國 立 交 通 大 學

企業管理碩士學位學程

碩 士 論 文

Reading Market Signal for Disruptive Innovation

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中華民國一百年八月

Definition of a Decision Making Process for Market Entry Strategy under a Transaction Cost Analysis and a Resource-Based View

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Chinese Abstract

當科技快速進步時，公司在市場地位和產品研發上和很難與之保持同等速率，對於目前上櫃公司，若要保持不墜之地，更是得花一番努力。當公司遇到改革創新的問題時，通常都以新的子公司來處理。

本文旨在提供新的思維，讓目前的公司能不必另起爐灶來面對創新改革的必然趨勢。本篇以矩形理論爲基礎，提供明確的市場訊號給公司處理改革創新的棘手問題；目前的科技和創新科技斜線圖呈現直角時，公司就必須關注創新科技。目前有三種不同的當前與創新理論分析市場訊號，其總結爲當直角點出現於交叉線附近時，但時間早於交叉點時，則此直角點爲最佳市場訊號。

English Abstract

As the speed of technology development increases, it becomes more and more difficult for a firm to keep its market position and update products with the same pace. The competition becomes very tough especially for incumbent firms. When an incumbent firm is facing disruptive innovation historical evidence shows that it cannot handle it within the organization. Usually it sets up a separate entity for handling disruptive innovation.

This study aims to help incumbent firms handle disruptive innovation within the organization by proposing a potential market signal for them to switch from incumbent technology to disruptive one. If there is a way for incumbent firms to read this market signal before disruption dominates then it can not only save its market position but also release it from non-necessary waste allied with opening a new branch or establishing a new company.

This study applies orthogonality theory to propose a market signal for a disruptive technology. The theory leads to our proposal that when the slopes of incumbent technology and disruptive technology become orthogonal it is a market signal for an incumbent firm to focus on disruptive technology. Three pairs of incumbent-disruptive technology were investigated for the market signal. The findings show that an orthogonal point occurs around an intersection point, earlier than an intersection point for two out of three technologies. It suggests that orthogonal point is more sufficient market signal than intersection.

Acknowledgement

First of all, I would like to give my thanks to my advisor Professor Jin-Su Kang by whom this thesis was carefully guided and inspired. I give her a gratitude for professionalism and understanding as well as challenging me when it was needed.

Also I want to thank my internal and external reviewers for their comments and valuable feedback.

Last but not least, I want to thank my family and especially my brother Dima for the patience and support.

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I. Introduction

1.1.Research Background and Motives.

The past century was depicted by precipitate development of technology. It penetrates into all levels of people's life, starting from home appliances and health to professional and personal networking. When the debate about classic "chicken-egg" question "Is technology changing us or it is we who change technology" is still going on, there are no doubts that speed of technological development forever changed the way of doing business. Constant innovativeness is doubly tight built into organization's daily routine. With the rapid development of technology both customers and companies learned the importance of innovation. If before tradition was the synonym of stability in our new society constant innovation pulled this connotation. And the speed with which organization should response to fast technological change is increasing as well. Capability of organization to handle the innovation has found a lot of interest in economic literature. (e.g. Chandler, 2002; Christensen, 1997; Hannan & Freeman, 1984; Nelson & Winter, 1982; Porter, 1980; Staw, Sandelands & Dutton, 1981; Tushman & Romanelli, 1985, etc.)

Innovation within one organization undoubtedly requires a lot of resources. Therefore, it would be quite logical to assume that the more resources (money, equipment, qualified specialists etc.) organization has the more successful it will be in terms of innovativeness. But history is full of the contradictious examples. For example, Digital Equipment and Data General were leaders of minicomputers market, but they absolutely missed the opportunity to capture desktop computer market, although both company developed prototypes of desktop computers way before they were introduced by Apple, Commodore and Tandy and later by IBM. Still they failed to get this eventually niche market and later, when desktop computer outperformed minicomputer, they lost their initial market. And there are many examples like that. One might say that the reason behind this phenomenon is bad management or poor organizational structure or insufficient R&D department. But the failed companies very often had very high-qualified management teams and invested a lot in R&D and listened to their customers, in other words did all the "right" things.

Recently this topic was brought into a very wide discussion by Harvard Business School professor Clayton M. Christensen. He argued that the reason why the firms from above example failed is because they faced a disruptive innovation. An established company looking at a disruptive opportunity is looking at something its customers do not want and that is less profitable than its alternative investment opportunities. It is not irrational for established firms to ignore disruptive opportunities. On the contrary – the decision not to compete in disruptive technology is often a sound business decision. It only looks disastrously wrong in retrospect. [6]

The health care industry is a classic disruptive opportunity. Patients have to pay for more technology than they actually need. In some circumstances, those who are ill certainly do need the best experts, high tech tests and specialists – but not in all cases. Today's system offers patients one-size-fits-all health care. Disruptive innovations would allow low-cost, low-margin and less technically sophisticated care to be provided at the low end of the market, or to be offered to current non-consumers of health care. Successful disruptors will push health care from treatment to prevention.[6]

The theory of disruptive innovation explains that companies fail because they do the right things – things that are correct in the context of their value networks. A company whose capabilities equip it perfectly for one value network may be utterly incapable of operating in another value network. The biggest problem however, is to recognize potential threat of being disrupted on time and to respond accordingly. Distinguishing between sustaining innovations and disruptive innovations can be difficult, but doing so is essential for strategic success.

The main issues related to disruptive innovation is to define the right time for company to start to invest in disruptive innovation and also to find the way to handle it without loosing mainstream dimension of company's activities. (Lange, Boivie, Henderson 2009, O'Reilly and Tushman, 2004, Van Looy, Martens, Debackere, 2005, etc.)

The way that incumbent firms were handling the disruptive innovation is usually by splitting out into two organizations or establishing an independent company to handle this innovation and buying it out later. [26]

If the organization is capable to see the potentially disruptive technology before it actually disrupts, only in this case it has necessary time to build it in its organizational processes and to get a significant competitive advantage within one organization.

1.2.Research Objectives.

Based on the research background from previous section the importance of defining the right market signal of disruptive innovation is quite obvious. The theory of disruption is relatively new and there are many areas for research in this topic. Disruptive technology is difficult especially for incumbents to handle since it requires a significant resource allocation. There are multiple examples of handling this kind of innovation by establishing a branch of organization or even a separate company for dealing with disruptive technology while incumbent firm keep putting its main resources in currently dominant technology. [26]

There is also another way to handle disruptive innovation. If the timing when disruptive technology starts to develop independently from incumbent technology can be found sufficiently than firm would know when it should give up its dominant technology and focus on disruptive one without loosing its profit.

The research objective of current study is to define the proper market signal for potentially disruptive technology which will let incumbent company know that it is time to switch to a disruptive technology. In other words, to interpret market signal given by technology development in terms of disruption.

If such market signal is found the company will not have to establish a new entity for handling disruptive innovation anymore, it can be done within one organization. This will drastically reduce cost of handling new technology for a firm.

II. Literature Review

2.1. Definition of Disruptive Innovation.

Since the economists realized the importance of innovation it became one of the most widely used and recognized terms. Almost every significant economic theory contains a chapter on innovation. With technology development rapid acceleration innovation and its implementation became crucial for business to survive. It is not a matter of choice anymore but rather the matter of time and a kind of innovation. A research and development department has become a norm in a company long ago and their primary task is to make product better on a continuous basis. It is so called sustaining innovation. Sustaining innovations support established industries but often reach a point at which they outstrip market needs.

But some innovations are radical or discontinuous which means they are drastically different either in technology or in market or even in the method. Implementation of radical innovation leads to creative destruction. [21]. According to Schumpeter the process of creative destruction is required in order to change from traditional way of how the things are to a new way to implement the innovation. Consequently, creative destruction is quite difficult for incumbent firms to survive and keep their market positions [21].

The term Disruptive Innovation was coined by Christensen (1995, Disruptive Technologies: Catching the Wave). Undoubtedly Christensen is the most referred person in the matter of this subject. He wrote number of article in this matter (Christensen and Bower, 1996; Christensen and Rosenbloom, 1995; Christensen, 1996; Christensen et al, 1998) and books The Innovator's Dilemma (1997), The Innovator's Solution (2003), Seeing What's Next (2004), Disrupting Class(2008), Innovator's Prescription (2008) the last four books were co-authored with remarkable professionals in the field of study.

