國 立 交 通 大 學

財務金融研究所

碩 士 論 文

考量總體波動之風險中立信用移轉矩陣

Risk-neutral Model for Credit Migration Conditioned on Macroeconomic Fluctuations

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摘要

本論文探討總體因素的波動下對馬可夫信用移轉矩陣的調整，修改 Wei (2003) 所提出的模型，發展出隨景氣循環波動的信用移轉矩陣，來評價信用衍生性商品，然而 Wei (2003) 只簡單的利用數值的方法處理景氣的循環，並沒有實際的探究景氣波動背後的因素。因此，採用 Kim (1999) 所提出的信用循環指標法發展 AR(1) 模型，進一步探討不同的總體變數對景氣循環波動的影響，本論文提出四個總體經濟因子估計景氣循環的波動並提出一套更完整的過程去估計風險中立的信用移轉矩陣。

關鍵字:馬可夫、信用移轉矩陣、總體經濟因子

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ABSTRATE

This paper is intended as an investigation of a Markov model for credit migration with macroeconomic factors. The model modifies Wei (2003), developing the time-varying transition matrices which depend on the business cycle, to value credit derivatives. However, Wei (2003) adjusts the macroeconomic fluctuations according to numerical method. The question about which factors influence the fluctuations remains unsettled. Alternatively, we follow Kim (1999), constituting the credit cycle index, to develop AR (1) model. It offers the key to an understanding of the identification of the different impacts on macroeconomic variables. In this paper, we refine four macroeconomic variables to explain the business cycle deviation. It is noteworthy that this paper develops a calibration process which is better for estimating the risk-neutral credit migration.

Keyword：Markov、Credit migration、Macroeconomic factors

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1. Introduction

In recent years, investors have faced the fluctuation of credit unrest on account of the status that more and more corporations had failed to meet their obligations. There has been renewal of interest in the credit management. In 1988, Basel Committee drafted a series of regulations to prevent banks from default and to require the minimum capital adequacy ratio to control the credit risk in the business. However, this problem not only exists persistently but also makes us bear more loss. In early 1990s, Mexican peso extremely depreciated because of the balance of trade deficit. Some of the Latin- American businesses failed to make their promised payments. In 1997, the same tragedy happened again in Asia. In the next year, Long-term capital management (LTCM), which had been a famous hedge fund in America at that time, failed to keep its business. Sequentially, the subprime crisis has invaded the credit market around the globe since 2008. It has been the most severe economic hurricane since Great Depression in 1929. Although many scholars have dedicated to study this subject for a long time, most often fail to grasp the credit risk and must proceed to implement the work.

It will be useful to begin with making a distinction between two kinds of credit risk management. The first one is the structural model, which discriminates the defaulters from non-defaulters based on the default probabilities generated from information of the equity market. The second one is the reduced form model, which derives the implied default probabilities from the market value in lieu of the evaluation with the financial status. In this paper, we would like to focus on the combination of these two parts. For valuation, we develop the mechanism to transfer the actual probabilities of the credit migration to the implied probabilities.

The development on the structural model has been full of vitality since 1966. Beaver (1966) developed the single-discrimination model to classify default and non-default companies. The method which adopts one financial ratio to category the firms is too simple to work accurately. Altman (1968) then proposed the multi-discriminations. He selected five financial ratios to constitute Z-score. Whether the company defaults or not is decided by the magnitude of the Z-score. However, both of the methods just distinguish defaulters from non-defaulters. The default probabilities remain unknown. Ohlson (1980) proposed the logit model to predict the default probabilities and Zmijewski (1984) improved the idea a little further. He researched financial distress prediction models on nonrandom samples and adopted the probit-model for predicting. A great deal of efforts has been made on distinguishing default businesses. What seems lacking, however, is the consideration that the credit status will change over time. The credit matrices illustrate the information that the ratings change in the certain period. Altman and Kao (1992) observed the data of the credit migration and discovered the BB-rated or above bonds were expected to change at least once in ten years. A-rated bonds are more stable than AAA-rated and BB-rated bonds are the least stable in the same period. Belkin (1998a) discussed the credit risk of loan portfolio. He divided the credit risk of loan portfolio into system credit risk and idiosyncratic credit risk. Continually, Belkin (1998b) assumed the migration process in certain period follows the normal distribution. The credit change indicator is derived from inverse normal distribution. An agglomeration of the literatures is proposed by Kim (1999). He used probit model to estimate the credit cycle index, which presents the business cycle stage. Subsequently, he followed Belkin (1998b) estimate the correlation coefficient between credit change indicator and credit cycle index. Thus, the historical average probabilities of the credit migration can be adjusted as the conditional probabilities of the credit migration under the business cycle.

There is a heavy disadvantage to use the structural model to predict the default probabilities. It is difficult for us to observe the asset value. Let us return to the reduced model. Jarrow et al. (1997) proposed the implied default probabilities via the market price of the bonds. The variables, market price of the bonds, the risk-free rate and the coupon, can be easily observed and help us derive the implied default probabilities. The risk premium illustrates the relationship between the historical average default probabilities and the implied default probabilities. The probabilities of the credit migration under the equivalent martingale measure are the product of the historical average probabilities of the credit migration and the risk premiums. Kijima and Komoribayashi (1998) modified the Jarrow-Turnbull model to estimate the risk premiums for the empirical application. Wei (2003) contends that the probabilities of the credit migration vary with the business cycle. He developed the credit matrices under the business cycle in terms of the numerical method.

