國立交通大學

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碩 士 論 文

應用於相關性通道具位元配置有限回授系統之傳送器設計

Design of statistical precoder for correlated MIMO channel with limited feedback of bit allocation

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摘要

在本篇論文中，我們提出一個多輸入多輸出有限回授具位元配置系統在萊斯通道。首先我們推算出能使系統達到最小錯誤率的最佳位元與功率配置。接著我們根據相關性通道的統計特性設計出最佳的統計傳送器達到最小化錯誤率上限。我們考慮線性接收器與判定回授接受器。模擬結果我們顯示我們所提出的系統可以用較少的回授位元達到低錯誤率。

致謝

感謝指導教授 林源倍老師在兩年研究過程中，有耐心給予我專業領域上的教導，在研究上遇到困難時也會適時給我我幫助，讓我能順利的完成碩士論文，遇到老師真的是我學生生涯中最幸運的一件事。另外也要感謝林清安教授和蔡尚澕教授在百忙之中抽空參加我的口試，並且在論文上所提出的建議，使我的論文更完善。

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Design of statistical precoder for correlated MIMO channel with limited feedback of bit allocation

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Abstract

In this thesis, we design statistical precoder for precoded MIMO systems over correlated Ricean channels with limited feedback of bit allocation. We assume a reverse link of very low rate is available so that the receiver can send back the index of BA vector chosen from a codebook known to both transmitter and receiver. Furthermore we assume the correlated channel is slow fading and the statistics of the channel are known to the transmitter. Based on statistical of the channel, we derive the optimal statistical precoder so that bounds of the BER averaged over the random correlated channel is minimized. We will consider both linear and decision feedback receivers in the design of bit allocation codebook. The distribution of the bit allocation is taken into consideration. As a result, a nice tradeoff between performance and feedback rate can be achieved for correlated channels. Simulations show very good performance can be achieved when optimal precoder is used.

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Chapter 1

Introduction

MIMO systems with limited feedback have received great interest recently [1]-[10]. The system performance in terms of transmission rate or error rate can be improved significantly with limited amount of feedback from the receiver through a reverse channel [1]. It is generally assumed that the transmitter has no knowledge of the forward link channel and only the receiver has knowledge of the channel state information. The feedback of the complete channel information to the transmitter will require infinite number of bits. In practice the reverse channel can support only a limited transmission rate and it is desirable to have feedback rate as low as possible.

Recently precoded spatial multiplexing systems with finite-rate feedback have been investigated extensively [2]- [10]. The receiver chooses the optimal precoder from a codebook and sends the index back to the transmitter. Optimal codebook designs of unitary precoders using Grassmannian subspace packing for different criteria are developed in [2]. In [3], randomly generated codebooks known to the transmitter and receiver a priori is proposed. The optimal unitary precoder for minimizing BER (bit error rate) using infinite feedback rate, i.e., full channel state information available to the transmitter, is given in [4] and generalized Lloyd algorithm is used for constructing codebooks. Capacity loss due to quantized feedback is thoroughly analyzed in [5]. A special form of precoding system is the antenna selection system [6] that chooses the best subset of transmit antennas to minimize BER. In this case the transmitter enjoys low complexity as the precoder is a submatrix of the identity matrix. In multimode antenna selection [7], the number of substreams M or mode is allowed to vary with the channel and the bits are uniformly allocated to the M substreams. It is shown in [7] that with M_t bits of feedback, multimode antenna selection can achieve full diversity order M_rM_t, where M_t and M_r are respectively the number of transmit and receive antenna. In multimode precoding [8], the number of substreams M can also vary with the channel. In addition, a precoder codebook is designed for each possible M. The design of codebooks for multimode transmission over spatially correlated channels is developed in [9]. Generalized Lloyd algorithm is used in [10] to design capacity maximizing codebooks for multimode transmission.

Wireless communication over correlated fading be considered in [11]- [13]. The transmitter optimization be propose and determine a necessary and sufficient condition for maximize capacity in [11] and the special case that is used single antenna at receiver in [12]. In [13], a approximate minimum average symbol error rate precoder is designed for spatial multiplexing system with power allocation in Ricean channel.

A particular useful class of spatial multiplexing transceiver is the V-BLAST system that employs successive interference cancellation at the receiver [14]. The conventional V-BLAST system uses uniform bit/power allocation and thus no feedback is needed. It has been extended by incorporating power allocation or bit allocation when there is feedback [15]- [21]. In [15], approximate minimum BER power allocation was derived and the feedback is the power allocation information. An efficient algorithm for per antenna power and rate control of VBLAST system is developed in [16]. Joint optimization of bit allocation and detection ordering for minimizing outage probability is given in [17]. Successive quantization of power and bit allocation is proposed in [18]. Through the feedback of power and bit allocation, considerable gain can be achieved. Rate and power are optimized for uncoded error probability in [19]. As the receiver feedbacks only the ordering of detection to the transmitter, only a low feedback rate is needed. Average error probability is analyzed in [20] when power and bit allocation are taken into consideration. The optimal bit allocation is obtained by exhausting all possible constellations subject to a sum rate constraint. Several optimal designs of MIMO transceivers with decision feedback and bit loading are proposed in [21]. These optimal designs have similar performance when the channel state information is available to the transmitter. For the case of limited feedback, the use of identity precoder combined with feedback of only bit allocation is suggested therein as it intuitively requires less feedback. In earlier works of V-BLAST systems with bit allocation and a sum rate constraint [16] [18] [21], an exhaustive listing of all possible constellation combinations is used and thus a moderate feedback rate may be needed. Using capacity as a criterion statistical bit loading is considered in [22]. When the channel statistics are available to the transmitter but not the current state of the channel, the precoder can be designed according to the channel statistics. For example, optimal beamforming for maximizing average capacity of correlated channels has been designed in [23] [24]. There have also been a lot of research on designing statistical precoders of various design criteria for spatial multiplexing. Precoder for minimizing error probability are derived in [25] [26] [27]. The optimal precoder that minimizes the sum of mean squared error is given in [28]. A unified framework for solving a number of transceiver design problems for correlated channel is presented in [31]. The method can be applied when the cost belongs to a useful class of functions of subchannel mean squared error. In these works, a uniform bit allocation is assumed. Optimization of precoders with a fixed bit allocation vector have been considered in [29] [30].

In [45] the so called the BA system is proposed for the transmission over uncorrelated MIMO channels with feedback of bit allocation. For a given channel, a bit allocation vector is chosen from a codebook whose codewords (bit allocation vectors) satisfy the target transmission rate. The index of the selected codeword is feedback to the transmitter. The transmitter allocates bits to the modulation symbols according to the bit allocation vector and perform spatial multiplexing (precoding) using a precoder known to the transmitter and receiver a priori. In [45] it is shown that a uncorrelated channel the optimal precoder can be an arbitrarily unitary matrix for a uncorrelated channel and the BA system can achieve full diversity order.

In this thesis, we consider the transmission for Ricean channel (mean and covariance information) with feedback of bit allocation. Linear and decision feedback receiver be considered. We assume transmitter knows statistics of the correlated channel via a feedback link. We derive the optimal statistical precoder to minimize the bounds of BER averaged over the random channel. Simulations will show the BER performance is improved with optimal statistical precoder and detection order.

1.1 Outline

- Chapter 2: General system model is presented.
- Chapter 3: Previous works are reviewed in this chapter. In section 3.1 we review a spacial case of GTD based that is QR based system by P.P Vaidyanathan and C.C. Wang. Section 3.2 introduces a BER criterion and optimal unitary precoder for precoded spatial multiplexing system with infinite feedback rate proposed by S. Zhou and B. Li.
- Chapter 4: The proposed BA system over correlated channel for both covariance feedback and mean feedback are presented in this chapter. The optimal bits and power allocation are derived in 4.1. optimal statistical precoders are designed in 4.2. Feedback of bits allocation using a codebook in 4.3. In 4.4, we show that BA system can achieve full diversity.
- Chapter 5: Simulation examples are presented in this chapter.
- Chapter 6: A conclusion is given in this chapter.

1.2 Notations

1. Bold face upper case letters represents matrices. Bold face lower case letters represents matrices. The notation $\mathbf{A}^\dagger$ denotes transpose-conjugate of $\mathbf{A}$. The notation $\mathbf{A}^T$ denotes transpose of $\mathbf{A}$.

2. The function $E[y]$ denotes the expect value of a random variable y.

3. The notation $\mathbf{I}_m$ is used to represent the $m \times m$ identity matrix.

4. The notation $C(n, k)$ is used to denote the chosen function of n and k.

Chapter 2

General System Model

Consider the wireless system with M_t transmit antenna and M_r receiver antenna in Figure 2.1. The channel is modeled by an $M_r \times M_t$ memoryless matrix with

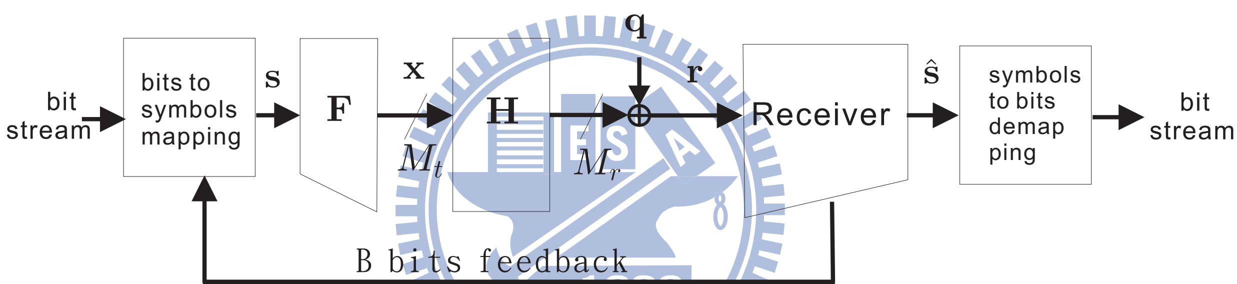


Figure 2.1: MIMO system with limited feedback

channel noise vector $\mathbf{q}$ of size $M_r \times 1$. The noise vector $\mathbf{q}$ is assumed to be additive white Gaussian with zero mean and variance N_0. Suppose the system can process M substreams where $M \leq \min(M_r, M_t)$. The input vector $\mathbf{s}$ is an $M \times 1$ vector which consists of M modulation symbols. The symbols s_k are assumed to be zero mean and uncorrelated, hence the autocorrelation matrix $\mathbf{R}_s = E[\mathbf{s}\mathbf{s}^\dagger]$ is a diagonal matrix. Assume the total transmission power is P_t and $\mathbf{F}$ is an unitary $M_r \times M$ matrix. The total transmission power can be written as $E[\mathbf{x}^\dagger \mathbf{x}] = E[\mathbf{s}^\dagger \mathbf{F}^\dagger \mathbf{F} \mathbf{s}] = \sum_{k=0}^{M-1} \sigma_{s_k}^2$, where we have used the fact that $\mathbf{F}^\dagger \mathbf{F} = \mathbf{I}_M$. We will consider linear and decision feedback receiver in this paper. Define the error vector at the output of receiver as $\mathbf{e} = \hat{\mathbf{s}} - \mathbf{s}$. When the receiver is linear and zero forcing, the receiver output $\hat{\mathbf{s}} = \mathbf{G}\mathbf{r}$, where the $M \times \ M_r$ receiver matrix is $\mathbf{G} = (\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger$ [32]. The error vector at the output of $\mathbf{G}$ has autocorrelation matrix $\mathbf{R}_e = E[\mathbf{e}\mathbf{e}^\dagger]$ given by [32]

$$\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1} \tag{2.1}$$

When there is decision feedback at the receiver, the part from previously detected symbols are subtracted from the received signal and this is also called successive interference cancellation. The decision feedback receiver can be described as a recursive procedure [14]. First initializes $\mathbf{r}_0 = \mathbf{r}$, $\mathbf{A}_0 = \mathbf{H}\mathbf{F}$ and $i = 0$. The steps in the recursions are as follows. (1) Let $\mathbf{G}_i$ be the Moore-Penrose inverse of $\mathbf{A}_i$. Find the row vector of $\mathbf{G}_i$ that has the smallest 2-norm. Call the index of the row vector $\mathbf{w}_i$. (2) Compute $y_i = \mathbf{w}_i^T\mathbf{r}_i$, apply symbol detection on y_i, and call the output $\hat{s}_i$. (3) Subtract from $\mathbf{r}_i$ the contribution of the k_ith subchannel, $\mathbf{r}_{i+1} = \mathbf{r}_i - \hat{s}_{ki}\mathbf{a}_{ki}$, where $\mathbf{a}_{ki}$ is the k_ith column of $\mathbf{A}_0$ and zero the k_ith column of $\mathbf{A}_i$ to obtain $\mathbf{A}_{i+1}$. When all the subchannels are of the same constellation, the post detection SNR of the k_ith subchannel is $\rho_{ki} = \frac{P_t/M}{N_0\|w_i\|^2}$. In this case, the above procedure is optimal in the sense that the worst subchannel error rate is minimized.

Assuming the inputs s_k are b_k-bit QAM symbols, the kth symbol error rate is well approximated by [42].

$$SER_k = 4(1 - \frac{1}{2^{b_k/2}})Q\left(\sqrt{\frac{3\sigma_{s_k^2}}{(2^{b_k}-1)\sigma_{e_k^2}}}\right), \tag{2.2}$$

where $Q(y) = \frac{1}{\sqrt{2\pi}}\int_y^\infty e^{-t^2/2}dt, \quad y \geq 0$. Note that for the decision feedback receiver $\frac{\sigma_{s_k}}{\sigma_{e_k}}$ is the post detection SNR and (2.2) is the error rate assuming there is no error in detecting previous symbols. When Gray code is used, the BER can be approximated by $BER_k \approx SER_k/b_k$. Using this approximation, the BER for a given channel $\mathbf{H}$ can be computed using

$$BER \approx \frac{1}{R_b}\sum_{k=0}^{M-1} b_k BER_k = \frac{1}{R_b}\sum_{k=0}^{M-1} SER_k \tag{2.3}$$

For a given channel $\mathbf{H}$, the BER depends on the bit allocation and power allocation, which will be optimized to minimize BER in chapter 4. The channel is well known Ricean model [13] or mean information model [11]. In the Ricean model, the flat fading channel is composed of a line-of-sight(LOS) component and a Rayleigh component. We can express $\mathbf{H}$ as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{sp} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w\mathbf{R}_t^{1/2}, \tag{2.4}$$

where K is Ricean factor defined as the power ratio of LOS signal to diffused scattered signal, $\mathbf{H}_w$ is an $M_r \times M_t$ matrix of i.i.d, zero mean, unit variance complex Ganussion random variable and $\mathbf{R}_t$ is the $M_t \times M_t$ correlation matrix. $\mathbf{H}_{sp}$ can be expressed as

$$\mathbf{H}_{sp} = \mathbf{a}_r \times \mathbf{a}_t^T,$$

where

$$\mathbf{a}_r = \begin{bmatrix} 1 & e^{j2\pi d_r \sin\theta_r} & \cdots & e^{j2\pi d_r (M_r-1)\sin\theta_r} \end{bmatrix}^T$$
$$\mathbf{a}_t = \begin{bmatrix} 1 & e^{j2\pi d_r \sin\theta_t} & \cdots & e^{j2\pi d_t (M_t-1)\sin\theta_t} \end{bmatrix}^T$$

are the line-of-sight(LOS) array responses at receiver and transmitter with angle of arrival θ_r and angle of departure θ_t respectively and a Uniform Linear Array is considered. If K is large then a pure LOS channel in environment. Such a model assumes correlation only exists at transmitter, this assume is useful for downlink transmission [33]. We also discuss two special case for (2.4) as follow.

1) No line of sight ($K = 0$)

In a Environment full of obstacles, the multipath components is enough then ricean factor K will approach 0, thus the channel model becomes to

$$\mathbf{H} = \mathbf{H}_w\mathbf{R}_t^{1/2}. \tag{2.5}$$

It is well known covariance information model [11].

