國立交通大學

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碩 士 論 文

具關連性多輸入多輸出系統之位元分配與傳送器設計

Joint Design of Statistical Precoder and Statistical Bit Allocation for Correlated MIMO Channels

研 究 生：蕭君維

指導教授：林源倍 博士

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摘要

在本篇論文中，我們提出對於輸入端具關連性多輸入多輸出系統，其位元配置與傳送器的設計方法。首先我們根據具關連性通道之錯誤變異係數的統計特性推導出最佳位元配置與平均錯誤率下限，接著我們設計出能最小化平均錯誤率下限之傳送器與最適合的通道數目。模擬結果顯示我們所提出的方法可以有效降低具關連性多輸入多輸出系統之錯誤率。

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Joint Design of Statistical Precoder and Statistical Bit Allocation for Correlated MIMO Channels

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Abstract

In this thesis we consider the design of statistical precoder and statistical bit allocation for multi-input multi-output (MIMO) systems over correlated channels. We assume the correlated channel is slow fading and full channel state information is available at the receiver, while only the statistics of the correlated channels is assumed to be known at transmitter. We will first derive the statistical bound of bit error rate (BER) and the corresponding optimal real bit allocation. Based on this statistical BER bound, the optimal unitary statistical precoder is derived both for linear and decision feedback receiver. Second, the statistical integer bit allocation is designed the greedy algorithm. Finally, different number of substreams will be considered and selected by statistical BER bound. Simulations show lower BER can be achieved when optimal number of substreams is selected for correlated channels.

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Chapter 1

Introductions

In recent years, MIMO wireless communication systems have received substantial attention. When full channel state information (CSI) is available at transmitter and receiver, many criteria have been considered in the transceiver designs, e.g., [1]- [19]. When the receiver is linear, optimal transceivers that maximize the mutual information are proposed in [1]- [5], transceiver designs that minimize mean square error are considered in [6]- [9] and optimal transceivers that minimize the BER are derived in [10]- [13]. When the receiver is nonlinear, the Vertical Bell Labs Layered Space-Time (V-BLAST) scheme [14] in which uncoded data streams are transmitted from each transmitter antenna, and detected at the receiver using nulling and successive interference cancellation. To minimize the probability of error, the order of detection in V-BLAST is based on the post detection signal-to-noise ratio (SNR) the calculation of which renders the reception procedure computationally demanding. In [15] and [16], a framework is developed for jointly designing channel-dependent ordered decoding at the receiver and decoding order-dependent rate/power allocation at the transmitter. In [14]- [16], the designs are without precoding techniques. The precoding techniques for nonlinear receiver are considered in [17]- [19]. Precoder designs that minimizes mean square error (MSE) are derived in [17]. Several precoder design criteria are considered in [18], e.g., minimizing MSE or transmission power. In [19], the precoder is jointly optimized with the bit allocation.

However, full CSI at the transmitter is often not possible. Instead, partial CSI at the transmitter could be available by numerical channel realizations or already known channel statistics, like approximate capacity distribution in [20] and [21] or averaged mean square error in [22] and [24]. Based on this situation, there have been lots of research on optimizing system performance with statistical transceiver or bit allocation design. In [25], the precoder design with linear receiver by minimizing the upper bound of statistical joint error probability. In [29] and [30], a unified frame work considers the design problem when imperfect CSI consists of the channel mean and covariance matrix or, the channel estimate and the estimation error covariance matrix. The transceiver design is based on a general cost function of the average MSE as well as a design with individual MSE based constraints. In [31], for given long term statistical channel information feedback and rank deficiency MIMO channel, the precoder is constructed based on the criterion of minimizing the pair-wise error probability bound. Also precoder for minimizing error probability is derived in [32]. In [33], the precoder designed with decision feedback receiver is also based on minimizing sum of statistical MSEs. Then the precoder design in [33] has been extended to more general version in [35]. In [35], other kinds of design problems consist of minimizing statistical symbol error rate, statistical bit error rate and statistical outage probability error are considered and a closed form solution for the statistical bit allocation weighted MSEs is derived to reduce the complexity. In these precoder designs, an uniform or given bit allocation is assumed and sophisticated convex optimization techniques or inequality properties are needed in [36], [39], [37] and [38]. There are also some statistical bit allocation designs without precoding technique. In [40], the statistical bit allocation is considered by minimizing the statistical outage probability errors obtained by numerical channel realizations. In [42], the optimal transmission antenna subset is derived by maximizing the statistical minimum substream SNR. After choosing the transmit antenna set, the bit allocation is obtained using numerical channel realizations.

In this thesis, we consider the statistical design problem at the transmitter. We assume the statistics of transmit correlation matrix is available at transmitter and full CSI is available at receiver. Previous works have shown the methods of statistical precoder design or statistical bit allocation design. Our goal is jointly designing the optimal precoder, bit allocation and number of substreams. Linear and decision feedback receiver are both considered. For given number of substreams M, we derive the statistical BER bound $\phi_M(\mathbf{b})$ first. The statistical BER bound $\phi_M(\mathbf{b})$ achieves the minimum $\Omega_M(\mathbf{b})$ while the optimal real bit allocation is used. Applying this optimal bit allocation, the optimal precoder is derived by minimizing $\Omega_M(\mathbf{b})$. In practical applications, the bit allocation should be nonnegative integers.

After the convex property of statistical objective functions are examined, greedy algorithm can be used to find the optimal nonnegative integer bit allocation. It is clearly that the optimal nonnegative integer bit allocation is close to the optimal real bit allocation. Also the statistical BER bound $\phi_M(\mathbf{b})$ is useful for determining the optimal number of substreams. Thus the optimal precoder, bit allocation and number of substreams are obtained. Finally, simulations show our design have well performance.

1.1 Outline

- Chapter 2: The system model is presented.
- Chapter 3: Previous works are reviewed in this chapter. Optimum statistical precoder with linear receiver [25] is reviewed in sec3.1. Optimum statistical precoders with decision feedback receiver [33] [35] are given in sec3.2. Statistical bit allocations for decision feedback receiver [40] [42] are reviewed in sec3.3.
- Chapter 4: The proposed staP-BA system for correlated channel is given. The statistical BER bound and the optimal bit allocation are derived in sec4.1. The optimal statistical precoder under transmit power constraint is developed in sec4.2. Various statistical integer bit allocation design methods are presented in sc4.3.
- Chapter 5: Simulation examples are presented in this chapter.
- Chapter 6: A conclusion is given in this chapter.

1.2 Notations

- Boldfaced lower case letters represent vectors and boldfaced upper case letters are reserved for matrices. The notation $\mathbf{A}^\dagger$ denotes transpose-conjugate of $\mathbf{A}$.
- The function $E[\mathrm{y}]$ denotes the expected value of a random variable y.
- The notation $\mathbf{I}_m$ is used to represent the $m \times m$ identity matrix.
- The notation $\mathrm{tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$.

Chapter 2

General System Model

2.1 Statistical Bit Allocation System Model

Consider the MIMO system with M_t transmit antennas and M_r receive antennas in Figure 2.1. The channel is modeled by an $M_r \times M_t$ memoryless matrix $\mathbf{H}$ with $M_r \times 1$ channel noise vector $\mathbf{q}$. We assume the channel is slow fading so that the channel does not change during each channel use. The noise vector $\mathbf{q}$ is assumed to be additive white Gaussian with zero mean, unit variance and $E[\mathbf{q}\mathbf{q}^\dagger] = N_0\mathbf{I}_{M_r}$. The channel considered in this thesis is of the form

$$\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2}, \tag{2.1}$$

where $\mathbf{H}_w$ is an $M_r \times M_t$ matrix whose elements are independent Gaussian random variables with zero mean and unit variance. The matrix $\mathbf{R}_t$, of dimensions $M_t \times M_t$, is called the transmit correlation matrix. In this case, the rows of $\mathbf{H}$ are independent and each has autocorrelation matrix equal to $\mathbf{R}_t$. Let the eigen decomposition of $\mathbf{R}_t$ be

$$\mathbf{R}_t = \mathbf{U}_t^\dagger \mathbf{\Lambda}_t \mathbf{U}_t,$$

where $\mathbf{\Lambda}_t$ is a diagonal matrix and the diagonal elements $\lambda_{t,i}$ are the eigenvalues of $\mathbf{R}_t$. Let the diagonal elements of $\lambda_{t,i}$ be ordered such that $\lambda_{t,0} \geq \lambda_{t,1} \geq \ldots \geq \lambda_{t,M_t-1}$ and assume $\lambda_{t,M-1} > 0$.

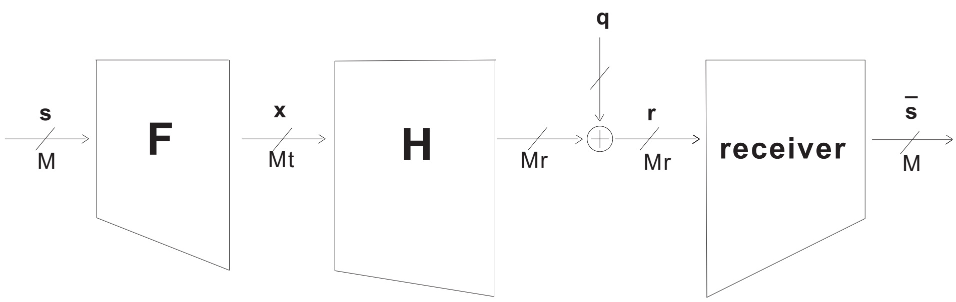


Figure 2.1: The system model of MIMO transmission system.

Suppose the transmitter and receiver can process M substreams of symbols, where $M \leq min(M_t, M_r - 1)$. The spatial multiplexing precoder $\mathbf{F}$ is an $M_t \times M$ matrix with orthonormal columns. The input vector $\mathbf{s}$ is an $M \times 1$ vector consisting of modulation symbols s_k, for $k = 0, 1, \ldots, M-1$, carrying b_k bits. The number of bits transmitted per channel use is thus

$$R_b = \sum_{k=0}^{M-1} b_k.$$

The symbols s_k are assumed to be uncorrelated with zero mean and unit variance, $E[\mathbf{s}\mathbf{s}^\dagger] = \mathbf{I}_M$. The total transmission power is constrained to be P_t. Thus, we have $tr(E[\mathbf{x}\mathbf{x}^\dagger]) = tr(E[\mathbf{F}\mathbf{s}\mathbf{s}^\dagger\mathbf{F}^\dagger]) = tr(\mathbf{F}\mathbf{F}^\dagger) \leq P_t$.

2.2 Receiver Design

In this thesis, we will consider two types of zero forcing receivers, linear and decision feedback receivers.

2.2.1 Linear receiver

When the receiver is linear and zero forcing, the receiver is an $M \times M_r$ matrix. We denote it as $\mathbf{G}$,

$$\mathbf{G} = (\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger.$$

[23] The receiver output is $\bar{\mathbf{s}} = \mathbf{G}\mathbf{r}$. Define the error vector at the output of the receiver as $\mathbf{e} = \bar{\mathbf{s}} - \mathbf{s}$. In this case, the error vector $\mathbf{e}$ has autocorrelation

matrix $\mathbf{R}_e = N_0 E[\mathbf{e}\mathbf{e}^\dagger]$ given by [23]

$$\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}.$$

We can obtain the kth average subchannel error variance $\bar{\sigma}^2_{e_k}$ by averaging the above error covariance matrix over the correlated channel. Let the ith column of $\mathbf{H}^{dag}$ be $\mathbf{g}_i$, then the autocorrelation matrix of $\mathbf{g}_i$ is equal to $\mathbf{R}_t$. It is known that $\mathbf{H}^\dagger\mathbf{H} = \sum_{k=0}^{M-1}\mathbf{g}_i\mathbf{g}_i^\dagger$ has a complex Wishart distribution with M_r degrees of freedom, denoted as $\mathcal{W}_{M_t}(R_t, M_r)$ [24]. If $\mathbf{F}$ is a nonsingular matrix then the matrix $\mathbf{R}_e^{-1} = N_0^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}$ has a Wishart distribution $\mathcal{W}_M(N_0^{-1}\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}, M_r)$. It has been shown in [22] that when a matrix $\mathbf{C}$ is of Wishart distribution $\mathcal{W}_p(\mathbf{A}, t)$ with $t > p$, then $E[\mathbf{C}^{-1}] = \frac{1}{t-p}\mathbf{A}^{-1}$. Using this result, $\overline{\mathbf{R}}_e = E[\mathbf{R}_e]$ is given by

$$\overline{\mathbf{R}}_e = \frac{N_0}{M_r - M}(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F})^{-1}, \text{ where } \mathrm{M_r} > \mathrm{M}. \tag{2.2}$$

2.2.2 Decision feedback receiver

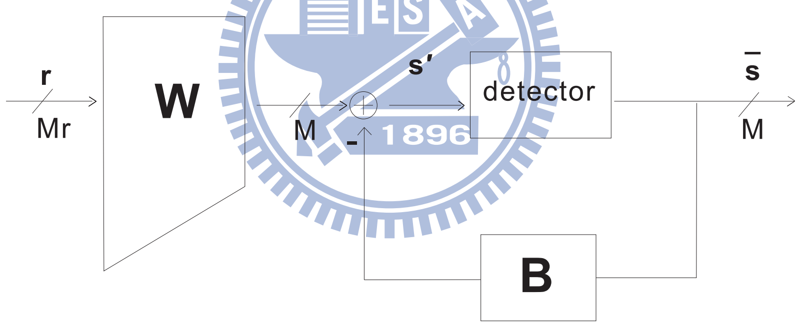


Figure 2.2: Block diagram of the decision feedback receiver based on QR decomposition.

To consider a decision feedback receiver, we can use the receiver structure in Figure 2.2 based on the QR decomposition of $\mathbf{HF}$ [33]. This corresponds to the case of a reverse detection ordering which detects from the Mth to the 1st subchannel. Let the QR decomposition of $\mathbf{HF}$ be $\mathbf{QR}$, where $\mathbf{Q}$ is an $M_r \times M$ matrix with orthonormal columns and $\mathbf{R}$ is an $M \times M$ upper triangular matrix with $[\mathbf{R}]_{ii} = r_{ii}$. The feedforward matrix $\mathbf{W}$ and feedback matrix $\mathbf{B}$ are given respectively by

$$\mathbf{W} = \text{diag}\left(r_{00}^{-1}, r_{11}^{-1}, \ldots, r_{M-1,M-1}^{-1} \right) \mathbf{Q}^{\dagger},$$

$$\mathbf{B} = \text{diag}\left(r_{00}^{-1}, r_{11}^{-1}, \ldots, r_{M-1,M-1}^{-1} \right) \mathbf{R} - \mathbf{I}_M.$$

Assuming there is no error propagation, the kth subchannel error $e_k = s'_k - s_k$ has variance $\sigma_{e_k}^2 = N_0 r_{kk}^{-2}$, for $k = 0, 1, \ldots, M-1$. The error variance averaged over the random channel is $\overline{\sigma}_{e_k}^2 = E[\sigma_{e_k}^2] = N_0 E[r_{kk}^{-2}]$. The value $E[r_{kk}^{-2}]$ has been shown to be related to the Cholesky decomposition of $\mathbf{F}^{\dagger}\mathbf{R}_t\mathbf{F}$ in [33] when $M_r > M$. Let the Cholesky decomposition of $\mathbf{F}^{\dagger}\mathbf{R}_t\mathbf{F}$ be $\mathbf{LDL}^{\dagger}$, where $\mathbf{L}$ is a lower triangular matrix with unity diagonal elements and $\mathbf{D}$ is diagonal. Then

$$\overline{\sigma}_{e_k}^2 = N_0 E[r_{kk}^{-2}] = \frac{N_0 d_{kk}^{-1}}{M_r - k - 1}, \; k = 0, 1, \ldots, M-1, \tag{2.3}$$

where d_{kk} is the kth diagonal element of $\mathbf{D}$.

