

國立交通大學

電信工程研究所

碩士論文

正交分頻多工存取下載系統在非完全通道資訊下之  
資源分配及中繼器選取

Downlink Resource Allocation and Relay Selection  
for OFDMA Networks With Imperfect Channel  
Information

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中華民國 一 百 年 八 月

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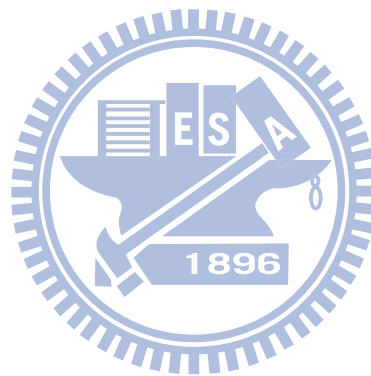
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## 中文摘要

正交分頻多工存取(OFDMA)系統可以利用簡單的通道等化器來有效的避免頻率選擇性衰減並使用最小(Nyquist)次載波距(subcarrier spacing)來提高頻譜效能。這種多工技術也容許很有彈性的資源分配，以充分利用不同用戶的通道狀況，一則可滿足個別用戶傳送速率的要求，另則可最大化整個系統的通道容量。此外，由於電磁信號傳播的統計特性，蜂巢式行動通訊系統之收訊強度無法定量保證，其中難免有部分用戶收訊品質不佳。一個經濟有效的解決方案便是中繼站的設立，適當的佈建中繼站不但有助於提升其信號品質，並可擴增基地台的覆蓋範圍與容量。

本論文主要在研究有多座中繼站的OFDMA下鏈系統之資源分配及中繼站選取的問題。我們同時考慮傳送(基地台)與接收(用戶)端皆使用單一天線(SISO)以及兩端皆有多根天線(MIMO)兩種架構，而下傳時雖有多個中繼站可以挑選，但各通道狀況的資訊卻有不確定性；亦即，基地台只有通道狀況的估計值以及誤差機率分佈。我們使用在此不確定情況下的通道容量下限來作為系統效能的評估標準。對上述SISO與MIMO兩種架構下，我們都推導出傳送端與中繼站的最佳功率比來達到最大的通道容量下

限。根據這個功率比我們分別提出了幾近最佳的次載波及其能量（功率）分配的演算法。我們的演算法複雜度不高，電腦模擬結果也顯示它能兼顧個別用戶的公平性，滿足其傳送速率要求也可以令系統的整體傳送速率達到最大值。





# Downlink Resource Allocation and Relay Selection for OFDMA Networks With Imperfect Channel Information

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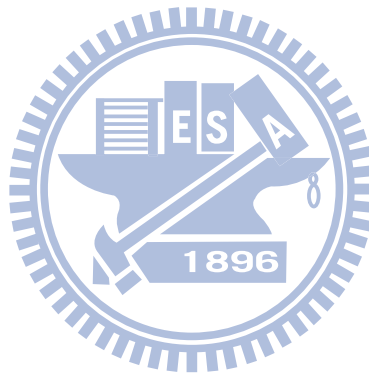
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## Abstract

The Orthogonal Frequency Division Multiple Access (OFDMA) scheme is an efficient anti-fading transmission scheme which renders high spectral efficiency and simple channel equalization. It also allows flexible resource allocation (RA) to meet various user requirements and achieve maximum network capacity. With the help of relays, link quality at cell edge can be improved and both network capacity and the coverage area can therefore be improved.

In this thesis, we consider the problem of RA and relay selection for downlink transmission in both single-input single-output (SISO) and multiple-input multiple-output (MIMO) OFDMA based cellular networks. We assume the availability of multiple cooperative relay stations but not the perfect channel state information (CSI). Instead, the base station knows only the estimated channel (link) gain and the associated error distribution. We use a tight capacity lower bound (CLB) for a link with imperfect CSI as the performance metric. In SISO networks, we derive the optimal source and relay power allocation ratio that maximizes the CLB of a cascaded source-relay-destination link. Based on this optimal power ratio, we propose a simple suboptimal algorithm that assigns power, subcarriers and cooperative relays to each serving mobile station. We

then derive the optimal power ratio for MIMO networks. Using the proposed subcarrier assignment algorithm for SISO network, we present the optimal and a suboptimal power allocation schemes. To reduce the computation complexity, we derive a near-optimal power ratio, assuming both source-to-relay and relay-to-destination links have the same rank. Simulation results show that our algorithm not only meets the users' rate constraints with very high probabilities but yields an excellent sum rate (CLB) performance.



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# Chapter 1

## Introduction

The Orthogonal Frequency Division Multiple Access (OFDMA) scheme enjoys the advantages of an OFDM based transmission system, i.e., high spectral efficiency, simple and robust equalization against frequency selective fading; it also offers flexibility in radio resource allocation for meeting various rate requirements. Due to OFDMA transmits a wide band signal on multiple orthogonal subcarriers, in which the channel condition of one subcarrier is independent of one another, which means a subcarrier in deep fading for one user may have good condition for another user. Thus, by proper scheduling and resource block assignment, OFDMA can exploit multi-user diversity [1] in a time-varying frequency-selective fading channel. As a result, the OFDMA scheme has been selected as the air interface standard by two major 4G campaigns, namely, the IEEE802.16m and 3GPP's LTE-A downlink.

Recent researches have found that, a base station (BS) can cooperate with relay stations using a Time Division Duplex (TDD) based Decode-and-Forward (DF) or Amplify-and-forward (AF) scheme to enhance the network capacity. In a typical two-phase cooperative system, the transmitter sends its signal to a relay node (RN) and the destination node (DN) in the first half of a transmission frame and the relay then sends the re-generated signal to the destination in the second half [3]. By combining the two copies received in both phases, the DN increases the link's capacity.

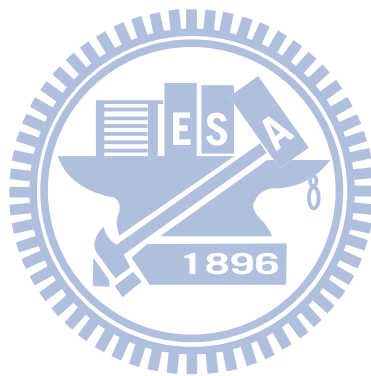
To maximize the sum rate, a BS in a relay based cooperative network must dynamically allocate its resources, namely, power, subcarriers and cooperative RNs to various DNs (i.e., mobile stations) according to the conditions of the BS-DN, BS-RN, and RN-DN links. The problem of resource allocation in either conventional OFDMA or relay-aided OFDMA systems has been intensively studied [4], [5]. But in these works, a common assumption is that the channel state information (CSI)—the gains of all links—of the system is perfectly known by the BS.

However, the channel information is estimated by dividing the demodulated pilot pattern with the known symbol. Due to the additive noise in demodulating the received preamble, the channel estimation error can be assumed as a gaussian random variable [11]. Moreover, due to feedback delays, channel estimation errors in transmitter are unavoidable. For the feedback delay error, since the channel is modeled as a Gaussian random process, the channel gain and its delayed version then can be a jointly Gaussian [10]. Thus, perfect CSI assumption leads to underuse or overuse of component links and are likely (especially in relay-aided links) to results in transmission outages [12]. [13] considered optimal resource allocation for maximizing the ergodic sum rate of an OFDMA system with imperfect CSI. A suboptimal algorithm for goodput maximization was given in [14]. To our knowledge, [15] is the only work which investigates the issue of joint relay selection and resource allocation in a cooperative relay network with imperfect CSI. However, the authors used a mean rate to characterize the CSI uncertainty which may lead to different interpretations.

In this thesis, we consider a problem similar to that studied in [14] under a different performance metric. As the channel capacity in the presence of imperfect CSI is not known [16], we use a tight capacity lower bound as the performance metric and derive the corresponding optimal source (BS) and RN power ratio if a given RN is to be used for relaying the source signal to a DN. We then present a low-complexity resource allocation scheme with an aim to not only maximize the total sum rate (lower bound) but also

meet the users' (DNs') rate and power constraints. The scheme includes link (direct link only or a relay is needed) selection, subcarriers assignments(SA) and power allocation (PA).

This thesis is organized as follows: In following chapter, we describe the system model and related assumptions for the problem of concern. In chapter 3, we proposed algorithms to solve the problem we face. Moreover, we extend the resource allocation problem to the case for multiple-input and multiple-output(MIMO) case in chapter 4. Finally, we conclude our work in chapter 5.



# Chapter 2

## Relay System and Cooperative Transmission

### 2.1 Relay Networks

In a wireless communication system, one of the most important problems is the fading effect. While in recent years, cooperative communications have been used to exploit the spatial diversity in multiuser wireless systems without the need of multiple antennas at each node, which is not practical to employ in a mobile station (MS) due to the receive and transmit antennas should be separated far enough. Moreover, the term cooperative communications typically means a system where users share and coordinate their resources to enhance the transmission quality.

In a basic cooperative communication system, it consists a source node, a relay node and a destination node. Depending on the condition of the component links between source node and relay node, relay node and destination node, and source node and destination node, the source node can choose to whether use the relay or not. If the source uses relaying, destination combines the two copies from source node and relay node, the cooperative diversity can be utilized. Furthermore, for a much general cooperative communication system, there are multiple source nodes, relay nodes, and destination nodes. Thus, by opting to transmit a data stream to the appropriate destination node

from an appropriate relay node, the source node gains the multiuser diversity.

## 2.2 Relay Strategies

Many cooperation techniques have been proposed based on the concept of relaying [2], the most commonly used strategies in these methods are decode-and-forward (DF) and amplify-and-forward (AF). For a two-hop relaying we'll use in our scenario, the source node broadcasts its message to both the relay node and the destination. If the relay node employs the DF scheme, it will decode and regenerate a new message to the destination in second phase. At the destination, it employs a maximum-ratio-combining detector to the signals from both the source and the relay paths. Otherwise, if the AF scheme is used, the relay node just simply amplifies the received signal and forwards it directly to the destination. No decoding of the message is needed in AF scheme.

Moreover, in [6], it compares the performance of DF with the performance of AF scheme. It shows that the distance between the relay node, the source node, and the destination node is the most important point to influence the performance of each relaying scheme. When the distance between relay node and source node is lower than the distance between relay node and destination node, the relay node has a higher received signal-to-noise ratio (SNR). Thus, DF is a better scheme for the relay node to employ. Otherwise, when the distance between relay node and source node is higher than the distance between relay node and destination node, the relay node has a higher probability of the decoding error. Then, we'll choose the AF scheme for relaying.

However, the reliability of interuser channels also relates to the performance of relay cooperation. In the DF scheme, the relay node retransmits the signal from the source only if the signal is well decoded. Similarly, for the AF scheme, due to both the signal and noise being amplified by the relay node, if the quality of the source-relay link is bad, the performance at the destination node will decrease. Therefore, we need to decide whether to use the relays or not according to the source-relay channel.

## 2.3 Capacity of Cooperative Transmissions

In [7], the capacity of basic cooperative transmissions has been introduced. If we denote  $X_s$ ,  $X_r$ ,  $Y_r$  and  $Y_d$  the transmitted signals from source and relay, the received signal at relay and destination, respectively. The capacity of a relay channel with channel transition probability  $p(y_r, y_d|x_s, x_r)$  is

$$C \leq \sup_{p(x_s, x_r)} \min \{I(X_s, X_r; Y_d), I(X_s; Y_r, Y_d|X_r)\} \quad (2.1)$$

where the sup is over all joint distributions  $p(x_s, x_r)$ .

Under the derived capacity (2.1), if we denote  $SNR_{SR}$ ,  $SNR_{RD}$  and  $SNR_{SD}$  the SNR of source-relay, relay-destination and source-destination path, the capacity of a single user, single relay and single destination cooperative communication system can be formulated as

$$C \leq \min \{\log_2(1 + SNR_{SR}), \log_2(1 + SNR_{SD} + SNR_{RD})\} \quad (2.2)$$

where the second term is due to the maximum-ratio-combining detector at the destination node.



## Chapter 3

# Downlink Resource/Relay Allocation for SISO OFDMA Networks

This chapter considers the scenario for a downlink single-input single-output OFDMA system as shown below. We represent resource allocation schemes that maximize the total capacity with each user's minimum rate constraint and the overall total power constraint while facing the channel estimated errors. By taking the channel estimated errors into account, we derive a tight capacity lower bound as the performance matrix. Then we propose a simple suboptimal algorithm that assigns power, subcarriers and cooperative relays to each mobile station.

### 3.1 System Model and Assumptions

We consider the downlink of an  $N$ -subcarrier OFDMA cooperative network which contains a BS,  $M$  fixed relay nodes, and  $K$  MS's equipped with one antenna and randomly distributed within a cell. Similar to the conventional relay-based cooperative communication systems, we assume a two-phase (time-slot) transmission scheme with perfect timing synchronization among all network users. Each subcarrier suffers from slow flat Rayleigh fading and there is no change of the channel state during a two-phase period. A data stream from a source user must be carried by the same subcarrier no



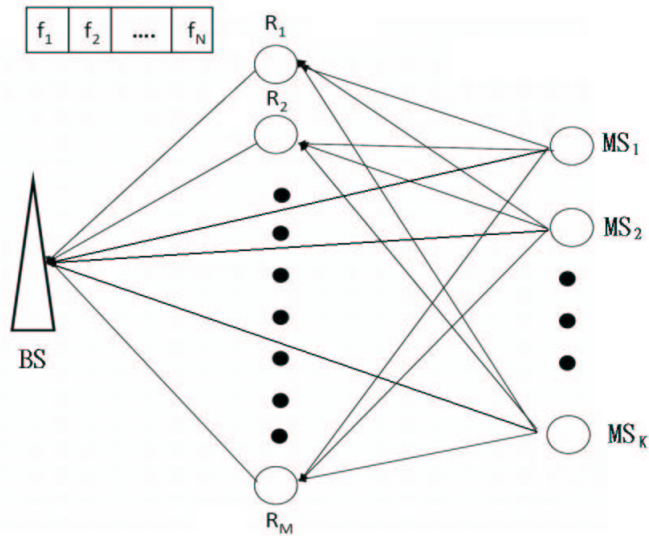


Figure 3.1: SISO system model

matter it is transmitted by a source node or a relay node. As mentioned in the previous section, our protocol offers both relay-aided (RA) and non-relay-aided (NRA) modes. We consider the decode-and-forward (DF) cooperative relay scheme only and assume that the maximum-ratio-combining detector is employed by the destination (user) node, assuming perfect decoding at the relays. Although the system model presented below describes a downlink setup, it can be easily converted into an uplink scenario with all the results obtained remain valid.

We assume the mobile user obtains the channel information by MMSE estimator and the feedback of the estimate is instantaneous and perfect to the BS. We also assume that the phase of the channel gain can be perfectly acquired while having channel estimation error which pertains to the amplitude of the correct channel gain. As a result, the same channel information about the channel gain with an estimation error is available simultaneously to both the transmitter and the receiver. Based on the imperfect CSI from all users and the minimum rate requirement of each MS, the BS acts as a central control device to carry out all resource allocation related operations which include collecting link information, appropriating resources, and informing MS' about their assigned resources.

## 3.2 Achievable rates for RA and NRA modes

For the source-to-destination link, the matched filter output at the destination node can be expressed as

$$r = (\hat{h} + \tilde{h})x + w \quad (3.1)$$

where  $\hat{h}$  denotes the estimated complex channel gain,  $\tilde{h}$  is the estimation error which can be modelled as a zero-mean complex Gaussian random variable with variance  $\sigma_h^2$ ,  $w$  is the complex white Gaussian noise with variance  $\sigma_n^2$ .

Although a closed-form expression for the capacity of the above link is not known, a tight lower bound  $C_{LB}$  (in bits/sec/Hz) is given by [16]

$$C_{LB} = \log_2 \left[ 1 + \frac{P|\hat{h}|^2}{P\sigma_h^2 + \sigma_n^2} \right] \quad (3.2)$$

where  $P = E\|x\|^2$ . For brevity, we will refer to link capacity and its lower bound interchangeably in the subsequent discourse unless there is danger of ambiguity. For the same reason, we denote by  $h_{SD}(k, n)$  the estimated (component) link gain between the BS and the  $k$ th MS, by  $h_{RD}(k, m, n)$  that between the  $m$ th relay and the  $k$ th MS, and by  $h_{SR}(m, n)$  that between BS and relay  $m$ , all on subcarrier  $n$ . The corresponding transmit powers are denoted by  $p_S(n, k)$ ,  $p_{RD}(k, m, n)$ ,  $p_{SR}(m, n)$ .

