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STABILITY ANALYSIS OF A NONLINEAR COUPLED-CORE REACTOR CONTROL SYSTEM

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ABSTRACT-In this paper, stability-equation method is applied to the analysis of a large coupled-core reactor control system having multiple nonlinearities and adjustable parameters. The characteristics of the limit-cycle and the asymptotically stable regions can be easily defined in a parameter plane. A numerical example is given and comparisons with other methods in current literature are made.

I. INTRODUCTION

In current literature, several methods have been applied to the analysis of large coupled-core reactor control systems[1-3]. Raju and Stone[1] have derived an analytical model and investigated system stability using the describing function approach; Raju and Josselson[2] have obtained conditions of stability using the Popov criterion; Tsouri and Rootenberg[3] have applied the Tsypkin locus method for limit cycle and stability analysis.

In the above mentioned methods, all the systems are considered symmetrical, and it is assumed that each system can be reduced into two single-input, single-output systems [1,3], then the single-output systems are analyzed. In this paper, a general method based upon the stability-equation method[4,5] is proposed. The considered systems need not be symmetrical and reduced. In addition, the systems may have both nonlinearities and adjustable parameters. The main approach of the proposed method is to analyze system stability and the existence of limit cycles by finding the simultaneous solutions of both the stability-equations[4,5] and the harmonic-balance equations[6-13].

II. AN ANALYTICAL MODEL OF THE COUPLED-CORE REACTOR

The linearized equations of the coupled-core reactor control system considered in this paper are as follows[1-3]:

$$\dot{n}_1=-\frac{D}{A}n_1+\frac{D}{A}n_2-\frac{b}{A}n_1+\frac{rh}{A}T_1+\frac{h}{A}p_1 \quad \text{(1-a)}$$

$$\dot{n}_2=-\frac{D}{A}n_2+\frac{D}{A}n_1-\frac{b}{A}n_2+\frac{rh}{A}T_2+\frac{h}{A}p_2 \quad \text{(1-b)}$$

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$$\dot{c}_1=\frac{b}{A}n_1-\lambda c_1 \quad \text{(1-c)}$$

$$\dot{c}_2=\frac{b}{A}n_2-\lambda c_2 \quad \text{(1-d)}$$

$$T_1=Kn_1-aT_1 \quad \text{(1-e)}$$

$$T_2=Kn_2-aT_2 \quad \text{(1-f)}$$

where:

- n_1,n_2 -deviations of power in core#1 and core#2, respectively,and n is taken as proportional to neutron flux.
- c_1,c_2 -deviations in average concentration of delayed neutrons in core#1 and in core#2, respectively.
- T_1,T_2 -deviations in temperature for core#1 and core#2, respectively.
- K -proportionality constant between power and temperature.
- D -power coupling coefficient between cores.
- λ -effective delayed neutron decay-time constant.
- b -fraction of neutrons delayed.
- A -prompt neutron generation time.
- r -reactivity-temperature coefficient.
- a -heat removal coefficient.
- h -steady-state power level.
- p -reactivity.

Taking the Laplace transformation of Eq.(1), the block diagram of the model including the controller[1-3] is shown in Fig.1(a), where

$$G_1(S)=G_2(S)=\frac{h}{AS(1+T_mS)} \quad \text{(2-a)}$$

$$G_{11}(S)=G_{22}(S)=\frac{(S+a)(S+\lambda)}{(S+\frac{D}{A})(S+\lambda)(S+a)+\frac{b}{A}S(S+a)+\frac{rKh}{A}(S+\lambda)} \quad \text{(2-b)}$$

$$G_{12}(S)=G_{21}(S)=\frac{D}{A} \quad \text{(2-c)}$$

T_m is the time constant of the control-rod drive motor; N_1 and N_2 represent the on-off

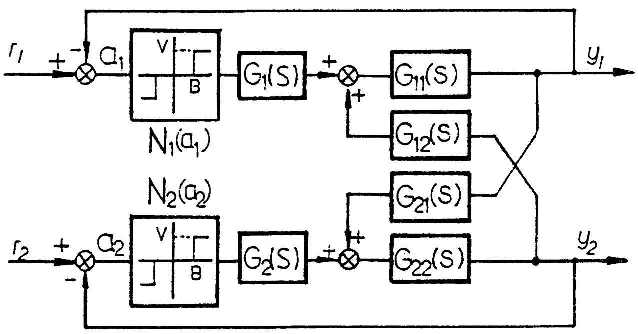

Fig.1(a). Block diagram of the control system for a large coupled-core reactor.