2) $\mathbf{R}_t = \mathbf{I}_{M_t}$

No correlation at transmitter assumption, the $\mathbf{H}$ becomes

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{sp} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w. \tag{2.6}$$

For transmitter correlation matrix $\mathbf{R}_t$, we consider a uniform linear array of M_t antennas with spacing d_t. The plane wave departure directions of these signals span an angular spread θ_t and uniformly distributed, we find [34] [35].

$$[\mathbf{R}_t]_{m,k} = \frac{1}{S} \sum_{i=-(S-1)/2}^{i=(S-1)/2} e^{-2j\pi(k-m)d_t \cos(\frac{\pi}{2}+\theta_{t,i})} \tag{2.7}$$

where S is the number of scatterers with corresponding directions of arrival $\theta_{t,i}$

$$\theta_{t,i} = \frac{1}{S-1}\theta_t \times i, \qquad i = -(S-1)/2 \ldots\ldots (S-1)/2. \tag{2.8}$$

when θ_t or d_t is large, $\mathbf{R}_t$ will converge to the identity matrix which is uncorrelated fading. When θ_t or d_t is small,the correlation matrix becomes rank deficient which is full correlated fading.

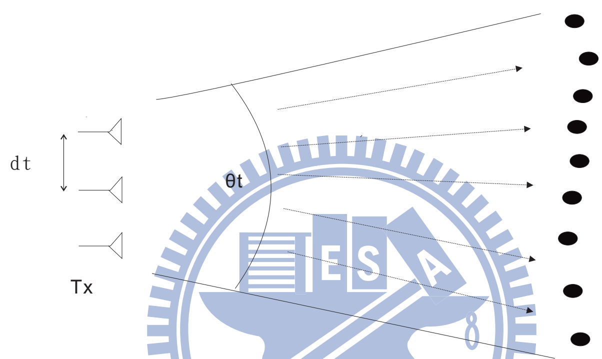


Figure 2.2: Propagation scenario for fading correlation.

Chapter 3

Previous Works

In this chapter, we review two referred works in the literature. Section 3.1 presents a GTD based system for optimal transceiver design and a special case is the QR based system proposed in [21]. Section 3.2 presents a limited feedback precoder system with BER selection criterion and codebook design proposed in [4].

3.1 GTD Based System

3.1.1 Formulating the Power Minimization Problem and Solution

The generalized triangular decomposition (GTD), proposed in [21]. Let $\mathbf{H}$ be a $M \times N$ rank-K matrix with singular values $\sigma_1, \sigma_2 \ldots \sigma_K$ in descending order. let $\mathbf{h} = [\sigma_1, \sigma_2 \ldots \sigma_K]$ and $\mathbf{r} = [r_1, r_2 \ldots r_K]$ be a given vector which satisfies

$$\hat{\mathbf{r}} \prec_{\times} \mathbf{h} \tag{3.1}$$

Then there exist matrices $\mathbf{R}, \mathbf{Q}, \mathbf{P}$ such that

$$\mathbf{H} = \mathbf{QRP}^{\dagger} \tag{3.2}$$

where $\prec_{\times}$ is multiplicative majorization [36], $\mathbf{R}$ is a $K \times K$ upper triangular matrix with diagonal terms equal to r_k, and $\mathbf{Q} \in C^{M \times K}$, $\mathbf{P} \in C^{N \times K}$ both have orthonormal columns.

The problem of minimizing the transmitted power subject to the specified BER and total bit rate constraints, and the ZF constraint can be written as follows:

$$\min_{\mathbf{F},\mathbf{G},\mathbf{B},\mathbf{b}_k} \mathbf{P}_T = \sum_{k=1}^{M} c_k 2^{\mathbf{b}_k} [\mathbf{F}^{\dagger}\mathbf{F}]_{kk} [\mathbf{G}\mathbf{G}^{\dagger}]_{kk}. \tag{3.3}$$

constrainted by $\sum_{k=1}^{M} b_k = R_b$ and $\mathbf{GHF}-\mathbf{B} = \mathbf{I}$. Where $c_k = \frac{N_0}{3}(Q^{-1}(P_e(k)/4))^2$. The following are solution for GTD-based method to construct the transceiver matrices $\mathbf{F}$,$\mathbf{G}$,$\mathbf{B}$ [21].

$$\mathbf{F} = [\mathbf{P}]_{M_t \times M} \tag{3.4}$$

$$\mathbf{G} = (diag([\mathbf{R}]_{M\times M}))^{-1} [\mathbf{Q}^{\dagger}]_{M \times M_r} \tag{3.5}$$

$$\mathbf{B} = (diag([R]_{M\times M}))^{-1} [\mathbf{R}]_{M\times M} - \mathbf{I} \tag{3.6}$$

$$b_k = \log_2\left(\frac{c_k}{M} 2^{R_b/M} \left(\frac{1}{\prod_{k=1}^{M}}\right)^{1/M}\right) - \log_2 c_k + \log_2([\mathbf{R}]_{kk}^2) \tag{3.7}$$

With above choice, the minimum transmission power can be achieve.

3.1.2 QR Transceiver ZF-VBLAST

The QR Transceiver is a special case of GTD based system. Based on the general system model at chapter 2, the system in [21] has decision feedback receiver and precoder is identity. Assume the number of subchannels $\mathbf{M}$ are used. This system has bit allocation, the optimal power loading is equally that $\mathbf{R}_s = \frac{P_0}{M}\mathbf{I}_M$. Because the precoder matrix is identity and only bit allocation vector need to be known. The channel matrix be written as $\mathbf{H} = \mathbf{QR}$, where $\mathbf{Q}$ has orthonormal columns, and $\mathbf{R}$ is upper triangular. $|\mathbf{R}(k,k)|^{-2}$ is error variance corresponding to kth subchannel. The receiver can compute $\{b_k\}$ from [21]

$$b_k = \frac{1}{M}\sum_{l=1}^{M} \log_2[\mathbf{G}\mathbf{G}^{\dagger}]_{ll} - \log_2[\mathbf{G}\mathbf{G}^{\dagger}]_{kk} + \frac{R_b}{M} \tag{3.8}$$

(3.8) is called the optimal bit loading formula. we will quantize it to the bit allocation vector nearest to the vector in pre-determined codebook C_b, and feed back the index of that vector to the transmitter.

3.2 Precoder System

3.2.1 System Model

Based on the general system model at chapter 2, the system in [4] assumes the number of subchannels M is fixed and all M subchannels are used. The system is without bit allocation design. Thus, the bit loading is uniform and the target bit rate R_b is divisible for M. Each symbol carries $\frac{R_b}{M}$ bits. The power is also equally allocated for each symbols, $\mathbf{R_s} = \frac{P_0}{M}\mathbf{I}_M$. For the reverse channel, it is constrained to send B bits. In this paper, the feedback information is the precoder matrix. Therefore, a precoder codebook $\mathcal{C}_F$ of size 2^B is prepared. After the estimation of forward channel, a precoder matrix is selected using a BER-based selection criterion from $\mathcal{C}_F$ and the corresponding index is fed back to the transmitter. The BER-based selection criterion will be reviewed as follows.

BER selection criterion. Under the assumption of uniform bit allocation, the average BER for each precoder matrix in $\mathcal{C}_F$ can be computed by (2.3). The BER-base selection criterion is

$$\widehat{\mathbf{F}} = \arg\min_{\mathbf{F}\in\mathcal{C}_F} BER(\mathbf{F}, \mathbf{H}). \tag{3.9}$$

To choose a precoder matrix by BER selection criterion, we need to compute the BER formula (2.3) for each precoder matrix in $\mathcal{C}_F$. Therefore, 2^B computations of (2.3) are required to complete BER selection criterion.

3.2.2 Optimal Precoder for infinite-feedback rate

With infinite feedback bits, it can be assumed that the transmitter has full channel knowledge. The optimal precoder $\mathbf{F}_{opt}$ with BER-based criterion can be derived directly from $\mathbf{H}$. The optimal precoder $\mathbf{F}_{opt}$ can provide a benchmark performance for finite-rate precoder feedback system. Assuming the singular value decomposition of $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^{\dagger}$, where $\mathbf{U}$ and $\mathbf{V}$ are respectively $M_r \times M_r$ and $M_t \times M_t$ unitary matrices. The $M_r \times M_t$ matrix $\mathbf{\Lambda}$ is a diagonal matrix whose diagonal elements are the singular values of $\mathbf{H}$ in a nonincreasing order. And let β_k be the k-th largest subchannel SNR. The optimal precoders for zero forcing and MMSE receiver are given respectively as follows.

Zero-forcing case. Consider a rectangular/square QAM constellation with size M is applied for $\bar{\mathbf{b}}$. Constellation-specific threshold Γ_{th} is shown in table 3.2.2.

1. When $\beta_1 \le \Gamma_{th}$, $\mathbf{F}_{opt} = \mathbf{V}_M$, where $\mathbf{V}_M$ is the $M_t \times M$ matrix obtained by keeping the first M columns of $\mathbf{V}$.

2. When $\beta_M \ge \Gamma_{th}$, $\mathbf{F}_{opt} = \mathbf{V}_M\mathbf{Q}_M$, where $\mathbf{Q}_M$ is an $M \times M$ unitary that has equal magnitude property, i.e., $|[\mathbf{Q}]_{m,n}| = 1/\sqrt{M}$, for $0 \le m, n \le M-1$.

3. When conditions in 1 or 2 do not hold, the optimal precoder $\mathbf{F}_{opt}$ can't be found analytically. Suppose that K_1 subchannels' SNR are larger than Γ_{th}. Then one suboptimal precoder that is better than $\mathbf{V}_M$ can be constructed as

$$\mathbf{F} = \mathbf{V}_M \begin{bmatrix} \mathbf{Q}_{K_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-k_1} \end{bmatrix} \tag{3.10}$$

MMSE case. Consider a rectangular/square QAM constellation with size M is applied for $\bar{b}$. Two constellation-specific thresholds $\Gamma_{th,l}$, $\Gamma_{th,h}$ are shown in table 3.2.2.

1. When $\Gamma_{th,l} \le \beta_M$ and $\beta_1 \le \Gamma_{th,h}$, $\mathbf{F}_{opt} = \mathbf{V}_M$.

2. When $\beta_1 \le \Gamma_{th,l}$ or $\beta_M \ge \Gamma_{th,h}$, $\mathbf{F}_{opt} = \mathbf{V}_M\mathbf{Q}_M$.

3. When conditions in 1 or 2 do not hold, the optimal precoder $\mathbf{F}_{opt}$ can't be found analytically. Suppose that K_1 subchannels' SNR are larger than $\Gamma_{th,h}$ and K_2 subchannel SNRs are smaller than $\Gamma(th, l)$. Then one suboptimal

precoder that is better than $\mathbf{V}_M$ can be constructed as

$$\mathbf{F} = \mathbf{V}_M \begin{bmatrix} \mathbf{Q}_{K_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-K_1-K_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_{K_2} \end{bmatrix} \tag{3.11}$$

M	2	4	8	16	32	64	128	256
Γ_{th}	1.5	3	9.01	14.93	38.46	62.50	166.7	250.0

Table 3.1: Table of Γ_{th}

M	2	4	8	16	32	64	128	256
$\Gamma_{th,l}$	0	0	0.579	0.247	0.326	0.264	0.330	0.271
$\Gamma_{th,h}$	0	0	7.621	13.72	37.46	61.50	165.7	249.0

Table 3.2: Table of $\Gamma_{th,l}$ and $\Gamma_{th,h}$

3.2.3 Codebook construction

From [2] it is shown that the precoder codebook design problem can be related to Grassmanian subspace packing. Thus, in [4], generalized Lloyd algorithm is used to construct a precoder codebook by minimizing a chordal distance cost function. The chordal distance between two unitary M_t by M matrices, $\mathbf{F}_i$ and $\mathbf{F}_j$ is

$$d_c(\mathbf{F}_i, \mathbf{F}_j) = \frac{1}{\sqrt{2}} \left\| \mathbf{F}_i\mathbf{F}_i^\dagger - \mathbf{F}_j\mathbf{F}_j^\dagger \right\|_F, \tag{3.12}$$

where $\|\cdot\|_F$ denotes Frobenius norm. Suppose that $\mathbf{V}$ is an isotropically distributed $M_t \times M$ matrix. The following algorithm quantizes $\mathbf{V}$ to 2^B matrices. Starting with an initial codebook $\mathcal{C}_F = \{\mathbf{F}_0, \mathbf{F}_1, \cdots, \mathbf{F}_{2^B-1}\}$ (obtained from random computer search or using the currently best codebook if available), the codebook design steps are as follows.

1. Generate a training set with N_{tr} samples $\{\mathbf{V}_n\}_{n=1}^{N_{tr}}$.
2. Iterate following steps until it converges.
   (a) Assign $\mathbf{V}_n$ to one of the regions $\{\mathcal{R}_i\}_{i=0}^{2^B-1}$ using the rule

$$\mathbf{V}_n \in \mathcal{R}_i, \quad \text{if} \quad d_c(\mathbf{V}_n, \mathbf{F}_i) < d_c(\mathbf{V}_n, \mathbf{F}_j), \forall j \neq i. \tag{3.13}$$

(b) For each region $\mathcal{R}_i$, find the centroid as

$$\mathbf{F}_i^{centroid} = \arg\min_{\mathbf{F}} \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} d_c^2(\mathbf{V}_n, \mathbf{F}) \tag{3.14}$$

$$= \arg\min_{\mathbf{F}} \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} trace(\mathbf{I}_M - \mathbf{F}^\dagger \mathbf{V}_n \mathbf{V}_n^\dagger \mathbf{F}) \tag{3.15}$$

$$= \arg\max_{\mathbf{F}} trace(\mathbf{F}^\dagger \mathbf{R} \mathbf{F}) \tag{3.16}$$

where $\mathbf{R}$ is defined as

$$\mathbf{R} = \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} \mathbf{V}_n \mathbf{V}_n^\dagger. \tag{3.17}$$

Let the eigendecomposition of $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{U}_R \mathbf{\Lambda}_R \mathbf{U}_R^\dagger. \tag{3.18}$$

$\mathbf{\Lambda}_R$ is a diagonal matrix whose diagonal elements are in nonincreasing order. It is easy to show that $\mathbf{F}_i^{centroid}$ is a $M_t \times M$ matrix obtained by keeping the first M columns of $\mathbf{U}_R$.

(c) Set $\mathcal{C}_F = \{\mathbf{F}_i^{centroid}\}_{i=1}^{2^B-1}$. During each iteration, The codebook will be record if the minimum chordal distance of $\mathcal{C}_F$

$$\min_{0 \le i < j \le 2^B-1} d_c(\mathbf{F}_i, \mathbf{F}j)$$

is larger than the currently best.

3. Go back to 1, generate another training set, then execute the next steps. The algorithm will stop if there is no further improvement on the minimum chordal distance.

Chapter 4

The proposed BA system

In this chapter we propose the design of statistical precoder for correlated MIMO channel with limited feedback of bit allocation . The proposed system will be termed a BA system. We will derive optimal unconstrained bit allocation and statistical precoders for both linear and decision feedback receiver for minimizing BER. We also show that proposed BA system can achieve full diversity.

4.1 Optimal Bit and Power Allocation

In this section, we will consider the BA system when there is no integer constraint on bit allocation. For a given precoder, we will derive the optimal bit allocation that minimizes the BER. We will see that the solution has the same form as that given in [21] in which the bit allocation is optimized for minimum transmission power. The BER obtained with optimal bit allocation will be used in the next section to design statistical precoders for minimum BER. The results derived in this chapter are valid at linear and decision feedback receivers for correlated channel in chapter 2. The optimal bit allocation will also be useful in chapter 5 for efficient codeword selection in practical applications where feedback rate is limited.

Assume the transmission rate is high and the number of bits loaded on the kth subchannel b_k is large enough so that $1 - 2^{-b_k/2} \approx 1$ and $1 - 2^{-b_k} \approx 1$, then the symbol error rate expression in (2.2) can be approximated by

$$SER_k \approx 4Q\left(\sqrt{\frac{3}{2^{b_k}}\frac{\sigma_{s_k}^2}{\sigma_{e_k}^2}}\right) \tag{4.1}$$

For the convenience of derivation, we define the function

$$f(y) = Q(1/\sqrt{y}), \quad y \geq 0, \tag{4.2}$$

The function $f(y)$ is monotone increasing and it can be verified that $f(y)$ is convex for $y \leq 1/3$ and concave for $y \geq 1/3$. Using $f(\cdot)$, we have $SER_k \approx 4f(2^{b_k}\sigma_{e_k}^2/(3\sigma_{s_k}^2))$. Therefore the BER in (2.3) can be written as

$$BER \approx \frac{4}{R_b}\sum_{k=0}^{M-1} f\left(2^{b_k}\sigma_{e_k}^2/(3\sigma_{s_k}^2)\right) \tag{4.3}$$

Let us consider the high SNR case in which the convexity of $f(\cdot)$ holds and the low SNR case in which the concavity of $f(\cdot)$ holds.