In Taiwan, Yeh (1998) analyzed the empirical result concerning the relationship between the refunded rate and the business cycle. Lai (2002) followed Kim (1999) construct the credit cycle index. Shen and Chang (2005) adopted the credit portfolio view method proposed by Wilson (1997) to estimate the default probabilities. Lee, Wang and Liu (2008) research the empirical result of the credit risk in relation to the business cycle. They compared Kim's (1999) method and Wilson's (1997) to estimate the default rate under the business cycle.

The purpose here is to explore a little further into the mechanism which help us convert the credit migration matrices from the physical measure to risk-neutral measure under the business cycle. We follow Kim (1999) construct the AR(1) probit model to estimate the credit cycle index and present the probabilities of the credit migration under the macroeconomic circumstances. For explanation of the business cycle fluctuations, one must identify the business cycle variables. In addition, we also empirically derive the risk-free probabilities of the credit migration from the risk premiums, which illustrate the relationship between the probabilities of the actual credit migration and of the risk-free credit migration for valuation. The next section presents the converting process from the physical measure to the risk-neutral measure. Section 3 illustrates the sample information. Section 4 contains the empirical result. The last section presents the constraints and the conclusions.

2. Model

According to Jarrow et al. (1997), the distribution of the credit migration is modeled by a time-homogeneous Markov chain process. Let Ω stand for the set of all possible state. We assume that P_{ij} denote the probability that the i-rated bonds will transfer to the j-rated in next period. A time-homogeneous Markov chain process of the credit migration can be represented by

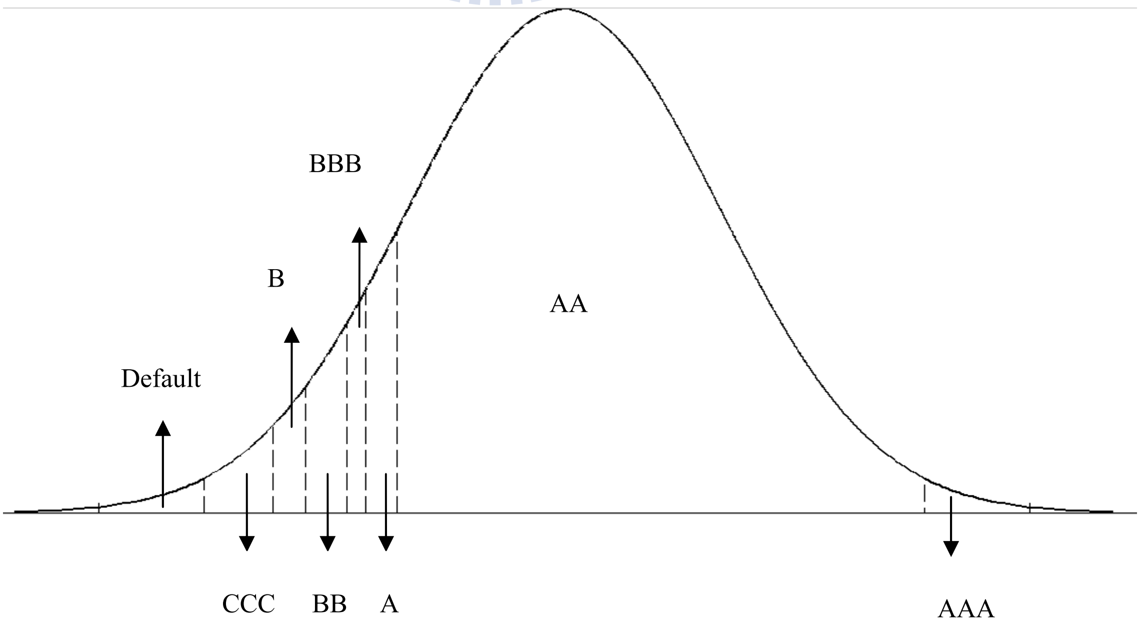
$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k-1,1} & p_{k-1,2} & \cdots & p_{k-1,k} \\ 0 & 0 & \cdots & 1 \end{pmatrix} \qquad (1)$$

where $P_{ij} \geq 0$, $\sum_{j=1}^{k} p_{ij} = 1$ for i = 1, 2, …, k-1. The raw probabilities are illustrated as the historical average of the data. There are two problems we must deal with before calculating the risk-neutral credit migration under the equivalent martingale measure. First, the probabilities of the credit-rating transition do not vary with the business cycle.

Intuitively, the default probability will be lower when the economy is growing rapidly and vice versa. Wei (2003) only used the numerical method to deal with the problem but didn't identify which macroeconomic factors have an effect on the degree of the deviation. In this paper, we modify Kim (1999), constituting the credit cycle index which can reflect the economic situations, to use AR(1) model proposed to adjust the credit probabilities under the business cycle. Second, we must transfer the credit-rating migration to the risk-neutral credit matrices under the equivalent martingale measure.