Using the above notations, we express the capacity (lower bound) of a basic cooperative network with imperfect CSI as

$$r_m(k, n) = \min \left\{ \log(1 + G_1|h_{SR}(m, n)|^2), \right. \\ \left. \log(1 + G_2|h_{SD}(k, n)|^2 + G_3|h_{RD}(k, m, n)|^2) \right\} \quad (3.3)$$

where

$$G_1 = \frac{\alpha_m p_{SR}(m, n)}{\alpha_m p_{SR}(m, n) \sigma_h^2 + \sigma_n^2}, \\ G_2 = \frac{\alpha_k p_{SR}(m, n)}{\alpha_k p_{SR}(m, n) \sigma_h^2 + \sigma_n^2}, \\ G_3 = \frac{\alpha_{k,m} p_{RD}(k, m, n)}{\alpha_{k,m} p_{RD}(k, m, n) \sigma_h^2 + \sigma_n^2}, \quad (3.4)$$

and  $\alpha_m, \alpha_k, \alpha_{k,m}$  are the path losses of source to relay (SR), source to destination (SD), and relay to destination (RD) links, respectively. For simplicity, we will henceforth denote  $h_{SR}(m, n)$  by  $h_{SR}$ ,  $h_{SD}(k, n)$  by  $h_{SD}$ , and  $h_{RD}(k, m, n)$  by  $h_{RD}$ . For a given sum power,  $P = p_{SR}(m, n) + p_{RD}(k, m, n)$ , the optimal power distribution ratio  $\Gamma(k, m, n, P)$  that maximizes the capacity is given by

$$\Gamma(k, m, n, P) = \frac{p_{RD}(k, m, n)}{p_{SR}(m, n)} = (x_1)^{\frac{1}{3}} + (x_2)^{\frac{1}{3}} + \frac{a}{3} \quad (3.5)$$

where

$$x_1 = \frac{1}{2} \left( \frac{2a^3}{27} + \frac{ab}{3} + c + \sqrt{\left(\frac{2a^3}{27} + \frac{ab}{3} + c\right)^2 - \frac{4(3b+a^2)^3}{729}} \right),$$

$$x_2 = \frac{1}{2} \left( \frac{2a^3}{27} + \frac{ab}{3} + c - \sqrt{\left(\frac{2a^3}{27} + \frac{ab}{3} + c\right)^2 - \frac{4(3b+a^2)^3}{729}} \right),$$

and

$$a = \alpha_{k,m} \sigma_n^2 |h_{RD}|^2,$$

$$b = P [\alpha_m \alpha_{k,m} \sigma_h^2 \sigma_n^2 |h_{SR}|^2 - \alpha_k \alpha_{k,m} \sigma_h^2 \sigma_n^2 |h_{SD}|^2 - \alpha_{k,m} \alpha_m \sigma_h^2 \sigma_n^2 |h_{RD}|^2 - \alpha_{k,m} \alpha_k \sigma_h^2 \sigma_n^2 |h_{RD}|^2]$$

$$+ [\alpha_m \sigma_n^4 |h_{SR}|^2 - \alpha_k \sigma_n^4 |h_{SD}|^2 - 2\alpha_{k,m} \sigma_n^4 |h_{RD}|^2],$$

$$c = P^2 \sigma_h^4 \alpha_m \alpha_k \alpha_{k,m} [|h_{SR}|^2 - |h_{SD}|^2 - |h_{RD}|^2]$$

$$+ P \sigma_h^2 \sigma_n^2 [\alpha_m \alpha_k |h_{SR}|^2 + \alpha_m \alpha_{k,m} |h_{SR}|^2$$

$$- \alpha_k \alpha_m |h_{SD}|^2 - \alpha_k \alpha_{k,m} |h_{SD}|^2 - \alpha_{k,m} \alpha_m |h_{RD}|^2 - \alpha_{k,m} \alpha_k |h_{RD}|^2]$$

$$+ \sigma_n^4 [2\alpha_m |h_{SR}|^2 - 2\alpha_k |h_{SD}|^2 - \alpha_{k,m} |h_{RD}|^2],$$

$$d = P \sigma_h^2 \sigma_n^2 [\alpha_m \alpha_k |h_{SR}|^2 - \alpha_k \alpha_m |h_{SD}|^2] + \sigma_n^4 [\alpha_m |h_{SR}|^2 - \alpha_k |h_{SD}|^2]$$

The corresponding maximum achievable rate is

$$r_m(k, n, P) = \log_2 \left[ 1 + \frac{\frac{P}{1+\Gamma(k,m,n,P)} \alpha_m |h_{SR}(m, n)|^2}{\frac{P}{1+\Gamma(k,m,n,P)} \alpha_m \sigma_h^2 + \sigma_n^2} \right] \quad (3.6)$$

Since in NRA mode, we allow the source to be active for both phases, a fair comparison on the achievable rate should be measured with respect to the total consumed energy.

The resulting link capacity over two OFDM symbols is

$$r_D(k, n, P) = 2 \log_2 \left[ 1 + \frac{\frac{P}{2} \alpha_k |h_{SD}(k, n)|^2}{\frac{P}{2} \alpha_k \sigma_h^2 + \sigma_n^2} \right] \quad (3.7)$$

### 3.3 Problem Formulation

To begin with, we define  $\rho_{(k,n,m)}$  as the subcarrier assignment and link selection indicator so that  $\rho_{(k,n,m)} = 1$  and  $m > 0$  indicates subcarrier  $n$  is allocated to user  $k$  who options for RA mode using relay node  $m$  while  $m = 0$  indicates the user options for the NRA mode. Otherwise, user  $k$  does not have access to subcarrier  $n$  over the  $m$ th link. Suppose the available total transmission power is  $P_T$ , then the problem of maximizing the total system rate under the users' rate constraints is equivalent to

$$\max \sum_{k=1}^K R_k = \max \sum_{k=1}^K \sum_{n=1}^N \sum_{m=0}^M \rho_{(n,k,m)} r_m(k, n, p_m(k, n))$$

subject to

$$\begin{aligned} R_k &\geq R_{k,min} \quad \forall k \\ \sum_{k=1}^K \sum_{m=0}^M \rho_{(n,k,m)} &\leq 1 \quad \forall n \\ \rho_{(n,k,m)} &\in \{0, 1\} \quad \forall n, k, m \\ \sum_{n=1}^N \sum_{k=1}^K \sum_{m=0}^M \rho_{(n,k,m)} p_m(k, n) &\leq P_T \end{aligned} \quad (3.8)$$

where  $R_k$  and  $R_{k,min}$  are the achievable rate and the minimum rate requirement for user  $k$ , and  $r_0(k, n, P) = r_D(k, n, P)$ . The above formulation is an NP-hard mixed integer programming problem. Instead of finding the optimal solution, we propose a low-complexity suboptimal algorithm in the following section.

### 3.4 Proposed Resource Allocation Schemes

We decompose the task of joint subcarrier/power assignment and the corresponding link selection into a three-stage process which can be described by  $\rho_{(n,k,m)} = \delta_{(n,k)} \beta_m(n, k)$ , where  $\delta_{(n,k)}$  and  $\beta_m(n, k)$  represent the subcarrier assignment and link selection operations, respectively, i.e.,  $\delta_{(n,k)} = 1$  implies that subcarrier  $n$  is allocated to user  $k$ , and  $\beta_m(n, k) = 1$  means user  $k$  sends its data over subcarrier  $n$  with the help of relay  $m$ . Obviously, the only alternate value for these two functions is zero.

The original problem is thus divided into three subproblems: **P1**–link selection subproblem for deciding  $\{\beta_m(n, k)\}$ , **P2**–subcarrier assignment for deciding  $\{\delta_{(n,k)}\}$ , and **P3**–power allocation. In the first stage, we select for each DN the best link among  $(M + 1)$  candidate relay links. Based on the selected relay links, the BS then allocate subcarriers to each DN according to its link gain and minimum rate requirement. The BS tries to maximize the user diversity gain under the minimum rate constraints. In the third stage, we proceed to allocate power by taking into account the channel estimation error.

### 3.4.1 Relaying/Direct link selection rule

Since the optimal relay/BS power ratio  $\Gamma$  depends on the available transmit power and the final power allocation is still unknown, we assume a fair power distribution  $\frac{P_T}{N}$  for all DNs. We then determine if a relay is needed for user  $k$  if subcarrier  $n$  is available by

$$m^* = \arg \max_{0 \leq m \leq M} r_m(k, n, P_T/N) \quad (3.9)$$

When  $m^* = 0$ , DN  $k$  should use only the direct link if it was given subcarrier  $n$ . We then set  $\beta_{m^*}(n, k) = 1$  and  $\beta_m(n, k) = 0$ , for  $0 \leq m \leq M, m \neq m^*$ .

### 3.4.2 Subcarrier assignment

Many suboptimal subcarrier assignment algorithm has been proposed but most of them seldom consider the user rate constraint in this step. They usually assign a subcarrier to the DN who has the best channel gain on this subcarrier [8]. Instead of choosing the user  $k^*$  having the largest weighted rate [9],  $w_k \times r(k, n)$  with  $w_k \stackrel{def}{=} \frac{R_{k,min} - R_k}{R_{k,min}}$  and  $R_k$  being the current rate for DN  $k$ , we choose  $(k^*, n^*)$  which gives the maximum weighted rate over all users and unassigned subcarriers. The process repeats until either all the rate requirements are met or all subcarriers are assigned. For the former case, we assign each of the remaining subcarriers to the one having best rate on it. If the latter case

occurs, we need a rate-balancing step after power allocation. The complete subcarrier assignment algorithm is summarized in Table (3.1).

<p>Given <math>r(k, n)</math> and <math>\delta_{(n,k)} = 0</math>, for <math>1 \leq k \leq K, 1 \leq n \leq N</math>  Set <math>U = \{1, 2, \dots, N\}</math>, and <math>r_k = 0 \quad \forall k</math>  while (<math> U  \geq 1</math>)      <math>W_k = (R_{k,min} - R_k) / R_{k,min} \quad \forall k</math>      if (<math>\max_{1 \leq k \leq K} W_k &gt; 0</math>)          <math>(n^*, k^*) = \arg \max_{n \in U, 1 \leq k \leq K} W_k r(k, n)</math>      else          <math>(n^*, k^*) = \arg \max_{n \in U, 1 \leq k \leq K} r(k, n)</math>      end      <math>U = U \setminus \{n^*\}</math>, <math>\delta_{(n^*, k^*)} = 1</math>, <math>R_{k^*} = R_{k^*} + r(k^*, n^*)</math>  end</p>
---

Table 3.1: Subcarrier assignment algorithm.

### 3.4.3 Power allocation

We now suggest a nearly optimal power allocation scheme. We first assign equal power  $P_T/N$  to subcarriers using relaying and allocate the remaining power by a modified water-filling (mwf) on subcarriers using no relay. Based on (3.7), the water-filling solution can be obtained by the quadratic formula

$$p(k, n) = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (3.10)$$

where

$$\begin{aligned} a &= \frac{\alpha_k \sigma_h^2}{2\sigma_n^2} \left( \frac{\sigma_h^2}{|h_{SD}(k,n)|^2} + 1 \right), \\ b &= \frac{2\sigma_h^2}{|h_{SD}(k,n)|^2} + 1, \\ c &= - \left( \frac{2}{\lambda \ln 2} - \frac{2\sigma_n^2}{\alpha_k |h_{SD}(k,n)|^2} \right)^+ \end{aligned}$$

where  $(t)^+ = \max(0, t)$ . When there is no estimation error ( $\sigma_h^2 = 0$ ), the optimal power allocation is similar to the conventional water-filling solution, i.e.  $p(k, n) = \left( \frac{2}{\lambda \ln 2} - \frac{2\sigma_n^2}{\alpha_k |h_{SD}(k,n)|^2} \right)^+$ . By iteratively modifying  $\lambda$  to satisfy the total power constraint (3.8), we obtain the optimal power allocation for subcarriers using only direct link.

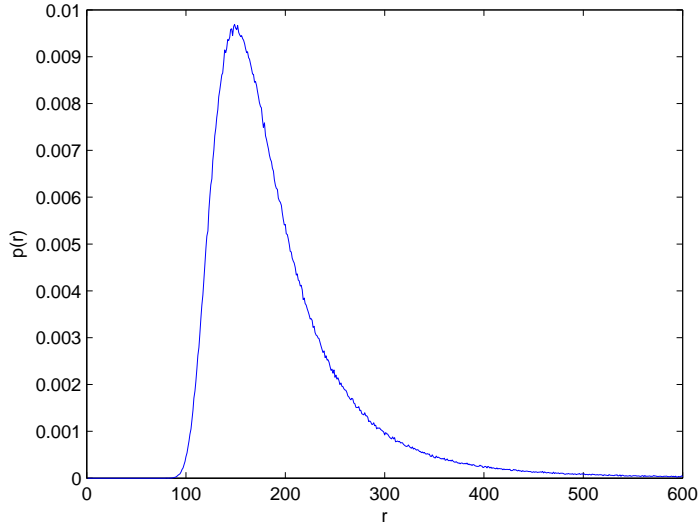


Figure 3.2: The probability density function of the user location distribution;  $r_0 = 150$  m.

As mentioned before, a rate balancing step is needed if the user rate constraints are not satisfied. We divide the DNs into two groups: Group I consists of DNs whose rate requirements have been met and members of Group II include all other DNs. We first select the DN, say DN  $i$ , from Group II whose rate allocation is the lowest and the one, say DN  $j$  from Group I having the largest surplus rate ( $R_j - R_{j,min}$ ). Among the subcarriers which have been assigned to DN  $j$ , we reassign to DN  $i$  the one which is the best for it if the reassignment does not make DN  $j$  become a member of Group II. The rate requirements for almost all DNs in Group II can be met through such a reassignment. In order not to make the process too complicated, we give up on DN  $i$  when the above reassignment is not allowed. The rate-balancing process is sequentially applied to all DNs in Group II in descending order of the allocated rate.

### 3.5 Numerical Results and Discussions

The simulation results shown in this section assume a single-cell network with multiple DNs that are randomly distributed within a 120-degree section of the 600-meter radius

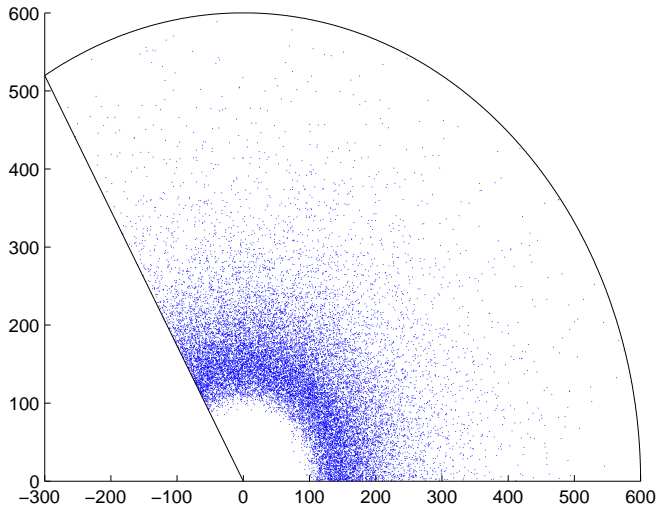


Figure 3.3: The user location distribution;  $r_0 = 150$  m.

circle centered at the BS. The RNs are placed on a circle with a 150-meter radius with an equal angular spacing. As shown in Fig. 3.2 and 3.3, the probability density function (pdf) of the DN locations is given by [20]

$$P = \frac{r_0^4}{r^5} \exp \left[ -\frac{5}{4} \left( \frac{r_0}{r} \right)^4 \right]. \quad (3.11)$$

where  $r > 0$  is the radius and  $r_0 = 150$  m. We also assume each subcarrier suffers from independent Rayleigh fading in any direct or relay link with a path loss exponent 3.5. For the convenience of comparison, we normalize each link gain with respect to the worst-case gain corresponding to the longest link distance.

We compare the performance of our subcarrier assignment (SA) algorithm (**P2**-solver) with two subcarrier assignment schemes which we refer to as greedy SA and weighted SA algorithms, respectively. These two algorithms were modified from those presented in [8] and [9], respectively. As the originally schemes were designed with perfect CSI assumption and have different RN selection criterion, we use our **P1** and **P3** solutions but keep that for P2 intact for the sake of fair comparison.

In Figs. 3.4–3.5, we assume there are 8 DNs, 3 RNs in a 32-subcarrier OFDMA cell



with a total system power of  $P_T = 3.2$  W (i.e. the average transmitted power for each subcarrier is 0.1 W) and varying minimum rate requirements. The first figure shows that the sum rate of our algorithm is closer to the greedy SA than the weighted SA does. For a given subcarrier assignment scheme, we compare two power allocation methods—equal power allocation and the proposed modified water-filling power allocation for the direct link. As expected, our scheme of [4.34] performs better than the equal power allocation approach. The second figure depicts the rate failure probability behavior, i.e., the probability that the algorithm fails to meet a user’s rate requirement. Obviously, our solution has a much lower rate failure probability than those achievable by either greedy SA or weighted SA scheme. In Fig. 3.6 and Fig. 3.7 we compare the sum rate

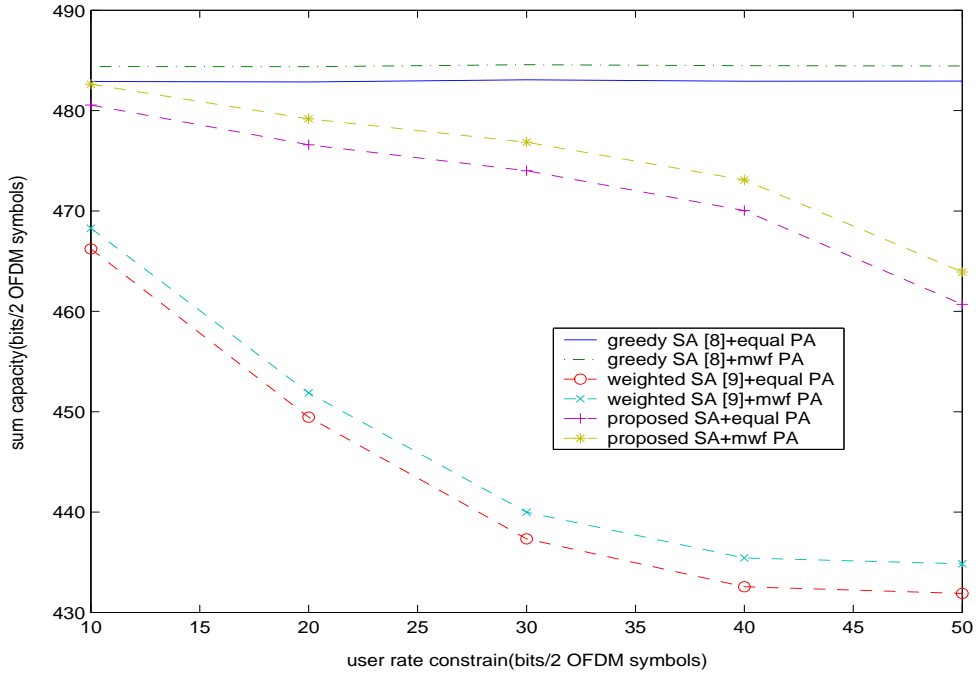


Figure 3.4: Sum rate v.s. user rate constraint; 8 DNs, 3 RNs and 32 subcarriers with  $P_T = 3.2$ .

and required rate failure probability (i.e., the probability that an algorithm fails to meet the rate requirement) performance as a function of the user number in a 32-subcarrier OFDMA network having 3 RNs,  $P_T = 3.2$  and a minimum user rate requirement of

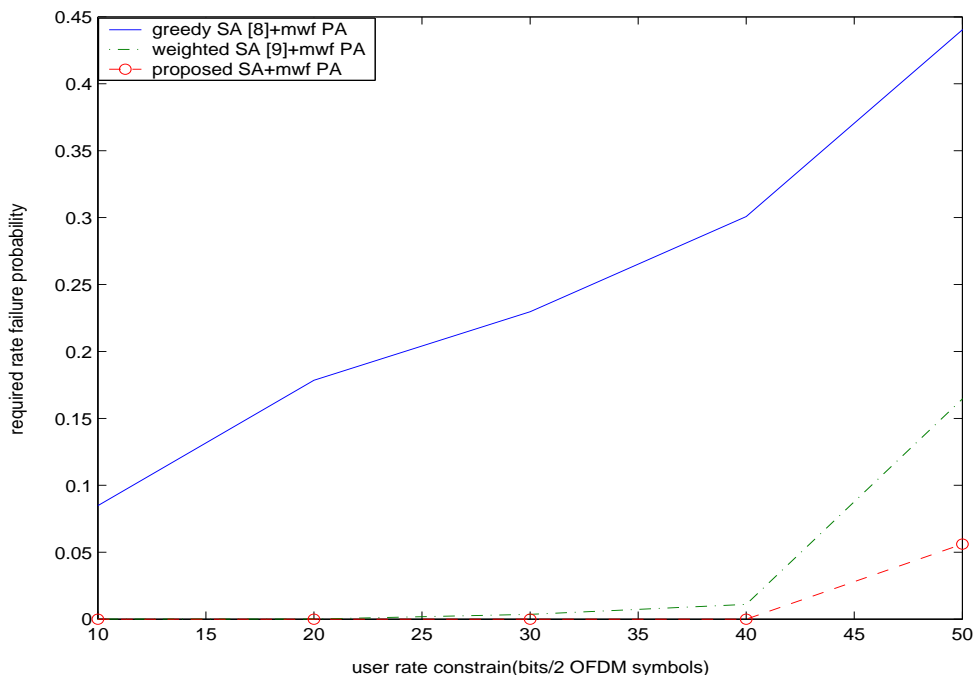


Figure 3.5: Rate failure probability v.s. user rate constraint; 8 DNs, 3 RNs and 32 subcarriers with  $P_T = 3.2$ .

