國 立 交 通 大 學

電 信 工 程 研 究 所

碩 士 論 文

兩微帶天線邊緣耦合效應之研究

Analysis of Edge Coupling of Two Patch Antennas

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摘　要

本論文主要是在理論上探討兩個微帶天線之間的電磁耦合機制，希望能有類似於 Yagi-Uda 型天線陣列或微帶天線的頻寬增進等方面的應用。

致　謝　Acknowledgement

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最後引用某位名人說過的話：「要感謝的人實在太多了,那就感謝天吧。」

Analysis of Edge Coupling of Two Patch Antennas

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Abstract

This study is aimed at exploring theoretically the electromagnetic coupling mechanism of two patch antennas attempted for applications in areas such as Yagi-Uda type patch arrays or bandwidth enhancement of the patch antennas.

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Chapter 1

Introduction

The purpose of this thesis is to study the edge coupling mechanism by proximity of two patch antennas. One of the applications is to explore the possibility of construction of patch arrays through edge coupling in a similar manner of passive coupling for a Yagi-Uda array. The other aims at bandwidth enhancement of patch antennas considering the coupled patches resonance at adjacent frequencies. For a passive array application, for example, it is desirable to make the non-radiating edge of an active patch to couple energy to the adjacent passive one. To simplify the analysis, we make use of the cavity model with the gap between the coupling edges modelled as a slot so that the equivalence principle can be applied to the analysis of the composite structure.

Chapter two starts the analytic formulation of the problem with introduction of basic theories and assumptions. It follows the derivation of analytic solution in Chapter three. The numerical solutions by moment method are given in Chapter four. The discussion and conclusion follow at the end.

Chapter 2

Formulation

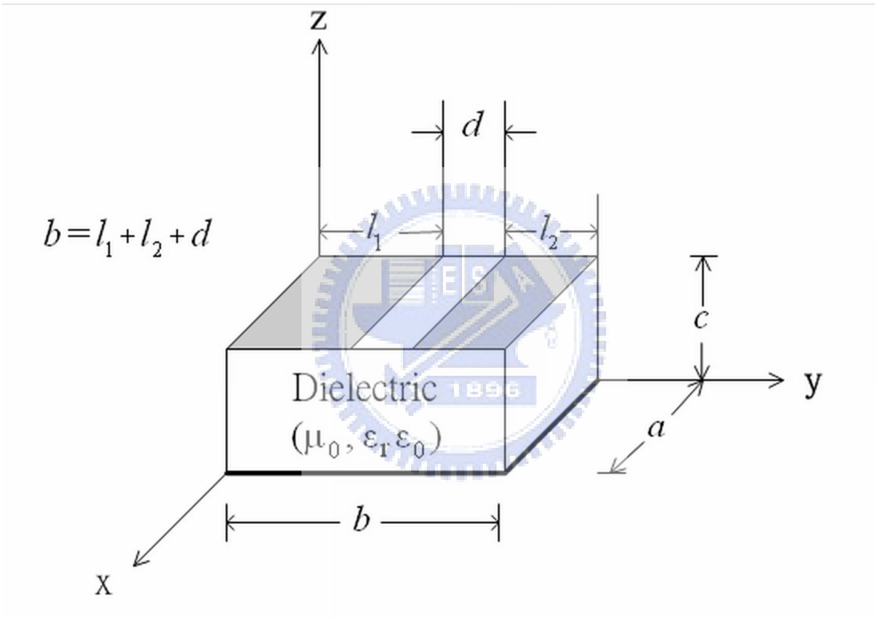

Fig. 1. Configuration of two edge-coupled patches

2.1 Equivalent Problem

Two patch antennas of width a, and edge lengths l_1 and l_2, respectively, are shown in Fig. 1. The first patch is excited by an electric current $\mathbf{J}$, while the other is coupled from the first through the edges of the two patches separated by a distance d, which is close enough to provide the desired coupling. The problem can be analyzed by full-wave integral equation approach, with the unknown current on the patches to be solved. In this study, however, we adopt the simpler cavity-model approach with the assumption that the patches are surrounded by magnetic walls at the nonadjacent edges between the patches and the ground plane, which amounts to neglecting the fringing field under the edges. The gap between coupling edges of the two patches is modelled as a slot in the composite structure. The problem is thus reduces to one of cavity with slot coupling to the free space. This type of problem has been studied extensively in the literature [1, 2]. By equivalence principle the coupling effect of the slot is accounted for by an equivalent magnetic current $\mathbf{M}$. With the aid of this equivalent current the field problem can be casted into two separate parts; that interior and exterior to the cavity.

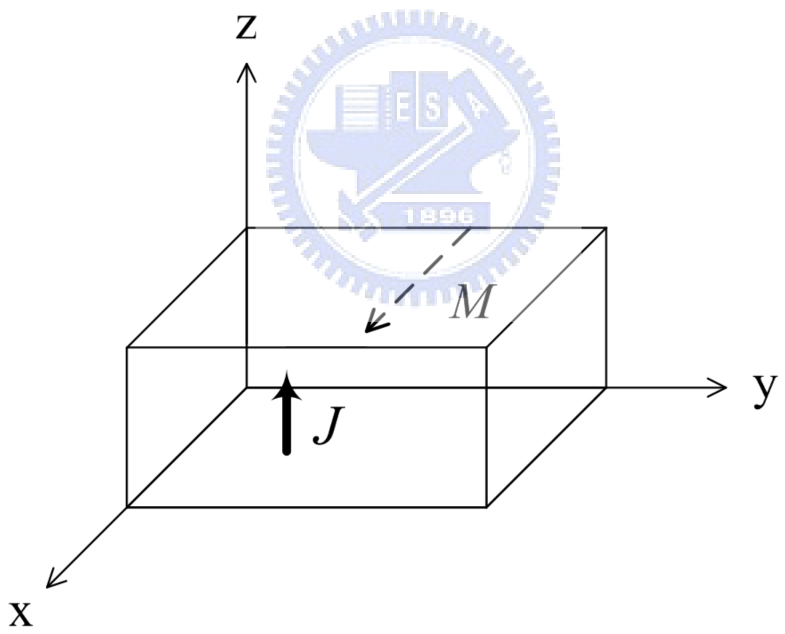

Fig. 2. Geometry for the interior problem

Interior Problem The equivalence problem interior to the cavity is shown in Fig. 2 where the equivalent magnetic current, in terms of the unknown aperture electric field,

$$\mathbf{M} = \mathbf{E} \times \hat{\mathbf{n}} = \hat{\mathbf{z}} \times \mathbf{E} \tag{2.1}$$

is present underneath the slot with the slot now closed by perfect electric conductor. The fields inside the cavity can be solved by normal mode expansion to be described in the following sections.

