

Experimental and theoretical analysis of microjet droplet behavior

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Abstract

A model for predicting fluid flow with forming droplet in a microjet fluidic device is described. The model considers the approach of non-linear autoregressive moving average model with exogenous inputs (NARMAX) to analyze the dynamic flow behavior of droplet that can be helpful to design microjet geometry in bionanotechnology application. The study in microjet controls a droplet size that is arrival at nanoliter volumes. The working fluids in experimental measuring process are the solution of gold nanoparticles and oil acid. This study focuses on a real time for droplet behavior with the variety of pressure in the channel. Based on identifying both time and frequency domain models, the behavior of droplet flow could be analyzed in microfluidic on-chip system.

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Keywords: Non-linear model; Microjet; Droplet; Frequency response; Microfluidic device

1. Introduction

A chip-based microfluidic device is an assembly of microstructure on a common substrate that used for manipulation of small fluids flow [1]. Recently the exploitation of microfluidic chip-based systems for biomedical or biological reaction studies is attracting broad interest in MEMS/NEMS research field [2]. So, developing a model method can be useful to improve the different designs of microfluidic device system. Analysis and prediction for non-linear system have been an important research topic. The related theories for non-linear system also have been developed and applied [3]. Early researchers utilizing the Volterra and Wiener kernels non-linear system identification approach have been comprehensively reviewed [4,5]. And subsequently, some researches have been done on the efficient computation of the kernels of physical systems [6,7]. Today, the results of kernel-based identification tech-

niques have been published [8–10]. Simultaneously, these have been used to model single-input single-output (SISO) systems, but few people paid attention to them in contrast with the Volterra modeling of multi-input multi-output (MIMO). Marmarelis have done on multi-input Volterra modeling [11] and Westwick and Kearney modeled a multi-input system via the Wiener theory [12].

The problem of droplet control the dynamic droplets in a microjet are quite complicated due to unstable flow fluids. In the recent years, the development of MEMS/NEMS technology, people have started to use a lot of methods to qualitative and quantitative analyses of biomolecular or biomedical sample conducted [13,14]. In this present study, we use the NARMAX modeling technique [15,16,17] to build-up appropriate models with measuring the inlet and outlet pressure. The models can describe and predict a unstable microfluidic system. The working fluids herein are used the solution of gold nanoparticles and oil acid because of biological applications. Then, the polydimethylsiloxane (PDMS) microjet fluidic device can be fabricated. We demonstrated that a novel concept in microjet, its dynamics droplet flow, can be controlled by means of model analysis.

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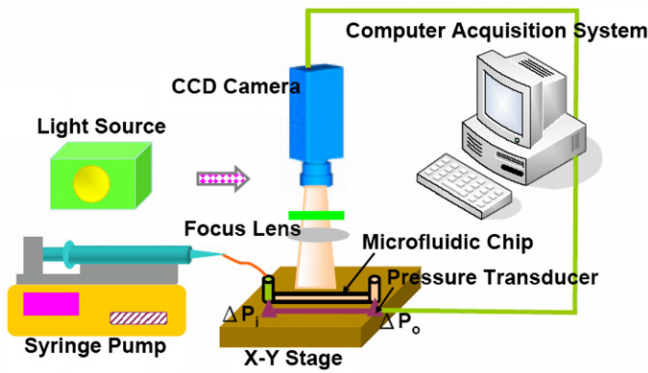


Fig. 1. Schematic of the experimental setup.

2. Experiment

A schematic diagram of the experiment setup is shown in Fig. 1. In this study, the solution of gold nanoparticles can be synthesized by using chemical reduction method [18] and is measured by a UV–vis spectrometer (UV–vis model: U3310, Hitachi) to determine the peak of absorption spectrum that is related with the size of particle. The microfluidic device used is fabricated using polydimethylsiloxane (PDMS) replica and the soft lithography technology [19]. Briefly, the microjet is made by casting PDMS onto a negative SU-8 (SU-8 2100; MicroChem Corp.) structure that is fabricated on a glass slide. The spin

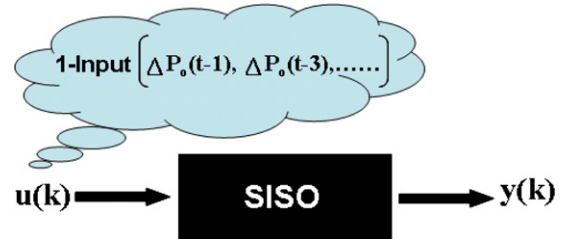


Fig. 2. Schematic view of the SISO model.

speed parameters of SU-8 2001 for the layer height is 100 μm at 500 rpm/60 s, 2750 rpm/30 s and 500 rpm/10 s. After the wafer was removed from the rinsed and dried, a prepolymer PDMS (Sylgar 184; Dow Corning Corp.) and curing agent in a ratio of 10:1 ratio can be dispensed on the wafer and cured for 1 h under vacuum at 60 $^{\circ}\text{C}$. Then, PDMS is peeled off the SU-8 structure and holes are drilled through PDMS using a carbide drill bit to define the inlet and outlet.

