國立交通大學

資訊科學與工程研究所

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Symmetry-aware Partial Shape Correspondence

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中 華 民 國 九 十 九 年 九 月

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Symmetry-aware Partial Shape Correspondence

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對稱性感知之局部形體對應

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摘 要

我們提出一個全自動的局部形體對應演算法，可以多樣種類的形體間進行語義式對應(semantic correspondence)，即使在複合式物體上，也能有合理的表現。藉由對稱性偵測的幫助，我們可以取得更高品質的對應結果，在拓普結構的小差異與形體上的小特徵上能有比較準確的判斷。利用物體對稱性為基礎對形體做結構性的診斷，藉此我們的演算法比較不受拓普雜訊影響，並且於複合式物體的局部對應有很大的突破。我們利用統計式的方式來推算出局部對應的結果，並利用完整的對應結果做拓普結構與表面特徵的檢驗以推算更合理的對應。

Symmetry-aware Partial Shape Correspondence

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ABSTRACT

We propose a fully automatic partial correspondence algorithm that allows matching of a wide variety of shapes with semantically similar, which can solve the topological noise causing by composite object. Through the symmetry detection, we obtain the high-quality correspondence on small topological difference or small feature. Use statistical method to synthesize a partial correspondence, but also evaluate the rationality of final correspondence in global structure of shape to avoid topology inconsistency. Our algorithm is insensitive to topology noise, and it takes a step forward on partial shape correspondence between composite objects.

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I would like to thank my advisors, Professor Jung-Hong Chuang and Sai-Keung Wong, for their guidance, inspirations, and encouragement. I am also grateful to my senior colleague, Tan-Chi Ho and Jau-An Yang, for their advices and suggestions on my thesis research. And then I want to express my gratitude to all my colleagues in CGGM lab: Chien-Kai Lo, Chien Yun, Ying-Yi Chou and Yu-Chen Wu discusses with me on research issues and helps me figure out the questions and encourage me. Tsung-Shian Huang help me understand much rendering knowledge, and all my junior colleagues' kind assistances. Lastly, I would like to thank my parents for their love, and support. Without their love, I could not pass all the frustrations and pain in my life.

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CHAPTER 1

Introduction

What is the shape correspondence problem? According to the survey paper [vKZHCO10] which publish by Oliver et al., the correspondence problem can be generally stated as "*given input shapes $S_1, S_2, ..., S_N$, establish a meaningful relation between their elements.*" Figure 1.1 presents an example of a correspondence. The correspondence problem has been researched widely in computer vision and image analysis. However, computing a correspondence still a hard problem in the 3D computer graphic, since it involve with not only the geometric comparison, need to consider the sematic meaning of shape. Sematic correspondence is an ill-posed problem, and the characteristic may at local and global levels, it's difficult to analyze and compare the structure of shapes.

Shape correspondence is an important work in many applications of digital geometry processing, such as shape retrieval, shape registration, shape morphing and information transfer. **Shape retrieval :** Given a query object, the identity of the object is inferred from the best match to one of the models in a database. **Shape registration:** Given a number of scans in arbitrary initial positions, the goal of registration is to match regions that correspond across the scans, so that the scans can be aligned and the target object can be fully reconstructed. **Shape morph-**

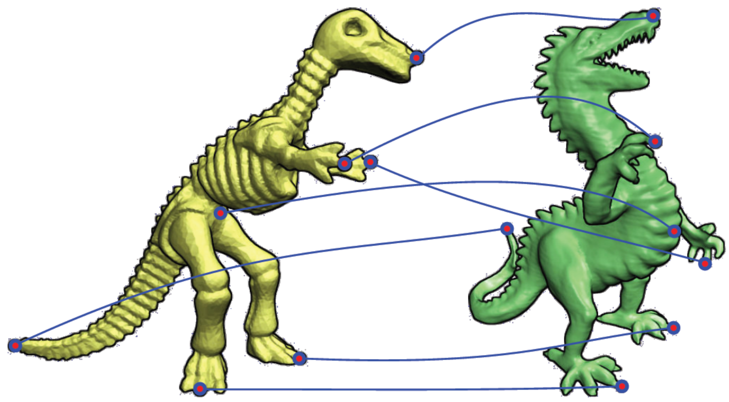


Figure 1.1: An example of meaningful correspondence, where the elements are feature points on the surface of the shapes and the relation links pairs of these points. [vKZHCO10]

ing: Give two different models, and changes one model into another through during a seamless transition. **Information transfer :** Transfer information from a 3D object to a target 3D object, especially to reuse of attributes or motion information associated to the source shape. Since the shape correspondence is recurrent problem in numerous geometry processing application, improving the quality of correspondences is a popular research topic in recent years, the survey paper [vKZHCO10] present by Oliver et al discuss the different forms of the correspondence problem and review the main solution methods, the current methods have many opportunities for improvement and there are several open problems still unsolved.

The two geometric shapes may be very different, only the sub-parts are similar. So we have to find a subset of shape elements for a meaningful correspondence can be computed, this called partial correspondence problem. The problem is defined as finding a subset of shape elements for which a meaningful correspondence can be computed. Partial correspondence problem are difficult than full correspondence problem, since it have to matching the subsets. In previous literatures related to shape correspondence, focus on addition features which do not exist in both shapes. The addition features is usually a small part, otherwise the results may be inaccurate. The current approaches for partial shape correspondence only suit for the two shapes which is similar to each other. Consider the maximal matching of feature nodes, and the rests is the unmatched part. The correspondence can be accurate only when the two shapes are semantically similar. However, when the source and target shape are very semantically different, may lead to large errors. Ex: Figure 1.2, for the approaches which support partial correspondence, it's still a challenging case. Furthermore, the accuracy of correspondence is not good enough. here is still a room for improvement in many different cases, for instance, the shape with different semantics but similar geometry, the shape with small topological difference, and global flipping problem. How to improve correspondence quality is a question which worthwhile to think.

Our algorithm represents a fully automatic correspondence algorithm that allows matching of a wide variety of shapes with semantically similar, which can solve the topological noise causing by composite object, Such as the Neptune in Figure 1.2. The most correspondence approaches is purely geometric-based, lack of the semantic information, in some situation will cause large error. High-level shape signature is needed to improve the correspondence quality. In this thesis, we use symmetry information to enhance the quality of correspondence and determine the structural of shape. We use curve skeleton to assist symmetry detection and shape correspondence, since curve-skeleton is an effective descriptor to convey the characteristics of 3D shape. It has advantage to resist different surface details. But curve-skeleton don't utilize enough surface information, it influence the quality of correspondence. We use the intersection configuration of surface to inspect and adjust the result of correspondence, to find the high-quality correspondence. Use local correspondence statistically to achieve the partial correspondence, and estimate the rationality of global correspondence on shape surface to find the final correspondence.

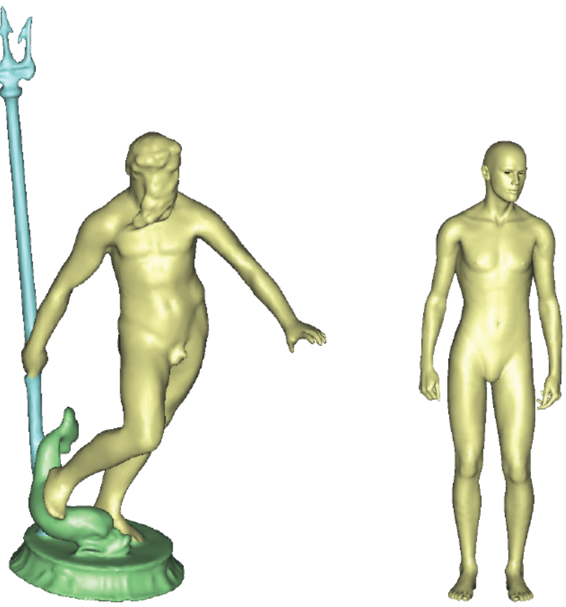


Figure 1.2: An example of partial correspondence between parts of shapes: the goal here is to find a correspondence between Neptune's statue (left) and the human (right), by relating the parts in yellow and ignoring the extra parts shown in green and blue. [vKZHCO10]

1.1 Contribution

The contributions of the thesis can be summarized as follows:

- Present a fully automatic partial correspondence algorithm.
  - Allows matching of a wide variety of shapes with semantically similar structures but different geometric details.
  - Produces meaningful partial correspondence for composite objects.
  - Produces reasonably good correspondence between shapes with group-similarity.
  - Is insensitive to topological noise

1.2 Organization of the thesis

The following chapters are organized as follows. Chapter 2 gives the literature review and the background knowledge. Chapter 3 describes our correspondence algorithm, including the overview, the symmetry detection, the structure diagnosing and the correspondence searching. Chapter 4 shows the experimental results of our correspondence algorithm and comparison to electors voting algorithm [ATCO$^+$10]. Finally, we summarize our correspondence algorithm and discuss its limitations and future works in Chapter 5.

CHAPTER 2

Related Work

In this chapter, we classify extensive previous work related to shape matching and feature correspondence. We also review existing symmetry detection approaches in the context of digital geometry processing.

2.1 Shape correspondence

In this section, we give the literature review and background of different classifications of correspondence methods, included the rigid alignment approaches (Subsection 2.1.1), non-rigid alignment approaches (Subsection 2.1.2) and similarity-based CP approaches (Subsection 2.1.3). Then we review the curve skeleton correspondence problem and its solution methods (Subsection 2.1.4). Since computing a partial correspondence is an important specialization in our thesis approach, we introduce the previous work of partial correspondence (Subsection 2.1.5).

2.1.1 Rigid alignment approaches

Rigid alignment is a common approach for shape matching, The basic idea is executing a rigid transformation for alignment to derive from a set of sample of points. The rigid alignment approaches used in the computer vision first, [HU90] simply *sample* a transformation and *verify* the quality in aligning the shapes. This method also used in shape retrieval [AMCO08], which pre-process the invariances of sample and verify method for efficient retrieval. The verification step can simplify by voting on the transformation, and the verifying transformations also can applies to other contexts, such as [LF09], utilized the Möbius transformations and the voting method to establish the correspondence.

2.1.2 Non-rigid alignment approaches

The idea of non-rigid alignment approaches is about the same with rigid alignment approaches, but it usually applies to nonrigid shape and the transformation may be nonlinear. This approach develops well in shape registration and deformation transfer [ACP03, SP04, PMG$^+$05] use the different transformations to find the best transformations. [LSP08] execute the two steps transformations, applied a roughly global rigid transformation first, then per-vertex affine transformations for full alignment. The non-rigid alignment can also achieve by deforming one shape into the other, [dATSS07] utilized a 3D optical flow and [dAST$^+$08] enforced by Laplacian deformation.

2.1.3 Similarity-based CP approaches

This approach estimates the similarity between pairs of shape elements or feature points and derives a correspondence from those estimates. The similarity evaluated by the objective function which designed for different shape descriptors, such as points from 3D scans [CU08], deforming surfaces [AKS$^+$05] and skeleton [BMSF06]. Similarity-based correspondence approach also called the *feature matching approach*. There are many solution approaches to pairwise assignment between feature points, which called correspondence search. We classify them roughly into two categories, optimization and tree-based search.