Christensen's theory posits that disruption occurs when a technology that is superior on a new dimension that appeals to a niche, but inferior on a dimension that appeals to the mass market, improves on the latter dimension to meet the needs of the mass market. The theory suggests that such a lower attack is potentially "disruptive," because managers of incumbent firms may ignore or belittle a seemingly inferior technology. [23] The customers for disruptive innovations are usually at the low end of the market,or may not yet be in the market at all. [6].

There are also several kinds of disruption : technology disruption, firm disruption and demand disruption. [23]

- Technology disruption occurs when the new technology crosses the performance of the dominant technology on the primary dimension of performance.
- Firm disruption occurs when the market share of a firm whose products use a new technology exceeds the market share of the largest firm whose products use the highest-share technology.
- Demand disruption occurs when the total share of products in the market based on a new technology exceeds the share of products based on the dominant technology.

In terms of performance disruptive technology when launched is considered to be worse than domain technology and it doesn't satisfy needs of mainstream customers. It survives by selling to niche market but then after disruptive technology advances it becomes sufficient enough to satisfy needs of mainstream market that's where the companies who kept domain technology and ignored disruptive one get into threat of losing its business.

On the first stage of this process when disruptive technology is still underperforms we often hear word "Never.." in prediction towards its development.

As an example, famous prediction made in 1977 by co-founder of Digital Equipment Corp., Ken Olsen. He said: "there is no reason for any individual to have a computer at his home". Now we have over a billion people having computers at home.[Internet Source, 1] Out of the three computing giants of the late 80s, only IBM is still standing. Digital Equipment was swallowed by HP and Wang was bought by Getronics. Minicomputers were considered as a toy in the late 1970s and they were predicted never become a prevalent computing platform.

In an interview in February 2005, Amstrad chief, Sir Alan Sugar said, "Next Christmas, the iPod will be dead, finished, gone, kaput". It proved wrong as 174 million units of the media player have been sold through out the world and it lasted for more than a Christmas.

Disruptive innovation is a complex process and a relatively young subject in academic literature. An attempt to bring some light on this issue was made by Sood and Tellis (2010). They tried to give definitions for types of technologies, types of attacks, and domains of disruption, and explain the dynamics of competition between new and dominant technologies. They derive seven testable hypotheses, which they test using historical data on 36 platform technologies from seven markets. [23] Among other interesting findings, their study showed that disruptive strategies that used low prices are not that common as was thought before. According to Sood and Tellis there is an upper attack—where an entering new technology performs better than the dominant technology. And there is a lower attack—where an entering new technology performs worse than the dominant technology on the primary dimension of performance.

The term “disruptive technology” has been attributed to technologies entering via a lower attack. But, although 47% of all technologies adopt a lower attack, only 16% of all technologies cause technology disruption and only 14% of all technologies cause firm disruption via a lower attack. [23]

The definition of disruptiveness is still debated. The main problem here is that disruptiveness is a relevant phenomenon. To be more specific, innovation may be sustaining towards one technology but disruptive towards another. For instance, the camera in mobile phones is a sustaining technology for mobile phones, but it disrupts digital cameras market. Other issue that Sood and Tellis pointed out is the timing of disruption. In other words, it is not clear from which point of time technology considered to be disruptive, because due to the complex definition technology can gain its disruptiveness in the process of market penetration.

To sum up, Christensen gave definition for both sustaining and disruptive innovation in his book “Seeing What’s Next”:

> Sustaining innovations – These are improvements to existing products that enhance performance in dimensions traditionally valued by mainstream customers. They make existing products and services better.
>
> Disruptive innovations – These innovations change the value proposition. Initially, disruptive innovations under-perform mainstream products but offer some advantages of cost and ease of use. They cause fundamental changes in the marketplace.

For the purpose of this thesis we are going to use Govindarajan and Kopalle’s definition of disruptive innovation. Since Christensen himself acknowledged that this definition is better than his, because it captures broader scope. According to Govindarajan and Kopalle’s:

Disruptive innovation is an innovation which introduces a different set of features, performance, and price attributes relative to the existing product, an unattractive combination for mainstream customers at the time of product introduction because of inferior performance on the attributes these customers value and/or a high price—although a different customer segment may value the new attributes. [11]

2.2. Main problems for organization facing disruptive innovation.

Disruptive innovation in general is quite difficult for firm to handle. Cooper and Schendel (1976) concluded that drastically new technologies usually penetrated into new markets, rather than satisfied needs of already existing markets moreover this type of technologies were usually implemented to the market by newly established firms. Foster (1986) noted that at points when new technologies enter an industry, entrants seem to enjoy an 'attacker's advantage' over incumbent firms. Henderson and Clark (1990) posited that entrant firms enjoyed a particular advantage over incumbents in architectural technology change. Sood and Tellis (2010) though discovered that actually incumbents caused 50% of all technology disruptions and 62% of all firm disruptions. However, in all markets, even though incumbents introduced more technologies and caused more disruption than entrants, many incumbents lost market dominance and subsequently failed. [23].

Burgelman and Siegel (2007) points out that strategic changes required by innovation usually cannot keep up with rapid development of innovation itself. He argues that customers are slowing down the innovation since they don't want to change to a new products, therefore the task of shaping and predicting customers preferences lies down on a firm itself. [3]

Innovative activities, by their very nature, display dual and paradoxical requirements in terms of interaction. These polarities, pertaining to the social dynamics in which exploitation versus exploration unfold, can be seen as one of the root causes of the complex nature of organizing innovation at the firm level.[26] Disruptive innovation requires even more from the firm business process. According to March, innovation requires two kinds of activities from firm to be made at the same time: exploration and exploitation. [12] The social dynamics in which both types of activities are embedded not only expose characteristics of a different, but even of an opposing nature. Exploitation benefits from homogeneous relational fields, whereas exploration presupposes more heterogeneous ones; exploration implies conflict and a redefinition of identities, while exploitation thrives on consensus and can be seen as identity confirming [26].

Naturally, scholars were working mostly on solving difficulties with the re-allocation firms resources and capabilities when facing disruptive innovation (Tushman and Anderson, 1986; O'Reilly and Tushman, 2007, Lange Bowie Henderson, 2009).

One of the common things for disruption is inability of incumbent firms to hold their market positions when entrants are thought to be quite successful. Interesting findings shows Sood and Tellis's research. According to them, incumbents cause 50% of all technology disruptions and 62% of all firm disruptions. However, in all markets, even though incumbents introduced more technologies and caused more disruption than entrants, many incumbents lost market dominance and subsequently failed. Hence, there is no room for complacency. Entrants do disrupt. And for entrants to account for many disruptions, often without the expertise, market knowledge, or resources of the incumbents, is quite impressive.[23]

One of the ways to solve the problem, which found a big support among academic works, is to establish a separate branch for handling disruptive innovation. Since it is very difficult to maintain both sustaining and disruptive innovation within one organization it was argued that it is possible to do it in a so-called ambidextrous organization. (Tushman and O'Reilly, 1997, He and Wong, 2004). Later O'Reilly and Tushman (2007) argued that ambidexterity is more like a capability rather than structural change of organization. Ambidexterity requires a coherent alignment of competencies, structures and cultures to engage in exploration, a contrasting congruent alignment focused on exploitation, and a senior leadership team with the cognitive and behavioral flexibility to establish and nurture both.[18] Therefore this is a point of view on handling disruption from internal side, when Christensen is focused more on external causes.

When organization faces disruptive innovation it encounters several problems. First of all disruptiveness of innovation contemplates different marketing strategy for the product. Since the target market is usually different all the marketing mix should be changed. For company it means the structural change within the corporate culture and business process. In other words it should change in most of the cases its internal and external networks. [22]

When Christensen points out the importance of external variables for firm handling disruption Sood and Tellis conclude that it is internal culture that is a key factor responsible for disruption rather change of technology or market itself. [23].

The special cases of disruptive innovation were studied over last two decades. Henderson and Clark (1990) studied shift from vacuum tube radios to transistor radios. A comprehensive study was made by Jörnmark (1993) about steel mill manufacturers in Europe.