High SNR case

Assume SNR is large enough so that the arguments of $f(\cdot)$ are smaller than 1/3 and the convexity of $f(\cdot)$ holds. Using the convexity of $f(\cdot)$, we have

$$\begin{aligned}
BER &\approx \frac{4}{(R_b/M)}\frac{1}{M}\sum_{k=0}^{M-1} f\left(2^{b_k}\sigma_{e_k}^2/(3\sigma_{s_k}^2)\right) && (4.4)\\
&\geq \frac{4}{(R_b/M)} f\Big(\frac{1}{3M}\sum_{k=0}^{M-1} 2^{b_k}\sigma_{e_k}^2/\sigma_{s_k}^2\Big) && (4.5)\\
&\geq \frac{4}{(R_b/M)} f\left(\frac{2^{R_b/M}}{3}\Big(\prod_{k=0}^{M-1}\sigma_{e_k}^2\Big)^{1/M}\Big(\prod_{k=0}^{M-1}\frac{1}{\sigma_{s_k}^2}\Big)^{1/M}\right) && (4.6)\\
&\geq \frac{4}{(R_b/M)} f\left(\frac{2^{R_b/M}}{3P_t/M}\Big(\prod_{k=0}^{M-1}\sigma_{e_k}^2\Big)^{1/M}\right) && (4.7)\\
&\triangleq BER_0 && (4.8)
\end{aligned}$$

The second inequality is obtained by using the fact that $\sum_{i=0}^{M-1} b_i = R_b$ and the AM-GM (arithmetic mean-geometric mean) inequality and also using the monotone increasing property of $f(\cdot)$. We can obtain the third inequality using the

AM-GM inequality $(\prod_{k=0}^{M-1}\sigma_{s_k}^2)^{1/M} \leq 1/M\sum_{k=0}^{M-1}\sigma_{s_k}^2 = P_t/M$ and the monotone increasing property of $f(\cdot)$. Notice that the lower bound BER_0 in (4.8) is independent of bit allocation and power allocation. The optimal bit allocation and power allocation are such that the three inequalities in (4.8) become equalities. Due to the convexity of $f(\cdot)$, the first equality in (4.5) holds if and only if $2^{b_k}\sigma_{e_k}^2/\sigma_{s_k}^2$ are of the same value for all k. The same set of conditions is also necessary and sufficient for equality to hold in the second inequality as $f(\cdot)$ is monotone increasing. The third equality is achieved if $\sigma_{s_0}^2 = \sigma_{s_1}^2 = \ldots = \sigma_{s_{M-1}}^2 = P_t/M$. The optimal bit allocation for minimizing the BER is thus

$$b_k = \frac{1}{M}\sum_{l=0}^{M-1}\log_2(\sigma_{e_l}^2) - \log_2(\sigma_{e_k}^2) + \frac{R_b}{M} \tag{4.9}$$

With the above optimal bit allocation and uniform power allocation, the lower bound BER_0 is achieved. We can see that the symbols with smaller error variances $\sigma_{e_k}^2$ are allocated with more bits. When bits are allocated as in (4.9), $2^{b_k}\sigma_{e_k}^2/\sigma_{s_k}^2$ are the same for all k. This means the symbol error rates are equalized for all transmitted symbols. The bit allocation formula in (4.9), derived using the criterion of minimum BER, has the same form as that designed for minimum transmission power in [21].

Low SNR case

Assume **S**NR is low enough so that the arguments of $f(\cdot)$ are larger than 1/3 and the concavity of $f(\cdot)$ holds.

$$BER \approx \frac{4}{(R_b/M)}\frac{1}{M}\sum_{k=0}^{M-1} f\left(2^{b_k}\sigma_{e_k}^2/(3\sigma_{s_k}^2)\right) \leq \frac{4}{(R_b/M)} f\left(\frac{1}{3M}\sum_{k=0}^{M-1}2^{b_k}\sigma_{e_k}^2/\sigma_{s_k}^2\right) \tag{4.10}$$

The inequality follows from the concavity of $f(\cdot)$. Similar to the high SNR case, the quantity on the right hand side is minimized if uniform power allocation is used and bit allocation is chosen according to (4.9). In this case the upper bound on the right hand side is equal to BER_0 and at the same time the inequality in (4.10) becomes an equality, ie., $BER \approx BER_0$

Summarizing, for both high and low SNR regions the BER with optimal bit allocation and uniform power loading is approximately BER_0. The results can be used for both linear and decision feedback receivers. The quantity BER_0 is different for different types of receivers. We will use BER_0 in the next section to derive optimal statistical precoder of BA system over correlated channel.

4.2 Design of statistical precoders for minimum BER

In this section we consider the design of optimal statistical precoders over correlated channels model described in chapter 2. Assume $M_r > M$. To consider the average **BER** performance, we average BER_0 computed in (4.8) over the random channel $\mathbf{H}$,

$$\overline{BER_0} = E[BER_0] = E\left[\frac{4}{R_b/M} f\left(c\prod_{k=0}^{M-1}\sigma_{e_k}^{2/M}\right)\right] \tag{4.11}$$

where we have used $c = \frac{2^{R_b/M}}{3(P_t/M)}$. To simplify the expression further, we define the geometric mean function

$$h(\mathbf{y}) = \prod_{i=0}^{M-1} y_i^{1/M} \tag{4.12}$$

$\mathbf{y} = \begin{bmatrix} y_0 & y_1 & \cdots & y_{M-1} \end{bmatrix}^T$ and $y_i > 0$. Let $y_i = c\sigma_{e_i}^2$, then $BER_0 = \frac{4}{(R_b/M)} f(h(\mathbf{y}))$. To analyze $\overline{BER_0}$, we first derive the Hessian matrix of $f(h(\mathbf{y}))$, which is an $M \times M$ matrix with the (i,j)th entry given by $[\mathbf{H}_{ess}]_{i,j} = \partial^2 f(h(\mathbf{y}))/\partial y_i \partial y_j$, for$0 \leq i, j < M$. We can verify that $[\mathbf{H}_{ess}]_{ij}$ is given by

$$[\mathbf{H}_{ess}]_{i,j} = \begin{cases} 0.5/M^2 f'(h(\mathbf{y}))y_i^{-1}y_j^{-1}(1-h(\mathbf{y}))) & ,i \neq j \\ 0.5/M^2 f'(h(\mathbf{y}))y_i^{-2}(1-(1+2M)h(\mathbf{y})), & .i = j \end{cases} \tag{4.13}$$

It is derive in appendix. It is known that [37] a function is convex (concave) if and only if the Hessian matrix is positive (negative) semi definite. In the following we discuss the behavior of $\overline{BER_0}$ for the high and low SNR cases.

High SNR case

Consider $P_t/N_0 \gg 1$ such that the arguments of $f(\cdot)$ are much smaller than unity, ie., $h(\mathbf{y}) = \frac{2^{R_b/M}}{3(P_t/M)} \prod_{k=0}^{M-1} \sigma_{e_k}^{2/M} \ll 1$. We can approximate the ith diagonal element of the Hessian matrix in (4.13) as $1/(2M^2)f'(h(\mathbf{y}))y_i^{-2}$. Defining the $M \times 1$ vector $\mathbf{u}$ with ith element $u_i = 1/y_i$, we have $\mathbf{H}_{ess} \approx 1/(2M^2)f'(h(\mathbf{y}))\mathbf{u}\mathbf{u}^T$, which is a positive semidefinite matrix. Applying Jensens inequality, we get $E[f(h(\mathbf{y}))] \gtrsim f(h(E[\mathbf{y}]))$. Therefore we have

$$E[BER_0] = E\left[\frac{4}{R_b/M} f(c\prod_{k=0}^{M-1} \sigma_{e_k}^{2/M})\right] \geq \frac{4}{R_b/M} f\left(c\prod_{k=0}^{M-1} \bar{\sigma}_{e_k}^{2/M}\right) \triangleq \overline{BER}_{bd} \tag{4.14}$$

where $\bar{\sigma}_{e_k}^2 = E[\sigma_{e_k}^{2/M}]$ is the kth error variance averaged over the channel $\mathbf{H}$. The right hand side $\overline{BER}_{bd}$ is a lower bound of $\overline{BER}_0$.

Low SNR case

A property of $f(h(\mathbf{y}))$ that is useful for studying $E[BER_0]$ in low SNR region is presented in the following lemma.

Lemma 1. *Let $f(x)$ and $h(\mathbf{y})$ be as defined in (4.2) and (4.12), respectively. Then the composite function $f(h(\mathbf{y}))$ for $y_i > 0$ is concave when $h(\mathbf{y}) \geq 1/3$.*

Proof. The Hessian matrix in (4.13) can be rewritten as

$$\mathbf{H}_{ess} = 1/M^2 f'(h(\mathbf{y}))h(\mathbf{y})[0.5(1/h(\mathbf{y}) - 1)\mathbf{u}\mathbf{u}^T - M\mathbf{D}]$$

, where $\mathbf{u}$ is $M \times 1$ with ith element $u_i = 1/y_i$, and $\mathbf{D}$ is a diagonal matrix with $[\mathbf{D}]_{ii} = 1/y_i^2$. We examine the quadratic form $\mathbf{v^T H v}$ for an arbitrary $M \times 1$ vector $\mathbf{v}$. It can be rearranged as

$$\mathbf{v^T H v} = \frac{1}{M^2} f'(h(\mathbf{y}))h(\mathbf{y})[(\mathbf{v^T u u^T v} - M\mathbf{v^T D v}) + 0.5(1/h(\mathbf{y}) - 3)\mathbf{v^T u u^T v}].$$

The first term in the bracket $\mathbf{v^T u u^T v} - M\mathbf{v^T D v}$ is equal to $(\sum_{k=0}^{M-1} v_k u_k)^2 - M\sum_{k=0}^{M-1} v_k^2 u_k^2$, which is always non positive due to Cauchy-Schwartz inequality. The second term in the bracket, equal to $0.5(1/h(\mathbf{y}) - 3)(\sum_{k=0}^{M-1} v_k u_k)^2$, it is non positive if $(1/h(\mathbf{y}) - 3) \leq 0$ ie., $h(\mathbf{y}) \geq 1/3$. Therefore we can conclude that when $h(\mathbf{y}) \geq 1/3$, the Hessian matrix of $f(h(\mathbf{y}))$ is negative semidefinite and thus $f(h(\mathbf{y}))$ is concave. ■

The above lemma means that BER_0 is concave in $\sigma_{e_i}^2$ when

$$h(\mathbf{y}) = \frac{2^{R_b/M}}{3(P_t/M)} \left(\prod_{k=0}^{M-1} \sigma_{e_k}^2 \right)^{1/M} \geq 1/3, \tag{4.15}$$

which holds in low SNR case, ie., small P_t/N_0. When $f(h(\mathbf{y}))$ is concave, we can apply Jensens inequality $E[f(h(\mathbf{y}))] \leq f(h(E[\mathbf{y}]))$ to obtain

$$E[BER_0] \leq \frac{4}{R_b/M} f\left(c \prod_{k=0}^{M-1} \bar{\sigma}_{e_k}^{2/M} \right) \triangleq \overline{BER}_{bd}. \tag{4.16}$$

Now $\overline{BER}_{bd}$ becomes an upper bound of $\overline{BER_0}$. In both high SNR and low SNR regions, we would like to have the bound $\overline{BER}_{bd}$ minimized, which is discussed for linear receivers and decision feedback receivers for Ricean channel.

4.2.1 Optimal precoders design with Ricean channel

Suppose $\mathbf{A}$ is a $M_r \times M_t$ matrix each row of which is independently drawn from a M_t-variate normal distribution with zero mean each row of $\mathbf{A}$ is independently and let the ith column of $\mathbf{A}^\dagger$ be $\mathbf{g}_i$, then the autocorrelation matrix of $\mathbf{g}_i$ is equal to $\mathbf{R}_t$. It is known that $\mathbf{A}^\dagger\mathbf{A} = \sum_{i=0}^{M_r-1} \mathbf{g}_i\mathbf{g}_i^\dagger$ has a complex Wishart distribution with M_r degrees of freedom, denoted as $\mathcal{W}_{M_t}(\mathbf{R}_t, M_r)$ [38]. When $\mathbf{B}$ has a Wishart distribution, we say $\mathbf{B}^{-1}$ has inverse Wishart distribution. For Ricean channel model, the channel be considered as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{sp} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w\mathbf{R}_t^{1/2}. \tag{4.17}$$

It is known $\mathbf{H}^\dagger\mathbf{H}$ has a complex non-central Wishart distribution $\mathcal{NW}_{M_t}(\mathbf{R}_t, \mathbf{M}, M_r)$ [39], where $\mathbf{M} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{sp}$, is called non-centrality parameter matrix means the expectation of $\mathbf{H}$. M_r is degree of freedom and $\mathbf{R}_t$ is the autocorrelation matrix of $\mathbf{H}_w\mathbf{R}_t^{1/2}$. This non-central Wishart distribution can be approximated by a

Wishart distribution [39]

$$\mathcal{NW}_{M_t}(\mathbf{R}_t, \mathbf{M}, M_r) \sim \mathcal{W}_{M_t}(\widehat{\mathbf{R}}_t, M_r), \tag{4.18}$$

where $\widehat{\mathbf{R}}_t = \mathbf{R}_t + \mathbf{M}^\dagger\mathbf{M}/M_r$.

Linear receiver We can obtain $\overline{\sigma}^2_{e_k}$ by averaging the error covariance matrix $\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}$ over the channel. If $\mathbf{F}$ is nonsingular matrix then the matrix $\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}$ is $\mathcal{W}_M(\mathbf{F}^\dagger\widehat{\mathbf{R}}_t\mathbf{F}, M_r)$ and so the matrix $\mathbf{R}_e^{-1} = 1/N_0\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}$ has a Wishart distribution $\mathcal{W}_M(N_0^{-1}\mathbf{F}^\dagger\widehat{\mathbf{R}}_t\mathbf{F}, M_r)$. Then $\mathbf{R}_e$ has an inverse Wishart distribution. It has been shown in [43] that when a matrix $\mathbf{B}$ is Wishart distribution $\mathcal{W}_p(\mathbf{A}, r)$ with $r > p$, then $E[\mathbf{B}^{-1}] = 1/(r-p)\mathbf{A}^{-1}$. Using this result and assuming $M_r > M$, $\overline{\mathbf{R}}_e = E[\mathbf{R}_e]$ is given by

$$\overline{\mathbf{R}}_e = \frac{N_0}{M_r - M}(\mathbf{F}^\dagger\widehat{\mathbf{R}}_t\mathbf{F})^{-1}. \tag{4.19}$$

Let the eigen decomposition of $\widehat{\mathbf{R}}_t$ be $\widehat{\mathbf{R}}_t = \widehat{\mathbf{U}}_t\widehat{\mathbf{\Lambda}}_t\widehat{\mathbf{U}}_t^\dagger$, where $\widehat{\mathbf{\Lambda}}_t$ is a diagonal matrix and the diagonal elements$\lambda_{t,i}$ are the eigen value of $\widehat{\mathbf{R}}_t$. Let the diagonal elements of $\widehat{\mathbf{\Lambda}}_t$ be ordered such that $\lambda_{t,0} \geq \lambda_{t,1} \geq \ldots \geq \lambda_{t,M_t-1}$ and assume $\lambda_{t,M_t-1} > 0$

Theorem 1. *For the linear receiver, the BER bound $\overline{BER}_{bd}$ in (4.14) satisfies*

$$\overline{BER}_{bd} \geq \overline{BER}_{bd,lin}, where \overline{BER}_{bd,lin} = \frac{4M}{R_b}Q\left(\sqrt{\frac{3P_t/M}{2^{R_b/M}N_0}(M_r - M)\prod_{i=0}^{M-1}\lambda_{t,i}^{1/M}}\right) \tag{4.20}$$

The inequality becomes an equality when $\mathbf{F} = \widehat{\mathbf{U}}_{t,M}$, where $\widehat{\mathbf{U}}_{t,M}$ is the submatrix of $\widehat{\mathbf{U}}_t$ that consists of the first M column vectors of $\widehat{\mathbf{U}}_t$.

Proof. Majorization theorem [36] will be used to prove the theorem. For completeness, some related definitions are given below.