Optimization : The correspondence search can achieve by optimization . In Linear Assignment Problem (LAP), a weighted bipartite graph can solved by Hungarian algorithm in $O(n^3)$ [PS82]. If the objective function compares both linear and quadratic terms, then the problem become a Quadratic Assignment Problem (QAP) is a NP-hard problem [PRW94]. There are several techniques to approximate to NP-hard assignment problems, such as [vKHZW07], which achieved by ant colony optimization.

Tree-based search : Build the combinatorial tree to explore the possible correspondences, each node represents a partial solution. Search for the correspondence by using some tree-based search techniques, such as branch-and bound and priority search. [GMGP05, FS06, ZSCO$^+$08, ATCO$^+$10]. The pruning test the difference between pairwise assignments, quantifying the distortion in the Euclidean [GMGP05, FS06] or quantifying the difference of geodesic distances [ZSCO$^+$08, ATCO$^+$10]. [ZSCO$^+$08] estimated difference by deforming source into target, [ATCO$^+$10] estimated difference in the spatial configuration of the shapes.

2.1.4 Skeleton correspondence

Curve skeleton is a common shape descriptor. Since skeleton is an effective descriptor to convey the characteristics of 3D shape, it has vantage to estimates similarity between two shape. Since our algorithm use curve-skeleton as a shape descriptor, we brief review the recent paper which utilized skeleton or reeb graph to solve the matching problem [SSGD03, SKK04, BMSF06, XWLB09, ATCO$^+$10]. [SSGD03] uses graph matching techniques to match the skeletons, since skeleton is easy to estimate topological similarity and perform part-matching on skeleton. [BMSF06] derived heuristic for partial shape-matching, which is able to recognize similar sub-parts of objects represented as 3D polygonal meshes. The graph matching techniques also found in 2D shapes, [XWLB09] match the critical points on skeleton graphs by comparing the geodesic paths between end points and junction points of the skeleton for image matching.

[ATCO$^+$10] performed multidimensional scaling on skeleton to achieve pose- normalizing and used a statistical correspondence algorithm called electors voting to find the correspondence efficiently. These papers have in common is that correspondence result relies on skeleton quality. The important problem is whether shape skeleton represents shape appropriately.

2.1.5 Partial correspondence

Computing a partial correspondence is an important specialization of shape correspondence, it is a more difficult problem than computing a full correspondence, since searching for the correct assignment for subset increases the complexity. One approach estimates the objective function, and search for sharp difference. The sharp difference becomes watershed between outlier points and matched points [GMGP05, ZSCO$^+$08]. Alternatively, the other approach estimates the number of outlier feature as an optimization problem first. This process decrease the number of feature nodes, and only the remained feature nodes appear in the computed correspondence [MC03, BBM05, OEK08]. The voting methods naturally have the ability to perform partial matching. The voting methods compute many candidates correspondence and cast a vote on the assignment of candidate [LF09, ATCO$^+$10]. The heuristic algorithms for skeleton matching also perform partial matching [SKK04, BMSF06], which is inherent intuitiveness. However, when two 3D shapes are very semantically different in whole shape, only sub parts are semantic similar, this current approaches may lead to large errors. The situation often occurs at composite object, as Figure 1.2 showed. Partial correspondence performs well in image research area, [BBM05, OEK08] matched two image which outlier feature are more than matched feature. It's difficult to compute a partial correspondence between 3D shapes when they are very semantically different, since large semantic difference often lead to large topological difference.

2.2 Symmetry detection

It's hard to obtain a high-quality sematic correspondence only using pure geometric information. If the parts of source and target have different semantics but similar geometry, the correspondence could have large errors (e.g.2.1). So we use the symmetry information as a high-level shape signature to enhance the quality of correspondence.

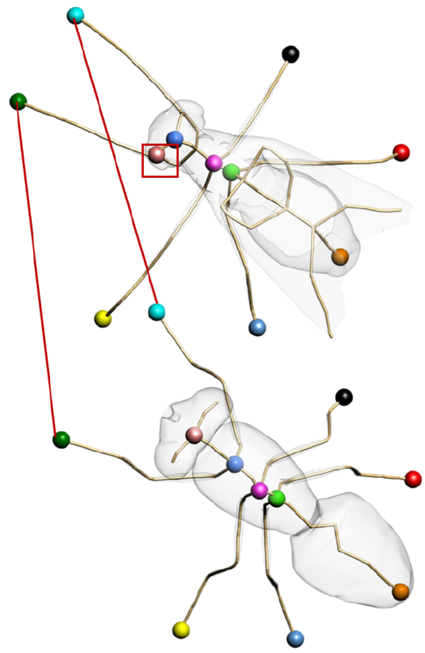


Figure 2.1: The correspondence that computed by[ATCO+10] are semantically incorrect, since components have different semantics but similar geometry.

The symmetry detection algorithm can classify to compute extrinsic and intrinsic symmetry. Extrinsic symmetry are symmetric to a line or a plane, it's a rigid symmetry. Intrinsic symmetry is symmetric to an axis or a surface, it's a nonrigid symmetry. Some algorithms find one sole symmetry axis for a shape , which is called global symmetry, and the others may find several symmetry axes for each part of shape, which is called partial symmetry. According to Kai Xu et al. published in 2009 [XZT+09], we classify the symmetry detection algorithm into four categories, *global extrinsic symmetry*, *partial extrinsic symmetry*, *global intrinsic symmetry* and *partial intrinsic symmetry*. **Global extrinsic symmetry.** [ZW97] detected approximate symmetry by defining a symmetry measure for a given transformation as **d**istance of a shape to the closest symmetric shape. [KFR04] represents a 3D model as a collection of spherical functions by signal processing techniques, called symmetry descriptor. **Partial extrinsic symmetry.**

[PSG$^+$06] developed a transform that measures the reflective symmetries of a 3D shape with respect to all possible lines in the plane. [SS06] proposed an approach is capable of detecting local approximate planar symmetries, defining a shape as a hierarchical union of planar symmetric parts. [MP06] proposed to detect approximate partial symmetries for shapes based on transform-space voting and clustering, while a similar approach. **Global intrinsic symmetry.** [MS06] proposed a method for recovering global symmetry of 3D shapes based on generalized moments, analyzing the extrema and spherical harmonic coefficients. [RK07] measures the reflective symmetries of a 3D shape to all possible lines in the plane and spatial distribution of the object's asymmetry. [OSG08] presented a purely algebraic algorithm for detecting global intrinsic symmetries of shapes and classifying symmetries into discrete classes. **Partial intrinsic symmetry.** [XZT$^+$09] developed a voting scheme to obtain an intrinsic reflectional symmetry axis transform, which is a scalar field over the mesh that accentuates prominent symmetry axis of the shape.

Sematic correspondence is an ill-posed problem, it compare intrinsically similar for correspondence. And the input models usually are non-rigid shape, so we have to use intrinsic symmetry information of shape. Furthermore, the shape may be composited by several components, and each component have a symmetry axis. Therefore we need apply a partial intrinsic symmetry detection algorithm, and [XZT$^+$09] is suit for our requirement. Unfortunately, it's very slow to compute the partial intrinsic symmetry in current approaches. Therefore we purposed a fast approximation for partial intrinsic symmetry detection, the specific will be discussed in section 3.3.

CHAPTER 3

Algorithm

We develop a fully automatic correspondence algorithm that allows matching of a wide variety of shapes with semantically similar, which can solve the topological noise causing by complex shape. The algorithm uses symmetry information to enhance the accuracy of correspondence search and diagnose the structure of shape, estimates both local and global information during the correspondence process. This chapter introduces our symmetry-aware partial shape correspondence algorithm in detail, included overview(section 3.1), symmetry detection(section 3.2), structure diagnosing(section 3.3) and correspondence search(section 3.4).

3.1 Overview

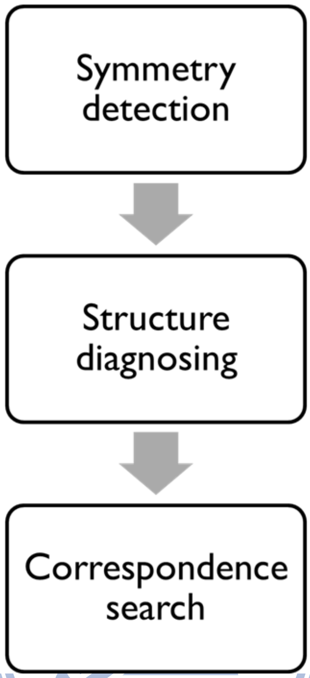


Figure 3.1: System overview

We give an overview of our method in this section, the main steps show in Figure 3.1. We take non-manifold mesh and curve skeleton as the input. First, we applied symmetry detection and estimated structure of shape. Then we build the combinatorial tree for correspondence search, each tree node represent a matching between feature nodes and each path from root to leaf represents a local correspondence and use the branch-and-bound technique to reduce the number of tree node. After the correspondence search, we find many candidate local correspondences. Using intersection configuration to estimate the rationality for each local correspondences, and accumulate correspondence score for the matching between source and target. Then, we obtain a correspondence score from every reasonable local correspondence. Finally, utilizing the correspondence score, we execute the SA iteration to compute the final correspondence.

3.2 Symmetry detection

In order to obtain the corresponding results with higher quality, we need the high-level shape signature. Symmetry is ubiquitous in nature and man-made artifacts, and it's helpful to compare different shape. Furthermore, symmetry is useful to diagnose structure of shape. Therefore, we perform symmetry detection for both source shape and target shape. Since the semantic correspondence is an ill-posed problem, we must attain intrinsic symmetry rather than extrinsic symmetry. The shape often don't have a meaningful global symmetry, some shapes even have many symmetry axes, so the partial symmetry algorithm is necessary for our correspondence algorithm. The current papers that support partial intrinsic symmetry are rare. [XZT$^+$09] did excellent work to estimate partial intrinsic symmetry, which conformed to our needs. Unfortunately, the time cost of partial intrinsic symmetry algorithm is high. The worst-case time complexity of [XZT$^+$09] is $O(n^4)$, a mesh with 10K vertices and with 3K samples, the process takes about 2 hours on an Intel Core 2 Duo 6300, 1.86 GHz machine with 1GB memory. Since we use the symmetry information only for the shape correspondence, we seek for efficient algorithm for the approximation of partial intrinsic symmetry. We propose an efficient algorithm to estimate partial intrinsic symmetry on curve skeleton.

Algorithm 1 Voting for intrinsic symmetry axis

Give a set J to represent junction nodes on S
  for j ∈ J do
    Given a set L, for all links of j ∈ L
    for x, y ∈ L do
      for s ∈ x, b ∈ y do
        if Similarity-Filter(a, b) passed **and**
          Distance-Filter(a, b) passed **and**
          Locality-Filter(a, b, x, y) passed **then**
          JunctionSpanVoting(L, x, y)
        end if
      end for
    end for
  end for

Algorithm 2 JunctionSpanVoting

Given a set L to represent links of j
  for x ∈ L
    for a ∈ x
      T(a) = T(a) + symmetryScore
    end for
  end for

Similarity-Filter(a,b). This filter ensure the skeleton node a and b are similar geometrically. We use the skeleton node radius value as a similar property, which denote the radius of insphere. Radius of skeleton node roughly represents the breadth of shape.