Bit Error Rate The inputs s_k are b_k-bits QAM symbols, the kth symbol error rate can be approximate by [47]

$$SER_k = 4(1 - 2^{-b_k/2}) Q\left(\sqrt{\frac{3}{(2^{b_k} - 1)\sigma_{e_k}^2}} \right). \tag{2.4}$$

When gray code is used, then BER can be approximated by $BER_k \approx SER_k/b_k$. Then

$$BER \approx \frac{1}{R_b} \sum_{k=0}^{M-1} b_k BER_k \approx \frac{1}{R_b} \sum_{k=0}^{M-1} SER_k. \tag{2.5}$$

Other Detection Orderings

- **Forward ordering**
  Similar to the reverse ordering, the detection order of forward ordering is fixed but detection starts from the 1st to the Mth subchannel.

The receiver structure is similar to that in Figure 2.2 and we use QL decomposition to replace the QR decomposition. Let the QL decomposition of $\mathbf{HF}$ be $\mathbf{QL}_q$, where $\mathbf{Q}$ is an $M_r \times M$ matrix with orthonormal columns and $\mathbf{L}_q$ is an $M \times M$ lower triangular matrix with $[\mathbf{L}_q]_{ii} = l_{q,ii}$. The feedforward matrix $\mathbf{W}$, feedback matrix $\mathbf{B}$ and $\overline{\sigma}^2_{e_k}$ in [35] are given by

$$\mathbf{W} = \mathbf{Q}\ \mathrm{diag}\Big(l^{-1}_{q,00}, l^{-1}_{q,11}, \ldots, l^{-1}_{q,M-1M-1}\Big),$$

$$\mathbf{B} = \mathrm{diag}\Big(l^{-1}_{q,00}, l^{-1}_{q,11}, \ldots, l^{-1}_{q,M-1M-1}\Big)\ \mathbf{L} - \mathbf{I}_M$$

Let the Cholesky decomposition of $\mathbf{F}^{\dagger}\mathbf{R}_t\mathbf{F}$ be $\mathbf{L}_1^{\dagger}\mathbf{D}_1\mathbf{L}_1$, where $\mathbf{L}_1$ is a lower triangular matrix with unity diagonal elements and $\mathbf{D}_1$ is diagonal. And the error variance averaged over the random channel is

$$\overline{\sigma}^2_{e_k} = N_0 E[l^{-2}_{kk}] = \frac{N_0 d^{-1}_{1,kk}}{M_r - M + k + 1},\ k = 0, 1, \ldots, M-1, \qquad (2.6)$$

where $d_{1,kk}$ is the kth diagonal element of $\mathbf{D}_1$.

- **VBLAST ordering** [14]
  In the VBLAST system, the detection ordering is not fixed. It is based on the current channel. The decision feedback receiver can be described as a recursive procedure [14]. First initializes $\mathbf{r}_0 = \mathbf{r}$, $\mathbf{A}_0 = \mathbf{HF}$ and $i = 0$. The steps in the recursions are as follows. (1) Let $\mathbf{G}_i$ be the Moore-Penrose inverse of $\mathbf{A}_i$. Find the row vector of $\mathbf{G}_i$ that has the smallest 2-norm. Call the index of the row vector n_i and the row vector $\mathbf{w}_i$. (2) Compute $y_i = \mathbf{w}_i\mathbf{r}_i$, apply symbol detection on y_i, and call the output $\overline{s}_{n_i}$. (3) Subtract from $\mathbf{r}_i$ the contribution of the n_ith subchannel, $\mathbf{r}_{i+1} = \mathbf{r}_i - \overline{s}_{n_i}\mathbf{a}_{n_i}$, where $\mathbf{a}_{n_i}$ is the n_ith column of $\mathbf{A}_0$, and zero the n_ith column of $\mathbf{A}_i$ to obtain $\mathbf{A}_{i+1}$. ρ_{n_i} is maximized in the nth stage and the post detection SNR of the n_ith subchannel is $\rho_{n_i} = \frac{P_t}{MN_0||\mathbf{w}_i||^2}$, hen all the subchannels are of the same constellation. In this case, the above procedure is optimal in the sense that the worst subchannel error rate is minimized.

- **Rate-Normalized-SNR ordering**
  The detection procedure of rate-normalized-SNR ordering is similar to

that of VBLAST [14]. The only difference is that the rate normalized SNR

$$\rho_{n_i} = \frac{P_t}{MN_0(2^{b_{n_i}} - 1)||\mathbf{w}_i||^2}$$

is maximized.

- **Greedy QR ordering** [15]
  The algorithm is proposed in [15] for efficient ordering using QRE decomposition. The recursive procedure starts by minimizing the error variance of last detection layer by a permutation matrix $\mathbf{\Gamma}_1$ and unitary matrix $\mathbf{Q}_1$. Then we repeat the same procedure to find $\mathbf{\Gamma}_2, \ldots, \mathbf{\Gamma}_{M_t}$ and $\mathbf{Q}_2, \ldots, \mathbf{Q}_{M_t}$. At last, this ordered QR decomposition of $\mathbf{H}$ has the following form $\mathbf{H\Gamma} = \mathbf{QR}$, where $\mathbf{\Gamma} = \mathbf{\Gamma}_1\mathbf{\Gamma}_2 \cdots \mathbf{\Gamma}_{M_t}$ is a permutation matrix, $\mathbf{Q} = \mathbf{Q}_1\mathbf{Q}_2 \cdots \mathbf{Q}_{M_t}$ is an unitary matrix with orthonormal columns and $\mathbf{R}$ is an upper triangular matrix. The permutation matrix $\mathbf{\Gamma}$ determines the detection ordering.

Chapter 3

Previous Works of Statistical Designs

In this chapter, previous works on statistical precoder and bit allocation are reviewed. Section 3.1 recaps the statistical prcoder design with linear zero forcing receiver proposed in [25] and [26]. In section 3.2, we present the method of designing optimal statistical zero forcing decision feedback precoder are proposed in [33] and [35] for equal or given bit allocation. In section 3.3, the designs of statistical bit allocation without precoder were proposed in [40] and [42]. In all sections, full knowledge of channel state information is available at receiver. At the transmitter, however, only statistical characteristics of the channel are available.

3.1 Statistical Precoders for Linear Receivers

In this subsection, we review the statistical precoder proposed in [25] for linear zero forcing receiver. In [25], the system model is the same as that in section 2.2.1. The bit allocation is uniform and the target bit rate is a multiple of M. Each symbol carries R_b/M bits. Using the results from [27] and [28], the average probability of a symbol error $P_{e,k}$ on subchannel k can be bounded by

$$P_{e,k} \leq \beta \left(N_0 [(\mathbf{F}^\dagger \mathbf{R}_t \mathbf{F})^{-1}]_{kk} \right)^{M_r - M + 1},$$

where β is a constant depending on the modulation. In order to solve the optimal $\mathbf{F}$ that optimizes the cumulative error probability of all subchannels, the equivalent constrained optimization problem is formulated as

$$\begin{array}{ll} \underset{\mathbf{F}}{\text{minimize}} & \sum_{k=0}^{M-1}\left([(\mathbf{F}^{\dagger}\mathbf{R}_t\mathbf{F})^{-1}]_{kk}\right)^{M_r-M+1} \\ \text{subject to} & \text{tr}(\mathbf{F}\mathbf{F}^{\dagger}) = \text{Pt} \end{array}$$

The optimal statistical precoder is of the form

$$\mathbf{F}_{KSVR} = \sqrt{\frac{P_t}{Tr(\mathbf{\Lambda}_{t,M}^{-1/2})}}\mathbf{U}_{t,M}\mathbf{\Lambda}_{t,M}^{-1/4}\mathbf{W}_M, \tag{3.1}$$

where $\mathbf{W}_M$ is a normalized $M \times M$ DFT matrix and $\mathbf{U}_{t,M}$ and $\mathbf{\Lambda}_{t,M}$ are respectively submatrices of $\mathbf{U}_t$ and $\mathbf{\Lambda}_t$ consist of the first M columns of $\mathbf{U}_t$ and $\mathbf{\Lambda}_t$.

3.2 Statistical Precoders for Decision Feedback Receivers

In this subsection, we introduce the statistical precoder design method for zero forcing decision feedback systems that use convex optimization technique. The design method was first proposed in [33] considered equal bit allocation and minimization of MSE. More general method was proposed in [35] that considered different types of objective functions for a given bit allocation. The system in [33] requires $M = M_t < M_r$ and the system in [35] requires $M \leq min(M_t, M_r - 1)$.

In [33], the optimal statistical precoder is designed by minimizing the average arithmetic mean-squared error (MSE) subject to a constraint on the total transmission power. The bit allocation here is uniform and the optimization problem shown as follows:

$$\begin{array}{ll} \underset{\mathbf{F}}{\text{minimize}} & E_{\mathbf{H}}\left[\sum_{k=0}^{M_t-1} r_{kk}^{-2}\right] \\ \text{subject to} & \text{tr}(\mathbf{F}^{\dagger}\mathbf{F}) \leq \text{Pt}, \end{array}$$

where r_{kk}^{-2} is the same as that in (2.3). Using the relationships between p_k ,

$\lambda_{t,k}$ and $E[r_{kk}^{-2}]$, the equivalent optimization problem is as follows.

$$\begin{array}{ll} \underset{\overline{d}_k, \overline{p}_k}{\text{minimize}} & \sum_{k=0}^{M_t-1} \frac{1}{M_r-k+1} e^{-\overline{d}_k} \\ \text{subject to} & \left\{ \begin{array}{c} \sum_{k=0}^{M_t-1} e^{\overline{p}_k} = \text{Pt} \\ \overline{p}_1 \geq \overline{p}_2 \geq \ldots \geq \overline{p}_{M_t} \\ \sum_{k=0}^{M_0-1} \overline{d}_k \leq \sum_{k=0}^{M_0-1} \overline{p}_k + \sum_{k=1}^{M_0-1} \overline{\lambda}_{t,k}, \quad 1 \leq M_0 < M_t \\ \sum_{k=0}^{M_t-1} \overline{d}_k = \sum_{k=0}^{M_t-1} \overline{p}_k + \sum_{k=0}^{M_t-1} \overline{\lambda}_k \end{array} \right. \end{array},$$

where $\overline{d}_k = \log d_k, \overline{p}_k = \log p_k$ and d_k is as the same as d_{kk} in (2.3). The above design problem is called Geometric Programming and is a convex optimization problem that can be efficiently solved using an interior point method [38]. Let $\mathbf{D}_p = diag(p_0^{1/2}, p_1^{1/2}, \ldots, p_{M_t-1}^{1/2})$ and

$$\mathbf{L}_g = diag(p_0\lambda_{t,0}, p_1\lambda_{t,1}, \ldots, p_{M_t-1}\lambda_{t,M_t-1}).$$

Let the generalized triangular decomposition [34] of $\mathbf{L}_g^{1/2}$ be $\mathbf{N\Gamma S}^\dagger$, where $\mathbf{N}$ and $\mathbf{S}$ are two $M_t \times M_t$ matrices with orthonormal columns and $\mathbf{\Gamma}$ is an upper triangular matrix with $diag(\mathbf{\Gamma}) = diag(d_0, d_1, \ldots, d_{M_t-1})$. The optimal statistical precoder has the following form

$$\mathbf{F}_{LZW} = \mathbf{U}_t\mathbf{D}_p\mathbf{S}. \tag{3.2}$$

In [35], the optimal statistical precoder is designed by minimizing a convex cost function subject to a constraint on the total transmission power. The convex cost function can be the sum of average MSE, average joint error probability and average outage probability function. The bit allocation is not necessarily uniform. The case of is sum of average MSE is reviewed below. The optimization problem is formulated as

$$\begin{array}{ll} \underset{\overline{\eta}_k, \overline{p}_k}{\text{minimize}} & \sum_{k=0}^{M-1} e^{-q(\overline{\eta}_k - \overline{\alpha}_k)} \\ \text{subject to} & \left\{ \begin{array}{c} \sum_{k=0}^{M-1} e^{\overline{p}_k} = \text{Pt} \\ \overline{p}_1 \geq \overline{p}_2 \geq \ldots \geq \overline{p}_{M} \\ \sum_{k=0}^{M_0-1} \overline{\eta}_k \leq \sum_{k=0}^{M_0-1} \overline{p}_k + \sum_{k=1}^{M_0-1} \overline{\lambda}_{t,k}, \quad 1 \leq M_0 < M \\ \sum_{k=0}^{M-1} \overline{\eta}_k = \sum_{k=0}^{M-1} \overline{p}_k + \sum_{k=0}^{M-1} \overline{\lambda}_{t,k} \end{array} \right. \end{array}$$

,where q denotes the qth norm of average mean square error, $\overline{\alpha}_k = \log_2\left(\frac{2^{b_k}-1}{M_r-M+k-1}\right)$, $\overline{\lambda}_k = \log_2 \lambda_k$ and $\overline{\eta}_k = \log_2 \eta_k$. As in [35], this optimization problem can be solved by the interior point method [38]. Let

$$\widetilde{\mathbf{\Lambda}} = diag(p_0\lambda_{t,0}, p_1\lambda_{t,1}, \ldots, p_{M-1}\lambda_{t,M-1}),$$

$$diag(\mathbf{R}_1) = diag(\eta_0, \eta_1, \ldots, \eta_{M-1})$$

and take the generalized triangular decomposition on $(\widetilde{\mathbf{\Lambda}}^{1/2})^\dagger = \mathbf{Z}\mathbf{R}_1\mathbf{I}^\dagger$, where $\mathbf{Z}$ and $\mathbf{I}$ are two $M \times M$ matrices with orthonormal columns and $\mathbf{R}_1$ is an upper triangular matrix. Then the optimal statistical precoder is given by

$$\mathbf{F}_{JOJ} = \mathbf{U}_{t,M}\mathbf{D}_p\mathbf{Z}. \tag{3.3}$$

3.3 Statistical Bit Allocation with Decision Feedback Receiver

In this subsection, we recaps two methods for finding statistical bit allocation in [40] and [42] . In [40], the bits are allocated based on statistics for an i.i.d. channel. In [42], the number of transmit antennas together with the transmit symbol constellations are determined using the knowledge of channel correlation matrices.