One of the biggest unsolved problems is when the company should recognize and respond to a thread of being disrupted. As many scholars pointed out it is indeed not that easy to define whether the innovation is disruptive or not prior to the actual fact of disruption.

III. Research Methodology

3.1. Diffusion and Substitution Model.

Diffusion Model of technology adoption was first introduced by Frank Bass in 1969. It is one of the most frequently used and referenced model in Marketing Science. Over last four decades model has been reviewed and extended. Still the basic three-parameter model gives accurate results.

In this study the Norton and Bass model for diffusion and substitution of successive generations of products will be used. [17]

Norton and Bass (1987) were first to capture both substitution and diffusion effect of successive generations, moreover they applied model for aggregated data whereas before model was used just to forecast one generation of technology. They combined the original Bass diffusion model and Fisher and Pry(1971) technology substitution model and applied it for 4 generations of DRAM technology and 3 generations of SRAM technology. The fitting is showed in Figure 1.

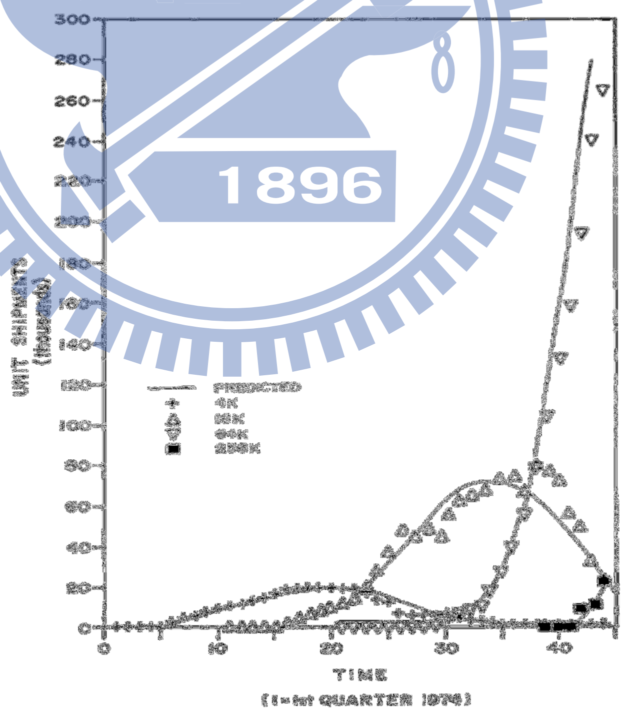

Figure 1. Norton and Bass Model Fit to DRAM data.

Source: Norton and Bass. A Diffusion Theory Model of Adoption and Substitution for Successive Generations of High-Technology Products.

There are several reasons to choose particularly this extension of original Bass diffusion model. Since the disruptive innovation eventually substitutes dominant technology it's important to capture the substitution effect, which the model mentioned above proved to do quite accurately. We can treat disruptive innovation as successive generation. Although the technology often differs, but the customers' needs that it satisfies in the end are the same as a product based on previous technology. Norton and Bass model showed good fitting with an actual data so we find it sophisticated to use for this study.

Model developed by Norton and Bass is based on three parameters that have to be estimated. The model itself looks like this:

$$S_1(t) = F_1(t)m_1(1 - F_2(t - \tau)),\ \ for\ t > 0 \quad (1)$$

$$S_2(t) = F_2(t - \tau)(m_2 + F_1(t)m_1),\ \ for\ t > \tau > \tau \quad (2)$$

Where:

- S_1 and S_2 are sales amounts at time t of product 1 and 2 respectively;
- m_1 and m_2 represent number of people estimated to eventually adopt the new product 1 and 2 respectively;
- τ is a time when second generation is introduced;
- F_1 and F_2 are fractions of ultimate potential which has adopted by time *t*;

Functions F_1 and F_2 in the original Norton and Bass model are expressed by the same equation, since they assumed that coefficients p and q are constant through the generations. In this case the fraction of ultimate potential adopted by time t will be:

$$F_i = \frac{1-e^{-b_i t}}{1+a_i e^{-b_i t}} \quad (3)$$

Where:

$a_i = \frac{q_i}{p_i}$

$b_i = q_i + p_i$;

p_i- coefficient of innovation, which represents number of initial adopters of the product i

q_i- coefficient of imitation, which is associated with the word-of-mouth effect of the product i

Since τ is the time when second generation is introduced, then $F_2(t-\tau)=0$ for $t<\tau$;

Figure2 provides graphical explanation of the Norton and Bass model.

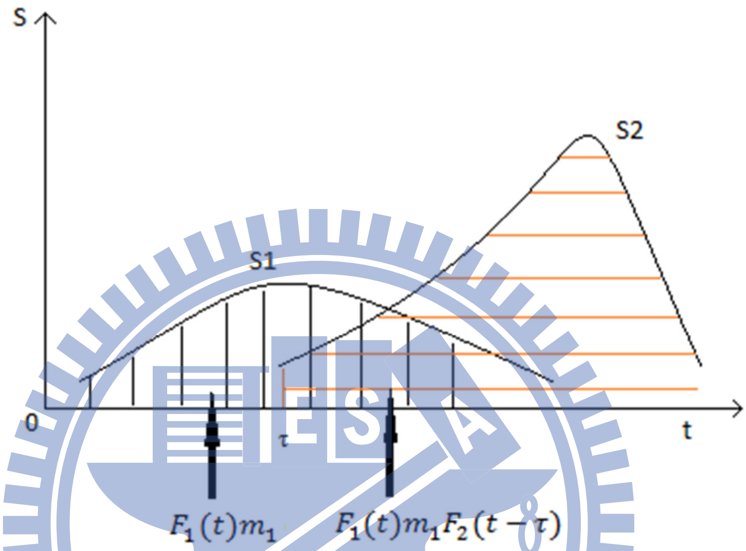

Figure 2. Sales of two generations of technologies.

This is a pictorial representation of the sales. The region before time τ represents sales before second generation is introduced and equals to $F_1(t)m_1$ (fraction of already adopted market multiply by total potential market size). But as soon as second generation is introduced the picture changes, now the part of market m_1 is taken by second generation so the sales which are taken by second generation equals to $F_1(t)m_1F_2(t-\tau)$. The same logic is used to the second generation.

The basic model was extended in a different ways including in the model more variables. Bin Jun and S. Park developed a model based on consumer choice behavior that simultaneously

captures the diffusion and the substitution processes for a multi-generation product. Their model captures effects of exogenous variables such as price are incorporated into the diffusion processes through the choice behavior of the consumer. Besides they included the time variables to account for the effects of the unavailable variables as well as to reflect the diffusion effects of multigenerational products. [15]

3.2. Coefficients.

The key role in Norton and Bass model mentioned above play three coefficients. The understanding of nature of these coefficients is crucial for comprehending the model itself.

When new product is launched some customers tend to adopt them earlier than others. The first individuals to adopt new product are so-called innovators. [21] They are thought to be willing to take risk and to have more secure financial position then other categories of this theory. The complete Rogers adopt categories are presented in Figure 3

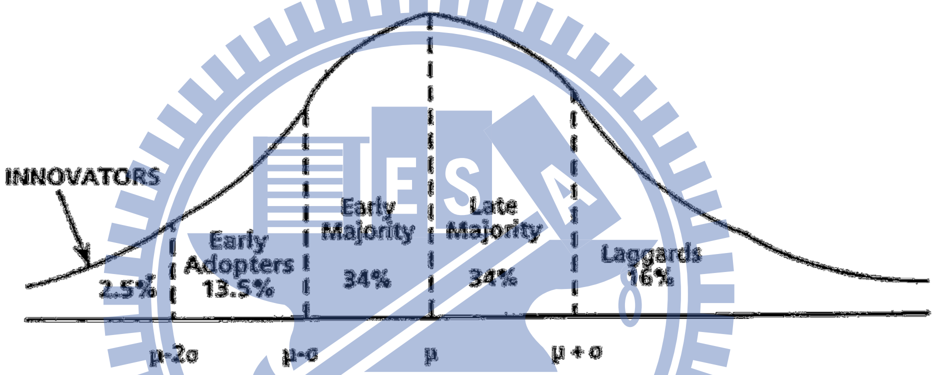

Figure 3. Rogers adopt categories.

Source: Rogers. Diffusion of Innovations.