Given that the each row follows normal distribution, we can cumulate the probability from default state. After recording the cumulative probabilities which represent the threshold of the credit-rating transition in each state (The highest rating is always equal to one), we convert the probabilities into Z-score. To depict it, we assume a probability that the AA bond will be transited to AAA, AA, A, BBB, BB, B, CCC, and default are 0.01, 0.85, 0.04, 0.02, 0.03, 0.02, 0.02, 0.01, respectively. The threshold of the credit-rating transition can be represented by

Figure 1 The Threshold of Credit Ratings Transition



In each rating, we can construct the probability distribution as Figure 1. By combining all ratings, the historical probabilities of the credit migration are transited to Z-score as

$$L = \begin{pmatrix} L_{12} & L_{13} & \cdots & L_{1k} \\ L_{22} & L_{23} & \cdots & L_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ L_{k-1,2} & L_{k-1,3} & \cdots & L_{k-1,k} \end{pmatrix} \qquad (2)$$

According to Belkin (1998), we must adjust the historical probabilities of the credit migration under the business cycle. In this paper, we develop the probit-AR(1) model to solve the heteroskedasticity as

$$p_t = \Phi\left[\beta_0 + \sum_{i=1}^{h} \beta_i X_{i,t-1} + \varepsilon_t\right] \qquad (3)$$

$$\varepsilon_t = \eta \varepsilon_{t-1} + u_t \qquad (4)$$

$$u_t \sim N[0,1]$$

where p_t is the refunded rate collected from Central Bank of Republic of China at time *t*. β_i is the coefficient we need to estimate. $X_{i,t-1}$ is the macroeconomic variable. $\Phi[\bullet]$ represents cumulative normal distribution. We use the model to forecast the Z-score of the default probability at time *t+1*. Sequentially, the credit cycle index at each time can be represented by

$$Z_t = -\frac{\Phi^{-1}(p_t) - \mu_{\Phi^{-1}(p_t)}}{\sigma_{\Phi^{-1}(p_t)}} \qquad (5)$$

where Z_t is the credit cycle index at time t. $\Phi^{-1}(p_t)$ is inverse normal distribution of the default rate predicted by Eq (3). $\mu_{\Phi\text{-}1(pt)}$ and $\sigma_{\Phi\text{-}1(pt)}$ stand for the expected value and the standard deviation, respectively. If Z_t is negative, the economy status is located on the recession stage and vice versa. According to Belkin (1998), the credit-change indicator can be represented by

$$L_t = \gamma Z_t + \sqrt{1-\gamma^2}\,\varepsilon_t \qquad (6)$$

where L_t is the credit-change indicator. γ is the coefficient which measures the correlation between the credit-change indicator and the credit cycle index. ε_t is the error term which represents the unsystematic risk. Then, calculate the probability of default as

$$prob(\varepsilon_t < \frac{L_t - \gamma Z_t}{\sqrt{1-\gamma^2}}) \qquad (7)$$

Sequentially, the probability that the i-rated bond transfer to the j-rated at the next time can be represented by

$$p_t(i, j \mid Z_t) = \Phi(\frac{L^i_{j+1} - \gamma Z_t}{\sqrt{1-\gamma^2}}) - \Phi(\frac{L^i_j - \gamma Z_t}{\sqrt{1-\gamma^2}}) \qquad (8)$$

However, γ is unknown. There are two methods proposed by Kim (1999) to estimate γ. The first one is to estimate the different γ in each rating. Theoretically, different ratings have different sensitivity to market risk, but it works well subject to large sample size. The second one is to divide the ratings into investment and speculative grade. Empirically, it is more convenient for us to implement although the results are rougher than the method of estimating γ in each rating. Thus, we follow Kim (1999) choose the latter method to estimate γ which can be represented by

$$\min_{Z_t} \sum_i \sum_j \frac{n_{t,i}[p_t(i,j) - p_t(i,j \mid Z_t)]^2}{p_t(i,j \mid Z_t)[1 - p_t(i,j \mid Z_t)]} \tag{9}$$

where $n_{t,j}$ is the number of the i-rated companies which transfer to the j-rated at time t. $p_t(i,j)$ is the actual probability which the i-rated company transfers to j-rated at time t. $p_t(i,j|Z_t)$ is the probability we estimate the i-rated company transfers to j-rated under the macroeconomic condition.

We must, in turn, transfer the credit-rating migration to the risk-neutral credit migration matrices. Let Q denote the risk-neutral credit migration which are derived from the market price of the bond as

$$q = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1k} \\ q_{21} & q_{22} & \cdots & q_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ q_{k-1,1} & q_{k-1,2} & \cdots & q_{k-1,k} \\ 0 & 0 & \cdots & 1 \end{pmatrix} \tag{10}$$

where conditions for Eq. (1) must be satisfied here. We can derive the risk-neutral default probability in each rating by

$$V_i(t,T) = [1 - q_{ik}]V_0(t,T) + q_{ik}\delta V_0(t,T) \tag{11}$$

where $V_i(t,T)$ is the price of the i-rated bond matured at time T. q_{ik} is the probability that the i-rated bond will default at next time. $V_0(t,T)$ is the price of the riskless bond matured at time T. δ is the recovery rate. According to Jarrow et al. (1997), we develop the variable π_i to connect the actual default probability and the risk-neutral default probability as