3.3.1 Statistical bit allocation for i.i.d. channel

For a flat-fading MIMO channel $\mathbf{H}_w(M_t \leq M_r)$ assumed in (2.1), the relationship between transmitted symbol $\mathbf{s}$ and received symbol $\mathbf{r}$ is

$$\mathbf{r} = \mathbf{H}_w\mathbf{s} + \mathbf{q},$$

where $\mathbf{q}$ is the noise vector which has the same definition in Chapter 2. Assuming that each transmit antenna has equal transmit power and forward detection ordering is used, the instantaneous transmission rate to be allocated to transmit antenna l is

$$C_l = \log_2 \det(\mathbf{I}_{M_r} + \frac{P_t}{M_t N_0}\mathbf{H}_{l-1}\mathbf{H}_{l-1}^\dagger) - \log_2 \det(\mathbf{I}_{M_r} + \frac{P_t}{M_t N_0}\mathbf{H}_l\mathbf{H}_l^\dagger),$$

where $\mathbf{H}_l = [\mathbf{h}_{l+1}\mathbf{h}_{l+2}\ldots\mathbf{h}_{M_t}]$ and P_t is the total transmission power. It has been observed that the distribution of the capacity of a MIMO Rayleigh channel can be accurately approximated by a Gaussian distribution at medium and high SNR [20] [21], denoted as $C_l \sim N(\eta_l, \sigma_l)$, where η_l and σ_l are the mean and variance of the lth transmission rate. In order to select data rates for different antennas, the outage probability of the whole system is minimized. It is equivalent to maximizing the probability when no subchannel have transmission rate greater than the respective subchannel capacities and the optimization problem is as follows

$$\begin{array}{ll} \underset{b_l \in R}{\text{minimize}} & \prod_{l=0}^{Mt-1} \int_{b_l}^{\infty} \frac{1}{\sqrt{2}\sigma_{K,l}} e^{\frac{-(t-\eta_l)^2}{2\sigma_{K,l}^2}} dt \\ \text{subject to} & \sum_{l=0}^{Mt-1} b_l = \mathrm{R_b} \end{array},$$

where $\sigma_{K,l} = \sigma_l / K$, K is the number of independent channel realizations. Then, the optimum bit allocation for the lth antenna is

$$b_l \approx \eta_l + \frac{\sigma_l}{\sum_{m=1}^{M_t} \sigma_m} (R_b - \sum_{m=1}^{M_t} \eta_m). \tag{3.4}$$

3.3.2 Statistical bit allocation for correlated channel

In [42], the author proposed a selection criteria to choose the optimal transmit antenna set υ which maximize the minimum subchannel SNR given by

$$(M_t, \upsilon) = \arg\max_{(\widetilde{M}_t, \widetilde{\upsilon})} \left[\frac{1}{\widetilde{M}_t} \left(\ln\det(\mathbf{R}_{t(\widetilde{M}_t, \widetilde{\upsilon})}) + \sum_{j=1}^{\widetilde{M}_t} \sum_{i=1}^{M_r - j} \frac{1}{i} - R_b \ln 2 \right) - \ln \widetilde{M}_t \right], \tag{3.5}$$

where $\widetilde{M}_t$ is the number of active antennas of the transmit antenna set $\widetilde{\upsilon}$. After the optimal transmit antenna set υ is chosen, the real bit allocation is derived by assuming post detection SNR are all the same and the statistical bit allocation is obtained by the below equation

$$b_{i|\mathbf{H}_w} = \frac{R_b}{M_t} + 2\log|\lambda_i(\mathbf{R})| - \frac{1}{M_t}\log_2\det(\mathbf{R}^{\dagger}\mathbf{R}), \tag{3.6}$$

where $\mathbf{R}$ is obtained from the QR decomposition of $\mathbf{H}_w = \mathbf{QR}$ and $\lambda_i(\mathbf{R})$ denotes the ith largest eigenvalue of $\mathbf{R}$.

Chapter 4

Statistical Precoder Design

For a given precoder, we will first derive the statistical BER bound and the optimal real bit allocation. Based on the BER bounds, we can obtain the optimal precoder under the transmission power constraint. At last, we will discuss methods for finding statistical integer bit allocation.

4.1 Derivation of Statistical BER Bound and Optimal Real Bit Allocation

The BER can be computed using (2.4) and (2.5). For the convenience of derivation, we define the function

$$f(y) = Q(1/\sqrt{y}), y > 0.$$

The function $f(y)$ is monotone increasing and it can be verified that $f(y)$ is convex for $y \leq 1/3$ and concave for $y \geq 1/3$.

High SNR case Let us consider the high SNR case in which the convexity of $f(.)$ holds. Using $f(.)$, we have $SER_k = 4(1 - \frac{1}{2^{b_k/2}})f\left(\frac{(2^{b_k}-1)\sigma_{e_k}^2}{3}\right)$ and $BER = \frac{4}{R_b}\sum_{k=0}^{M-1}(1-\frac{1}{2^{b_k/2}})f\left(\frac{(2^{b_k}-1)\sigma_{e_k}^2}{3}\right)$. The averaged BER is given by

$$E[BER] \approx \frac{1}{R_b}\sum_{k=0}^{M-1} 4(1-\frac{1}{2^{b_k/2}})E[f\left(\frac{(2^{b_k}-1)\sigma_{e_k}^2}{3}\right)] \quad (4.1)$$

$$\geq \frac{4}{R_b}\sum_{k=0}^{M-1}(1-\frac{1}{2^{b_k/2}})f\left(\frac{(2^{b_k}-1)\bar{\sigma}_{e_k}^2}{3}\right) \quad (4.2)$$

$$\triangleq \phi_M(\mathbf{b}), \quad (4.3)$$

where $\mathbf{b} = [b_0\ b_1 \ldots b_{M-1}]$ is the bit allocation vector. The inequality in (4.1) is obtained using Jensen's inequality. Assume b_k is large enough so that $(1-2^{-b_k/2}) \approx 1$ and $2^{b_k}-1 \approx 2^{b_k}$, then

$$\phi_M(\mathbf{b}) \approx \frac{4}{R_b}\sum_{k=0}^{M-1} f(\frac{2^{b_k}\bar{\sigma}_{e_k}^2}{3})$$

$$\geq \frac{4}{R_b/M} f\left(\frac{1}{M}\sum_{k=0}^{M-1}\left(\frac{2^{b_k}\bar{\sigma}_{e_k}^2}{3}\right)\right) \quad (4.4)$$

$$\geq \frac{4}{R_b/M} f\left(\frac{1}{3}\left(\prod_{k=0}^{M-1} 2^{b_k}\bar{\sigma}_{e_k}^2\right)^{1/M}\right) \quad (4.5)$$

$$= \frac{4}{R_b/M} f\left(\frac{2^{R_b/M}}{3}\left(\prod_{k=0}^{M-1}\bar{\sigma}_{e_k}^2\right)^{1/M}\right)$$

$$\triangleq \Omega_M.$$

The inequality in (4.4) is due to the convexity of $f(.)$. The inequality in (4.5) is due to AM-GM (arithmetic mean-geometric mean) inequality and the monotone increasing property of $f(.)$. Due to the convexity of $f(.)$, the inequality in (4.4) holds if and only if $2^{b_k}\bar{\sigma}_{e_k}^2$ are of the same value for all k and $\sum_{k=0}^{M-1} b_k = R_b$. The optimal bit allocation for minimizing $\phi_M(\mathbf{b})$ is thus

$$b_k = \frac{R_b}{M} - \log_2(\bar{\sigma}_{e_k}^2) + \frac{1}{M}\sum_{j=0}^{M-1}\log_2(\bar{\sigma}_{e_j}^2),\ k = 0, 1, \ldots, M-1. \quad (4.6)$$

For convenience, we call the above optimal real bit allocation as $\mathbf{b}_{real,M}$.

Low SNR case Using a similar procedure, we can derive the statistical BER bound for the low SNR case in which the concavity of $f(.)$ holds.

$$E[BER] \leq \frac{4}{R_b}\sum_{k=0}^{M-1}(1-\frac{1}{2^{b_k/2}})f\left(\frac{(2^{b_k}-1)\bar{\sigma}_{e_k}^2}{3}\right) = \phi_M(\mathbf{b}) \qquad (4.7)$$

In this case, $\phi_M(\mathbf{b})$ is an upper bound of averaged BER. Assume as in the high SNR case that each subchannel transmission rate b_k is high enough so that $(1-2^{-b_k/2}) \approx 1$ and $2^{b_k}-1 \approx 2^{b_k}$, then

$$\phi_M(\mathbf{b}) \approx \frac{4}{R_b}\sum_{k=0}^{M-1} f\left(\frac{2^{b_k}\bar{\sigma}_{e_k}^2}{3}\right) \geq \frac{4}{R_b/M} f\left(\frac{1}{M}\sum_{k=0}^{M-1}(\frac{2^{b_k}\bar{\sigma}_{e_k}^2}{3})\right) = \Omega_M. \qquad (4.8)$$

That is $\phi_M(\mathbf{b})$ has minimum equal to Ω_M. The minimum achieved when bit allocation is as given in (4.6). In this case, the equality in (4.8) becomes an equality and we have $\phi_M(\mathbf{b}) \approx \Omega_M$ as in the high SNR case. Therefore Ω_M is a lower bound of averaged BER in the high SNR case and a upper bound in the low SNR case. For both high and low SNR cases, we would like to have the bound $\phi_M(\mathbf{b})$ minimized. The minimization of the lower bound in high SNR case is also important because the averaged BER can not be small if the lower bound is large.

4.2 Precoder Design

Our goal now is to find the optimum precoder $=F$ that to minimize Ω_M. This optimization problem is as follows:

$$\begin{aligned} &\underset{\mathbf{F}}{\text{minimize}} \quad f\left(\frac{2^{R_b/M}}{3}\left(\prod_{k=0}^{M-1}\bar{\sigma}_{e_k}^2\right)^{1/M}\right) \\ &\text{subject to} \quad Tr(\mathbf{F}^\dagger\mathbf{F}) \leq P_t. \end{aligned} \qquad (4.9)$$

- **Decision feedback receiver**:
  Since $f(.)$ is a monotone increasing function, minimizing (4.9) is equivalent to minimizing $(\prod_{k=0}^{M-1}\bar{\sigma}_{e_k}^2)^{1/M}$. Here, we will find the precoder $\mathbf{F}$ which minimizes $(\prod_{k=0}^{M-1}\bar{\sigma}_{e_k}^2)^{1/M}$.

Using (2.3), we have $\prod_{k=0}^{M-1}\bar{\sigma}_{e_k}^2 = \prod_{k=0}^{M-1}\frac{N_0 d_{kk}^{-1}}{M_r-k-1}$. Note that $\prod_{k=0}^{M-1} d_{kk}^{-1} = \det(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F})$. Let the singular decomposition of $\mathbf{F}$ be

$$\mathbf{F} = \mathbf{U}_f\mathbf{\Sigma}_f\mathbf{V}_f^\dagger,$$

where $\mathbf{U}_f$ is $M_t \times M$ with $\mathbf{U}_f^\dagger\mathbf{U}_f = \mathbf{I}_M$, $\mathbf{V}_f$ is $M\times M$ unitary with $\mathbf{V}_f^\dagger\mathbf{V}_f = \mathbf{I}_M$ and $\mathbf{\Sigma}_f$ is an $M\times M$ diagonal matrix. Then

$$\det(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}) = \det(\mathbf{\Sigma}_f)\det(\mathbf{U}_f^\dagger\mathbf{R}_t\mathbf{U}_f).$$

It follows that

$$\begin{aligned}\prod_{k=0}^{M-1}\bar{\sigma}_{e_k}^2 &= \frac{1}{\det(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F})}\prod_{k=0}^{M-1}\frac{N_0}{(M_r-k-1)}\\ &= \frac{1}{\det(\mathbf{\Sigma}_f)\det(\mathbf{U}_f^\dagger\mathbf{R}_t\mathbf{U}_f)}\prod_{k=0}^{M-1}\frac{N_0}{(M_r-k-1).}\end{aligned}$$

As $\mathbf{U}_f$ has orthonormal columns, we can apply the Poincare separation theorem [43] to bound $\det(\mathbf{U}_f^\dagger\mathbf{R}_t\mathbf{U}_f)$ using the eigenvalues of $\mathbf{R}_t$. Poincare separation theorem says $\lambda_i(\mathbf{B}) \geq \lambda_i(\mathbf{C}^\dagger\mathbf{B}\mathbf{C})$, $i = 0, 1, \ldots, r-1$, for any $n\times n$ Hermitian matrix $\mathbf{B}$ and any $n\times r$ unitary matrix $\mathbf{C}$ with $\mathbf{C}^\dagger\mathbf{C} = \mathbf{I}_r$. Using this theorem, we have

$$\det(\mathbf{U}_f^\dagger\mathbf{R}_t\mathbf{U}_f) \leq \prod_{i=0}^{M-1}\lambda_{t,i}.$$

On the other hand

$$\det(\mathbf{\Sigma}_f^2) = \prod_{i=0}^{M-1}[\mathbf{\Sigma}_f^2]_{ii} \leq \left(tr(\mathbf{\Sigma}_f^2)/M\right)^M = \left(tr(\mathbf{F}^\dagger\mathbf{F})/M\right)^M \leq (P_t/M)^M.$$

Thus $\det(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}) \leq (P_t/M)^M\prod_{i=0}^{M-1}\lambda_{t,i}$ and the product of averaged error variance satisfies

$$\prod_{k=0}^{M-1}\bar{\sigma}_{e_k}^2 \geq \frac{1}{\det(\mathbf{\Sigma}_f)}\prod_{k=0}^{M-1}\frac{N_0}{\lambda_{t,i}(M_r-k-1)} \geq \prod_{k=0}^{M-1}\frac{MN_0}{P_t\lambda_{t,i}(M_r-k-1)}. \tag{4.10}$$

Because the choice of unitary matrix $\mathbf{V}_f$ does not effect the value $\det(\mathbf{F}^\dagger \mathbf{R}_t \mathbf{F})$, we choose $\mathbf{V}_f = \mathbf{I}_M$ without loss of generality. The equality (4.10) holds if $\mathbf{F} = \mathbf{U}_{t,M}$ and the diagonal element of $\mathbf{\Sigma}_f$ are identical. Therefore the optimal precoder

$$\mathbf{F} = \sqrt{\frac{P_t}{M}} \mathbf{U}_{t,M}. \tag{4.11}$$

- **Linear receiver**:
  From [36], we know an increasing function of a Schur-concave function is Schur-concave. Because $(\prod_{k=0}^{M-1} \bar{\sigma}_{e_k}^2)^{1/M}$ is a Schur-concave function and $f(.)$ is monotone increasing function, the objective function in (4.9) is Schur-concave. Using the result of [29], the optimal precoder for Schur-concave objective function under total transmission power constraint has the form $\mathbf{F}_{op} = \mathbf{U}_{t,M} \mathbf{\Sigma}_p$, where $\mathbf{U}_{t,M} \in \mathbf{C}^{M_t \times M}$ has as columns the eigenvectors of $\mathbf{R}_t$ corresponding to the M largest eigenvalues in decreasing order and $\mathbf{\Sigma}_p = diag(p_1, p_2, \ldots, p_M) \in \mathbf{C}^{M \times M}$. In this case, the product of the averaged error variance is given by

$$\prod_{k=0}^{M-1} \bar{\sigma}_{e_k}^2 = \left(\frac{N_0}{M_r - M} \right)^M \prod_{k=0}^{M-1} \lambda_{t,k}^{-1} \prod_{k=0}^{M-1} p_k^{-2} \tag{4.12}$$

  Furthermore the constraint in (4.9) can be simplified as follows:

$$Tr(\mathbf{F}_{op}^\dagger \mathbf{F}_{op}) = Tr(\mathbf{\Sigma}_p^\dagger \mathbf{U}_{t,M}^\dagger \mathbf{U}_{t,M} \mathbf{\Sigma}_p) = \sum_{k=0}^{M-1} p_k^2 \leq P_t.$$

  Since $f(.)$ is a monotone increasing function, minimizing (4.9) is equivalent to minimizing $(\prod_{k=0}^{M-1} \bar{\sigma}_{e_k}^2)^{1/M}$, which, due to (4.12), is equivalent to minimizing $\prod_{k=0}^{M-1} p_k^{-2}$. The original problem can be reduced to the equivalent one shown below.

$$\begin{aligned} \underset{p_k}{\text{minimize}} \quad & \prod_{k=0}^{M-1} p_k^{-2} \\ \text{subject to} \quad & \sum_{k=0}^{M-1} p_k^2 \leq P_t. \end{aligned}$$

By using AM-GM inequality, the above problem can be solved easily and the optimal power allocation is $p_k = \sqrt{\frac{P_t}{M}}, k = 1, 2, \ldots, M$. Therefore we obtain the optimal precoder as

$$\mathbf{F}_{opt} = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M},$$

which is the same as the optimal precoder for the decision feedback receiver.