Distance-Filter(a,b). This filter reject skeleton node a and b, if their geodesic distance difference on curve skeleton exceed threshold. By this filter, we find the a and b which is isometric to skeleton junction node.

Locality-Filter(a, b, x, y). We reject skeleton node a and b, if the link x and y are not continuously symmetric. We estimate the density of symmetric node to determine whether the symmetry is continuous, the idea is similar to [EKSX96].

Symmetry axes finding. After the voting process, we obtain a symmetry axis score on each skeleton node. We use symmetry score to find symmetry axes on skeleton. The link between junction nodes called central link. In accordance with symmetry score of central links, order of the link is set to symmetry axis. During the process, check if the central link cause symmetry axis branch, if so, we pass this link. Until all the central links processed or the score of the rest central links are lower than the threshold.

Symmetry pair finding. After the symmetry axes finding, we search the symmetry pair on the skeleton of shape. For every link that connects to symmetry axes, we estimate symmetry score that contribute to the symmetry axes. For every link that connect to symmetry axes, traverse the node of the link from the side connected to the symmetry axis, estimate whether the node contribute symmetry score to the symmetry axes, until all the node are estimated or the contribution node compose more than two clusters. The end of the first cluster is the symmetry node, and we can find a node as symmetry pair on the other link. We use density-based clustering which is similar to DBSCAN [EKSX96]. Since the clustering is on a skeleton, the process can simplify to just measure the continuity of symmetric density.

Group similarity. The phenomenon is similar to intrinsic symmetry, called *group similarity*, such as Figure 3.2. There are many sub-part are intrinsically symmetric to each other, we can't assign a symmetry pair for it, we label it to group similarity. The two shapes both with group similarity is hard for most correspondence algorithm. This information is meaningful to our algorithm, since we use it to enhance correspondence. The specific method will introduce in 3.4.1 and 3.4.2.

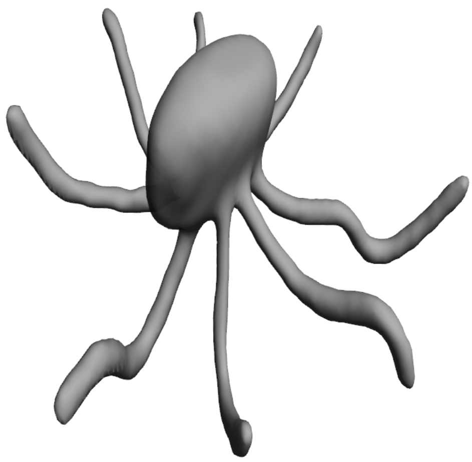


Figure 3.2: Group similarity on octopus model.

3.3 Structure diagnosing

The shape may composite by several components, such as Neptune holding a trident or two people holding hands. Many characteristics of shape are vary a lot in these case, such as centricity, shape topology, it lead to large error. We solve this problem by diagnosing the structure of the shape before the correspondence search. The symmetry axis on skeleton, which is a central of shape, we called *core path*. And the junction node on a core path are called *core node*. Each core path represents a center of sub-part of shape, it's useful to diagnose the structure of shape. Figure 3.3 show core paths of Neptune.

We obtain symmetry pairs during symmetry detection process which we introduced in 3.2, but symmetry pairs are usually on regular nodes. Since we use only feature node to compute the correspondence between shapes, it's necessary to move symmetric pair from regular node to feature node, the process called *symmetric pair moving*. In addition to the symmetry information, the structure of shape is important for correspondence, since it's prone to misunderstanding when shape have topology changes. The topology changes may arise due to acquisition imperfections, so-called *topological noise*. The topology noise is a common phenomenon in computer graphics, since the 3D models often obtain by three-dimensional scanners, or recon-

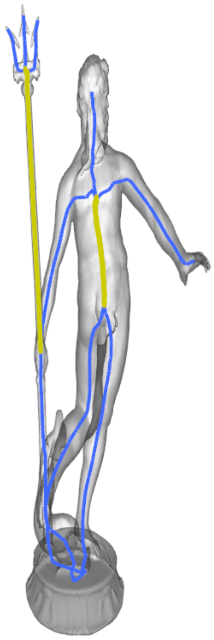
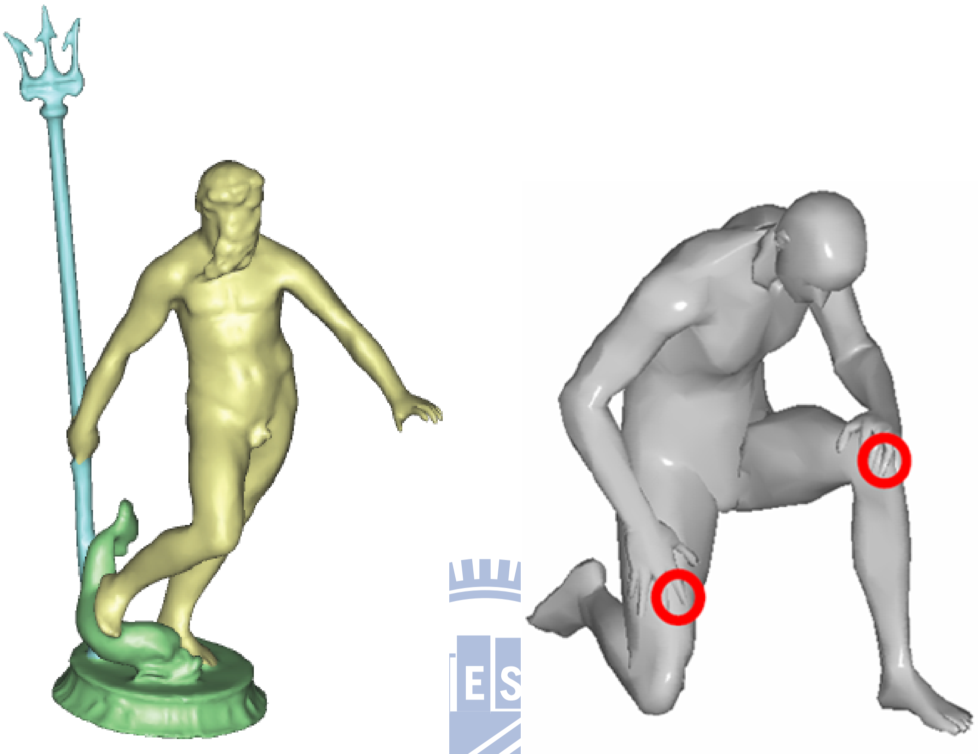


Figure 3.3: Core path of Neptune.

structed from volumetric data. The topology noise may be caused by surface gluing or partially data missing(holes). Unfortunately, intrinsic similar is sensitive to topology changes. Since the influence of mesh hole is slight in correspondence, we only handle the topology noise causing by surface gluing. We classify topology noise that changes topology structure to two categories, *composite object* and *self-overlap* (e.g. Neptune and human in Figure 3.4).

3.3.1 Diagnose composite object

Symmetric pair moving. Since our algorithm use feature nodes for correspondence, the symmetric pair information have to allocate to feature nodes. We move the symmetric pairs on the opposite direction to the core path and find the nearest feature node. Sometime, there are two different core paths have same links as those symmetry link pairs. In this situation, the symmetric pairs will meet during the symmetric pairs moving. It means the link is dominated by two different structural components of shape. The two nodes confront with each other for their moving direction by symmetry score and the length to their core path. The result of confrontation



(a) Neptune. [vKZHCO10] (b) Human. [BBK09]

Figure 3.4: Topology noise can causing by composite object and self-overlap. The composite object is shown as the human model in (a), The self-overlap is shown as the Neptune model in (b).

determine the node as a *critical connected node*, set the connected point and the node nearby connected point as symmetry nodes , and associate to different core nodes. The critical connected node and the symmetry node next to it become feature nodes and set the connective link between two nodes as *weak connecter*. This process solves the case of topology noise causing by composite obect and the connected node is on symmetry link, such as 3.5. If the symmetry node moves to a junction node after symmetric pairs moving, we set the node as a symmetry node, and set the connective link between symmetry nodes as *weak connecter*.

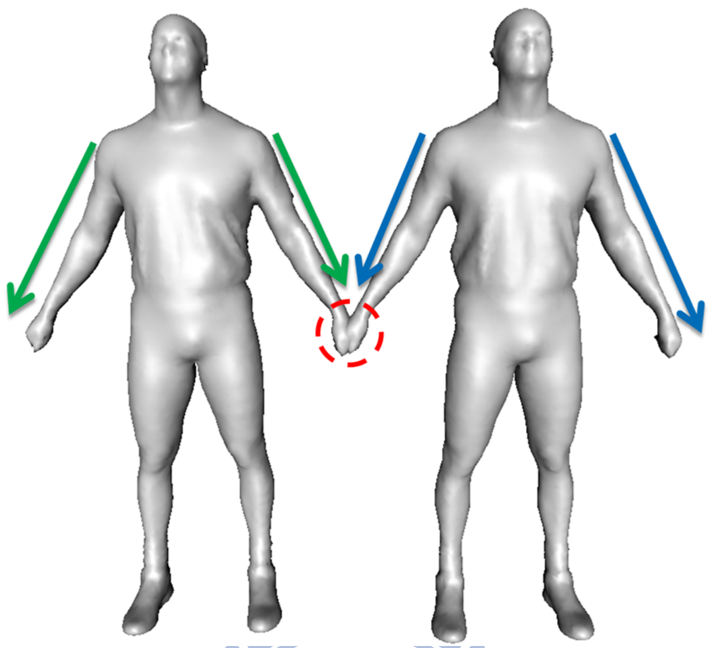
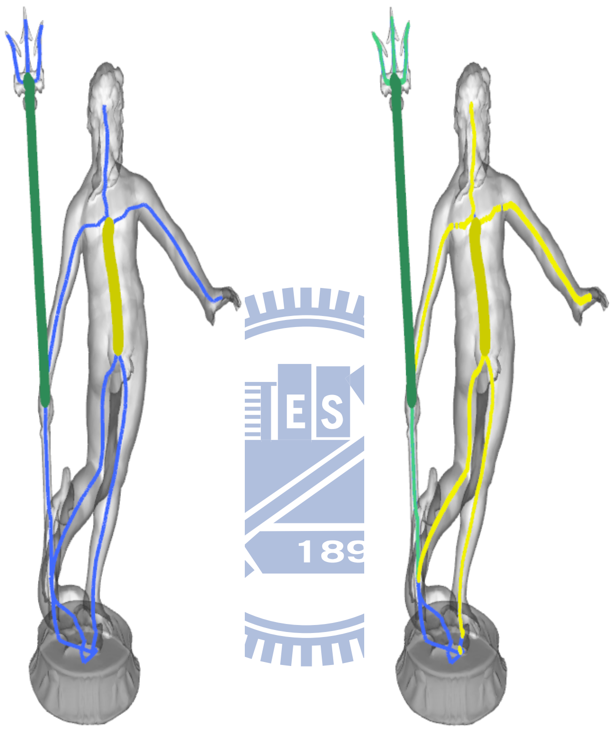


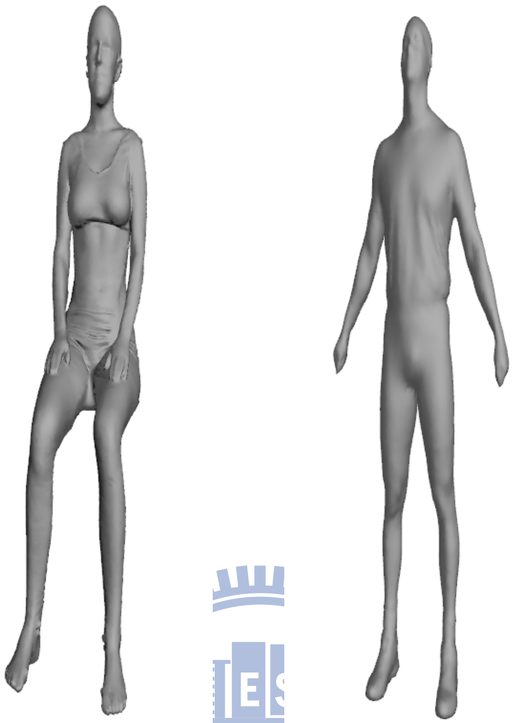
Figure 3.5: The critical connected node marks by red circle. Symmetric pairs confront with each other on symmetry link and determine the location of critical connected node.