Exterior Problem For the fields outside the cavity the equivalent magnetic current is given as

$$-\mathbf{M} = \mathbf{E} \times \hat{\mathbf{z}} \tag{2.2}$$

which takes into account of the continuity of the tangential electric field on the slot. Away from the surrounding edges the current can be considered sitting on top of an large ground plane so that the method of images can be applied to simplify the problem. As a result there is an equivalent current $-2\mathbf{M}$ radiating in free space as the exterior fields is concerned. The resultant fields can be readily obtained by the well-known potential-integral solution.

In the above formulation both the fields inside and outside the cavity are derived in terms of $\mathbf{M}$, or the unknown tangential electric field on the slot. Equating the tangential magnetic fields across the slot thus leads to coupled field equations for $\mathbf{M}$

2.2 Fields Inside the Cavity

Fields in the waveguides or cavities can be expanded in terms of normal modes for given boundary conditions. By modal analysis [3] each mode can be represented by an equivalent transmission line with specific propagation constant and characteristic impedance, and propagates in the waveguide independent of others. For a given source distribution there corresponds equivalent driving voltage and current sources for each transmission line representing this particular mode. The field problem is thus reduced to a circuit problem from which the solution can be readily obtained by circuit means. Finally the total field is synthesized by contribution from all modal fields. For the rectangular cavity there exists no TEM mode. Only $TE(H)$ and $TM(E)$ modes need to be considered.

The derivation parallels essentially the modal analysis for a metallic waveguide[3], albeit with a different boundary condition associated with the magnetic wall. The result is summarized in the following.

2.2.1 Modal Representation of the Fields

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_t(\mathbf{r}) + \mathbf{E}_z(\mathbf{r}) \tag{2.3}$$

$$E_z(\mathbf{r}) = \frac{1}{j\omega\epsilon}\nabla_t \cdot (\mathbf{H}_t \times \hat{\mathbf{z}}) - J_z \tag{2.4}$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_t(\mathbf{r}) + \mathbf{H}_z(\mathbf{r}) \tag{2.5}$$

$$H_z(\mathbf{r}) = \frac{1}{j\omega\mu}\nabla_t \cdot (\hat{\mathbf{z}} \times \mathbf{E}_t) - M_z \tag{2.6}$$

$$\mathbf{E}_t(\mathbf{r}) = \sum_i V_i^{'}(z)\mathbf{e}_i^{'}(\boldsymbol{\rho}) + \sum_i V_i^{''}(z)\mathbf{e}_i^{''}(\boldsymbol{\rho}) \tag{2.7}$$

$$\mathbf{H}_t(\mathbf{r}) = \sum_i I_i^{'}(z)\mathbf{h}_i^{'}(\boldsymbol{\rho}) + \sum_i I_i^{''}(z)\mathbf{h}_i^{''}(\boldsymbol{\rho}) \tag{2.8}$$

For E modes:

$$(\nabla_t^2 + k_{ti}^{'2})\psi_i^{'} = 0 \tag{2.9}$$

$$\mathbf{e}_i^{'} = -\frac{1}{k_{ti}^{'}}\nabla_t\psi_i^{'} \tag{2.10}$$

$$\mathbf{h}_i^{'} = \widehat{\mathbf{z}} \times \mathbf{e}_i^{'} \tag{2.11}$$

$$Z_i^{'} = \frac{\kappa^{'}}{\omega\epsilon} \tag{2.12}$$

For H modes:

$$(\nabla_t^2 + k_{ti}^{''2})\psi_i^{''} = 0 \tag{2.13}$$

$$\mathbf{h}_i^{''} = -\frac{1}{k_{ti}^{''}}\nabla_t\psi_i^{''} \tag{2.14}$$

$$\mathbf{e}_i^{''} = \mathbf{h}_i^{''} \times \widehat{\mathbf{z}} \tag{2.15}$$

$$Z_i^{''} = \frac{\omega\mu}{\kappa^{''}} \tag{2.16}$$

The boundary conditions for the transverse eigenvalue problems are assciated with the electric and magnetic walls enclosing the cavity and will be specified in the analysis to follow. For both E and H modes the dispersion relation is

$$\kappa = \sqrt{k^2 - k_{ti}^2} \tag{2.17}$$

where k_{ti} is found from the transverse eigenvalue problem. In the derivation of the above expressions the Maxwell's equations have been decomposed into transverse and longitudinal components such that

$$\mathbf{r} = \boldsymbol{\rho} + \mathbf{z} \tag{2.18}$$

$$\nabla = \nabla_t + \hat{\mathbf{z}}\frac{\partial}{\partial z} \tag{2.19}$$

All eigenmodes are normalized to observe the orthonormal conditions as in [3].

2.2.2 Transmission Line Representation of Modal Fields

The modal amplitudes V_i and I_i can be shown to satisfy the transmission line equations with characteristic impedance Z_i as given above. The equivalent transmission line representation for each mode is shown in Fig. 3. The corresponding voltage source v_i and current source i_i are related to $\mathbf{J}$ and $\mathbf{M}$ according to

$$v_i(z) = \int \mathbf{M} \cdot \mathbf{h}_i^\star \, dS + Z_i^\star \int \mathbf{J} \cdot \mathbf{e}_{zi}^\star \, dS \tag{2.20}$$

$$i_i(z) = \int \mathbf{J} \cdot \mathbf{e}_i^\star \, dS + Y_i^\star \int \mathbf{M} \cdot \mathbf{h}_{zi}^\star \, dS \tag{2.21}$$

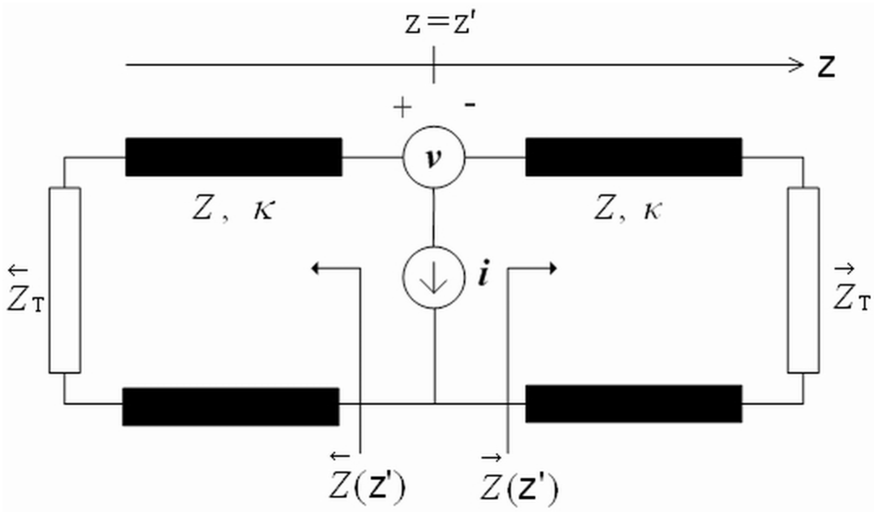

Fig. 3. Equivalent circuit for waveguide mode

For point sources $v(z) = v\,\delta(z - z')$ and $i(z) = i\,\delta(z - z')$ at $z = z'$ the solution is given for $z > z'$