48 bits/2 OFDM symbols. It is clear that our algorithm outperforms the weighted SA scheme. In Fig. 3.8, we examine the conditional average achievable rate ratio  $\gamma$  defined as  $\gamma = E[R_k/R_{k,min}|R_k < R_{k,min}]$ , if  $P(R_k < R_{k,min}) \neq 0$ ; otherwise  $\gamma = 1$ . We observe that our algorithm is far better than the greedy algorithm and when the user number is large, outperform the weighted SA algorithm of [9]. For fair comparison, all three algorithms employ the proposed mwf power loading scheme. In Fig. 3.9, we show the average probability of doing the load balancing step. While the user number increases, users are more likely not to satisfy their rate constraints, and thus the probability of doing the load balancing step increases.

In Figs. 3.10–3.11, we consider another scenario in which there are 8 DNs, 3 RNs in a 128-subcarrier OFDMA cell with a total system power of  $P_T = 12.8$  W. They also shows that the sum rate of our algorithm not only has the lower probability that users fail to meet their rate requirements but is closer to the greedy SA than the weighted SA does.

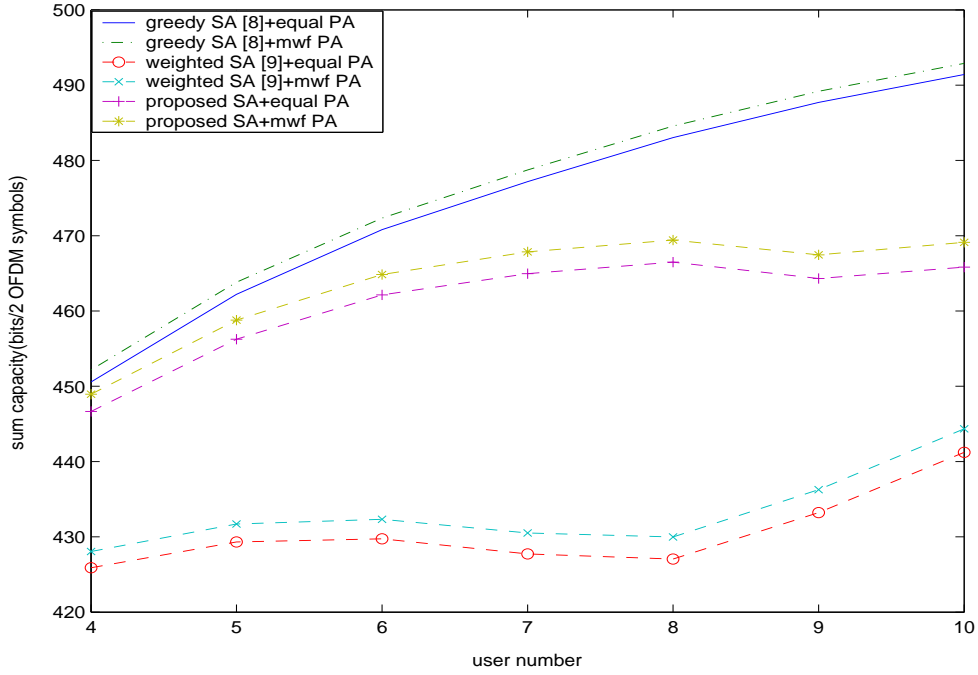


Figure 3.6: Sum rate v.s. user number, 3 relay nodes and 32 subcarriers with  $P_T = 3.2$  and the minimum user rate requirement is 48 bits/2 OFDM symbols.

Moreover, the performance enhancement by the proposed modified water-filling power allocation for the direct link. Then in Fig. 3.12 and Fig. 3.13 we compare the sum rate and required rate failure probability performance as a function of the user number in a 128-subcarrier OFDMA network having 3 RNs,  $P_T = 12.8$  and a minimum user rate requirement of 48 bits/2 OFDM symbols. Our algorithm also outperforms the weighted SA scheme.

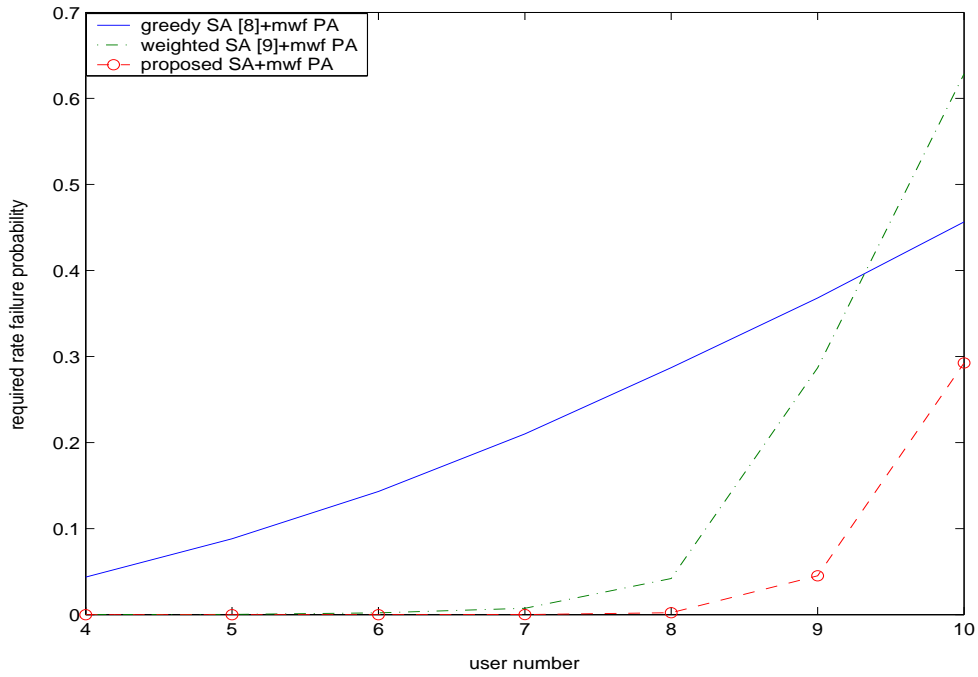


Figure 3.7: Rate failure probability v.s. user number; 3 relay nodes and 32 subcarriers with  $P_T = 3.2$  and the minimum user rate requirement is 48 bits/2 OFDM symbols.

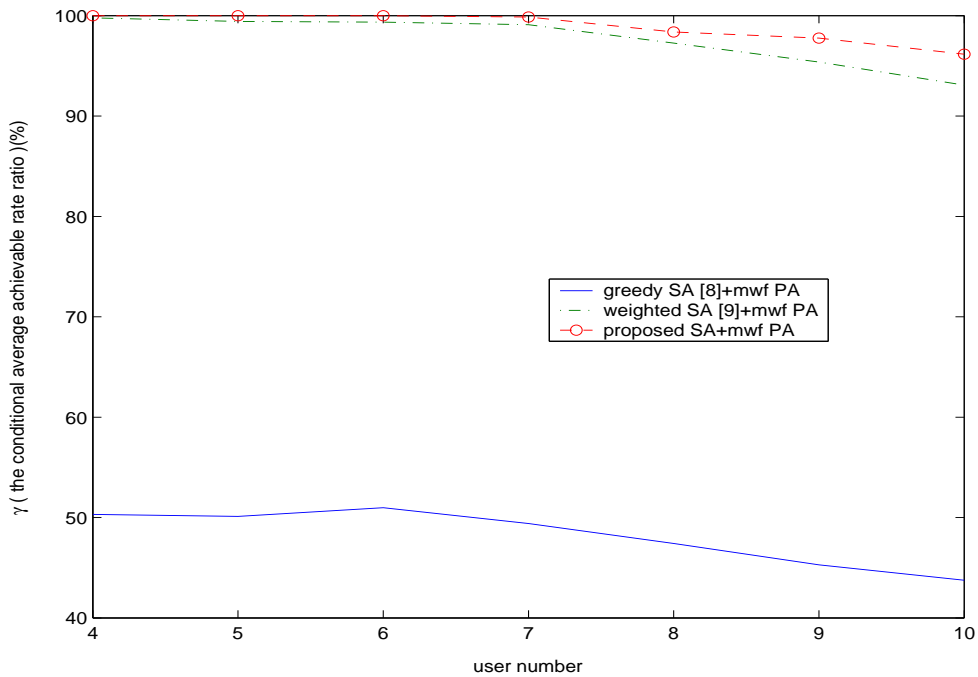


Figure 3.8: Average achievable rate ratio for the rate failure event v.s. user number; 3 relay nodes and 32 subcarriers with  $P_T = 3.2$  and the minimum user rate requirement is 48 bits/2 OFDM symbols.

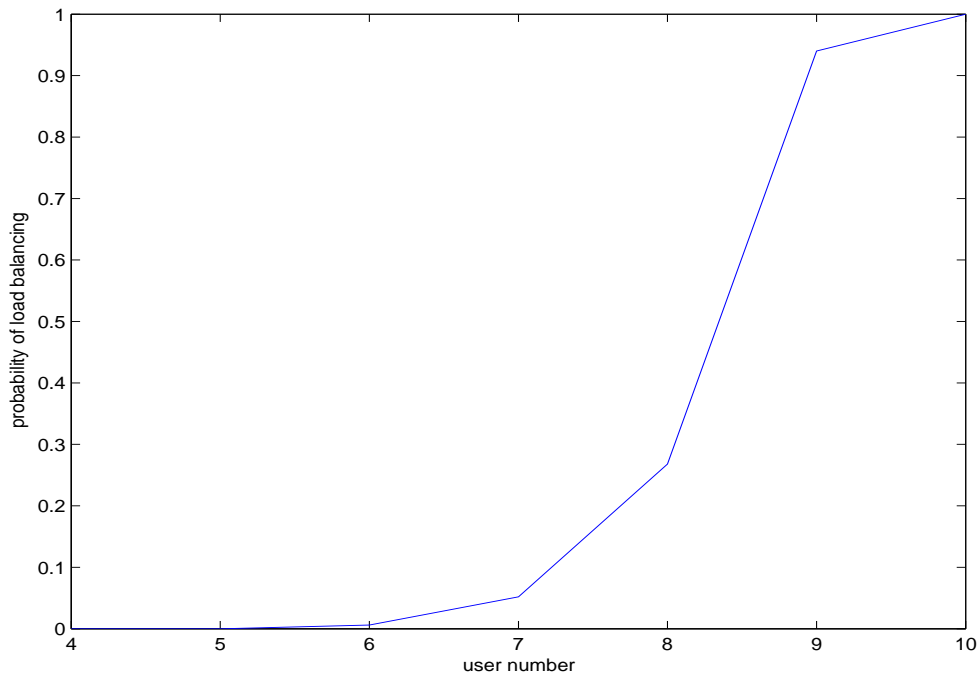


Figure 3.9: Load balance probability v.s. user number; 3 relay nodes and 32 subcarriers with  $P_T = 3.2$  and the minimum user rate requirement is 48 bits/2 OFDM symbols.

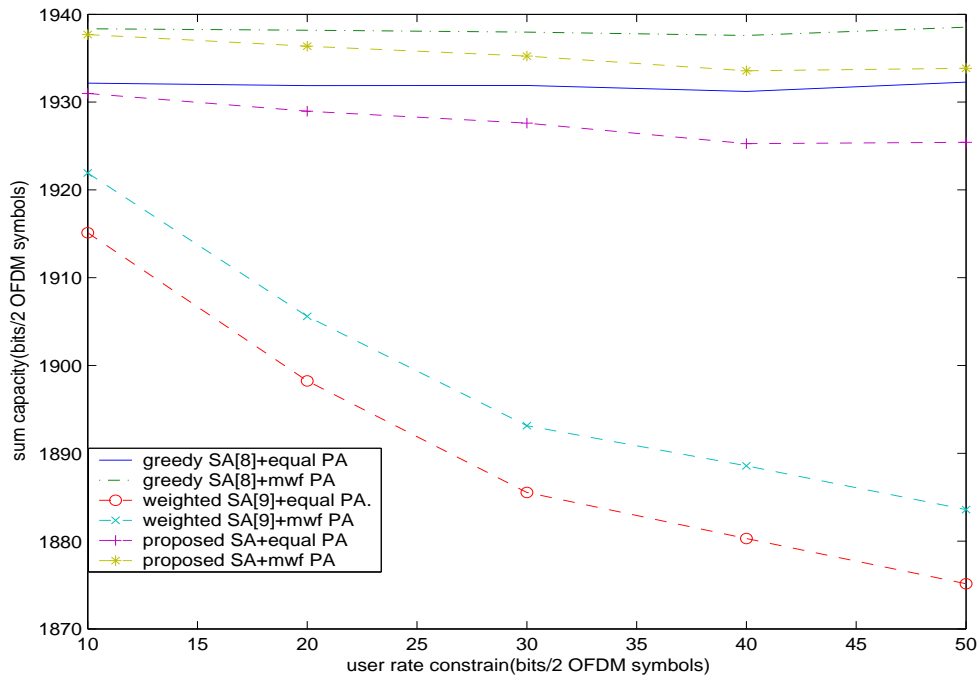


Figure 3.10: Sum rate v.s. user rate constraint; 8 DNs, 3 RNs and 128 subcarriers with  $P_T = 12.8$ .

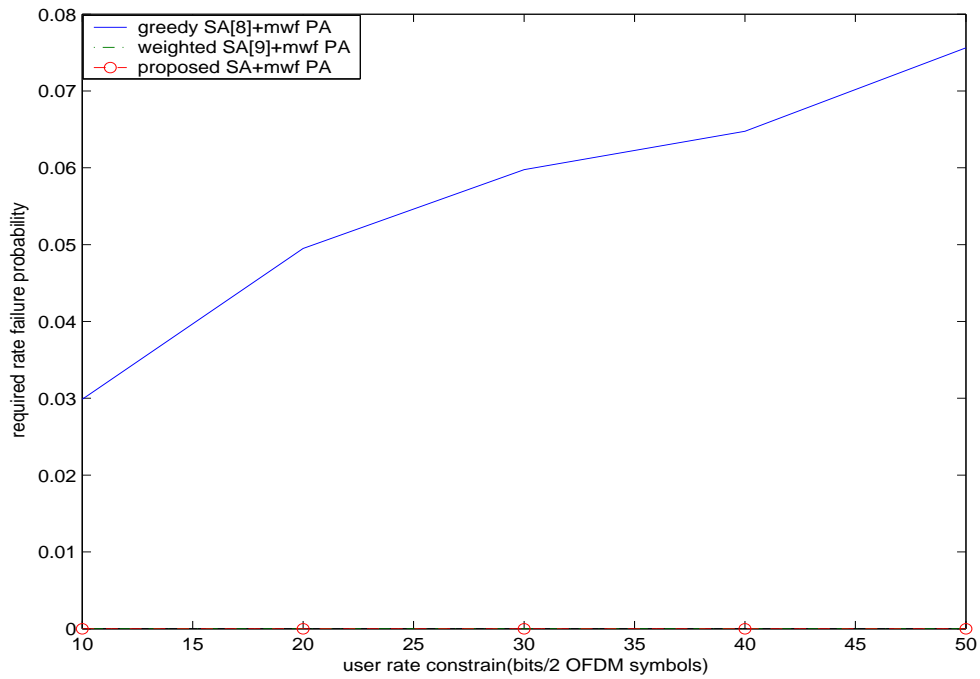


Figure 3.11: Rate failure probability v.s. user rate constraint; 8 DNs, 3 RNs and 128 subcarriers with  $P_T = 12.8$ .

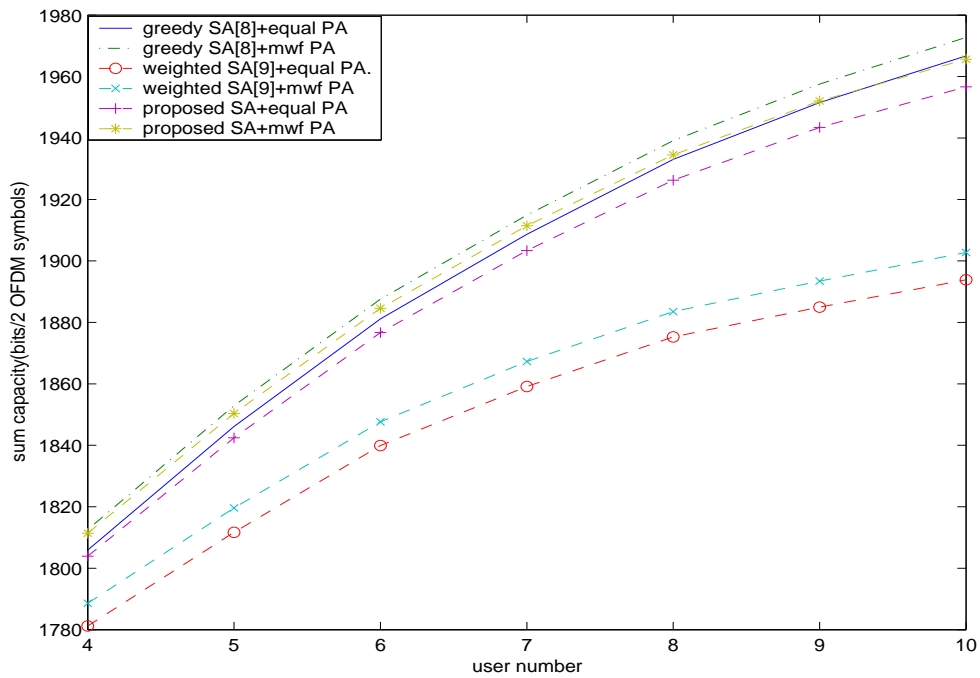


Figure 3.12: Sum rate v.s. user number; 3 relay nodes and 128 subcarriers with  $P_T = 12.8$  and the minimum user rate requirement is 48 bits/2 OFDM symbols.