Components scoping. Each core path represents a center of sub-part. The core path dominates a subgraph of skeleton and the nodes of every subgraph can't repeat. First, label every node on core path label to their core path ID. Then, we label every link that connects to core path to their core path ID, except for junction node, since the junction node share by many links. Final, we find every junction without label, and connected to more than one core path. The nodes set to *critical connected node*, and the nearby node which label with core path ID is also a *critical connected node*. The connected link between critical connected nodes set as *weak connecter*. The critical connected nodes which adjacent to each other are disconnection. After this process, the core path dominates every node that connected nodes. Figure 3.6 presents an example on Neptune.



(a) (b)

Figure 3.6: Components scoping on Neptune model. (a) The core path of Neptune. (b) Each core path dominates a subgraph. The color of skeleton nodes represents it belong to the core with same color, blue mean it don’t dominated by any one



(a) Woman (b) Man

Figure 3.7: (a) A woman model which have topology noise between hand and leg. (b) A man model without topology noise. The two shapes are semantically similar but topologically very different. The Figure (a) is genus 0, Figure (b) is genus 2.

3.3.2 Diagnose topology noise

The shape topology could be very different when the shape surface gluing on itself. It may lead to large error in shape correspondence process. Figure 3.7 show the topology noise causing by self-overlap leads to change shape topology. We use *unity adjusting* and *genus reduction* to handle this problem.

Unity adjusting. We utilize the unity of symmetry path to find the *gluing point*, which is the critical point of topology noise. For each symmetry path, we compare with its symmetric pair, and find the topology difference on junction node between two paths. The junction node that cause topology difference are set to *gluing point*, and become a feature point. After this process, the topology is unity between every symmetry path. The connected edge which causes gluing is *weak connecter*.

Genus reduction. Since the topology noise causing by self-overlap will create genus holes, we can use this property to diagnose *gluing point*. The genus hole is represented in curve skeleton as a loop. A loop contain core node are called *core loop*, for each core loop, we choose a feature node to be a *gluing point* by estimating the length to core path and the shape breadth on gluing point. The connected edge which causes gluing is *weak connecter*.

3.4 Correspondence search

We present an algorithm which statistically analyze the rationality of local correspondence, and adjust the global correspondence by simulated annealing. This method find the correspondence with large different between two shapes, such as topological difference, structural difference, and enhance the symmetry consistence. Furthermore, it obtains a better correspondence with group similar. We build the combinatorial tree for correspondence search, each tree node represents a matching between feature nodes and each path from root to leaf represents a local correspondence and use the branch-and-bound technique to reduce the number of tree node. After the correspondence search, we find many candidate local correspondences. Using intersection configuration to estimate the rationality for each local correspondences, and accumulate correspondence score for the matching between source and target. Then, we obtain a correspondence score from every reasonable local correspondence. Finally, utilize the correspondence score, we execute the SA iteration to compute the final correspondence.

3.4.1 Combinatorial search

The combinatorial tree search we used is similar to Au et al. [ATCO$^+$10]. However, unlike [ATCO$^+$10] which expanding tree node by a predefined ordering that all junction feature nodes preceding non-junction nodes, we predefine the ordering by considering the structure of shape.

Furthermore, we adjust the pruning test to overcome the structural difference and enhance the symmetry consistency.

We expand the tree in a depth-first search manner, the predefined ordering defined by the cluster of sub-skeleton, for each cluster, all junction nodes preceding non-junction nodes. This accelerate the combinatorial search process, since the irrational feature matching between different sub-skeleton are pruned early. Each path from the root to any leaf represents a correspondence between two shapes, and the correspondence is uniquely represented in a search tree.

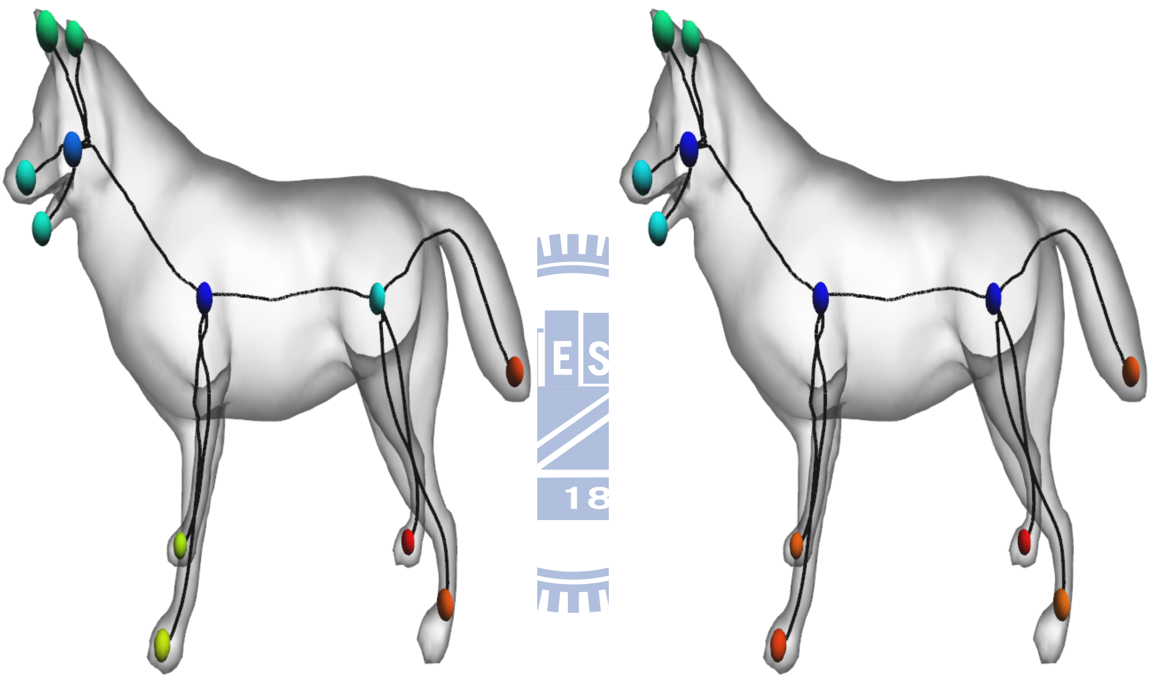
Node-Centricity. We define the centricity C_p of feature node p as the average geodesic distance form p to all other skeleton nodes which is connected without *weak connecter* : $C_p = \frac{1}{P}\sum_{p_i \in P} geo(p, p_i)$, which $con(p)$ denotes a set of feature nodes that connected to p without using *weak connecter* and $geo(\cdot, \cdot)$ is the geodesic distance between p and p_i along skeletal paths. The centricity are normalized by dividing it by the maximum centricity value of connected node without *weak connecter*. If the centricity are large than the threshold ε_C, we reject the feature pair (p_k, q_k).

$$\frac{|C_{p_k} - C_{q_k}|}{(C_{p_k} + C_{q_k})/2} > \varepsilon_C \tag{3.1}$$

Trunk-Distance. We define a property are similar to centricity, called *trunk-distance*. It represent the distance between the feature and its core path: $TD_p = \min_{t_i \in T(p)} geo(p, t_i)$. $T(p)$ denote the core nodes which dominate p (The dominate process describe in subsection 3.3.1). If the trunk-distance are large than the threshold ε_T, we reject the feature pair (p_k, q_k). The trunk-distance are normalized by dividing it by the maximum trunk-distance of its core path. The significance of *trunk-distance* are very different to *centricity*, since it consider the structure of shape. Figure 3.8 show the difference between centricity and trunk-distance.

$$\frac{|TD_{p_k} - TD_{q_k}|}{(TD_{p_k} + TD_{q_k})/2} > \varepsilon_T \tag{3.2}$$

Path-Length and Path-Radius. As the reasonable corresponding skeleton have similar



(a) Centricity on dog model. (b) Trunk-distance on dog model.

Figure 3.8: The difference between centricity and trunk-distance. Warm colors denote longer distance, Cool colors denote shorter distance

lengths and average path radius, we reject (p_k, q_k) if equation 3.3 or equation 3.4 is true.

$$\frac{|geo(p_k, p_j) - geo(q_k, q_j)|}{(geo(p_k, p_j) + geo(q_k, q_j))/2} > \varepsilon_C, \forall (p_j, q_j) \in \Omega \tag{3.3}$$

$$\frac{|rad(p_k, p_j) - rad(q_k, q_j)|}{(rad(p_k, p_j) + rad(q_k, q_j))/2} > \varepsilon_C, \forall (p_j, q_j) \in \Omega \tag{3.4}$$

The $geo(\cdot, \cdot)$ and $rad(\cdot, \cdot)$ is the geodesic distance and the average path radius of two feature nodes along skeletal paths, which don't contain *weak connecter*.

Topology Consistency. It's a local topological test for subgraph of the final correspondence. Let p_i and q_j denote the closest node junction node of p_k and q_k. If p_i and q_j are both exist in Ω, but p_k is not matched with q_k(i.e., $i \neq j$), we reject (p_k, q_k).

Symmetry Consistency. If p_k and q_k are both symmetry node, then we applied symmetry pair and symmetry core testing. **symmetry pair testing:** Let p_i denote the symmetry pair of p_k, q_j denote the symmetry pair of q_k. If p_i and q_j are both exist in Ω, but they are not matched(i.e., $i \neq j$), we reject (p_k,q_k). **symmetry core testing:** Let p_i denote the symmetry core of p_k, q_j denote the symmetry pair of q_k. If p_i and q_j are both exist in Ω , but they are not matched(i.e., $i \neq j$), we reject (p_k,q_k).

