

# 使用三維信號的柵狀編碼調變技術之效能分析

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## 摘要

以傳統的柵狀編碼調變技術來說，二維的信號，比如說像是  $M$ -PSK 或是  $M$ -QASK (其中  $M = 2^N$ )，通常都會被用在許多傳輸系統當中。在這篇論文裡，我們將會介紹所謂「三維信號」的想法，以及「將三維信號視為二維信號的延伸」的概念。以傳統上使用二維信號的柵狀編碼調變技術為參考，我們會敘述建立三維信號的方式、將三維信號分解(partition)為小組(subset)的方式、以及將二進位的資料對應到這些信號的方式。相對於發送端的編碼，接收端使用了一種叫做「軟性決定維特比演算法」的方式來解碼。我們會舉出一些二維的以及三維的例子，並且將這些例子做分析比較。我們可以由結果來推斷，使用三維信號的柵狀編碼調變技術的錯誤品質會比使用二維信號的來得好。

# Performance Analysis On Trellis-Coded Modulation Schemes Using Three-Dimensional Signal Sets.

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## Abstract

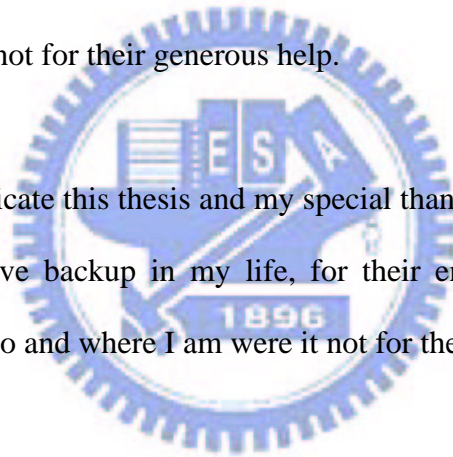
In conventional trellis-coded modulation (TCM) schemes, two-dimensional constellations such as  $M$ -PSK or  $M$ -QASK, where  $M = 2^N$ , are generally used in many transmission systems. In this thesis, the idea of three-dimensional constellations is introduced and the concept of treating three-dimensional constellations as an expansion of two-dimensional constellations is presented. Conventional TCM schemes using two-dimensional constellations are taken as references. The method of constructing three-dimensional constellations and partitioning the signal points into subsets and mapping information bits into those signal points are described. Soft-decision Viterbi algorithm (SOVA) is applied for the decoding process of the presented TCM schemes. A number of examples are given and comparisons are made. We conclude that three-dimensional TCM schemes perform better than conventional two-dimensional TCM schemes.

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