# 國立交通大學

## 電子工程學系 電子研究所碩士班

## 碩士論文

以聯盟賽局理論之合作式感知無線網路 功率控制及時間分配 ES Coalitional Game Theoretic Power Control and Time

Allocation in Cooperative Cognitive Radio Networks

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在本篇論文,我們以賽局理論的角度來研究合作式感知無線網路功率控制及時 間分配。我們考慮一個由多主要服務者(primary service)和多次要服務者(secondary service)所構成的感知無線網路。在此無線網路架構下,主要服務者和次要服務者採 取合作式通訊,主要服務者是此賽局的頻譜擁有者,透過主動租借頻帶給次要服務 者使用,藉以獲取通訊品質提升;次要服務者藉由協助主要服務者傳輸,換取租借 頻帶的時間,得到頻帶使用權。我們以聯盟賽局理論的模型來分析每個玩家之間合 作的行為,並轉化為系統的最佳化問題,在符合條件下求得核心解(core)。主要考慮 兩種情況,首先是只考慮功率控制,其次是同時考慮功率控制跟時間分配,並分別 在兩種情況下對玩家的合作行為做分析。此外,我們在第二種情況下提出演算法, 保證可以求得最佳解,而此最佳解符合聯盟賽局中核心解定義,所有玩家會合作形 成大聯盟。在模擬中,也比較我們的作法和前人的作法,並在玩家的利益上探討賽 局的收斂行為。另外,也分析系統架構下,我們提出的演算法會收斂到核心解,而 在此平衡點,對所有玩家都是有利的,所以保證整體系統配置穩定。



# **Coalitional Game Theoretic Power Control and Time Allocation in Cooperative Cognitive Radio Networks**

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## Abstract

In this thesis, we study the problem of power control and time allocation in cooperative cognitive radio networks (CCRNs) from a game theoretic approach. Particularly, we consider a CCRN with multiple primary users (PUs) and multiple secondary users (SUs), where all players exploit cooperative communication. In the game, the spectrum is licensed for PUs, through leasing the spectrum to SUs for a fraction of time in exchange for improving transmission rates. On the other hand, SUs have opportunities to access the spectrum due to assist primary transmission. We apply the coalitional game to model the cooperative interactions among players and we formulate the problem as an optimization problem and achieve the core under certain conditions. We mainly focus on two cases, which the first case only considers power control and the other one considers power control and time allocation problems. We analyze players' cooperative interactions in the two cases and we propose a novel algorithm to solve the second case. The proposed algorithm is guaranteed convexity and achieves the equilibrium in the core. According to the definition of the core, all the players in the system will form grand coalition. In the simulations, we compare the proposed approach with other approaches and numerically study the players' payoffs in the coalitional game. Furthermore, we also show that proposed algorithm converges to the core, which

guarantees that the payoff allocation is stable in the system.



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# Contents

1	Introduction						
	1.1	Motivation	1				
	1.2	Why Coalitional Game?	2				
	1.3	Related Work and Our Approach	2				
	1.4	Contributions of the Research	3				
2	Cognitive Radio and Game Theory Preliminary						
	2.1	Cognitive Radio	5				
		2.1.1 Cooperative Cognitive Radio Network	6				
	2.2	Game Theory	8				
		2.2.1 Basic Definitions of Game Theory	8				
		2.2.2 Coalitional Game	0				
3	Coa	itional Game in Cooperative Cognitive Radio Networks	3				
	3.1	Problem Setup	3				
	3.2	Coalitional Game Formulation	5				
	Problem Formulation for Power Control Case	6					
		3.3.1 Utility Model	6				
		3.3.2 Optimization Problem Formulation	9				
		3.3.3 The Core	0				
	3.4	Problem Formulation for Power Control and Time Allocation Case 2	1				
		3.4.1 Utility Model	1				
		3.4.2 Optimization Problem Formulation	3				

		3.4.3	Algorithm for Solving Problem Formulation	24
	3.5	Algori	thm Converges to the Core	27
4	Simu	ulations		29
	4.1	Simula	tion Setup	29
	4.2	Numer	ical Results	30
		4.2.1	Power Control Case	30
		4.2.2	Power Control and Time Allocation Case	34
		4.2.3	Comparison between Different Approaches	39
		4.2.4	Algorithm Converges to the Core	42
5	Con	clusion	and Future Work	44
	5.1	Conclu	sion	44
	5.2	Future	Work	45
Bil	oliogr	raphy		47

# **List of Figures**

2.1	Simplified cognition cycle.	6
2.2	System model for cooperative cognitive radio networks	7
3.1	The CCRN network with $N_p=2$ and $N_s=2$ : (a) primary transmission (b)	
	cooperative transmission (c) secondary transmission	14
4.1	The performance of CCRN in power control case, $u_s = 8$ : (a) The payoff	
	of PUs (b) The payoff of SUs.	31
4.2	SU's power allocation in power control case	32
4.3	The impact of access gain factor $(u_s)$ in power control case	32
4.4	The performance of CCRN in power control case, $u_s = 12$ : (a) The payoff	
	of PUs (b) The payoff of SUs.	33
4.5	The performance of CCRN in power control and time allocation case, $u_s$	
	= 8: (a) The payoff of PUs (b) The payoff of SUs	35
4.6	The impact of access gain factor $(u_s)$ in power control and time allocation	
	case with $\alpha = 0.642$ , $\beta = 0.217$	36
4.7	SU's power allocation in power control and time allocation case	36
4.8	The impact of fraction of time $\alpha$ with $\beta$ fixed	38
4.9	The impact of fraction of time $\alpha$ with $\beta$ variation	38
4.10	Comparison the performance between different approaches: (a) The pay-	
	off of PUs (b) The payoff of SUs	40
4.11	Comparison the sum payoff between different approaches	41
4.12	Proposed algorithm converges to the core	42

# Chapter 1

# Introduction

## 1.1 Motivation

Recently, the demand for wireless communication services has grown significantly. The spectrum is a scare resource under unbalanced utilization. The recent FCC report [1] indicated that some frequency bands are unoccupied most of the time, whereas others are heavily used. To improve the spectrum utilization, the concept of cognitive radio has been proposed [2] [3]. Cognitive radio [4] is a novel technique which allows unlicensed users to access unoccupied frequency bands. In the CR network, licensed users are called primary users (PUs) and unlicensed users are called secondary users (SUs). Exploiting CR technique, the spectrum efficiency is improved significantly. On the other hand, a new paradigm of CR network is proposed in [5], termed *cooperative cognitive radio network* (CCRN). The idea of cooperation has been applied in many disciplines, such as economics and political science. Whereas, in wireless communication area, the concept of cooperative communication is applied in the CR network. In the CCRN scenario, PUs and SUs may increase their interests from cooperation. Therefore, cooperative CR is a promising technology which not only can improve the spectrum efficiency, but also can increase users' interests in the system.

## **1.2 Why Coalitional Game?**

Traditional Media Access Control (MAC) theory is based on optimization operation to optimize the system utility. Although some problems can be decomposed into optimizing network utility and user's utility separately by dual-prime method [6], the solution of the problem does not always satisfy each user's utility.

In contrast to optimization method, game theory is a mathematical tool to analyze each user's utility. Game theory is much effective to analyze each user's behavior. Although the network utility may not be optimized, the solution lets each user's utility optimized individually. On the other hand, a coalitional game is a branch of game theory. A coalitional game is a powerful tool to analyze user's cooperative behavior. Also, the considered problem formulation is about users' cooperative interactions. Hence, a coalitional game is a powerful tool to model the problem.

# 1.3 Related Work and Our Approach

The idea of cooperative communication was first proposed by [5]. According to the property-right model in [5], the spectrum is leased to SUs in exchange for remuneration. In other words, PUs involve SUs as cooperative relays, so PUs' transmission rates are improved. Hence, PUs are willing to lease spectrum to SUs for a fraction of time. [7] also applys the idea of cooperative transmission, and terms the scenario as cooperative cognitive radio networks (CCRNs). The other two works [8] [9] are similar, also applying cooperative spectrum leasing in the CR network. And [5] [7] [8] [9] all adopt a Stackelberg game to model the problem. However, a Stackelberg game is called as a leader-follower game, which implies that PUs make decisions first, then SUs. Hence, a Stackelberg game can not let PUs and SUs make decision at the same time. [10] is the first work to apply a coalitional game in CCRNs and [11] is an extended version. The rationale behind this is that coalitional game is more effective to analyze users' cooperative interactions. The other contribution of [11] is that the authors consider *multiple* PUs and *multiple* SUs in the system. Whereas, [5] and [7] only consider one PU and multiple SUs. [12] also considers multiple PUs and multiple SUs with multiple subchannels avail-

able in a sub time slot and allocates optimal fractions of time to sub time slots in order to maximize system utility.

In our approach, we consider multiple PUs and SUs in the CCRN. Under the cooperative scenario, PUs involve SUs as relays to improve transmission rates. In return, SUs are licensed a fraction of time for accessing the spectrum. Our work considers two problems, *i.e.* power control and time allocation. Channel allocation in the scenario is specially designed to ensure that each SU can access multiple channels. We propose a novel algorithm to tackle the problem iteratively in two steps, each step only considering one issue. The rationale behind this is that proposed algorithm is guaranteed convexity by solving the problem in two steps. As a result, the algorithm can achieve the solution in the core, which is a basic solution concept in a coalitional game.