(1)Given a sequence $a_{[0]}, a_{[1]}, \ldots, a_{[M-1]}$,the notation $a_{[k]}$ refers to the permuted sequence such that $a_{[0]} \geq a_{[1]} \geq \ldots \geq a_{[M-1]}$. (2) Given two real vectors $\mathbf{a} = \begin{bmatrix} a_0 & a_1 & \cdots & a_{M-1} \end{bmatrix}^T$ and $\mathbf{b} = \begin{bmatrix} b_0 & b_1 & \cdots & b_{M-1} \end{bmatrix}^T$, we say that $\mathbf{a}$ majorizes $\mathbf{b}$ if the following two conditions are satisfied: $\sum_{k=0}^{M-1} a_k = \sum_{k=0}^{M-1} b_k$

and $\sum_{k=0}^{n} a_{[k]} \geq \sum_{k=0}^{n} b_{[k]}$, $0 \leq n \leq M-2$. Let $g(\mathbf{y})$ be a real-valued function of a real vector $\mathbf{y}$. We say that $g(\mathbf{y})$ is Schur-concave if $g(\mathbf{a}) \leq g(\mathbf{b})$ whenever $\mathbf{a}$ majorizes $\mathbf{b}$.

The function $g(\mathbf{x}) = \prod_{i=0}^{M-1} x_i$, for $x_i > 0$ is known to be Schur concave [41]. As $\overline{\sigma}_{e_i}^2$ are the diagonal elements of $\overline{\mathbf{R}}_e$, the sequence $\{\overline{\sigma}_{e_i}^2\}_{i=0}^{M-1}$ is majorized by $\{\lambda_i(\overline{\mathbf{R}}_e)\}_{i=0}^{M-1}$, where we have used $\lambda_i(\mathbf{A})$ to denote the i-th largest eigen value of $\mathbf{A}$. So $\prod_{i=0}^{M-1} \overline{\sigma}_{e_i}^2 \geq \prod_{i=0}^{M-1} \lambda_i(\overline{\mathbf{R}}_e)$ and the equality holds when $\overline{\mathbf{R}}_e$ is a diagonal matrix. The matrix $\overline{\mathbf{R}}_e^{-1}$ is the inverse of $\overline{\mathbf{R}}_e$, their eigen values are related by $\lambda_i(\overline{\mathbf{R}}_e) = 1/\lambda_{M-1-i}(\overline{\mathbf{R}}_e^{-1})$. As $\overline{\mathbf{R}}_e^{-1} = \frac{M_r-M}{N_0}\mathbf{F}^\dagger\widehat{\mathbf{R}}_t\mathbf{F}$ and $\mathbf{F}$ is unitary, we can apply the Poincare separation theorem to bound the eigen values of $\overline{\mathbf{R}}_e^{-1}$ using the eigen values of $\widehat{\mathbf{R}}_t$. Poincare separation theorem says $\lambda_i(\mathbf{B}) \geq \lambda_i(\mathbf{C}^\dagger\mathbf{B}\mathbf{C})$,$i = 0, 1, \ldots, r-1$, for any $n \times n$ Hermitian matrix $\mathbf{B}$ and any $n \times r$ unitary matrix $\mathbf{C}$ with $\mathbf{C}^\dagger\mathbf{C} = \mathbf{I}_r$. Using this theorem, we have $\prod_{i=0}^{M-1} \frac{M_r-M}{N_0}\lambda_i(\widehat{\mathbf{R}}_t) \geq \prod_{i=0}^{M-1} \frac{M_r-M}{N_0}\lambda_i(\overline{\mathbf{R}}_e^{-1})$. Thus

$$\prod_{i=0}^{M-1} \overline{\sigma}_{e_i}^2 \geq \prod_{i=0}^{M-1} \lambda_i(\overline{\mathbf{R}}_e) = \prod_{i=0}^{M-1} \frac{1}{\lambda_i(\overline{\mathbf{R}}_e^{-1})} \geq \prod_{i=0}^{M-1} \frac{N_0}{M_r - M} \frac{1}{\lambda_i(\widehat{\mathbf{R}}_t)} \tag{4.21}$$

In (4.20),The lower bound $\prod_{i=0}^{M-1} \frac{N_0}{M_r-M} \frac{1}{\lambda_i(\widehat{\mathbf{R}}_t)}$ can be achieved by choosing $\mathbf{F} = \widehat{\mathbf{U}}_{t,M}$. Using the above inequality and the monotone increasing property of $f(\cdot)$, we can establish the inequality in (4.20). Therefore, to minimize the BER bound $\overline{BER}_{bd}$ the optimal precoder is $\mathbf{F} = \widehat{\mathbf{U}}_{t,M}$. ■

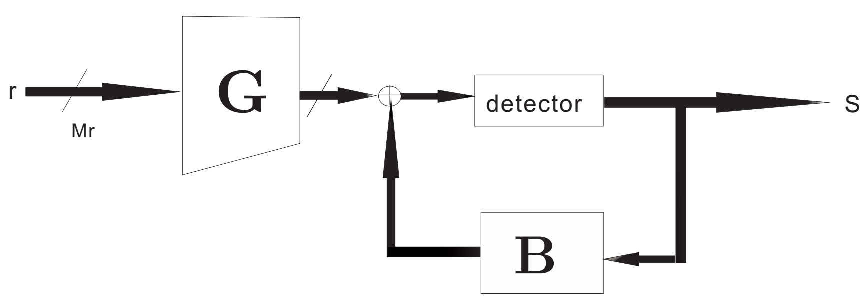


Figure 4.1: Block diagram of the decision feedback receiver.

Decision feedback receiver To consider the precoder design for a decision feedback receiver, we can use the receiver structure in Fig .4.1 based on the

QR decomposition of **HF** [21] [28]. This corresponds to the case of a reverse detection ordering. Let the QR decomposition of **HF** be **QR**, where **Q** is an $M_r \times M$ unitary matrix and **R** is an $M \times M$ upper triangular matrix with diagonal element $[\mathbf{R}]_{ii} = r_{ii}$. The feedforward matrix **G** and feedback matrix **B** are given, respectively, by [32]

$$\mathbf{G} = \begin{pmatrix} r_{00}^{-1} & r_{11}^{-1} & \cdots & r_{M-1,M-1}^{-1} \end{pmatrix} \mathbf{Q}^\dagger \tag{4.22}$$

$$\mathbf{B} = \begin{pmatrix} r_{00}^{-1} & r_{11}^{-1} & \cdots & r_{M-1,M-1}^{-1} \end{pmatrix} \mathbf{R} - \mathbf{I}_M \tag{4.23}$$

Assuming there is no error propagation, the kth subchannel error $e_k = \widehat{s}_k - s_k$ has variance $\sigma_{e_k}^2 = N_0 r_{kk}^{-2}$, $k = 0, 1, \ldots, M-1$. The average error variance is $\overline{\sigma}_{e_k}^2 = N_0 E[r_{kk}^{-2}]$. The value $E[r_{kk}^{-2}]$ as been shown to be related to the Cholesky decomposition of $\mathbf{F}^\dagger \widehat{\mathbf{R}}_t \mathbf{F}$ in [28]. The result is summarized in the following lemma.

Lemma 2. *[28] When $K = 0$, $\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2}$, the following result was derived. Let the Cholesky decomposition of $\mathbf{F}^\dagger \mathbf{R}_t \mathbf{F}$ be $\mathbf{LDL}^\dagger$ where $\mathbf{L}$ is a lower triangular matrix with unity diagonal elements and $\mathbf{D}$ is diagonal. For $M_r > M$, $E[r_{kk}^{-2}] = d_{kk}^{-1}/(M_r - k - 1)$, for $k = 0, 1, \ldots, M-1$ where d_{kk} is the kth diagonal element of $\mathbf{D}$.*

Using lemma 2 and the approximation in (4.18), the results in Lemma 2 allows us to establish the following bound for $\overline{BER}_{bd}$

Theorem 2. *For the decision feedback receiver with $M_r > M$, the BER bound $\overline{BER}_{bd}$ in (4.14) satisfies*

$$\overline{BER}_{bd} \geq \overline{BER}_{bd,df},$$

$$\overline{BER}_{bd,df} = \frac{4M}{R_b} Q\left(\sqrt{\frac{3P_t/M}{2^{R_b/M} N_0} \prod_{k=0}^{M-1} (M_r - k - 1)^{1/M} \prod_{k=0}^{M-1} \lambda_{t,k}^{1/M}}\right), \tag{4.24}$$

where $\lambda_{t,k} = \lambda_k(\mathbf{F}^\dagger \widehat{\mathbf{R}}_t \mathbf{F})$. The inequality becomes an equality when $\mathbf{F} = \widehat{\mathbf{U}}_{t,M}$, where $\widehat{\mathbf{U}}_{t,M}$ is the submatrix of $\widehat{\mathbf{U}}_t$ that consists of the first M column vectors of $\widehat{\mathbf{U}}_t$.

Proof. Using Lemma 2, we can obtain $\overline{\sigma}^2_{e_k} = N_0 d_{kk}^{-1}/(M_r - k - 1)$. Thus $\prod_{k=0}^{M-1} \overline{\sigma}^2_{e_k} = \prod_{k=0}^{M-1} N_0 d_{kk}^{-1}/(M_r - k - 1)$. Note that $\prod_{k=0}^{M-1} d_{kk} = \prod_{k=0}^{M-1} \lambda_k(\mathbf{F}^\dagger \widehat{\mathbf{R}}_t \mathbf{F})$. The $\overline{BER}_{bd}$ in(4.14) can be expressed as

$$\overline{BER}_{bd} = \frac{4M}{R_b} f\left(cN_0 \prod_{k=0}^{M-1} (M_r - k - 1)^{-1/M} \prod_{k=0}^{M-1} \lambda_{t,k}^{-1/M} (\mathbf{F}^\dagger \widehat{\mathbf{R}}_t \mathbf{F})\right) \tag{4.25}$$

Applying the Poincare separation theorem (also stated in the proof of Theorem 1), we have the inequality

$$\overline{BER}_{bd} = \frac{4M}{R_b} f\left(cN_0 \prod_{k=0}^{M-1} (M_r - k - 1)^{-1/M} \prod_{k=0}^{M-1} \lambda_{k}^{-1/M} (\mathbf{F}^\dagger \widehat{\mathbf{R}}_t \mathbf{F})\right) \tag{4.26}$$

$$\geq \frac{4M}{R_b} f\left(cN_0 \prod_{k=0}^{M-1} (M_r - k - 1)^{-1/M} \prod_{k=0}^{M-1} \lambda_{t,k}^{-1/M}\right) \tag{4.27}$$

$$= \overline{BER}_{bd,df} \tag{4.28}$$

The lower bound $\overline{BER}_{bd,df}$ can be achieved when $\mathbf{F} = \widehat{\mathbf{U}}_{t,M}$ ∎

$\mathbf{R}_t = \mathbf{I}$ case

In this special case, we assume no correlation at transmitter and receiver. Channel is considered as $\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{sp} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w$, where $\mathbf{H}_{sp} \triangleq \mathbf{a}_r \mathbf{a}_t^T$, $\mathbf{a}_r$ and $\mathbf{a}_t$ are LOS array response at transmitter and receiver described in chapter 2. Let $\mathbf{H}_{sp}^\dagger \mathbf{H}_{sp} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^\dagger$, we have

$$\widehat{\mathbf{R}}_t = \mathbf{I}_{N_t} + c\mathbf{H}_{sp}^\dagger \mathbf{H}_{sp} = \mathbf{V}(\mathbf{I}_{N_t} + c\mathbf{\Lambda})\mathbf{V}^\dagger, \tag{4.29}$$

where $c = \frac{K}{(K+1)M_r}$. Note that

$$\mathbf{H}_{sp}^\dagger \mathbf{H}_{sp} = \mathbf{a}_t^* \mathbf{a}_r^\dagger \mathbf{a}_r \mathbf{a}_t^T = \|\mathbf{a}_r\|^2 \|\mathbf{a}_t\|^2 \widetilde{\mathbf{a}}_t^* \widetilde{\mathbf{a}}_t^T = \lambda_0 \mathbf{v}\mathbf{v}^\dagger,$$

where $\lambda_0 = \|\mathbf{a}_r\|^2 \|\mathbf{a}_t\|^2$, $\widetilde{\mathbf{a}}_t = \frac{\mathbf{a}_t}{\|\mathbf{a}_t\|}$, $\mathbf{v} = \widetilde{\mathbf{a}}_t^*$, we can see that λ_0 is the only nonzero eigenvalue, the other eigenvalues are 0 and the eigenspaces of λ_0 and 0 are othogonal. Because $\mathbf{H}_{sp} = \mathbf{a}_r \mathbf{a}_t^T = [a_{r_0}\mathbf{a}_t \quad a_{r_1}\mathbf{a}_t \dots a_{r_{N_r}}\mathbf{a}_t]^T$, we can see that when we take the hermitian of the first row and normalize, it is equal to $\widetilde{\mathbf{a}}_t^*$. When $\mathbf{R}_t = \mathbf{I}$, the first column of optimal precoder $\mathbf{F}$ is the hermitian of the first row of $\mathbf{H}_{sp}$ with normalization, the other columns of $\mathbf{F}$ can be arbitrarily chosen, except for the restriction that the columns of $\mathbf{F}$ are orthonormal. That is

$$\mathbf{F} = \begin{bmatrix} \frac{\mathbf{a}_t}{\|\mathbf{a}_t\|} & \mathbf{f}_2 & \cdots & \mathbf{f}_M \end{bmatrix}, \tag{4.30}$$

where $\mathbf{f}_2, \cdots \mathbf{f}_M$ are arbitrarily vectors such that $\mathbf{F}$ is unitary. Such a precoder has been shown in [11] to maximize capacity of a beamforming system.

No line of sight $(K = 0)$ case

In this case, we consider the special case that $K = 0$ such that $\mathbf{H}$ in (4.17) becomes to

$$\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2}$$

it is known $\mathbf{H}^\dagger\mathbf{H}$ has a complex Wishart distribution with M_r degree of freedom, denoted as $\mathcal{W}_{M_t}(\mathbf{R}_t, M_r)$ instead of complex non-central Wishart distribution so we don't need take approximate. Let the eigen decomposition of $\mathbf{R}_t$ be $\mathbf{R}_t = \mathbf{U}_t\mathbf{\Lambda}_t\mathbf{U}_t$, where $\mathbf{\Lambda}_t$ is a diagonal matrix and the diagonal elements$\lambda_{t,i}$ are the eigen value of $\mathbf{R}_t$. Let the diagonal elements of Λ_t be ordered such that $\lambda_{t,0} \geq \lambda_{t,1} \geq \ldots \geq \lambda_{t,M_t-1}$ and assume $\lambda_{t,M_t-1} > 0$.

Linear receiver Using the property of Wishart distribution , the matrix $\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}$ is $\mathcal{W}_M(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}, M_r)$ and we have

$$\overline{\mathbf{R}}_e = \frac{N_0}{M_r - M}(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F})^{-1} \tag{4.31}$$

Using the proof of theorem 1 , we have

$$\prod_{i=0}^{M-1} \overline{\sigma}_{e_i}^2 \geq \prod_{i=0}^{M-1} \lambda_i(\overline{\mathbf{R}}_e) = \prod_{i=0}^{M-1} \frac{1}{\lambda_i(\overline{\mathbf{R}}_e^{-1})} \geq \prod_{i=0}^{M-1} \frac{N_0}{M_r - M}\frac{1}{\lambda_i(\mathbf{R}_t)},$$

thus

$$\overline{BER}_{bd} = \frac{4M}{R_b} f(c \prod_{k=0}^{M-1} \overline{\sigma}_{e_k}^{2/M}) \tag{4.32}$$

$$\geq \frac{4M}{R_b} f\left(c \prod_{k=0}^{M-1} \lambda_i^{1/M}(\overline{\mathbf{R}}_e)\right) \tag{4.33}$$

$$\geq \frac{4M}{R_b} f\left(c \frac{N_0}{M_r - M} \prod_{k=0}^{M-1} \frac{1}{\lambda_i^{1/M}(\mathbf{R}_t)}\right) \tag{4.34}$$

$$= \overline{BER}_{bd,lin}. \tag{4.35}$$

When $\mathbf{F} = \mathbf{U}_{t,M}$, the lower bound $\overline{BER}_{bd,lin}$ can be achieved .

Decision feedback receiver We can use lemma 2 and theorem 2. Let the Cholesky decomposition of $\mathbf{F}^\dagger \mathbf{R}_t \mathbf{F}$ be $\mathbf{LDL}^\dagger$, thus we have

$$\overline{BER}_{bd} = \frac{4M}{R_b} f\left(cN_0 \prod_{k=0}^{M-1} (M_r - k - 1)^{-1/M} \prod_{k=0}^{M-1} \lambda_k^{-1/M}(\mathbf{F}^\dagger \mathbf{R}_t \mathbf{F})\right) \tag{4.36}$$

$$\geq \frac{4M}{R_b} f\left(cN_0 \prod_{k=0}^{M-1} (M_r - k - 1)^{-1/M} \prod_{k=0}^{M-1} \lambda_{t,k}^{-1/M}\right) \tag{4.37}$$

$$= \overline{BER}_{bd,df} \tag{4.38}$$

With the same result as mean feedback, then $\mathbf{F} = \mathbf{U}_{t,M}$ the bound $\overline{BER}_{bd}$ can be minimized.