$$q_{ij} = \pi_i(t)p_{ij}, q_{ii} = 1 - \sum \pi_i p_{ij}, i \neq j \tag{12}$$

And

$$\pi_i(t) = \frac{V_0(t,T) - V_i(t,T)}{(1 - \delta)V_0(t,T)p_{ik}} \tag{13}$$

However, there is a serious problem that π_i is meaningless when p_{ik} is equal to zero. Kijima and Komoribayashi (1998) developed the method in which the risk-neutral credit migration probability in each column is the product of the risk premium and the actual probability of the credit migration except the default column. The summation of the credit-migration probability in each row must be equal to one. The process that we transit the actual credit-migration probabilities to the risk-neutral credit-migration probabilities must leave one entry which adjusts the credit-migration probabilities to satisfy the condition. The difference between Jarrow et al. (1997) and Kijima and Komoribayashi (1998) is the column we choose to satisfy the condition. The former selects the diagonal entry but the latter chooses the default column. According to Kijima and Komoribayashi (1998), π_i can be modified by

$$p_i(t) = \frac{v_i(t,T) - dv_0(t,T)}{(1 - d)v_0(t,T)(1 - p_{ik})} \tag{14}$$

Eq. (14) solves the problem that the actual default probability is equal to zero. In this paper, we follow Kijima and Komoribayashi (1998) estimate the risk premiums which connect the actual and risk-neutral probabilities of the credit migration.

3. DATA

We consider the subject under three points: (1) the macroeconomic factors (2) the probability of the credit migration (3) the yield curve of bonds. First, we talk about how to select the macroeconomic factors. Yeh (1998) proposed four parts which may influence the credit-migration probabilities: (1) economic circumstances, (2) lending constraint, (3) type of lending, (4) seasonal factors. Lee, Wang, and Liu (2008) developed the AR(1)-GARCH(1,1) to predict the refunded rate. They chose six dependent variables: real growth rate of GDP, leading indicator, exchange rate (NTD/USD), annual growth rate of stock index, annual growth rate of CPI, and annual growth rate of M1B money supply. Chen (2010) follow Kim (1998) construct the cycle index and the macroeconomic factors are categorized according to economic indicator, foreign exchange indicator, money indicator, industry behavior, and others. Based on our observation from the previous research, we follow the classified framework which Chen (2010) proposed and select eleven variables: the real growth rate of GDP, leading indicator, exchange rate (NTD/USD), annual growth rate of stock index, commercial paper interest rate (91~180 days), annual growth rate of M1B money supply, annual growth amount of commercial paper, annual growth inventory of manufacturing, export orders, annual growth rate of CPI, and lending spread. The drop in the real growth rate of GDP (or leading indicator) reflects the worst economy and the refunded rate may increase. Economic indicator has a negative effect on refunded rate. Exchange rate depreciation will increase the firms' output and decrease th e default probabilities. We expect a negative impact. The increase of annual growth rate of stock index, annual growth rate of M1B money supply, annual growth amount of commercial paper and export orders reflect the better economy. We expect the negative relationships between these factors and refunded rate. Commercial paper interest rate (90~180) and lending spread accumulate while the economy has the tendency of recession. We expect the positive relationships. Table 1 presents the expected relationships between these macroeconomic factors and the refunded rate. For refining the variables, this paper devises a simple mechanism. First, the observation of correlation gives us a measure. Chen (2010) proposes the standard which is that the samples are taken away if the correlation between dependent and independent variable is lower than 0.25. Second, we adopt the stepwise method. Third, we inspect the collinearity between any two variables. Full discussion will be present in the next paragraph.

This paper selects these variables from 1996-2008 quarterly. Table 2 demonstrates the correlation between these macroeconomic factors and refunded rate. The correlation of the real growth rate of GDP is -0.04924, of lending spread is 0.11674, and of export orders is -0.16028, respectively. These are less than 0.25, which is the threshold required by Chen (2010), so that we draw out these three factors from our AR(1) model which predicts the refunded rate. Sequentially, variables are sieved by the stepwise method. There are three variables eliminated by this process: annual growth rate of CPI, annual growth rate of M1B money supply and annual growth amount of commercial paper. Finally, we test the collinearity of the remainders. Table 3 presents the VIF value. We will delete the macroeconomic factors if the VIF value is larger than three. There is a serious collinearity between the leading indicator and the commercial paper interest rate. Because of the higher correlation between the commercial paper interest rate and refunded rate, we make a choice to delete the leading indicator to solve the collinearity problem. The remaining variables will be applied to the model to predict the refunded rate. As a result, there are four independent variables—exchange rate (NTD/USD), annual growth rate of stock index, commercial paper interest rate and annual growth inventory of manufacturing.