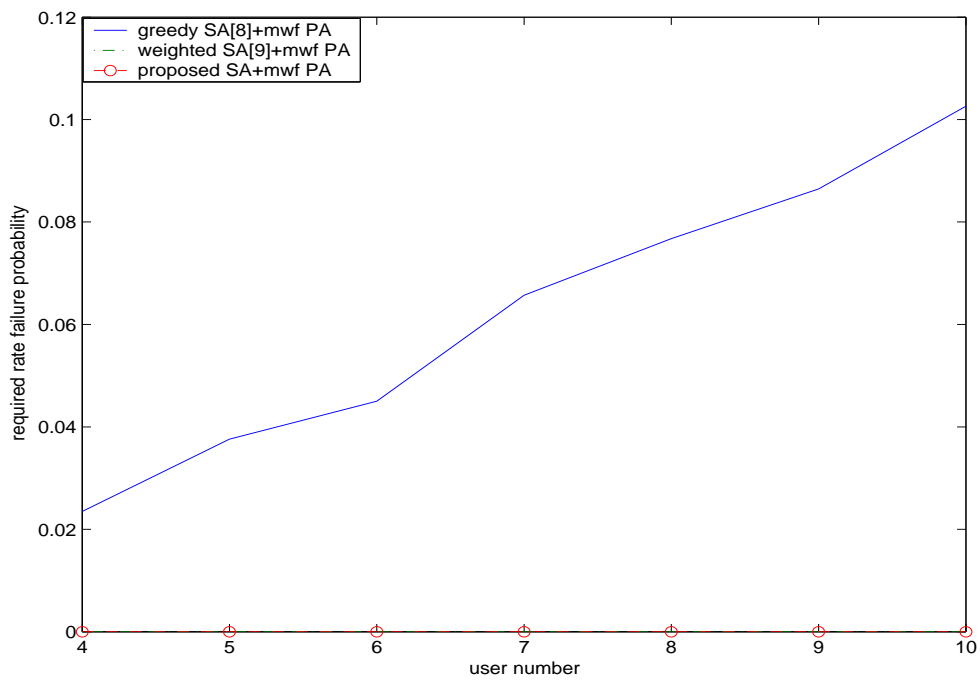


Figure 3.13: Rate failure probability v.s. user number; 3 relay nodes and 128 subcarriers with  $P_T = 12.8$  and the minimum user rate requirement is 48 bits/2 OFDM symbols.

# Chapter 4

## Downlink Resource/Relay Allocation for MIMO OFDMA Networks

In this chapter, we consider the scenario for a downlink multiple-input multiple-output OFDMA system. We also use the capacity lower bound and propose resource allocation schemes that assigns power, subcarriers and cooperative relays to each mobile station. However, we propose not only suboptimal power allocation but the optimal one.

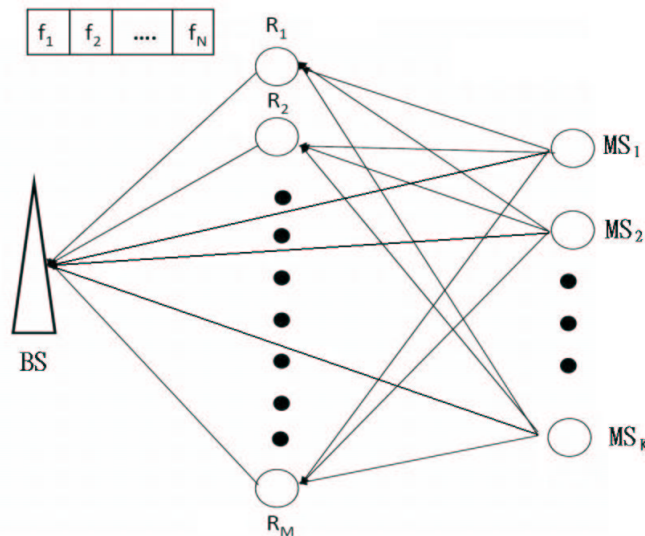


Figure 4.1: MIMO system model



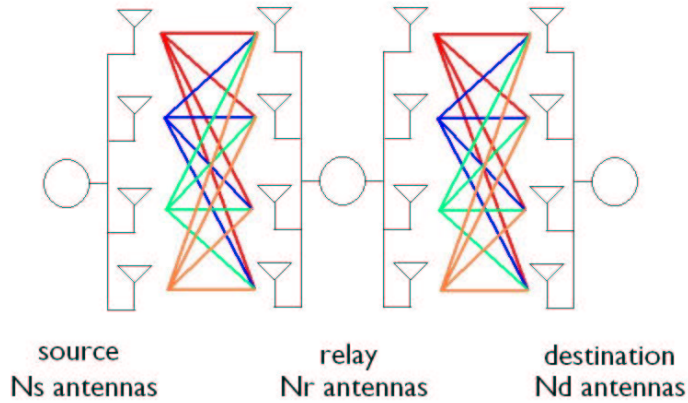


Figure 4.2: One cooperative path in Fig.4.1

## 4.1 System Model and Basic Assumptions

As shown in Fig. 4.1 and Fig. 4.2, we consider the downlink of an  $N$ -subcarrier OFDMA cooperative network which contains a BS,  $M$  fixed relay nodes, and  $K$  MS's equipped with  $N_s$ ,  $N_r$  and  $N_d$  antennas, respectively. We also consider the decode-and-forward (DF) cooperative relay scheme and assume a two-phase (time-slot) transmission scheme with perfect timing synchronization among all network users. Perfect decoding at the relays is assumed. Each subcarrier suffers from slow flat Rayleigh fading and there is no change of the channel state during a two-phase period. A data stream from a source user must be carried by the same subcarrier no matter it is transmitted by a source node or a relay node. However, we do not employ the maximum-ratio-combining detector for this MIMO system for simplification.

We also assume the mobile user estimate the channel information by MMSE estimator and the feedback of the estimate is instantaneous and perfect to the BS, then the same channel information about the channel gain with an estimation error is available simultaneously to both the transmitter and the receiver. In this chapter, the BS also acts as a central control device to allocate resources based on the imperfect CSI from all users and the minimum rate requirement of each MS.

## 4.2 Capacity Lower Bound for MIMO Channels

First, for a source-to-destination link, we denote the estimated gain matrix of MIMO channels, estimated error matrix of MIMO channels, and noise matrix as  $\hat{H}$ ,  $\tilde{H}$  and  $W$ . Than we assume the entries of  $\tilde{H}$  and  $W$  are independent, identically distributed (i.i.d.) and zero-mean circularly symmetric complex Gaussian (ZMCSCG) with variance  $\sigma_h^2$  and  $\sigma_n^2$ , respectively.

Moreover, given the singular value decomposition(SVD) of the estimated channel matrix be  $\hat{H} = UDV^*$ , we can find that  $U$  and  $V$  are unitary matrixes and  $D$  is a diagonal matrix whose diagonal entries are the singular values of  $\hat{H}$ . Thus, by multiplying the transmitted signal vector by the vector  $V$  before transmitted and multiplying the received signal by the vector  $U^*$  at destination, the processed received signal vector  $Y$  then can be expressed as

$$Y = U^*(\hat{H} + \tilde{H})VX + U^*W \quad (4.1)$$

where  $X$  is the transmitted signal vector. And we replace  $\hat{H}$  by  $UDV^*$ , (4.1) becomes

$$Y = DX + U^*\tilde{H}VX + U^*W \quad (4.2)$$

Assume  $\tilde{H} = \begin{bmatrix} h_{11} & \cdots & h_{1N_S} \\ \vdots & \ddots & \vdots \\ h_{N_R1} & \cdots & u_{N_R N_S} \end{bmatrix}$ ,  $V = \begin{bmatrix} v_{11} & \cdots & v_{1N_S} \\ \vdots & \ddots & \vdots \\ v_{N_S1} & \cdots & v_{N_S N_S} \end{bmatrix}$ ,

$U^* = \begin{bmatrix} u_{11} & \cdots & u_{1N_R} \\ \vdots & \ddots & \vdots \\ u_{N_R1} & \cdots & u_{N_R N_R} \end{bmatrix}$ ,  $X = \begin{bmatrix} x_1 \\ \vdots \\ x_{N_S} \end{bmatrix}$ ,

then

$$U^*\tilde{H} = \begin{bmatrix} \sum_{i=1}^{N_R} u_{1i}\tilde{h}_{i1} & \cdots & \sum_{i=1}^{N_R} u_{1i}\tilde{h}_{iN_S} \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{N_R} u_{N_R i}\tilde{h}_{i1} & \cdots & \sum_{i=1}^{N_R} u_{N_R i}\tilde{h}_{iN_S} \end{bmatrix} \quad (4.3)$$

$$U^*\tilde{H}V = \begin{bmatrix} \sum_{l=1}^{N_S} \sum_{i=1}^{N_R} u_{1i}\tilde{h}_{il}v_{l1} & \cdots & \sum_{l=1}^{N_S} \sum_{i=1}^{N_R} u_{1i}\tilde{h}_{il}v_{lN_S} \\ \vdots & \ddots & \vdots \\ \sum_{l=1}^{N_S} \sum_{i=1}^{N_R} u_{N_R i}\tilde{h}_{il}v_{l1} & \cdots & \sum_{l=1}^{N_S} \sum_{i=1}^{N_R} u_{N_R i}\tilde{h}_{il}v_{lN_S} \end{bmatrix} \quad (4.4)$$

$$U^* \tilde{H} V X = \begin{bmatrix} \sum_{k=1}^{N_S} \sum_{l=1}^{N_S} \sum_{i=1}^{N_R} u_{l1} \tilde{h}_{il} v_{lk} x_k \\ \vdots \\ \sum_{k=1}^{N_S} \sum_{l=1}^{N_S} \sum_{i=1}^{N_R} u_{N_R l} \tilde{h}_{il} v_{lk} x_k \end{bmatrix} \quad (4.5)$$

Since  $U^*$  is an unitary matrix, which means  $\sum_{i=1}^{N_R} u_{t_1 i} u_{t_2 i}^*$  equals to 1 if  $t_1 = t_2$  and otherwise equals to zero, the entries outside the main diagonal in the covariance matrix of  $U^* \tilde{H} V X$  are zero. Moreover, for the  $t$ th diagonal entries in the covariance matrix of  $U^* \tilde{H} V X$ , we need to calculate

$$E \left[ \left( \sum_{k_1=1}^{N_S} \sum_{l_1=1}^{N_S} \sum_{i_1=1}^{N_R} u_{t i_1} \tilde{h}_{i_1 l_1} v_{l_1 k_1} x_{k_1} \right) \left( \sum_{k_2=1}^{N_S} \sum_{l_2=1}^{N_S} \sum_{i_2=1}^{N_R} u_{t i_2} \tilde{h}_{i_2 l_2} v_{l_2 k_2} x_{k_2} \right)^* \right] \quad (4.6)$$

In (4.6), we can notice that

1. Due to  $x_{k_1}$  and  $x_{k_2}$  are independent,  $E \left[ \left( \sum_{l_1=1}^{N_S} \sum_{i_1=1}^{N_R} u_{t i_1} \tilde{h}_{i_1 l_1} v_{l_1 k_1} x_{k_1} \right) \left( \sum_{l_2=1}^{N_S} \sum_{i_2=1}^{N_R} u_{t i_2} \tilde{h}_{i_2 l_2} v_{l_2 k_2} x_{k_2} \right)^* \right]$

for all  $k_1 \neq k_2$  are equal to zero.

2. Due to  $\tilde{h}_{i_1 l_1}$  and  $\tilde{h}_{i_2 l_2}$  are independent,  $E \left[ \left( \sum_{l_1=1}^{N_S} \sum_{i_1=1}^{N_R} u_{t i_1} \tilde{h}_{i_1 l_1} v_{l_1 k_1} x_{k_1} \right) \left( \sum_{l_2=1}^{N_S} \sum_{i_2=1}^{N_R} u_{t i_2} \tilde{h}_{i_2 l_2} v_{l_2 k_2} x_{k_2} \right)^* \right]$

for all  $l_1 \neq l_2$  or  $i_1 \neq i_2$  are equal to zero.

Equation (4.6) than can be rewritten as

$$E \left[ \left( \sum_{k_1=1}^{N_S} \sum_{l_1=1}^{N_S} \sum_{i_1=1}^{N_R} u_{t i_1} u_{t i_1}^* \tilde{h}_{i_1 l_1} \tilde{h}_{i_1 l_1}^* v_{l_1 k_1} v_{l_1 k_1}^* x_{k_1} x_{k_1}^* \right) \right] \quad (4.7)$$

which equals

$$\sum_{i_1=1}^{N_R} u_{t i_1} u_{t i_1}^* \sum_{l_1=1}^{N_S} E \left[ \left( \tilde{h}_{i_1 l_1} \tilde{h}_{i_1 l_1}^* \right) \right] \sum_{k_1=1}^{N_S} (v_{l_1 k_1} v_{l_1 k_1}^*) E \left[ x_{k_1} x_{k_1}^* \right] \quad (4.8)$$

By (4.8), since  $V$  and  $U^*$  are unitary matrix, the diagonal entries of the covariance matrix of  $U^* \tilde{H} V X$  are all  $\sum_{k=1}^{N_S} \|x_k\|^2 \sigma_h^2$ . Thus, we can find that  $U^* \tilde{H} V X$  is still a zero-mean complex gaussian vector where each entry's variance is  $\sum_{k=1}^{N_S} \|x_k\|^2 \sigma_h^2$ .

Then a tight lower bound for the capacity of the above link  $C_{LB}$  (in bits/sec/Hz) can be formulated as

$$C_{LB} = \sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{p_i \lambda_i}{\sum_{i=1}^{N_t} p_i \sigma_h^2 + \sigma_n^2} \right) \quad (4.9)$$

where  $N_t$  is the number of eigen-channels,  $p_i$  is the power on the  $i$ th eigen-channel, and  $\lambda_i$  is the square to  $i$ th singular value of MIMO channel. We can notice that the power on each eigen-channel has the influence on all eigen-channels' rates. For brevity, we will refer to link capacity and its lower bound interchangeably in the subsequence discourse unless there is danger of ambiguity.

### 4.2.1 Achievable Rates for RA and NRA Modes

For a decode and forward relaying scheme cooperative transmission, denote the number of antennas on source, relay, and destination as  $N_s$ ,  $N_r$ , and  $N_d$ . Under DF scheme, given  $\min\{N_s, N_r\} = N_{t1}$  and  $\min\{N_r, N_d\} = N_{t2}$ , the achievable rate can be formulated as

$$r_m(k, n, P) = \min\left\{\sum_{i=1}^{N_{t1}} \log_2\left(1 + \frac{p_{s,i}(k, m, n)\alpha_m \lambda_{s,i}(k, m, n)}{\sum_{i=1}^{N_{t1}} p_{s,i}(k, m, n)\alpha_m \sigma_h^2 + \sigma_n^2}\right), \sum_{i=1}^{N_{t2}} \log_2\left(1 + \frac{p_{r,i}(k, m, n)\alpha_{k,m} \lambda_{r,i}(k, m, n)}{\sum_{i=1}^{N_{t2}} p_{r,i}(k, m, n)\alpha_{k,m} \sigma_h^2 + \sigma_n^2}\right)\right\} \quad (4.10)$$

where  $\sum_{i=1}^{N_{t1}} p_{s,i} = p_s$ ,  $\sum_{i=1}^{N_{t2}} p_{r,i} = p_r$ ,  $p_s + p_r = P$  and  $\lambda_{s,i}$  and  $\lambda_{r,i}$  are denoted the square to the  $i$ th singular values of the source-relay and relay-destination MIMO channels sorted in descending order and  $\alpha_s$  and  $\alpha_r$  are the path losses of the source-relay and relay-destination paths.

Moreover, for the NRA mode, we allow the source to be active for both phases, a fair comparison on the achievable rate should be measured with respect to the total consumed energy. The resulting link capacity over two OFDM symbols is

$$r_D(k, n, P) = 2 \sum_{i=1}^{N_t} \log_2\left(1 + \frac{\frac{p_i(k, n)}{2} \alpha_k \lambda_{s,i}(k, n)}{\sum_{i=1}^{N_t} \frac{p_i(k, n)}{2} \alpha_k \sigma_h^2 + \sigma_n^2}\right) \quad (4.11)$$

where  $p_i(k, n)$  is the power on the  $i$ th eigen-channel,  $N_t = \min\{N_s, N_d\}$  and  $\alpha_k$  is the path loss of the source to destination (SD) link.

### 4.3 Problem Statement

We also use the subcarrier assignment and link selection indicator  $\rho_{(k,n,m)}$  mentioned before. When  $\rho_{(k,n,m)} = 1$  and  $m > 0$ , it indicates subcarrier  $n$  is allocated to user  $k$  who options for RA mode using relay node  $m$  while  $m = 0$  it indicates the user options for the NRA mode. Otherwise, user  $k$  does not have access to subcarrier  $n$  over the  $m$ th link. We also suppose the available total transmission power is  $P_T$ , then the problem of maximizing the total system rate under the users' rate constraints is equivalent to

$$\begin{aligned}
 \max_{\sum_{k=1}^K R_k} R_k &= \max \sum_{k=1}^K \sum_{n=1}^N \sum_{m=0}^M \rho_{(k,n,m)} r_m(k, n, P(k, m, n)) \\
 \text{subject to} & \\
 R_k &\geq R_{k,\min} \quad \forall k \\
 \sum_{k=1}^K \sum_{m=0}^M \rho_{(k,n,m)} &\leq 1 \quad \forall n \\
 \rho_{(k,n,m)} &\in \{0, 1\} \quad \forall n, k, m \\
 \sum_{n=1}^N \sum_{k=1}^K \sum_{m=0}^M \rho_{(k,n,m)} P(k, m, n) &\leq P_T \\
 P(k, m, n) &\geq 0 \quad \forall n, k, m, i
 \end{aligned} \tag{4.12}$$

where  $R_k$  and  $R_{k,\min}$  are the achievable rate and the minimum rate requirement for user  $k$ , and  $r_0(k, n, P) = r_D(k, n, P)$ .