4.3 Feedback of bit allocation

In this proposed BA system, only bit allocation is adapted according to the varying random channel. The precoder is chosen as $\mathbf{F} = \mathbf{U}_{t,M}$. based on the the results in the previous section. Such a precoder depends only on the channel statistics and the information of the precoder need not be fed back to the transmitter frequently. The transmission power is uniformly distributed among the subchannels loaded with nonzero bits. When we consider bit allocation in practical applications, the bits assigned to the symbols are typically integer-valued. The components of the bit allocation vector $\mathbf{b}$ satisfy the sum rate constraint

$b_0 + b_1 + \ldots + b_{M-1} = R_b$ where $b_i \in \mathcal{Z}^+$ and $\mathcal{Z}^+$ denotes the set of nonnegative integers. The number of such nonnegative integer bit allocation vectors is $C(R_b + M - 1, R_b)$, where $C(\cdot, \cdot)$ denotes the choose function. This requires $B_0 = \lceil \log_2 C(R_b + M - 1, R_b) \rceil$ bits, where $\lceil x \rceil$ denotes the smallest integer larger than or equal to x. For example $R_b = 8, M = M_t = 4$, the required number of feedback bits is 8. The approach of using all possible constellation combinations is adopted in earlier works that employs bit allocation subject to a sum rate constraint [20] [21]. To reduce the feedback rate, the codebook is trimmed by imposing some constraints on the vectors [21].

Codeword selection. Suppose we are given B feedback bits and a codebook $\mathcal{C}_b$ of 2^B bit allocation vectors. The vectors in $\mathcal{C}_b$ satisfy the sum rate constraint so that the number of bits transmitted for each channel use is R_b. The BER expression in (2.3) is a function of bit allocation vector. For a given channel $\mathbf{H}$, we can choose the best bit allocation vector $\widehat{\mathbf{b}} \in \mathcal{C}_b$ that minimizes the BER, $\widehat{\mathbf{b}} = \arg\min_{b \in \mathcal{C}_b} BER(\mathbf{b}, \mathbf{H})$, where $BER(\mathbf{b}, \mathbf{H})$ denotes the BER when the channel is $\mathbf{H}$ and the bit allocation vector is $\mathbf{b}$. To make codeword selection more efficient, we can choose (suboptimal) codewords based on the optimal bit allocation given in (4.9). The criterion of minimizing the largest subchannel error rate will be considered. Suppose the optimal bit allocation vector computed from (4.9) is $\mathbf{b}^*$. Given a bit allocation vector $\mathbf{b} \in \mathcal{C}_b$, the kth subchannel symbol error rate associated with $\mathbf{b}$ is

$$SER_k \approx 4Q(\sqrt{\frac{3}{2^{b_k}} \frac{\sigma_{s_k}^2}{\sigma_{e_k}^2}}) = 4Q(\sqrt{\frac{3}{2^{b_k^*}} \frac{\sigma_{s_k}^2}{\sigma_{e_k}^2} 2^{b_k^* - b_k}}) \tag{4.39}$$

As shown in Sec. 4.1 the optimal bit allocation $\mathbf{b}^*$ equalizes the quantity $3\sigma_{s_k}^2/(2^{b_k^*}\sigma_{e_k}^2)$. Let us call this subchannel independent quantity A. Then we have $SER_k \approx 4Q(\sqrt{A 2^{b_k^* - b_k}})$. Therefore the largest subchannel error rate can be minimized by choosing the bit allocation vector $\mathbf{b} \in \mathcal{C}_b$ that has the largest $\min_k(b_k^* - b_k)$.The optimal bit allocation is derived under the assumption that all M subchannels are loaded with nonzero bits. To remove the assumption, we can compute BER_0 in (4.8) for each M_0 with $0 \leq M_0 \leq M$ where M_0 is the number of subchannels used, and choose the M_0 that has the smallest BER_0. We can then apply quantization

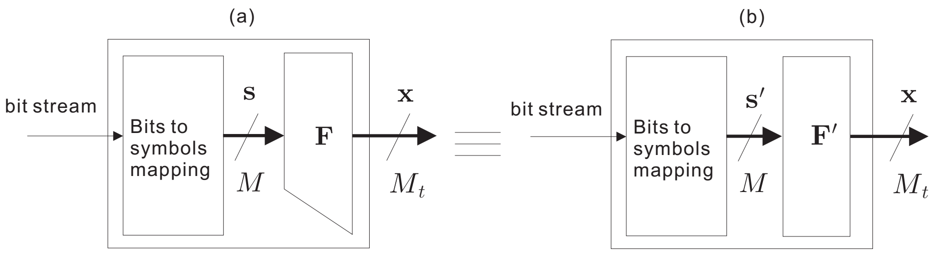


Figure 4.2: The transmitter of the BA system with (a) precoder F, and (b) augmented precoder F'.

on the corresponding optimal bit allocation using the above maximin criterion $\max_{\mathbf{b}\in\mathcal{C}_b} \min_k (b_k^* - b_k)$ Such a suboptimal selection criterion does not require the computation of BER for each bit allocation in the codebook. Simulations in chapter5 will demonstrate that the use of the suboptimal maximin criterion leads to only a minor degradation compared to the optimal BER criterion

Augmented precoding [45]. We have used a fixed $M_t \times M$ matrix $\mathbf{F}$ as the precoder. When $M < M_t$ and the channel matrix is such that the column space of $\mathbf{F}$ is contained in the null space of $\mathbf{H}$, then there is zero signal power at the receiver. This can be avoided by starting off with an augmented initial precoder $\mathbf{F}'$ of size $M_t \times M_t$. For a given M, we can choose M columns out of $\mathbf{F}'$ to form the actual $M_t \times M$ precoder $\mathbf{F}$, i.e., $(M_t - M)$ columns of $\mathbf{F}'$ are removed. The corresponding augmented input vector $\mathbf{s}'$ and bit allocation vector $\mathbf{b}'$ are of size $M_t \times 1$.The entries of $\mathbf{s}'$ and $\mathbf{b}'$ corresponding to the removed columns of $\mathbf{F}'$ are all equal to zero so that the transmitter output $\mathbf{F}'\mathbf{s}'$ is equal to $\mathbf{Fs}$. As we choose M columns from $\mathbf{F}'$, there are $C(M_t, M)$ possible choices for precoder $\mathbf{F}$. The transmitter with the augmented precoding scheme is shown in Fig. 4.2(b). The augmented bit allocation vector $\mathbf{b}'$ satisfies $b_0' + b_1' + \ldots + b_{M-1}' = R_b$, $b_i' \in \mathcal{Z}^+$, with the additional constraint that at most M of the components can be nonzero as it is assumed that the transmitter and receiver can process at most M substreams.It can be verified that the total number of possible integer bit allocation vectors satisfying the sum rate constraint is

$$\sum_{k=M_t-M}^{M_t-1} C(M_t,k)C(R_b-1,M_t-1-k) \tag{4.40}$$

As in the non augmented case we can design a smaller codebook $\mathcal{C}'_b$ b to have a smaller feedback rate. There is no need to feedback the information of the actual precoder $\mathbf{F}$ used. The information is embedded in the augmented bit allocation vector $\mathbf{b}'$. For $i = 0, 1, \ldots, M_t - 1$, the transmitter removes the ith column from $\mathbf{F}'$ if $b_i = 0$. The transmitter can then use the resulting $M_t \times M_0$ submatrix as the precoder, where M_0 is the number of nonzero entries in $\mathbf{b}'$. Note that for a given channel, using augmented precoder $\mathbf{F}'$ is not guaranteed to be better than using a fixed $\mathbf{F}$ because the codebooks are different.Suppose $\mathbf{F}$ is a submatrix of $\mathbf{F}'$. Let us consider the special case that the codewords of $\mathcal{C}'_b$ is obtained by inserting appropriate zeros in the codewords of $\mathcal{C}_b$.Then the system with augmented precoder has the same performance as the one with a fixed precoder, but not better. Nonetheless the simulations in chapter5 will demonstrate that when $M < M_t$ the system of augmented precoder outperforms the one with a fixed precoder for the same number of feedback bits.

Optimal detection ordering for decision feedback receiver. When all the subchannels use the same constellation, the optimal detection ordering for the decision feedback receiver is to maximize the post detection SNR ρ_i in each recursion [14]. Such an approach minimizes the worst subchannel error rate. It is not same for the case with bit allocation and bit allocation needs to be taken into consideration. Suppose the bit allocation is given. In the second step of the recursive procedure we need to choose the nonzero row vector of $\mathbf{G}_i$ to maximize

$$\mu_{k_i} = \frac{1}{(2^{b_{k_i}}-1)\|\mathbf{w}_i\|^2}, \quad for \quad k_i \in S, \tag{4.41}$$

where $S = \{j : b_j > 0\}$ is the collection of subchannels that are used for transmission. This can be proved by following a procedure similar to that in [14]. The maximization of μ_i (also called rate-normalized SNR) in each recursion has been shown to minimize the outage probability in [17]. Note that there is no need for the receiver to feedback the detection order; the transmitter only needs to know the bit allocation but not the detection ordering. For each bit allocation in the codebook, we can perform the recursive procedure to maximize the rate-normalized SNR. Then the best bit allocation and corresponding detection order can be selected.

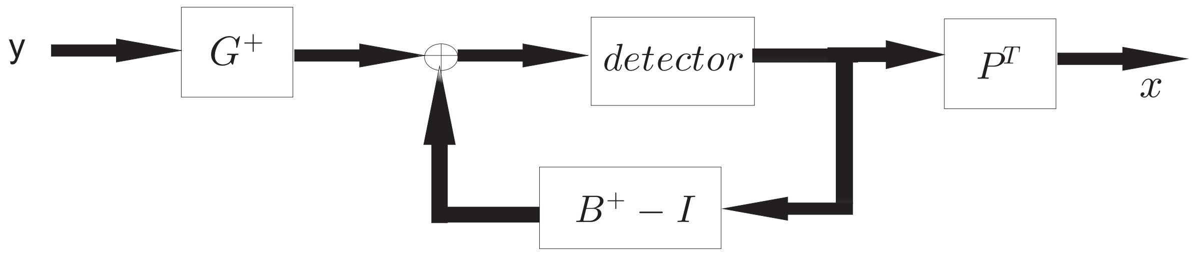


Figure 4.3: Block diagram of the decision feedback receiver based on cholesky decomposition.

Reduce complexity for optimal ordering. Above detection ordering, we need to take Moore-penrose inverse after each detected. It will raise complexity. In [44] V-BLAST is proposed to reduce the complexity by applying cholesky decomposition with symmetric permutation. It derive new algorithm based on a specific receiver structure in Figure4.3, where $\mathbf{G}$ is feedforward matrix, $\mathbf{B}$ is feedback matrix and $\mathbf{P}$ is permutation matrix that recover original ordering. Let the cholesky decomposition of $\mathbf{R}_e$ be $\mathbf{LDL}^\dagger$, where $\mathbf{L}$ is a $M_t \times M_t$ unit lower triangular matrix and $\mathbf{D}$ is a $M_t \times M_t$ diagonal matrix with diagonal element $[\mathbf{D}]_{ii} = d_{ii}$ and d_{ii} is the error variance of the ith detected of subchannel input x_i. the feedforward matrix $\mathbf{G}$ and feedback matrix $\mathbf{B}$ are given, respectively, by [44]

$$\mathbf{B}^\dagger = \mathbf{L}^{-1} \tag{4.42}$$

$$\mathbf{G}^\dagger = \mathbf{DL}^\dagger\mathbf{PH}^\dagger\mathbf{R}_e^{-1} \tag{4.43}$$

where $\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{HF})^{-1}$.
The algorithm with maximizing rate-normalized SNR is shown as follow

- step 1: $\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{HF})^{-1}$, $\mathbf{P} = \mathbf{I}_{M_t}$, $\mathbf{D} = \mathbf{0}_{M_t}$

- step 2: for $i = M_t, \dots, 1$
  $q = \arg\min_{q'} \mathbf{R}_e(q', q')(2^{b_{q'}} - 1)$
  $\mathbf{P}_i = \mathbf{I}_{M_t}$, whose ith and qth rows are exchanged
  $\mathbf{P} = \mathbf{P}_i\mathbf{P}$, $\mathbf{R}_e = \mathbf{P}_i\mathbf{R}_e\mathbf{P}_i^T$, $\mathbf{b} = \mathbf{P}_i\mathbf{b}$
  $\mathbf{D}(i,i) = \mathbf{R}_e(i,i)$, $\mathbf{R}_e(i : N_t, i) = \mathbf{R}_e(i : N_t, i)/\mathbf{D}(i,i)$
  for $j = i+1, \dots, M_t$
  $\mathbf{R}_e(j : M_t, j) = \mathbf{R}_e(j : M_t, j) - \mathbf{R}_e(j : M_t, i)\mathbf{R}_e^*(j,i)\mathbf{D}(i,i)$
  $\mathbf{R}_e(j, j : M_t) = \mathbf{R}_e(j : M_t, j)^\dagger$
  $\mathbf{L} = tril(\mathbf{R}_e)$

- step 3: $\mathbf{B}^\dagger = \mathbf{L}^{-1}$, $\mathbf{G}^\dagger = \mathbf{D}\mathbf{L}^\dagger\mathbf{P}\mathbf{H}^\dagger\mathbf{R}_e^{-1}$

By using this algorithm, we don't need to take matrix inverse after each detection so we can successfully reduce the complexity.

4.4 Diversity Gain of BA system [45]

we show that the BA system can achieve diversity order M_rM_t for a system with M_r receive antennas and M_t transmit antennas if the codebook is properly designed and has at least M_t codewords. Let the initial precoder $\mathbf{F}'$ be an $M_t \times M_t$ unitary matrix ($\mathbf{F}' = \mathbf{F}$ and $M = M_t$). The number of bits to be transmitted in each channel use is R_b, which is distributed among M symbols ($M \leq \min(M_t, M_r)$). The augmented bit allocation vector $\mathbf{b}'$ is of size $M_t \times 1$. It has at most M nonzero entries and $\sum_{i=0}^{M_t-1} b'_i = R_b$. Suppose the bit allocation codebook is $\mathcal{C}'_b$. The minimum achievable BER is

$$BER_{\min}(\mathbf{H}) = \min_{\mathbf{b}' \in \mathcal{C}'_b} BER(\mathbf{b}', \mathbf{H}), \tag{4.44}$$

where $BER(\mathbf{b}', \mathbf{H})$ is the BER in (2.3) Assume the bit allocation codebook $\mathcal{C}'_b$ contains the set of codewords

$$\mathcal{C}^*_b = \{R_b\mathbf{e}_0, R_b\mathbf{e}_1, \cdots, R_b\mathbf{e}_{M_t-1}\}, \tag{4.45}$$

where $\mathbf{e}_i$ are standard vectors of size $M_t \times 1$, i.e., $[\mathbf{e}_i]_i = 1$ and $[\mathbf{e}_i]_j = 0$ for $j \neq i$. The following lemma shows that the BA system can achieve full diversity order using the bit allocation vectors in $\mathcal{C}_b^*$. Therefore to achieve a diversity order of M_rM_t we can use a codebook of size M_t, which requires only $\log_2 M_t$ feedback bits.