Bass refers to these categories in a more general way. He divides all adopters into innovators, which is the same as Rogers' theory, and imitators. Imitators are individuals that base their choice on someone else experience of adoption of a new product. It is highly interacts with a so-called word-of-mouth effect.

In their study Norton and Bass made several assumptions. One of the original assumptions was particularly questionable. Specifically, they assumed that p – coefficient of innovation and q – coefficient of imitation are constant through all the generations of technology. However it showed good fitting with real data for DRAM and SRAM still this assumption is being debated a lot and was recognized as a limitation by Norton and Bass themselves.

There were number of studies trying to estimate p and q. Sultan Farley and Lehmann (1990) did a meta-analysis of 213 coefficients targeting different industries, empirical evidence showed that coefficient of innovation doesn't vary that much from case to case. In the same time coefficient of imitation varies depending on type of innovation, type of estimation procedure and presence of other coefficients. Their empirical analysis also showed that coefficient of innovation average value is p=0.03 and coefficient of imitation average value is q=0.38.[24]

Potential market is also a parameter to be estimated. Usually it is being estimated from previous sales data. However this approach had some critics. It has been argued that potential market should be treated as variable, with a particular growth rate. But the results of using this estimation proved to be insignificant. [2]

Therefore the possible ways to estimate potential market is to consult with third party such as marketing consultant or analyst forecast, also managerial opinion can be quite valuable for this purpose. For current study we use estimation from previous sales data.

Franses and van Dijk (2000) introduced several ways of estimating parameters more precisely. Franses and Dijk Model for Diffusion parameters is described by following equations:

$$m_t = m_1 + \sum_{j=2}^{k} m_j \frac{1}{1 + \exp[-\gamma_1 (t - t_j^*)]} \quad (4)$$

$$p_t = p_1 + \sum_{j=1}^{k-1} p_j^* \frac{1}{1 + \exp[-\gamma_1 (t - t_{j+1}^*)]} \quad (5)$$

$$q_t = q_1 + \sum_{j=1}^{k-1} q_j \frac{1}{1 + \exp[-\gamma_1 (t - t_{j+1}^*)]} \quad (6)$$

For estimating parameters in this way sufficient amount of data should be presented. [10] Otherwise it is recommended to let only market potential vary over time.

3.3. Potential Market Signal.

Obvious market signal for firm is intersection point of two curves representing sales of dominant and successive technology. At this point sales of successive technology start to overcome sales of dominant technology. However, at this point the market of successive technology becomes very attractive for new entrants, so the competition increases. There is another point which can be potential market signal. It is the point when slopes of dominant and successive technology becomes orthogonal.

Orthogonality occurs when two things can vary independently, they are uncorrelated, or they are perpendicular. In mathematics orthogonality means perpendicularity or when the angle is 90°. Curves or functions in the plane are orthogonal if their tangent lines are perpendicular at their point of intersection.

Two intersecting curves in the plane are orthogonal if, near the point of intersection, the two curves together almost form a plus sign. Supposedly, two curves are given by differentiable functions f and g from the real numbers to the plane such that $f(0) = g(0)$ is the point of intersection. Then the lines consisting of all points of the form $f'(0)t + f(0)$ and $g'(0)t + g(0)$ for all real numbers t, are called the tangent lines to f and g (at the point of intersection) respectively. Here f' and g' are the derivatives of f and g. If these tangent lines intersect at right angles, then the curves are said to be orthogonal.

In economics and statistics orthogonality corresponds with independence of variables. The independent variables which impact dependent variable are said to be orthogonal if they are uncorrelated. [19]

If disruptive technology at some point of time starts to develop independently from incumbent technology than orthogonality should be applicable to the curves that represent sales of those two technologies. The proposed idea is illustrated in a Figure4.

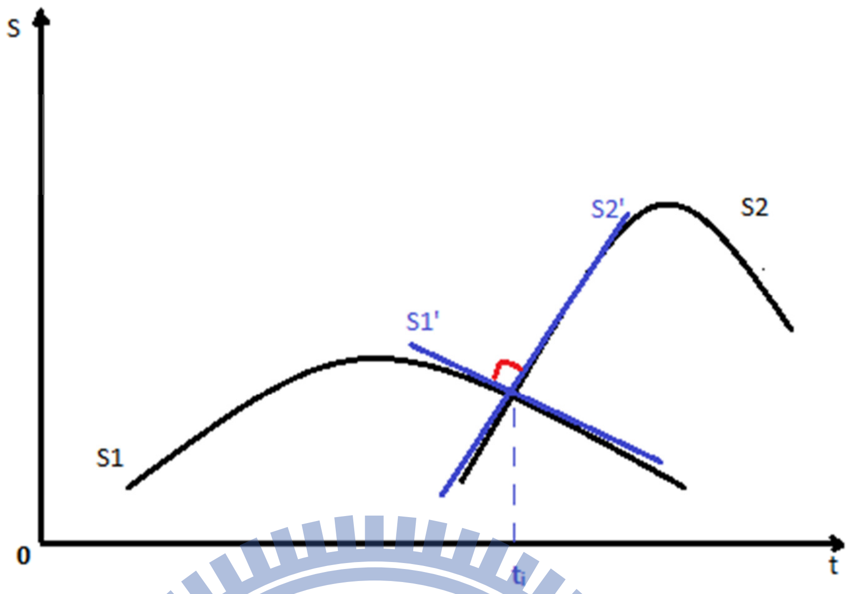

Figure 4. Orthogonal point and Intersection point of dominant and disruptive technology.

Vertical axes represent sales of technology. Horizontal axes is timeline.

Downward curve S1 stands for sales of dominant or incumbent technology in the market, whereas S2 is a curve for sales of disruptive technology.

S1' and S2' are their slopes (in other words their first derivatives from time). The intersection point of two curves is marked as ti. The orthogonal point is the intersection of S1' and S2'.

According to the meaning of orthogonality mentioned above, it appeals for us that orthogonal point could be a market signal for the incumbent firm to switch to the disruptive technology if it is close enough to the intersection point. Since after intersection point disruptive technology is not dependent on incumbent technology anymore (at that point it is already outnumbered sales of dominant technology and started to develop independently), than if the orthogonal point occurs earlier than intersection it could predict disruption and let incumbent firm to take some actions.

3.4. Model

Observing two technologies is basically working with two curves on a plate. The equation that describes those curves are (1), (2) and (3).

Let us write the formula (3) in terms of p and q, considering them to be different for 2 generations:

$$F_1 = \frac{1-e^{-(p_1+q_1)t}}{1+\frac{q_1 e^{-(p_1+q_1)t}}{p_1}}, for\ t > 0 \quad (7)$$

$$F_2 = \frac{1-e^{-(p_2+q_2)(t-\tau)}}{1+\frac{q_2 e^{-(p_2+q_2)(t-\tau)}}{p_2}}, for\ t > \tau \quad (8)$$

Here we release the constant assumption for p and q across the technologies, however we assume them not to vary along the process of diffusion for one technology.

Then to apply the orthogonality concept the first derivative of both equations should be found, therefore we will find a tangle of two curves. The result of multiplication of first derivatives should be -1 for tangles to be orthogonal.

$$\left(\frac{dS_1(t)}{dt}\right)\left(\frac{dS_2(t)}{dt}\right) = -1 \quad (9)$$

If sophisticated data is available it is easy to calculate time for which slopes of two technologies will be orthogonal.

Since orthogonality has a meaning of independence in economics and disruptive technology has a characteristics of developing independently from incumbent technology than we expect that orthogonal point will occur in the area of intersection point if technology is potentially disruptive.

According to the Abel-Ruffini theorem there is no general algebraic solution—that is, solution in radicals— to polynomial equations of degree five or higher. [9] Therefore it is impossible to find a general solution for intersection point and orthogonal point. Though, it doesn't mean that there is no solution at all. According to the mentioned above theorem it is easy to find solution by using numeric methods. Therefore, for the purpose of this study we are going to use Maple software.

The model will be applied to three pairs of technology: Film camera and Digital camera, VCR and DVD player, 4kb and 16kb DRAM. Each couple represents dominant and successive technology, in each case we assume the successive technology to be disruptive.