4.3 Design of Integer Bit Allocation

The optimal bit allocation compared in (4.6) has real entries. In practical applications, b_k should be nonnegative integers. In this section, we will present methods for designing integer bit allocation with given precoder $\mathbf{F} = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M}$. First, we will introduce two numerical methods of integer bit allocation design. Second, we will present two methods based on the statistical BER bound. At last, we will consider the optimal number of subchannel used. The methods given in this section can be applied for both linear and decision feedback receiver. All bit allocations discussed here have positive entries.

4.3.1 Minimum BER bit allocation ($\mathbf{b}_{mber}$)

We denote $\mathcal{C}_{b,M}$ as a bit allocation codebook which contains all length equal to or less than M positive integer bit allocation vectors such that $\sum_{k=0}^{M-1} b_k = R_b$. For each bit allocation in codebook $\mathcal{C}_{b,M}$, we calculate its corresponding average BER performance using a large number of random channels. The vector that has the smallest average BER performance is denoted as $\mathbf{b}_{mber}$ and it represents the best bit allocation in BER performance.

4.3.2 Most probable bit allocation ($\mathbf{b}_{prob}$)

First, we generate a large number of training channels. For each training channel, we can find the corresponding BER-minimizing bit allocation vector in $\mathcal{C}_{b,M}$. The vector $\mathbf{b}_{prob}$ is the most probable bit allocation.

4.3.3 Bit allocation using greedy algorithm ($\mathbf{b}_{gr,M}$)

When the transmission power P_t is large enough so that all argument of $f(.)$ in $\phi_M(\mathbf{b})$ are smaller than $1/3$ and $f(.)$ is operating at the convex region. We can use the greedy algorithm to solve this allocation problem efficiently. The solution of this kind of optimization problem was first proposed in [44]. This algorithm is also called "marginal allocation" or "incremental" algorithm. More detailed analysis was proposed in [45].

In [45], the allocation problem has the following form.

$$\begin{array}{cc} \underset{x_k \in \{0,1,\ldots,M\}}{\text{minimize}} & \sum_{k=0}^{M-1} F_k(x_k) \\ \text{subject to} & \sum_{k=0}^{M-1} x_k = N \end{array},$$

where N is a positive integer and the function $F_k(.)$ is convex in x_k.

The optimal $\mathbf{x}$ is determined as follows.

- **Step 1** Let $\mathbf{x} = (0, 0, \ldots, 0)$ be a $M \times 1$ vector and $r = 0$.
- **Step 2** Let

  $$v = \underset{j \in \{0,1,\ldots,M\}}{\arg\min} \Big(F_j(x_j + 1) - F_j(x_j) \Big),$$

  then $x_v = x_v + 1$.
- **Step 3** If $r = N$, then stop. The current $\mathbf{x}$ is an optimal solution. Otherwise, $r = r + 1$ and return to step 2.

Ignoring the factor $(1 - 2^{-b_k/2})$, then $\phi_M(\mathbf{b})$ in (4.3) can be approximated as

$$\phi_M(\mathbf{b}) = \frac{4}{R_b} \sum_{k=0}^{M-1} F_k(b_k),$$

where $F_k(b_k) = f\left(\frac{(2^{b_k}-1)\bar{\sigma}_{e,k}^2}{3}\right)$. Suppose SNR is high so that the argument in $f(.)$ is smaller than $1/3$. In this case, $F_k(b_k)$ is convex and the greedy algorithm can be used. We denote this bit allocation as $\mathbf{b}_{gr,M}$.

Remark In Section 4.1, we have obtained a optimal real bit allocation $\mathbf{b}_{real,M}$. Then we can use the quantization method referred in [46] to get a positive integer bit allocation denoted as $\mathbf{b}_{i,M}$. Because $\mathbf{b}_{real,M}$ is also derived by the convex assumption, $\mathbf{b}_{i,M}$ is the same as $\mathbf{b}_{gr,M}$ in our simulations.

4.3.4 Bit allocation for minimizing statistical bound ($\mathbf{b}_{d,M}$)

Here, we introduce the statistical integer bit allocation that minimizes $\phi_M(\mathbf{b})$. We denote this statistical bit allocation as $\mathbf{b}_{d,M}$ and is given by

$$\mathbf{b}_{d,M} = \underset{\mathbf{b} \in \mathcal{C}_{b,M}}{\arg\min} \quad \phi_M(\mathbf{b}). \tag{4.13}$$

The vector $\mathbf{b}_{d,M}$ can be obtained by an exhausted search. In Section 4.3.2, we have shown greedy integer bit allocation is also optimal so these two statistical integer bit allocations are the same in high SNR region. However, if the arguments of $f(.)$ for the optimal bit allocation are not all located at convex or concave region, then the optimal bit allocation can not be found by the greedy algorithm. In the simulations, the performance of $\mathbf{b}_{d,M}$ is very close to that of $\mathbf{b}_{mber}$. It can be used for all SNR region.

4.3.5 Optimal number of substream M_{opt}

The optimal precoder derived in Sec 4.2 is $\mathbf{F} = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M}$, which is obtained under the high bit rate assumption $b_k \gg 1$. Implicity M substreams are transmitted. We can also consider the transmission of fewer substreams. Let M_0 be the number of substreams transmitted. When we reduce M_0, each subchannel will be allocated more transmission power but more bits. Conversely, each subchannel is allocated less transmission power but fewer bits when we choose larger M_0. The tradeoff between different M_0 has become an interesting issue. Recently, there are several researchers (e.g. [42], [48], [49] and [50]) mentioned that wireless MIMO system could have better performance when the number of subchannels used is variable. In these papers, the M_0 selection function plays an important role on system performance. Here, we choose $\phi_{M_0}(\mathbf{b})$ we have derived earlier as our selection function. We can find the best bit allocation to minimize $\phi_M(\mathbf{b})$ for each M_0 and choose the best one.

- **Step 1** For each $M_0 \in \{1, 2, \ldots, M\}$, the corresponding optimal precoder is $\mathbf{F} = \sqrt{\frac{P_t}{M_0}}\mathbf{U}_{t,M_0}$. Then we apply the greedy algorithm in Sec 4.3.3. to find $\mathbf{b}_{gr,M_0}$ such that the statistical BER bound $\phi_{M_0}(\mathbf{b}_{gr,M_0})$ in (4.3) is minimized.

- **Step 2** The optimum number of substreams is given by

$$M_{opt} = \underset{i \in \{1,2,\ldots,M\}}{\arg\min} \quad \phi_i(\mathbf{b}_{gr,i}). \tag{4.14}$$

  and the corresponding optimal bit allocation is $\mathbf{b}_{gr,M_{opt}}$.

In a similar manner, we can obtain the $\mathbf{b}_{d,M_0}$ for each $M_0 \in \{1, 2, \ldots, M\}$. Then we can also use $\phi_{M_0}(\mathbf{b})$ as the selection function to find the best integer bit allocations $\mathbf{b}_{d,M_{opt}}$.

Chapter 5

Simulation Result

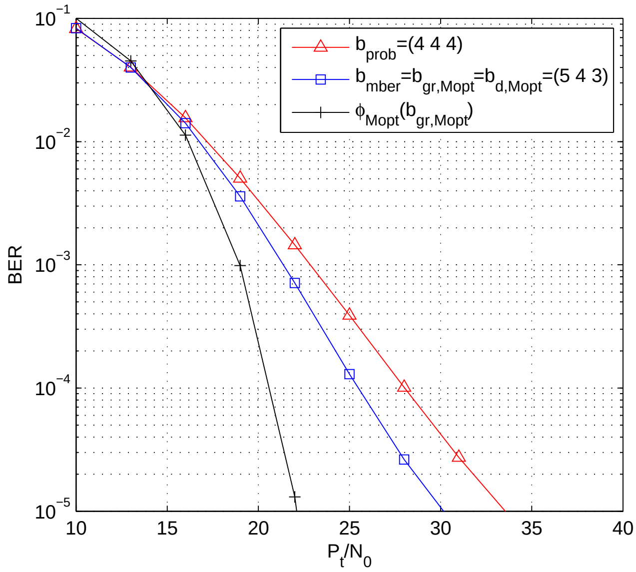
In this chapter, we present simulation results of our statistical precoder and statistical bit allocation (staP-BA) system. In our simulations, we use the exponential model for the channel correlation matrix $\mathbf{R}_t$. A 4×4 example of $\mathbf{R}_t$ is given below

$$\mathbf{R}_t = \begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho^* & 1 & \rho & \rho^2 \\ \rho^{*2} & \rho^* & 1 & \rho \\ \rho^{*3} & \rho^{*2} & \rho^* & 1 \end{pmatrix},$$

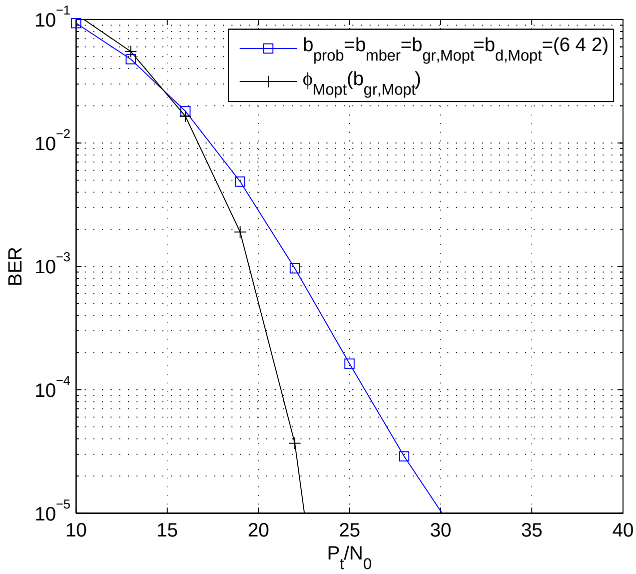
where ρ^* denotes the complex conjugate of ρ. We denote the uniform bit allocation as $\mathbf{b}_{uni}$. In all examples, $\mathbf{b}_{prob}$ and $\mathbf{b}_{mber}$ are found at the high SNR region using 10^5 training channels. We have used 10^6 channels in the simulation examples.

Example 1. Comparison of different integer bit allocation schemes. In this example, we discuss the BER performance between different statistical bit allocation methods proposed in Chapter 4 and we use the proposed precoder $F = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M}$ and reverse ordering detected from the Mth to the 1st substream.

A. For $M_r = 4$, $M_t = 3$, $M = 3$, $R_b = 12$, we show BER plots for different ρ in Figure 5.1. We can see $\mathbf{b}_{gr,M_{opt}}$ is the same as $\mathbf{b}_{d,M_{opt}}$ and $\mathbf{b}_{mber}$ for the different correlation parameters. Also shown is the statistical BER bound $\phi_{M_{opt}}(\mathbf{b}_{gr,M_{opt}})$ a lower bound in high SNR region and an upper bound in low SNR region shown in Sec 4.1.

(a) 

BER
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10 15 20 25 30 35 40
P_t/N_0
b_{prob}=(4 4 4)
$b_{mber}=b_{gr,Mopt}=b_{d,Mopt}$=(5 4 3)
$\phi_{Mopt}(b_{gr,Mopt})$

(b) 

BER
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10 15 20 25 30 35 40
P_t/N_0
$b_{prob}=b_{mber}=b_{gr,Mopt}=b_{d,Mopt}$=(6 4 2)
$\phi_{Mopt}(b_{gr,Mopt})$

(c)

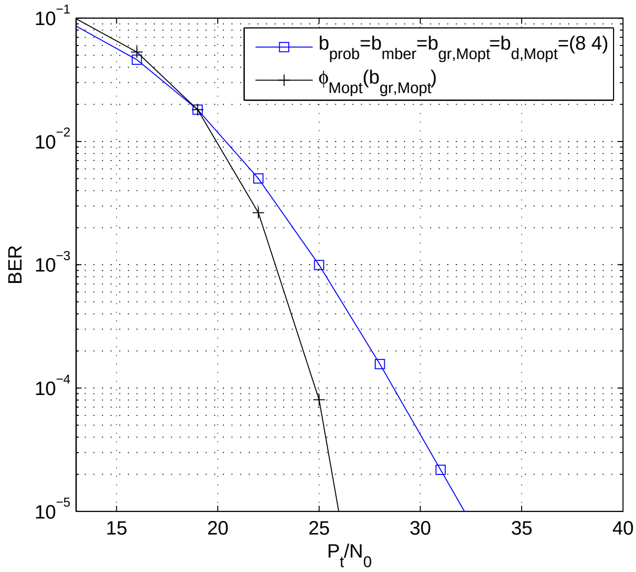
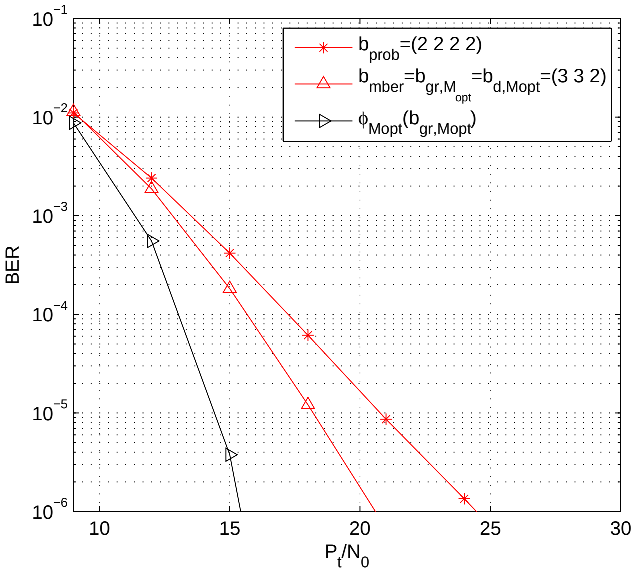


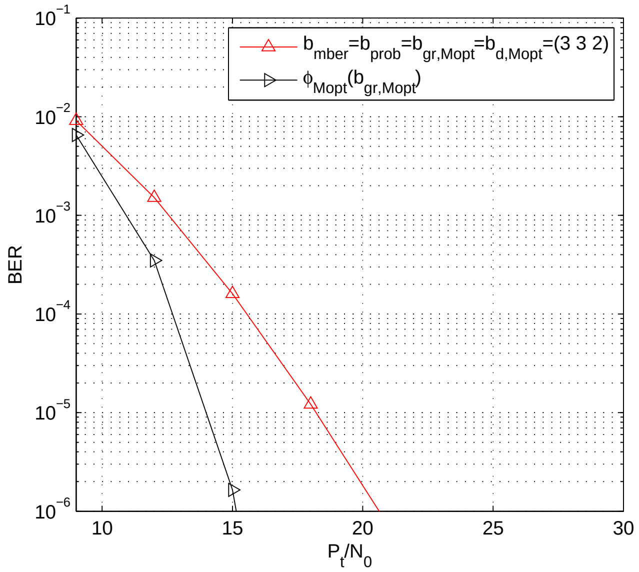
Figure 5.1: Example 1.A. Different Integer Bit Allocation schemes ($M_r = 4,\ M_t = 3,\ M = 3,\ R_b = 12$) for (a) ρ =0, (b) ρ =0.5, and (c) ρ =0.9.