### **1.4** Contributions of the Research

Dealing with power control and time allocation problems in the CCRN, we propose an algorithm to solve the problem iteratively in two steps. First, we allocate fractions of time for sub time slots. Given the time allocation coefficients, we optimize SU's power levels for relaying and accessing. Then, we repeat the two steps iteratively until we achieve the maximum utility. The algorithm can obtain the optimal power levels and time coefficients to achieve the solution in the core. Detailed description is introduced in the following section. On the other hand, special design of channel allocation ensures that each user can access multiple channels in a sub time slot. With the special channel allocation, it is more beneficial for SUs to stay in the CCRN. In the simulations, we compare proposed algorithm with other approaches and show that proposed algorithm converges to the core.

As a final remark, we emphasize that our work considers power control and time allocation two problems. Other works often consider one problem, *e.g.* [7] and [11] only consider time allocation. In the related works, we only see one work [5] considering time allocation and power control problems. However, [5] is modeled as a Stackelberg game. Our work is the first one applying a coalitional game on power control and time allocation problems. We also proof that the considered problem has a nonempty core,

which ensures the stability of the system. In the simulations, we also show that proposed algorithm converges to the core.



# Chapter 2

# **Cognitive Radio and Game Theory Preliminary**

## 2.1 Cognitive Radio

Cognitive radio (CR) was first proposed by Joseph Mitola in 2000's doctoral dissertation [4]. It is a software-defined wireless communication system that is capable of achieving highly reliable communication by adjusting its transmission parameters according to the radio environment it senses. CR is called "cognitive", because its structure supporting a cognition cycle consisting of Observe, Orient, Plan, Decide, and Act phases as shown in Fig. 2.1<sup>1</sup>. The figure has been widely used to understand the cognitive radio or analyze the performance of cognitive networks. Recently, the unbalanced utilization of spectrum urges the need for intelligent spectrum management technique. For realistic implementation, CR is built on software based radio and wide-band RF front end to achieve the functionality. There are some prototypes of CR already built, such as the first prototype CR1 by Mitola [4], and CR and networking by Virginia tech [13].

Although the original purpose of cognitive radio is not utilized to improve the spectrum efficiency, now it is viewed as a novel technique to tackle the problem of spectrum under-utilization. CR can be used to detect the spectrum holes or actively negotiate with

<sup>&</sup>lt;sup>1</sup>This figure is adapted from Mitola, "Cognitive Radio: An Integrated Agent Architecture for Soft-ware Defined Radio", Ph.D. dissertation, Royal Inst. Technol. (KTH), pp. 48, 2000



Figure 2.1: Simplified cognition cycle.

primary users to access the spectrum. In recent years, there are lots of researches on CR-related topics. These researches can be classified into three fundamental tasks [3]:

- 1. Radio-scene analysis, which includes estimation of interference of the radio environment and detection of spectrum holes.
- 2. Channel state information and predictive channel modeling, which encompasses estimation of channel-state information (CSI) and prediction of channel capacity for the use by the transmitter.
- 3. Transmitter power control and dynamic spectrum management.

Our work is based on the transmitter power control and dynamic spectrum management. We apply cooperative CR technique to tackle the problem.

#### 2.1.1 Cooperative Cognitive Radio Network

The idea of cooperative communication was first proposed by [5] in 2008, which introduced a property-right model of cognitive radio, also called as spectrum leasing. PUs are



Figure 2.2: System model for cooperative cognitive radio networks

aware of the existence of SUs and actively lease the spectrum for a fraction of time to SUs by charging at a certain price. In the scenario, PUs can gain additional revenue by spectrum leasing. Motivated by the idea of spectrum leasing, some recent works [5] [7] incorporated cooperative communication into CR networks, which is termed as *cooperative cognitive radio network* (CCRN).

A typical CCRN scenario is shown in Fig.  $2.2^2$ . In Fig. 2.2(a), PUs transmit signals to SUs through primary transmission. Then, in Fig. 2.2(b), both SUs and PUs transmit signals to primary access point (PAP). In this sub time slot, SUs are served as relays to assist primary transmission. In the last sub time slot, SUs can access the spectrum for its own traffic. Hence, under the cooperative scenario, PU's rate can be improved by exploiting cooperative diversity. In return, SUs gain opportunities to access the spectrum for a fraction of time, in which SUs can transmit its own traffic to secondary access point (SAP). Hence, both PUs and SUs can increase their interests in the CCRN scenario, achieving a "win-win" situation.

The scenario of CCRN is a new cognitive radio paradigm. SUs are served as cooperative relays for primary transmission, so PU's transmission rate increases significantly

<sup>&</sup>lt;sup>2</sup>This figure is adapted from Y. Yi, et al., "Cooperative Communication-Aware Spectrum Leasing in Cognitive Radio Networks", in IEEE Proc. DySPAN, pp. 1–11, 2010

by exploiting cooperative diversity. The received SNR terms can be summed up by the technique of maximum ratio combining (MRC). For secondary systems, SUs are licensed a fraction of time to access the spare spectrum. Hence, both PUs and SUs can benefit from the cooperative scenario. However, for some PUs, when the required traffic demands are satisfied, primary systems are not interested to increase their transmission rates. They want to gain some certain benefits, *e.g.* payment, which is more interesting to PUs. Hence, there are many researches discussing about cooperative interactions between PUs and SUs. We adopt a game theoretic approach to formulate the problem.

## 2.2 Game Theory

Game theory is a mathematical tool to predict the result of rational decision makers. Predicting the decisions made by players has great merit in many field, such as card game, gambling, economics, politics, diplomatics, and also wireless communication. Although, sometimes the explicit model is difficult to be defined (*e.g.* politics) or too complex to derive the winning strategy (*e.g.* chess game). Game theory is still a powerful tool to provide a solution to simplified problems. For wireless communication network, applying game theory to predict and further to regulate the players' behavior are anticipated since the increasing complexity of the wireless network results in significant interference and foreseeable dynamics of users in cognitive radio network.

In this section, we introduce some basic knowledge of noncooperative game for understanding game theory more easily, while interested readers can refer to [14] or [15] for detail description.

#### 2.2.1 Basic Definitions of Game Theory

A game in essence is that multiple players and each player possess its own strategy, which it can freely adjust and the objective function, which depends on its and other players' strategy. From a mathematical viewpoint, a game is defined as **Definition 1** A game  $\Gamma$  is

$$\Gamma = \left\langle \mathcal{N}, \ \{A_x\}_{x \in \mathcal{N}}, \ \{u_x\}_{x \in \mathcal{N}} \right\rangle, \tag{2.1}$$

where  $\mathcal{N} \equiv \{1, 2, \dots, N\}$  is the set of players,  $A_x$  is the set of actions available for player x, and we denote the available actions for all players as  $A = A_1 \times A_2 \times \dots \times A_N$ . An action, a.k.a. strategy, taken by player x is  $a_x \in A_x$ , and the action profile of all players is denoted by  $\mathbf{a} = a_1 \times a_2 \times \dots \times a_N \in A$ . For notational simplicity, we denote  $\mathbf{a}_{-x}$  as the action profile taken by all players except player x.  $u_x$  is utility function of player x, which is a function of  $a_x$  and  $\mathbf{a}_{-x}$ .

We introduce some assumptions in game theory. First, each player is rational and selfish, so each player aims to maximize its own utility. We should mind that "selfish" does not mean "malicious". A selfish player cares about its own utility, while a malicious player may harm other players. It is also assumed that all players know the rules of the game. In other words, each player knows all players' action set and utilities, so the action profile is perfectly observed by all players. Actually, the scenario is too ideal due to these assumptions, so other kinds of game models are developed to make the scenario more practical, *e.g.* coalitional game, which we apply in this thesis. A coalitional game is one kind of the cooperative games. The characteristic property of cooperative game is that players may cooperate to maximize their own utilities. The players in a coalitional game cooperate to form cooperative groups, *i.e.* coalitions, which are the basic units in a coalitional game. The details of a coalitional game are introduced in the latter section. We still go on the basics of game theory.

**Definition 2** The best response  $b_x(\mathbf{a}_{-x})$  of player x to the action profile  $\mathbf{a}_{-x}$  is an action  $a_x$  such at:

$$b_x(\mathbf{a}_{-x}) = \arg \max_{a_x \in A_x} u_x(a_x, \mathbf{a}_{-x}).$$
(2.2)

Since best response is the best action for player x, player x would like to stick to it. We know that each player would take the best response, so the result of the game is the action

profile that is the best response for all players, if it exists. This mutual best response point is well-known as Nash Equilibrium (NE), which was found by the John Forbes Nash. NE is an equilibrium point, because every player would stick to it. The formal definition of NE is given as

**Definition 3** The pure strategy profile  $\mathbf{a}^*$  constitutes a Nash Equilibrium (NE), if for each player x:

$$u_x(a_x^*, \mathbf{a}_{-x}^*) \ge u_x(a_x, \mathbf{a}_{-x}^*), \forall a_x \in A_x$$
(2.3)

NE is viewed as a *solution concept, i.e.* the rule how the game will be played, of a static game with complete information. However, the existence of NE needs to satisfy certain conditions and theorems and detailed explanation can be found in [15]. It's notable that there are different solution concepts for different kinds of games, *e.g.* the *core* for a coalitional game.