IV. Data

4.1.Sales Data

Digital camera's technology is a typical example of disruptive innovation. It was introduced in 1975 by Eastman Kodak's engineer Steven Sasson. Eastman Kodak was a leader of photography market at that time and invested a lot in R&D. The camera weighed 8 pounds (3.6 kg), recorded black and white images to a cassette tape, had a resolution of 0.01 megapixels (10,000 pixels), and took 23 seconds to capture its first image in December 1975. The prototype camera was a technical exercise, not intended for production.[Internet Source, 2]

During the 1990s, digital cameras achieved only limited market penetration; the vast majority of photographic images were still captured on traditional film. The critical advantages of digital imaging were in image manipulation and image transmission. The main reason why digital photography didn't get much attention in the beginning was poor quality of its images.

Digital cameras did not offer the same sharpness of resolution as conventional photography. However, digital imaging offered the potential for image manipulation and transmission that were quite beyond traditional photography.[Internet Source, 2] Eastman Kodak, being the market leader in photography and having huge retailing and photofinishing labs chain, didn't see the threat of digital photography. Ultimately, digital imaging had the potential to bypass retailers and photofinishers completely. See Figure 5.

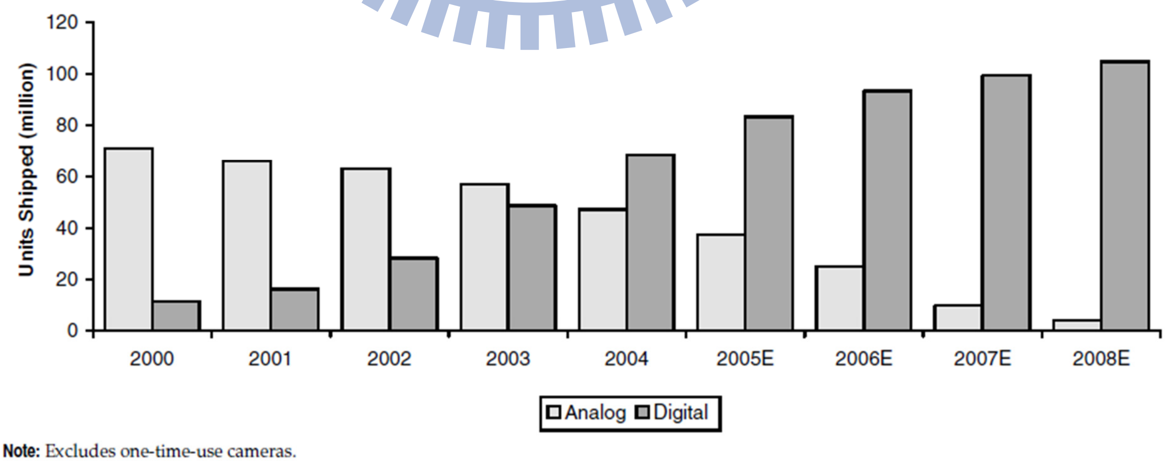

Figure 5. Ananlog vs. Digital cameras shipments.

Source: IDC Bernstein Research

Eventually Kodak the same as Fuji entered the market only in 2000 to protect themselves from digital imaging cannibalizing photographic film products. The sales data by vendor is presented in the next Figure 6 and Figure 7.

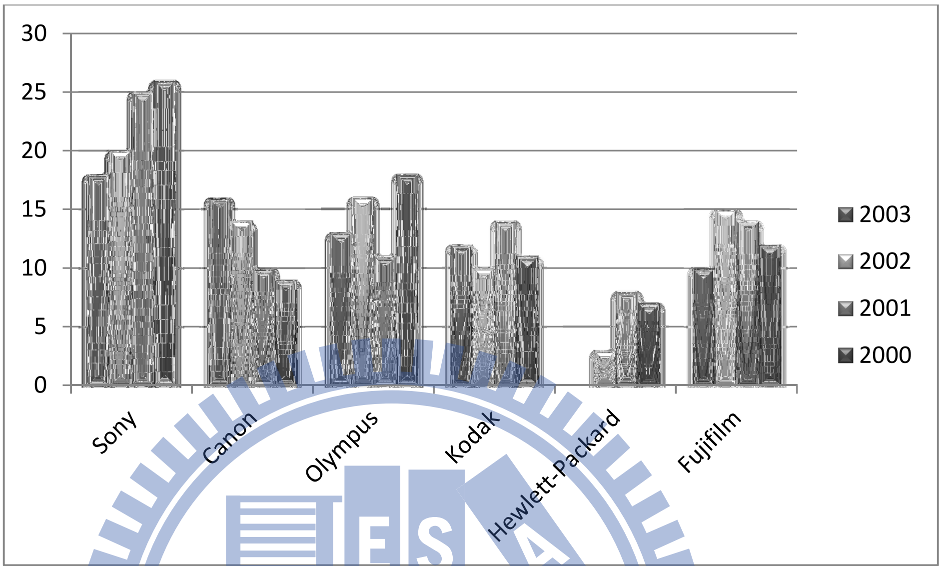

Figure 6. Digital Cameras. Market Share by Vendor in US market.

Source: IDC Bernstein Research

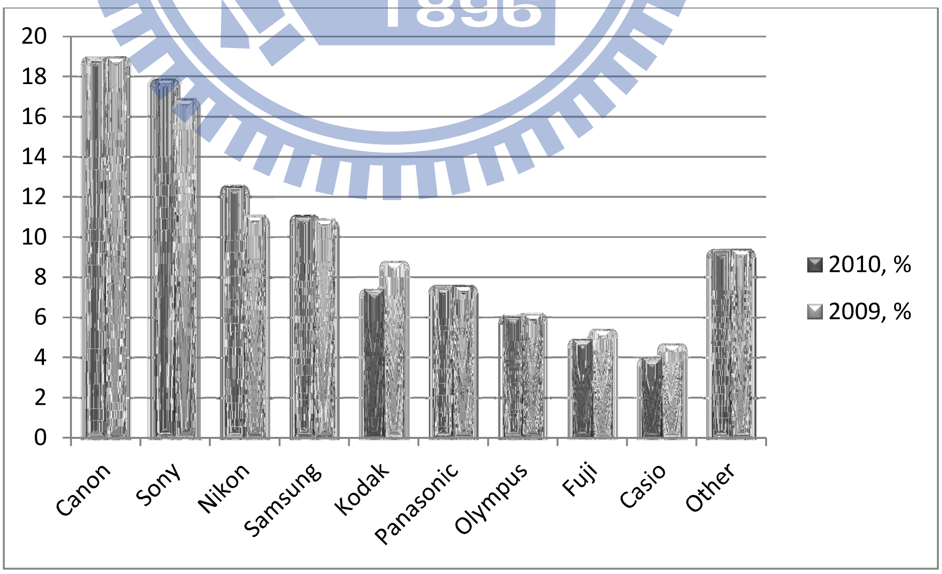

Figure 7. Digital Cameras. World's market share by Vendor.

Source: Bloomberg published data from IDC Japan

The sales data of film and digital camera products is used from Photo Marketing Association International. We used the Digital Camera shipment volume worldwide and Analog Camera shipment volume worldwide. As for the Digital Camera twelve years of data were available, from 1999 to 2010. Film camera's market is much older, so the shipment data is very limited. Therefore we used the last generation of film cameras - Analog camera, and assumed the peak of it in 2000. Since most of the technology repeats bell-shaped curve we assumed the data to be symmetric relative to the point of 1999. Since there is no data available after 2008 (the shipment volume is too small to consider) in the end we got data for seventeen years. Table 1 and Table 2 show the shipment and cumulative shipment for both products.

Table 1. Digital Camera Shipments

Digital Camera, in millions of units		
year	Shipments S□	Cumulative Shipments C□
1	5.5	5.5
2	11	16.5
3	18.5	35
4	30.5	65.5
5	43.4	108.9
6	59.77	168.67
7	64.76	233.43
8	78.98	312.41
9	100.37	412.78
10	119.76	532.54
11	105.86	638.4
12	121.5	759.9

Source: PMA annual market reports.

Table 2. Film Camera Shipments.

Film Camera, in millions of units		
Year	Shipments S□	Cumulative
1	3	3
2	10	13
3	22	35
4	38	73
5	47	120
6	50	170
7	63	233
8	67	300
9	71	371
10	66	437
11	63	500
12	50	550
13	47	597
14	38	635
15	22	657
16	10	667
17	3	670

Source: PMA annual market reports.