B. For $M_r = 6,\ M_t = 4,\ M = 4,\ R_b = 8$, we can see the BER plots in Figure 5.2. $\mathbf{b}_{gr,M_{opt}}$ is still the same as $\mathbf{b}_{d,M_{opt}}$ and $\mathbf{b}_{mber}$ for different correlation parameters. This corroborate our earlier observation that when the transmission power is high enough so that all $f(.)$ is operating at convex region, $\mathbf{b}_{d,M_{opt}}$ and $\mathbf{b}_{gr,M_{opt}}$ should be the same. Figure 5.1 and 5.2 confirm this conclusion.

(a)



(b)



(c)

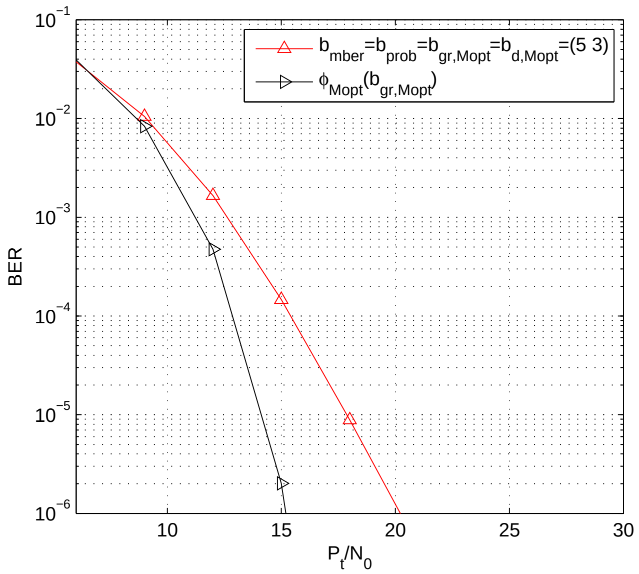


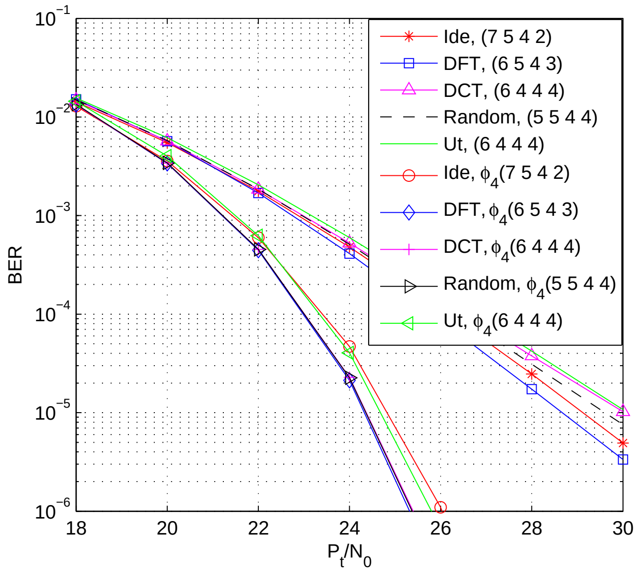
Figure 5.2: Example 1.B. Different Integer Bit Allocation schemes (M_r = 6, $M_t = 4$, $M = 4$, $R_b = 8$) for (a) ρ =0, (b) ρ =0.3, and (c) ρ =0.7.

Example 2. Performance for different $\mathbf{V}_f$. In this example, we discuss the BER performance between different $\mathbf{V}_f$ chosen in section 4.2. The parameter settings are $M_r = 6,\ M_t = 4,\ M = 4,\ R_b = 18$. The proposed precoder is $F = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M}\mathbf{V}_f$ and the reverse ordering is used. Five different $\mathbf{V}_f$ are used

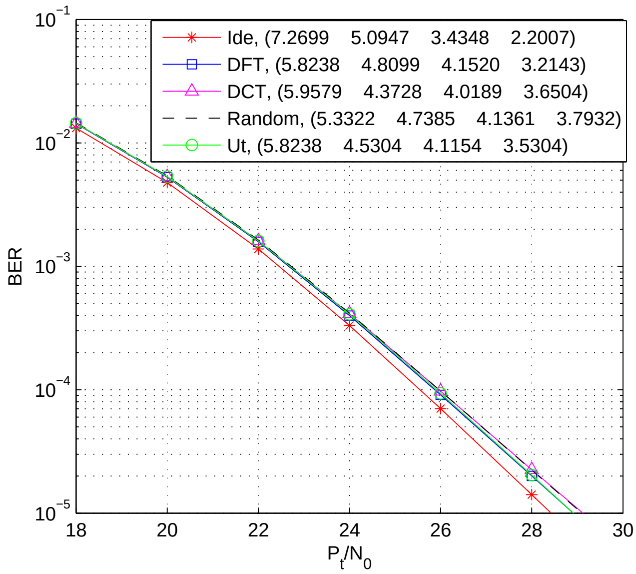
- $\sqrt{\frac{P_t}{M}}\mathbf{I}_{Mt,M}$.
- DFT matrix.
- DCT matrix.
- Random unitary matrix.
- $\sqrt{\frac{P_t}{M}}\mathbf{U}'_{t,M}$.

For each $\mathbf{V}_f$, the corresponding $\mathbf{b}_{real,M}$ and $\mathbf{b}_{gr,M}$ can be found in Section 4.1 and 4.3.3. The BER performances and $\phi_M(\mathbf{b})$ are shown in Figure 5.3. In Figure 5.3 (a), we can find there are only a little difference between different $\mathbf{V}_f$. In Figure 5.3 (b), the BER difference become smaller when $\mathbf{b}_{real,M}$ is used. Now we assume b_k is large enough so that $(1 - 2^{-b_k/2}) \approx 1$ and $2^{b_k} - 1 \approx 2^{b_k}$, then the BER formula used in (2.5) for simulations is turned to be $BER \approx \frac{4}{R_b}\sum_{k=0}^{M-1} f\left(\frac{2^{b_k}\sigma_{e_k}^2}{3}\right)$. The result is shown in Figure 5.3 (c). We can find the BER performance between different $\mathbf{V}_f$ are all the same. For convenience, we will all use $\mathbf{V}_f = \sqrt{\frac{P_t}{M}}\mathbf{I}_{Mt,M}$ in the later examples.

(a)


BER
P_t/N_0
Ide, (7 5 4 2)
DFT, (6 5 4 3)
DCT, (6 4 4 4)
Random, (5 5 4 4)
Ut, (6 4 4 4)
Ide, φ_4(7 5 4 2)
DFT, φ_4(6 5 4 3)
DCT, φ_4(6 4 4 4)
Random, φ_4(5 5 4 4)
Ut, φ_4(6 4 4 4)
18 20 22 24 26 28 30
10^-1 10^-2 10^-3 10^-4 10^-5 10^-6

(b)


BER
P_t/N_0
Ide, (7.2699 5.0947 3.4348 2.2007)
DFT, (5.8238 4.8099 4.1520 3.2143)
DCT, (5.9579 4.3728 4.0189 3.6504)
Random, (5.3322 4.7385 4.1361 3.7932)
Ut, (5.8238 4.5304 4.1154 3.5304)
18 20 22 24 26 28 30
10^-1 10^-2 10^-3 10^-4 10^-5

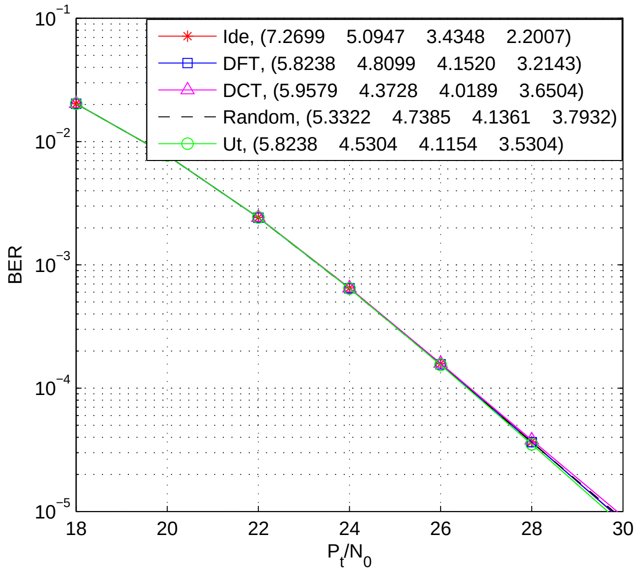
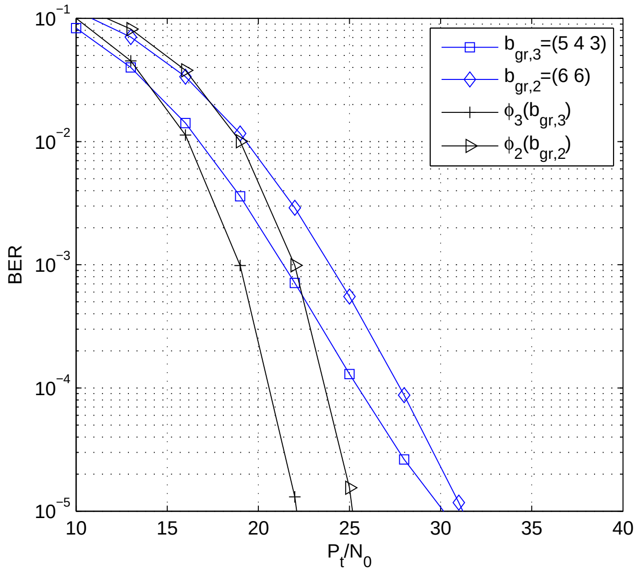
(c) 

Figure 5.3: Example 2.A. Performance for different $\mathbf{V}_f$ ($M_r = 6$, $M_t = 4$, $M = 4$, $R_b = 18$) for (a) $\mathbf{b}_{gr,4}$, (b) $\mathbf{b}_{real,4}$ and (c) $\mathbf{b}_{real,4}$ with high bit rate assumption.

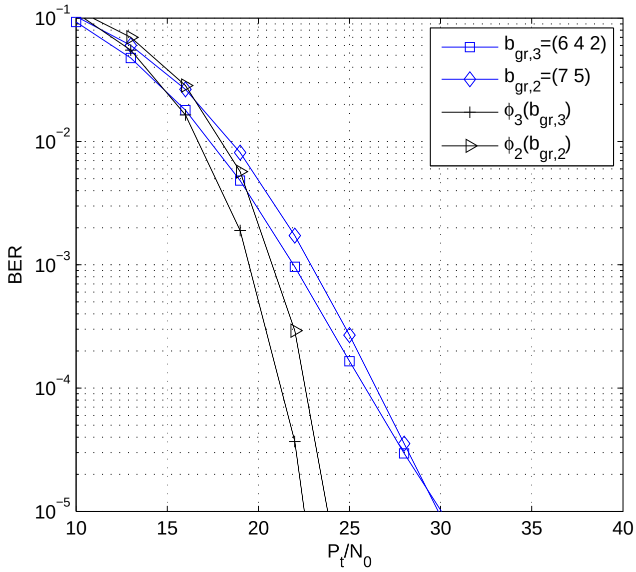
Example 3. Performance for different M_0. In this example, the precoder used is $\mathbf{F} = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M}$. For a given M_0, we find the optimal integer bit allocation that minimizes $\phi_{M_0}(\mathbf{b})$ using the greedy algorithm in Sec 4.3.3. By calculating $\phi_{M_0}(\mathbf{b})$ for each M_0, we obtain the optimal number of substream M_{opt} which has minimum $\phi_{M_0}(\mathbf{b})$ in Sec 4.3.5.

A. For $M_r = 4$, $M_t = 3$, $M = 3$, $R_b = 12$, the BER plots are shown in Figure 5.4. In this case, M_{opt} is 3 for $\rho = 0,\ 0.5$. We can see in Figure 5.4 (a) (b), the use of 3 substreams given a better performance for both reverse ordering. For $\rho = 0.9$, the optimal number of substream M_{opt} is 2 and Figure 5.4 (c) shows that using 2 substreams yields a lower error rate. This demonstrates that the statistical lower bound $\phi_{M_0}(\mathbf{b})$ provides a useful reference.

(a)



(b)



(c)

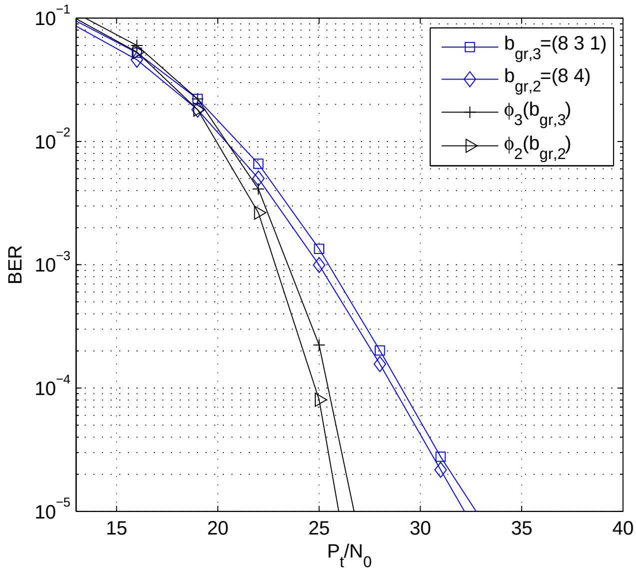
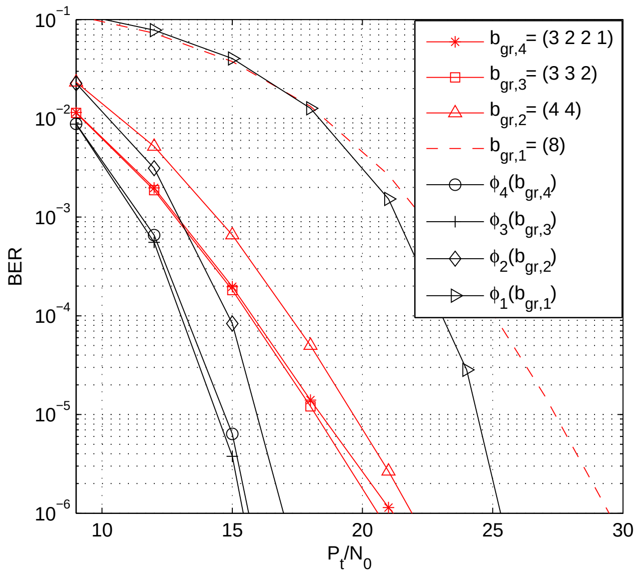
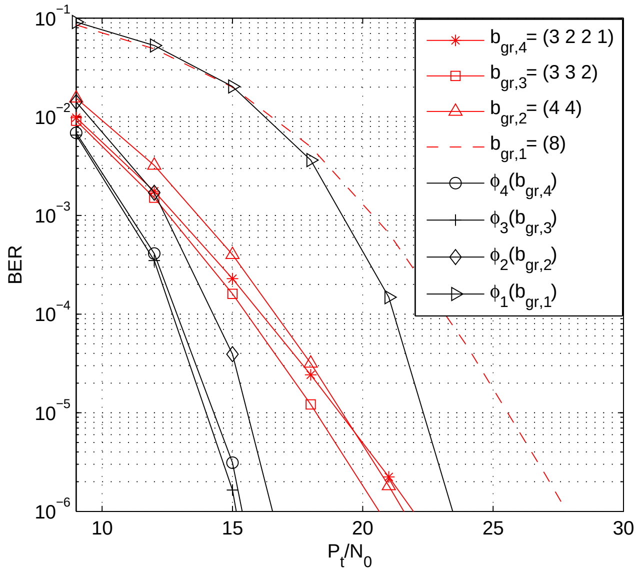


Figure 5.4: Example 3.A. Performance for different M_0 ($M_r = 4$, $M_t = 3$, $M = 3$, $R_b = 12$) for (a) $\rho = 0$, $M_{opt} = 3$, (b) $\rho = 0.5$, $M_{opt} = 3$ and (c) $\rho = 0.9$, $M_{opt} = 2$.