### 2.2.2 Coalitional Game

In general, game theory can be divided into two branches: noncooperative and cooperative games [16]. The main branch of cooperative games describes the formation of cooperative groups of players, called as coalitions. In the section, we focus on the coalitional game, because we applies a coalitional game to solve the problem in the thesis. A coalitional game focuses on how the players cooperate with each other in the system, in which a coalition is the basic unit. Players in the same coalition have some agreements about forming cooperative group. Hence, the notable issue is how to choose the players to cooperate with. Whereas, the value of a coalition is quantified by the coalition value, which is generated by all the players in the coalition. The players in the system have incentives to join the coalition that increases their own utilities. The assumptions in a coalitional game are different from the basic game described in previous section, so we introduce the coalitional game formulation in the following section.

#### **Game Formulation**

A coalitional game  $\Gamma$  can be formulated as follows

$$\Gamma = \left\langle \mathcal{N}, \{v(\mathcal{S})\}_{\mathcal{S}\subseteq\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}} \right\rangle, \tag{2.4}$$

where  $\mathcal{N} \equiv \{1, 2, \dots, N\}$  is the set of players,  $v(\mathcal{S})$  is the coalition value of coalition  $\mathcal{S}$ , and  $u_i$  is the utility function of player *i*. In a coalitional game, the coalition value is the most important element, which is generated by the players in the coalition. Coalition value can be defined in different forms, *e.g.* rate, power, or payment, according to the game formulation.  $u_i$  defines the utility of player *i* and coalition value is generated by the players' utilities in the coalition. Hence, the coalition value  $v(\mathcal{S})$  is highly related to utility function  $u_i$ . The player's utility received in the coalition is called *payoff*.

In a coalitional game, a coalition is the basic unit and how to divide the players into coalitions is the crucial issue. In this thesis, we consider a special class of coalitional games that all the players would form one coalition, *i.e.* grand coalition. In the grand coalition, all the players will cooperate with each other with certain agreements, so the whole system is stable. Another assumption of the coalitional game that we are concerned is transferrable utility (TU) [17]. TU property implies that the total utility represented as a real number can be divided in any manner between the coalition members. The utility that a player *i* received from the division of v(S) constitutes the player's payoff denoted as  $x_i$ . Whether the payoff allocation is stable or not, we can examine it by the solution concept introduced in the following section.

#### **Solution Concept**

In the coalitional game theory [16] [18], the most renowned solution concept of a coalitional game is the *core*. The relationship between the core and a coalitional game is similar to Nash Equilibrium and a noncooperative game. The core is directly related to the stability of grand coalition. In other words, the existence of the core implies that the whole system is stable. Due to the superadditivity property, players have incentives to form grand coalition  $\mathcal{N}$ , consisting of all players. The definition of superadditivity is as

follows

**Definition 4** The two coalitions have the property of superadditivity if S and Z are disjoint coalitions  $S \cap Z = \emptyset$ , then  $v(S) + v(Z) \leq v(S \cup Z)$ . If two disjoint coalitions satisfy the above equation, they are called superadditive.

Hence, if two coalitions are superadditive, they will merge together to form a new coalition. If we discover that all the coalitions in the system are superadditive, all the players will join to form grand coalition. Also, players can increase their payoffs in the grand coalition, so players have no incentive to leave the grand coalition. In other words, the grand coalition is stabilized. The formal definition of the core is given as

**Definition 5** A payoff vector x is stable in a coalition S if  $\sum_{i \in S} x_i \ge v(S)$ , i.e. the player i has an incentive for the proposed payoff  $x_i$ . The set of stable payoff allocation, i.e. the core is defined as:

$$C = \left\{ x \in R^{|\mathcal{N}|} : \sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}) \text{ and } \sum_{i \in \mathcal{S}} x_i \ge v(\mathcal{S}), \ \forall \mathcal{S} \subseteq \mathcal{N} \right\}.$$
 (2.5)

We can see that from the definition of the core, which needs to satisfy two conditions. The first condition is called as group rational, which the total sum of players' payoffs is equal to the coalition value of grand coalition  $\mathcal{N}$ . The second condition is related to the individually rational. A payoff vector is individually rational if every player can obtain a benefit no less than acting alone, *i.e.*  $x_i \ge v(i)$ ,  $\forall i \in \mathcal{N}$ . Hence, the second condition can be viewed as the sum payoff at least the same with the coalition value  $v(\mathcal{S})$ .

With the definition of the core, we can examine whether the payoff allocation is stable or not. As mentioned above, the core is directly related to the stability of the system. However, the core is not always guaranteed to exist in a coalitional game. Actually, the core set is empty in many coalitional games, so the grand coalition cannot be guaranteed stabilized. In these situations, we may consider alternative solution concepts, but they are not the main topic in this thesis. Interested readers can refer to [16] [18] for detailed description. In the next chapter, we focus on applying a coalitional game in CCRN.

# Chapter 3

# **Coalitional Game in Cooperative Cognitive Radio Networks**

## 3.1 Problem Setup

We consider a CDMA based cooperative cognitive radio network (CCRN) consisting of  $N_p$  PUs and  $N_s$  SUs, where  $N_p = \{1, 2, \dots, N_p\}$  and  $N_s = \{1, 2, \dots, N_s\}$ . It is an uplink transmission scenario, which PUs aim to transmit signals to primary base station (PBS) whereas SUs want to access the spectrum to transmit data to secondary access point (SAP). There are  $N_b$  available channels licensed to PUs in the system, where  $N_b = \{1, 2, \dots, N_b\}$ . There is an example of CCRN with  $N_p = 2$ ,  $N_s = 2$ , and  $N_b = 2$  as shown in Fig. 3.1. The channels are licensed to PUs, so only PUs have the legal rights to use the channels. If SUs want to access the channels, they need to cooperate with PUs or give payment for channels' accessing. We apply the idea of *cooperative communication* in the CCRN. SUs are served as cooperative relays for primary transmission, so PUs' transmission rates are improved by exploiting cooperative diversity. In return, SUs gain the opportunities to access spare channels. SUs are licensed fractions of time to transmit their own traffic. Hence, both PUs and SUs can benefit from the cooperative scenario. By the assistance of SUs, the transmission rates of PUs increase by exploiting the technique of maximum ratio combining (MRC) at the receiver.

For the time scheduling of channels, we consider a time slot set as t, which is divided



Figure 3.1: The CCRN network with  $N_p=2$  and  $N_s=2$ : (a) primary transmission (b) cooperative transmission (c) secondary transmission

into three sub time slots. The first sub time slot used by PUs is set as  $(1-\alpha)t$  and the other sub time slots are set as  $\beta t$  and  $(\alpha-\beta)t$  for SU's relaying and SU's accessing, respectively. For notational simplicity, we normalize theses sub time slots by t, so the fractions of time for sub time slots are  $1 - \alpha$ ,  $\beta$ ,  $\alpha - \beta$ , respectively. For PU's channel allocation, each channel is licensed for one PU, *e.g.* PU 1 can only uses channel 1 for transmission. In the first sub time slot, PUs transmit their traffic to SUs and PBS simultaneously using the broadcast nature of wireless communication. Then, SUs help to relay the traffic received from PUs to PBS in the second sub time slot, which is for SU's relaying. In the last sub time slot, each SU can access multiple channels for its own transmission. The reason why each SU can access multiple channels in a sub time slot is explained in the next section.

In the channels allocation of SUs, we exploit a special design to enable that each SU can access multiple channels in a sub time slot. In the relay mode, each SU selects the channel with the best channel quality to assist PUs' transmission. In other words, each PU's traffic is relayed by SU's best channel to increase capacity. Each SU helps to relay the traffic from PUs in the coalition, so it is reasonable that each SU can access available channels in the coalition. On the other hand, the system assigns unique spreading code to each SU in order to differentiate SUs when accessing the same channel. In the next section, we introduce the coalitional game formulation in the CCRN.

## **3.2** Coalitional Game Formulation

In this section, we describe the general coalitional game formulation. The complete problem formulation for power control is introduced in latter section. We formulate the cooperative cognitive radio network as a coalitional game

$$\Gamma = \left\langle \mathcal{N}, \{v(\mathcal{S})\}_{\mathcal{S}\subseteq\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}, \{x_i\}_{i\in\mathcal{N}} \right\rangle, \tag{3.1}$$

where  $\mathcal{N} \triangleq N_p \bigcup N_s$  is the set of players with  $N_p = \{1, 2, \dots, N_p\}$  being the set of all PUs and  $N_s = \{1, 2, \dots, N_s\}$  being the set of all SUs,  $v(\mathcal{S})$  is the coalitional value of coalition  $\mathcal{S}$  with  $\mathcal{S} \subseteq \mathcal{N}$ ,  $u_i$  is player *i*'s utility function, and  $x_i$  is the payoff of player *i*. Coalition value is generated by the PUs and SUs in the coalition. The player's payoff is characterized by the utility function  $u_i$ , so the last three elements in a coalitional game are mutually dependent.

The formal definition of a coalition in CCRN is given as

**Definition 6** A coalition S is a set of players (e.g. PUs and SUs) that cooperate with each other. For a coalition S, we denote the set of PUs and SUs in the coalition S as  $S_p$  and  $S_s$ , respectively. Let  $S_b$  represent the available channels in the coalition S. The grand coalition is denoted as  $N_p \bigcup N_s$ , consisting of all PUs and SUs.