relays. The equivalent system block diagram of Fig.1(a) is shown in Fig.1(b), where

$$W_{11}(S)=G_1(S)G_{11}(S)/\Delta(S) \quad (3\text{-a})$$

$$W_{21}(S)=G_1(S)G_{11}(S)G_{21}(S)G_{22}(S)/\Delta(S) \quad (3\text{-b})$$

$$W_{12}(S)=G_2(S)G_{11}(S)G_{12}(S)G_{22}(S)/\Delta(S) \quad (3\text{-c})$$

$$W_{22}(S)=G_2(S)G_{22}(S)/\Delta(S) \quad (3\text{-d})$$

and $\Delta(S)=1-G_{11}(S)G_{21}(S)G_{21}(S)G_{22}(S)$.

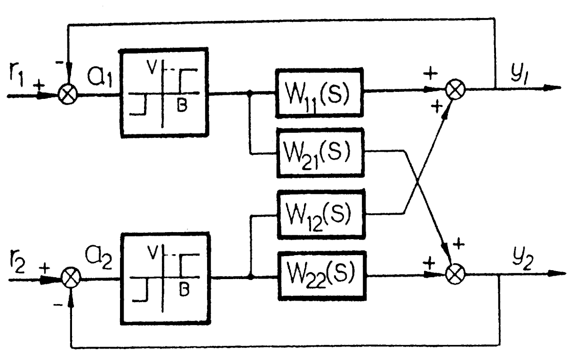

Fig.1(b). Equivalent block diagram of the system shown in Fig.1(a).

III. THE BASIC APPROACH

Consider the system shown in Fig.1(b). Assume that the input signals to the nonlinearities N_1 and N_2 are

$$a_1=A_1\exp[j(\omega t+\theta_1)] \quad (4\text{-a})$$

and

$$a_2=A_2\exp[j(\omega t+\theta_2)] \quad (4\text{-b})$$

respectively, where A_1 and A_2 are the amplitudes; θ_1 and θ_2 are the phase angles. Consider a_1 as the reference signal; i.e., to let θ_1 to be zero, then the harmonic-balance equations [6-13] of loop-1 and loop-2 are

$$A_1N_1(a_1)W_{11}(j\omega)+A_2e^{j\theta_2}N_2(a_2)W_{12}(j\omega)=-A_1 \quad (5\text{-a})$$

and

$$A_1N_1(a_1)W_{21}(j\omega)+A_2e^{j\theta_2}N_2(a_2)W_{22}(j\omega)=-A_2e^{j\theta_2} \quad (5\text{-b})$$

respectively, where θ_2 is the phase angle of the input signal to the nonlinearity N_2 with a_1 as the reference signal; $N_1(a_1)$ and $N_2(a_2)$ are the describing functions (or equivalent gains [14,15]) of the nonlinearities N_1 and N_2, respectively.

From Eq.(5-a), one has

$$e^{j\theta_2} = -\frac{A_1[1+N_1(a_1)W_{11}(j\omega)]}{A_2N_2(a_2)W_{12}(j\omega)} \quad (6)$$

Similarly, Eq.(5-b) gives

$$e^{j\theta_2} = -\frac{N_1(a_1)W_{21}(j\omega)}{A_2[1+N_2(a_2)W_{22}(j\omega)]} \quad (7)$$

Equating Eqs.(6) and (7), one has

$$F(j\omega)=1+N_1(a_1)W_{11}(j\omega)+N_2(a_2)W_{22}(j\omega) +N_1(a_1)N_2(a_2)[W_{11}(j\omega)W_{22}(j\omega) -W_{12}(j\omega)W_{21}(j\omega)]=0 \quad (8)$$

which is the characteristic equation of the considered system. Note that $N_1(a_1)$ and $N_2(a_2)$ are considered as varying parameters.

Since N_1 and N_2 are two single-valued nonlinearities [1,2], Eq.(8) can be decomposed into two stability-equations [4,5] as

$$F_e(\omega)=B_1(\omega)+N_1(a_1)C_1(\omega)+N_2(a_2)D_1(\omega) +N_1(a_1)N_2(a_2)E_2(\omega)=0 \quad (9)$$

and

$$F_o(\omega)=B_2(\omega)+N_1(a_1)C_2(\omega)+N_2(a_2)D_2(\omega) +N_1(a_1)N_2(a_2)E_2(\omega)=0 \quad (10)$$

From Eqs.(9), one has

$$N_2(a_2)=-\frac{B_1(\omega)+N_1(a_1)C_1(\omega)}{D_1(\omega)+N_1(a_1)E_1(\omega)} \quad (11)$$