For VCR and DVD player we used partially shipment data, partially coefficient estimated by Lilien at al. in the paper Diffusion models: Managerial application and software. [15]. In this paper authors estimated parameters in several product categories based on penetration data and long data series. For DVD player the data from USA is used (as the biggest world economy they represent the trend). The data from 1997 (the year DVD player was introduced to USA market) to 2007 was used. It presented the shipments of DVD player manufacturers to USA customers only.[Internet Source, 5] Therefore we've got eleven data for DVD player. For VCR the data also was estimated based on USA sales.

As for the DRAM 4kb and 16kb data, we have used the data from Norton and Bass (1987), since their parameters showed good fitting. [17]

4.2.Parameters Estimation.

In this study we used three sets of technologies. Therefore six sets of parameters should be estimated. Since the previous studies estimated parameters quite accurately we are going to use those estimations for the purpose of this thesis. The DRAM 4kb and 16kb according to Norton and Bass are showing good fitting with p and q being constant through both generations. So we are going to use this data for our study. The parameters are shown in Table 3 and Table 4.

Table 3. Coefficient of innovation and imitation and potential market for DRAM 4kb and 16kb.

	DRAM	
Parameters	4Kb	16Kb
p	0.0037	0.0037
q	0.34	0.34
m	22523.24	59789.5

Table 4. Coefficient of innovation and imitation and potential market for VCR

Parameters	VCR
p	0.01
q	0.28
m	289.65

Nevertheless, the parameters for Digital camera and for Film camera markets still should be estimated, as well as DVD players market. We use nonlinear least squares method for the Bass forecasting model. The following nonlinear optimization problem is set up:

$$\text{Min}\sum_{t=1}^{N} \varepsilon_t^2 \qquad (10)$$

s.t.

$$S_t = \left[p + q\left(\frac{C_{t-1}}{m}\right)\right](m - C_{t-1}),\ \ t=1,\ldots,N \qquad (11)$$

$$\varepsilon_t = S_t - V_t,\ t=1,\ldots,N \quad (12)$$

Where:

ε_t– error;

C_{t-1}- cumulative sales at time t-1

S_t-forecasted sales at time t;

V_t - actual sales data (in our case shipment volume);

p- coefficient of innovation;

q- coefficient of imitation;

m- ultimate potential market;

The set of constraints and model can be found in Appendix3.

We used LINGO software to estimate parameters. [1]

The LINGO solution for this optimization problem is given in a Table 5 and Table 6.

Table 5. LINGO estimation results for Digital and Film cameras.

Parameter	Digital camera	Film camera
p	0.02	0.04
q	0.42	0.29
m	1104.64	730.60

Table 6. LINGO estimation result's for DVD players

Parameter	DVD player
p	0.01
q	0.34
m	980.80

Also we have to estimate parameter τ, since we don't have a full range of primary data for all the technologies.

DRAM case is quite easy because we have all the quarterly sales volumes for this technology. Based on this data the time of introduction 16Kb DRAM is τ =11.

Digital and Film camera is a more difficult case. Because we don't have the full historical data for Film camera, for analysis we chose the last generation of Film cameras – Analog cameras, which use rolls of film for taking images but have a small screen for previewing before taking the image. Based on the few sources which are internet articles we estimated the τ to be 7 for this type of technology. It means the year 1999 was picked as year of introduction the digital camera. Although the digital cameras were introduced much earlier but only from 1999 it started to be affordable for both professionals and high-end consumers (it cost was under $6,000).[Internet Source, 11] Also the shipment volume for digital cameras are available only from 1999 based on the reason mentioned above.

For VCR and DVD player market, it is even more difficult to define τ, because the development of VCR market is very complex and identified with video-format war which had many stages. We consider the previous before last generation of VCRs (since the last one combines DVD and VHS types into one technological device). So for current study we assume those VCRs that were introduced in the late 1980s, or VHS-compatible VCRs. [Internet Source, 9]. DVD players were introduced in 1997. [Internet Source, 10]. Therefore we used τ=10 for this technology.

After all the parameters are estimated, the MAPLE numeric method could be run and we can find intersections and orthogonal points and compare them. The input to MAPLE for three technologies can be found in Appendix2. Table 7 contains MAPLE solution for the proposed model.

Table 7. Findings.

	Intersection point	Orthogonal point
VCR vs. DVD	15.74	15.91
Digital vs. Film camera	11.99	10.33
4kb vs. 16kb DRAM	20.41	18.77

The table above contains solutions for the model found by MAPLE. Due to the complexity of the original equations the solution for orthogonal point consist from many estimated points given in Appendix4. Due to the nature of the data we would use only real points for each set of data there are two solutions, but only one of them makes sense, since another one is a time which is much later than available data (actually at this point of time the previous generation was already removed from the market or stopped being produced anyway) therefore we take into consideration real numbers which are within the range of available data.

Since it is difficult to see on the graph if the found point is really orthogonal or not (see Figure 1 for example), we have checked it in algebraic way.

Indeed the multiplication of two slopes is close to -1. For example for DRAM the multiplication of first derivatives (that are slopes) in the found orthogonal point is very close to -1:

$$S_1'=-0.0003068$$

$$S_2'=3279.581526$$

$$S_1'*S_2'=-1.006175612$$

Therefore, we have found that in all three cases orthogonal point was quite close to intersection point and it also occurred earlier than intersection point. The interpretations of this result can be explained in next chapter.

V. Conclusion, Limitations and Further Research.

5.1. Conclusion.

The theory of disruptive technology is relatively new field of study in academic science but it draw a lot of attention from business world due to the problems it caused for incumbent firms especially. Empirical evidence showed that it is difficult, if not even impossible, for incumbent firm to handle disruptive technology within one organization due to the nature of this kind of technology. However, some scholars think it is possible for incumbent firm to switch to disruptive innovation within one organization if it would be possible to get a market signal about disruption on time. [6]

Intersection point is one of the obvious points for market signal, because after intersection the successive technology (which is disruptive in our case) sales surplus those of previous technology (incumbent technology). However, at this point it is already too late for firm to change its strategy or to enter the market, because at this point market becomes too attractive for others so the competition eliminates first entrant's advantage. In other words, there should be another market signal to let the firm know that it is time to invest in a disruptive technology so it can catch the initial market share.

The proposed market signal in this study is related to algebraic meaning of orthogonality and definition of disruptiveness. The point of time at which slopes of two curves representing incumbent and disruptive technology become orthogonal (in other words form the angle of 90°) appeals to the independency of two technologies.

We found that orthogonal point was very close to intersection point (15.74099128 and 15.90730883 for VCR and DVD; 11.98987755 and 10.3288613 for digital and film cameras; 20.41196977 and 18.76694276 for 4kb and 16kb DRAMs).

Out of three pairs of technologies that were studied in two out of three cases orthogonal point occurred earlier than intersection point. This alone shows that the point at which slopes of sales of technologies become orthogonal can be a better market signal than intersection point. Also in all three cases the points are very close. This can be a way to distinguish a threat from disruptive technology for incumbent firms.

Figure 8 and 9 illustrates findings for Cameras market and for DRAMs market.

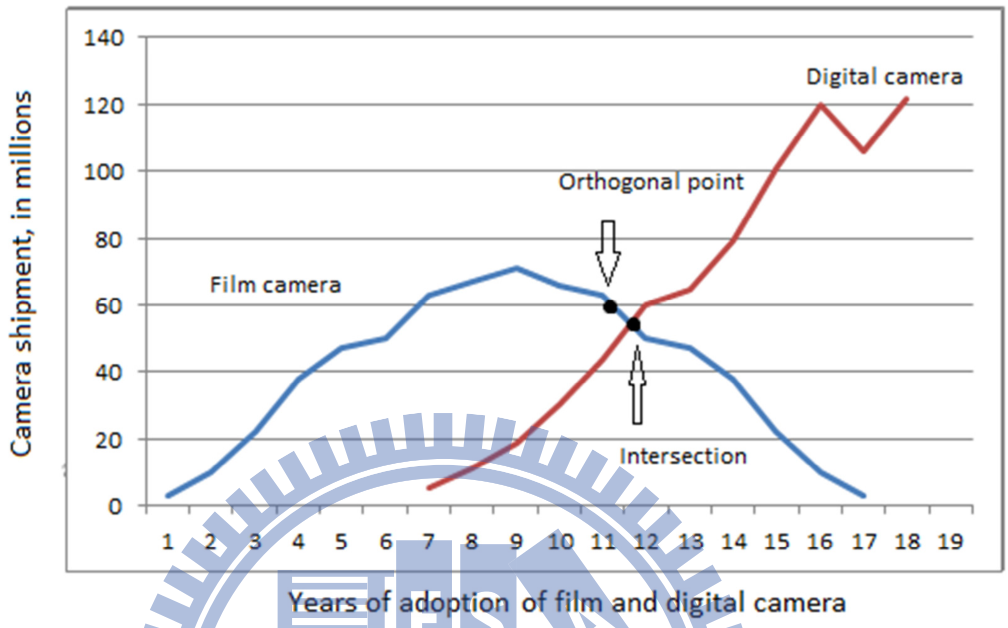

Figure 8. Orthogonal point and intersection point for Digital and Film cameras.