Lemma 3. *For a finite-rate feedback MIMO channel with M_r receive antennas and M_t transmit antennas, the BA system with an $M_t \times M_t$ augmented unitary precoder $\mathbf{F}'$ achieves diversity order M_rM_t if the bit allocation codebook $\mathcal{C}_b'$ contains the M_t vectors in (4.44).*

Proof. As $\mathcal{C}_b^*$ is a subset of $\mathcal{C}_b'$, we have

$$BER_{\min}(\mathbf{H}) = \min_{\mathbf{b}' \in \mathcal{C}_b'} BER(\mathbf{b}', \mathbf{H}) \leq \min_{\mathbf{b}' \in \mathcal{C}_b^*} BER(\mathbf{b}', \mathbf{H}). \tag{4.46}$$

The average BER is bounded by

$$\overline{BER} \leq E[\min_{\mathbf{b}' \in \mathcal{C}_b^*} BER(\mathbf{b}', \mathbf{H})].$$

When the bit allocation $\mathbf{b}'$ is chosen from $\mathcal{C}_b^*$,all the R_b bits are allocated to the same symbol and this system becomes a beamforming system. For example, when $\mathbf{b}' = [\ R_b \ \ 0 \ \ \cdots \ \ 0\ ]^T$, the beamforming vector is the 0-th column of $\mathbf{F}'$. When we choose $\mathbf{b}' \in \mathcal{C}_b^*$ to minimize the BER, we are actually choosing the best beamforming vector from the columns of $\mathbf{F}'$ to maximize the received SNR. In other words, the equivalent codebook of beamforming vectors is $\mathcal{C}_f = \{\mathbf{f}_0', \mathbf{f}_1', \cdots, \mathbf{f}_{M_t-1}'\}$, where $\mathbf{f}_i'$ is the i-th column of $\mathbf{F}'$. From [40], we know such a beamforming system has diversity order equal to M_rM_t if the span of $\mathcal{C}_f$ is equal to $\mathbb{C}^{M_t}$. Because $\mathbf{F}'$ is an $M_t \times M_t$ unitary matrix, the span of $\mathcal{C}_f$ is the same as $\mathbb{C}^{M_t}$.Therefore the BA system has diversity order M_rM_t when codebook $\mathcal{C}_b'$ contains the vectors in (4.44). ∎

Chapter 5

Simulations

In our simulations, the channel is of the form

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{sp} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w\mathbf{R}_t^{1/2} \quad \text{for Ricean channel.}$$

and

$$\mathbf{H} = \mathbf{H}_w\mathbf{R}_t^{1/2} \quad \text{for no line of sight.}$$

and

$$\mathbf{H} = \mathbf{H}_w \qquad \text{for uncorrelated channel.}$$

Consider different channel case as following

Channel I Uncorrelated channel.

Channel II No line of sight with low correlation for $d_t = 2$, $\theta_t = 40°$.

Channel III No line of sight with high correlation for $d_t = 2$, $\theta_t = 8°$.

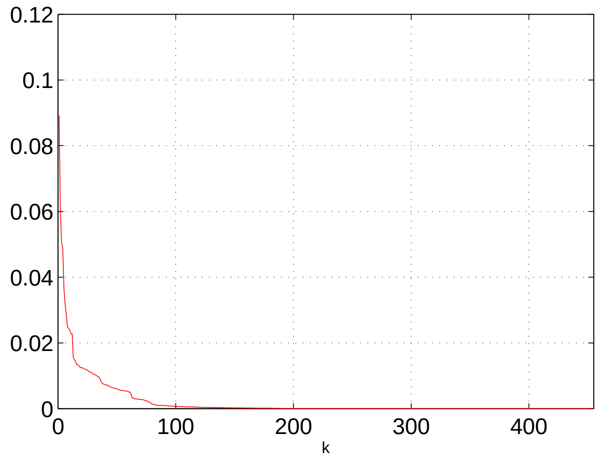
Channel IV Ricean channel with low correlation for $d_t = 2$, $\theta_t = 40°$, $d_r = 1$, $\theta_r = 20°$, $K = 5$.

Channel V Ricean channel with high correlation for $d_t = 1$, $\theta_t = 20°$, $d_r = 1$, $\theta_r = 10°$, $K = 3$.

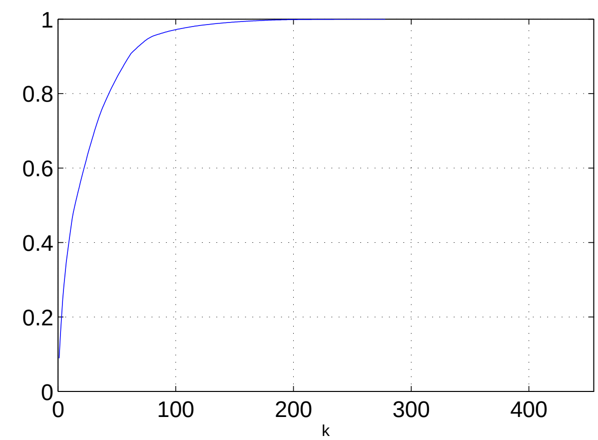
We have used 10^6 channel realizations in the Monte Carlo simulations. The error rates are computed using (2.3) for both linear and decision feedback receivers. For the decision feedback receiver, the detection order is determined using the criterion of maximizing the rate-normalized SNRs mentioned in Sec 4.3. Antennas with spacing d_t, d_r and plane-wave span an angular spread of θ_t, θ_r at transmitter and receiver respectively.

Example 1. Distribution of bit allocation vectors.

In this example, the Channel I is considered. The number of receive antennas M_r is 5, and the number of transmit antennas M_t is 4. we compute the empirical distribution of bit allocation vectors. For a given channel realization, the best bit allocation vector in the codebook is chosen using the BER criterion. The number of bits transmitted per channel use is $R_b = 12$ and the number of substreams that the transmitter and receiver can process is $M = 4$. The corresponding optimal precoder $\mathbf{F}$ is the identity matrix and the receiver is linear. The number of possible integer bit allocation vectors is 455. We include in the codebook all 455 integer bit allocation vectors. Fig. 5.1(a) shows the distribution of the bit allocation vectors, where the indexes of the vectors are ordered so that the probabilities are in decreasing order. The cdf (cumulative distribution function) is shown in Fig 5.1(b). We can see that some bit allocation vectors are far more probable than others. The probability of the 53 most probable bit allocation vectors is more than 99%. The distribution of the bit allocation vectors is highly skewed, rather than uniform. In all following examples with quantize bit allocation, we will choose the most probable 2^B bit allocation vectors obtained in experiments like this example and use them as codewords when the number of feedback bits is B.



(a)

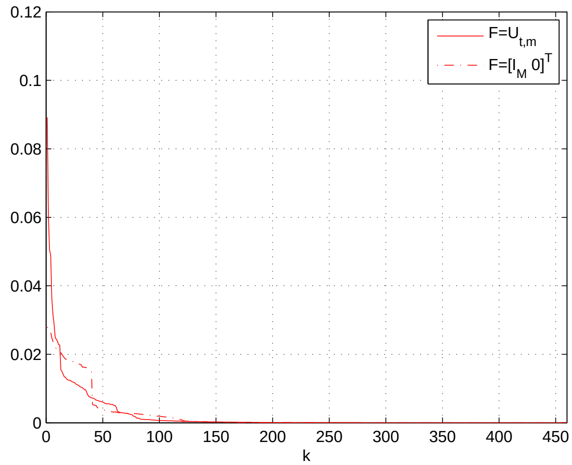


(b)

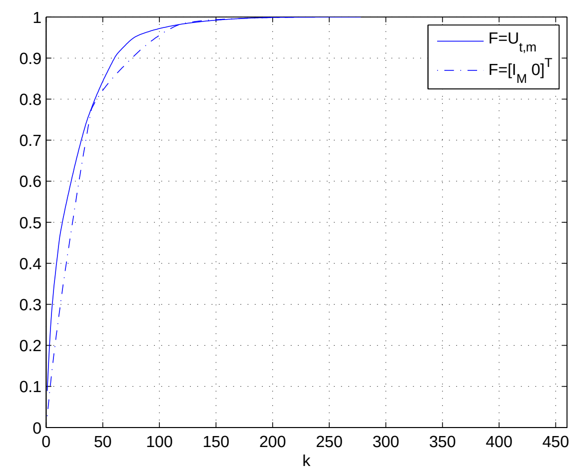
Figure 5.1: (a) Probability mass function of the bit allocation vectors for Channel I, $M_r = 5$, $M_t = 4$, $M = 4$, and Rb = 12; (b) corresponding cumulative distribution function.

Example 2. Precoder and distribution of bit allocation.

The correlated Channel II with zero mean is considered for $M_r = 4$, $M_t = 5$, $M = 4$. The number of bits transmitted per channel use is $R_b = 8$. We condider two type of the precoder $\mathbf{F} = \mathbf{U}_{t,M}$ and $\mathbf{F} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$ used, the receiver is linear. The number of possible integer bit allocation vector is 460. The codebook contains all 460 integer bit allocation vectors. Fig. 5.2(a) shows the distribution of the bit allocation vectors. The cumulative distribution function (cdf) is shown in Fig.



(a)



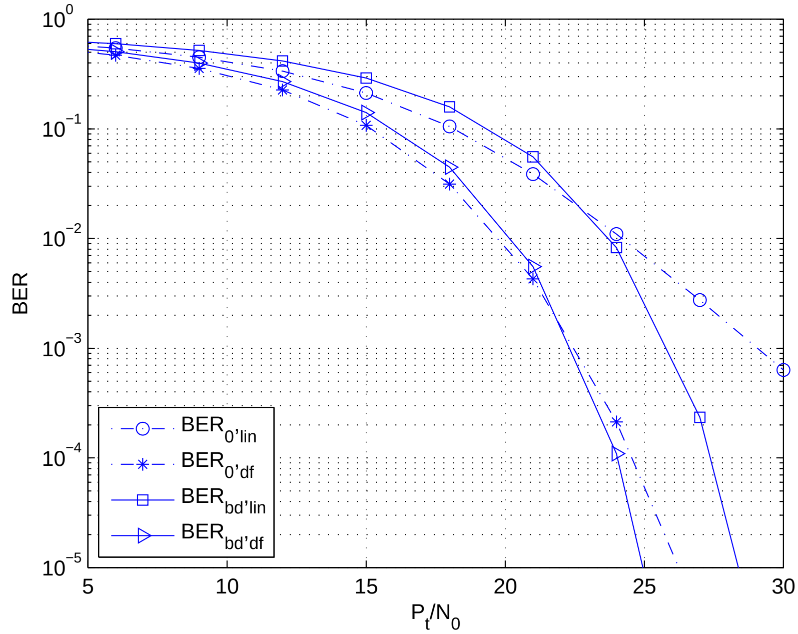
(b)

Figure 5.2: (a) Probability mass function of bit allocation vectors for Channel II, $M_r = 4, M_t = 5, M = 4$ and$R_b = 8$; (b) Corresponding CDF.

5.2(b). From Fig. 5.2(a) we can see when $\mathbf{F} = \mathbf{U}_{t,M}$ is used, the distribution of bit allocation vectors is more concentrated.

Example 3. BER bound.

In Fig. 5.3(a), Channel II is used. $M_r = 4$, $M_t = 5$, $M = 3$, $R_b = 10$, the precoder is $\mathbf{F}' = \mathbf{U}_t$. We show the BER bounds $\overline{BER}_{bd,lin}$ and$\overline{BER}_{bd,df}$. We have also computed BER_0 in (4.8) over 10^6 channels for a linear receiver and for a decision feedback receiver. The results are called, respectively, $\overline{BER}_{0,lin}$ and $\overline{BER}_{0,df}$. The gap between $\overline{BER}_{bd,lin}$ and $\overline{BER}_{bd,df}$ is around 3.5dB. We can see that the curve $\overline{BER}_{bd,df}$ is an upper bound for $\overline{BER}_{0,df}$ in low SNR and a lower bound for $\overline{BER}_{0,df}$ in high SNR, consistent with what we have shown in Sec. 4.2. The same can be observed for the case of linear receiver. In Fig. 5.3(b) Channel IV with both mean and covariance information is used. $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$. We use the approximation in (4.18) and choose $\mathbf{F}' = \widehat{\mathbf{U}}_t$. In. 5.4 shows the same set of curves. We can have conclusions similar to those for correlated Channel II with zero mean.



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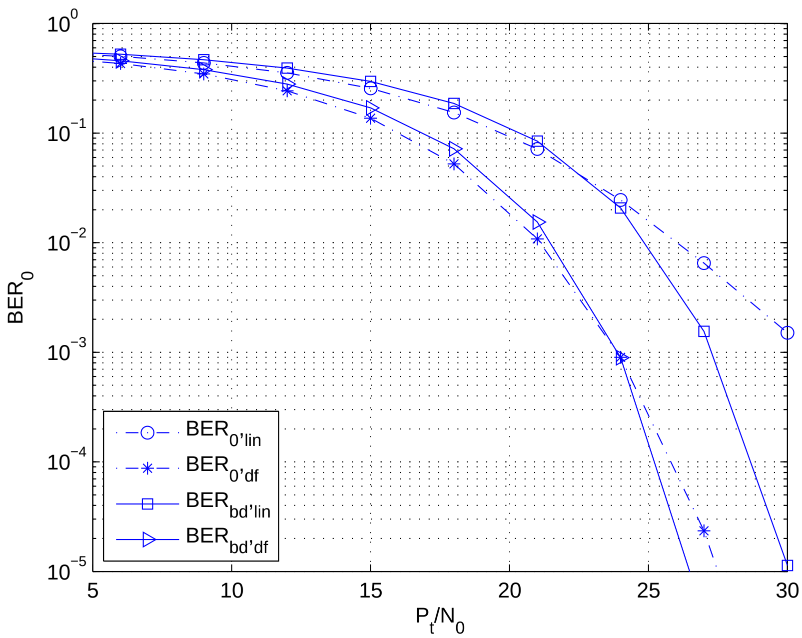
(a)

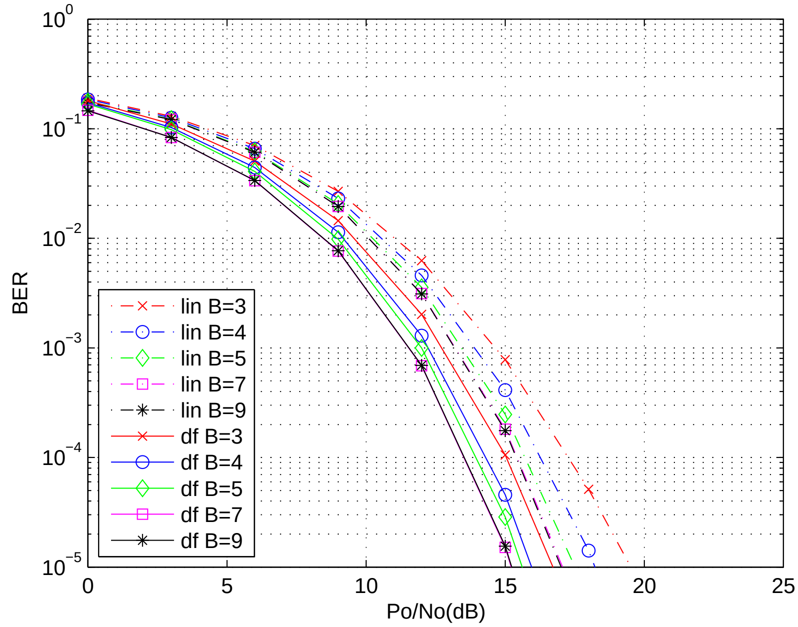


(b)

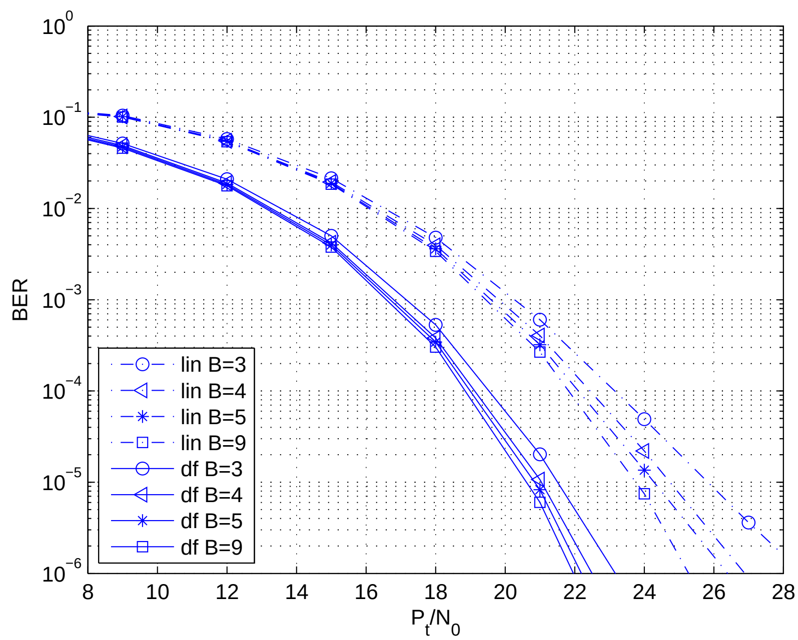
Figure 5.3: (a) BER bound for $M_r = 4$, $M_t = 5$, $M = 3$ and $R_b = 10$ for Channel II (b) BER bound for $M_r = 5$,$M_t = 4$,$M = 4$ and $R_b = 12$ for Channel IV

Example 4. BER for different feedback bits.