Similarly, Eq.(10) gives

$$N_2(a_2)=-\frac{B_2(\omega)+N_1(a_1)C_2(\omega)}{D_2(\omega)+N_1(a_1)E_2(\omega)}, \quad (12)$$

Equating Eqs.(11) and (12), one has

$$[C_1(\omega)E_2(\omega)-C_2(\omega)E_1(\omega)]N_1(a_1)^2+[C_2(\omega)D_1(\omega)+B_2(\omega)E_1(\omega)-C_1(\omega)D_2(\omega)-B_1(\omega)E_2(\omega)]\times N_1(a_1)+[B_2(\omega)D_1(\omega)-B_1(\omega)D_2(\omega)]=0 \quad (13)$$

For specified values of frquency(ω), the values of $N_1(a_1)$ can be found by solving Eq.(13), then the corresponding values of $N_2(a_2)$ can be found from Eq.(11) or Eq.(12).

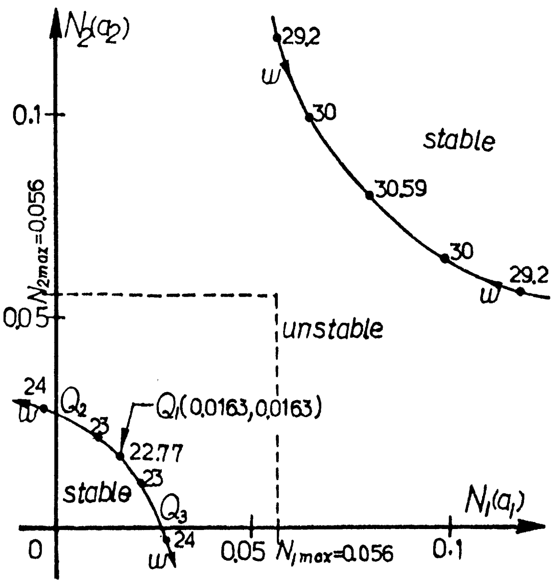

Fig.2. Root-loci of the stability-equations for Case 1 with M=22.

For a number of suitable values of ω, the real solutions(roots) of $N_1(a_1)$ and $N_2(a_2)$ can be plotted in a $N_1(a_1)$ vs. $N_2(a_2)$ plane. The typical root loci for a latter case are shown in Fig.2.

By use of Fig.2, the conditions of having a limit cycle are explained as follows:

(i) Every point on the curves as shown in Fig.2 represents a set of $N_1(a_1)$, $N_2(a_2)$ and ω which can satisfy the condition that a limit cycle may exist if the roots ω_{ei} and ω_{oj} of the even and odd stability-equations $F_{ei}(\omega)$ and $F_{oj}(\omega)$, respectively, are all real and alternating in sequence. There is an exception, however, when one root pair is equal to the other(i.e., $\omega_{ei}=\omega_{oj}=\omega$) [4,5].But unfortunately for nonlinear multivariable systems, there are infinite number of solutions which can satisfy this condition[13]. This is quite different from that of the single-input, single-output systems.

(ii) If the root-loci shown in Fig.2 separate the stable and unstable regions, then a limit cycle may exist. The reason is that, if the system becomes stable(unstable) when the amplitudes A_1 and A_2 increase(decrease), a stable limit cycle may exist at the stability boundary[4,5,16].

(iii) A limit cycle may exist only if the corresponding values of $N_1(a_1)$ and $N_2(a_2)$ of the root-loci are less than the maximal gains (N_{1max} and N_{2max}) of the nonlinearities. For example, in Fig.2 only the section between points Q_2 and Q_3 can give a limit cycle.

(iv) A limit cycle may exist only if the roots $N_1(a_1)$ and $N_2(a_2)$ satisfy both Eqs.(5-a) and (5-b). From Eqs.(5-a) and (5-b), the possible simultaneous solution can be found by equating the real and imaginary parts of Eqs.(6) and (7), respectively; i.e.,

$$\begin{vmatrix} e^{j\theta_{26}} & -e^{j\theta_{27}} \end{vmatrix} = 0 \quad (14)$$

where θ_{26} and θ_{27} represent the phase angles found from Eqs.(6) and (7), respectively.

If the considered nonlinear system can satisfy all the above four conditions, a limit cycle may exist. The three parameters A_1, A_2 and ω of the limit cycle are defined by Eqs.(9), (10) and (14). Additional explanations are given in the following section.