Table 1
Relationships between macroeconomic variables and refunded rate

	Variable	Direction
Economic indicator	Real growth rate of GDP	—
	Leading indicator	—
Foreign exchange indicator	Exchange rate (NTD/USD)	—
Money indicator	Annual growth rate of stock index	—
	Commercial paper interest rate(90~180)	+
	Annual growth rate of M1B money supply	—
	Annual growth amount of commercial paper	—
Industry behavior	Annual growth inventory of manufacturing	?
	Export orders	—
Others	Annual growth rate of CPI	?
	Lending spread	+

Table 2

Correlation between variables and refunded rate

	Variable	Correlation Coefficient
Economic indicator	Real growth rate of GDP	-0.04924
	Leading indicator	-0.35225
Foreign exchange indicator	Exchange rate (NTD/USD)	-0.41781
Money indicator	Annual growth rate of stock index	-0.34334
	Commercial paper interest rate(90~180)	0.57845
	Annual growth rate of M1B money supply	-0.47939
	Annual growth amount of commercial paper	0.34835
Industry behavior	Annual growth inventory of manufacturing	0.27772
	Export orders	-0.16028
Others	Annual growth rate of CPI	0.33602
	Lending spread	0.11674

Table3

Collinearity

Variables	R-square	VIF
Leading indicator	0.6948	3.27654
Exchange rate for NT to US	0.4845	1.939864
Annual growth rate of stock index	0.4966	1.986492
Commercial paper interest rate(90~180)	0.7365	3.795066
annual growth inventory of manufacturing	0.3614	1.565925

The probabilities of the credit migration are in turn acquired by TCRI dataset published by TEJ monthly. The credit status can be classified into ten grades. Grade 1-4 stand for investment grades; grades 5-9 stand for speculative grades. The last column (row) is the default grade. We take the historical average probabilities of the credit migration as the unconditional credit migration probabilities. Table 4 illustrates the data in the second quarter in 2009. The data of the bond yields are published by the Gre Tai Securities Market (GTSM). In this paper, we collect the 3-month and one year bond yields since Q2 2009. Table 5 reports the bond yields. The bond yields data published by GTSM measure the ratings according to the Taiwan Ratings. But the ratings of the credit migration are derived from the system published by TEJ. We must elaborate the relationship between these two different systems. TEJ classify grade 1-4 as the investment grade. The ratings above tw-BBB are regarded as the investment grade by the Taiwan Ratings. Thus, grade 1-4 classified by TEJ are equal to tw-AAA, tw-AA, tw-A, tw-BBB classified by the Taiwan Ratings, respectively. Because of the smaller trade volume of the speculative grade bonds in Taiwan, dealers do not quote in the exchange house. There is no information about the market data of the bond yields of the speculative grade. We infer the bond yields of the speculative grade from the data of the investment grade bonds. According to Wei (2003), we assume second-order deviations in each rating are the same and extrapolate the bonds yields of the speculative grade. Table 5 demonstrates the conclusion we estimate the bond yields in 5-9 ratings. We also collect the 3-month and one year T-bill yields as the risk-free rate. Table 6 illustrates the data.

Table 4
The historical mean of seasonal credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.94	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.94	0.04	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.93	0.04	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.04	0.89	0.05	0.00	0.01
8	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.88	0.05	0.02
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.90	0.05

Table 5
Bond yields of the different ratings in different maturities since Q2 2009

Taiwan Ratings	twAAA	twAA	twA	twBBB					
TEJ	1	2	3	4	5	6	7	8	9
3-month	0.0105	0.0121	0.0158	0.0196	0.0236	0.0277	0.0320	0.0364	0.0410
1-year	0.0131	0.0148	0.0183	0.0227	0.0279	0.0339	0.0407	0.0483	0.0568

Table 6
The risk-free rate bond yields since Q2 2009

Maturity	3-month	1- year
Yields	0.000469	0.002552

4. Empirical result

In this section, we empirically derive the risk-neutral probabilities of credit migration under the equivalent martingale measure. First, the physical probabilities of credit migration can be translated into the unconditional credit-change indicator as Eq. (2). Table 7 illustrates the result. Furthermore, we estimate the regression coefficient with AR(1) model in order to solve the heteroskedasticity. According to Table 8, exchange rate (NTD/USD) is not significant—the same result as Lee, Wang, and Liu (2008) had. Annual growth rate of stock index have a significantly negative effect on the refunded rate. The result corresponds with our expectation that the refunded rate decreases as the annual growth rate of stock index increases. Commercial paper interest rate have a significantly positive relation with the refunded rate, which indicates that the more banks increase interest rate, the more stressfully firms run their businesses. When firms gradually fail to keep their businesses, the refunded rate will increase. The fitted value of the inverse normal distribution of the refunded rate can be derived from the regression coefficient estimation as Table 8 shows. The credit cycle index, which represents macroeconomic circumstance, can be easily acquired by Eq. (5). Figure 2 illustrates the inverse relationship between the refunded rate and the credit cycle index.

We further predict the credit cycle index in the first and second quarter in 2009. The credit cycle indexes are -0.611 in the first quarter and 0.4194 in the second, respectively. It represents that the economy experiences that recession in the first quarter but booms in the second quarter in 2009. Chen (2010) predicts that the credit cycle index in the second quarter is 0.91. Identically, the economy was in an expansion status in the second quarter of 2009.