B. For $M_r = 6,\ M_t = 4,\ M = 4,\ R_b = 8$, the BER performance is shown in Figure 5.5, M_{opt} is 3 for $\rho = 0$, 0.3 and M_{opt} is 2 for $\rho = 0.7$. Again, we can see that using the statistical bound $\phi_{M_0}(\mathbf{b})$ is a useful reference to determine the number of substreams transmitted.

(a) 

(b)



(c)

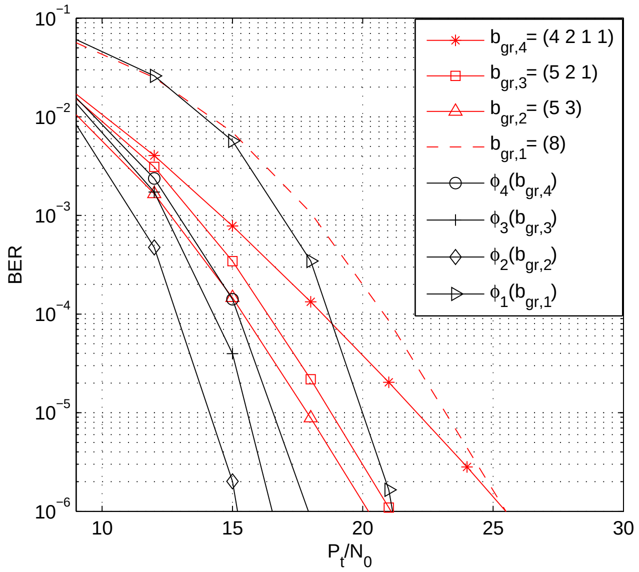


Figure 5.5: Example 3.B. Performance for different M_0 ($M_r = 6$, $M_t = 4$, $M = 4$, $R_b = 8$) for (a) $\rho = 0$, $M_{opt} = 3$, (b) $\rho = 0.3$, $M_{opt} = 3$ and (c) $\rho = 0.7$, $M_{opt} = 2$.

Example 4. Different Precoders. In this example, we compare the performance between different precoders for the same bit allocation and fixed detection ordering. The parameter settings are $M_r = 4$, $M_t = 3$, $M = 3$, $R_b = 18$. Four different precoders are used

- $\sqrt{\frac{P_t}{M}}\mathbf{U}_t$, the precoder given in (4.11) for minimizing the BER statistical bound.

- $\mathbf{F}_{JOJ}$, the optimal statistical precoder derived in [35] for a given bit allocation.

- $\mathbf{F}_{LZW}$, the MSE-minimizing statistical precoder given in [33] for uniform bit allocation.

- $\sqrt{\frac{P_t}{M}}\mathbf{I}_{M_t,M}$, which consist of first M columns of $\mathbf{I}_{M_t}$.

Reverse ordering is used for all precoders except $\mathbf{F}_{JOJ}$, for which forward ordering is applied as $\mathbf{F}_{JOJ}$ is designed for forward ordering. We show the results for two different bit allocation, uniform bit allocation in Figure 5.6 and $\mathbf{b}_{gr,M}$ in Figure 5.7. The vector $\mathbf{b}_{gr,M}$ is computed using the greedy algorithm in Sec 4.3.3. when the precoder is $\sqrt{\frac{P_t}{M}}\mathbf{U}_t$. We can see that for uniform bit allocation, $\mathbf{F}_{JOJ}$ and $\mathbf{F}_{LZW}$ are better than the other two. As correlation parameter ρ increases, the optimal bit allocation become more nonuniform and $\mathbf{F}_{LZW}$ does not perform as well. This is because $\mathbf{F}_{LZW}$ is designed for uniform bit allocation. Notice that $\mathbf{F}_{JOJ}$ performs better because its design considers bit allocation.

(a)

(b)

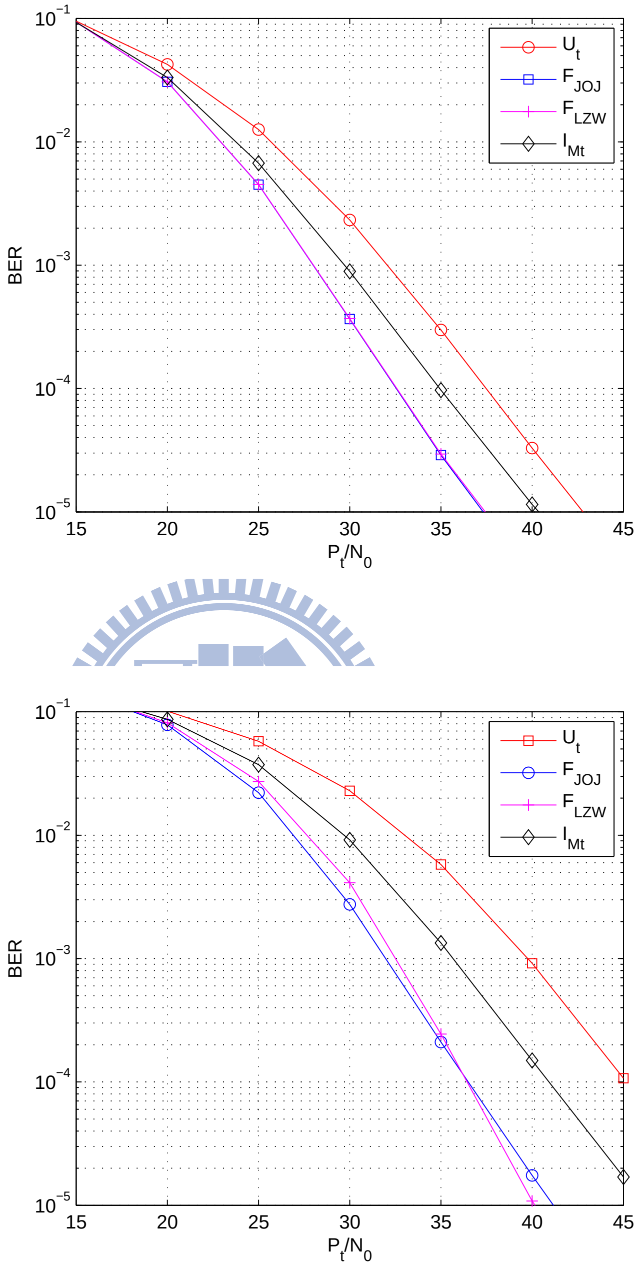
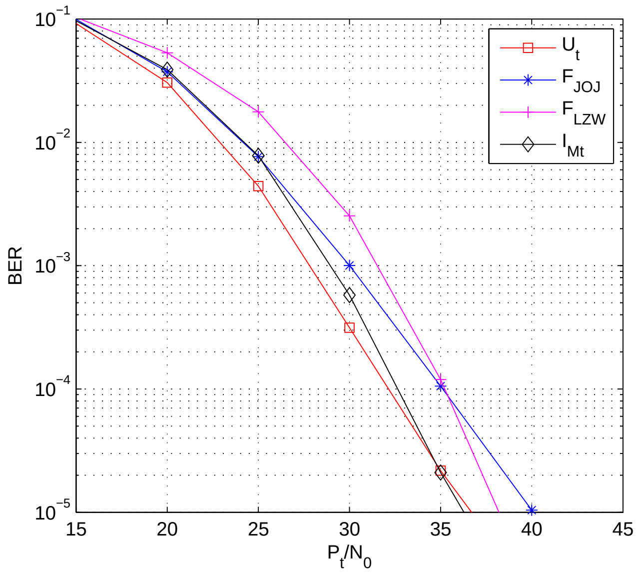


Figure 5.6: Example 4.A. Performance of different precoders for uniform bit allocation. (a) ρ =0.5; (b) ρ =0.9.

When the bit allocation is $\mathbf{b}_{gr,M}$ in Fig 5.7, the precoder $\sqrt{\frac{P_t}{M}}\mathbf{U}_t$ become the best of the four.

(a)



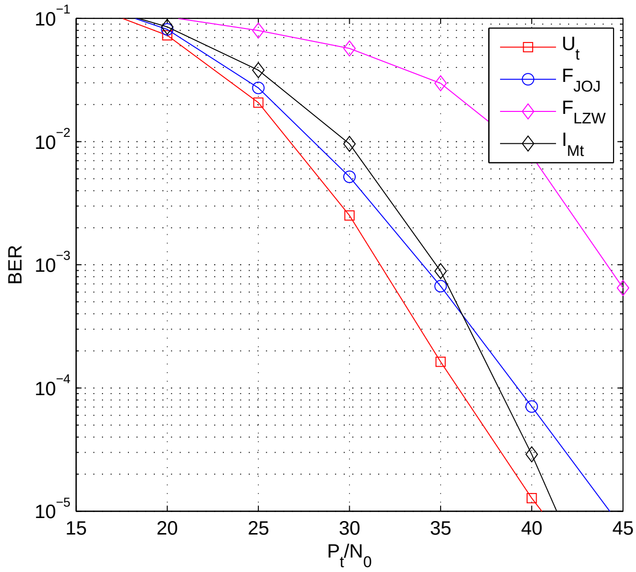
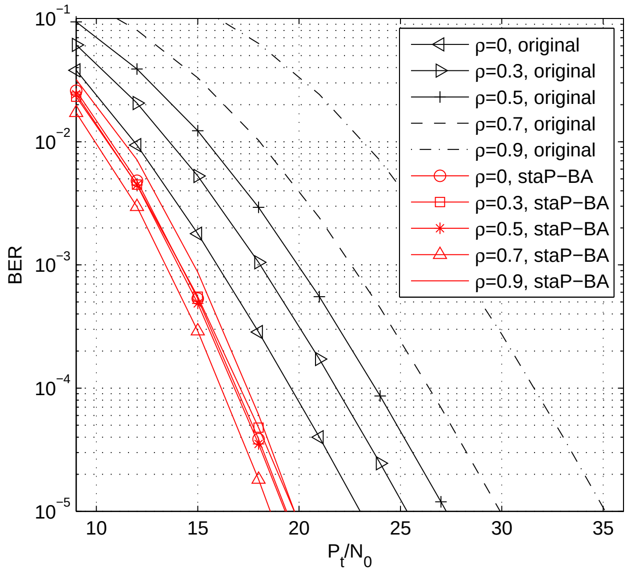
(b) 

Figure 5.7: Example 4.B. Performance of different precoders when the bit allocation is $\mathbf{b}_{gr,M}$. (a) ρ =0.5, $\mathbf{b}_{gr,M}$ = [8 6 4]; (b) ρ =0.9, $\mathbf{b}_{gr,M}$ = [10 5 3].

Example 5. Performance for different ρ. In this example, we compare our staP-BA system with the original system for different ρ. The parameter settings are $M_r = 6$, $M_t = 4$, $M = 4$. The original system is assumed without precoding ($\mathbf{F} = \mathbf{I}_{M_t}$) and uniform bit allocation. The results are shown in Figure 5.8 and 5.9. In Figure 5.8, the BER performance become closer after applying our method. In Figure 5.9, we define the improvement gain as the SNR difference between these two system. Then we can find the improvement gain becomes larger when ρ increases. This demonstrates our staP-BA system has better improvement for highly correlated channels.

(a)



(b)

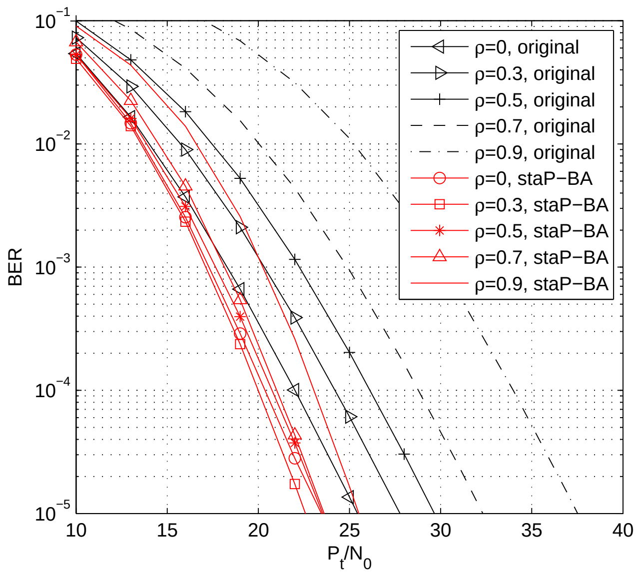
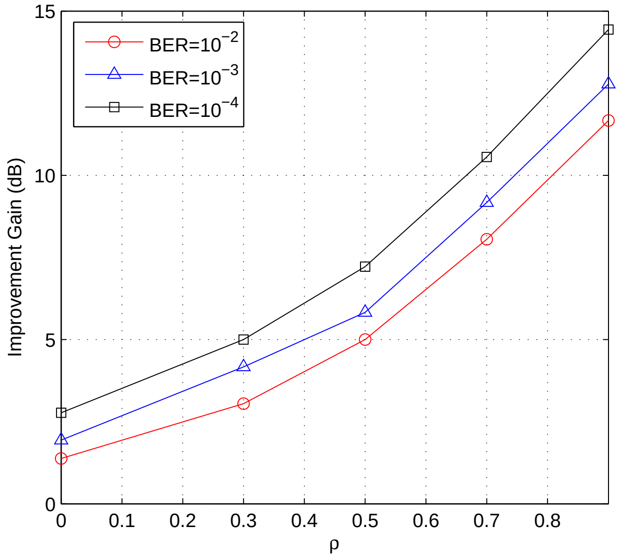


Figure 5.8: Example 5.A. Performance for different ρ ($M_r = 6,\ M_t = 4,\ M = 4$) for (a) $R_b = 8$, and (b) $R_b = 12$.

(a)



(b)

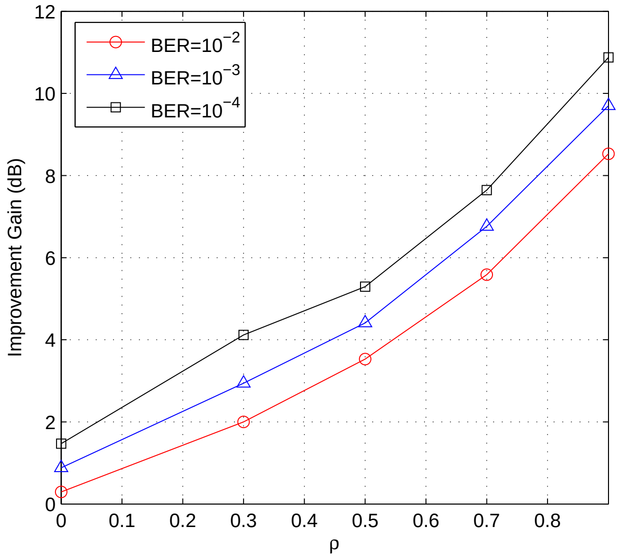


Figure 5.9: Example 5.B. Comparison of improvement gain ($M_r = 6,\ M_t = 4,\ M = 4$) for (a) $R_b = 8$, and (b) $R_b = 12$.