IV. ANALYSIS OF THE CONTROL SYSTEM

CASE 1: Assume that the numerical values of the parameters of the system considered are at b=.0064,A=0.1,λ=.001sec,K=10 F/MW.sec, a=10/sec, r=.001/ F,h=40MW,D=.015,B=.25MW, T =0.07sec,and V=Mx10^{-3} δk/k.sec[3]. For M=22 and for a number of frequencies(ω),the simultaneous solutions of Eqs.(9) and (10) are shown in Fig.2 where the stability of each region has been checked. At every point on the root loci, it has been checked that the roots ω_{ei} and ω_{oj} of the stability-equations $F_{ei}(\omega)$ and $F_{oj}(\omega)$, respectively, are all real and alternating in sequence except that one root pair is equal to the other(i.e., $\omega_{ei}=\omega_{oj}=\omega$).

By inspecting the root-loci shown in Fig.2, the section between points Q_2 and Q_3 can satisfy Conditions (i) to (iii). Solving Eq.(14) along the section between points Q_2 and Q_3, the point Q_1 (0.0163,0.0163) with oscillating frequency ω=22.77 rad/sec and amplitudes $A_1=A_2$=1.703 can satisfy condition (iv). Therefore, a limit cycle may exist at point Q_1. This fact is supported by checking the roots ω_{ei} and ω_{oj} of the stability-equations in the neighborhood of point Q_1[16]. Fig.3 shows the ω_{ei} and ω_{oj} loci for $N_1(a_1)$ is fixed at 0.0163(i.e., A_1=1.703) while $N_2(a_2)$ is varying. From Fig.3(a), one can see that if the value of $N_2(a_2)$ is less than 0.0163 (i.e., A_2= 1.703), the roots ω_{ei} and ω_{oj} are alternative in sequence, then the corresponding system is stable[4,5,16]. If the value of $N_2(a_2)$ is larger than 0.0163,the corresponding system is unstable. A similar result can be obtained when $N_2(a_2)$ is fixed at 0.0163(i.e.,A_2=1.703) and $N_1(a_1)$ is varying. Therefore, a stable limit cycle will exist at the stability boundary where $N_2(a_2)$=0.0163; i.e., A_2=1.703.

In Fig.2, for another branch of the root-loci, the corresponding values of $N_1(a_1)$ and $N_2(a_2)$ are larger than the maximal gains of $N_1(a_1)$ and $N_2(a_2)$, respectively; therefore, no limit cycle can exist.

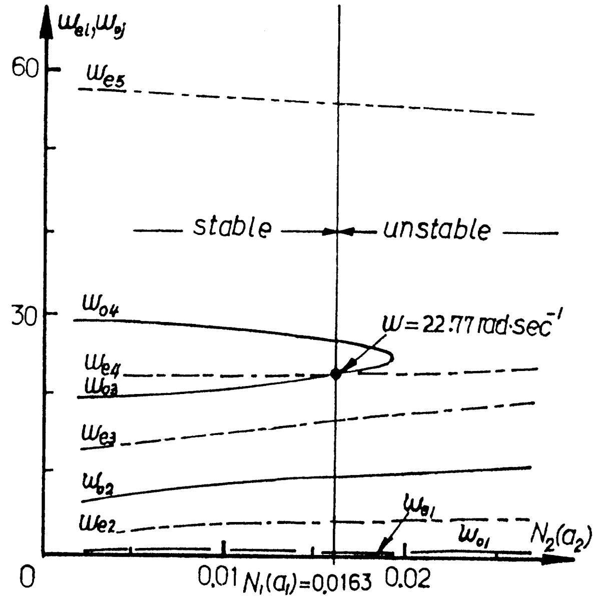

Fig.3. Root-loci of ω_{ei} and ω_{oj} of the stability-equations with fixed $N_1(a_1)$ and varying $N_2(a_2)$.

Note that the root-loci can also be plotted in the A_1 vs. A_2 plane by solving Eqs.(11), (12) and (13) which $N_1(a_1)$ and $N_2(a_2)$ directly relate to the describing functions of the nonlinearities N_1 and N_2; i.e.,

$$N_i(a_i)=\frac{4V}{\pi A_i}\left(1-\frac{B^2}{A_i^2}\right)^{1/2} \qquad i=1,2 \qquad (15)$$

The result is given in Fig.4 where point Q_4 represents a stable limit cycle. In this case condition (iii) is not necessary for analysis.