$$V(z,z') = \frac{-1}{2}\left[v\frac{\overrightarrow{Z}(z')+Z}{\overleftrightarrow{Z}(z')} + iZ\frac{\overrightarrow{Y}(z')+Y}{\overleftrightarrow{Y}(z')}\right]\left[e^{-j\kappa(z-z')} + \overrightarrow{\Gamma}(z')e^{+j\kappa(z-z')}\right] \quad (2.22)$$

$$I(z,z') = \frac{-1}{2}\left[i\frac{\overrightarrow{Y}(z')+Y}{\overleftrightarrow{Y}(z')} + vY\frac{\overrightarrow{Z}(z')+Z}{\overleftrightarrow{Z}(z')}\right]\left[e^{-j\kappa(z-z')} - \overrightarrow{\Gamma}(z')e^{+j\kappa(z-z')}\right] \quad (2.23)$$

for $z < z'$

$$V(z,z') = \frac{-1}{2}\left[-v\frac{\overleftarrow{Z}(z')+Z}{\overleftrightarrow{Z}(z')} + iZ\frac{\overleftarrow{Y}(z')+Y}{\overleftrightarrow{Y}(z')}\right]\left[e^{+j\kappa(z-z')} + \overleftarrow{\Gamma}(z')e^{-j\kappa(z-z')}\right] \quad (2.24)$$

$$I(z,z') = \frac{-1}{2}\left[-i\frac{\overleftarrow{Y}(z')+Y}{\overleftrightarrow{Y}(z')} + vY\frac{\overleftarrow{Z}(z')+Z}{\overleftrightarrow{Z}(z')}\right]\left[e^{+j\kappa(z-z')} - \overleftarrow{\Gamma}(z')e^{-j\kappa(z-z')}\right] \quad (2.25)$$

where

Z:	characteristic impedance	Y:	characteristic admittance
v:	driving voltage source	i:	driving current source
κ:	propagation constant	Γ:	reflection coefficient

and

$$\overleftrightarrow{Z}(z') = \overleftarrow{Z}(z') + \overrightarrow{Z}(z'); \quad \overleftrightarrow{Y}(z') = \frac{1}{\overleftrightarrow{Z}(z')}; \quad \overleftarrow{Y}(z') = \frac{1}{\overleftarrow{Z}(z')}; \quad \overrightarrow{Y}(z') = \frac{1}{\overrightarrow{Z}(z')}; \quad (2.26)$$

The modal index has been omitted for simplicity. For arbitrary source distribution the fields are obtained by integration of the above results.

2.2.3 Analytic Solutions

In the following the fields due to $\mathbf{J}$ and $\mathbf{M}$ are considered separately.

Fields due to Electric Current J

Fig. 2 is the cross-sectional view of the patch. The eigenvalue problem is defined in $z - x$ cross-sectional plane with y taken as the longitudinal coordinate.

E-Mode Solution The wave function ψ_i' satisfies the wave equation

$$(\nabla_t^2 + k_{ti}'^2)\, \psi_i' = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \psi_i' = 0 \tag{2.27}$$

with boundary conditions:

1. $e'_{ix} = \hat{\mathbf{x}} \cdot \left[\frac{-1}{k'_{ti}} (\nabla_t \psi'_{t_i}) \right] = \frac{-1}{k'_{ti}} \left[\frac{\partial}{\partial x} \psi'_{ix}(x) \right] \psi'_{iz}(z) = 0$ at $z = 0, c$

   for vanishing tangential electric field on PEC

2. $h'_{iz} = \hat{\mathbf{z}} \cdot \left[\hat{\mathbf{y}} \times \mathbf{e}_i' \right] = \frac{-1}{k'_{ti}} \left[-\frac{\partial}{\partial x} \psi'_{ix}(x) \right] \psi'_{iz}(z) = 0$ at $x = 0, a$

   for vanishing tangential magnetic field on PMC

where ψ_{ix} and ψ_{ix} are functions due to separation of variables

$$\psi_i' = \psi'_{ix}(x)\psi'_{iz}(z). \tag{2.28}$$

As a result

$$\psi'_{mn}(x) = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{c}\right), \quad \text{m=0, 1, 2, 3..., n=1, 2, 3...} \tag{2.29}$$

where the normalization factor

$$A_{mn} = \begin{cases} \sqrt{\dfrac{2}{ac}}, & m = 0 \\ \sqrt{\dfrac{4}{ac}}, & m \neq 0 \end{cases} \tag{2.30}$$

The modal index i has been replaced by the double index mn for the two-dimensional eigenvalue problem. The eigenvalue for the mnth mode is found to be

$$k'_{tmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{c}\right)^2} \tag{2.31}$$

and hence the dispersion relation

$$\kappa'_{mn} = \sqrt{k^2 - k'^2_{tmn}} \tag{2.32}$$

***H*-Mode Solution** The wave function ψ''_i satisfies the wave equation

$$(\ \nabla_t^2 + k''^2_{ti})\ \psi''_i = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\psi''_i = 0 \tag{2.33}$$

with boundary conditions:

1. $h''_{iz} = \hat{\mathbf{z}} \cdot \left[\frac{-1}{k''_{ti}} \nabla_t \psi''_{ti}\right] = \frac{-1}{k''_{ti}}\left[\frac{\partial}{\partial z}\psi''_{iz}(z)\right]\psi''_{ix}(x) = 0 \ \text{ at } x = 0, a$

   for vanishing tangential magnetic field on PMC

2. $e''_{ix} = \hat{\mathbf{x}} \cdot \left[\mathbf{h}''_i \times \hat{\mathbf{y}}\right] = \frac{-1}{k''_{ti}}\left[-\frac{\partial}{\partial x}\psi''_{ix}(x)\right]\psi''_{iz}(z) = 0 \ \text{ at } z = 0, c$

   for vanishing tangential electric field on PEC where by separation of variables

$$\psi_i'' = \psi_{ix}''(x)\psi_{iz}''(z) \tag{2.34}$$

As a result

$$\psi_{mn}''(x) = A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi x}{c}\right), \quad \text{m=0, 1, 2, 3..., n=1, 2, 3...} \ , \tag{2.35}$$

where the normalization factor

$$A_{mn} = \begin{cases} \sqrt{\dfrac{2}{ac}}, & n = 0 \\ \sqrt{\dfrac{4}{ac}}, & n \neq 0 \end{cases} \tag{2.36}$$