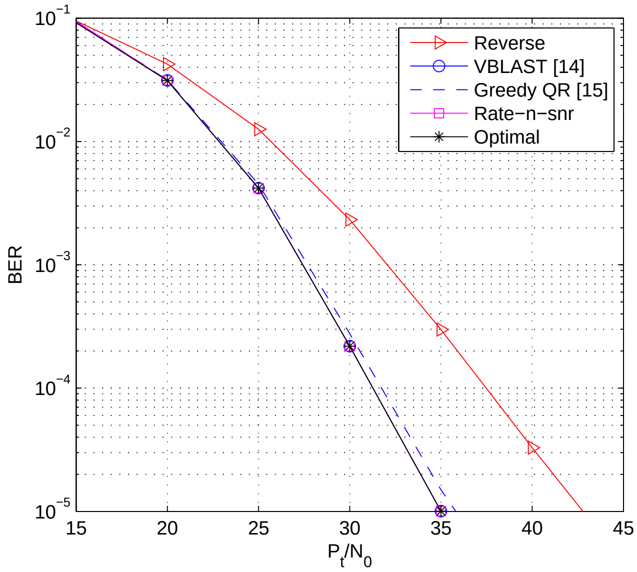
Example 6. Comparison of Different Detection Ordering. In this example, we use the precoder $\mathbf{F} = \sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M}$ and compare the performance of different detection orderings for the same bit allocation (uniform bit allocation or $\mathbf{b}_{gr,M_{opt}}$). The detection orderings considered are

- Reverse ordering
- VBLAST ordering [14]
- Greedy QR ordering [15]
- Rate-normalized-SNR ordering
- Optimal ordering, which is obtained by an exhausted search of all detection orderings for minimum BER.

A. $M_r = 4,\ M_t = 3,\ M = 3,\ R_b = 18,\ \rho = 0.5$. The BER plots are given for uniform bit allocation is in Figure 5.10 (a) and for $\mathbf{b}_{gr,M_{opt}}$ in Figure 5.10 (b). For uniform bit allocation, rate-normalized-SNR ordering and VBLAST ordering are the same and the performance is indistinguishable form the optimal ordering. The greedy QR ordering is slightly worse. The selection criteria of greedy QR ordering is minimizing the error variance of the last detected symbol for each recursive procedure and is different from the VBLAST ordering. The reverse ordering has the worst performance because it is a fixed detection ordering. We can not change the detection order to improve the error rate when a subchannel has low SNR.

When $\mathbf{b}_{gr,M_{opt}}$ is used. We see in Figure 5.10 (b) that the BER performance of rate-normalized-SNR ordering is very close to the optimal ordering but the VBLAST ordering is not. This demonstrates the importance of taking bit allocation into consideration in determining detection ordering when nonuniform bit allocation is used. In this case, reverse ordering performs better than VBLAST and greedy QR ordering because $\mathbf{b}_{gr,M_{opt}}$ is designed based on the reverse ordering.

(a)



(b)

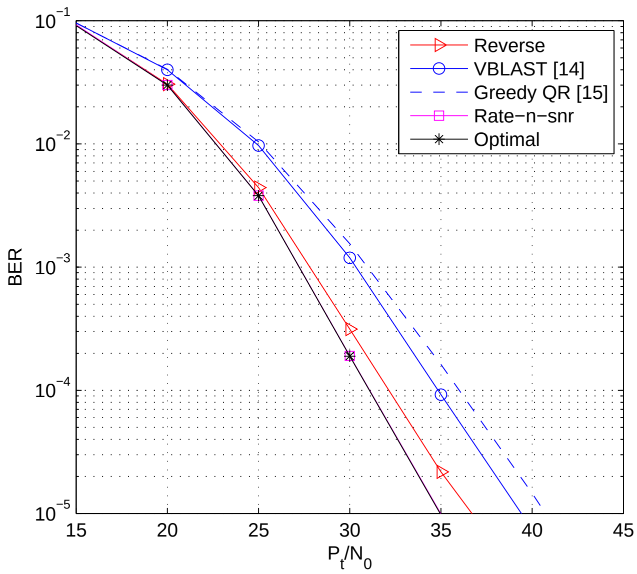
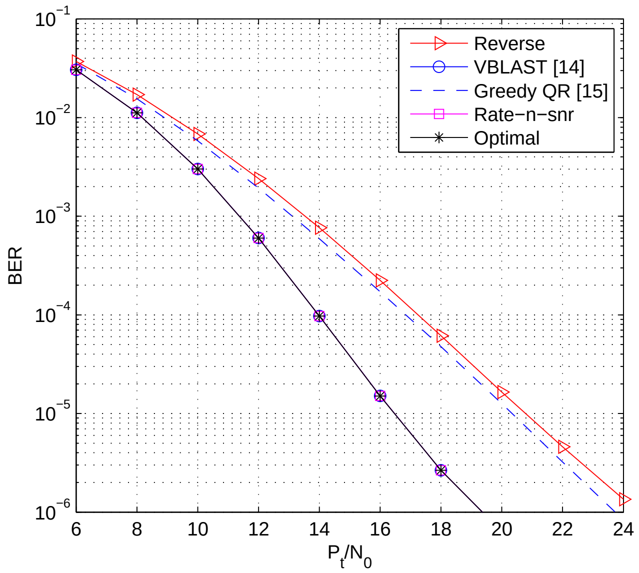


Figure 5.10: Example 6.A. Comparison of different detection orderings for $(M_r = 4,\ M_t = 3,\ M = 3,\ R_b = 18,\ \rho = 0.5)$ (a) uniform bit allocation, and (b) $\mathbf{b}_{gr,M_{opt}} = [8\ 6\ 4]$.

B. $M_r = 6$, $M_t = 4$, $M = 4$, $R_b = 8$, $\rho = 0$. The BER plot is given in Figure 5.11. For uniform bit allocation in Figure 5.11 (a), rate-normalized-SNR and VBLAST ordering are the same and the performance is indistinguishable for the optimal ordering. The greedy QR and reverse ordering performs worse. For $\mathbf{b}_{gr,M_{opt}}$ in Figure 5.11 (b), the BER performance of Rate-normalized-SNR ordering is also very close to the optimal ordering. Comparing with the previous case in Figure 5.10 (b), the optimal bit allocation $\mathbf{b}_{gr,M_{opt}}$ is more uniform. The VBLAST ordering has less performance loss. In this case, reverse ordering performs better than greedy QR ordering.

(a)



(b)

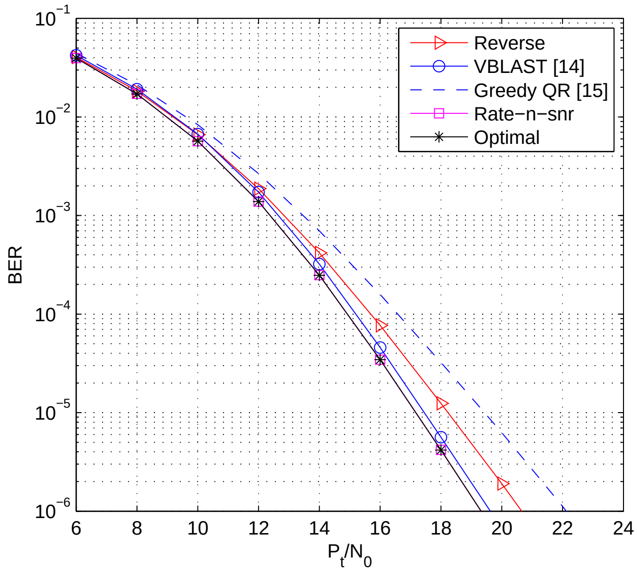


Figure 5.11: Example 6.B. Comparison of different detection ordering ($M_r = 6,\ M_t = 4,\ M = 4,\ R_b = 8,\ \rho = 0$) for (a) uniform bit allocation, and (b) $\mathbf{b}_{gr,M_{opt}} = [3\ 3\ 2]$.

Example 7. Comparison with other related works. In this example, we compare our proposed staP-BA system with the methods reviewed in Chapter 3. The following is a list of systems in the comparison.

- Vertical Bell Laboratories Layered Space-Time (V-BLAST) system [14]. It is a novel Multi-Input Multi-Output (MIMO) antenna scheme and it focuses on the detection algorithms.

- Statistical bit allocation system (staBA_{Du}) (reviewed in Sec 3.3.1). It is designed by minimizing the outage probability error [40]. The integer bit allocation used is obtained quantizing the bit allocation in (3.4). The quantization method used is introduced in [46]. The reverse ordering is used in staBA_{Du}.

- Statistical bit allocation system ($\text{staBA}_{\text{Ravi}}$) (reviewed in Sec 3.3.2). It designed by selecting the optimal antenna set [42]. The integer bit allocation used is rounding the bit allocation in (3.6) The greedy QR ordering is used in $\text{staBA}_{\text{Ravi}}$.

- Statistical precoder system (staP_{LZW}) (reviewed in Sec 3.2). It is designed by minimizing MSE for uniform bit allocation [33] and the reverse ordering is used in staP_{LZW}.

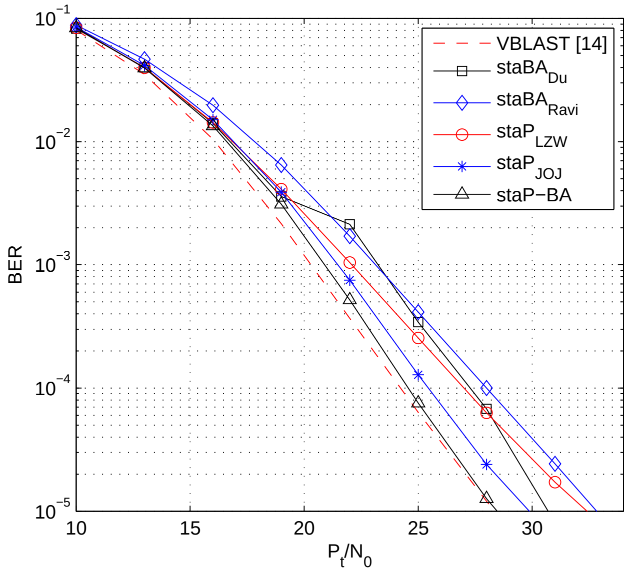
- Statistical precoder system (staP_{JOJ}) (reviewed in Sec 3.2). It is designed by minimizing bit allocation weighted MSE for a given bit allocation [35]. We use the optimal bit allocation $\mathbf{b}_{mber}$ which is introduced in Section 4.3.1 to minimize BER for the precoder in staP_{JOJ}. The forward ordering is used in staP_{JOJ}.

- Our proposed system (staP-BA), in which the precoder, bit allocation and detection ordering used are $\sqrt{\frac{P_t}{M}}\mathbf{U}_{t,M}$, $\mathbf{b}_{gr,M_{opt}}$ and rate-normalized-SNR ordering.

The simulation results are shown in Figure 5.12, 5.13 and 5.14.

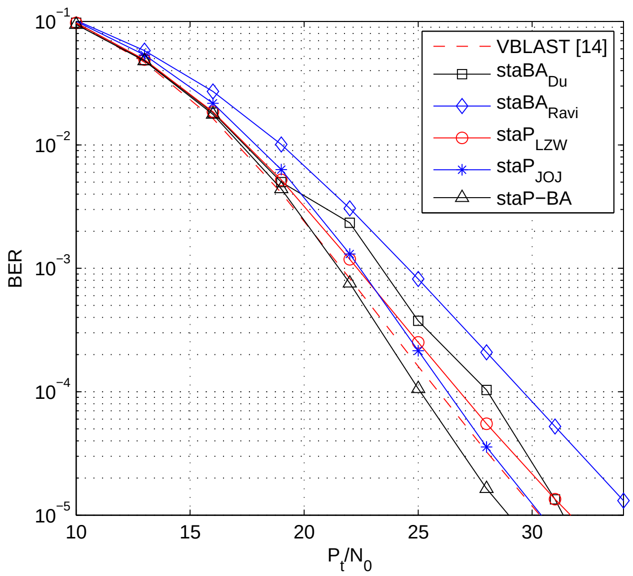
A. Figure 5.12 shows the BER performance when $M_r = 4$, $M_t = 3$, $M = 3$, $R_b = 12$.. Note that the bit allocation in staBA_{Du} is SNR-dependent and performs better for higher SNR. All the other bit allocations does not change with SNR. We can see that the proposed combination of statistical bit allocation and statistical precoder has the smallest BER for correlated channel. VBLAST system performs best for i.i.d. channels and only worse than our staP-BA system for correlated channels. This phenomenon will be shown again with other channel parameters in the later two examples.

(a)



VBLAST [14]
staBA$_{Du}$
staBA$_{Ravi}$
staP$_{LZW}$
staP$_{JOJ}$
staP−BA
BER
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10
15
20
25
30
P_t/N_0

(b)



VBLAST [14]
staBA$_{Du}$
staBA$_{Ravi}$
staP$_{LZW}$
staP$_{JOJ}$
staP−BA
BER
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10
15
20
25
30
P_t/N_0

(c)

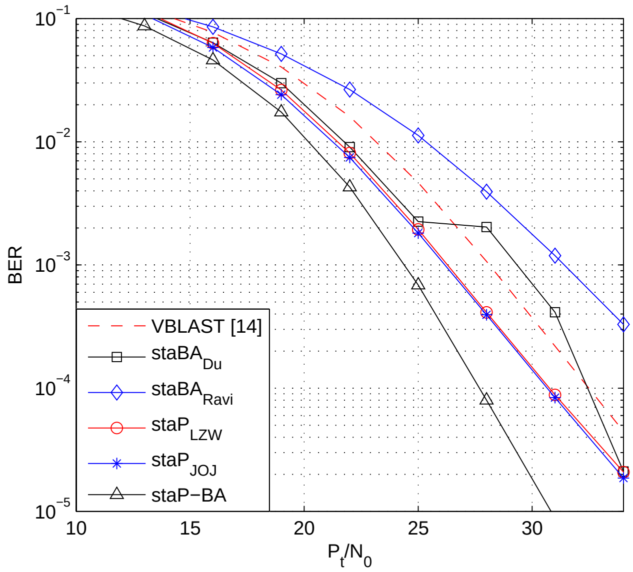
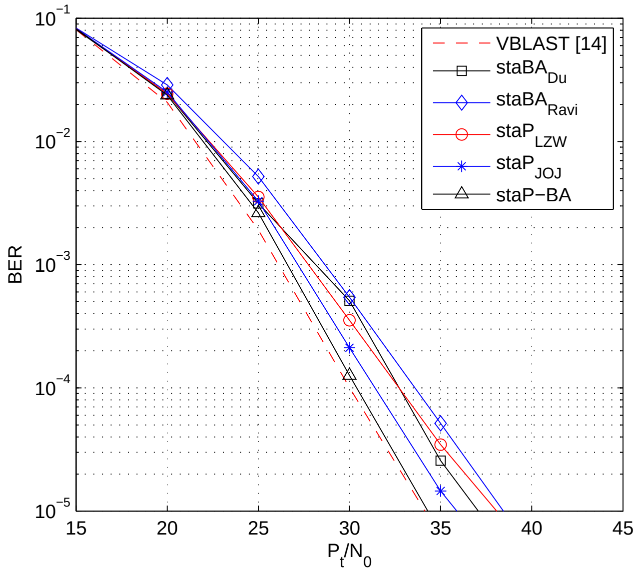
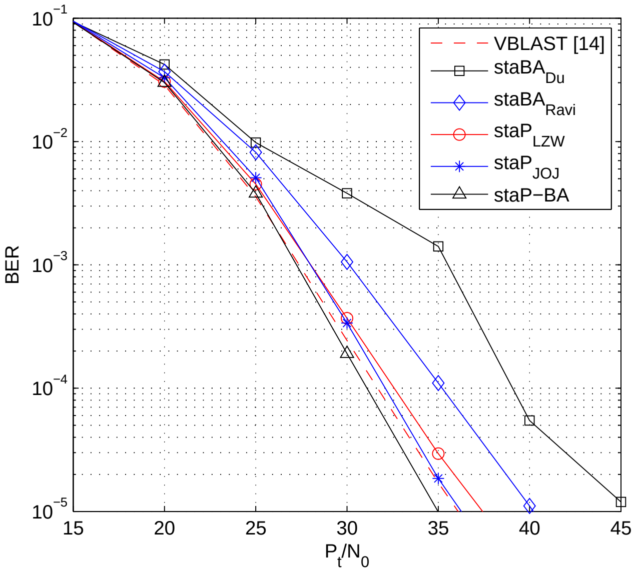


Figure 5.12: Example 7.A. Comparison with other related works ($M_r = 4,\ M_t = 3,\ M = 3,\ R_b = 12$) for (a) ρ =0, $\mathbf{b}_{gr,M_{opt}}$=(5 4 3) (b) ρ =0.5, $\mathbf{b}_{gr,M_{opt}}$=(6 4 2) and (c) ρ =0.9, $\mathbf{b}_{gr,M_{opt}}$=(8 4).