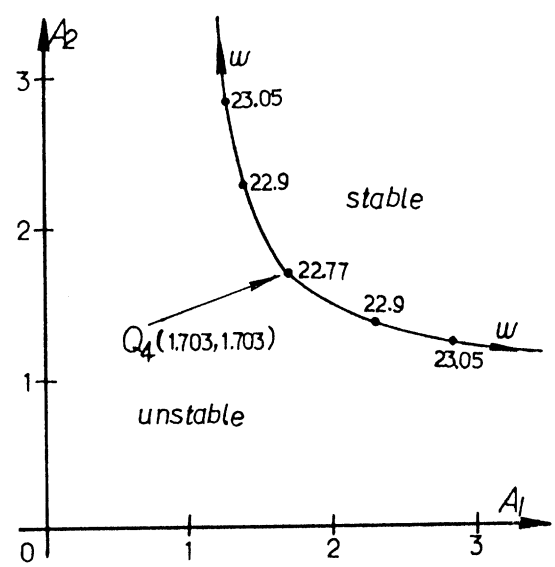

Fig.4. Root-loci of the stability-equations for Case 1 with M=22.

By computer simulation, Fig.5 shows the limit cycle of the system for M=22. The simulated result is quite close to that obtained by calculation.

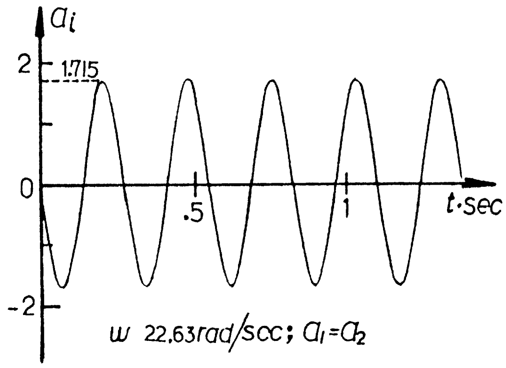

Fig.5. The simulated limit-cycle of Case 1 for M=22.

CASE 2: For the system considered in Case 1, assume that the nonlinearities N_1 and N_2 are replaced by two double-valued nonlinearities[3] as shown in Fig.6. Then the describing functions $N_1(a_1)$ and $N_2(a_2)$ of Eqs.(5)-(8) are replaced by

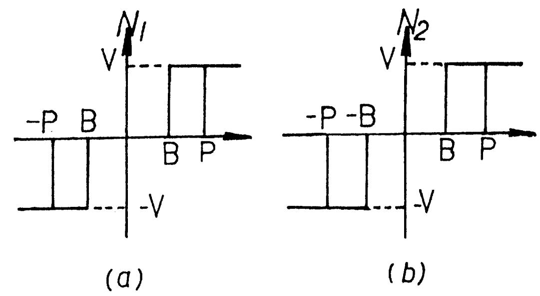

Fig.6. Nonlinearities of Case 2.

$$N_k(a_k)=N_{kr}(a_k)+jN_{ki}(a_k) \qquad k=1,2 \qquad (16)$$

where $$N_{kr}=\frac{2V}{\pi A_k}\left[\left(1-\frac{B^2}{A_k^2}\right)^{1/2}+\left(1-\frac{P^2}{A_k^2}\right)^{1/2}\right]$$

$$N_{ki}=-\frac{2V(P-B)}{\pi A_k^2}, \qquad A_k \geq B.$$

By using the same approach as in Case 1, Eq.(8) is decomposed into

$$\begin{aligned}F_e(\omega)=&B_1(\omega)+N_{1r}(a_1)C_1(\omega)-N_{1i}(a_1)C_2(\omega)\\&+N_{2r}(a_2)D_1(\omega)-N_{2i}(a_2)D_2(\omega)\\&+[N_{1r}(a_1)N_{2r}(a_2)-N_{1i}(a_1)N_{2i}(a_2)]\times\\&E_1(\omega)-[N_{1r}(a_1)N_{2i}(a_2)+N_{1i}(a_1)\times\\&N_{2r}(a_2)]E_2(\omega)=\emptyset \qquad (17)\end{aligned}$$

and

$$\begin{aligned}F_o(\omega)=&B_2(\omega)+N_{1r}(a_1)C_2(\omega)+N_{1i}(a_1)C_1(\omega)\\&+N_{2r}(a_2)D_2(\omega)+N_{2i}(a_2)D_1(\omega)+[N_{1r}(a_1)\times\end{aligned}$$

$$N_{2r}(a_2)-N_{1i}(a_1)N_{2i}(a_2)]E_2(\omega)$$
$$+[N_{1r}(a_1)N_{2i}(a_2)+N_{1i}(a_1)\times$$
$$N_{2r}(a_2)]E_1(\omega)=0 \qquad (18)$$

where $B_i(\omega)$, $C_i(\omega)$, $D_i(\omega)$ and $E_i(\omega)$ are the same as those of Eqs.(9) and (10). For M=22, B=0.15MW and P=0.25MW, and for a number of frequencies(ω), the root locus of the stability-equations is plotted as shown in Fig.7. It has been checked that every point on this root-locus can satisfy conditions (i) and (ii). Solving Eq.(14) along this root-locus, the point Q_5 (1.752,1.752) with an oscillating frequency ω=22.526 rad/sec represents a stable limit cycle.