The eigenvalue for the mnth mode is

$$k_{t_{mn}}'' = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{c}\right)^2} \tag{2.37}$$

and hence the dispersion relation

$$\kappa_{mn}'' = \sqrt{k^2 - k_{tmn}''^2} \tag{2.38}$$

Modal Source Currents The probe is assumed to have the current distribution

$$\mathbf{J} = \hat{\mathbf{z}}\, I \sin\left[k_0(h - z)\right]\delta(x - x_s)\delta(y - y_s) \tag{2.39}$$

which contributes for each mnth mode the driving source current

$$i_{mn}(y) = \int \mathbf{J}\cdot\mathbf{e}_{mn}^{*} dS \tag{2.40}$$

with the integration taken over the area of cross section $0 \leq x \leq a,\ 0 \leq z \leq c$. Thus

$$i_{mn}'(y) = \frac{-1}{k_{t_{mn}}'} A_{mn}\frac{n\pi}{c}\cos\frac{m\pi x_s}{a}\frac{k_0}{k_0^2 - (\frac{n\pi}{c})^2}\left[\cos\frac{n\pi h}{c} - \cos k_0 h\right]\delta(y - y_s) \tag{2.41}$$

$$i_{mn}''(y) = \frac{-1}{k_{t_{mn}}''} A_{mn}\frac{m\pi}{a}\cos\frac{m\pi x_s}{a}\frac{k_0}{k_0^2 - (\frac{n\pi}{c})^2}\left[\cos\frac{n\pi h}{c} - \cos k_0 h\right]\delta(y - y_s) \tag{2.42}$$

No driving voltage source exists in this case. The corresponding circuit configuration is shown in Fig. 4 with open-circuit termination at both ends loading with perfect magnetic

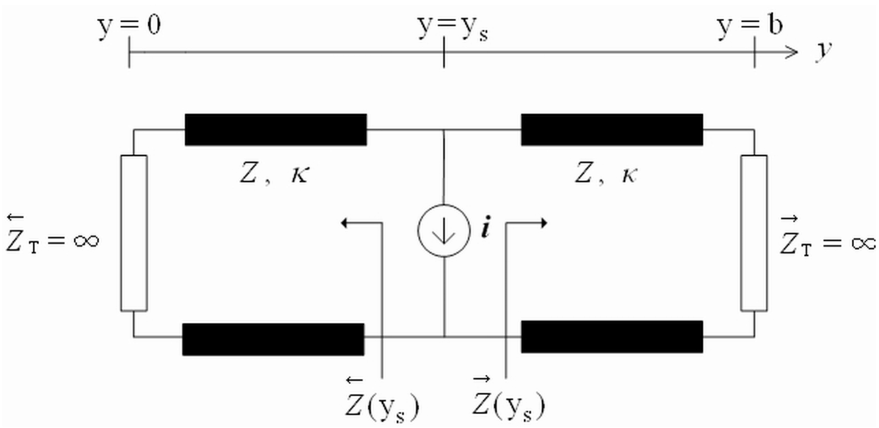

Fig. 4. Equivalent circuit for a electric current excitation

wall. The solution is

for $y > y_s$

$$V_{mn}(y, y_s) = \frac{-1}{2}\left[v_{mn}\frac{\overrightarrow{Z}_{mn}(y_s) + Z_{mn}}{\overleftrightarrow{Z}_{mn}(y_s)} + i_{mn}Z_{mn}\frac{\overrightarrow{Y}_{mn}(y_s) + Y_{mn}}{\overleftrightarrow{Y}_{mn}(y_s)}\right] \times \left[e^{-j\kappa_{mn}(y-y_s)} + \overrightarrow{\Gamma}_{mn}(y_s)e^{+j\kappa_{mn}(y-y_s)}\right] \tag{2.43}$$

$$I_{mn}(y, y_s) = \frac{-1}{2}\left[i_{mn}\frac{\overrightarrow{Y}_{mn}(y_s) + Y_{mn}}{\overleftrightarrow{Y}_{mn}(y_s)} + v_{mn}Y_{mn}\frac{\overrightarrow{Z}_{mn}(y_s) + Z_{mn}}{\overleftrightarrow{Z}_{mn}(y_s)}\right] \times \left[e^{-j\kappa_{mn}(y-y_s)} - \overrightarrow{\Gamma}_{mn}(y_s)e^{+j\kappa_{mn}(y-y_s)}\right] \tag{2.44}$$

for $y < y_s$

$$V_{mn}(y, y_s) = \frac{-1}{2}\left[-v_{mn}\frac{\overleftarrow{Z}_{mn}(y_s) + Z_{mn}}{\overleftrightarrow{Z}_{mn}(y_s)} + i_{mn}Z_{mn}\frac{\overleftarrow{Y}_{mn}(y_s) + Y_{mn}}{\overleftrightarrow{Y}_{mn}(y_s)}\right] \times \left[e^{+j\kappa_{mn}(y-y_s)} + \overleftarrow{\Gamma}_{mn}(y_s)e^{-j\kappa_{mn}(y-y_s)}\right] \tag{2.45}$$

$$I_{mn}(y, y_s) = \frac{-1}{2}\left[-i_{mn}\frac{\overleftarrow{Y}_{mn}(y_s) + Y_{mn}}{\overleftrightarrow{Y}_{mn}(y_s)} + v_{mn}Y_{mn}\frac{\overleftarrow{Z}_{mn}(y_s) + Z_{mn}}{\overleftrightarrow{Z}_{mn}(y_s)}\right] \times \left[e^{+j\kappa_{mn}(y-y_s)} - \overleftarrow{\Gamma}_{mn}(y_s)e^{-j\kappa_{mn}(y-y_s)}\right] \tag{2.46}$$

where

$$Z'_{mn} = \frac{\kappa'_{mn}}{\omega\epsilon} = \frac{1}{Y'_{mn}}, \quad Z''_{mn} = \frac{\omega\mu}{\kappa''_{mn}} = \frac{1}{Y''_{mn}}; \tag{2.47}$$

$$|\overleftarrow{\Gamma}_{mn}| = 1,\ |\overrightarrow{\Gamma}_{mn}| = 1 \text{ due to open circuit at y=0, b} \tag{2.48}$$

$$\overleftarrow{\Gamma}_{mn}(y_s) = e^{-j2\kappa_{mn}y_s}; \overrightarrow{\Gamma}_{mn}(y_s) = e^{-j2\kappa_{mn}(b-y_s)}; \tag{2.49}$$

$$\overleftarrow{Y}_{mn}(y_s) = jY_{mn}\tan(\kappa_{mn}y_s) = \frac{1}{\overleftarrow{Z}_{mn}(y_s)}; \tag{2.50}$$

$$\overrightarrow{Y}_{mn}(y_s) = jY_{mn}\tan(\kappa_{mn}(b-y_s)) = \frac{1}{\overrightarrow{Z}_{mn}(y_s)}; \tag{2.51}$$

$$\overleftrightarrow{Z}_{mn}(y_s) = \overleftarrow{Z}_{mn}(y_s) + \overrightarrow{Z}_{mn}(y_s);\ \ \overleftrightarrow{Y}_{mn}(y_s) = \overleftarrow{Y}_{mn}(y_s) + \overrightarrow{Y}_{mn}(y_s); \tag{2.52}$$

and $\overleftarrow{\Gamma}_{mn} = \overleftarrow{\Gamma}'_{mn}$ or $\overleftarrow{\Gamma}''_{mn}$, $\overrightarrow{\Gamma}_{mn} = \overrightarrow{\Gamma}'_{mn}$ or $\overrightarrow{\Gamma}''_{mn}$, $\kappa_{mn} = \kappa'_{mn}$ or κ''_{mn}, respectively.