Spatial Configuration. A semantic correspondence is a ill-pose problem, we rely on intrinsic similar between two shapes. The spatial information usually is considered as extrinsic information, which is sensitive to the pose of shape. But it's hard to identify local flips, without considering any spatial information of skeletons. Oscar Au et al. present a effective approach in [ATCO$^+$10] to utilize spatial information by performing least-squares multidimensional scaling (MDS) on skeleton and use rotation distortion to estimate difference. We use the symmetry and structure information to improve this approach for higher correspondence quality. Least-squares MDS is a spatial embedding [EK03], which can span out the branches of the skeletons,

Figure 3.9 show the pose-normalized results of Least-squares MDS. However, MDS is sensitive to topology difference, and work terrible on composite object. Figure 3.10 and Figure 3.11 show the bad results of least-squares MDS. We solve this problem by breaking weak connecters during the least-squares MDS process. First, we build a duplicate skeleton and break every weak connecter. Then the duplicate skeleton are break into several sub skeletons for every connected components. Final, we apply least-squares MDS to each sub skeleton. After least-squares MDS, the spatial configuration test performs on pose-normalization sub skeleton. We use a 3×3 matrix A_{pq} as a linear transformation matrix, to represent the non-translational transformation between two matching feature node sets p_k and q_k. Different form [ATCO+10], the A_{pq} maps the vectors $\{p'_s - p'_k,\ p'_c - p'_k,\ p'_{k-1} - p'_k\}$ to $\{q'_s - q'_k,\ q'_c - q'_k,\ q'_{k-1} - q'_k\}$, where p' and q' are pose-normalized positions of the feature nodes, p_s is the symmetry pair of p_k, q_s is the symmetry pair of q_k, and p_c, q_c are core nodes. This way improve the accuracy of spatial configuration, since the vectors intersect on core of shape are much representative. Especially on the small branch of skeleton, since it usually become very close to other branch after MDS process. As [ATCO+10], we use polar decomposition to find the rotation component R_{pq}. For the effect of global flipping, we multiply R_{pq} with -1 when $det(R_{pq}) = -1$ The rotation distortion is defined as $RD_{pq} = \|A_{pq} - R_{pq}\|_F$, where $\|\cdot\|$ is the Frobenius norm. Let $RD_\Omega = max\{R_{pq}, R_{qp}\}$, which represents the rotation distortion. If $RD_\Omega > \varepsilon_{RD}$, we reject this correspondence.

3.4.2 Weighted voting

A path from the root to any leaf represents a partial correspondence between two shapes. First, we estimate the rationality of every path. Gromov-Hausdorff distance is useful property to measure intrinsic similarity. The base idea is to compute the Hausdorff distance in canonical form, which is isometric embedding. [SCF10] present a concept called *fuzzy geodesics*, which are stable with respect to the Gromov-Hausdorff distance. The pairwise fuzzy geodesics between feature points form a set of fuzzy ropes, as Figure 3.12 showed. The crossing pattern

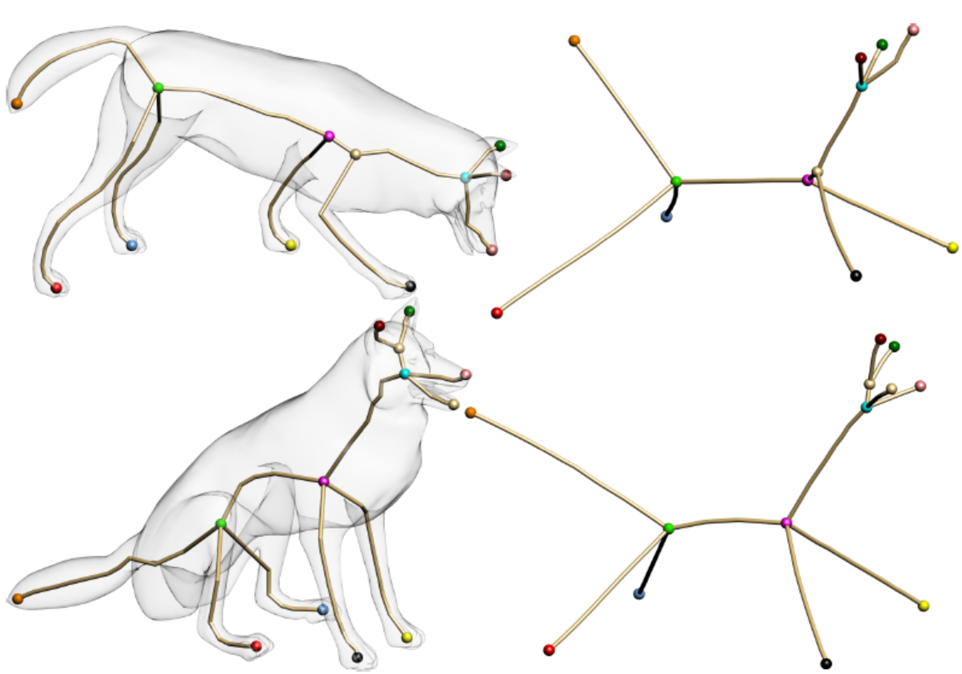
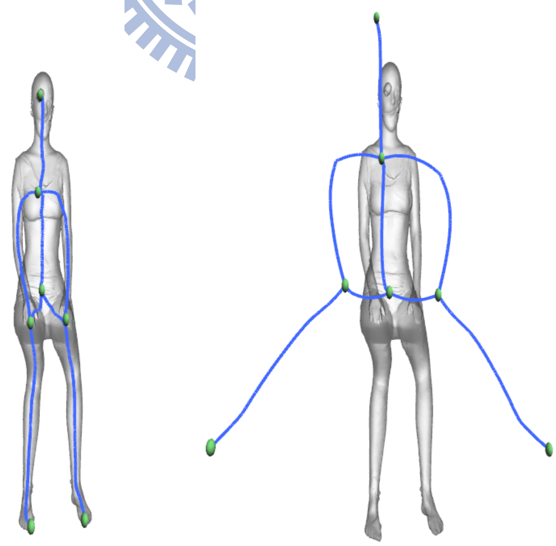
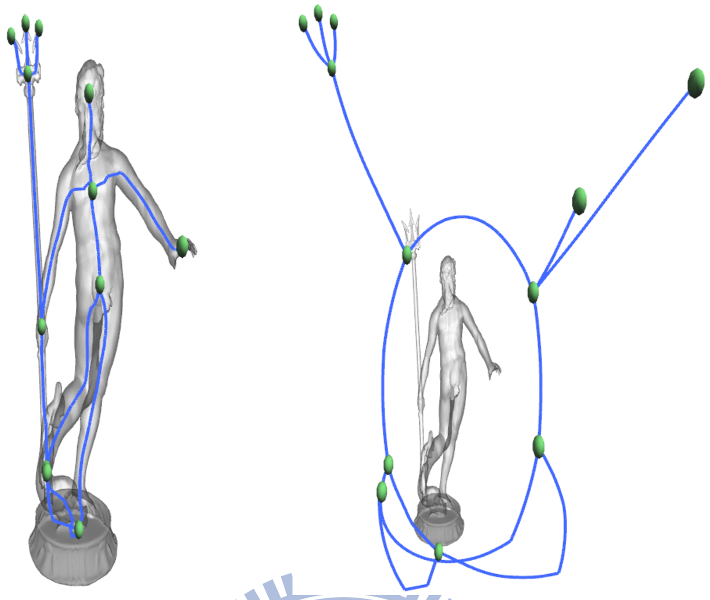


Figure 3.9: The shape are pose-normalized by performing least-square MDS. [ATCO+10]



(a) Origin. (b) After MDS.

Figure 3.10: Shape with topology noise.



(a) Origin. (b) After MDS.

Figure 3.11: Composite object.

of the ropes called *intersection configuration* (ICD)[SCF10]. We use intersection configuration distance between two shapes to measure the consistency of correspondence. We compute the reciprocal of intersection configuration distance as a consistent score for each terminal node of tree paths. Then compute the matching score for arbitrary feature pair by weighted voting. The tree path is called candidate correspondence, if the path is length and consistent score reach the threshold. Use the consistent score as weight, each candidate correspondence vote for its matching pairs. The weighted voting enhance topology consistence of local correspondence. Furthermore, intersection configuration on mesh surface obtains more information, which can't be conscious on curve skeleton, such as the starfish. In order to understand how to compute ICD between two shapes, we brief the definition of [SCF10] in this paragraph.

Definition 3.1 *Given a parameter* $\sigma > 0$, *the fuzzy geodesic between any two points* p

$and\ q\ on\ a\ Riemannian\ manifold\ M\ is\ defined\ as\ a\ function\ G^{\sigma}_{p,q} : M \rightarrow \Re :$

$$G^{\sigma}_{p,q}(x) = exp\left(-\frac{d_M(x,p) + d_M(x,q) - d_M(p,q)}{\sigma}\right).$$

Definition 3.2 $For\ any\ two\ pairs\ (p,q)\ and\ (u,v)\ on\ a\ Riemannian\ manifold\ M,$ $the\ intersection\ of\ the\ fuzzy\ geodesics\ G_{p,q}\ and\ G_{u,v},\ denoted\ I^{u,v}_{p,q},\ is\ their$ $point-wise\ multiplication,\ namely\ I^{u,v}_{p,q} : M \rightarrow \Re :$

$$I^{u,v}_{p,q} = G_{p,q}(x) \cdot G_{u,v}(x)$$

Definition 3.3 $Given\ a\ set\ of\ points\ S\ = \{s_i\}^{m}_{i=1} in\ X,\ its\ intersection\ configuration,$ $denoted\ IC_S,\ is\ a\ four\ dimensional\ square\ matrix\ of\ size\ m\ here\ the\ entry\ (i,j,k,l)$ $is\ the\ p-norm\ of\ the\ intersection\ of\ the\ fuzzy\ geodesics\ G_{s_i,s_j}\ andG_{s_k,s_l},\ namely$ $IC_S(i,j,k,l) =\ \|I^{S_k,S_l}_{S_i,S_j}\|_p$

Definition 3.4 $Given\ sparse\ correspondence\ h : S = \{s_i\}^{m}_{i=1} \rightarrow Y\ from\ X\ to\ Y,$ $define\ the\ intersection\ configuration\ distance\ as$

$$ICD(h) = \sum_{i,j,k,l} |IC_S(s_i, s_j, s_k, s_l) - IC_{h(s)}(h(s_i), h(s_j), h(s_k), h(s_l))|$$

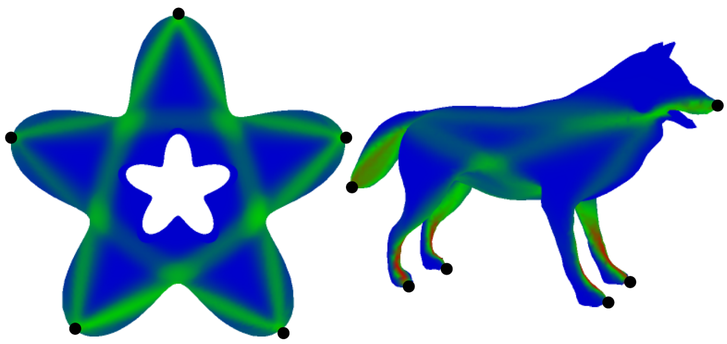


Figure 3.12: The sum of the fuzzy geodesics between all pairs of marked points [SCF10].