3. Model

A wide class of discrete time multi-variable non-linear stochastic systems can under appropriate assumptions be represented by the NARMAX model. Herein, a SISO system is illustrates in Fig. 2. Based on the estimation of parameters of a NARMAX model, Lang and Billings [20]

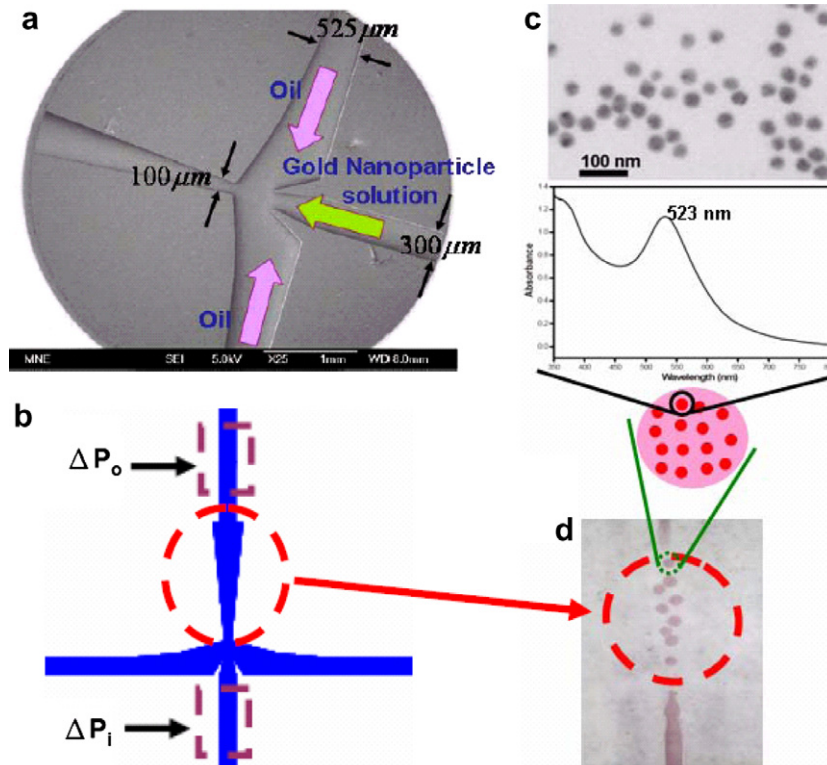


Fig. 3. (a) FE-SEM photograph of the microjet. (b) Schematic of the measured pressure spots. (c) TEM photograph of the gold nanoparticle ($\sim\pm 12$ nm) and UV–vis absorption spectrum (523 nm). (d) Optical microscope image of the dynamic droplet flow.