Table 7

Unconditional credit-change indicator

	1	2	3	4	5	6	7	8	9	Default
1	[∞,-1.89]	[-1.89, -∞]								
2	[∞, 2.82]	[2.82,-1.76]	[-1.76,-2.69]	[-2.69,-3.04]	[-3.04,-∞]					
3		[∞, 2.28]	[2.28,-1.70]	[-1.70,-3.01]	[-3.01,-∞]					
4		[∞, 3.54]	[3.54,2.35]	[2.35,-1.85]	[-1.85,-2.76]	[-2.76,3.54]	[3.54,-∞]			
5				[∞, 2.07]	[2.07,-1.75]	[-1.75,-2.78]	[-2.78,-3.54]	[-3.54,-3.54]	[-3.54,-3.55]	[-3.54,-∞]
6				[∞, 3.35]	[3.35,1.87]	[1.87,-1.72]	[-1.72,-2.78]	[-2.78,-3.09]	[-3.09,-3.19]	[-3.19,-∞]
7				[∞, 3.54]	[3.54,3.12]	[3.12,1.70]	[1.70,-1.52]	[-1.52,-2.33]	[-2.33,-2.51]	[-2.51,-∞]
8						[∞,2.90]	[2.90,1.61]	[1.61,-1.54]	[-1.54,-2.16]	[-2.16,-∞]
9						[∞,3.29]	[3.29,2.39]	[2.39,1.71]	[1.71,-1.62]	[-1.62,-∞]

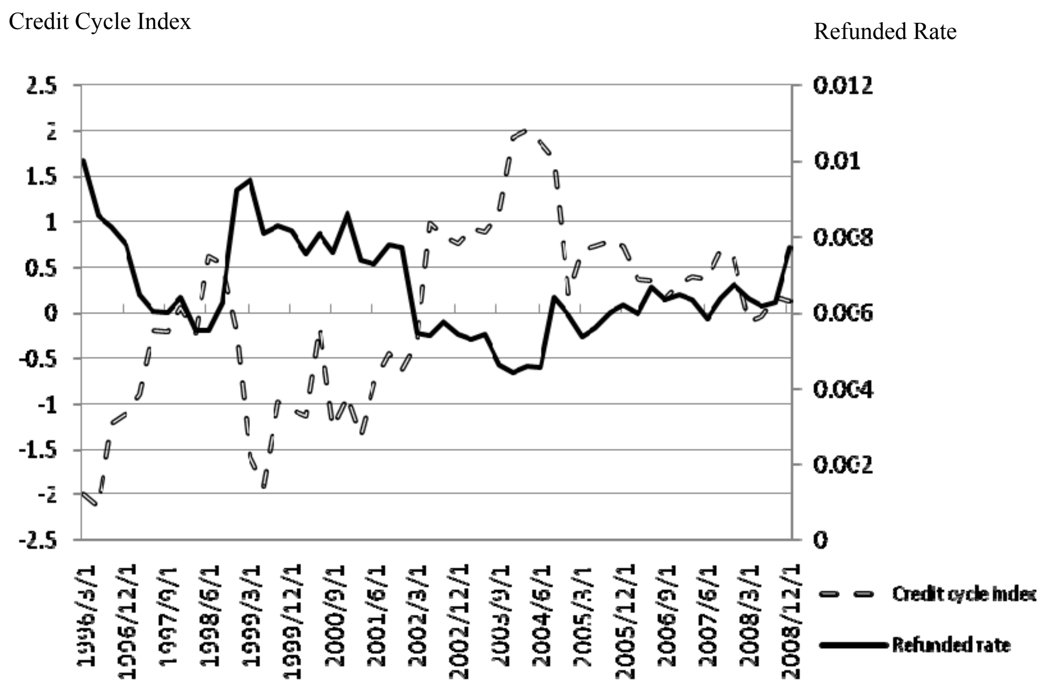
Table 8

Probit-AR(1) Model Estimation

Variable	DF	Estimate	Standard E	t-Value	p-Value
Intercept	1	-2.1973 ***	0.2092	-10.5	< 0.0001
Exchange rate	1	-0.01	0.006117	-1.64	0.1075
Annual growth rate of stock	1	-0.0877 **	0.0412	-2.13	0.0386
commercial paper interest rate	1	1.4404 **	0.7117	2.02	0.0488
Annual growth inventory of manufacturing	1	-0.1611	0.1614	-1	0.3233
AR1		0.6399 ***	0.1338	4.78	< 0.0001
MSE	0.00163		Root MSE		0.04032
SEC	-168.517		AIC		-180.225
Dubin-Watson	1.8471		R-square		0.6968

*** ,** mean significances at 10%, 5%, 1% ,respectively.

Figure 2 Relationships between Credit Cycle Index and Refunded Rate



Through Eq. (6), (7) and (8), the conditional probabilities of the credit migration in each rating can be derived. However, γ is an unknown. Kim (1998) estimated that the value is 0.0537 in the investment grade and 0.3384 in the speculative grade[1]. We adopt the value Kim (1998) estimated and calculate the conditional probabilities of the credit migration. Table 9 illustrates the estimation.

Table 9
The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.96	0.02	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.94	0.02	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.05	0.91	0.03	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.90	0.03	0.01
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.92	0.03

The risk premiums link the physical probabilities with the probabilities of the credit migration under the risk-free equivalent martingale measure. Table 10 illustrates the zero-coupon bond price which can be computed by the yield rate. However, the recovery rate is an unknown. Shan (2003) used the TEJ dataset to estimate the recovery rate in Taiwan. He found that the recovery rate in Taiwan was about forty percent so that we can use the information to estimate the risk premium. According to Kijima and Komoribayashi (1998) as Eq. (14), the risk premium in each rating can be derived as Table 11.