In the coalition, PUs and SUs can exploit cooperative communication. In this scenario, SUs serve as cooperative relays to assist PUs' transmission, in exchange for opportunities to access the spare channels. Hence, both PUs and SUs can benefit from the cooperative scenario. From the channel allocation aspect, a channel owned by PU m can be used by SU k only if they are in the same coalition. Whereas, a SU can assist a PU's transmission when they stay in the same coalition. In the next section, we introduce the problem formulation considering power control.

### **3.3** Problem Formulation for Power Control Case

#### 3.3.1 Utility Model

In this section, we only consider the problem with power control whereas the time coefficients are set fixed at  $\alpha = 0.5$  and  $\beta = 0.25$ . We assume that no PU uses the same channel with other PUs in a sub time slot, so each PU is assigned only one channel, *e.g.* channel 1 licensed for PU 1. The power control problem that we consider here is to allocate SU's power levels for relaying and accessing. On the other hand, PU's transmit power is set fixed. The noise variance is denoted as  $\sigma^2$ , assuming that all receivers have the same noise level for simplicity. The direct transmission rate of PU *m* can be represented by  $r_m = \log_2 \left(1 + \frac{P_{pm}|h_{pm}|^2}{\sigma^2}\right)$ , where  $P_{pm}$  is the transmit power of PU *m*, and  $h_{pm}$  is the channel gain from PU *m* to PBS. For the relay strategy, we employ decode-and-forward (DF) [19] for cooperative communication in the CCRN. According to the DF strategy, PU's transmission rate is determined by two stages, *i.e.* PU to SUs and SUs to PBS. For SU's relaying, SUs receive the traffic from PUs in the first sub time slot and in the second sub time slot, SUs decode the data and forward to PBS.

The channel gains of link pairs among PUs, SUs, the PBS, and the SAP are modeled as independent and identically distributed (i.i.d.) complex Gaussian random variables. The channels are assumed to be invariant within each sub time slot, but varying over sub time slots. We use the following notation to denote the instantaneous channel gains:  $h_{p_m,s_k}$  denotes the channel gain between PU *m* and SU *k*;  $h_{s_k,p}^{(j)}$  denotes the channel gain between SU *k* and PBS using channel *j*;  $h_{s_k}^{(j)}$  denotes the channel gain between SU *k* and its receiver SAP using channel *j*, for any  $m \in N_p, k \in N_s$ , and  $j \in N_b$ .

Due to consider power control, the transmit power can be adjusted in order to improve system utility. We denote  $P_{s_k,p_m}$  as the power of SU k for relaying PU m's traffic and  $P_{s_k}$ as the power of SU k for accessing. Hence, the power control focuses on how to allocate the SU's power for relaying and accessing. We assume that each SU allocates the same power to relay every PU's traffic. For example, SU 1's power for relaying PU 1's and PU 2's traffic is the same.

Suppose that each SU can assist PUs' transmission using multiple channels in a sub

time slot. In other words, there are multiple channels licensed for each SU in a sub time slot. According to the DF strategy, PU's data rate is determined by PU to SUs and SUs to PBS two stages. In order to detect the signals from PU *m* to SUs, the transmission rate is dominated by the worst channel  $|h_{p_m,s_k}|$  in the subset  $s_k \in S_s$  represented by

$$r_{p_m,s} = \frac{1}{2} \cdot \log_2 \left( 1 + \frac{P_{p_m} \min_{k \in S_s} |h_{p_m,s_k}|^2}{\sigma^2} \right), \ m \in S_p,$$
(3.2)

where  $\frac{1}{2}$  is fraction of time for PU's transmission. As shown in (3.2), the transmission rate from PU *m* to SUs is dominated by the worst channel, so that all the data transmitted by the PU can be decoded correctly. In the second sub time slot, SUs will decode and relay the data received from PUs to PBS. At the PBS, it also decodes the data received from PUs in the first sub time slot and sums up the received SNR with the technique of maximum ratio combining (MRC). At the receiver, the data transmitted from PU and SUs is the same, so that we can sum up the received SNR. Hence, the received SNR is enhanced by the assistance of SUs. The transmission rate from SUs to PBS for assisting PU *m* is represented by

$$r_{s,p_m} = \frac{1}{4} \cdot \log_2 \left( 1 + \frac{P_{p_m} |h_{p_m}|^2}{\sigma^2} + \sum_{k \in S_s} \frac{P_{s_k, p_m} \max_{j \in S_b} |h_{s_k, p}^{(j)}|^2}{\sigma^2} \right), \quad (3.3)$$

where  $\frac{1}{4}$  is the fraction of time for SU's relaying. In (3.3), the first term is obtained by PU *m*'s transmission to PBS through direct link, and the second term is the sum SNR achieved by each SU's best channel to relay PU *m*'s traffic. The SNR terms are summed up with the technique of MRC. Therefore, PU's transmission rate is improved by the assistance of SUs.

Hence, according to DF cooperative strategy, the overall transmission rate of PU m can be achieved by

$$r_{p_m} = \min\{r_{p_m,s}, r_{s,p_m}\}, \ m \in S_p.$$
(3.4)

In order to decode the data from a PU, the transmission rate is dominated by the minimum rate in the two stages. Under the cooperative scenario, each SU helps to relay the traffic from PUs in the coalition, so each SU gains the opportunities to access spare

channels in the coalition. Based on CDMA network, the system assigns each SU an unique spreading code. Then, the total access rate of SU k can be represented by

$$r_{s_k} = \frac{1}{4} \cdot \sum_{j \in S_b} \log_2\left(1 + \frac{P_{s_k} |h_{s_k}^{(j)}|^2}{\sigma^2}\right), \ k \in S_s,\tag{3.5}$$

where  $\frac{1}{4}$  is the fraction of time for SU's accessing. Each SU is licensed for multiple channels, and it can transmit different signals on the each channel. Hence, from (3.5), we can see that SU's access rate is achieved by the sum rate of multiple channels. If a SU does not join any coalition, they cannot access any channel. Hence, a SU is beneficial to join the coalition for accessing multiple channels. Also, this is a novel design for SUs to access multiple channels in the CCRN. Therefore, the total relaying energy consumption of SU *k* is

$$\zeta_k = c_s \cdot \frac{1}{4} \cdot \sum_{m \in S_p} P_{s_k, p_m}, \ k \in S_s, \tag{3.6}$$

where  $\frac{1}{4}$  is the fraction of time for SU's relaying and  $c_s$  is SU's cost per relaying energy; the summation of power levels for relaying PUs' traffic. We assume that the power of each SU for assisting every PU is the same. Due to consider power control, it is reasonable to define SU's cost as relaying energy consumption.

To summarize player's payoff allocated in the coalition, PU *m* generates data rate gain of  $F(r_{p_m})$ , where  $F(\cdot)$  is a concave increasing function. For example, utility function can be linear, *i.e.*  $F(r_{p_m}) = r_{p_m}$ . Payoff is the player's utility received in the coalition and it is a division of the coalition value v(S). Therefore, PU *m*'s payoff in the coalition is represented as

 $F(r_{p_m})$ 

On the other hand, SU k can be evaluated by utility function  $G(r_{s_k})$ , where  $G(\cdot)$  is a concave increasing function. Hence, SU k's payoff in the coalition is represented as

$$G(r_{s_k}) - \zeta_k$$

The utility function ensures the concavity of player's payoff. In the optimization problem, the problem's concave property is notable. The concave property of the utility function can help us to achieve the solution concept, which is introduced detailed in the following section. After defining the utility model, we continue to formulate the power control problem in the CCRN.

#### **3.3.2** Optimization Problem Formulation

In this section, we model the power control problem as a transferable utility (TU) game. The TU property implies that the total utility specified by a real number can be divided in any manner between the coalition members. The TU game is fully defined once the set of coalitions and the coalition value  $v(\cdot)$  are specified. In the TU game, coalition value is a real number that can be distributed arbitrarily [20]. We define v(S) in the CCRN as the maximum utility generated by PUs and SUs in the coalition S. The other assumption is that the coalition value does not depend on the actions of the PUs or SUs outside the coalition.

According to the utility model and the power constraints on SUs, the coalitional game in the CCRN can be formulated as follows: 1896

$$v(\mathcal{S}) \triangleq \mathbf{Maximize} : \sum_{m \in S_p} F(r_{p_m}) + \sum_{k \in S_s} \left( G(r_{s_k}) - \zeta_k \right)$$

Subject to :

(3.7)

(2)  $P_{s_k,p_m}, P_{s_k} \ge 0, m \in S_p, s \in S_s.$ 

(1)  $P_{s_k,p_m} + P_{s_k} \le P_{max}, \ m \in S_p, \ k \in S_s,$ 

The objective function is the coalition value v(S), which is the summation of all players' payoffs in the coalition. Constraint (1) sets SU's power level no more than the upper bound  $P_{max}$ . We can see that SU's transmit power is the summation of relaying power and accessing power. Note that each SU's power for assisting every PU is the same. Constraint (2) ensures that the power levels are inside feasible region. In (3.7),  $F(\cdot)$  and  $G(\cdot)$  are concave functions, so it is a concave problem. Then, we want to achieve maximum in a concave problem. However, how to solve the problem formulation with the core is a crucial issue. Hence, we introduce the solution concept detailed in the next section.