In next section, we propose a low-complexity suboptimal algorithm due to the above formulation is also an NP-hard mixed integer programming problem which is hard to find the optimal solution.

### 4.4 Resource Allocation Schemes in MIMO Channels

For the optimization problem in MIMO system, we also decompose resource allocation schemes into a three subproblems as mentioned in Chap.3: **P1**–link selection

subproblem for deciding  $\{\beta_m(n, k)\}$ , **P2**–subcarrier assignment for deciding  $\{\delta_{(n,k)}\}$ , and **P3**–power allocation.

#### 4.4.1 Relaying/Direct Link Selection Rule and Subcarrier Assignment

Since in the relaying/direct link selection step, the final power allocation is still unknown, we then assume a fair power distribution  $P = \frac{P_T}{N}$  for all DNs. Under such assumption, for a relay-assisted subcarrier  $n$  for user  $k$  relayed by relay  $m$ , we need to find not only the optimal power ratio of relay to source such that the power is efficiently used but the optimal power allocation on each eigen-channel, thus the problem can be formulated as

$$\begin{aligned}
& \max \quad r_m(k, n, P) \\
& \text{subject to} \\
& r_m(k, n, P) \leq \sum_{i=1}^{N_{t1}} \log_2 \left( 1 + \frac{p_{s,i}(k, m, n) \alpha_m \lambda_{s,i}(k, m, n)}{\sum_{i=1}^{N_{t1}} p_{s,i}(k, m, n) \alpha_m \sigma_h^2 + \sigma_n^2} \right) \\
& r_m(k, n, P) \leq \sum_{i=1}^{N_{t2}} \log_2 \left( 1 + \frac{p_{r,i}(k, m, n) \alpha_{k,m} \lambda_{r,i}(k, m, n)}{\sum_{i=1}^{N_{t2}} p_{r,i}(k, m, n) \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) \\
& \sum_{i=1}^{N_{t1}} p_{s,i}(k, m, n) + \sum_{i=1}^{N_{t2}} p_{r,i}(k, m, n) \leq \frac{P_T}{N} \\
& p_{s,i}(k, m, n) \geq 0, \quad p_{r,i}(k, m, n) \geq 0, \quad \forall i, k, m
\end{aligned}$$

By assuming  $P_s(k, m, n) = \sum_{i=1}^{N_{t1}} p_{s,i}(k, m, n)$  and  $P_r(k, m, n) = \sum_{i=1}^{N_{t2}} p_{r,i}(k, m, n)$ , we can decompose it into two sub-problems:

**Subproblem 1:**

$$\begin{aligned}
& \max r_m(k, n, P) \\
& \text{subject to} \\
& r_m(k, n, P) \leq \sum_{i=1}^{N_{t1}} \log_2 \left( 1 + \frac{p_{s,i}(k, m, n) \alpha_m \lambda_{s,i}(k, m, n)}{P_s(k, m, n) \alpha_m \sigma_h^2 + \sigma_n^2} \right) \\
& r_m(k, n, P) \leq \sum_{i=1}^{N_{t2}} \log_2 \left( 1 + \frac{p_{r,i}(k, m, n) \alpha_{k,m} \lambda_{r,i}(k, m, n)}{P_r(k, m, n) \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) \\
& P_s(k, m, n) + P_r(k, m, n) \leq \frac{P_T}{N} \\
& p_{s,i}(k, m, n) \geq 0, p_{r,i}(k, m, n) \geq 0, \forall i, k, m
\end{aligned}$$

**Subproblem 2:**

For SR path:

$$\begin{aligned}
& \max \sum_{i=1}^{N_{t1}} \log_2 \left( 1 + \frac{p_{s,i}(k, m, n) \alpha_m \lambda_{s,i}(k, m, n)}{P_s(k, m, n) \alpha_m \sigma_h^2 + \sigma_n^2} \right) \\
& \text{subject to} \\
& \sum_{i=1}^{N_{t1}} p_{s,i}(k, m, n) \leq P_s(k, m, n) \\
& p_{s,i}(k, m, n) \geq 0 \quad \forall i, k, m
\end{aligned} \tag{4.13}$$

For RD path:

$$\begin{aligned}
& \max \sum_{i=1}^{N_{t2}} \log_2 \left( 1 + \frac{p_{r,i}(k, m, n) \alpha_{k,m} \lambda_{r,i}(k, m, n)}{P_r(k, m, n) \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) \\
& \text{subject to} \\
& \sum_{i=1}^{N_{t2}} p_{r,i}(k, m, n) \leq P_r(k, m, n) \\
& p_{r,i}(k, m, n) \geq 0 \quad \forall i, k, m
\end{aligned} \tag{4.14}$$

For each subproblem 2, we can derive the optimal power allocated on subcarrier  $n$ 's SR and RD eigen-channels from the water-filling solution

$$p_{s,i}(k, m, n) = \left( \mu_{s,n} - \frac{\alpha_m P_s(k, m, n) \sigma_h^2 + \sigma_n^2}{\alpha_m \lambda_{s,i}(k, m, n)} \right)^+ \tag{4.15}$$

$$p_{r,i}(k, m, n) = \left( \mu_{r,n} - \frac{\alpha_{m,k} P_r(k, m, n) \sigma_h^2 + \sigma_n^2}{\alpha_{m,k} \lambda_{r,i}(k, m, n)} \right)^+ \quad (4.16)$$

where  $\mu_{s,n}$  and  $\mu_{r,n}$  are inner water-levels which mean the water-levels controlling the power of eigen-channels.

We now assume the number of the eigen-channels of subcarrier  $n$  on SR and RD path allocated positive power are  $\kappa_s(n)$  and  $\kappa_r(n)$ , respectively, then from (4.15) and (4.16), the inner water-level  $\mu_{s,n}$  and  $\mu_{r,n}$  that satisfies the subcarrier power constraint (4.13) and (4.14) can be expressed as

$$\mu_{s,n} = \frac{1}{\kappa_s(n)} \left( P_s(k, m, n) + \frac{\alpha_m P_s(k, m, n) \sigma_h^2 + \sigma_n^2}{\alpha_m} \sum_{i=1}^{\kappa_s(n)} \frac{1}{\lambda_{s,i}(k, m, n)} \right) \quad (4.17)$$

$$\mu_{r,n} = \frac{1}{\kappa_r(n)} \left( P_r(k, m, n) + \frac{\alpha_{m,k} P_r(k, m, n) \sigma_h^2 + \sigma_n^2}{\alpha_{m,k}} \sum_{i=1}^{\kappa_r(n)} \frac{1}{\lambda_{r,i}(k, m, n)} \right) \quad (4.18)$$

Thus, the rate of subcarrier  $n$  on SR/RD link can be formulated as

$$\begin{aligned} R_s(k, m, n) &= \sum_{i=1}^{N_{s1}} r_{s,i}(k, m, n) \\ &= \sum_{i=1}^{\kappa_s(n)} \log_2 \left( 1 + \frac{\alpha_m P_{s,i}(k, m, n) \lambda_{s,i}(k, m, n)}{\alpha_m P_s(k, m, n) \sigma_h^2 + \sigma_n^2} \right) \\ &= \sum_{i=1}^{\kappa_s(n)} \log_2 \left( \frac{\alpha_m \mu_{s,n} \lambda_{s,i}(k, m, n)}{\alpha_m P_s(k, m, n) \sigma_h^2 + \sigma_n^2} \right) \end{aligned} \quad (4.19)$$

$$\begin{aligned} R_r(k, m, n) &= \sum_{i=1}^{N_{t2}} r_{r,i}(k, m, n) \\ &= \sum_{i=1}^{\kappa_r(n)} \log_2 \left( 1 + \frac{\alpha_{k,m} P_{r,i}(k, m, n) \lambda_{r,i}(k, m, n)}{\alpha_{k,m} P_r(k, m, n) \sigma_h^2 + \sigma_n^2} \right) \\ &= \sum_{i=1}^{\kappa_r(n)} \log_2 \left( \frac{\alpha_{k,m} \mu_{r,n} \lambda_{r,i}(k, m, n)}{\alpha_{k,m} P_r(k, m, n) \sigma_h^2 + \sigma_n^2} \right) \end{aligned} \quad (4.20)$$



Moreover, the lagrange function of subproblem 1 is

$$\begin{aligned}
L &= r_m(k, n, P) \\
&+ \lambda_{n,s} \left( \sum_{i=1}^{N_{t1}} \log_2 \left( 1 + \frac{p_{s,i}(k, m, n) \alpha_m \lambda_{s,i}(k, m, n)}{P_s(k, m, n) \alpha_m \sigma_h^2 + \sigma_n^2} \right) - r_m(k, n, P) \right) \\
&+ \lambda_{n,r} \left( \sum_{i=1}^{N_{t2}} \log_2 \left( 1 + \frac{p_{r,i}(k, m, n) \alpha_{k,m} \lambda_{r,i}(k, m, n)}{P_r(k, m, n) \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) - r_m(k, n, P) \right) \\
&+ \lambda_P \left( \frac{P_T}{N} - P_s(k, m, n) - P_r(k, m, n) \right)
\end{aligned} \tag{4.21}$$

where  $\lambda_{n,s}$ ,  $\lambda_{n,r}$  and  $\lambda_P$  are lagrange multipliers. Based on (4.17) and (4.19), we differentiate the lagrange function (4.21) by  $P_s(k, m, n)$  and let the resulting function equal to 0, then the optimal solution of  $P_s(k, m, n)$  in subproblem1 can be obtained by the quadratic formula

$$P_s(k, m, n) = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{4.22}$$

where

$$\begin{aligned}
a &= \frac{\alpha_m \sigma_h^2}{\sigma_n^2} \left( 1 + \sigma_h^2 \sum_{i=1}^{\kappa_s(n)} \frac{1}{\lambda_{s,i}(k, m, n)} \right), \\
b &= 1 + 2\sigma_h^2 \sum_{i=1}^{\kappa_s(n)} \frac{1}{\lambda_{s,i}(k, m, n)}, \\
c &= - \left( \frac{\lambda_{n,s} \kappa_s(n)}{\lambda_p \ln 2} - \frac{\sigma_n^2}{\alpha_m} \sum_{i=1}^{\kappa_s(n)} \frac{1}{\lambda_{s,i}(k, n)} \right)^+
\end{aligned}$$

Also, based on (4.18) and (4.20), we can derive  $P_r(k, m, n)$  by differentiating the lagrange function (4.21) by  $P_r(k, m, n)$

$$P_r(k, m, n) = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{4.23}$$

where

$$\begin{aligned}
a &= \frac{\alpha_{k,m} \sigma_h^2}{\sigma_n^2} \left( 1 + \sigma_h^2 \sum_{i=1}^{\kappa_r(n)} \frac{1}{\lambda_{r,i}(k, m, n)} \right), \\
b &= 1 + 2\sigma_h^2 \sum_{i=1}^{\kappa_r(n)} \frac{1}{\lambda_{r,i}(k, m, n)}, \\
c &= - \left( \frac{\lambda_{n,r} \kappa_r(n)}{\lambda_p \ln 2} - \frac{\sigma_n^2}{\alpha_{k,m}} \sum_{i=1}^{\kappa_r(n)} \frac{1}{\lambda_{r,i}(k, n)} \right)^+
\end{aligned}$$

Besides, the relationship between  $\lambda_{n,s}$  and  $\lambda_{n,r}$  can be found by differentiating the la-

grange function (4.21) by  $r_m(k, n)$ :

$$\lambda_{n,s} + \lambda_{n,r} = 1 \quad (4.24)$$

The final step is to adjust all lagrange multipliers until the power constraint in subproblem 1 is satisfied and  $R_s(k, m, n) = R_r(k, m, n)$  for having the best power efficiency. The maximum  $r_m(k, n, \frac{P_T}{N})$  is denoted by  $r_m^*(k, n, \frac{P_T}{N})$ . Notice that the ratio of  $P_r(k, m, n)$  to  $P_s(k, m, n)$  is the optimal ratio when given  $P = \frac{P_T}{N}$ .

Moreover, considering about the direct link, the power allocation problem can be formulated as

$$\begin{aligned} \max \quad & r_D(k, n, \frac{P_T}{N}) \\ \text{subject to} \quad & \sum_{i=1}^{N_i} p_i(k, n) = \frac{P_T}{N} \\ & p_i(k, n) \geq 0 \quad \forall i, k \end{aligned} \quad (4.25)$$

we can derive the optimal power allocated on each of its eigen-channel is

$$p_i(k, n) = \left( \mu_n - \frac{\frac{\alpha_k P(k, n) \sigma_h^2 + \sigma_n^2}{2}}{\frac{\alpha_k}{2} \lambda_{s,i}(k, n)} \right)^+ \quad (4.26)$$

By tuning the water-level  $\mu_n$  to satisfy the total power constraint, we can find the maximum  $r_D(k, n, \frac{P_T}{N})$  and denote it by  $r_0^*(k, n, \frac{P_T}{N})$ .

We then determine the relay who can achieve the highest rate for user  $k$  on subcarrier  $n$ , which can be formulated as

$$m^* = \arg \max_{0 \leq m \leq M} r_m^*(k, n, \frac{P_T}{N}) \quad (4.27)$$

When  $m^* = 0$ , DN  $k$  should use only the direct link if it was given subcarrier  $n$ . We then set  $\beta_{m^*}(n, k) = 1$  and  $\beta_m(n, k) = 0$ , for  $0 \leq m \leq M, m \neq m^*$ .

However, for the subcarrier assignment in MIMO systems, we also use the subcarrier assignment algorithm summarized in Table (3.1).

## 4.4.2 Power Allocation

After the relaying/direct link selection and subcarrier assignment, we now suggest a nearly optimal power allocation scheme and the optimal power allocation scheme.

### ► Nearly Optimal Power Allocation

Similar to the SISO case, we first assign equal power  $P_T/N$  to subcarriers using relaying and then allocate the remaining power by a modified water-filling (mwf) on subcarriers using no relays. Thus, for NRA subcarriers, our power allocation problem can be formulated as

$$\begin{aligned} & \max \sum_{k=1}^K \sum_{n \in U_0} \rho_{(n,k,0)} r_0(k, n, p_1(k, 0, n), \dots, p_{N_t}(k, 0, n)) \\ & \text{subject to} \\ & \sum_{k=1}^K \sum_{n \in U_0} \rho_{(k,n,0)} \sum_{i=1}^{N_t} p_i(k, 0, n) \leq P_{NRA} \\ & p_i(k, 0, n) \geq 0 \quad \forall n, k, i \end{aligned} \quad (4.28)$$

where  $U_0$  is the set of NRA subcarriers and  $P_{NRA}$  is the remaining power for NRA subcarriers. However, by assuming  $\sum_{i=1}^{N_t} p_i(k, 0, n) = P(k, n)$ , we can decompose this problem into two subproblems:

#### Subproblem 1:

$$\begin{aligned} & \max \sum_{k=1}^K \sum_{n \in U_0} \rho_{(n,k,0)} r_0(k, n, P(k, n)) \\ & \text{subject to} \\ & \sum_{k=1}^K \sum_{n \in U_0} \rho_{(k,n,0)} P(k, n) \leq P_{NRA} \\ & P(k, n) \geq 0 \quad \forall n \end{aligned} \quad (4.29)$$

**Subproblem 2:** for each subcarrier  $n$  in  $U_0$

$$\begin{aligned}
& \max \quad \sum_{k=1}^K \rho_{(n,k,0)} r_0(k, n, p_1(k, 0, n), \dots, p_{N_t}(k, 0, n)) \\
& \text{subject to} \\
& \quad \sum_{k=1}^K \rho_{(k,n,0)} \sum_{i=1}^{N_t} p_i(k, 0, n) = P(k, n) \\
& \quad p_i(k, 0, n) \geq 0 \quad \forall i, k
\end{aligned} \tag{4.30}$$

For the subproblem 2 of each subcarrier  $n$  in  $U_0$ , we can derive the optimal power allocation on each of its eigen-channel

$$p_i(k, 0, n) = \left( \mu_n - \frac{\frac{\alpha_k}{2} P(k, n) \sigma_h^2 + \sigma_n^2}{\frac{\alpha_k}{2} \lambda_{s,i}(k, n)} \right)^+ \tag{4.31}$$

where  $\mu_n$  is the water level. Assume the number of the eigen-channel of subcarrier  $n$  allocated positive power is  $\kappa(n)$ , then from (4.31), the water-level  $\mu_n$  that satisfies the subcarrier power constraint (4.30) can be expressed as

$$\mu_n = \frac{1}{\kappa(n)} \left( P(k, n) + \frac{\frac{\alpha_k}{2} P(k, n) \sigma_h^2 + \sigma_n^2}{\frac{\alpha_k}{2}} \sum_{i=1}^{\kappa(n)} \frac{1}{\lambda_{s,i}(k, n)} \right) \tag{4.32}$$

Thus, the rate of subcarrier  $n$  is

$$\begin{aligned}
R_D(k, n, P(k, n)) &= \sum_{i=1}^{N_t} r_i(k, n) \\
&= \sum_{i=1}^{\kappa(n)} 2 \log_2 \left( 1 + \frac{\frac{\alpha_k}{2} p_i(k, 0, n) \lambda_{s,i}(k, n)}{\frac{\alpha_k}{2} P(k, n) \sigma_h^2 + \sigma_n^2} \right) \\
&= \sum_{i=1}^{\kappa(n)} 2 \log_2 \left( \frac{\frac{\alpha_k}{2} \mu_n \lambda_{s,i}(k, n)}{\frac{\alpha_k}{2} P(k, n) \sigma_h^2 + \sigma_n^2} \right)
\end{aligned} \tag{4.33}$$

Based on (4.32) and (4.33), the optimal solution of subproblem1 can be obtained by the quadratic formula

$$P(k, n) = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{4.34}$$

where

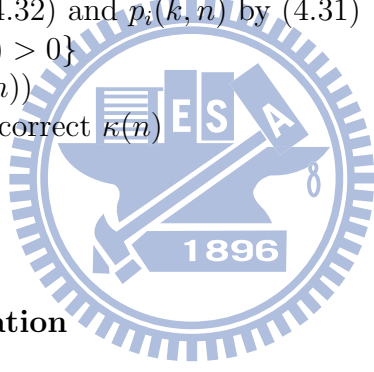
$$a = \frac{\alpha_k \sigma_h^2}{2 \sigma_n^2} \left( 1 + \sigma_h^2 \sum_{i=1}^{\kappa(n)} \frac{1}{\lambda_{s,i}(k, n)} \right),$$

$$b = 1 + 2\sigma_h^2 \sum_{i=1}^{\kappa(n)} \frac{1}{\lambda_{s,i}(k,n)},$$

$$c = - \left( \frac{2\kappa(n)}{\lambda \ln 2} - \frac{2\sigma_n^2}{\alpha_k} \sum_{i=1}^{\kappa(n)} \frac{1}{\lambda_{s,i}(k,n)} \right)^+$$

and  $(t)^+ = \max(0, t)$ . By iteratively modifying  $\lambda$  to satisfy the total power constraint (4.29), we obtain the optimal power allocation for subcarriers using only direct link. So, the proposed power allocation for NRA subcarriers can be summarized as :

Given  $P_{NRA}$ ,  $\rho_{(n,k,0)}$ ,  $P(k, n) = 0$  and  $p_i(k, n) = 0$  for all  $k, n, i$   
Set  $U_0 = \left\{ n \mid \sum_{k=1}^K \rho_{n,k,0} = 1 \right\}$ ,  $\kappa(n) = N_t$  for  $n \in U_0$   
while  $\left( \left| \sum_{k=1}^K \sum_{n \in U_0} P(k, n) - P_{NRA} \right| \geq \epsilon \right)$   
    modify  $\lambda$  and calculate  $P(k, n)$  by (4.34) for user  $k$  such that  $\rho_{(n,k,0)} = 1$   
    for ( $n = 1$  to  $|U_0|$ )  
        calculate  $\mu_n$  by (4.32) and  $p_i(k, n)$  by (4.31)  
         $U_n = \{ i \mid p_i(n, k) > 0 \}$   
        while ( $|U_n| \neq \kappa(n)$ )  
            modify to the correct  $\kappa(n)$   
        end  
    end  
end



### ► Optimal Power Allocation

For the optimal power allocation, the optimization problem can be formulated as

$$\max \sum_{k=1}^K R_k = \max \sum_{k=1}^K \sum_{n=1}^N \sum_{m=0}^M \rho_{(n,k,m)} r_m(k, n, P(k, m, n))$$

subject to

$$\sum_{n=1}^N \sum_{k=1}^K \sum_{m=0}^M \rho_{(k,n,m)} P(k, m, n) \leq P_T \quad (4.35)$$

$$P(k, m, n) \geq 0 \quad \forall n, k, m$$

The power allocated on the NRA subcarriers has been shown in nearly optimal power allocation, so we first introduce the power allocation on the RA subcarriers.