In Fig. 5.4(a), $M_r = 4$, $M_t = 5$, $M = 4$, $R_b = 8$, Channel II is considered, the precoder is $\mathbf{F}' = \mathbf{U}_t$. We shows the BER performance of the BA system for different number of feedback bits. The codewords are selected to minimize BER. The performance is shown for both linear and decision feedback receivers for different number of feedback bits. The BER is improved when the number of feedback bits B increases. We can see that BER of $B = 5$ is close to that of $B = 9$, in which case all the integer bit allocation codewords are used. Observe that the curves correspond to $B = 7$ and $B = 9$, are indistinguishable in the figure. We can understand this by examining the distribution plot in Fig. 5.2 The cdf is very close to one for $k \geq 150$. When we increase B from 7 to 8 to 9, the added codewords are almost never chosen so the performance has no improvement. Fig. 5.4(b) also shows BER of the BA system when Channel V is considered with $M_r = 5$, $M_t = 4$, $M = 4$ and $R_b = 12$. The precoder is chosen as $\mathbf{F}' = \widehat{\mathbf{U}}_t$. For the case $B = 9$ which considers all integer bit allocation codewords, the gain of the decision feedback receiver over the linear receiver is around 3.5dB, similar to the gap between $\overline{BER}_{bd,df}$ and $\overline{BER}_{bd,lin}$ observed in Fig. 5.3(a).



(a)

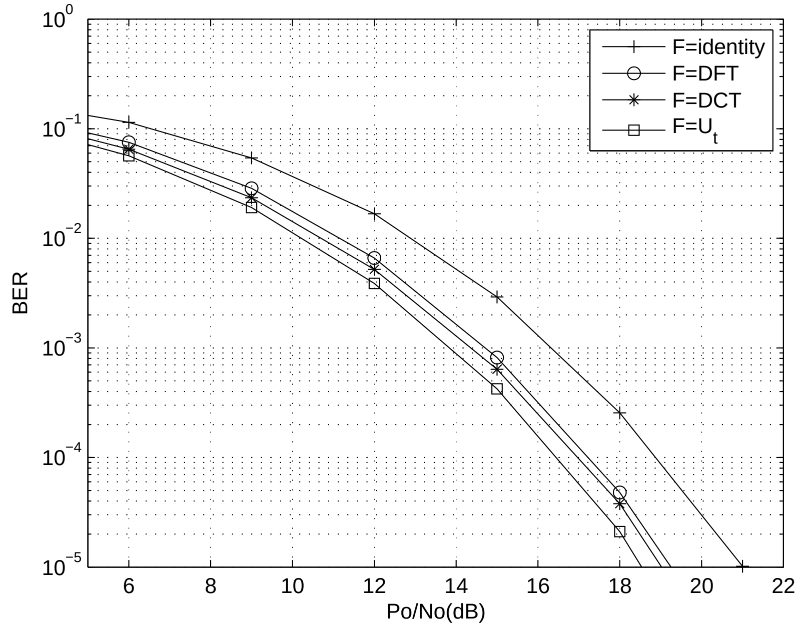


(b)

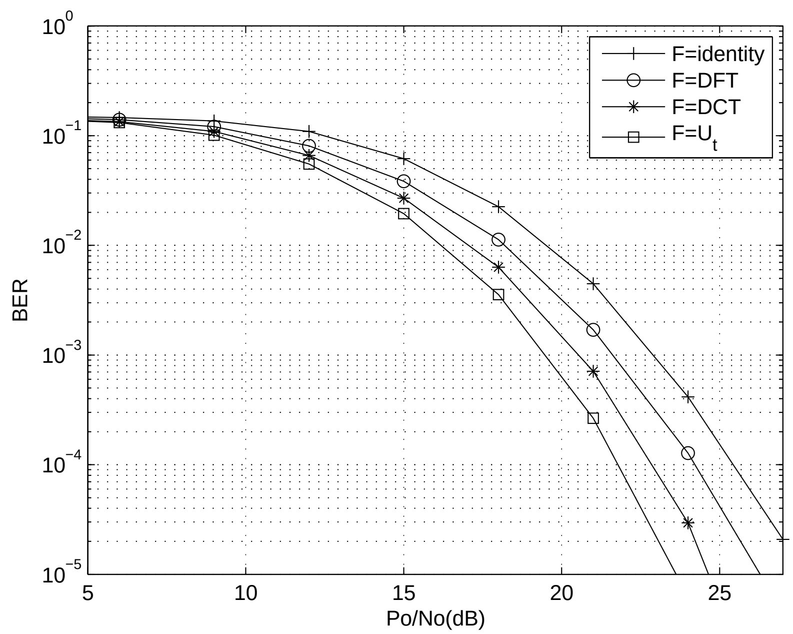
Figure 5.4: (a) Different feebback bits with $M_r = 4$, $M_t = 5$, $M = 4$, $R_b = 8$ for Channel II (b) Different feebback bits with $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ for Channel V

Example 5. BER for different Precoders.

In Fig. 5.5(a), $M_r = 4$, $M_t = 5$, $M = 4$, $R_b = 8$, $B = 9$ and channel III be considered. The BER plots are given for four different types of $M_t \times M_t$ precoders and decision feedback at receiver. (1) the identity matrix, (2) the normalized DFT matrix , (3) the DCT matrix and (4) $\mathbf{F} = \mathbf{U}_t$. We can see that $\mathbf{U}_t$ has the best performance among. Fig. 5.5(b) shows the same set of curves for four precoders with linear receiver. Channel IV be considered. It has the same result as covariance feedback case that optimal precoder is $\mathbf{F} = \widehat{\mathbf{U}}_t$.



(a)

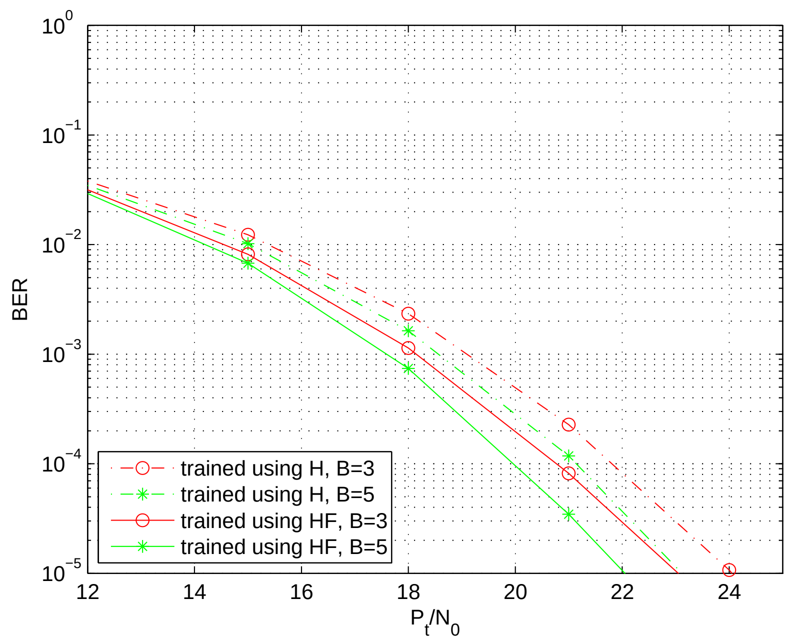


(b)

Figure 5.5: (a) BER for different precoder $M_r = 4$, $M_t = 5$, $M = 4$, $R_b = 8$ for Channel III. (b) BER for different precoder $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ for Channel IV.

Example 6. BER for different $\mathcal{C}_b$.

In Fig. 5.6, $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ and $\mathbf{F} = \mathbf{U}_t$, Channel II and linear receiver are considered. In this case we show BER for two codebook, one trained using $\mathbf{H}$ and one trained using $\mathbf{HF}$. Even though the precoder is chosen as $\mathbf{F} = \mathbf{U}_t$, the performance of the codebook trained using $\mathbf{HF}$ is better than the other for about 1dB for the same feedback rate. So we can conclude codebook training is important for system performance.

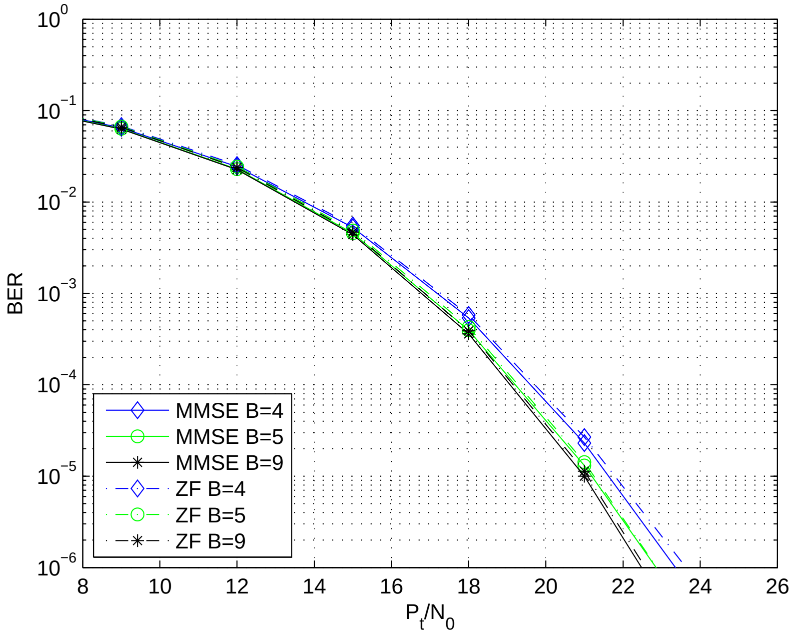


5.6

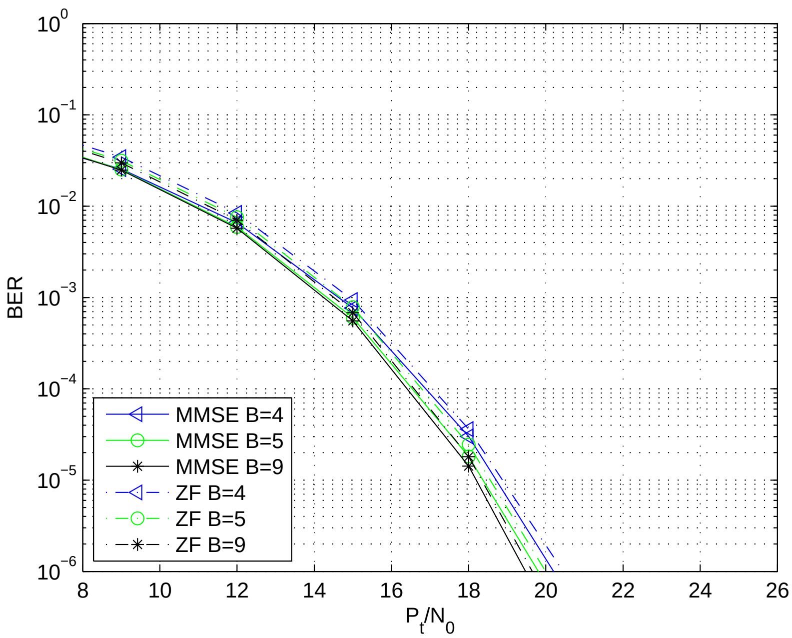
Figure 5.6: BER with different $\mathcal{C}_b$, $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ for Channel II

Example 7. BER for MMSE and ZF receivers.

In this case, $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ and $\mathbf{F} = \mathbf{I}_{M_t}$, Channel I is considered. We show the BER performance of MMSE and ZF receivers with linear and decision feedback receivers. Fig. 5.7(a) is linear receiver. In each case, the codebook is trained based the channel at receiver. From Fig. 5.7(a) we can see the ZF receiver is close to that of MMSE receiver. Fig. 5.7(b) show the two curves again when the receiver has decision feedback. We can draw conclusions similar to that for the linear receiver case.



(a)

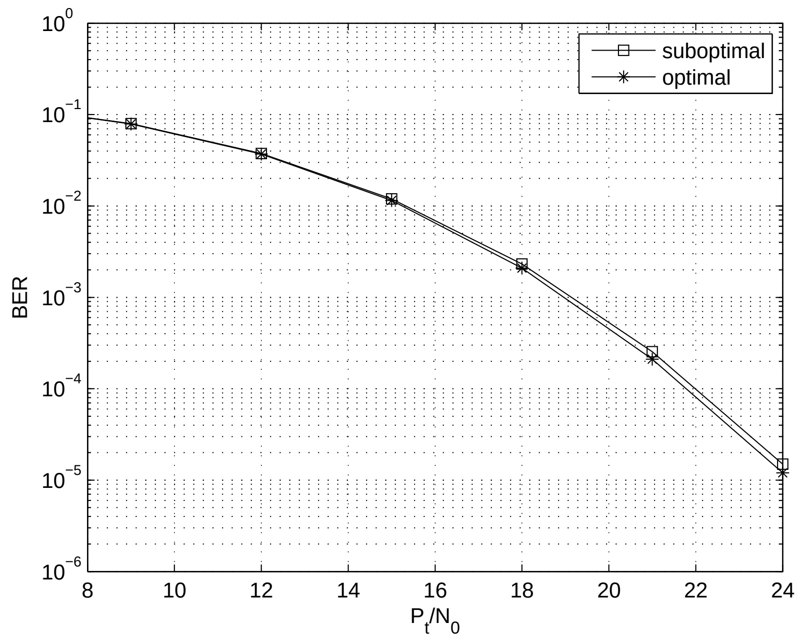


(b)

Figure 5.7: (a) BER for linear receiver, $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ for Channel I (b) BER for decision feedback receiver for Channel I.

Example 8. Codeword selection criterion.

In this example, $M_r = 3$, $M_t = 4$, $M = 3$, $R_b = 10$, Channel II is considered and linear receiver is used. We compare the results using the BER criterion and the maximin criterion. In the first case, the codeword in the codebook that leads to the minimum BER is chosen. In the second case, the suboptimal codeword is chosen by quantizing the optimal bit allocation vector using the maximin criterion $\mathbf{b} = \arg\max_{\widehat{\mathbf{b}}\in\mathcal{C}_b} \min(b_k^* - \widehat{b}_k)$ described in Sec. 4.3. The results for $B = 8$ are shown in Fig. 5.8. The BER using the suboptimal maximin criterion is close to that using the minimum BER criterion.

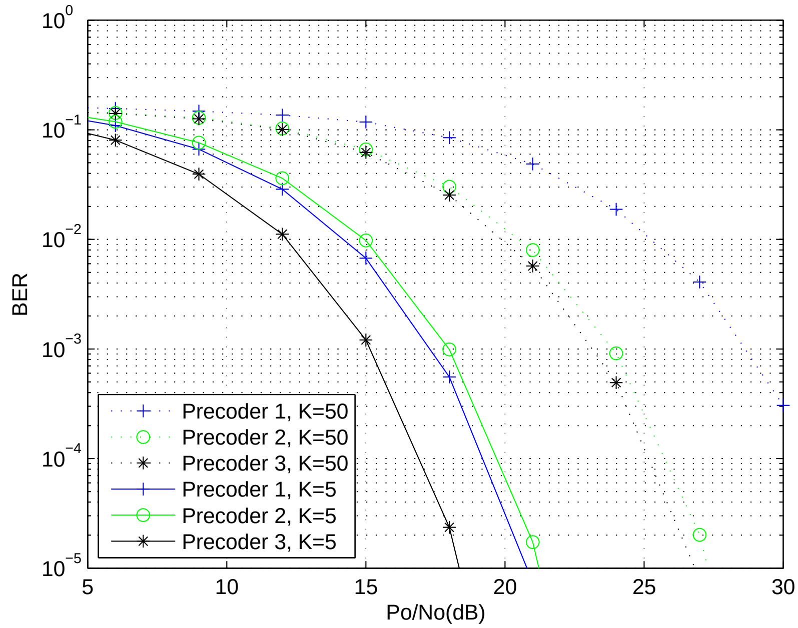


5.8

Figure 5.8: BER of BA system for $M_r = 3$, $M_t = 4$, $M = 3$, $R_b = 10$, Channel II is considered using the optimal BER criterion and suboptimal maximin criterion.