3.4.3 SA iteration

After the weighted voting, we obtain a correspondence score for each feature pair. The correspondence score represent the rationality of matching, therefore a correspondence that have the highest total score are a good correspondence. However, the correspondence which highest have total score may not preserve topology consistence, since the correspondence score only consider the partial matching, is lack for global information. Based on this consideration, we estimate the rationality for global correspondence by ICD. Since we can't estimate the quality before the global correspondence is determined, we find a correspondence solution, and refine it, like simulated annealing. The idea came from [CG99], which called this process *FT iteration*, since the iteration alternates between finding an optimal flow and an optimal transformation. We call the process *SA iteration*, which alternates between *supplanting* and *attempting*. Supplanting establish a structure and attempting find the optimal of this structure. We add tree paths to a initial correspondence by greedy strategy, until the paths contain every feature node. This process have to comply with 1-1 mapping rule, every feature node only match to one node. We calculate the correspondence score for initial correspondence. Then, execute supplant and attempting iteratively until reaching the iteration threshold. Specifically, the process is described in below. **Supplanting:** Supplant the path which contributed least. We estimate the rationality on final correspondence by ICD. **Attempting:** Calculate the correspondence score, if the score are higher, substitute this path for the supplanting path, else set this path as a failure, and attempt for the next path. The attempting step test all tree paths in the order of total correspondence score.

CHAPTER 4

Results

In this chapter, we present the experiment results of the correspondence, and comparison with related works. We also list the processing time of our algorithm, including symmetry detection, structure diagnosing, and correspondence search.

4.1 Results

In this section, we demonstrate some correspondence results. We have tested our symmetry-aware partial shape correspondence algorithm on various models, such as the four-legged animals, composite model, the shape with topology noise and the shape with group similarity. By using symmetry information, we avoid to confuse small branch of curve skeleton. The correspondence score is obtained by statistical method, naturally support partial correspondence. Since the SA iteration and structure diagnosing enhance the capability of partial correspondence, we can compute a meaningful partial correspondence on composite object. We can see the result between four-legged animals in Figure 4.1, the correspondence accuracy is reliable, especially at hear and mouth. For instance, the horse and cow turn their head to the right side, hear and mouth are easy to confuse, since it's similar in spatial configuration after the MDS process.

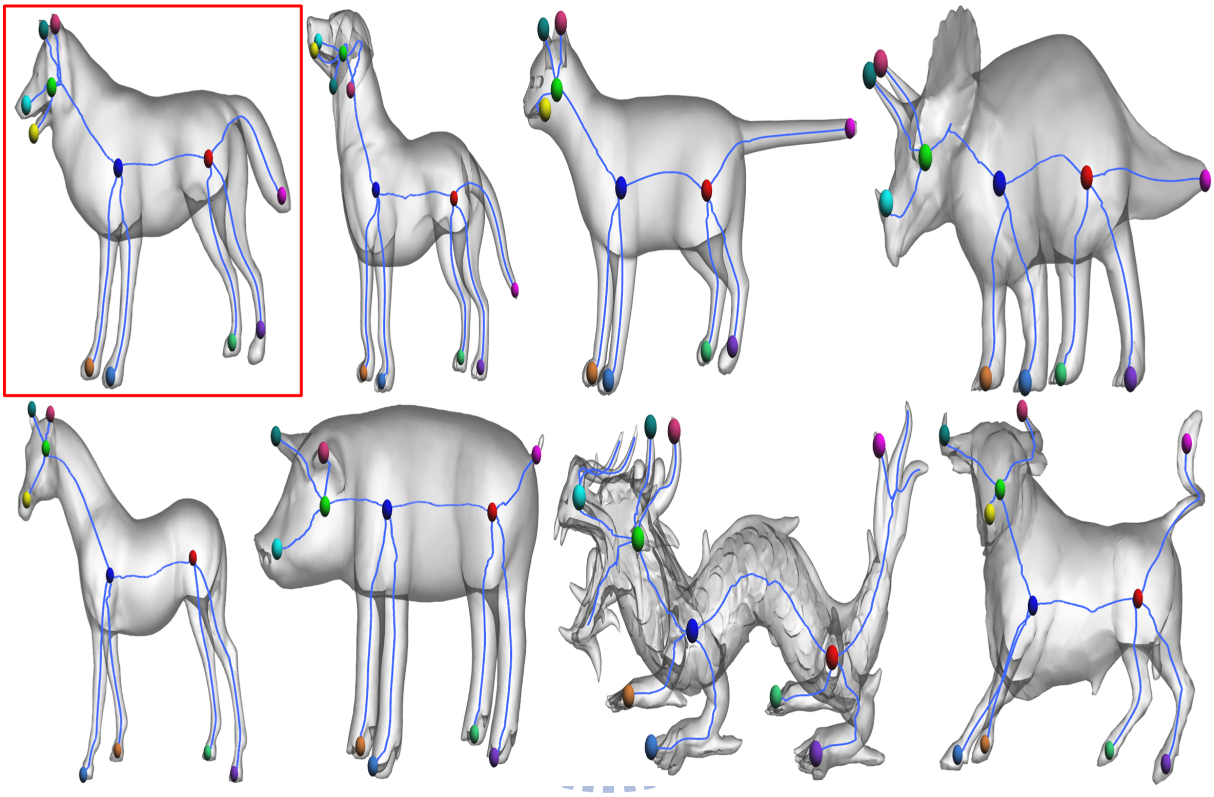
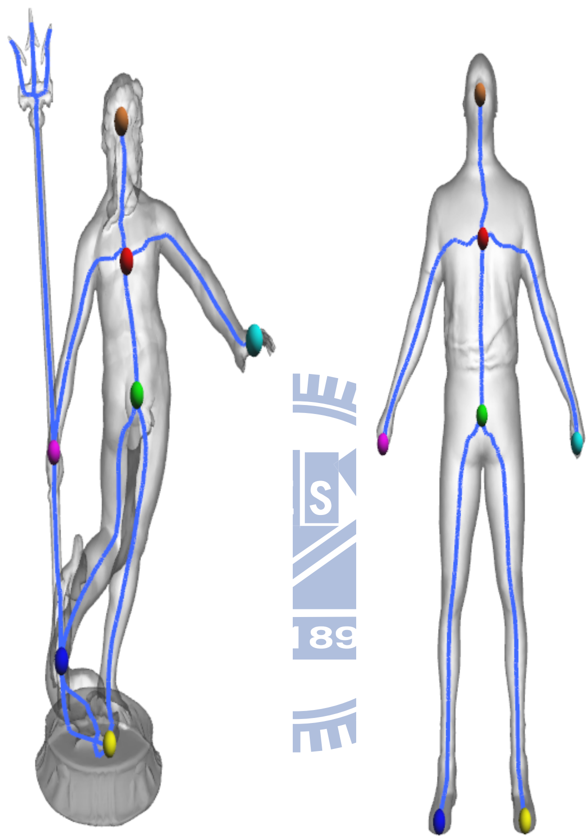


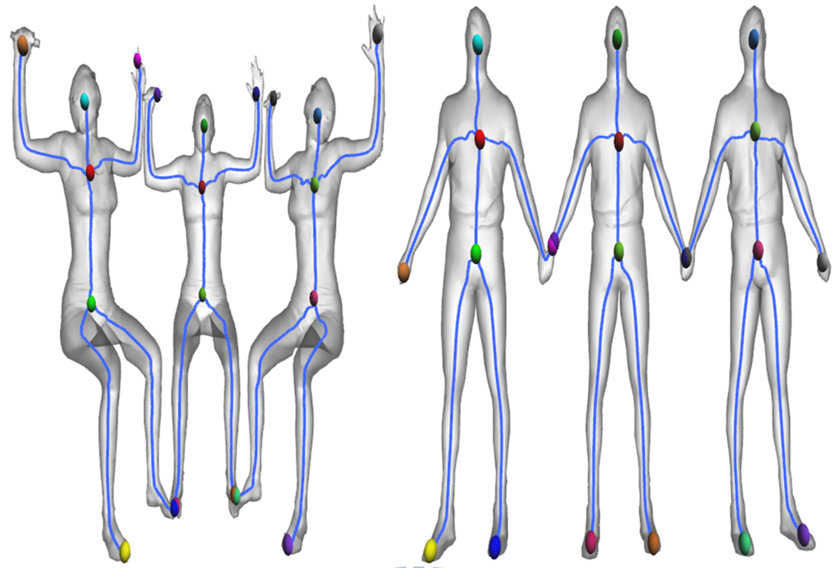
Figure 4.1: The correspondence results on four-legged animals, matching wolf to dog, cat, triceratops, horse, pig, dragon and cow.

By structure diagnosing, we have more information of shape composition. Therefore, our algorithm can work on composite objects and match sub-part statistically to find a partial correspondence. Figure 4.2 demonstrates an example for partial correspondence, the Neptune match to the man correctly, even if they have large topology difference. Figure shows another example, the two shapes both composite objects. The shape on the left is composite by three women step on their neighbors. The shape on the right is composite by three men lead by the hands. The two shapes are semantically similar but topologically different, and the connected points between human are not even a feature point before structure diagnosing. Our algorithm created four new feature nodes for each shape and computed a meaningful correspondence. There are two feature points on each connected point and the feature nodes match to the correct side, even they are almost in same position.

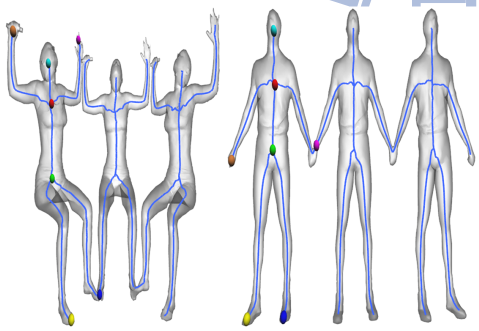


(a) Neptune. (b) Man.

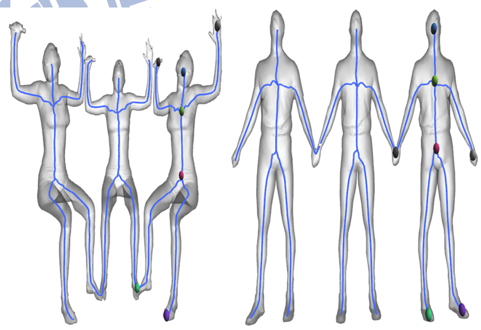
Figure 4.2: The correspondence between Neptune and a man. (a) Neptune is a composite objects, which composited by a trident, a human and a stage. (b) A simple man model.



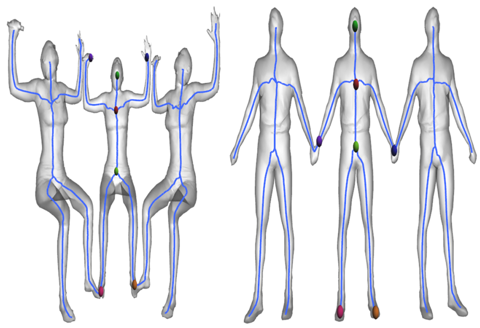
(a) The correspondence result.



(b) The correspondence on left part.