Summing all the modal fields we have for the transverse fields

$$\mathbf{E}_t = \sum_{m,n} V'_{mn}(y)\mathbf{e}'_{mn}(x,z) + \sum_{m,n} V''_{mn}(y)\mathbf{e}''_{mn}(x,z) \tag{2.53}$$

$$\mathbf{E}_t = \sum_{m,n} I'_{mn}(y)\mathbf{h}'_{mn}(x,z) + \sum_{m,n} I''_{mn}(y)\mathbf{h}''_{mn}(x,z) \tag{2.54}$$

The longitudinal fields are derived from the transverse fields to give

$$E_y = \frac{1}{j\omega\epsilon}\left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right)\cdot[(H_x\hat{\mathbf{x}} + H_z\hat{\mathbf{z}})\times\hat{\mathbf{y}}] \tag{2.55}$$

$$H_y = \frac{1}{j\omega\mu}\left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right)\cdot[\hat{\mathbf{y}}\times(E_x\hat{\mathbf{x}} + E_z\hat{\mathbf{z}})] \tag{2.56}$$

Fields due to Magnetic Current M

Fig. 2 is also the cross-sectional view of the patch that the eigenvalue problem is defined in $z - x$ plan with y taken as the longitudinal coordinate.

E**-Mode Solution** The wave function $\psi_i^{'}$ satisfies the wave equation

$$(\nabla_t^2 + k_{ti}^{'2})\ \psi_i^{'} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) \psi_i^{'} = 0 \tag{2.57}$$

with boundary conditions:

1. $h^{'}{}_{ix} = \hat{\mathbf{x}} \cdot \left[\hat{\mathbf{z}} \times \frac{-1}{k_{ti}^{'}} \nabla_t \psi_i^{'}\right] = \frac{1}{k_{ti}^{'}} \left[\frac{\partial}{\partial y}\psi_{iy}^{'}(y)\right] \psi_{ix}^{'}(x) = 0 \text{ at } y = 0, b$

   for vanishing tangential magnetic field on PMC

2. $h^{'}{}_{iy} = \hat{\mathbf{y}} \cdot \left[\hat{\mathbf{z}} \times \frac{-1}{k_{ti}^{'}} \nabla_t \psi_i^{'}\right] = \frac{-1}{k_{ti}^{'}} \left[-\frac{\partial}{\partial x}\psi_{ix}^{'}(x)\right] \psi_{iy}^{'}(y) = 0 \text{ at } x = 0, a$

   for vanishing tangential magnetic field on PMC

where by separation of variables

$$\psi_i^{'} = \psi_{ix}^{'}(x)\psi_{iz}^{'}(z) \tag{2.58}$$

As a result

$$\psi_{ix}^{'}(x) = A_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right), \quad \text{m=0, 1, 2, 3..., n=0, 1, 2, 3... ,} \tag{2.59}$$

where the normalization factor

$$A_{mn} = \begin{cases} \sqrt{\frac{1}{ab}}, & m = 0, n = 0 \\ \sqrt{\frac{2}{ab}}, & m = 0, n \neq 0 \\ \sqrt{\frac{4}{ab}}, & mn \neq 0 \end{cases} \tag{2.60}$$

The modal index has been replaced by the double index mn for the two-dimensional eigenvalue problem. The eigenvalue for the mnth mode is

$$k_{tmn}^{'} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{2.61}$$

and hence the dispersion relation

$$\kappa_{mn}^{'} = \sqrt{k^2 - k_{tmn}^{'2}} \tag{2.62}$$

H**-Mode Solution** The wave function ψ_i'' satisfies the wave equation

$$(\nabla_t^2 + k_{ti}''^2)\ \psi_i'' = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \psi_i'' = 0 \tag{2.63}$$

with boundary conditions:

1. $h''_{ix} = \hat{\mathbf{x}} \cdot \left[\frac{-1}{k_{ti}''} \nabla_t \psi_i'' \right] = \frac{1}{k_{ti}''} \left[\frac{\partial}{\partial x} \psi_{ix}''(z) \right] \psi_{iy}''(y) = 0 \text{ at } y = 0, b$

   for vanishing tangential magnetic field on PMC

2. $h''_{iy} = \hat{\mathbf{y}} \cdot \left[\frac{-1}{k_{ti}''} \nabla_t \psi_i'' \right] = \frac{1}{k_{ti}''} \left[\frac{\partial}{\partial y} \psi_{iy}''(z) \right] \psi_{ix}''(x) = 0 \text{ at } x = 0, a$

   for vanishing tangential magnetic field on PMC

where by separation of variables

$$\psi_i'' = \psi_{ix}''(x) \psi_{iz}''(z) \tag{2.64}$$

As a result the normalized wave function

$$\psi_{mn}''(x) = \sqrt{\frac{4}{ac}} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi z}{c}\right), \quad \text{m=1, 2, 3..., n=1, 2, 3... ,} \tag{2.65}$$

The eigenvalue for the mnth mode is

$$k_{t_{mn}}'' = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{c}\right)^2} \tag{2.66}$$

and hence the dispersion relation

$$\kappa_{mn}'' = \sqrt{k^2 - k_{tmn}''^2} \tag{2.67}$$

Modal Source Currents The equivalent magnetic current is assumed to be

$$\mathbf{M} = \hat{\mathbf{x}}\, M(x_s, y_s)\delta(z-c) \tag{2.68}$$

The equivalent source voltage for the mnth mode is found from

$$v_{mn}(z) = \int \mathbf{M}\cdot \mathbf{h}^*_{mn}\, dS \tag{2.69}$$

where $\mathbf{M}$ is the unknown to be solved. The integration is taken over the slot area covering the region $0 \le x \le a,\ l_1 \le y \le l_1 + d$. To solve $\mathbf{M}$ let it be expanded by a set of basis functions

$$\mathbf{M} = \hat{\mathbf{x}} \sum_{l=1}^{L} c_l\, p_l(x,y) \tag{2.70}$$

where

$$p_l(x,y) = \begin{cases} 1, & x_l \le x \le x_{l+1},\ y_l \le y \le y_{l+1} \\ 0, & \text{otherwise} \end{cases} \tag{2.71}$$

Hence

$$v'_{mn} = \sum_{l=1}^{L} c_l \int_{x_l}^{x_{l+1}} \int_{y_l}^{y_{l+1}} p_l(x,y)\, \hat{\mathrm{x}}\cdot \mathbf{h}'^*_{mn}\, dydx \tag{2.72}$$

$$v''_{mn} = \sum_{l=1}^{L} c_l \int_{x_l}^{x_{l+1}} \int_{y_l}^{y_{l+1}} p_l(x,y)\, \hat{\mathrm{x}}\cdot \mathbf{h}''^*_{mn}\, dydx \tag{2.73}$$