Note that the risk premiums developed by Jarrow (1997) just describe the relationship between probabilities under the equivalent measure and probabilities under the physical

1 Appendix A presents the results under the different γ.

Table 10 The corporation and risk-free rate bonds price

Panel A

The corporate bonds price

TCRI	3-month	One year
1	0.9974	0.9870
2	0.9970	0.9854
3	0.9961	0.9819
4	0.9951	0.9776
5	0.9941	0.9726
6	0.9931	0.9668
7	0.9921	0.9603
8	0.9910	0.9531
9	0.9899	0.9452

Panel B

The risk-free rate bond price

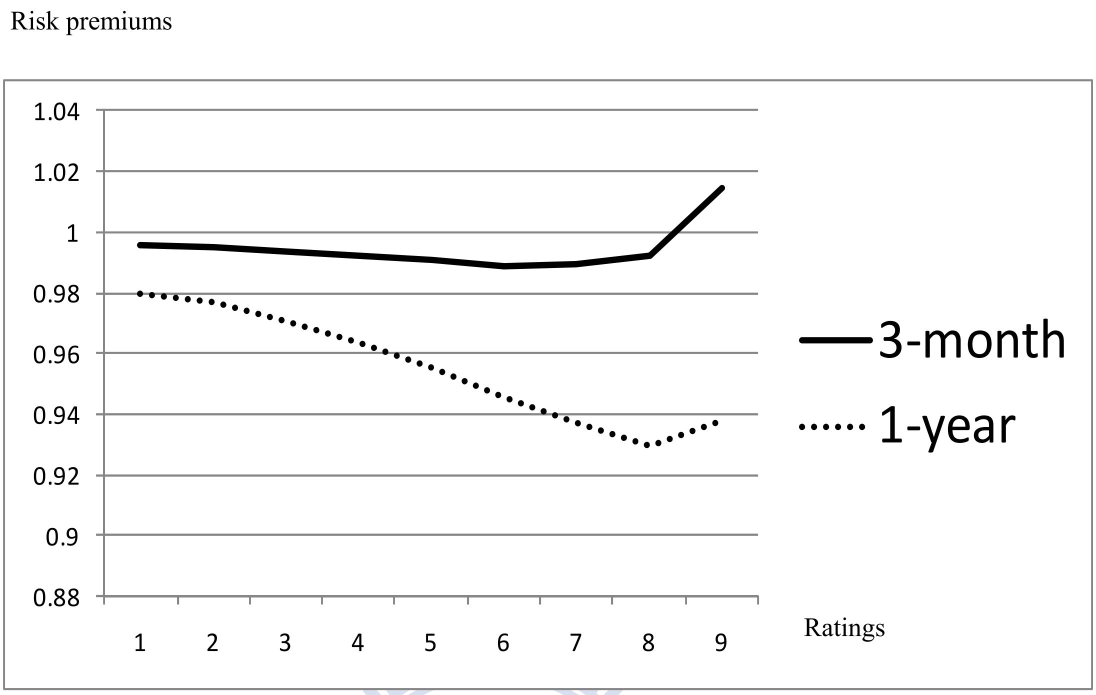
	3-month	One year
	0.9999	0.9994

Table 11

The risk premium

TCRI	3-month	One year
1	0.9759	0.9782
2	0.9949	0.9754
3	0.9934	0.9696
4	0.9918	0.9625
5	0.9902	0.9542
6	0.9887	0.9447
7	0.9892	0.9360
8	0.9920	0.9283
9	1.0139	0.9370

Figure 3 Relationships between risk premiums and ratings



measure. According to the definition, if the risk premium is unity, the default rate remains unchanged. The probabilities under the equivalent measure are higher than those under the physical measure when the risk premium is smaller than one and vice versa. We plot the risk premiums against the different ratings. Figure 3 illustrates the U-shaped curve. Both Kijima and Komoribayashi (1998) and Wei (2003) had the same result.

Panel A in Table 12 presents the implied probabilities for three-month. Theoretically, the regression coefficients should be estimated again in order to compute the implied probabilities of the credit migration for one year, but this method is subject to large sample size. Alternatively, we devise a new method. First, the credit cycle index prior to one year is derived from the average of seasonal credit cycle index. In order to predict the credit cycle index in the next year, some adjustment should be considered. We assume that the level of adjustment is identical with the difference between credit cycle index on certain quarter of the year and the subsequent year. Appendix A elaborates the process. Panel B in Table 12 presents the risk-neutral probabilities of the credit migration for one year.

Table 12
The risk-neutral probabilities of credit migration
Panel A 3-month transition matrix

	1	2	3	4	5	6	7	8	9	Default
1	0.9680	0.0278	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0042
2	0.0025	0.9555	0.0338	0.0022	0.0011	0.0000	0.0000	0.0000	0.0000	0.0048
3	0.0000	0.0119	0.9397	0.0408	0.0012	0.0000	0.0000	0.0000	0.0000	0.0064
4	0.0000	0.0002	0.0097	0.9521	0.0274	0.0025	0.0002	0.0000	0.0000	0.0079
5	0.0000	0.0000	0.0000	0.0202	0.9480	0.0213	0.0009	0.0000	0.0000	0.0096
6	0.0000	0.0000	0.0000	0.0003	0.0326	0.9325	0.0225	0.0006	0.0001	0.0113
7	0.0000	0.0000	0.0000	0.0002	0.0006	0.0478	0.9023	0.0343	0.0019	0.0130
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0017	0.0572	0.8964	0.0299	0.0148
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0004	0.0081	0.0398	0.9350	0.0167