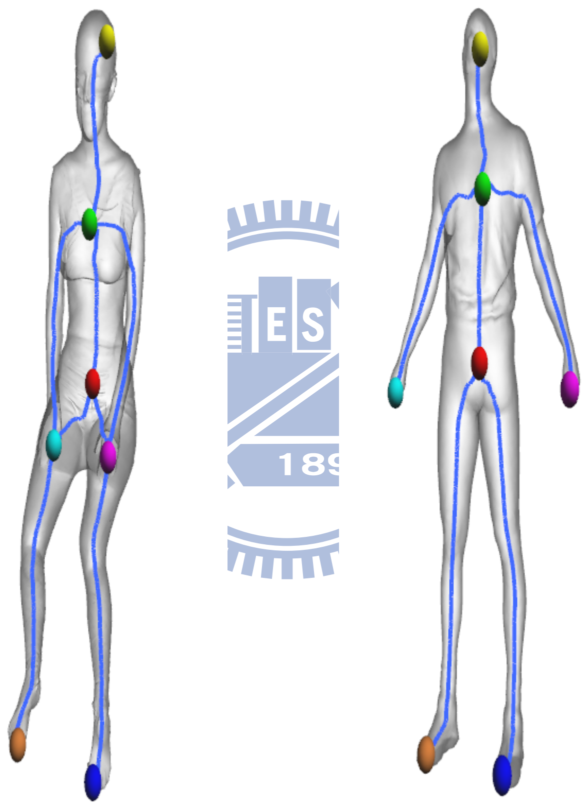
(c) The correspondence on right part.



(d) The correspondence on middle part.

Figure 4.3: The correspondence between two composite objects. (a) The correspondence between two shapes. (b)(c)(d) The correspondence on each sub skeleton.

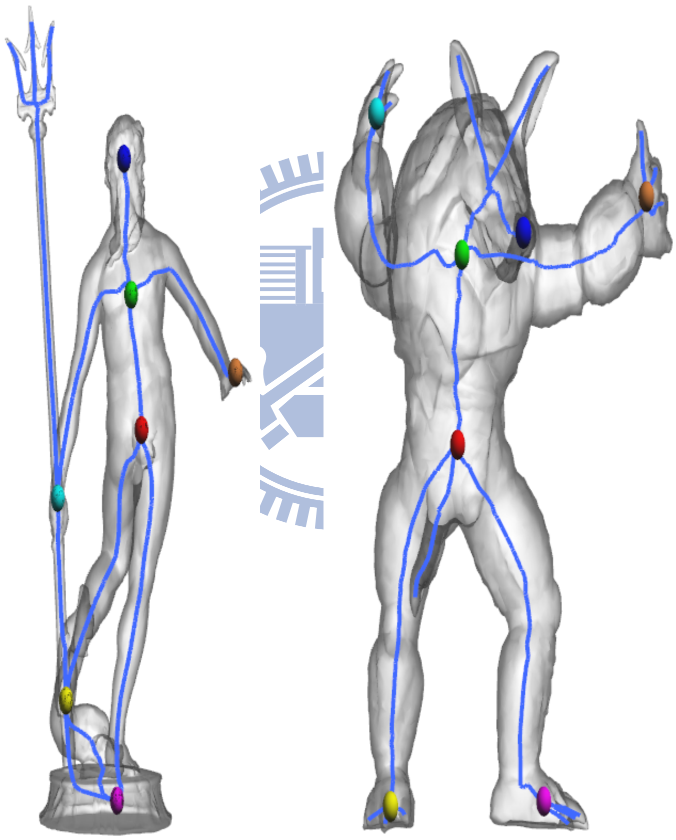
Our correspondence algorithm is insensitive to topology noise, since we can diagnose the topology noise through the help of symmetric information, such as unity adjusting and genus reduction. Figure 4.4 demonstrate an example for the capability of our method. Although the two shapes are very similar and it exist a full correspondence between them, it's still a challenge case for non-rigid correspondence. Because of the non-rigid correspondence is sensitive to topological difference.



(a) Woman. (b) Man.

Figure 4.4: The correspondence result between two semantically similar shapes. (a) A woman model which have topology noise between hands and legs. (b) A man model without topology noise. The two shapes are semantically similar but topologically very different.

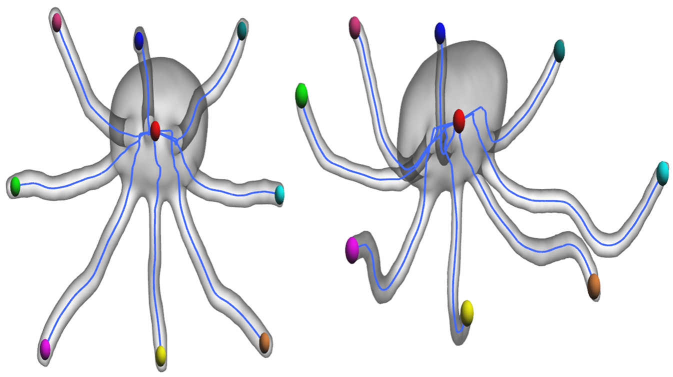
Generally speaking, most correspondence algorithm uses the geometrical information or topological information of shapes to estimate a meaningful correspondence. It will be a challenge if the two shapes are semantically similar but geometrically and topologically very different, as Figure 4.5 showed. Neptune and armadillo have different pose, different surface detail and different sub-part. Furthermore they are topologically very different, Neptune is genus 2 and armadillo is genus 0. Even so, our algorithm computed a partial correspondence between Neptune and armadillo effectively. There are 7 matched feature nodes, Neptune has 13 feature nodes and armadillo has 22 feature nodes.



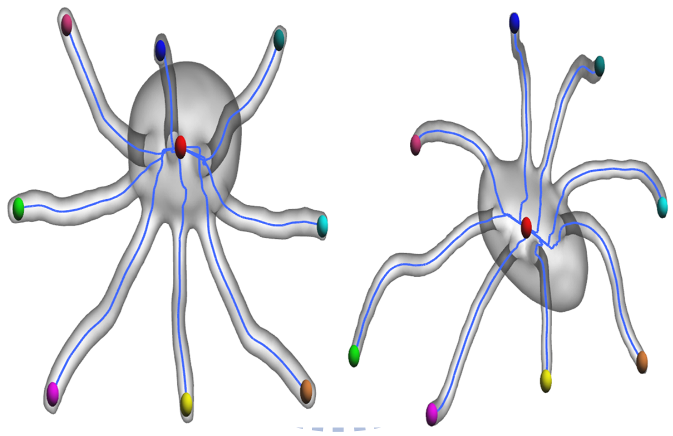
(a) Neptune. (b) Armadillo.

Figure 4.5: The partial correspondence between Neptune and armadillo. (a) Neptune have 13 feature nodes. (b) Armadillo model have 22 feature nodes.

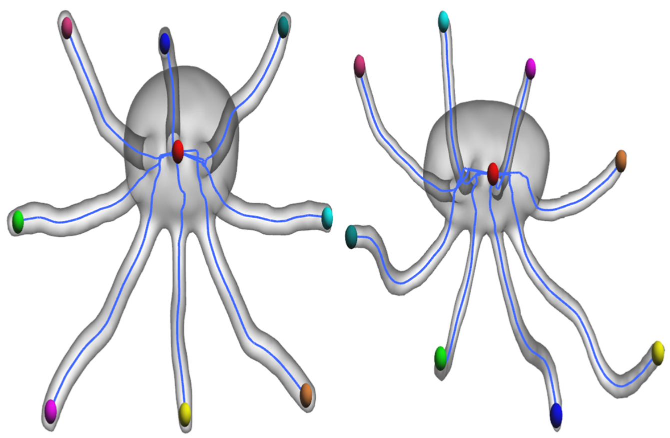
The similarity-based correspondence estimates the similarity between pairs of feature nodes and derives a correspondence. But in some situation, many feature points are very similar. We can't differentiate them by shape volume, surface characteristic, centricity, path-length or topology information. We called this group similarity, even symmetric configuration and spatial configuration are not effective in the case. Figure 4.6 demonstrate the effectiveness of our method on two shapes with group similarity, we have both advantages of intersection configuration[SCF10] and spatial configuration[ATCO^{+}10].



(a) Our approach.



(b) Without spatial configuration.

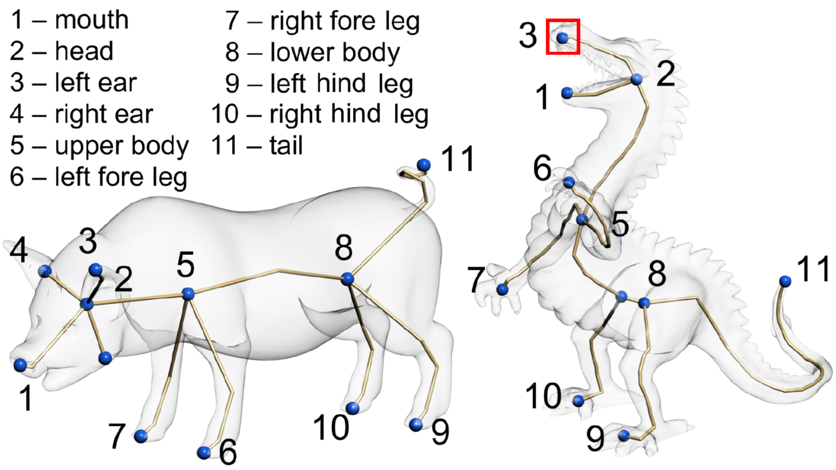


(c) Without intersection configuration.

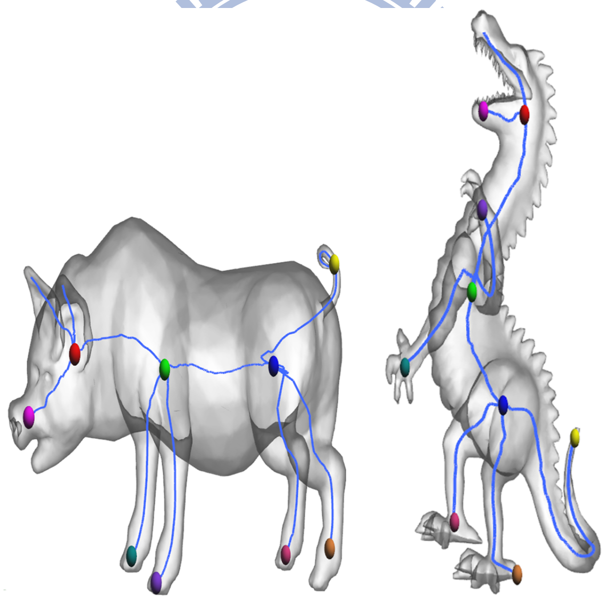
Figure 4.6: The correspondence between two octopuses.

4.2 Comparison

In this section, we compare our result to other related literature. Au et al. present a fast and fully automatic correspondence algorithm. [ATCO$^+$10], it was effective in a wide variety of shapes. Since [ATCO$^+$10] use curve skeleton as shape descriptor, it's very suitable to compare with our approach. Our algorithm is effective in variety of composite objects and insensitive to topology noise. In addition to this, the symmetry information increased the accuracy of correspondence. Figure 4.7 shows the advantage of symmetry consistency. The small branch on curve skeleton is easy to confuse after MDS. For instance, The horse turn its head causing error in [ATCO$^+$10], our algorithm are more accurate on small branch, as Figure 4.8 showed. Furthermore, our algorithm can differentiate small topological difference, as Figure 4.9 showed. Because of we checked the core-node of symmetric pair, to achieve topology consistence.