Substitute the modal fields we have

$$v'_{mn} = \sum_{l=1}^{L} \frac{-c_l A_{mn}}{k'_{t_{mn}}} \begin{cases} 0, & n=0 \\ -\left[\cos\dfrac{n\pi(l_1+d)}{b} - \cos\dfrac{n\pi l_1}{b}\right](x_{l+1}-x_l), & n\neq 0,\ m=0 \\ -\left[\cos\dfrac{n\pi(l_1+d)}{b} - \cos\dfrac{n\pi l_1}{b}\right] & \\ \times\left(\dfrac{b}{n\pi}\right)\left[\sin\dfrac{m\pi x_{l+1}}{a} - \sin\dfrac{m\pi x_l}{a}\right], & n\neq 0,\ m\neq 0 \end{cases} \tag{2.74}$$

and

$$
v''_{mn} = \sum_{l=1}^{L} \frac{-c_l A_{mn}}{k'_{t_{mn}}} \begin{cases} 0, & n = 0 \\ 0, & n \neq 0,\ m = 0 \\ -\left[\cos\dfrac{n\pi(l_1+d)}{b} - \cos\dfrac{n\pi l_1}{b}\right] & \\ \times\left(\dfrac{b}{n\pi}\right)\left[\sin\dfrac{m\pi x_{l+1}}{a} - \sin\dfrac{m\pi x_l}{a}\right], & n \neq 0,\ m \neq 0 \end{cases} \tag{2.75}
$$

No modal source current exists in this case. for $z < z_s = c$

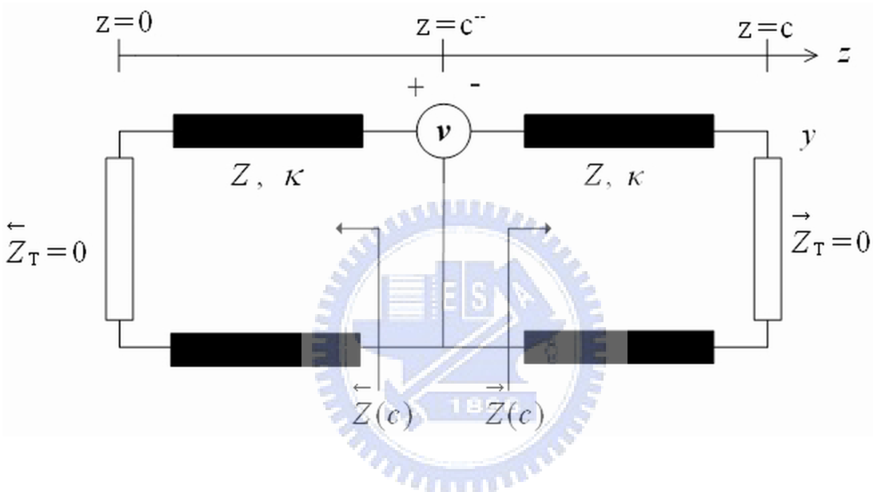

Fig. 5. Equivalent circuit for a magnetic current excitation

$$
\begin{aligned}
V_{mn}(z,z_s) &= \frac{-1}{2}\left[-v_{mn}\frac{\overleftarrow{Z}_{mn}(z_s)+Z_{mn}}{\overleftrightarrow{Z}_{mn}(z_s)}\right]\cdot\left[e^{+j\kappa_{mn}(z-z_s)} + \overleftarrow{\Gamma}_{mn}(z_s)e^{-j\kappa_{mn}(z-z_s)}\right] \\
I_{mn}(z,z_s) &= \frac{-1}{2}\left[v_{mn}Y_{mn}\frac{\overleftarrow{Z}_{mn}(z_s)+Z_{mn}}{\overleftrightarrow{Z}_{mn}(z_s)}\right]\cdot\left[e^{+j\kappa_{mn}(z-z_s)} - \overleftarrow{\Gamma}_{mn}(z_s)e^{-j\kappa_{mn}(z-z_s)}\right]
\end{aligned}
$$

where

$$
Z'_{mn} = \frac{\kappa'_{mn}}{\omega\epsilon} = \frac{1}{Y'_{mn}},\ Z''_{mn} = \frac{\omega\mu}{\kappa''_{mn}} = \frac{1}{Y''_{mn}}; \tag{2.76}
$$

$$
|\overleftarrow{\Gamma}_{mn}| = 1,\ |\overrightarrow{\Gamma}_{mn}| = 1 \text{ due to open circuit at z=0, c} \tag{2.77}
$$

$$\overleftarrow{\Gamma}_{mn}(z_s) = e^{j2\kappa_{mn}y_s}; \overrightarrow{\Gamma}_{mn}(z_s) = -1; \tag{2.78}$$

$$\overleftarrow{Z}_{mn}(z_s) = jZ_{mn}\tan(\kappa_{mn}z_s) = \frac{1}{\overleftarrow{Z}_{mn}(z_s)}; \ \overrightarrow{Z}_{mn}(z_s) = 0; \tag{2.79}$$

$$\overleftrightarrow{Z}_{mn}(z_s) = \overleftarrow{Z}_{mn}(z_s) + \overrightarrow{Z}_{mn}(z_s); \ \overleftrightarrow{Y}_{mn}(z_s) = \overleftarrow{Y}_{mn}(z_s) + \overrightarrow{Y}_{mn}(z_s); \tag{2.80}$$

$\overleftarrow{\Gamma}_{mn} = \overleftarrow{\Gamma}'_{mn}$ or $\overleftarrow{\Gamma}''_{mn}$, $\overrightarrow{\Gamma}_{mn} = \overrightarrow{\Gamma}'_{mn}$ or $\overrightarrow{\Gamma}''_{mn}$, $\kappa_{mn} = \kappa'_{mn}$ or κ''_{mn}, respectively.
Modal representation of transverse fields due to magnetic current source M:

$$\mathbf{E_t} = \sum_{m,n} V'_{mn}(z)\mathbf{e}'_{mn}(x,y) + \sum_{m,n} V''_{mn}(z)\mathbf{e}''_{mn}(x,y) \tag{2.81}$$

$$\mathbf{H_t} = \sum_{m,n} I'_{mn}(z)\mathbf{h}'_{mn}(x,y) + \sum_{m,n} I''_{mn}(z)\mathbf{h}''_{mn}(x,y) \tag{2.82}$$

The longitudinal fields are derived from the transverse fields as

$$\mathbf{E}_z = \frac{1}{j\omega\epsilon}(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y}) \cdot [(H_x\hat{\mathbf{x}} + H_y)\hat{\mathbf{y}}) \times \hat{\mathbf{z}}] \tag{2.83}$$

$$\mathbf{H}_z = \frac{1}{j\omega\mu}(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y}) \cdot [\hat{\mathbf{z}} \times (E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}})] \tag{2.84}$$

All the fields by magnetic current source M_2 are found.