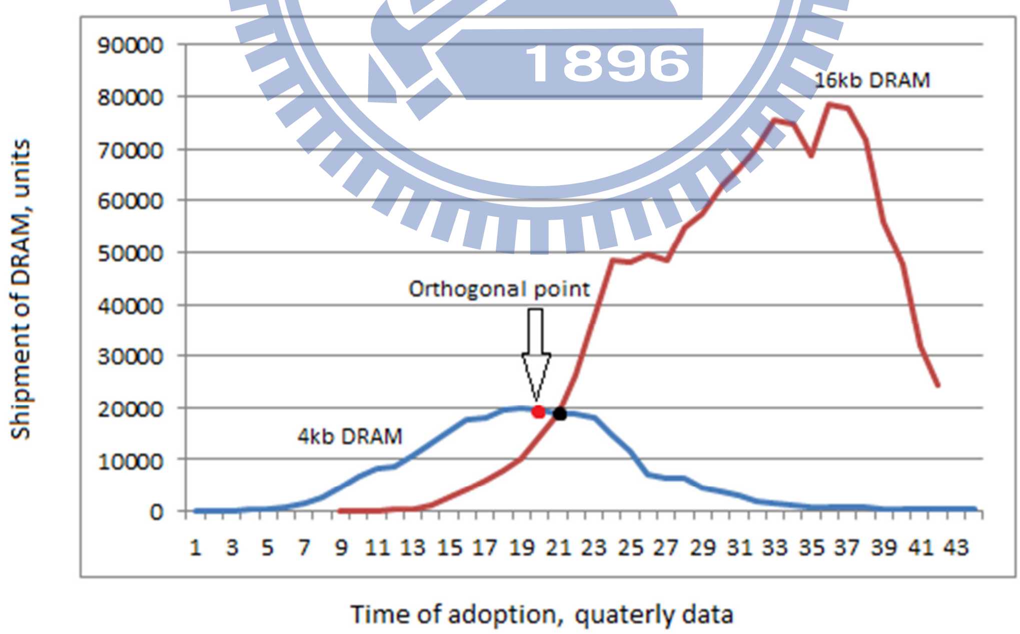

Figure 9. Orthogonal point and intersection point for 4kb and 16kb DRAMs.

From the previous chapter it is clear that orthogonal point and intersection point are very close for chosen technologies. Therefore, we can conclude that if those two points are close it can be a signal that chosen successive technology may disrupt incumbent technology.

Though, we recognize that much more research should be done in this matter according to the limitations of current study. The limitations and proposed research topics are presented in next two sections.

5.2. Limitations.

The biggest limitation of this study is data availability. Using coefficients from different studies is tricky because as Sultan, Farley and Lehmann pointed out in their research, coefficients vary from method of estimation. Although they reported the coefficients of innovation average .03 and the coefficients of imitation average .38 which is quite consistent with our findings. [24] Still the several methods of estimation should be used for more accurate results, which requires primary data from industries. This study is also limited to only three innovations, so it should be both deepen and broaden.

The data is very complex, since the technology development is very fast the speed of introducing new versions are so rapid it is difficult to keep up with research. For instance, digital cameras are being disrupted (according to some expert's opinion from 2007 already) by camera phones, and if few years ago that was still a potential threat now it is quite real. But because the nature of this two product is so different it is not easy to fit them into one model.

6.3. Recommendations for Future Research.

Current study suggests that orthogonal point of slopes of technologies one of which is disruptive and another is disrupted can be a market signal for companies to be aware of disruptive technology. According to the limitations mentioned in a previous section we propose next steps for future research:

- Using different estimation procedures and more sophisticated sets of data to estimate coefficients more precisely;
- Applying proposed model for sustaining technology and comparing results with disruptive technologies;
- For orthogonality could be checked the point of intersection of slopes of technologies. In this case we would get two different points of time, which may also be interpreted as a market signal;

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Appendix 1

General expression for the first derivatives of sales for technology 1 and 2 respectively.

$$
\mathit{d1} := \frac{1}{\left(p_1 + q_1\,\mathrm{e}^{-(p_1+q_1)\,t}\right)^2 p_2} \Big(p_1\,\mathit{m1}\,\mathrm{e}^{-(p_2+q_2)\,(t-\tau)} \big(p_2 - q_2\big) \Big(\mathrm{e}^{-(p_1+q_1)\,t} p_1^2 + 2\,p_1\,q_1\,\mathrm{e}^{-(p_1+q_1)\,t} + q_1^2\,\mathrm{e}^{-(p_1+q_1)\,t} - p_2\,p_1 - p_2\,q_1\,\mathrm{e}^{-(p_1+q_1)\,t} - q_2\,p_1 - q_2\,q_1\,\mathrm{e}^{-(p_1+q_1)\,t} + \mathrm{e}^{-(p_1+q_1)\,t} p_2\,p_1 + \mathrm{e}^{-2\,(p_1+q_1)\,t} p_2\,q_1 + \mathrm{e}^{-(p_1+q_1)\,t} q_2\,p_1 + \mathrm{e}^{-2\,(p_1+q_1)\,t} q_2\,q_1 \Big)\Big)
$$

$$
\mathit{d2} := -\frac{1}{p_2 \left(p_1 + q_1\,\mathrm{e}^{-(p_1+q_1)\,t}\right)^2} \Big(\mathrm{e}^{-(p_2+q_2)\,(t-\tau)} \big(p_2 - q_2\big)\,\mathit{m1} \Big(p_2\,m_2\,p_1^2 + 2\,p_2\,m_2\,p_1\,q_1\,\mathrm{e}^{-(p_1+q_1)\,t} + p_2\,m_2\,q_1^2\,\mathrm{e}^{-2\,(p_1+q_1)\,t} + p_2\,p_1^2 + p_2\,p_1\,q_1\,\mathrm{e}^{-(p_1+q_1)\,t} - p_2\,p_1^2\,\mathrm{e}^{-(p_1+q_1)\,t} - p_2\,p_1\,\mathrm{e}^{-2\,(p_1+q_1)\,t} q_1 + q_2\,m_2\,p_1^2 + 2\,q_2\,m_2\,p_1\,q_1\,\mathrm{e}^{-(p_1+q_1)\,t} + q_2\,m_2\,q_1^2\,\mathrm{e}^{-2\,(p_1+q_1)\,t} + q_2\,p_1^2 + q_2\,p_1\,q_1\,\mathrm{e}^{-(p_1+q_1)\,t} - q_2\,p_1^2\,\mathrm{e}^{-(p_1+q_1)\,t} - q_2\,p_1\,\mathrm{e}^{-2\,(p_1+q_1)\,t} q_1 - p_1^3\,\mathrm{e}^{-(p_1+q_1)\,t} - 2\,p_1^2\,\mathrm{e}^{-(p_1+q_1)\,t} q_1 - p_1\,\mathrm{e}^{-(p_1+q_1)\,t} q_1^2 \Big)\Big)
$$