#### 3.3.3 The Core

The power control problem satisfies the *time-sharing* condition in [21]. In our problem, the time coefficients are set fixed. Whereas, the objective function in (3.7) is concave with respect to  $P_{s_k,p_m}$  and  $P_{s_k}$  and the constraints are linear, which satisfies the time-sharing condition's two assumptions. As proven in [21], the problem's solution guarantees to exist and the duality gap is zero, which means that the problem can be solved in dual domain and achieve the same optimal value as in prime domain. This property is very effective to achieve the equilibrium point. Hence, we introduce a solution concept in a coalitional game known as the *core*. The core is the most renowned solution concept in coalitional games. The core implies that the payoff allocation is incentive for all the players in the system, so the grand coalition is stable. In other words, if a subset of PUs or SUs separates from the grand coalition, at least one player's payoff is worse off. The formal definition of the core is given as

**Definition 7** The core of a coalitional game in the CCRN is the set of feasible payoff allocation

$$C = \left\{ x \in R^{|N_p| + |N_s|} : \sum_{m \in N_p} x_{p_m} + \sum_{k \in N_s} x_{s_k} = v(N_p \bigcup N_s), \\ \sum_{m \in S_p} x_{p_m} + \sum_{k \in S_s} x_{s_k} \ge v(\mathcal{S}), \ \forall \mathcal{S} \subseteq (N_p \bigcup N_s) \right\},$$
(3.8)

where  $x_{p_m}$  and  $x_{s_k}$  are PU *m*'s payoff and SU *k*'s payoff in the coalition, respectively and  $|\cdot|$  denotes the cardinality of a set, *e.g.*  $|N_p|$  means the number of elements in  $N_p$ . The definition of the core in CCRN is similar to the general form in **Definition 5**. Nevertheless, we adjust the definition to fit the power control problem. The first condition in the core means that the total payoff equals to the coalition value of grand coalition. Then, the second condition represents that sum payoff received in grand coalition is at least the same with the coalition value v(S). This means that the players in the grand coalition

benefit most, so players have incentives to stay in the grand coalition. The definition also implies no player's payoff in a coalition  $S \subseteq (N_p \bigcup N_s)$  to make  $y_{p_m} > x_{p_m}$  for all  $m \in S_p$  or  $y_{s_k} > x_{s_k}$  for all  $k \in S_s$ . In other words, no subgroup of players will separate from the grand coalition. Hence, the core ensures the stability of the grand coalition. We have discussed the problem with power control, but the time coefficients are set fixed in this case. However, time allocation for players is significant to improve the system utility. Therefore, we introduce the problem considering power control and time allocation in the following section.

# 3.4 Problem Formulation for Power Control and Time Allocation Case

#### 3.4.1 Utility Model

In this section, we consider power control and time allocation problems. The problem setup is the same with the problem of power control case. A PU is licensed for only one channel, *e.g.* channel 1 for PU 1. With the special design of channel allocation, each SU is licensed for multiple channels for accessing. The noise variance is denoted as  $\sigma^2$ , assuming that all receivers have the same noise level. The power control is to allocate SU's power levels for relaying and accessing. On the other hand, the time allocation problem is to allocate the fractions of time for PU's transmission, SU's relaying, and SU's accessing. Whereas, the relay strategy is DF and channel gain's notation is the same with that in the power control case.

Applying the second derivative test [22] with respect to power levels and time coefficients, we discover that the second derivative test's result is zero, so the problem's optimal solution is not guaranteed to exist. Due to the result of the test, we cannot solve the problem directly, so we propose a novel algorithm to tackle the problem iteratively in two steps. In the first step, we allocate the fraction of time to each sub time slot. Then, we only consider power control in the second step to maximize the system utility with the allocated time coefficients. After solving the problem iteratively in two steps, the problem is guaranteed to achieve the equilibrium in the core. Therefore, we can solve the problem formulation with the proposed algorithm.

In the power control and time allocation problem, we modify the utility models as follows. The transmission rate from PU m to SUs is dominant by the worst channel  $|h_{p_m,s_k}|$  represented as

$$r_{p_m,s} = (1 - \alpha) \cdot \log_2 \left( 1 + \frac{P_{p_m} \min_{k \in S_s} |h_{p_m,s_k}|^2}{\sigma^2} \right), \ m \in S_p,$$
(3.9)

where  $(1 - \alpha)$  the fraction of time for PU's transmission. In the first sub time slot, PUs transmit to SUs and PBS using broadcast nature of wireless communication, so in order to detect the signals correctly, the transmission rate from PU *m* to SUs is dominated by the worst channel. Then, in the second sub time slot, SUs relay the traffic received from PUs to PBS. With the special design of channel allocation, each SU can access multiple channels in the coaltiion, so the received SNR is enhanced. The transmission rate from SUs to PBS for assisting PU *m* is achieved by

$$r_{s,p_m} = \beta \cdot \log_2 \left( 1 + \frac{P_{p_m} |h_{p_m}|^2}{\sigma^2} + \sum_{k \in S_s} \frac{P_{s_k,p_m} \max_{j \in S_b} |h_{s_k,p}^{(j)}|^2}{\sigma^2} \right),$$
(3.10)

where  $\beta$  is the fraction of time for SU's relaying. In (3.10), the first term is received from PU *m* to PBS through direct link and the second term is the sum SNR achieved by each SU's best channel to relay PU *m*'s traffic. These SNR terms are summed up with the technique of MRC. Hence, PU's rate is improved by the assistance of SUs in the coalition.

Hence, the overall transmission rate of PU m is achieved by

$$r_{p_m} = \min\{r_{p_m,s}, r_{s,p_m}\}, \ m \in S_p.$$
(3.11)

This equation is the same as (3.4) according to the DF strategy. In the cooperative scenario, each SU relays the traffic from PUs in the coalition, so each SU has opportunities to access available channels in the coalition. Therefore, the total access rate of SU *k* can be represented as

$$r_{s_k} = (\alpha - \beta) \cdot \sum_{j \in S_b} \log_2\left(1 + \frac{P_{s_k} |h_{s_k}^{(j)}|^2}{\sigma^2}\right), \ k \in S_s,$$
(3.12)

where  $(\alpha - \beta)$  is the fraction of time for SU's accessing and  $h_{s_k}^{(j)}$  is the channel gain of SU k using channel j for its own traffic. Each SU has opportunities to access multiple channels and it transmits different signals on each channel. Hence, the total access rate of SU k is the sum rate achieved by multiple channels. Then, the total cost of SU k is represented as

$$\zeta_k = c_s \cdot \beta \cdot \sum_{m \in S_p} P_{s_k, p_m} + w_s \cdot (\alpha - \beta), \ k \in S_s,$$
(3.13)

where  $c_s$  is SU's cost per relaying energy and  $w_s$  is SU's cost per access time;  $(\alpha - \beta)$  is SU's access time. We assume that the power of each SU for assisting every PU is the same. The first term in (3.13) is SU's relaying energy consumption and the second term is SU's access time cost. The second term can be viewed as SU's payment for spectrum accessing. The rationale behind SU's cost is that we consider power control and time allocation problems. To explain in detail, the energy consumption is the cost for power control and the access time cost is due to time allocation.

To sum up, PU *m*'s payoff in the coalition is  $F(r_{pm})$  and SU *k*'s payoff is represented as  $G(r_{s_k}) - \zeta_k$ , where  $F(\cdot)$  and  $G(\cdot)$  are concave increasing functions. The utility function design ensures the payoff's concavity, which can help us to achieve the equilibrium in the core. The optimization problem formulation for power control and time allocation is introduced in the following section.

#### **3.4.2** Optimization Problem Formulation

We model the CCRN considering power control and time allocation as a TU game. The problem formulation is similar to the power control case, but we also consider time allocation here. As the TU game's definition in [20], the coalition value is a real number, which can be divided in any manner between coalition members. We define coalition value v(S) as the maximum utility achieved by PUs and SUs in the coalition.

According to the modified utility model and constraints on power control and time allocation, the coalitional game can be formulated as:

$$v(S) \triangleq \text{Maximize} : \sum_{m \in S_p} F(r_{p_m}) + \sum_{k \in S_s} (G(r_{s_k}) - \zeta_k)$$
  
Subject to :  
(1)  $0 \le \beta \le \alpha \le 1$ ,  
(2)  $P_{s_k, p_m} + P_{s_k} \le P_{max}, \ m \in S_p, \ k \in S_s,$   
(3)  $P_{s_k, p_m}, \ P_{s_k} \ge 0, \ m \in S_p, \ s \in S_s.$   
(3)

The objective function of the problem formulation is the total payoff of all the players in the coalition according to the definition of v(S). For time coefficients,  $\alpha$  is the parameter to adjust the fraction of time for PUs' transmission. Then,  $\beta$  is the parameter to allocate the fraction of time for SU's relaying. In constraint (1), we confine the  $\alpha$  and  $\beta$ within 0 and 1 to ensure feasibility. The reason why  $\alpha$  is no less than  $\beta$  is to ensure SU's access time nonnegative. Constrain (2) sets the upper bound  $P_{max}$  to SU's power control and constraint (3) ensures the power levels are feasible. We assume that the power of each SU for assisting every PU is the same. From the problem formulation in (3.14), we can see that it is a problem considering power control and time allocation. However, according to the second derivative test, the problem is not guaranteed to achieve an optimal solution. In other words, we need to search for alternative methods to solve the problem. In the next section, we propose a novel algorithm to solve the problem, which guarantees to converge to the core.

