\infty}^{y} q\eta_d e^{\eta_d x} + p\eta_u e^{-\eta_u x} dx \\ &= \int_{-\infty}^{0} q\eta_d e^{\eta_d x} dy + \int_{0}^{y} p\eta_u e^{-\eta_u x} dx \\ &= \int_{-\infty}^{0} q\eta_d e^{-\eta_d x'} dy'(-1) + \int_{0}^{y} p\eta_u e^{-\eta_u x} dx \\ &= \int_{0}^{\infty} q\eta_d e^{-\eta_d x'} dy' + p\int_{0}^{y} \eta_u e^{-\eta_u x} dx \\ &= q + p(1 - e^{-\eta_u y}) \\ &= 1 - pe^{-\eta_u y} \end{aligned}$$

$$\therefore F(y) = \begin{cases} 1 - pe^{-\eta_u y} & ,\ y \geq 0 \\ qe^{\eta_d y} & ,\ y < 0 \end{cases}$$

Appendix B.

Let J_i be the logarithm of the ratio of the asset value after and before jump. We assume it is a double exponential distribution.

$$J_i = \ln A(t_i +) - \ln A(t_i -) = \ln Z_i = Y_i$$

We use the uniform distribution to generate the random jump size Y_i.

$x \sim Uniform[0,1]$

$$x = F(y) = \begin{cases} 1 - pe^{-\eta_u y} & , \ y \geq 0 \\ qe^{\eta_d y} & , \ y < 0 \end{cases}$$

1. If $x < q \Rightarrow y < 0$

$$\Rightarrow x = qe^{\eta_d y}$$

$$\Rightarrow y = \frac{1}{\eta_d}\ln(\frac{x}{q})$$

2. If $x \geq q \Rightarrow y \geq 0$

$$\Rightarrow x = 1 - pe^{-\eta_u y}$$

$$\Rightarrow y = \frac{-1}{\eta_u}\ln(\frac{1-x}{p})$$

$$\therefore y = \begin{cases} \frac{-1}{\eta_u}\ln(\frac{1-x}{p}) & x \geq q \\ \frac{1}{\eta_d}\ln(\frac{x}{q}) & x < q \end{cases}$$

Appendix C.

The moments of Merton jump-diffusion model and the double exponential jump diffusion model under risk-neutral measure.

Merton jump-diffusion model:

$$X(t) = \left(r - \frac{1}{2}\sigma^2 - \lambda k\right)t + \sigma W(t) + \sum_{i=1}^{N(t)} Y_i$$

$$Y_i \overset{iid}{\sim} N(\alpha, \beta), \ \ k = \exp\left(\alpha + \frac{\beta}{2}\right) - 1, \ \ N(t) \sim Possion(\lambda), \ \ W(t) \sim N(0,1)$$

$$EX = r - \frac{1}{2}\sigma^2 - \lambda k + \lambda\alpha$$

$$Var(X) = \sigma^2 + \lambda(\alpha^2 + \beta^2)$$

$$Skewness = \frac{\lambda\alpha^3}{\left(\sigma^2 + \lambda(\alpha^2 + \beta^2)\right)^{\frac{3}{2}}}$$

The double exponential jump diffusion model:

$$X(t) = \left(r - \frac{1}{2}\sigma^2 - \lambda\xi\right)t + \sigma W(t) + \sum_{i=1}^{N(t)} Y_i$$

$$EX = r - \frac{1}{2}\sigma^2 - \lambda\zeta + \lambda\left(\frac{p}{\eta_u} - \frac{q}{\eta_d}\right)$$

$$Var\left(X\right) = \sigma^2 + 2\lambda\left(\frac{p}{\eta_u^2} + \frac{q}{\eta_d^2}\right)$$

$$Skewness = \frac{6\lambda\left(\frac{p}{\eta_u^3} - \frac{q}{\eta_d^3}\right)}{\left(\sigma^2 + 2\lambda\left(\frac{p}{\eta_u^2} + \frac{q}{\eta_d^2}\right)\right)^{\frac{3}{2}}}$$