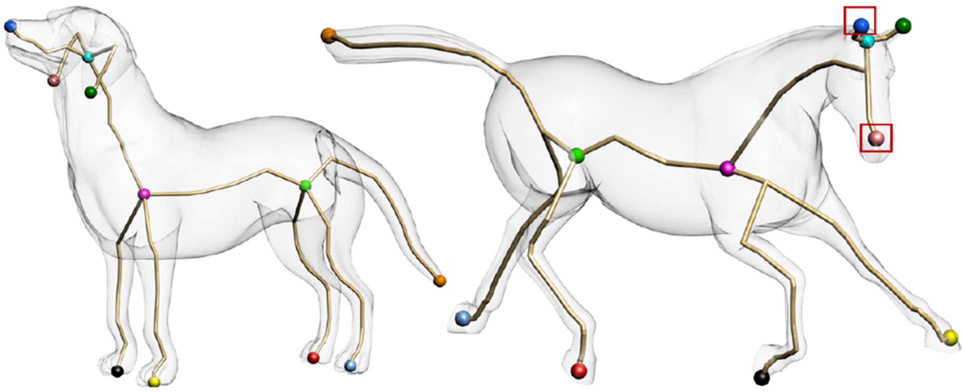


(a) [ATCO+10]

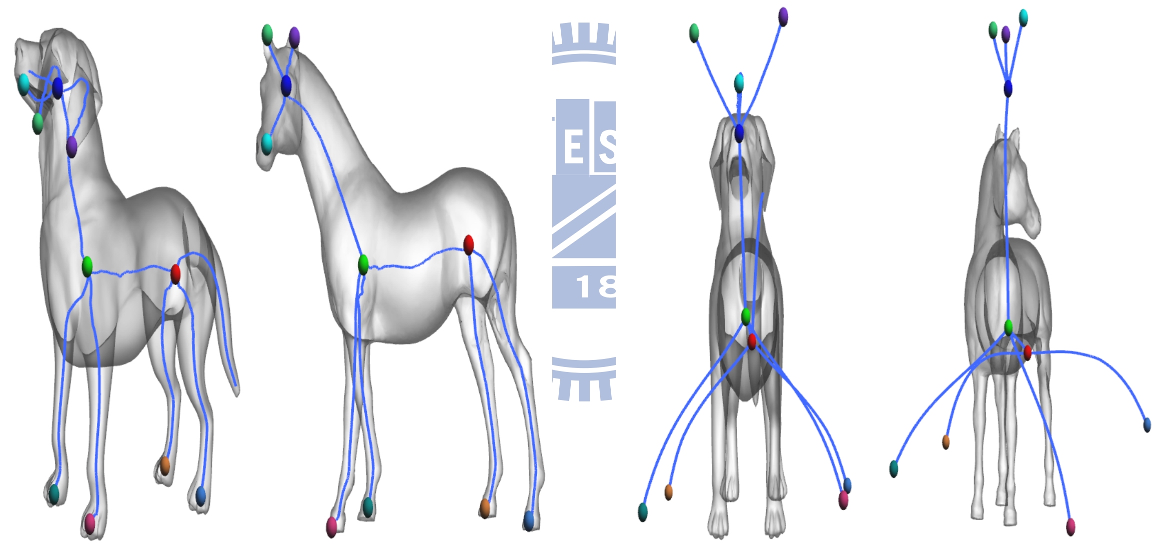


(b) Our approach.

Figure 4.7: The comparison with [ATCO+10] on pig and dragon.

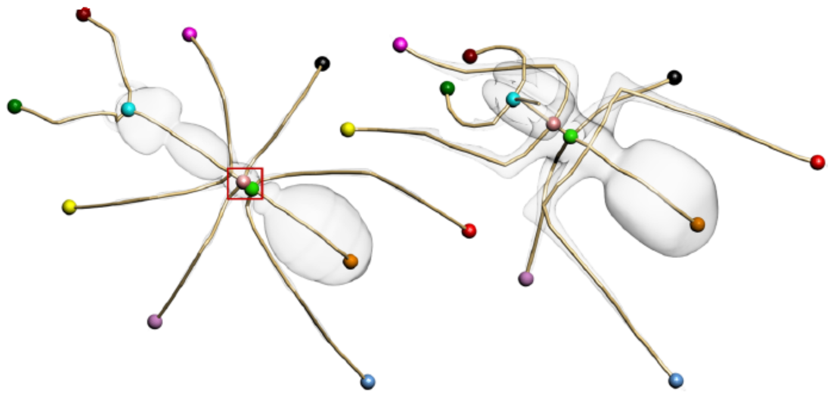


(a) [ATCO+10]

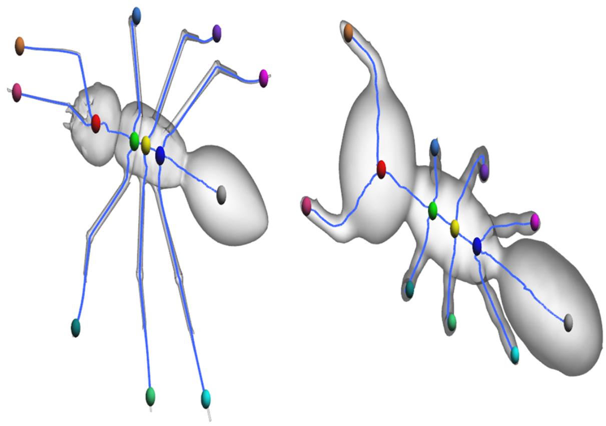


(b) Our approach. (c) Curve skeleton after MDS.

Figure 4.8: The comparison with [ATCO+10] on dog and horse.



(a) [ATCO$^+$10]



(b) Our approach.

Figure 4.9: The comparison with [ATCO$^+$10] on ants. (a) [ATCO$^+$10] produce semantically incorrect correspondence when two shapes have different semantics but similar geometry. (b) Our algorithm is conscious of small topological difference by using symmetry information.

4.3 Processing Time

In this section, we list the processing time of our algorithm. The processing can classify to shape process and correspondence process. Shape process compute for each shape, and correspondence process are compute for each pair of shapes. There are three shape processes in our algorithm, shape analysis, least-square MDS and intersection configuration. The shape analysis includes symmetry detection and structure diagnosing. Table 4.3 shows the computation time for each shape. Building the IC table cost most time, because the process have to compute geodesic distance between all pairs of mesh vertex. The process time of least-square MDS are depend on the iteration times of SAMCOF algorithm [BG97], we set the iteration times to 20 in the table. Since computing least-square MDS and calculating intersection configuration table take more processing time, we precomputed it for each shape. We only have to save the result of MDS and IC, not for the geodesic distance table on skeleton node and mesh vertex. The time cost of correspondence process are short, it take less than 2 seconds for all demonstration in this chapter. All experiments are performed on an Intel Core2 Duo E6750(2.66GHz) machine with 3G ram, using a single thread implementation.

Model	face num.	Shape Analysis(sec.)	MDS(sec.)	IC(sec.)
Wolf	9420	0.7	8.8	22.4
Dog	18112	1.6	19.9	122.9
Cat	6756	0.3	3.5	11.0
Triceratops	15764	0.8	11.8	85.1
Horse	19848	2.1	28.4	128.0
Pig	16818	2.5	35.1	87.4
Dragon	28198	2.9	14.6	313.7
Cow	4170	0.2	2.2	3.9
Octopuses1	14498	2.9	33.5	61.6
Octopuses2	12324	2.7	31.4	41.8
Neptune	19998	1.8	11.2	136.0
Armadillo.	20000	1.0	9.5	165.6

Table 4.1: The processing time for shape process step.

CHAPTER 5

Conclusions

We give a brief summary of our correspondence algorithm in this chapter. We also discuss the limitations of our system, and propose several directions of the future work.

5.1 Summary

We propose a fully automatic partial correspondence algorithm that allows matching of a wide variety of shapes with semantically similar structures but different geometric details and different topology. From the structure diagnosing, we have enough information to compute a high quality correspondence with the large structural difference or topology noise causing by gluing. And we use not only curve skeleton but also surface information to enhance correspondence, by this way, we can recognize the relative position between group similarity (e.g. tentacles of octopus). By the symmetry pairs, correspondence is more accurate with small topological difference, since we check the consistent between core nodes of symmetry pairs. Though using symmetry information, we have great progress in correspondence. We have a better way to deal with global flipping, through spatial configuration along the core path. And the topology consistence are estimated particularity in our SA iterative, even can handle group-wise correspondences, which have large topologically different.

5.2 Limitations

Our algorithm uses the symmetry information to enhance correspondence. The shape can be partial intrinsic symmetric or not symmetric at all, the symmetry are not necessary. However, if there is no symmetric property on the composite object or complex shape with topology noise, the structure of shape are hard to diagnose. Furthermore, it's have to handle the shape have very serious topology noise, which all part are glued together. Our algorithm is purely geometric-based, sometimes, the sematic correspondence need not only geometric information, but also base knowledge of the shape. Figure 5.1 show sematic correspondence can't arrive by purely geometric information. Finally, our algorithm use the curve skeleton adequately, the quality of correspondence rely on the skeleton, whether the skeleton represent the shape well, is critical to our correspondence algorithm.

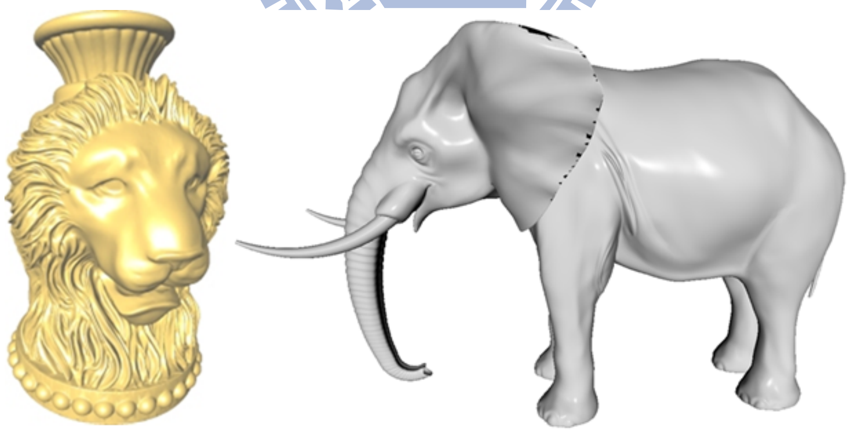


Figure 5.1: The sematic correspondence can't achieve by purely geometric information.

5.3 Future works

In the future, we would like to enhance our correspondence algorithm from two directions. The first direction is to reform the SA iteration, redesign the measure criterion to contain the topology noise information. Allow to change the topology of shape by supplanting and find the better structure by attempting. By this way, the SA iteration can adjust the correspondence on large topologically different.

The second direction is to build a knowledge database for different categories of shape. Diagnose the shape structure to several sub parts, and classify each sub part by shape retrieval technique. Use different arguments and different techniques to solve correspondence problem. For example, we apply face detection to the shape belong to human category, which often suffer from global flipping. By this way, we can utilize the knowledge of different categories of shape, and design the suit method to compute a correspondence.

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