#### 3.4.3 Algorithm for Solving Problem Formulation

The problem considering power control and time allocation cannot be solved directly, so we propose a novel algorithm to tackle the problem. The proposed algorithm solves the problem iteratively in two steps summarized in **Algorithm 1**.

The proposed algorithm solves the problem iteratively in two steps. At each iteration, the algorithm conducts two steps. In the first step, we allocate values on fractions of time  $\alpha$  and  $\beta$ . Then, with the allocated time coefficients, SU's relaying and access power is

Algorithm 1 Iterative algorithm for power control and time allocation

- 0: **Define**: Objective function in (3.14) at iteration *n* is denoted by  $v(\mathcal{S})^{(n)}$ . Maximum at iteration *n* is denoted by  $\Omega^{(n)}$ . 1: Initialize: Initialize  $\alpha^{(1)}$ ,  $\beta^{(1)}$ , and  $\Omega^{(1)}$ . 2: **Repeat**: Initialize  $P_{s_k,p_m}$  and  $P_{s_k}$ Condition 1: Check whether  $\alpha^{(n)}$ , and  $\beta^{(n)}$  are feasible. 3: **Repeat**: Optimize  $P_{s_k,p_m}, P_{s_k}$  to achieve  $v(\mathcal{S})^{(n)}$ 4: Condition 2: Check whether  $P_{s_k,p_m}$ , and  $P_{s_k}$  are feasible. 5: 6: **End repeat** If  $v(\mathcal{S})^{(n)} \ge \Omega^{(n-1)}$ 7:  $\Omega^{(n)} = v(\mathcal{S})^{(n)},$ Else 1896  $\Omega^{(n)} = \Omega^{(n-1)},$ 8: End if Update  $\alpha^{(n+1)} = \alpha^{(n)} + q^{(n)} (v(\mathcal{S})^{(n)} - \Omega^{(n-1)}),$ 9:  $\beta^{(n+1)} = \beta^{(n)} + t^{(n)} \left( v(\mathcal{S})^{(n)} - \Omega^{(n-1)} \right).$ 10: End repeat until  $\Omega$  converges
- 11: We obtain the optimal system utility  $\Omega^*$ , and the parameters

 $\alpha^*,\beta^*,P^*_{s_k,p_m}\text{, and }P^*_{s_k}\text{.}$ 

determined in the second step. The two steps are performed iteratively until the optimal solution is achieved. On the other hand, we update the values for  $\alpha$  and  $\beta$  with the subgradient method. The main purpose of updating time coefficients is to achieve the optimal point. As explained in the following section, a subgradient method is guaranteed to converge to the optimal point. Hence, the fractions of time  $\alpha$  and  $\beta$  are updated as follows:

$$\alpha^{(n+1)} = \alpha^{(n)} + q^{(n)} \left( v(\mathcal{S})^{(n)} - \Omega^{(n-1)} \right), \qquad (3.15)$$

$$\beta^{(n+1)} = \beta^{(n)} + t^{(n)} \left( v(\mathcal{S})^{(n)} - \Omega^{(n-1)} \right), \qquad (3.16)$$

where *n* is the iteration number and  $v(S)^{(n)}$  is the coalition value obtained at iteration *n*;  $\Omega^{(n-1)}$  is the maximum at the iteration *n*-1. The above subgradient update method is guaranteed to converge to the optimal  $\alpha^*$  and  $\beta^*$  as long as  $q^{(n)}$  and  $t^{(n)}$  are chosen to be sufficiently small [23]. When the norm of the subgradient is bounded, the choices that  $q^{(n)} = \mu/n$  and  $t^{(n)} = \delta/n$  for some constants  $\mu$  and  $\delta$  are guaranteed to converge to the optimal  $\Omega^*$ .

The reason why our proposed algorithm can solve the problem is that the algorithm solves the problem iteratively in two steps. In the first step, we allocate time coefficients, so the problem can be viewed as a linear combination of time coefficients without considering power control. Then, in the second step, we can solve the power optimization problem with the time coefficients allocated in the first step. The power optimization problem guarantees to be solved due to the time-sharing condition [21], which is introduced detailed in the next section. The two steps avoid the multiplicative terms of time and power parameters, so the problem is guaranteed to be solved in each iteration. As a result, the proposed algorithm converges to the core. In the next section, we proof that our proposed game has a nonempty core.

The complexity of proposed algorithm depends on two factors, *i.e.* no. of iterations denoted as  $|A_N|$  and the complexity of power optimization. At each iteration, we conduct power optimization to allocate SU's power levels for relaying and accessing. We denote the power optimization's complexity as  $\Theta_p$ , so the overall complexity of the algorithm can be represented as  $(|A_N| * \Theta_p)$ . In the next section, we show that the proposed algorithm

converges to the core.

## **3.5** Algorithm Converges to the Core

The problem formulation in (3.14) is not convex. That is, the problem cannot be guaranteed to achieve the optimal solution. Hence, we proposed the **Algorithm 1** to tackle the problem iteratively in two steps. In the first step, we only consider the problem of allocating fractions of time for  $\alpha$  and  $\beta$ . Then, in the second step, we substitute these allocated  $\alpha$ and  $\beta$  values into (3.14) to find SUs' optimal cooperative and access power. Since the objective function of the problem formulation in each step is concave, constraints are linear, so the duality gap is zero [21]. In other words, the problem can be solved in dual domain and achieve the same optimal value as in the prime domain. Therefore, the algorithm converges to the optimal time coefficients and power levels.

Since the algorithm is guaranteed to converge, we proof that the game has a nonempty core in the following. Firstly, we define the Lagrangian dual functions as

$$\begin{split} f_m(\pi) &= \max_{\substack{r_{p_m} \ge 0, \ m \in S_p}} \left( F(r_{p_m}) + \pi_m r_{p_m} \right) \\ g_k(\gamma, \tau) &= \max_{\substack{r_{s_k} \ge 0, \ \zeta_k \ge 0, \ k \in S_s}} \left( \frac{\mathbf{896}}{G(r_{s_k}) - \zeta_k} + \gamma_k r_{s_k} + \tau_k \zeta_k \right) \end{split}$$

Then, the dual problem is formulated as follows:

**Minimize** : 
$$\sum_{m \in S_p} \left( f_m(\pi) + \lambda_m \right) + \sum_{k \in S_s} \left( g_k(\gamma, \tau) + \sum_{j \in S_b} \eta_j \right)$$

Subject to :

$$\gamma_k, \ \tau_k \ge 0, \ k \in S_s,$$

$$\eta_j \ge 0, \ j \in S_b,$$

$$\pi_m, \ \lambda_m \ge 0, \ m \in S_p.$$
(3.17)

We formulate the dual problem by appropriately defining vectors  $\pi$ ,  $\lambda$ ,  $\gamma$ ,  $\tau$ , and  $\eta$ . Denote  $\mathcal{D}$  as the set of optimal solutions of the dual problem. Then, we define the core in the dual problem as

$$\begin{split} \mathcal{O} = & \Big\{ \mathbf{x}^* \in R^{|N_p| + |N_s|} : \\ & x_{p_m}^* = f_m(\pi^*) + \lambda_m^*, \ m \in N_p, \\ & x_{s_k}^* = g_k(\gamma^*, \tau^*) + \sum_{j \in S_b} \eta_j^*, \ k \in N_s, \\ & \text{for some } (\pi^*, \lambda^*, \gamma^*, \tau^*, \eta^*) \in \mathcal{D} \Big\}. \end{split}$$

Once  $\mathcal{O}$  has been constructed in dual problem, the core of the coalitional game in prime domain is nonempty. And,  $\mathcal{O} \subseteq \mathcal{C}$ .

*Proof*: Since the set  $\mathcal{O}$  is the subset of the set  $\mathcal{D}$ , and the set  $\mathcal{D}$  is nonempty. Thus, the set  $\mathcal{O}$  is nonempty. We show that for an arbitrary  $\mathbf{x}^* \in \mathcal{O}$ , then  $\mathbf{x}^* \in \mathcal{C}$ . We consider an arbitrary  $\mathbf{x}^* \in \mathcal{O}$  corresponding to  $(\pi^*, \lambda^*, \gamma^*, \tau^*, \eta^*) \in \mathcal{D}$ . Then,  $\sum_{m \in N_p} x_{p_m}^* + \sum_{k \in N_s} x_{s_k}^*$  is the optimal value of the objective function of the dual problem. Due to that  $F(\cdot)$  and  $G(\cdot)$  are concave functions, thus the objective function of the prime problem is a concave function in each step of the algorithm's iteration. Also, the constraints of the prime problem are linear. Then, the duality gap is zero as proven in [21]. Thus, we conclude that  $\sum_{m \in N_p} x_{p_m}^* + \sum_{k \in N_s} x_{s_k}^* = v(N_p \bigcup N_s)$ . According to the definition of the core, we only need to show that  $\sum_{m \in S_p} x_{p_m} + \sum_{k \in S_s} x_{s_k} \ge v(S)$  for each  $S \in (N_p \bigcup N_s)$ .