Appendix 2

MAPLE input for using Numeric Method to find the intersection and orthogonal point for Digital and Film cameras.

```
s1 := t→eval(S1(t), [p[1] = 0.04 , q[1] = 0.29, m[1] = 730.60, tau
      = 7, p[2] = 0.02, q[2] = 0.42]);
s2 := t→eval(S2(t), [p[1] = 0.04 , q[1] = 0.29, m[1] = 730.60, tau
      = 7, p[2] = 0.02, q[2] = 0.42, m[2] = 1104.64]);
s1(t);
s2(s);
solve(s1(t) = s2(t));
d1 := t→diff(s1(t), t);
d2 := s→diff(s2(s), s);
solve(diff(s1(t), t)·diff(s2(t), t) =−1)  assuming t > 7;
```

MAPLE input for using Numeric Method to find the intersection and orthogonal point for VCR and DVD players.

```
s1 := t→eval(S1(t), [p[1] = 0.01 , q[1] = 0.28, m[1] = 289.65, tau
      = 10, p[2] = 0.01, q[2] = 0.34]);
s2 := t→eval(S2(t), [p[1] = 0.01 , q[1] = 0.28, m[1] = 289.65, tau
      = 10, p[2] = 0.01, q[2] = 0.34 , m[2] = 980.8]);
s1(t);
s2(s);
solve(s1(t) = s2(t));
d1 := t→diff(s1(t), t);
d2 := s→diff(s2(s), s);
solve(diff(s1(t), t)·diff(s2(t), t) =−1)  assuming t > 10;
```

MAPLE input for using Numeric Method to find the intersection and orthogonal point for DRAM 4kb and 16kb.

```
s1 := t→eval(S1(t), [p[1] = 0.0037 , q[1] = 0.3369, m[1] = 22523, tau
      = 11, p[2] = 0.0037, q[2] = q[1]]);
s2 := t→eval(S2(t), [p[1] = 0.0037 , q[1] = 0.3369, m[1] = 22523,
      m[2] = 59789.50, tau = 11 , p[2] = p[1], q[2] = q[1]]);
s1(t);
s2(s);
solve(s1(t) = s2(t));
d1 := t→diff(s1(t), t);
d2 := s→diff(s2(s), s);
solve(diff(s1(t), t)·diff(s2(t), t) =−1)  assuming t > 11;
```

Appendix 3

LINGO optimization problem for Bass forecasting model.

```
MIN=E1^2+E2^2+E3^2+E4^2+E5^2+E6^2+E7^2+E8^2+E9^2+E10^2+E11^2+E12^2;
F1=(p)*m;
F2=(p+q*(5.5/m))*(m-5.5);
F3=(p+q*(16.5/m))*(m-16.5);
F4=(p+q*(35/m))*(m-35);
F5=(p+q*(65.5/m))*(m-65.5);
F6=(p+q*(108.9/m))*(m-108.9);
F7=(p+q*(168.67/m))*(m-168.67);
F8=(p+q*(233.43/m))*(m-243.3);
F9=(p+q*(312.41/m))*(m-312.41);
F10=(p+q*(412.78/m))*(m-412.78);
F11=(p+q*(532.54/m))*(m-532.54);
F12=(p+q*(638.4/m))*(m-638.4);
E1=F1-5.5;
E2=F2-11;
E3=F3-18.5;
E4=F4-30.5;
E5=F5-43.4;
E6=F6-59.77;
E7=F7-64.76;
E8=F8-78.98;
E9=F9-100.37;
E10=F10-119.76;
E11=F11-105.86;
E12=F12-121.5;
```

Appendix 4

MAPLE solution for orthogonal point. Digital and Film Cameras.

28.30670149, 10.32886130, .6354239600, -5.762784477, 28.29302539-7.227485307*I, 10.32974091-8.169187976*I, 7.621018269-9.078311564*I, -6.421316449-9.895574163*I, 3.633763014-13.14043186*I, 28.18910182-14.40835956*I, 10.31704524-16.32777643*I, -6.622456264-19.36136015*I, 28.10789532-21.51833634*I, 10.31203558-24.48716750*I, 6.280418774-27.39215951*I, 28.04868585-28.55993321*I, -6.891107455-28.55993321*I, 6.280418774+27.39215951*I, 10.31203558+24.48716750*I, 28.10789532+21.51833634*I, -6.622456264+19.36136015*I, 10.31704524+16.32777643*I, 28.18910182+14.40835956*I, 3.633763014+13.14043186*I, -6.421316449+9.895574163*I, 7.621018269+9.078311564*I, 10.32974091+8.169187976*I, 28.29302539+7.227485307*I

MAPLE solution for orthogonal point. VCR and DVD players.

35.49405005, 15.90730883, .4775993978-4.784390018*I, 35.54304644-9.005445736*I, 15.99340465-9.832925604*I, 12.84374904-10.66313275*I, 8.652420558-11.45848288*I, .2936756961-14.23102855*I, 35.46693464-17.99904624*I, 15.91172779-19.61273206*I, 0.3621428444e-1-24.17682653*I, 35.49110831-26.98682882*I, 15.97411669-29.49180849*I, 12.63991073-32.14655404*I, 9.709431617-33.35444995*I, -.3013491013-33.92996271*I, 35.41103871-35.95028263*I, 15.92630171-39.22978582*I, -.2697235598-44.03401414*I, 35.43930554-44.90612806*I, 15.94638523-49.13419140*I, 35.38536665-53.85080682*I, -.4956091021-54.03924079*I, 10.31224996-54.05117085*I, 12.35020381-54.22804447*I, 15.95099337-58.85803003*I, 35.44499690-62.79641291*I, -.2407308468-64.01171350*I, 15.92309866-68.76004532*I, 35.41993314-71.75705332*I, -.2237864042-74.11807854*I, 9.537625307-74.92367096*I, 12.69136563-76.17939128*I, 15.97829678-78.50342936*I, 35.50162023-80.72569543*I, .1125897562-83.81743848*I, 15.91034437-88.37596546*I, 35.47474909-89.71820814*I, .3900669675-93.68757418*I, 8.523631335-97.05227354*I, 12.85617943-97.61333455*I, 15.99490627-98.16435558*I, 35.54739622-98.71562608*I, .5158485738-103.1052699*I, 35.49323663-107.7211238*I, 15.90742708-107.9884506*I, .4204042142-112.6905575*I, 35.53712354-116.7256386*I, 15.99133974-117.8300021*I, 12.82589578-118.9438200*I, 8.811789884-119.9279615*I, .1854382359-122.1769821*I, 35.45822656-125.7148681*I, 15.91339542-127.6015416*I, -0.3472266135e-1-132.1841561*I, 35.48057243-134.6974566*I, 15.96969737-137.4865637*I, 12.58130273-140.4881840*I, 9.870670687-141.6015617*I, -.3661963283-141.9882589*I, 35.40305038-143.6560386*I, 15.92980557-147.2199285*I, -.2891379706-152.0828805*I, 35.43538826-152.6077585*I, 15.94191923-157.1259373*I, 35.38724332-161.5526023*I, -.4827134108-162.1169941*I, 10.25179617-162.1681054*I, 12.38933537-162.6706323*I, 15.95569011-166.8507470*I, 35.45225180-170.5001740*I, -.2023448130-172.0529416*I, 15.92020588-176.7496025*I, 35.42941947-179.4660888*I, -.1342063493-182.1528843*I, 9.357802173-183.2343069*I, 12.73539347-184.4924411*I, 15.98217044-186.4991219*I, 35.51177485-188.4400626*I, .1923712918-191.7966768*I, 15.90923324-196.3645752*I, 35.48144978-197.4367075*I, .4654978088-201.5825662*I, 8.439073375-205.5963453*I, 12.86350467-205.8869994*I, 15.99581805-206.1617755*I, 35.55005401-206.4372948*I, .5291426692-210.9831296*I, 35.49081711-215.4419772*I, 15.90778348-215.9769162*I, .3503332953-220.6154531*I, 35.52979206-224.4446907*I, 15.98874778-225.8268118*I, 12.80220534-227.2301773*I, 8.988814743-228.3540622*I, 0.7397267022e-1-230.1500775*I, 35.44887884-233.4291058*I, 15.91535853-235.5904902*I, -0.9875776732e-1-240.2026051*I, 35.47035711-242.4061952*I

MAPLE solution for orthogonal point. DRAM 4kb and 16kb.

52.45242734, 18.76694276, -8.708917374-4.612471927*I, 52.45322640-9.223701273*I, 18.75709087-9.223701273*I, 14.65178342-9.223701273*I, 10.30121024-9.223701273*I, -8.708917374+4.612471927*I