B. Figure 5.13 shows the BER performance when $M_r = 4$, $M_t = 3$, $M = 3$, $R_b = 18$. Comparing with the previous case, we only increase R_b to 18 and the optimal M_{opt} determined in (4.14) are all equal to M. We can see staBA$_{\text{Du}}$ performs the worst and our staP-BA system performs well.

(a) 

(b)



(c)

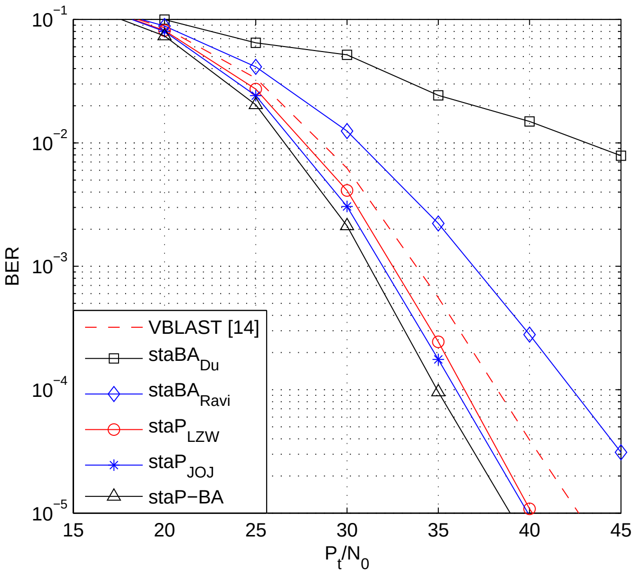
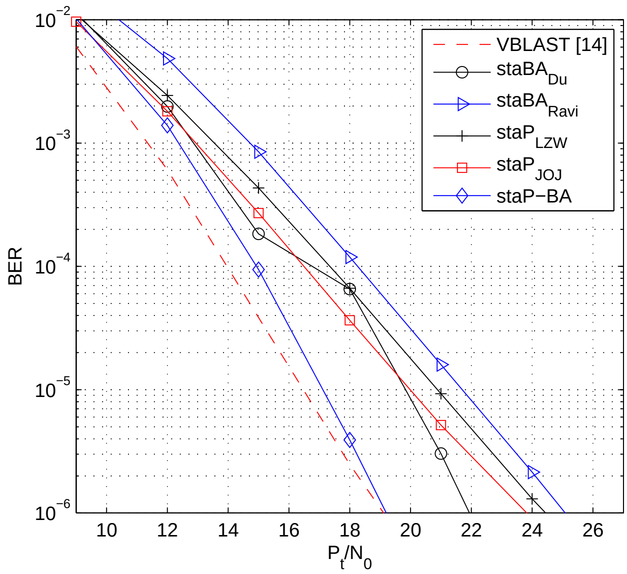


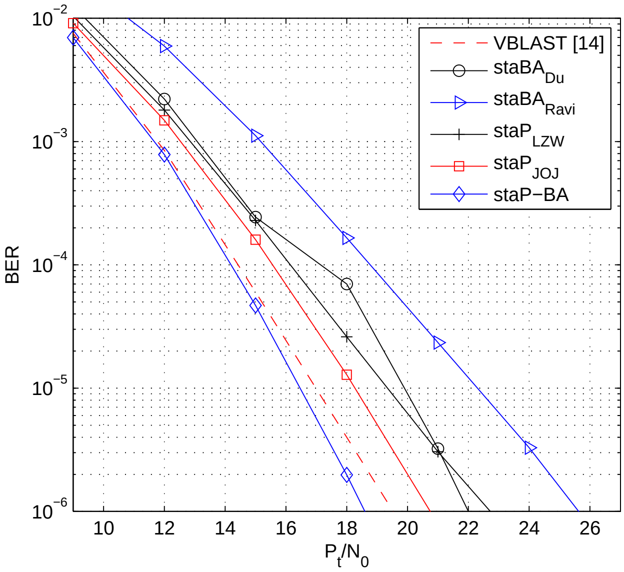
Figure 5.13: Example 7.B. Comparison with other related works (M_r = 4, M_t = 3, M = 3, R_b = 18) for (a) ρ =0, $\mathbf{b}_{gr,M_{opt}}$=(7 6 5), (b) ρ =0.5, $\mathbf{b}_{gr,M_{opt}}$=(8 6 4) and (c) ρ =0.9, $\mathbf{b}_{gr,M_{opt}}$=(10 5 3).

C. Figure 5.14 shows the BER performance when $M_r = 6$, $M_t = 4$, $M = 4$, $R_b = 8$. Similar to the previous case, staBA$_{\text{Du}}$ and staBA$_{\text{Ravi}}$ perform worse than other methods and the BER performance of staP$_{\text{LZW}}$ and staP$_{\text{JOJ}}$ are very close. Again, we can see staBA$_{\text{Du}}$ performs better for i.i.d channel. For $\rho = 0$ and 0.3 at BER=10^{-4}, staP$_{\text{JOJ}}$ performs best of the four related statistical works and our proposed staP-BA system is about 1.5dB better than staP$_{\text{JOJ}}$. For $\rho = 0.7$, staP$_{\text{JOJ}}$ performs best of the four statistical related works and our proposed staP-BA system is about 1.5dB better than staP$_{\text{JOJ}}$. The optimal bit allocation $\mathbf{b}_{mber}$ used for staP$_{\text{JOJ}}$ is (5 3) and is the same as $\mathbf{b}_{gr,M_{opt}}$ in staP-BA. In staP$_{\text{LZW}}$, the bit allocation used is (2 2 2 2). It has shown in Example 2.B. The optimal number of substream M_{opt} used is 2. We can see staP-BA and staP$_{\text{JOJ}}$ has larger improvement because M_{opt} is used.

(a)



(b)



(c)

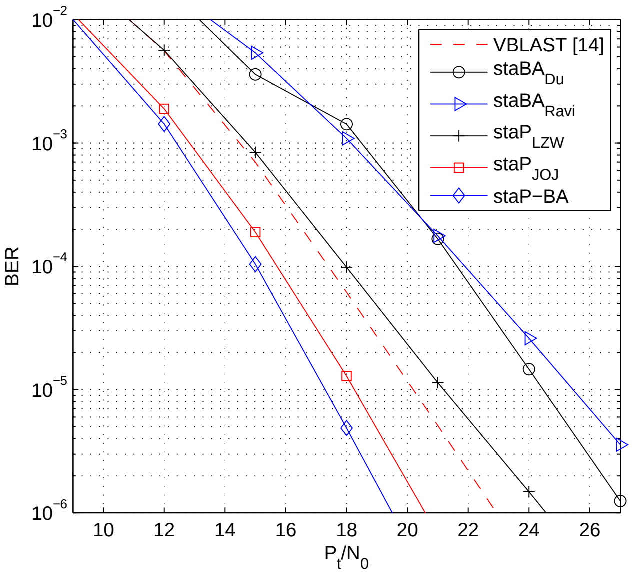
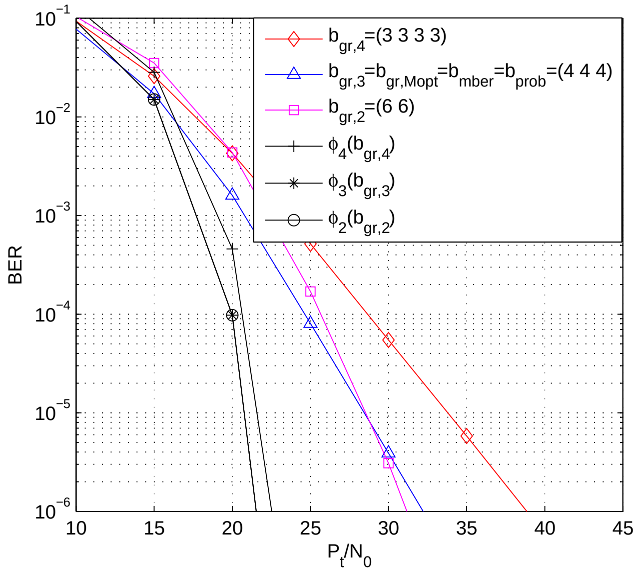


Figure 5.14: Example 7.C. Comparison with other related works (M_r = 6, M_t = 4, M = 4, R_b = 8) for (a) ρ =0, $\mathbf{b}_{gr,M_{opt}}$=(3 3 2), (b) ρ =0.3, $\mathbf{b}_{gr,M_{opt}}$=(3 3 2) and (c) ρ =0.7, $\mathbf{b}_{gr,M_{opt}}$=(5 3).

Example 8. Linear receiver. In this example, we present the result of staP-BA system with a linear receiver and compare it with the statistical precoder with linear receiver [25] mentioned in Section3.1. The result are shown in Figure 5.15 and 5.16 when $M_r = 5,\ M_t = 4,\ M = 4,\ R_b = 12$, ρ=0 and 0.7.

A. Figure 5.15 are given by different types of integer bit allocation, $\mathbf{b}_{mber}$, $\mathbf{b}_{prob}$, $\mathbf{b}_{gr,M}$ and $\mathbf{b}_{gr,Mopt}$ for statP-BA system. We can find the lowest BER achieved when $M_0 = M_{opt}$. Also we can find $\mathbf{b}_{gr,Mopt}$ is equal to $\mathbf{b}_{mber}$ in both correlated and uncorrelated channels.

(a)



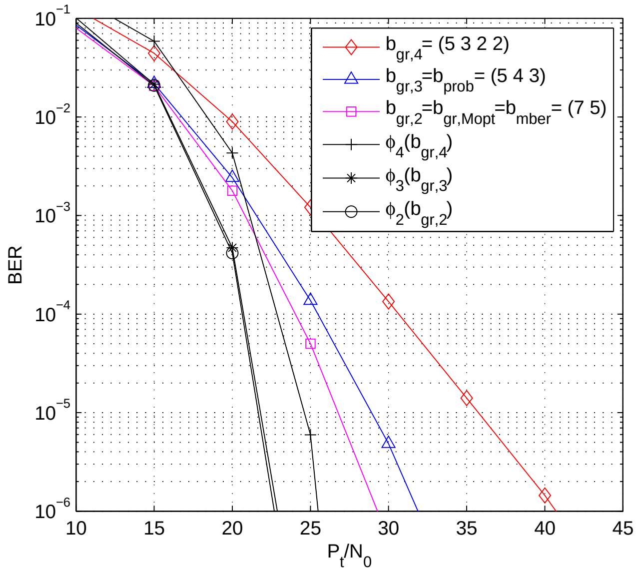
(b) 

Figure 5.15: Example 8.A. Linear receiver, Comparison of different integer bit allocations ($M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$) for (a) ρ =0, $M_{opt} = 3$ and (b) ρ =0.7, $M_{opt} = 2$.

Figure 5.16 shows the comparison of our staP-BA system with $\text{staP}_{\text{KSVR}}$ system introduced in Section 3.1. The statistical precoder system (staP_{LZW}), which is designed by minimizing the upper bound of statistical joint error probability [25] and the allocation is uniform in staP_{LZW}. We can find our staP-BA system perform better than staP_{LZW}. For $\rho = 0$, the proposed staP-BA system is about 2.5dB better than staP_{LZW} at BER=10^{-3} and this BER gap is about 5.5dB at BER=10^{-5}. For $\rho = 0.7$, the proposed staP-BA system is about 6dB better than staP_{LZW} at BER=10^{-3} and this BER gap is about 10dB at BER=10^{-5}.

(a)

(b)

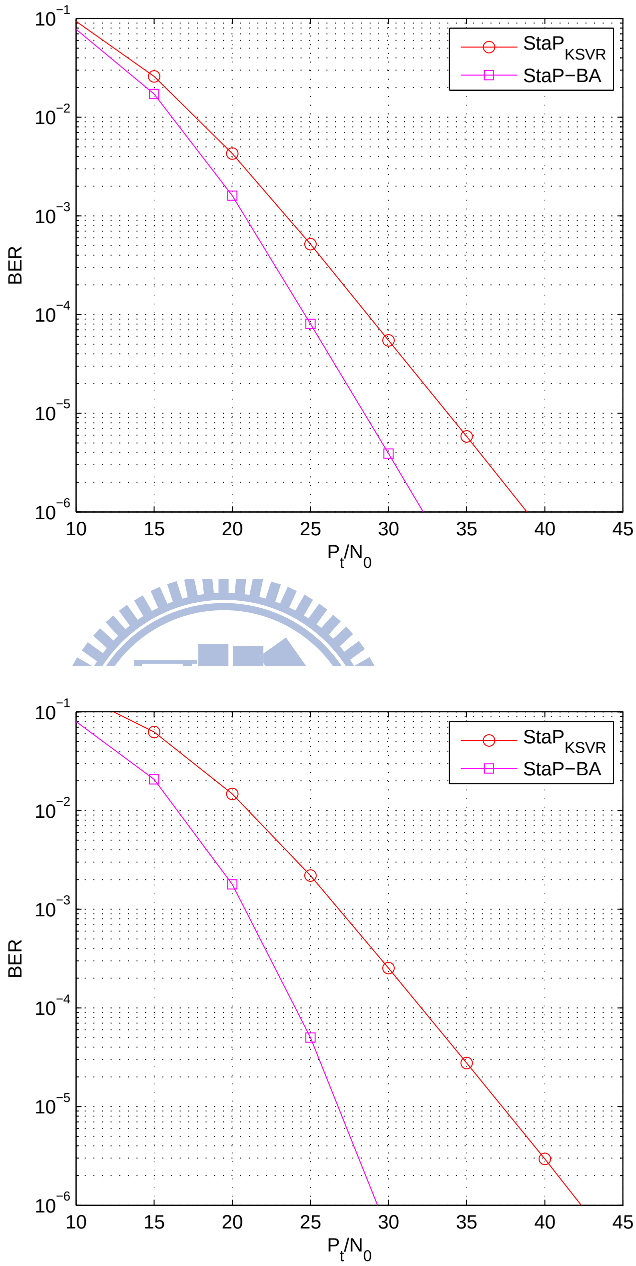


Figure 5.16: Example 8.B. Linear receiver, Comparison with other related works ($M_r = 5,\ M_t = 4,\ M = 4,\ R_b = 12$) for (a) ρ =0 and (b) ρ =0.7.

Chapter 6

Conclusions

In this thesis, we proposed statistical precoder and bit allocation for MIMO systems with correlated channel and this system is called the staP-BA system. We first derived the statistical BER bound for real bit allocation. Based on minimizing this bound, the optimal statistical precoder is derived both for linear and decision feedback receivers. Second, we proposed design methods for integer statistical bit allocation. In simulations, we have shown our statistical bit allocation performs well and the BER performance can be furthermore by using rate-normalized-SNR ordering.

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