For RA subcarriers, by assuming  $\sum_{i=1}^{N_{t1}} p_{s,i}(k, m, n) = P_s(k, m, n)$  and  $\sum_{i=1}^{N_{t2}} p_{r,i}(k, m, n) = P_r(k, m, n)$ , and simply  $r_m(k, n, P_s(k, m, n), P_r(k, m, n))$  by  $r_m(k, n)$ , we can also decom-

pose the optimization problem into two subproblems:

**Subproblem 1:**

$$\begin{aligned}
& \max \quad \sum_{k=1}^K \sum_{n \in U_1} \sum_{m=1}^M \rho_{(n,k,m)} r_m(k, n) \\
& \text{subject to} \\
& r_m(k, n) \leq \sum_{i=1}^{N_{t1}} \log_2 \left( 1 + \frac{p_{s,i}(k, m, n) \alpha_m \lambda_{s,i}(k, m, n)}{P_s(k, m, n) \alpha_m \sigma_h^2 + \sigma_n^2} \right) \quad \forall n, k, m \\
& r_m(k, n) \leq \sum_{i=1}^{N_{t2}} \log_2 \left( 1 + \frac{p_{r,i}(k, m, n) \alpha_{k,m} \lambda_{r,i}(k, m, n)}{P_r(k, m, n) \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) \quad \forall n, k, m \\
& \sum_{k=1}^K \sum_{n \in U_1} \sum_{m=1}^M \rho_{(k,n,m)} (P_s(k, m, n) + P_r(k, m, n)) \leq P_{RA} \quad (4.36) \\
& P_s(k, m, n) \geq 0, \quad P_r(k, m, n) \geq 0 \quad \forall n, k, m
\end{aligned}$$

where  $U_1$  is the set of NRA subcarriers and  $P_{RA}$  is the total power of NRA subcarriers.

The first and second constraints are due to the rate of the cooperative transmission is the minimum of the rate of SR and of RD link.

**Subproblem 2:** for each RA subcarrier  $n$  in  $U_1$

For SR path:

$$\begin{aligned}
& \max \quad \sum_{k=1}^K \sum_{m=1}^M \rho_{(k,n,m)} \sum_{i=1}^{N_{t1}} \log_2 \left( 1 + \frac{p_{s,i} \alpha_m \lambda_{s,i}(k, m, n)}{P_s \alpha_m \sigma_h^2 + \sigma_n^2} \right) \\
& \text{subject to} \\
& \sum_{k=1}^K \sum_{m=1}^M \rho_{(k,n,m)} \sum_{i=1}^{N_{t1}} p_{s,i}(k, m, n) = P_s(k, m, n) \quad (4.37) \\
& p_{s,i}(k, m, n) \geq 0, \quad \forall i, k, m
\end{aligned}$$

For RD path:

$$\begin{aligned}
& \max \quad \sum_{k=1}^K \sum_{m=1}^M \rho_{(k,n,m)} \sum_{i=1}^{N_{t2}} \log_2 \left( 1 + \frac{p_{r,i} \alpha_{k,m} \lambda_{r,i}(k, m, n)}{P_r \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) \\
& \text{subject to} \\
& \sum_{k=1}^K \sum_{m=1}^M \rho_{(k,n,m)} \sum_{i=1}^{N_{t2}} p_{r,i}(k, m, n) = P_r(k, m, n) \quad (4.38) \\
& p_{r,i}(k, m, n) \geq 0 \quad \forall i, k, m
\end{aligned}$$

For each subproblem 2, we have derived the optimal power allocated on subcarrier  $n$ 's SR and RD eigen-channels from the water-filling solution in (4.15) and (4.16). We now also assume the number of the eigen-channels of subcarrier  $n$  on SR and RD path allocated positive power are  $\kappa_s(n)$  and  $\kappa_r(n)$ , respectively. Moreover, the lagrange function of subproblem 1 can be expressed as

$$\begin{aligned}
L = & \sum_{k=1}^K \sum_{n \in U_1} \sum_{m=1}^M \rho_{(n,k,m)} \{ r_m(k, n) \\
& + \lambda_{n,s}(k, m, n) \left( \sum_{i=1}^{N_{i1}} \log_2 \left( 1 + \frac{p_{s,i}(k, m, n) \alpha_m \lambda_{s,i}(k, m, n)}{P_s(k, m, n) \alpha_m \sigma_h^2 + \sigma_n^2} \right) - r_m(k, n) \right) \\
& + \lambda_{n,r}(k, m, n) \left( \sum_{i=1}^{N_{i2}} \log_2 \left( 1 + \frac{p_{r,i}(k, m, n) \alpha_{k,m} \lambda_{r,i}(k, m, n)}{P_r(k, m, n) \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) - r_m(k, n) \right) \} \\
& + \lambda_P (P_{RA} - \sum_{k=1}^K \sum_{n \in U_1} \sum_{m=1}^M \rho_{(k,n,m)} (P_s(k, m, n) + P_r(k, m, n))) \tag{4.39}
\end{aligned}$$

where  $\lambda_{n,s}(k, m, n)$ ,  $\lambda_{n,r}(k, m, n)$  and  $\lambda_P$  are lagrange multipliers.

Based on (4.17) and (4.19), we differentiate the lagrange function (4.39) by  $P_s(k, m, n)$  and let the resulting function equal to zero, then the optimal solution of  $P_s(k, m, n)$  in subproblem1 is the same as (4.22). By the same way, the optimal  $P_r(k, m, n)$  is as expressed in (4.23). Moreover, by differentiating the lagrange function (4.39) by  $r_m(k, n)$ , the relationship between  $\lambda_{n,s}(k, m, n)$  and  $\lambda_{n,r}(k, m, n)$  is also the equation we showed in (4.24). We then need to adjust all lagrange multipliers until all power constraints are satisfied and  $R_s(k, m, n) = R_r(k, m, n)$  for all  $k, m, n$ . Finally, the resulting proposed optimal power allocation is summarized in Table 4.1.

## 4.5 Numerical Results and Discussions

The simulation results shown in this section assume a single-cell network with multiple DNs that are uniformly distributed within a 120-degree section of the 600-meter radius circle centered at the BS. The RNs are placed on a circle with a 150-meter radius with a equal angular spacing. We also assume each subcarrier suffers from independent Rayleigh

fading in any direct or relay link with a path loss exponent 3.5. Moreover, we set  $P_T$  with the value letting the edge user has a SNR equals to -5dB on a single subcarrier while the equal power is allocate, which means  $10 \log_{10} \frac{600^{-3.5} * P_T}{N \sigma_n^2} = -5$ . We also assume the number of antennas on source node and relay node is 4 and on MS is 2.

As in chapter 3, we also compare the performance of our subcarrier assignment (SA) algorithm (**P2**-solver) with the greedy SA and weighted SA algorithms. And for the sake of fair comparison, we use our **P1** and **P3** solutions but keep that for P2 intact.

First, we use the sub-optimal power allocation(PA1) to assign power on each subcarriers. In Figs. 4.3–4.4, we assume there are 8 DNs, 4 RNs in a 32-subcarrier OFDMA cell under various minimum rate requirements. Fig. 4.3 shows that the sum rate of our algorithm is closer to the greedy SA than the weighted SA does. Within the same subcarrier assignment scheme, we compare two power allocation methods—equal power allocation and the proposed nearly optimal power allocation(PA1). It obvious that the proposed algorithm performs better than the equal power allocation approach. While in Fig. 4.4, we depicts the rate failure probability which means the probability that the algorithm fails to meet a user’s rate requirement. It can be seem easily that our solution has the lowest rate failure probability. In Fig. 4.5 and Fig. 4.6 we also compare the sum rate and required rate failure probability while with various user number in a 32-subcarrier OFDMA network having 4 RNs. The minimum user rate requirement is set to 30 bits/2 OFDM symbols. It is clear that our algorithm outperforms the weighted SA scheme.

Moreover, for the following figures, the power allocation scenario used is the proposed optimal power allocation(PA2). In Figs. 4.7–4.8, we still assume in the 8 DNs, 4 RNs in a 32-subcarrier OFDMA cell under various minimum rate requirements. Fig. 4.7 shows that our algorithms have the best performance in the sum rate while Fig. 4.8 implies that the rate failure probability of our scheme is a little higher than weighted SA scheme. It’s due to our subcarrier assignment scheme has higher probability to assign user with



good channels at the beginning due to the overall selection while with bad channels at the ending for satisfying the rate constraints, which leads to a wider range of channel gains after subcarrier assignment. Thus, after the optimal power allocation, power on bad channels will be low, the rate failure probability then increases. In Fig. 4.9 and Fig. 4.10 we then show how user number infect the sum rate and required rate failure probability in a 32-subcarrier OFDMA network having 4 RNs with minimum user rate requirement 30 bits/2 OFDM symbols. We can notice that the result is the same as mentioned in the case of varying the minimum rate requirements.

For the comparison between the two power allocations we proposed, we show the sum rate of these two PAs and equal power allocation with our proposed SA considering various rate constraints and user numbers in Fig. 4.11 and Fig. 4.12, respectively. It is obviously that PA2 is the best power allocation scheme.

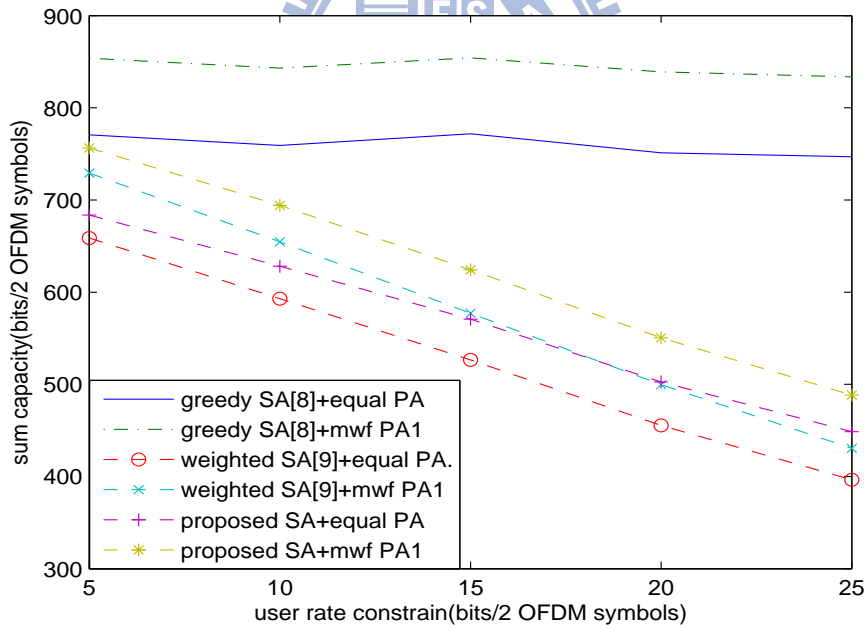


Figure 4.3: Sum rate v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers .

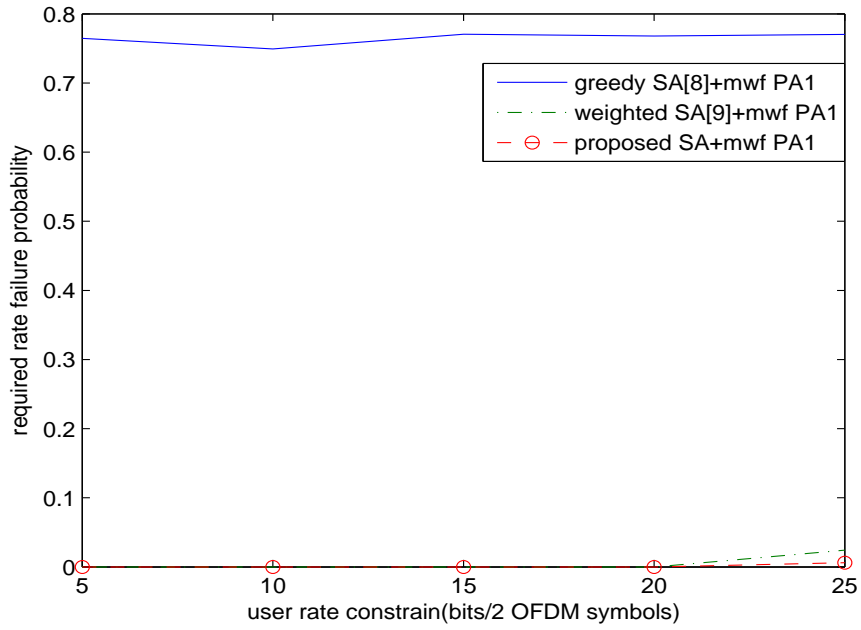


Figure 4.4: Rate failure probability v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers.

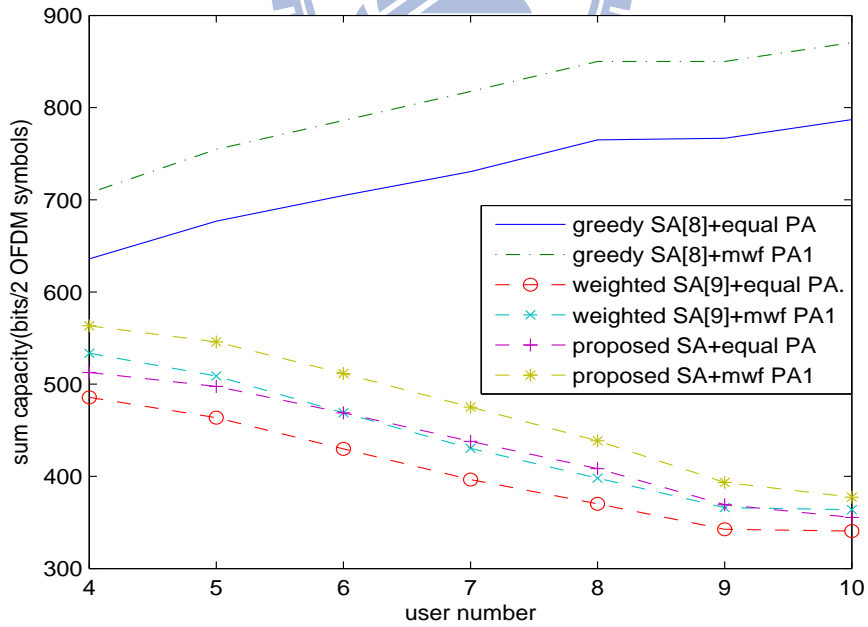
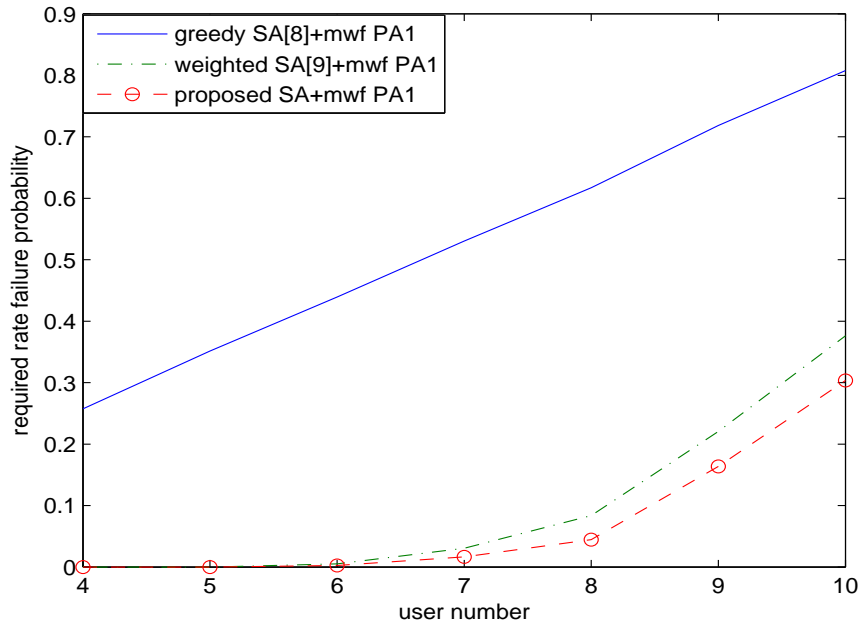


Figure 4.5: Sum rate v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.