By strong duality, v(S) equals to the optimal value of the objective function of the dual problem. The sub vectors  $(\pi_S, \lambda_S, \gamma_S, \tau_S, \eta_S)$  consisting of the components of  $(\pi^*, \lambda^*, \gamma^*, \tau^*, \eta^*)$ in S. Then,  $\sum_{m \in S_p} x_{p_m}^* + \sum_{k \in S_s} x_{s_k}^*$  is the value of the objective function of the dual problem with such variables. The optimal value of the objective function of the dual problem is a lower bound of  $\sum_{m \in S_p} x_{p_m}^* + \sum_{k \in S_s} x_{s_k}^*$ . Thus, we can conclude that  $\mathbf{x}^* \in C$ .

# Chapter 4

# Simulations

## 4.1 Simulation Setup

The problem formulation in Section 3.3 and Section 3.4 are simulated in this section. In the first section, we consider the power control case. Then, in the second section, we consider the power control and time allocation case and solve the problem with the proposed algorithm. In the third section, we compare the proposed algorithm with other approaches and show that proposed algorithm converges to the core in the last section.

We consider the scenario with  $N_p = 3$ ,  $N_s = 4$ , and  $N_b = 3$ . The noise variances at the receivers are the same and set to  $\sigma^2 = 10^{-2}$ . The channel gain between any two nodes is modeled as an i.i.d. complex Gaussian with CN(0,1) distribution. Let the utility functions be linear, *i.e.*  $F(r_{p_m}) = 10 \cdot r_{p_m}$  and  $G(r_{s_k}) = u_s \cdot r_{s_k}$ , where  $u_s$  is SU's access rate gain factor. The SU's cost per relaying energy is set as  $c_s = 8$  and SU's cost per accessing time is set as  $w_s = 8$ . The total power constraint for SUs is set as  $P_{max} = 2$ . The updating sizes of time coefficients are set as  $\mu = 0.009$  and  $\delta = 0.003$ . The optimization problem is simulated with the *cvx* tool [22]. Note that some parameters may change and will be specified in each simulation scenario.

## 4.2 Numerical Results

#### 4.2.1 **Power Control Case**

In this section, we simulate the coalitional game in the CCRN considering power control. The access rate gain parameter is set as  $u_s = 8$ . In the power control case, the time coefficients are set fixed at  $\alpha = 0.5$  and  $\beta = 0.25$  as defined in Section. 3.3, so the optimization problem only allocates SU's power levels for relaying and accessing. We show that the player's payoff in the grand coalition in Fig. 4.1. On the other hand, we also adjust the value of SU's access rate gain factor  $u_s$  to see its influence on the player's payoff in Fig. 4.3.

In Fig. 4.1, we can see the payoffs of PUs and SUs in the power control case. Fig. 4.1(a) shows that the payoff of PU 1 is the most beneficial among PUs due to the channel condition is better. For PUs, without the cooperation with SUs, the payoffs of direct transmission of PU 1, PU 2, and PU 3 are 14.28, 12.87, and 11.44, respectively; after joining the grand coalition, the payoffs increase to 23.78, 18.00, 20.44, respectively. Thus, we can see clearly that it is beneficial for PUs to join the grand coalition. For SUs, before joining any coalition, the payoffs of SUs are zero; after joining the grand coalition, the payoffs of SUs are zero; after joining the grand coalition, the payoffs of SUs are zero; after joining the grand coalition, the payoff of SU 1, SU 2, SU 3, and SU 4 increase to 20.77, 16.62, 18.07, 23.57, respectively. Therefore, it is beneficial for all the PUs and SUs to form grand coalition. The payoffs obtained from grand coalition lie in the core.

In Fig. 4.2, with total power constraint  $P_{max} = 2$ , we can see the optimal power allocation. For SUs, SU 3 allocates more power on relaying and less power on accessing. This results in the cost of SU 3 are the highest as shown in Fig. 4.1. The power allocation is influential to SU's payoff, so how to allocate the power levels to maximize the system utility is an important issue.

Fig. 4.3 show that we adjust the access rate gain factor  $u_s$  to see its influence on the data rate gain. As the  $u_s$  increases, the total data rate gain of SUs also increases. On the other hand, the total data rate gain of PUs decreases, because SUs allocate less power on relaying PUs' traffic. With the increasing of  $u_s$ , SUs allocate more power on accessing, so that the system can achieve better system utility. Whereas, SUs allocate less power on



Figure 4.1: The performance of CCRN in power control case,  $u_s = 8$ : (a) The payoff of PUs (b) The payoff of SUs.



Figure 4.2: SU's power allocation in power control case.



Figure 4.3: The impact of access gain factor  $(u_s)$  in power control case.



Figure 4.4: The performance of CCRN in power control case,  $u_s = 12$ : (a) The payoff of PUs (b) The payoff of SUs.

relaying, so the total data rate gain of PUs decreases. At  $u_s = 8$ , the data rate gain of SUs is higher than that of PUs, so increasing of  $u_s$  is more beneficial to SUs.

In the Fig. 4.4, we set the access rate gain factor as  $u_s = 12$ . Compare the result with the Fig. 4.1 with  $u_s = 8$ . PU's payoff in this figure is lower than that in the  $u_s = 8$  figure. The rationale behind is that SUs allocate less power on relaying, so the PUs' transmission rates decrease. For SUs, the payoffs of SU 1, SU 2, SU 3, and SU 4 are 31.30, 25.08, 25.36, and 35.51, respectively, which are about 1.5 times than those in the  $u_s = 8$  figure. There are two reasons behind this result. First, the increase of access rate gain factor is the most influential. Second, SUs allocate more power on accessing in this figure. With these two reasons, the payoffs of SUs increase dramatically in the CCRN.

#### 4.2.2 Power Control and Time Allocation Case

In this section, we simulate the problem formulation in (3.14) considering power control and time allocation. The payoff allocation is achieved by the proposed algorithm. In Fig. 4.5, we show the player's payoff obtained from grand coalition. The payoff allocation achieved by the proposed algorithm lies in the core.

Fig. 4.5 shows players' payoffs obtained from grand coalition. Comparing with Fig. 4.1, PU's payoff is lower in this figure. The main reason is that the fractions of time allocated for PU's transmission and SU's relaying decrease. The optimal time coefficients are  $\alpha = 0.642$  and  $\beta = 0.217$ ; the time coefficients in Fig. 4.1 are set as  $\alpha = 0.5$  and  $\beta = 0.25$ . On the other hand, players can adjust the time coefficients to maximize the system utility in this case. SU's data rate gain is higher than that in the power control case, because SUs allocate more fraction of time on accessing. Also, SU 3's cost is 4.29, the highest among SUs due to allocate more power on relaying. The payoff allocation is achieved by the proposed algorithm. Therefore, the figure shows the stable payoff allocation obtained by the proposed approach.

Fig. 4.6 shows the impact of  $u_s$  in the power control and time allocation case. We set the time coefficients are fixed at the optimal point with  $\alpha = 0.642$  and  $\beta = 0.217$ . Comparing with Fig. 4.3, total data rate gain of all SUs is higher in this case, *e.g.* at  $u_s = 6$ , increase of 70.86% comparing with the power control case. The main reason is that the



Figure 4.5: The performance of CCRN in power control and time allocation case,  $u_s = 8$ : (a) The payoff of PUs (b) The payoff of SUs.



Figure 4.6: The impact of access gain factor  $(u_s)$  in power control and time allocation case with  $\alpha = 0.642$ ,  $\beta = 0.217$ .



Figure 4.7: SU's power allocation in power control and time allocation case.

fraction of time allocated for SU's accessing increases. The total data rate gain of all SUs is higher than that of all PUs at  $u_s = 4$ , whereas in power control case, the  $u_s$  value is 8. Therefore, SUs can obtain higher payoffs in this case, which is more beneficial to them.

In Fig. 4.7, we can see that the SU's power allocation in the power control and time allocation case. The time coefficients are set the same with the Fig. 4.5. Each SU allocates most of the power on accessing and less power on relaying. SU 3 allocates more power on relaying, so its cost is the highest as shown in Fig. 4.5. Therefore, SU's power allocation effects SU's cost dramatically. Then, the system utility can be improved significantly by the power control.

In the Fig. 4.8, we can see the impact of fraction of time  $\alpha$  with  $\beta$  fixed at a specific value. As  $\alpha$  increases, the sum payoff also rises up. The reason is that with  $\alpha$  increasing, the fraction of time allocated for SU's accessing increases. Therefore, SU's payoff increases, so that the sum payoff rises up. However, this does not imply that the payoff allocation is stable. For example, when  $\beta$  is fixed at 0.2 and  $\alpha$  is 0.6, the payoff of each player is better than acting alone, so the proposed payoffs are incentive for players. However, when  $\alpha = 0.7$ , the payoffs of PU 1, PU 2, and PU 3 are 14.27, 13.11, and 12.26, respectively; while acting alone, their payoffs are 14.28, 12.87, and 11.44, respectively. PU 1's payoff is worse than acting alone. Hence, PU 1 has no incentive to join the grand coalition, so the payoff allocation is not stable. Therefore, proposed algorithm updates the time coefficients  $\alpha$  and  $\beta$  to achieve an equilibrium in the core, which not only considers the sum payoff, but also let players benefit most in the grand coalition.