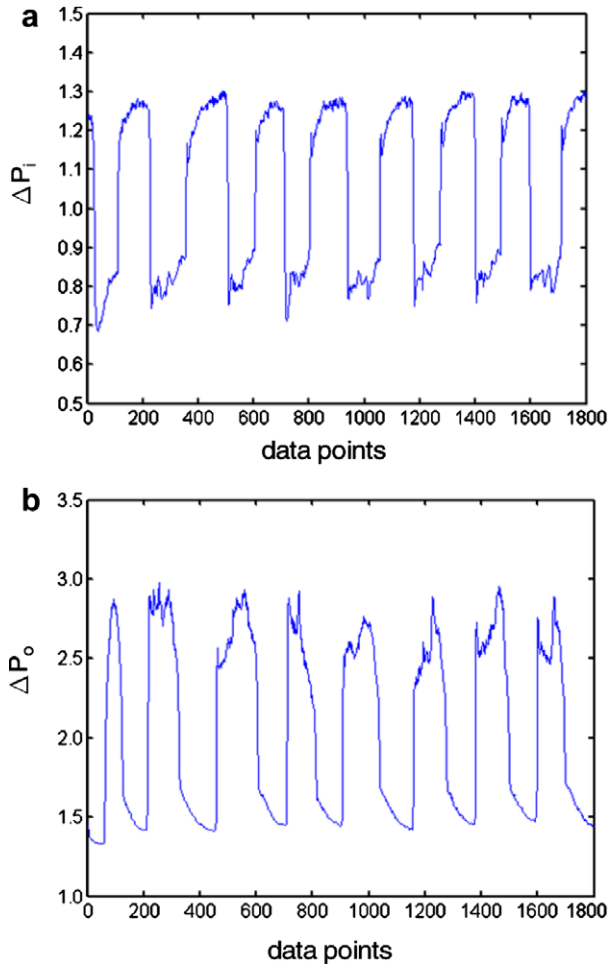


Fig. 4. I/O signals taken from the 1001th to 2800th samples: (a) pressure drop of inlet section (ΔP_i) and (b) pressure drop of outlet section (ΔP_o).

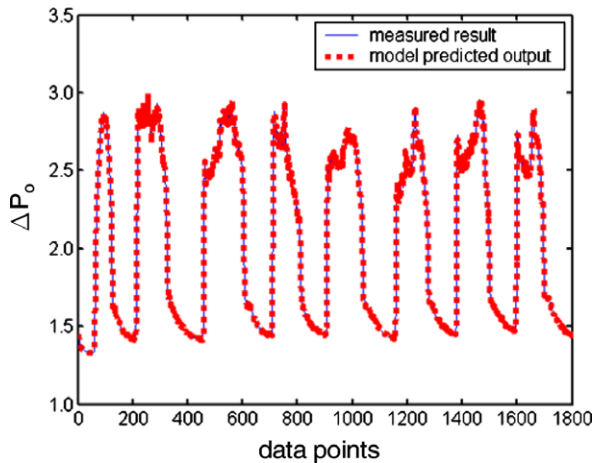


Fig. 5. Comparison of the model predicted output (---) with the original measurements (—).

has been proposed the generalized frequency responding function (GFRF).

The frequency responding functions come from NARMAX model can represent characteristics and be used to investigate the dynamic behavior of a real system.

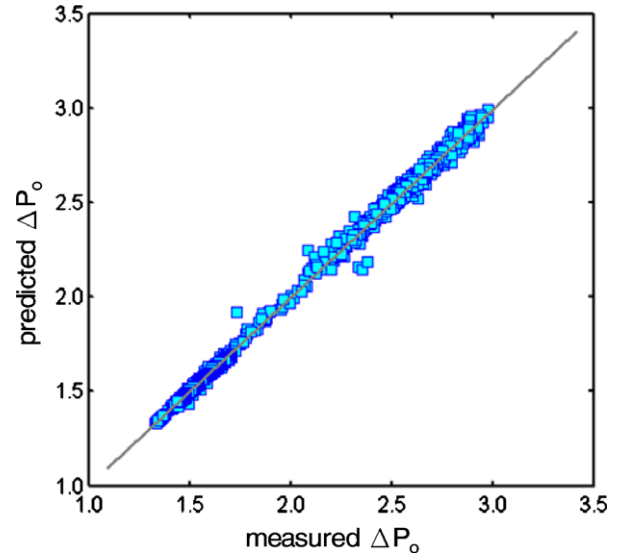


Fig. 6. Correlation of measured and predicted pressure drop from the present study.

4. Results and discussion

In this work, the experimental results of the dynamic droplets in the microjet are shown in Fig. 3a–d. In addition, the gold nanoparticles can be chosen, because they can be easily bound the biomolecule such as DNA or protein that can be applied to biological applications in the future. The working fluids can generate the droplets due to the liquid–oil interface. And the flow velocity can also be effected the droplets to take place. The inlet/outlet pressure drop can be measured in the channel as shown in Fig. 4. In the inlet section, the model predicted output is shown in Fig. 5. All the correlation tests are validated within a deviation under 5% and the comparison of measured and predicted ΔP_o is shown in Fig. 6. Besides, all the compared results are within a range of relative error in 9.5%, i.e.,

$$\sum_1^N \text{ABS}(\text{measured} - \text{predicted})/\text{measure}/N.$$

This demonstrates that utilization of the appropriate pressure drop models developed specifically for microjet is able to account for related dynamic droplet phenomena accurately.

Furthermore, the interpretation of non-linear effects in the frequency domain is done. The frequency functions computed using the models is written as

$$H(f) = \frac{0.12563 \times 10^3 \exp(-2j\pi(15f))}{1 - 0.23172 \times 10^3 \exp(-j2\pi f) - 0.26959 \times 10^3 \exp(-j2\pi(3f))}.$$

The GFRFs can be directly derived from NARMAX models. The frequency response function $H(f)$ is illustrated in Fig. 7. The resonances in $H(f)$ are found approximately at 0.17 Hz and the corresponding magnitude are 10 dB.