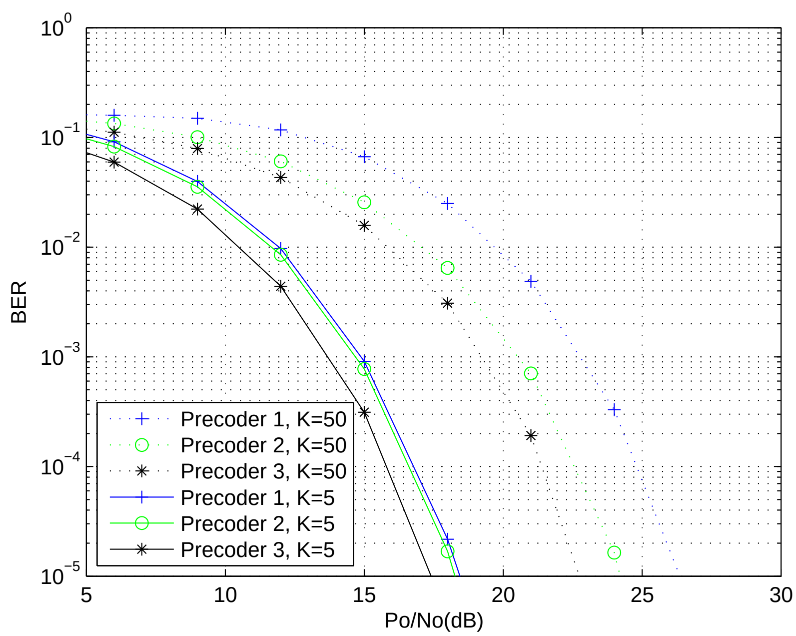
Example 9. BER for different K and precoder.

In this example $M_r = 3$, $M_t = 4$, $M = 3$, $R_b = 8$ and Channel IV with mean and covariance information is considered. The feedback bits is 8 and receiver is linear for Fig. 5.9. The BER plots are given for three types of $M_t \times M_t$ augmented precoders, (1) $\mathbf{F}' = eig(\mathbf{R}_t)$ which is the best precoder when there is no mean information. (2) The precoder is chosen as in (4.30), the optimal precoder when there is no correlation at transmitter, i,e. $\mathbf{R}_t = \mathbf{I}_M$ case in section 4.2.1. (3) $\mathbf{F}' = eig(\widehat{\mathbf{R}}_t)$. When the Ricean factor K is small, precoder 1 is better than precoder 2 and precoder is not as good for o large K. We also show the decision feedback receiver case in Fig. 5.10. The result is similar to Fig. 5.9. The BER performance is close for precoder 1 and precoder 2 when the Ricean factor K is small and precoder 2 is better than precoder 1 when the Ricean factor K is large. We can see the precoder 3 is better then the other two for small or large K.



5.9

Figure 5.9: $M_r = 3$, $M_t = 4$, $M = 3$, $R_b = 8$ with linear receiver for Channel IV.

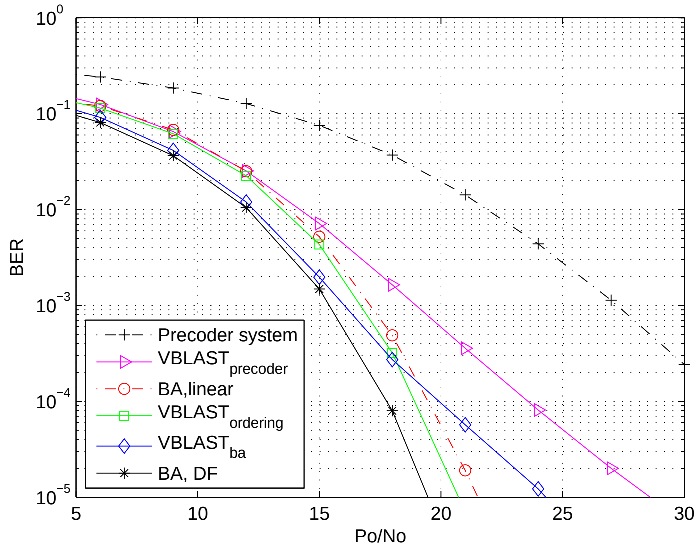


5.10

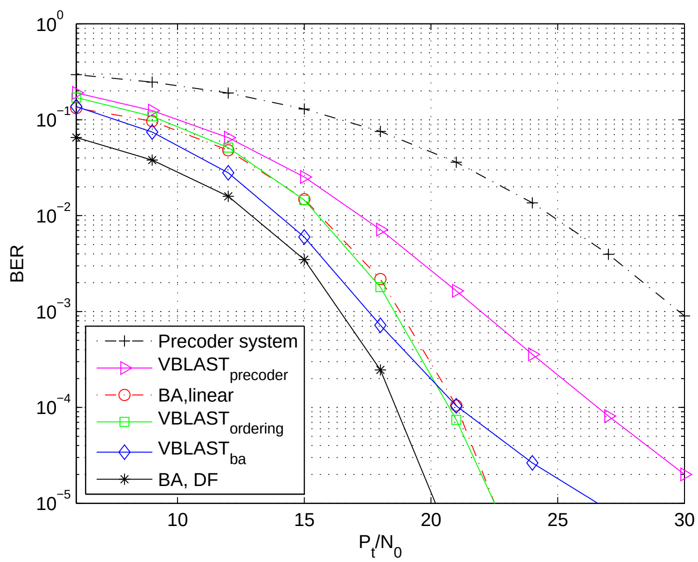
Figure 5.10: $M_r = 3$, $M_t = 4$, $M = 3$, $R_b = 8$ with decision feedback receiver for Channel IV.

Example 10. Comparisons of BER for $\mathbf{M}_t = \mathbf{M}$ case.

$M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ and Channel II is considered. In Fig. 5.11(a) we compare the BA system with the precoder system [4], in which the feedback is the index of the optimal precoder in the codebook and bits are uniformly loaded on all M symbols transmitted. In addition, we compare with the QR based system with bit allocation (VBLAST$_{ba}$) [21], the VBLAST system with feedback of ordering(VBLAST$_{ordering}$) [19]. The VBLAST$_{ordering}$ system in [19] feedback detection ordering for a fixed bit allocation. This is equivalent to having a codebook of all permutation of a single bit allocation vector. We also compare with VBLAST system with optimal precoder design (VBLAST$_{precoder}$) in [28]. The VBLAST$_{precoder}$ in [28] requires no instant feedback. It designs for precoder based on statistics of the channelfor minimizing MSE. We can see if system has no bit allocation i.e. VBLAST$_{precoder}$ and the precoder system, the BER performance is not as good. For VBLAST$_{ordering}$, the required number of feedback bits is $\log_2(4!) \approx 5$. The number of feedback bits is made as close to 5 as possible except VBLAST$_{precoder}$ system. For VBLAST$_{ba}$, the original codebook containing all integer vectors satisfying the sum rate constraint is trimmed by setting $b_1 \geq 2$ and $b_2, b_3, b_4 \geq 0$ as in [21], which results in a codebook of 35 codewords. For the precoder and BA systems, the codebook size is 32. The BER performance of the BA system with linear receiver is much better than of the the precoder system VBLAST$_{precoder}$ and is comparable to VBLAST$_{ordering}$ with decision feedback receiver. The VBLAST$_{ba}$ system has BER similar to the BA system with a decision feedback receiver in low SNR. In Fig. 5.11(b) we show the result for Channel V, high correlation case with mean information. We see that the BA system achieve a good performance due to statistical precoder design.



(a)

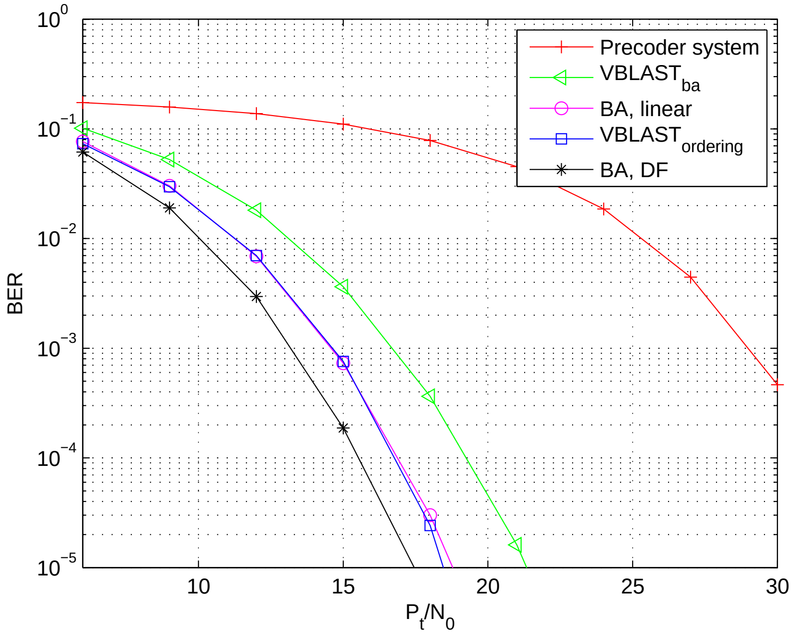


(b)

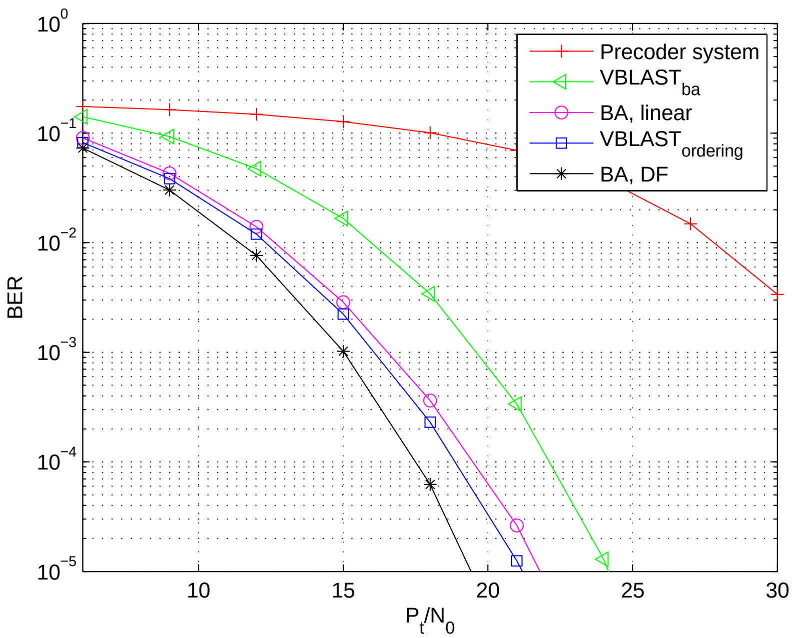
Figure 5.11: (a) Comparison of BER for $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ for channel II (b) $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$ for channel V.

Example 11. Comparisons of BER for $\mathbf{M}_t > \mathbf{M}$ case.

$M_r = 4$, $M_t = 4$, $M = 3$, $R_b = 8$ and Channel I is considered. In Fig. 5.12(a) we compare with VBLAST$_{ordering}$, VBLAST$_{ba}$ and precoder system. The VBLAST$_{precoder}$ system is not compared in this example because it can be used only when $M = M_t$. In the case, we use augmented precoder for BA system. For VBLAST$_{ordering}$, the required number of feedback bits is $\log_2(3!) \approx 3$. The number of feedback bits is made as close to 3 as possible for all other cases. For VBLAST$_{ba}$, the original codebook containing all integer vectors satisfying the sum rate constraint is trimmed by setting $b_1 \geq 2$ and $b_2, b_3 \geq 0$, which results in a codebook of 10 codewords. The BER performance of BA system with linear receiver is better than VBLAST$_{ba}$ due to the flexible codebook design and augmented precoder is used. We can also see the BA system with linear receiver is very close to VBLAST$_{ordering}$ which uses more complex decision feedback receiver in this case. Fig. 5.12(b) also show the high correlated case of Channel III. We can have conclusions similar to In Fig. 5.12(a).



(a)



(b)

Figure 5.12: (a) Comparison of BER for $M_r = 4$, $M_t = 4$, $M = 3$, $R_b = 8$ for channel I (b) $M_r = 4$, $M_t = 4$, $M = 3$, $R_b = 8$ for channel III.

Chapter 6

Conclusion

In this paper we considered the feedback of bit allocation for MIMO systems with limited feedback and the system is called a BA system. We first introduced system and channel model. Secondly, we derived the optimal unconstrained bit allocation for a given precoder. The optimal bit allocation is treated as a vector signal. Based on the results of optimal bit allocation and statistical of the channel, we can use a approximation distribution of statistical to design the statistical precoder for Ricean channel. For line of sight case, a non-approximate distribution of statistical to design the optimal precoder. Furthermore when the number of transmit antenna is larger than the number of symbols transmitted, augmented precoding improve the performance and the use of augmented precoding does not require additional feedback. We have also shown that the proposed BA system can achieve full diversity order. Simulations have demonstrated the proposed BA system achieves a nice good trade-off between performance and feedback rate.

Appendix

We now derive $[\mathbf{H}_{ess}]_{i,j}$ in (4.13).

Let $h(\mathbf{y}) = (\prod_{i=0}^{M-1} y_i)^{1/M}$ and Q(x) be defined as (2.2).

$$\frac{\partial}{\partial h(\mathbf{y})} f(h(\mathbf{y})) = \frac{\partial}{\partial h(\mathbf{y})} Q(1/\sqrt{\partial h(\mathbf{y})}) = \frac{\partial}{\partial h(\mathbf{y})} \int_{\frac{1}{\sqrt{h(\mathbf{y})}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

$$= \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2h(\mathbf{y})}} h(\mathbf{y})^{-3/2} \triangleq f'(h(\mathbf{y}))$$

we have $\frac{\partial f(h(\mathbf{y}))}{\partial y_i} = \frac{\partial f(h(\mathbf{y}))}{\partial h(\mathbf{y})} \frac{\partial h(\mathbf{y})}{\partial y_i}$, $\quad \frac{\partial h(\mathbf{y})}{\partial y_i} = \frac{1}{M} y_i^{-1} h(\mathbf{y})$

$$\frac{\partial f(h(\mathbf{y}))}{\partial y_i \partial y_j} = \frac{\partial}{\partial y_j}\left(\frac{\partial f(h(\mathbf{y}))}{\partial h(\mathbf{y})} \frac{\partial h(\mathbf{y})}{\partial y_i}\right) = \frac{\partial f(h(\mathbf{y}))}{\partial h(\mathbf{y}) \partial y_j} \frac{\partial h(\mathbf{y})}{\partial y_i} + \frac{\partial f(h(\mathbf{y}))}{\partial h(\mathbf{y})} \frac{\partial h(\mathbf{y})}{\partial y_i \partial y_j} \quad (6.1)$$

and $\frac{\partial f'(h(\mathbf{y}))}{\partial y_j} = f'(h(\mathbf{y})) \frac{\partial}{\partial y_j}(\frac{-1}{2} h(\mathbf{y})^{-1}) + (\frac{3}{-2M}) h(\mathbf{y})^{-2/5} y_j^{-1} h(\mathbf{y}) f'(h(\mathbf{y}))$

$$= f'(h(\mathbf{y}))(\tfrac{1}{2M} y_j^{-1} h^{-1}(\mathbf{y}) - \tfrac{3}{2M} y_j^{-1}), \qquad \text{substituting (6.1)}$$

$$\frac{\partial f(h(\mathbf{y}))}{\partial y_i \partial y_j} = f'(h(\mathbf{y}))\left(\frac{1}{2M^2} - \frac{3}{2M^2} h(\mathbf{y})\right) y_i^{-1} y_j^{-1} + f'(h(\mathbf{y})) \frac{1}{M^2} y_i^{-1} y_j^{-1} h(\mathbf{y})$$

$$= \frac{0.5}{M^2} f'(h(\mathbf{y})) y_i^{-1} y_j^{-1} (1 - h(\mathbf{y})) \qquad , j \neq i \qquad (6.2)$$

$$\frac{\partial f(h(\mathbf{y}))}{\partial y_i \partial y_j} = f'(h(\mathbf{y}))\left(\frac{1}{2M^2} y_i^{-1} h(\mathbf{y})^{-1} - \frac{3}{2M^2} y_i^{-1}\right) y_i^{-1} h(\mathbf{y}) + f'(h(\mathbf{y})) \frac{1}{M}\left(-y_i^{-2} h(\mathbf{y}) + \frac{1}{M} y_i^{-2} h(\mathbf{y})\right)$$

$$= f'(h(\mathbf{y})) \frac{1}{2M^2} y_i^{-2} (1 - 3h(\mathbf{y}) - 2Mh(\mathbf{y}) + 2h(\mathbf{y}))$$

$$= f'(h(\mathbf{y})) \frac{0.5}{M^2} y_i^{-2} (1 - (2M+1) h(\mathbf{y})) \qquad , j = i \qquad (6.3)$$

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