2.3 Exterior Fields

The configuration for the exterior field problem is shown in Fig. 6. For the purpose of evaluation of the exterior fields to match the boundary condition on the slot it is convenient to express the field quantities in cylindrical coordinates. The thin magnetic current has been expanded previously in terms of a finite number of small dipoles. Each dipole contributes the magnetic field components

$$H_\rho = Ml\frac{1}{4\pi}e^{-jk_0\rho}\left(\frac{2}{\eta_0\rho^2} + \frac{2}{j\omega\mu_0\rho^3}\right)\cos\phi \tag{2.85}$$

$$H_\phi = Ml\frac{1}{4\pi}e^{-jk_0\rho}\left(\frac{j\omega\epsilon_0}{\rho} + \frac{1}{\eta_0\rho^2} + \frac{1}{j\omega\mu_0\rho^3}\right)\sin\phi \tag{2.86}$$

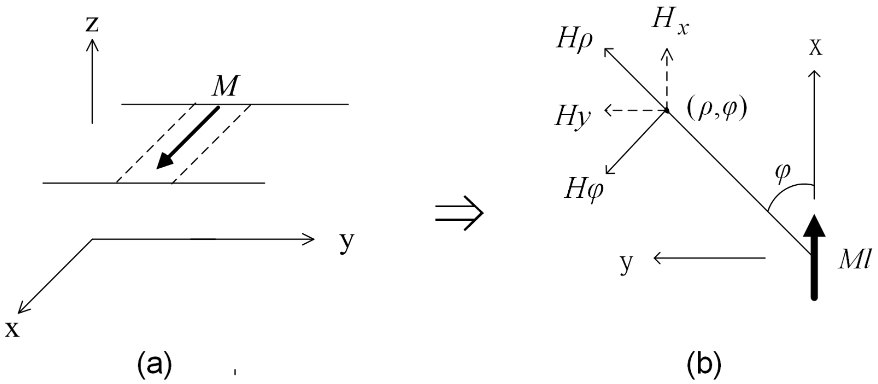

Fig. 6. (a) The field due to magnetic current on the xy-plane (b) Magnetic field in the plane of the patch at $z = c$

which is centered at the location of the dipole (x_s, y_s, c). Note that the two components H_ρ and H_ρ are the relevant tangential magnetic fields required for the matching of boundary condition on the slot. Summing contributions from all dipoles we obtain

$$H_\rho(\rho,\phi) = A\sum_{l=1}^{L} c_l \int_{\phi_l}^{\phi_{l+1}} \int_{\rho_l}^{\rho_{l+1}} \frac{2}{4\pi} e^{-jk_0\rho} \left(\frac{1}{\eta_0\rho} + \frac{1}{j\omega\mu_0\rho^2}\right) d\rho \cos\phi d\phi \tag{2.87}$$

$$H_\phi(\rho,\phi) = A\sum_{l=1}^{L} c_l \int_{\phi_l}^{\phi_{l+1}} \int_{\rho_l}^{\rho_{l+1}} \frac{1}{4\pi} e^{-jk_0\rho} \left(j\omega\epsilon_0 + \frac{1}{\eta_0\rho} + \frac{1}{j\omega\mu_0\rho^2}\right) d\rho \sin\phi d\phi \tag{2.88}$$

where A is the area of each basis. The above integrals can be greatly simplified by direction integration of

$$\int e^{-jk_0\rho}\left(\frac{1}{\eta_0\rho} + \frac{1}{j\omega\mu_0\rho^2}\right) d\rho = \frac{1}{\eta_0}\int e^{-jk_0\rho}\left(\frac{1}{\rho} + \frac{1}{jk_0\rho^2}\right) d\rho, \text{ where } \frac{1}{\omega\mu_0} = \frac{1}{k_0\eta_0}$$

$$\int e^{-jk_0\rho}\left(\frac{1}{\rho} + \frac{1}{jk_0\rho^2}\right) d\rho = \int [\cos(k_0\rho) - j\sin(k_0\rho)] \left(\frac{1}{\rho} + \frac{1}{jk_0\rho^2}\right) d\rho$$

$$= \int \frac{\cos(k_0\rho)}{\rho} d\rho + \int \frac{\cos(k_0\rho)}{jk_0\rho^2} d\rho - j\int \frac{\sin(k_0\rho)}{\rho} d\rho - j\int \frac{\sin(k_0\rho)}{jk_0\rho^2} d\rho$$

$$= \int \frac{\cos(k_0\rho)}{\rho} d\rho + j \left[\frac{\cos(k_0\rho)}{k_0\rho} + \int \frac{\sin(k_0\rho)}{\rho} d\rho \right] - j \int \frac{\sin(k_0\rho)}{\rho} d\rho$$

$$+ \left[\frac{\sin(k_0\rho)}{k_0\rho} - \int \frac{\cos(k_0\rho)}{\rho} d\rho \right] = j \frac{\cos(k_0\rho)}{k_0\rho} + \frac{\sin(k_0\rho)}{k_0\rho} = j \frac{e^{-jk_0\rho}}{k_0\rho} \tag{2.89}$$

In rectangular coordinates

$$H_x = H_\rho \cos\phi - H_\phi \sin\phi \tag{2.90}$$

$$H_y = H_\rho \sin\phi + H_\phi \cos\phi \tag{2.91}$$

which are to be used for matching the boundary condition with the interior magnetic field.

Chapter 3

Numerical Solution by Method of Moments

We have thus far been able to evaluate the fields, both interior and exterior to the cavity, in terms of the equivalent magnetic current $\mathbf{M}$. Now on the slot at $z = c$ the boundary condition requires that the tangential magnetic field is continuous across the slot. Put it in simple form,

$$[\mathbf{H}_i(\mathbf{r}, \mathbf{J})]_{tan} + [\mathbf{H}_i(\mathbf{r}, \mathbf{M})]_{tan} = [\mathbf{H}_o(\mathbf{r}, \mathbf{M})]_{tan} \tag{3.1}$$

or

$$[\mathbf{H}_o(\mathbf{r}, \mathbf{M})]_{tan} - [\mathbf{H}_i(\mathbf{r}, \mathbf{M})]_{tan} = [\mathbf{H}_i(\mathbf{r}, \mathbf{J})]_{tan} \tag{3.2}$$

where $\mathbf{H}_i(\mathbf{r}, \mathbf{J})$ and $\mathbf{H}_o(\mathbf{r}, \mathbf{J})$ denote respectively interior and exterior fields of the cavity. Write in terms of the expansion coefficients

$$[\mathbf{H}_o(\mathbf{r}, \mathbf{c})]_{tan} - [\mathbf{H}_i(\mathbf{r}, \mathbf{c})]_{tan} = [\mathbf{H}_i(\mathbf{r}, \mathbf{J})]_{tan} \tag{3.3}$$

where $\mathbf{c}$ denotes the coefficient vector $(c_1, c_2, ..., c_L)$.