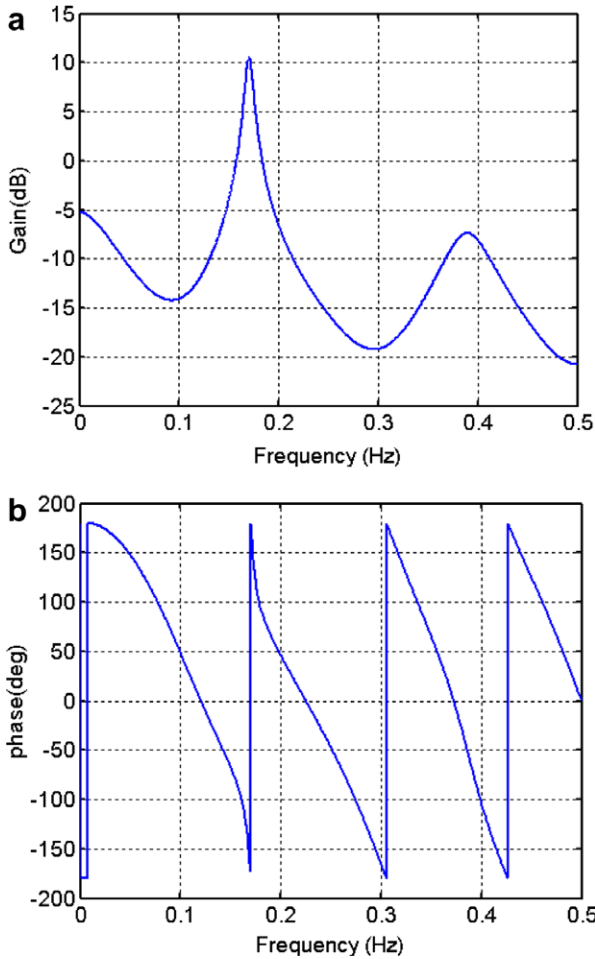


Fig. 7. (a) The frequency response function and (b) phase angle for pressure drop model.

The non-linear GFRFs are generated by the non-linear terms in the discrete-time models. From the results, the interpretation of the GFRFs has been comprehensively studied and non-linear effects have been related to the physical models of the microfluidic on-chip systems. This method for optimizing the design of microjet geometry can provide a useful prediction of microfluidic platform to control nanoparticle-based droplet.

Table 1
A pressure drop prediction model

Term	Coefficient	ERR
$\Delta P_o(t-1)^a$	0.22774E+003	0.9999
$\Delta P_o(t-3)^a$	0.27259E+003	0.8946E-006
$\Delta P_o(t-2) \Delta P_o(t-3)$	-0.18166E-001	0.2964E-006
$\Delta P_i(t-1) \Delta P_i(t-12)$	-0.71418E-003	0.3344E-006
$\Delta P_o(t-1) \Delta P_o(t-3)^a$	-0.89222E+001	0.8723E-006
$\Delta P_i(t-15) \Delta P_i(t-15)$	0.18409E-001	0.3688E-006
$\Delta P_o(t-3) \Delta P_i(t-15)$	-0.56569	0.1145E-006
$\Delta P_o(t-2) \Delta P_i(t-14)$	0.34144E-003	0.1447E-006
-	-0.68147E+004	0.4834E-007
$\Delta P_i(t-15)$	0.11674E+002	0.4171E-007

^a The most important terms.

5. Conclusion

In conclusion, combined time and frequency domain identification approach can be considered to analyze data from the flow behavior of dynamic droplet in the microjet. It has been successfully shown the microjet fluidic system with forming nanoparticle-based in different length can be build utilizing a series of discrete-time NARMAX models. Besides, they will provide the various conditions and constraints before our design and fabrication of microdevices in the future. Applying the NARMAX methodology for the type of microfluidic system is proved to be a good estimation method, and the novelty of the present results relates to the non-linear frequency domain analysis by the GFRFs which are derived for polynomial NARMAX models. The GFRFs reveal, for the microjet application, non-linear couplings which represent energy release and storage between input harmonic components taking place at low frequency and also on particular lines of frequency. Analytical expressions in MEMS/NEMS application by using GFRFs can provide a great deal of insight into the relationship between the time and frequency domain representations of non-linear systems.

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Appendix

The estimation results of pressure drop with forming droplets in the microjet for the I/O relationship can be represented in Table 1, where the model structure terms can be listed contribution.

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