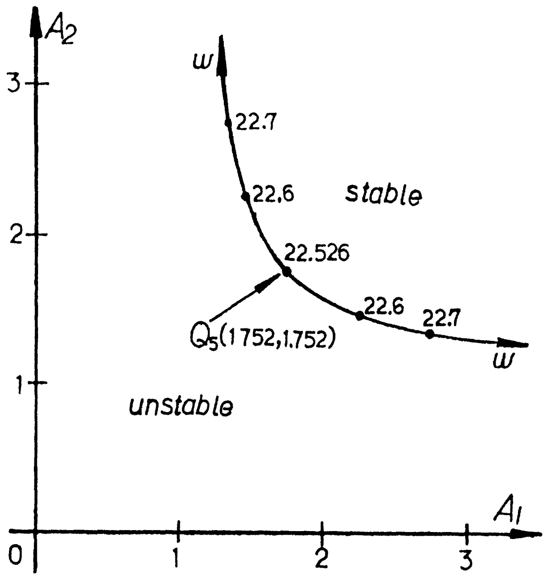

Fig.7. Root-loci of stability-equations for Case 2 with M=22,B=.15MW and P=.25MW.

By computer simulation, the limit cycle is shown in Fig.8, which is quite close to that obtained by calculation.

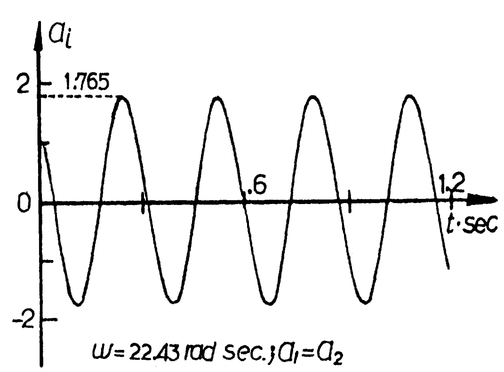

Fig.8. The simulated limit-cycle of Case 2 for M=22,B=0.15MW and P=0.25MW.

Note that, although Eqs.(17) and (18) are more complex than Eqs.(9) and (10), the approach is straightforward and the computations can be easily made on a computer.

V. CONSIDERATION OF PARAMETER ADJUSTMENT

In this section, control systems with adjustable parameters are considered. Assume that two adjustable parameters K_1 and K_2 are cascaded by the nonlinearities N_1 and N_2, respectively,then Eqs.(5-a) and (5-b) become

$$k_1A_1N_1(a_1)W_{11}(j\omega)+k_1A_2e^{j\theta_2}N_2(a_2)W_{12}(j\omega)=-A_1 \qquad (19\text{-a})$$

and

$$k_2A_1N_1(a_1)W_{21}(j\omega)+k_2A_2e^{j\theta_2}N_2(a_2)W_{22}(j\omega)=-A_2e^{j\theta_2} \qquad (19\text{-b})$$

respectively. Eq.(19-a) gives

$$e^{j\theta_2} = -\frac{A_1[1+k_1N_1(a_1)W_{11}(j\omega)]}{k_1A_2N_2(a_2)W_{12}(j\omega)} \qquad (20)$$

Similarly, Eq.(19-b) gives

$$e^{j\theta_2} = -\frac{k_2N_1(a_1)W_{21}(j\omega)}{A_2[1+k_2N_2(a_2)W_{22}(j\omega)]} \qquad (21)$$