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Figure 4.6: Rate failure probability v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

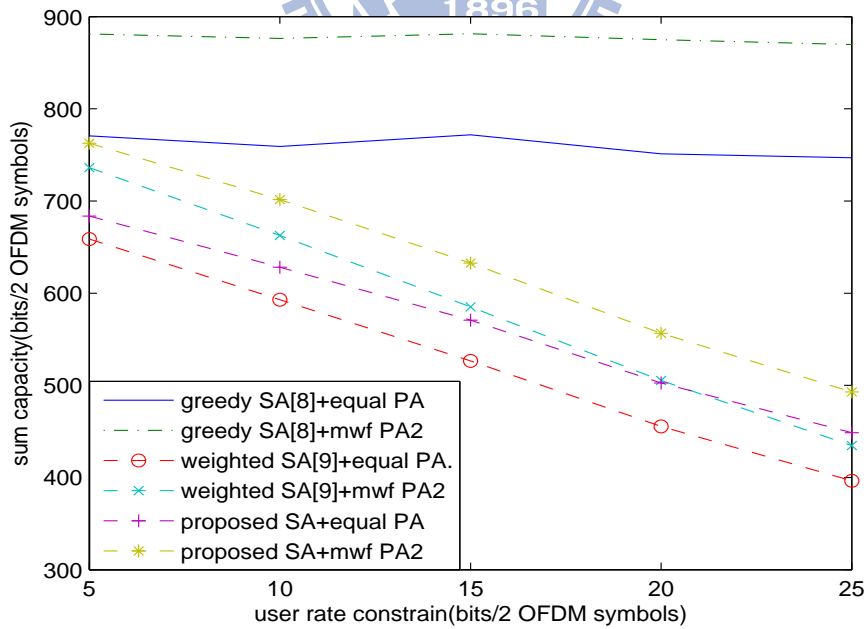


Figure 4.7: Sum rate v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers .

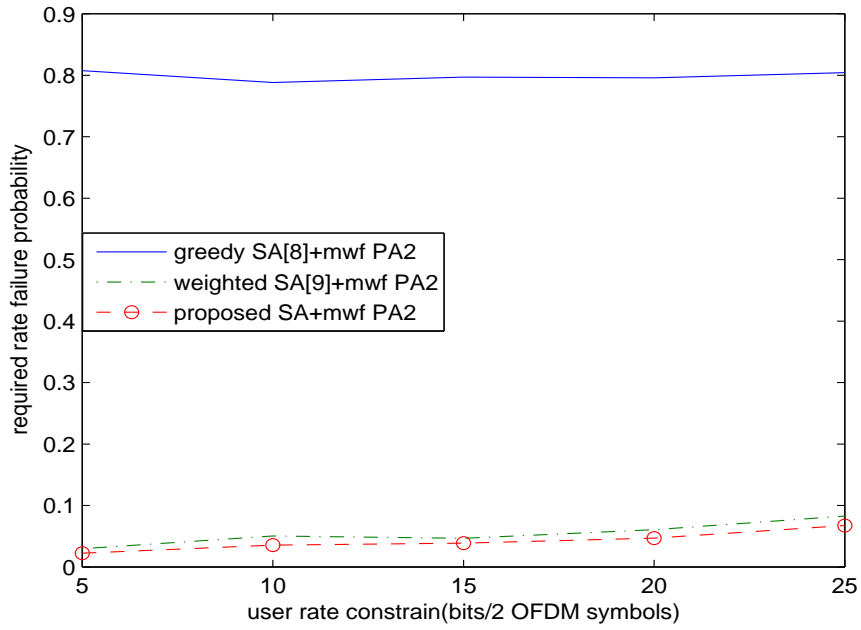


Figure 4.8: Rate failure probability v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers.

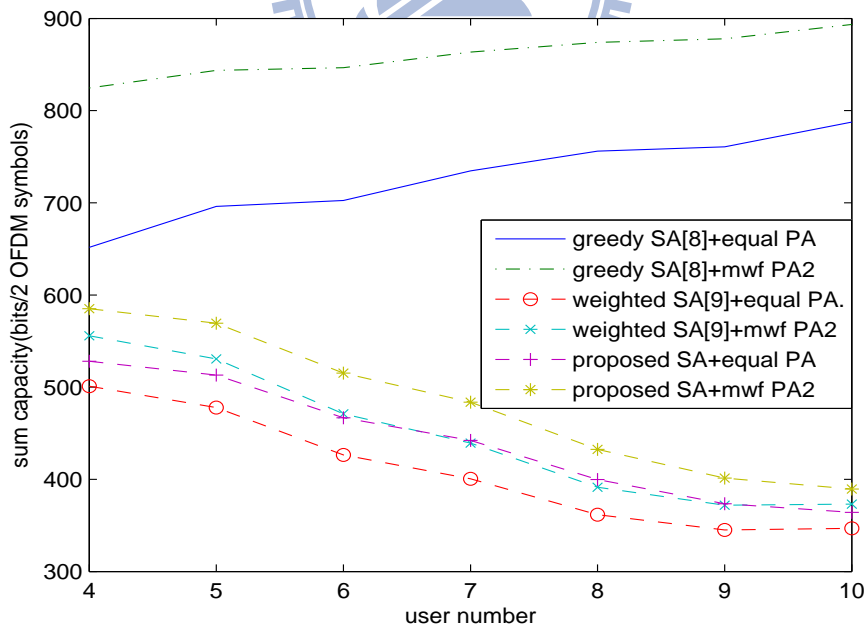


Figure 4.9: Sum rate v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

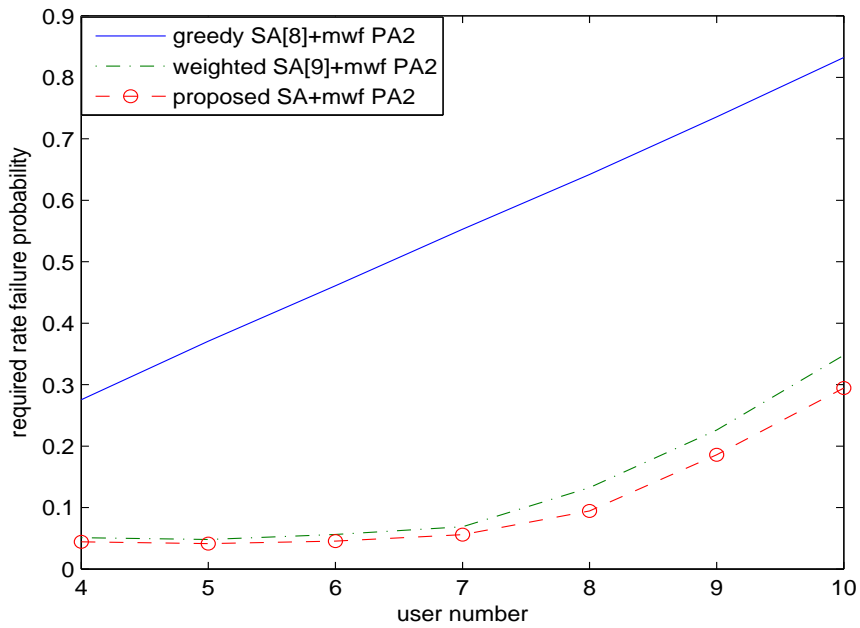


Figure 4.10: Rate failure probability v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

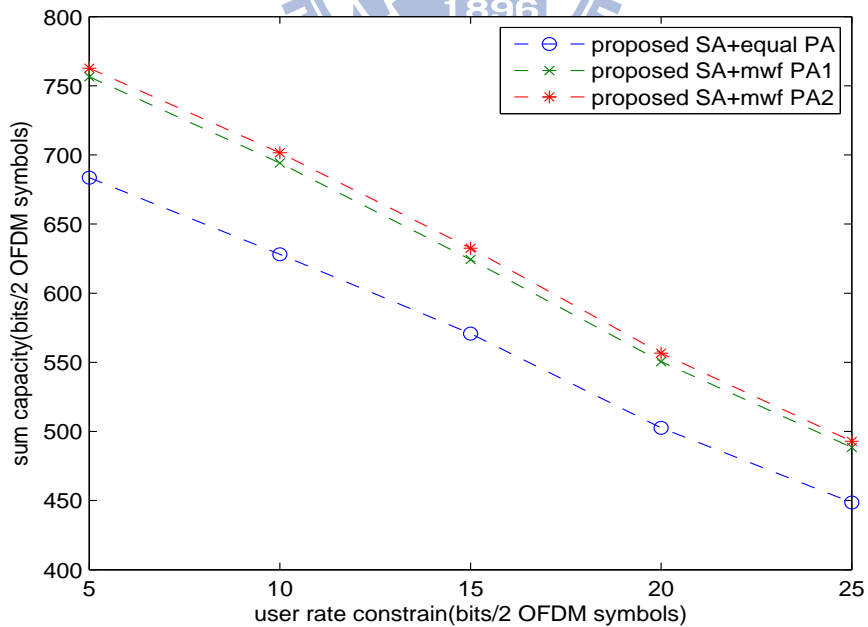


Figure 4.11: Sum rate v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers.

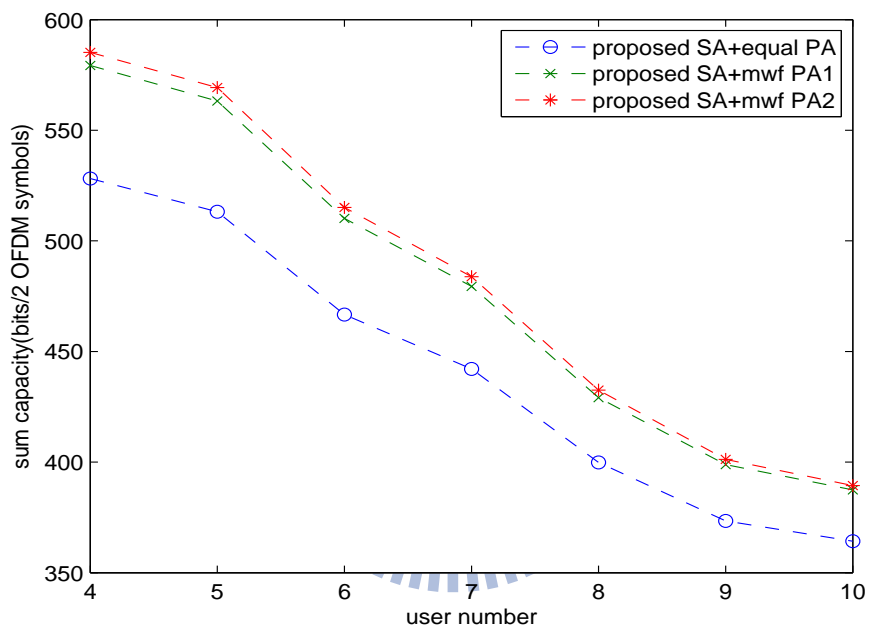


Figure 4.12: Sum rate v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

## 4.6 Same Rank of SR and RD Link

In this special case, we assume that  $N_t = \min(N_s, N_r) = \min(N_r, N_d)$  which mean the rank of SR and RD link are the same. Under DF scheme, we can re-transmit the total data received by a relay if

$$\sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{p_{s,i} \alpha_s \lambda_{s,i}}{p_s \alpha_s \sigma_h^2 + \sigma_n^2} \right) = \sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{p_{r,i} \alpha_r \lambda_{r,i}}{p_r \alpha_r \sigma_h^2 + \sigma_n^2} \right) \quad (4.40)$$

where  $\sum_{i=1}^{N_t} p_{s,i} = p_s$ ,  $\sum_{i=1}^{N_t} p_{r,i} = p_r$ ,  $\lambda_{s,i}$  and  $\lambda_{r,i}$  for  $i=1$  to  $N_t$  are denoted the square to singular values of the source-relay and relay-destination MIMO channels sorted in descending order and  $\alpha_s$  and  $\alpha_r$  are the path losses of the source-relay and relay-destination paths.

### 4.6.1 An Nearly Optimal Power Ratio

For the subcarrier assignment step we proposed, we now derive a nearly-optimal power ratio  $\Gamma = \frac{p_r}{p_s}$  such that the power allocated on the source node and relay node are efficient. First, we assume the SINR is high, so we can neglect the first term in the log function of both the right-hand and left-hand in (4.40). So we can rewrite (4.40) to

$$\sum_{i=1}^{N_t} \log_2 \left( \frac{p_{s,i} \alpha_s \lambda_{s,i}}{p_s \alpha_s \sigma_h^2 + \sigma_n^2} \right) = \sum_{i=1}^{N_t} \log_2 \left( \frac{p_{r,i} \alpha_r \lambda_{r,i}}{p_r \alpha_r \sigma_h^2 + \sigma_n^2} \right) \quad (4.41)$$

Moreover, given a sum power  $p$  such that  $p_s + p_r = p$ , we assume that  $p_{s,i} = \frac{p_s}{N_t}$  and  $p_{r,i} = \frac{p_r}{N_t}$  for all  $i$ . So, (4.41) can be rewritten as

$$\sum_{i=1}^{N_t} \log_2 \left( \frac{\frac{p_s}{N_t} \alpha_s \lambda_{s,i}}{p_s \alpha_s \sigma_h^2 + \sigma_n^2} \right) = \sum_{i=1}^{N_t} \log_2 \left( \frac{\frac{p_r}{N_t} \alpha_r \lambda_{r,i}}{p_r \alpha_r \sigma_h^2 + \sigma_n^2} \right) \quad (4.42)$$

From (4.42), we can achieve

$$\frac{1}{N_t} \sum_{i=1}^{N_t} \log_2 \left( \frac{\lambda_{s,i}}{\lambda_{r,i}} \right) = \log_2 \left( \frac{\sigma_h^2 + \frac{\sigma_n^2}{p_s \alpha_s}}{\sigma_h^2 + \frac{\sigma_n^2}{p_r \alpha_r}} \right) \quad (4.43)$$

Let  $c = 2^{\frac{1}{N_t} \sum_{i=1}^{N_t} \log_2 \left( \frac{\lambda_{s,i}}{\lambda_{r,i}} \right)}$ , (4.43) becomes

$$c = \frac{\sigma_h^2 + \frac{\sigma_n^2}{p_s \alpha_s}}{\sigma_h^2 + \frac{\sigma_n^2}{p_r \alpha_r}} \quad (4.44)$$

Since we assume the power ratio  $\Gamma = \frac{p_r}{p_s}$  and  $p = p_s + p_r$ , we replace  $p_s$  by  $\frac{p}{1+\Gamma}$  and  $p_r$  by  $\frac{\Gamma p}{1+\Gamma}$  in (4.44), then a quadratic equation of the power ratio  $\Gamma$  can be derived

$$\alpha_r \sigma_n^2 \Gamma^2 + (\alpha_r \sigma_n^2 + \alpha_r \alpha_s \sigma_h^2 p - \alpha_s \alpha_r c \sigma_h^2 p - \alpha_s c \sigma_n^2) \Gamma - \alpha_s c \sigma_n^2 = 0 \quad (4.45)$$

By the quadratic formula, the optimal power ratio can be formulated as

$$\Gamma = \frac{-b + \sqrt{b^2 + 4ad}}{2a} \quad (4.46)$$

where  $a = \alpha_r \sigma_n^2$ ,  $b = (\alpha_r - \alpha_s c) \sigma_n^2 + (1 - c) \alpha_s \alpha_r \sigma_h^2 p$ , and  $d = \alpha_s c \sigma_n^2$ . The add operator of the quadratic formula is chosen due to  $a$  and  $d$  are both positive. Using the above notations, we express the capacity (lower bound) of a basic MIMO cooperative network with imperfect CSI as

$$R = \min \left\{ \sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{\frac{p \alpha_s}{(1+\Gamma) N_t} \lambda_{s,i}}{\frac{p \alpha_s}{1+\Gamma} \sigma_h^2 + \sigma_n^2} \right), \sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{\frac{\Gamma p \alpha_r}{(1+\Gamma) N_t} \lambda_{r,i}}{\frac{\Gamma p \alpha_r}{1+\Gamma} \sigma_h^2 + \sigma_n^2} \right) \right\} \quad (4.47)$$

Then we define the achievable rate for user  $k$  relayed by relay  $m$  on subcarrier  $n$  is

$$r_m(k, n, P) = \min \left\{ \sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{\frac{P}{(1+\Gamma(k,m,n,P)) N_t} \alpha_m \lambda_{s,i}(k,m,n)}{\frac{P}{1+\Gamma(k,m,n,P)} \alpha_m \sigma_h^2 + \sigma_n^2} \right), \sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{\frac{\Gamma(k,m,n,P) P}{(1+\Gamma(k,m,n,P)) N_t} \alpha_{k,m} \lambda_{s,i}(k,m,n)}{\frac{\Gamma(k,m,n,P) P}{1+\Gamma(k,m,n,P)} \alpha_{k,m} \sigma_h^2 + \sigma_n^2} \right) \right\}$$

where  $\alpha_m$  and  $\alpha_{k,m}$  are the path losses of source to relay (SR), and relay to destination (RD) links, respectively.

Thus, while given the power allocated on each subcarrier is the equal power solution  $\frac{P_r}{N}$  we assuming in relaying/direct link selection scheme, now we can easily calculate the rate of each cooperative path without the iteratively computing of the lagrange multipliers. After the link selection step, the subcarrier assignment and power allocation is the same as mentioned before.

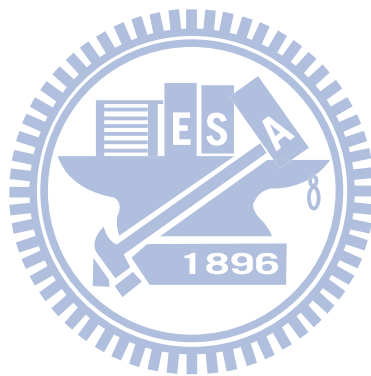
## 4.6.2 Simulations

The simulation results shown in this subsection are under the same model as preceding section while we assume the number of antennas on source node, relay node, and MS are all 4. For the purpose of comparison, we also compare the performance of our



subcarrier assignment (SA) algorithm (**P2**-solver) with the greedy SA and weighted SA algorithms using our **P1** and **P3**(PA1 and PA2) solutions. First, we use the sub-optimal power allocation(PA1) to calculate the power allocated on each subcarriers. We than assume there are 8 DNs, 4 RNs in a 32-subcarrier OFDMA cell under various minimum rate requirements in Fig. 4.13 and Fig. 4.14. Both of them also show that our scenario is the best method considering not only the rate constraints and total system capacity. In Fig. 4.15 and Fig. 4.16 we compare the sum rate and required rate failure probability with various user number in a 32-subcarrier OFDMA network having 4 RNs with the minimum user rate requirement 30 bits/2 OFDM symbols. It is clear that our algorithm outperforms all the other schemes. Otherwise, we also show the simulation results with the power allocation scenario changed to PA2. In Fig. 4.17 and Fig. 4.18, we show the sum rate and the rate failure probability in the 8 DNs, 4 RNs, 32-subcarrier OFDMA cell with various minimum rate requirements. In Fig. 4.19 and Fig. 4.20 we then show how user number infect the sum rate and required rate failure probability in a 32-subcarrier OFDMA network having 4 RNs with minimum user rate requirement 30 bits/2 OFDM symbols. The result is the same as mentioned in the preceding section. We than compare the two power allocations we proposed and equal power allocation in Fig. 4.21 and Fig. 4.22, both with our proposed SA while considering various rate constraints and user numbers respectively. It can be seem that PA2 is the best power allocation scheme. In addition, in Fig. 4.23, we show the comparison between the optimal power ratio, equal power ratio(power raiion equals to 1) and the derived nearly optimal power ratio by plotting their rate with various edge user SNR in a 1 DNs, 4 RNs and 1 subcarriers network. The derived nearly optimal power ratio and equal power ratio are both coupled with equal power allocation and water-filling power allocation on each subcarrier's eigen-channels. We can notice that the derived power ratio is much closer to the optimal power raio than the equal power ratio. And as the edge-user SNR becomes larger, the rate of the derived power ratio and of the optimal power raio are also closer.