Panel B 1-year transition matrix

	1	2	3	4	5	6	7	8	9	Default
1	0.9243	0.0757	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0055	0.8426	0.1170	0.0250	0.0099	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0526	0.8109	0.1160	0.0205	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0020	0.0436	0.8394	0.0937	0.0194	0.0019	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0983	0.7980	0.0884	0.0153	0.0000	0.0000	0.0001
6	0.0000	0.0000	0.0000	0.0042	0.1203	0.7745	0.0861	0.0118	0.0013	0.0018
7	0.0000	0.0000	0.0000	0.0017	0.0164	0.1931	0.6508	0.1041	0.0174	0.0165
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0343	0.2217	0.6200	0.0923	0.0318
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0066	0.0573	0.1468	0.6899	0.0994

5. Constraints and Conclusion

There are some problems we don't consider in the model. First, Jarrow and Turnbull model assumes the independence between the interest rate and the default rate. It means that the interest rate process have no effect on the default rate. Some scholars research the relationship between the credit risk and the interest rate risk, and others pay attention to the recovery rate under the credit circumstances. Second, we do not estimate the recovery rate and just follow Shen (2003)'s result that the recovery rate in Taiwan is about forty percent. Third, the GTSM did not collect the yields of the speculative grade and we only extrapolate it. Fourth, we do not estimate the coefficient γ, which presents the relationship between the credit cycle index and the credit change indicator, we follow Kim (1999)'s estimation instead.

In lieu of the numerical method, which Wei (2003) proposed to estimate the probabilities of the credit migration under the equivalent martingale measure, we follow Kim (1999) propose the AR(1) model to deal with the problem about the probabilities of the credit migration under the business cycle. We refine four variables, exchange rate (NTD/USD), annual growth rate of stock index, commercial paper interest rate and annual growth inventory of manufacturing, to keep track of the economic variations, and constitute the credit cycle index to adjust the physical probabilities of the credit migration into the conditional under the business cycle. For valuation, we follow Jarrow (1997) estimate the risk premiums to calculate the probabilities of the credit migration under the equivalent martingale measure. We combine Kim (1999) and Wei (2003) to propose a uniform framework for forecasting the risk-neutral probabilities of the credit migration. Further, the price of bonds under the different circumstances can be derived from the risk-neutral probabilities and help us illustrate the distribution of bond price with the credit status.

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Appendix A

Table 1 The results under the different γ

Investment grade $\gamma = 0.04$, Speculative grade $\gamma = 0.1$

The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.94	0.03	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.93	0.04	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.05	0.89	0.05	0.00	0.01
8	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.89	0.04	0.01
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.91	0.05

Investment grade $\gamma = 0.04$, Speculative grade $\gamma = 0.2$

The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.95	0.03	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.93	0.03	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.05	0.90	0.04	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.89	0.04	0.01
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.91	0.04

Investment grade $\gamma = 0.04$, Speculative grade $\gamma = 0.4$

The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.96	0.02	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.95	0.02	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.05	0.92	0.03	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.91	0.03	0.01
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.93	0.03

Investment grade $\gamma = 0.04$, Speculative grade $\gamma = 0.5$

The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.97	0.01	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.96	0.01	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.04	0.93	0.02	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.93	0.02	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.94	0.02

Investment grade γ = 0.06, Speculative grade γ = 0.1
The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.94	0.03	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.93	0.04	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.05	0.89	0.05	0.00	0.01
8	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.89	0.04	0.01
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.91	0.05

Investment grade γ = 0.06, Speculative grade γ = 0.2
The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.95	0.03	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.93	0.03	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.05	0.90	0.04	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.89	0.04	0.01
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.91	0.04

Investment grade γ = 0.06, Speculative grade γ = 0.4
The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.96	0.02	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.95	0.02	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.05	0.92	0.03	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.91	0.03	0.01
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.93	0.03

Investment grade γ = 0.06, Speculative grade γ = 0.5
The conditional probabilities of credit migration

	1	2	3	4	5	6	7	8	9	Default
1	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.01	0.95	0.04	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.01	0.96	0.03	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.02	0.97	0.01	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.03	0.96	0.01	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.04	0.93	0.02	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.93	0.02	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.94	0.02

γ reflects the sensitivity of the system risk. Because the economy booms in the second quarter of 2009, the default rate will decrease if γ is larger.

Appendix B

Table 2
Credit cycle index seasonally

Q2-08	Q3-08	Q4-08	Q1-09	Q2-09
-0.0457	0.1721	0.1248	-0.6114	0.4195

Table 2 illustrates the credit cycle index. First, we compute the credit cycle index from the second quarter of 2008 to the first quarter of 2009. It can be derived from the average of the credit cycle index and the value is -0.09. The level of adjustment is identical to the difference between the cycle credit index on the second quarter of 2008 and 2009. The adjustment term is 0.4651. The credit cycle index from the second quarter of 2009 to the first quarter of 2010 can be represent as -0.09 + 0.4651 = 0.3751.