Equating Eqs.(20) and (21), one has

$$F(j\omega)=1+k_1N_1(a_1)W_{11}(j\omega)+k_2N_2(a_2)W_{22}(j\omega)$$
$$+k_1k_2N_1(a_1)N_2(a_2)[W_{11}(j\omega)W_{22}(j\omega)$$
$$-W_{12}(j\omega)W_{21}(j\omega)]=0 \qquad (22)$$

which is the characteristic equation of the system under consideration. The stability-equations are

$$F_e(\omega)=B_1(\omega)+k_1[N_{1r}(a_1)C_1(\omega)-N_{1i}(a_1)C_2(\omega)]$$
$$+k_2[N_{2r}(a_2)D_1(\omega)-N_{2i}(a_2)D_2(\omega)]$$
$$+k_1k_2\{[N_{1r}(a_1)N_{2r}(a_2)-N_{1i}(a_1)\times$$
$$N_{2i}(a_2)]E_1(\omega)-[N_{1r}(a_1)N_{2i}(a_2)$$
$$+N_{1i}(a_1)N_{2r}(a_2)]\}E_2(\omega)=0 \qquad (23)$$

and

$$F_o(\omega)=B_2(\omega)+k_1[N_{1r}(a_1)C_2(\omega)+N_{1i}(a_1)C_1(\omega)]$$
$$+k_2[N_{2r}(a_2)D_2(\omega)+N_{2i}(a_2)D_1(\omega)]$$
$$+k_1k_2\{[N_{1r}(a_1)N_{2r}(a_2)-N_{1i}(a_1)\times$$
$$N_{2i}(a_2)]E_2(\omega)+[N_{1r}(a_1)N_{2i}(a_2)$$
$$+N_{1i}(a_1)N_{2r}(a_2)]E_1(\omega)=0 \qquad (24)$$

where $B_i(\omega)$, $C_i(\omega)$, $D_i(\omega)$ and $E_i(\omega)$ are the same as those in Eqs.(9) and (10). The condition defined in Eq.(14), now, becomes

$$\left| e^{j\theta_{220}} - e^{j\theta_{221}} \right| = 0 \qquad (25)$$

where θ_{220} and θ_{221} represent the phase angles found by Eqs.(20) and (21), respectively. The desirable solutions are A_1, A_2, K_1, K_2, and ω. Thus for specified values of A_1 and ω, one can find the solutions A_2, k_1 and k_2 by use of Eqs.(23)-(25). Now, for a specfied value of A_1 and a number of values of ω, a limit-cycle locus can be plotted in a K_1 vs. K_2 plane. For a number of constant-A_1 limit-cycle loci, the limit-cycle region and the asymptotically stable region can be found in the K_1 vs. K_2 plane[16]. Similarly, one can plot the constant-A_2 limit-cycle loci in the K_1 vs. K_2 plane for specified values of A_2 and a number of values of ω.

Case 3: Consider the system in Case 2. Assume that two adjustable parameters K_1 and K_2 are cascaded by nonlinearities N_1 and N_2 as shown in Fig.6, respectively. For M=22, B=0.15MW and P=0.25MW, following the above presented procedure the limit-cycle loci are plotted as shown in Fig.9, where

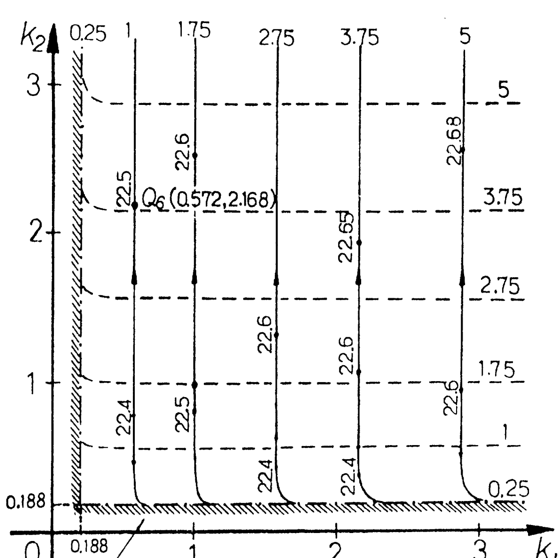

Fig.9. Limit-cycle loci of Case 3 for M=22, B=0.15MW and P=0.25MW.

the solid lines and dash lines are the constant-A_1 and the constant-A_2 limit-cycle loci, respectively; the shaded region shows the asymptoically stable region[16]. For illustration, the limit cycle represented by point Q_6(0.5718,2.1683),with amplitudes A_1=1, A_2=3.788, and with oscillating frequency ω=22.5rad/sec, has been simulated; the result is shown in Fig.10.