By point matching at $(x_i, y_i), i = 1, 2..., L$ in the above equation we have L linear independent equations for L unknowns $c_1, c_2, ..., c_L$

$$\sum_{j=1}^{L}[\mathbf{H}_o(x_i, c_j)]_{tan} - \sum_{j=1}^{L}[\mathbf{H}_i(x_i)]_{tan} = \sum_{j=1}^{L}[\mathbf{H}_i(\mathbf{r}, j_j)]_{tan},\ i = 1, 2...L \tag{3.4}$$

By matrix inversion we obtain the solution for $(c_1, c_2, ..., c_L)$.

Chapter 4

Numerical Results

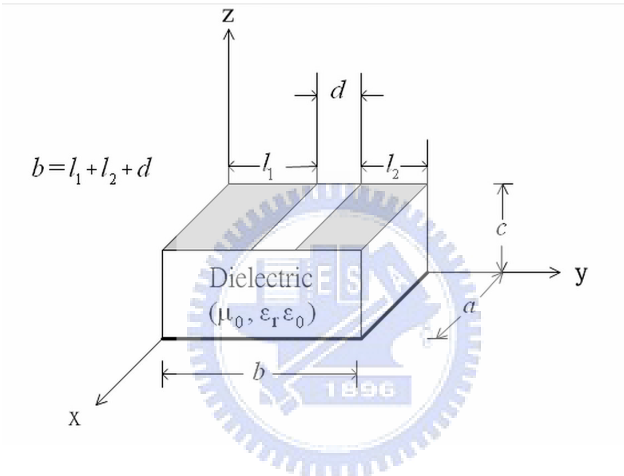

Fig. 7. Example

$l_1 = 31.6mm, l_2 = 24.6mm, d = 0.5mm, a = 31.6mm, c = 2.3mm, b = l_1+l_2+d, \epsilon_r = 2.35,$ and the height of feeding probe in the board h=1.15mm.

In the spherical coordinate,

elevation angle θ, $0° \leq \theta \leq 180°$ azimuthal angle ϕ, $0° \leq \phi \leq 360°$

Simulation software used is **Ansoft HFSS**.

The field distribution on the slot has been computed for the configuration as shown in Fig. 7. Three cases of frequencies 7, 11, and 15 GHz are considered. The results are shown respectively in Figs. 8-10. Also shown in the figures for comparison are that obtained by Ansoft HFSS simulator. As can be seen in the figures all cases show satisfactory agreement in general except that region near the ends of the slot, in particular for the case at 11 GHz. In our conjectures it is probably due to the simple assumption in the formulation that the slot is confined to the region between two adjacent metal edges, and ignores the region from the edge down to the ground plane near both end. However the field around such region is more complicated than we first suggest and which deserves further investigation.

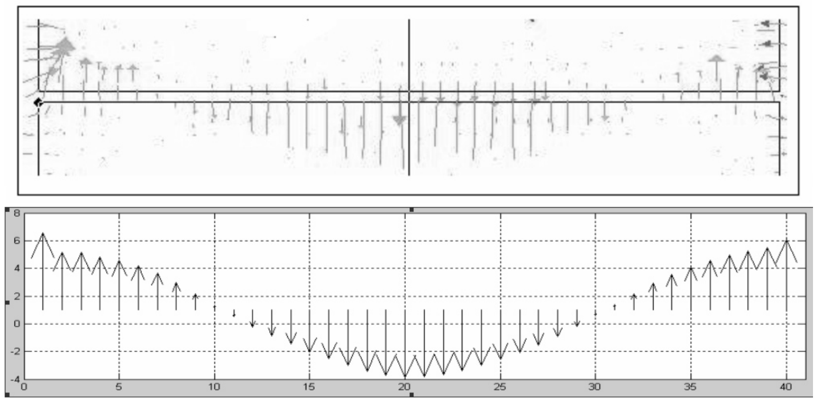

Fig. 8. f= 7GHz, the simulated fields by Ansoft HFSS and the computed fields in the slot.

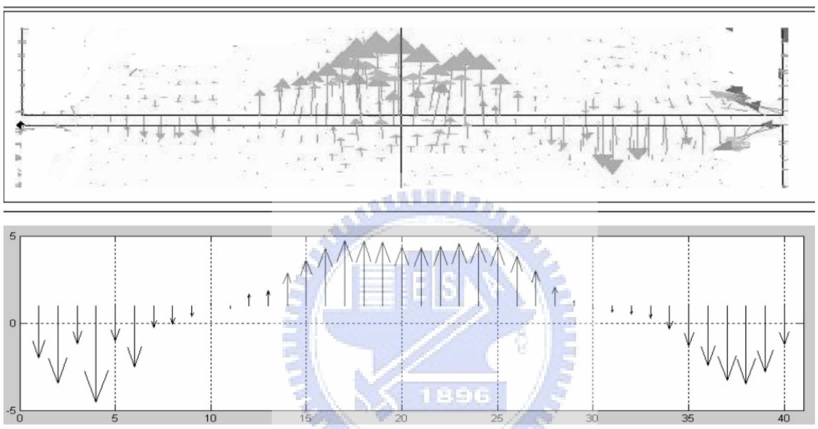

Fig. 9. f=11GHz, the simulated fields by Ansoft HFSS and the computed fields in the slot.

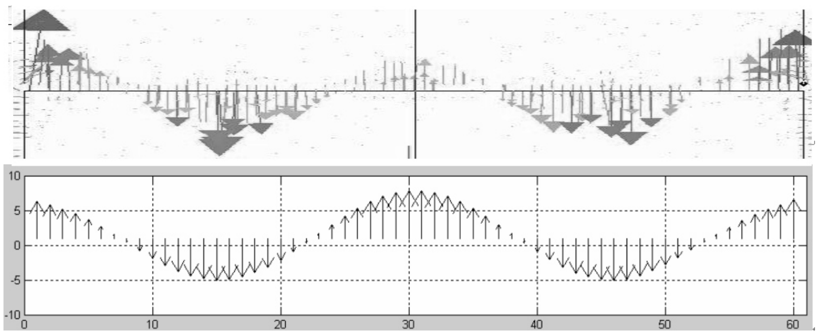

Fig. 10. f=15GHz, the simulated fields by Ansoft HFSS and the computed fields in the slot.

Chapter 5

Discussion and Conclusion

The edge coupling due to proximity of two patch antennas has been investigated in this thesis. The analysis is based on the cavity model of the patch with slot coupling so that the problem can be formulated by equivalence principle. Coupled field equations have been established and solved numerically by the method of moments. The results have been compared with that obtained by the commercial simulation software. The field distribution on the slot exhibits difference around the end of the slot which can be improved by extending the slot down to the ground plane at both ends.

The present analysis can be extended to the case of multiple patches which form a patch array of the Yagi-Uda type.

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