Fig. 4.9 shows that the impact of fraction of time  $\alpha$  with  $\beta$  variation. The  $\beta$  value changes with  $\alpha$  value and the difference between them is fixed. Clearly, the maximum sum payoff of different curves occur at different time points. For example, on the curve of  $\beta = \alpha - 0.4$ , the maximum 161.06 occurs at  $\alpha = 0.6$  and  $\beta = 0.2$ . Except achieving the maximum sum payoff, we also need to examine whether the payoff allocation is stable. At  $\alpha = 0.7$  and  $\beta = 0.3$ , PU 1's payoff is 14.27, whereas its payoff is 14.28 when acting alone. Thus, PU 1's payoff is worse off than acting alone. Hence, PU 1 has no incentive for the proposed payoff, so the equilibrium will not occur at this time point. Therefore, we can know more clear about how to achieve the equilibrium from this figure.



Figure 4.8: The impact of fraction of time  $\alpha$  with  $\beta$  fixed.



Figure 4.9: The impact of fraction of time  $\alpha$  with  $\beta$  variation.

#### 4.2.3 Comparison between Different Approaches

In the Fig. 4.10 with  $u_s = 8$ , we compare the performance between proposed approach and other approaches, which are specified as follows:

- 1. **No cooperation approach**: each PU only uses direct link to transmit signals to PBS without SU's help. Hence, SUs have no opportunity to access the channels.
- 2. Equal Power approach: there is no power control in the scenario, both PUs and SUs transmit with fixed power level.
- 3. **Proposed algorithm**: proposed algorithm based on coalitional game with considering power control and time allocation problems.

Fig. 4.10 shows performance in different approaches. First, we can see that SUs' payoffs are zero in the no cooperation approach. This is due to no channel for SU's accessing in this approach. Therefore, each PU only uses direct link to transmit signals to PBS without SU's assistance. The payoffs of PU 1, PU 2, and PU 3 in this approach are 14.28, 12.87, and 11.44, respectively. On the other hand, players can obtain higher payoffs in the equal power approach. Each player's payoff in the equal power approach is better than acting alone, *i.e.* no cooperation approach. Hence, the payoff allocation in the equal power approach is incentive for players, which implies that players have incentives to join the coalition.

Clearly, each SU's payoff is improved significantly in the proposed algorithm. The rationale behind this is that SUs allocate more power on accessing, so SUs' payoffs increase dramatically. However, PU 2's payoff is 18.63 in the equal power approach better than 14.67 in the proposed algorithm. The reason is that PU 2's rate is determined by the SUs to PBS part, while proposed algorithm allocates less power on SU's relaying. Hence, PU 2's rate is reduced in the proposed algorithm. Nevertheless, the coalition value is improved significantly by the proposed algorithm, so players are more incentive for the proposed algorithm. The coalition value is 170.30, whereas in the equal power approach, the coalition value is 122.49. Therefore, the payoff allocation of the proposed algorithm is stable.



Figure 4.10: Comparison the performance between different approaches: (a) The payoff of PUs (b) The payoff of SUs.



Figure 4.11: Comparison the sum payoff between different approaches.

Fig. 4.11 shows the comparison of the sum payoff between different approaches, where No. 1 is the no cooperation approach and No. 2 is the equal power approach; No. 3 is the proposed algorithm. The sum payoff of No. 1, No. 2, and No. 3 are 38.59, 122.49, and 170.30, respectively. The sum payoff is the lowest in the no cooperation approach among approaches, because each PU only uses direct link to transmit signals to PBS without SUs' help. Whereas, SUs have no chance to access channels to transmit their own traffic. Hence, both PUs and SUs perform poor in the no cooperation approach. For other two approaches, the sum payoff is also called as coalition value. In theses two approaches, all the players form the grand coalition. According to the definition of the v(S), the coalition value is the summation of players' payoffs in the coalition. However, the sum payoff in the no cooperation approach cannot be called as coalition value, because players just act alone and they do not form any coalition. The coalition value in the proposed algorithm increases 39.03% comparing with the equal power approach. The main reason is that SUs allocate more power on accessing, so SUs' payoffs improve significantly in the proposed algorithm. While, the equal power approach sets the same power levels to SU's relaying and SU's accessing. Hence, SUs are beneficial in the proposed algorithm. As a result, proposed algorithm can increase the coalition value dramatically.



Figure 4.12: Proposed algorithm converges to the core

#### 4.2.4 Algorithm Converges to the Core

Fig. 4.12 illustrates that the algorithm converges to the core in the aspect of time allocation. At first, the time coefficients are initialized at  $(\alpha, \beta) = (0.06, 0.03)$ . Then, at each allocated time point, we conduct optimal power control to maximize system utility. The time coefficients of  $\alpha$  and  $\beta$  are updated iteratively according to (3.15) and (3.16), respectively. In the figure, we can see that as the iteration increases, the updating step size decreases. Eventually, the algorithm converges to the equilibrium at (0.642, 0.217) with maximum coalition value 170.30. Then, we examine the payoff allocation obtained at this equilibrium with the definition of the core, which has two conditions. The first condition is guaranteed by the optimization problem's objective function, so we only need to examine the second condition of the definition. In the second condition, we have to show that players benefit most in the grand coalition. For example, if PU 1 acts alone, the payoff is 14.28; after joining the coalition with SU 1, the payoff increases to 17.01. Whereas, SU 1's payoff is zero before joining any coalition; after joining the coalition with PU 1, SU 1's payoff increases to 0.44. If PU 2 joins the coalition with PU 1 and SU 1, PU 2's payoff increases to 14.28. The more PUs joining the coalition, SUs can access the more channels, so SUs' payoffs increase. Also, the more SUs in the coalition, PUs' transmission

can be assisted by the more SUs. After comparing with different coalitions, we discover that player's payoff in the grand coalition is the most beneficial. Therefore, the payoff allocation achieved by the algorithm lies in the core. This implies that all the players have incentives for the proposed payoff allocation. As a result, proposed algorithm converges to the core.



# Chapter 5

# **Conclusion and Future Work**

## 5.1 Conclusion

We have applied a coalitional game to model the problem in the CCRN scenario. We consider the problem formulation in two cases, *i.e.* power control, and power control and time allocation case. In the power control case, the main purpose is to allocate the SU's power levels for relaying and accessing in order to maximize the system utility. On the other hand, in the power control and time allocation case, the problem is not guaranteed to be convex, so we proposed a novel algorithm to solve the problem iteratively in two steps. In the first step, we allocate the time coefficients and then, we conduct power control optimization int the second step. The proposed algorithm guarantees the problem solved in convex procedure. In addition, we have studied the convergence of the algorithm and the solution achieved by the algorithm lies in the core. Our problem formulation satisfies the time-sharing condition, which guarantees the problem's zero duality gap. We apply the time-sharing condition to proof that the core is nonempty. In the simulations, we have shown that the PUs' and SUs' payoffs lie in the core. We also discuss the relationship between time coefficients and coalition value. While, comparing between different approaches, our proposed algorithm can achieve a stable payoff allocation. Finally, the proposed algorithm converges to the equilibrium in the core.

## 5.2 Future Work

In this thesis, we have solved the problem considering power control and time allocation in CCRN scenario. We adopt a coalitional game to analyze the cooperative behavior of players. The proposed algorithm can solve the problem considering power control and time allocation iteratively in two steps. In the first step, we allocate the the time coefficients and the problem can be simplified as a power optimization problem. Hence, each step of the proposed algorithm is guaranteed to be convex. The time coefficients are updated with the subgradient method, which guarantees to converge to an equilibrium. However, there are still other methods to update time coefficients, which can be considered in the future work. Alternative methods for updating time coefficients can speed up the convergence of the algorithm.

In the network scenario, we consider multiple PUs and SUs based on CDMA network. When there are multiple users in the same channel, we use spreading codes to differentiate them. Whereas, the recent protocols, *e.g.* LTE, are all related to the OFDMA network. Hence, the future work can apply OFDMA network to the CCRN scenario. The most important part is the channel's scheduling for PU's and SU's usage. As in [24], the system model is CCRN based on OFDMA and authors also propose an algorithm to allocate the multi-channel cooperation. While, authors adopt a Nash Bargaining Game to model the problem in [24]. We have not seen any work applying OFDMA network to the CCRN with a coalitional game. Therefore, applying OFDMA network to the CCRN in a coalitional game approach is a practical direction for future work.

Another aspect for future work is to analyze the solution concept. The solution concepts of a coalitional game have been proposed in many years. However, few works discuss about the convergence region of the solution concept. In this thesis, we show that proposed algorithm converges to an equilibrium and examine that the equilibrium lies in the core. Future work can analyze the region of convergence (ROC) of the core. While, we have seen [25] discussion about the ROC of the core in a linear programming game. However, in our work, many parameters influence the ROC of the core. Hence, we can set the power levels fixed and analyze the ROC of the core in time coefficients domain.

If we can find the ROC of the core, this helps us a lot to achieve the optimal point inside the ROC. As a result, we can analyze the solution concepts of a coalitional game in many perspectives. This can helps us know more properties about the solution concepts.



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