CASE 4: Consider the system in Case 2. Assume that the nonlinearities N_1 and N_2 as shown in Fig.6 are followed by two adjustable parameters K_1 and K_2, respectively. This is

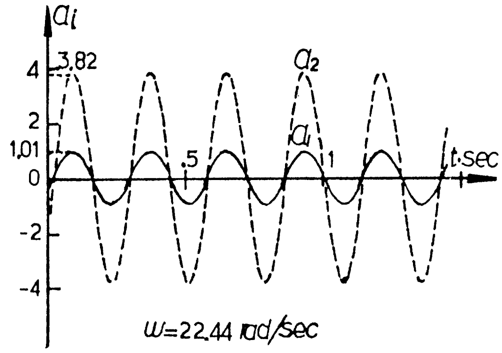

Fig.10. The simulated limit-cycle of Case 3 with K_1=0.5718 and K_2=2.6183.

equivalent to the case that the amplitudes of the nonlinearities N_1 and N_2 are adjustable. The harmonic-balance equations of the system are found as

$$k_1 A_1 N_1(a_1) W_{11}(j\omega) + k_2 A_2 e^{j\theta_2} N_2(a_2) W_{12}(j\omega) = -A_1 \qquad (26\text{-a})$$

and

$$k_1 A_1 N_1(a_1) W_{21}(j\omega) + k_2 A_2 e^{j\theta_2} N_2(a_2) W_{22}(j\omega) = -A_2 e^{j\theta_2} \qquad (26\text{-b})$$

Following the same procedure as indicated by Eqs.(22) to (25), the constant-A_1 limit-cycle loci are plotted as shown in Fig.11, where the shaded region is the asymptotically stable region.

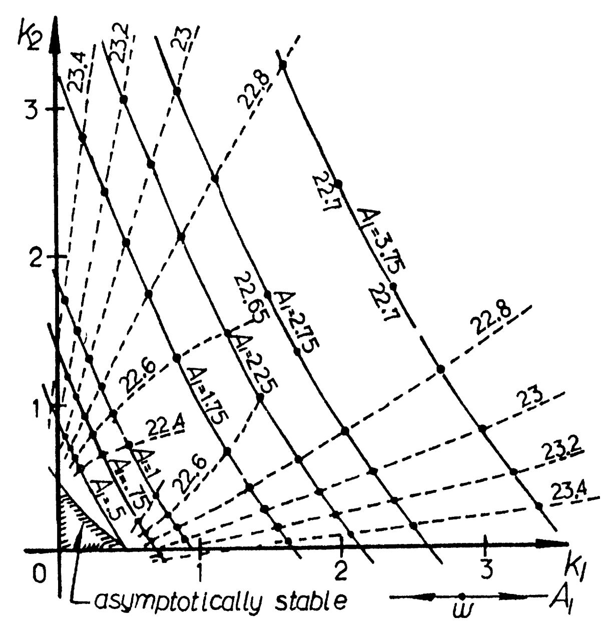

Fig.11. Limit-cycle loci of Case 4 for M=22, B=0.15MW and P=0.25MW.

Table 1 shows the calculated and simulated results of some points in Fig.11. It can be

Parameters		Calculated			Simulated		
k_1	k_2	ω	A_1	A_2	ω	A_1	A_2
0.3142	0.6555	22.4	0.75	0.9329	22.29	0.752	0.941
0.5729	0.1848	22.4	0.75	0.542	22.33	0.756	0.547
0.4958	0.7106	22.4	1.00	1.1147	22.32	1.008	1.124
0.6810	0.3715	22.4	1.00	0.8343	22.28	1.008	0.842
0.8364	1.3206	22.6	1.75	2.0051	22.46	1.768	2.017
1.6226	0.03463	23.4	1.75	0.934	23.27	1.771	0.972
2.0296	0.8124	22.8	2.75	2.115	22.77	2.777	2.143

Table 1. Calculated and simulated results of Case 4.

seen that the simulated results are quite close to those of calculated.

From Figs.9 and 11, one can see that the minimal values of K_1 and K_2 which give rise to a limit cycle are at $K_1=K_2=0.188$ for the symmetrical case[3]. Then the critical value of M for having a limit cycle is $M_c=K_1XM=0.188x22=4.1$, which is quite close to the result found by Tsouri and Rootenberg using the Tsypkin Locus Method[3].

Note that if the nonlinearities and the linear transfer fuctions are not symmetrical (such as $K_1=K_2$) the proposed method can be applied in the same way as for the symmetrical case. It is also worthwhile to point out that, by use of the asymptotically stable region, the limit cycle can be eliminated by adjusting the parameters in the system.

VI.CONCLUSIONS

In this paper, the stability-equation method has been applied for limit cycle analysis of a nonlinear coupled-core reactor control system.The proposed method is simpler than the other methods in current literature, and it has the potential to be applied to very complicated, nonlinear, symmetrical and asymmetrical systems.

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