Now we normalize all the lines in Fig. 4.23 with the rate of the optimal power ratio, than we show the rate ratio of the optimal power to each method in Fig. 4.24. It is much clear that the rate loss of the nearly optimal power ratio due to some approximation in our derivation is small, and with the larger edge-user SNR, it can be smaller.



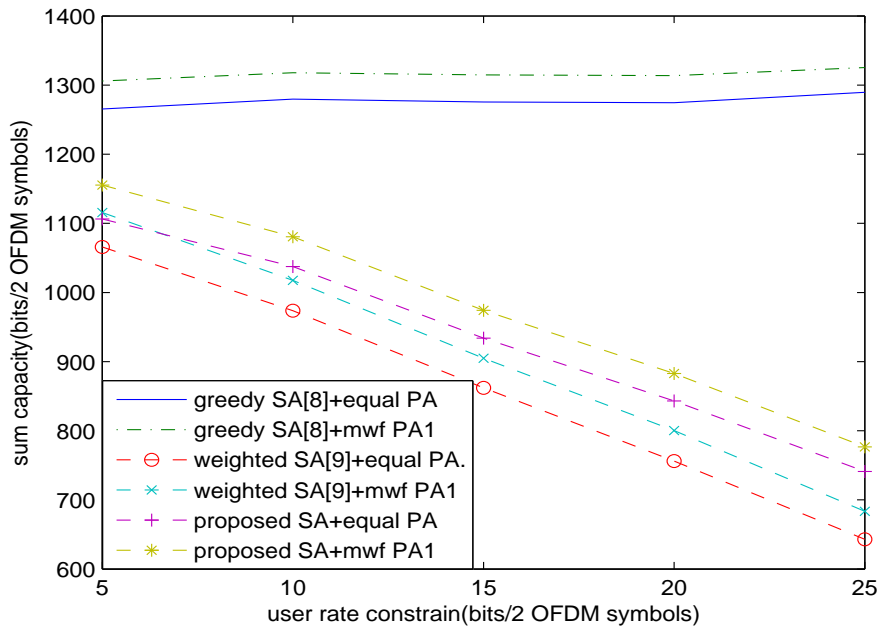


Figure 4.13: Sum rate v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers .

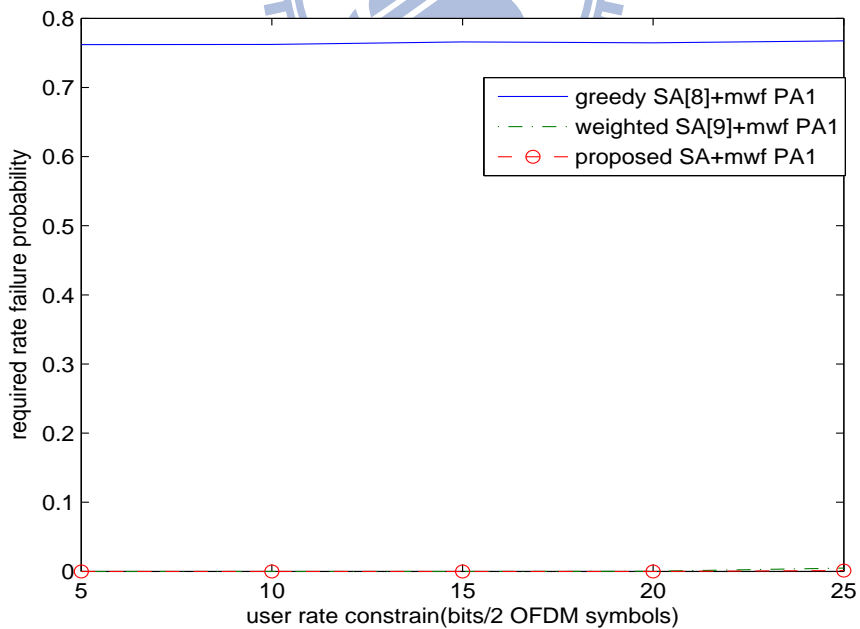


Figure 4.14: Rate failure probability v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers.

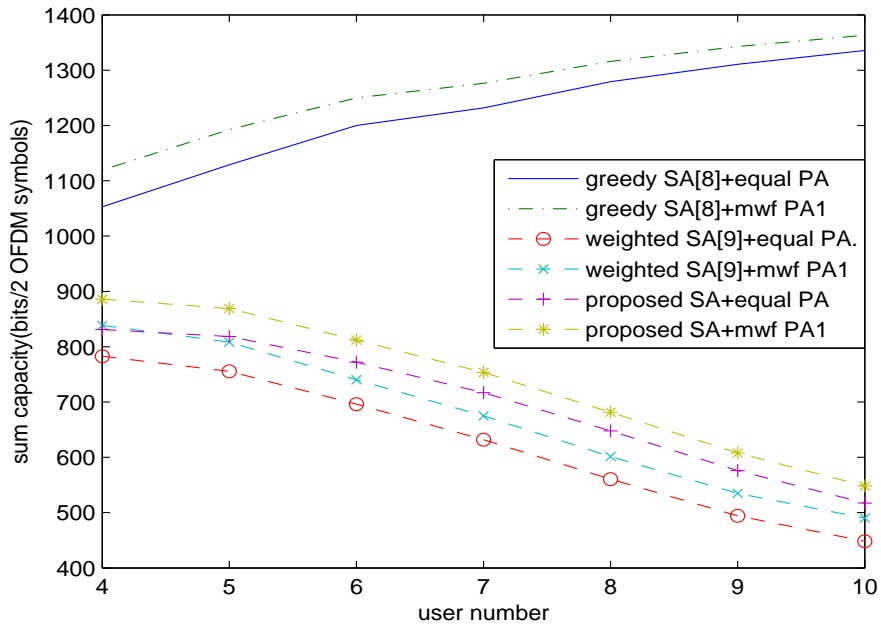
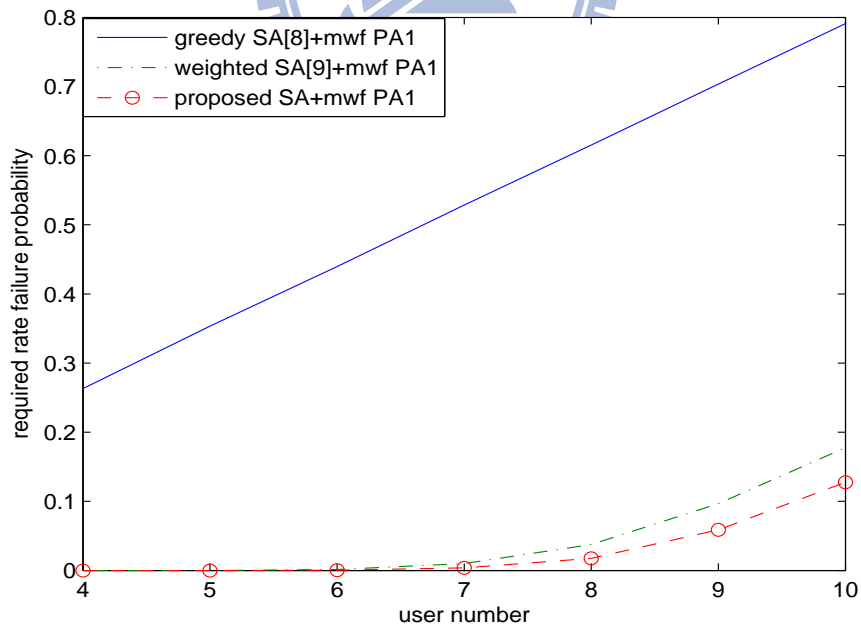


Figure 4.15: Sum rate v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.



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Figure 4.16: Rate failure probability v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

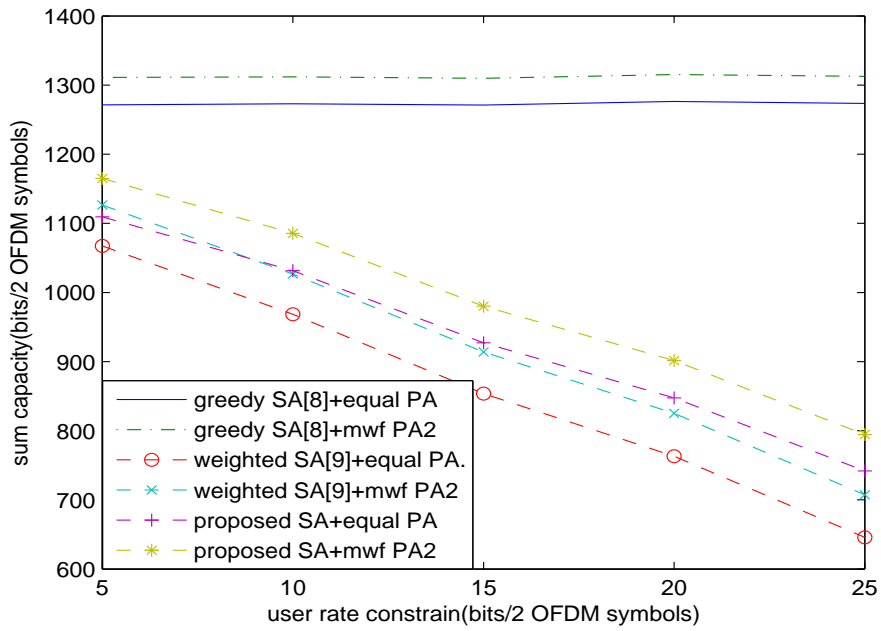


Figure 4.17: Sum rate v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers .

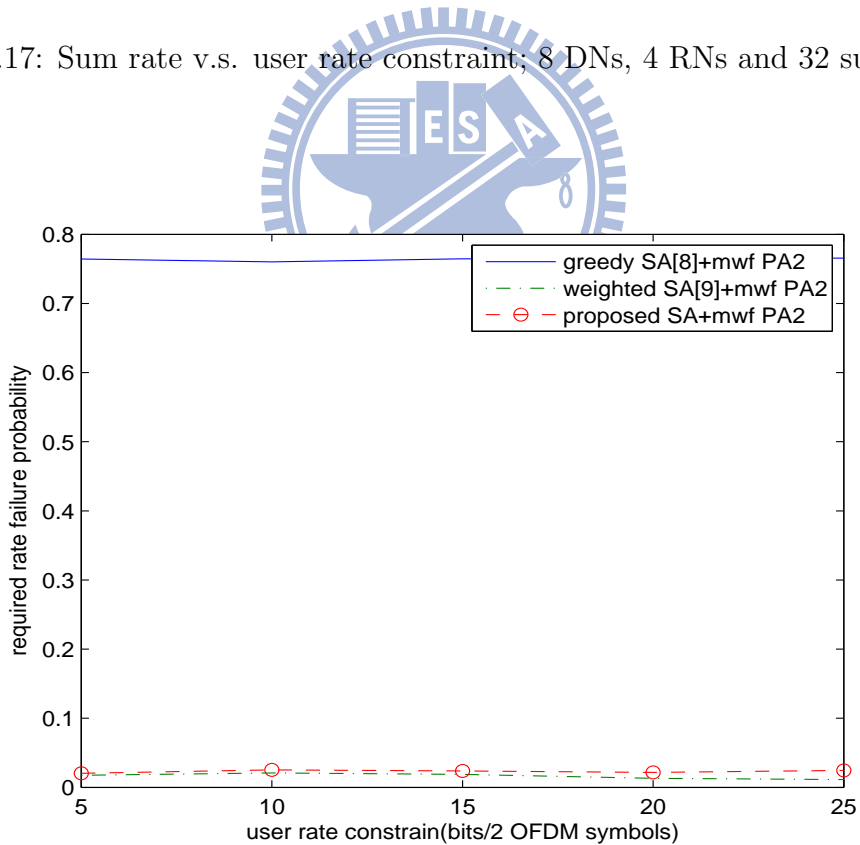


Figure 4.18: Rate failure probability v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers.

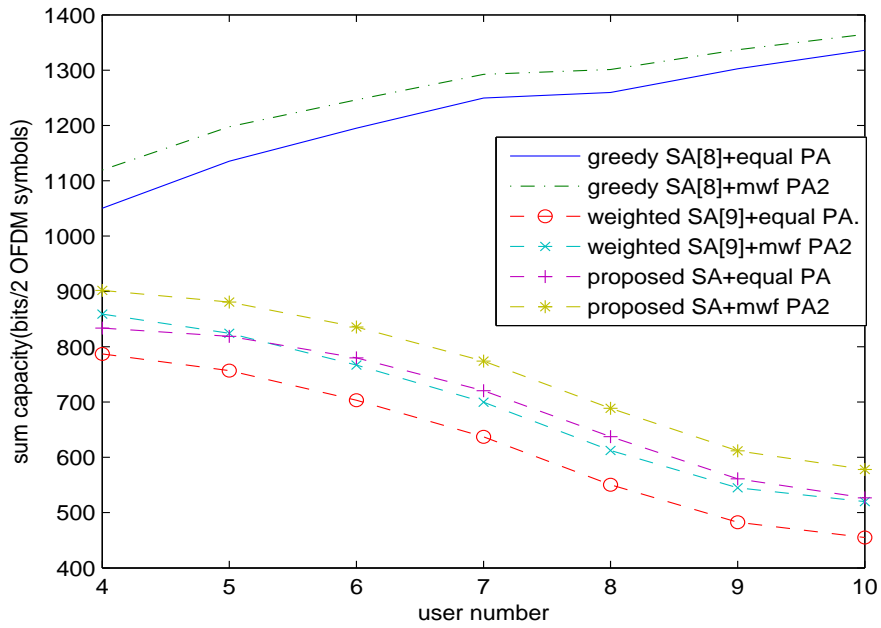


Figure 4.19: Sum rate v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

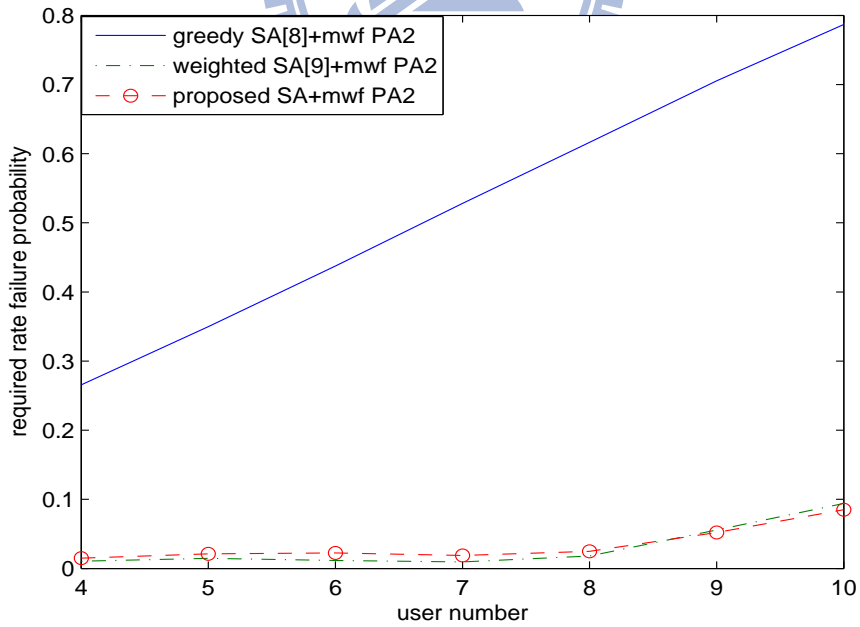


Figure 4.20: Rate failure probability v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

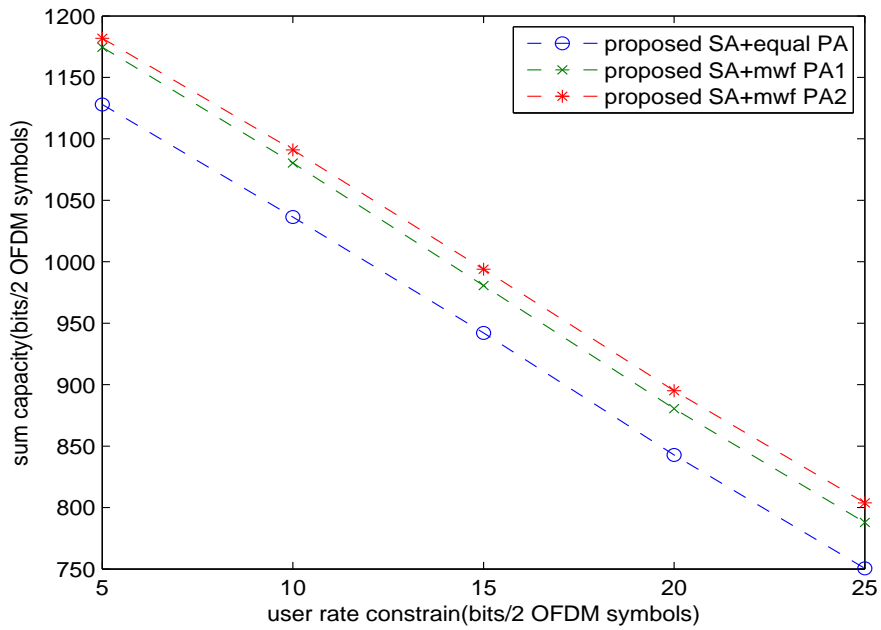


Figure 4.21: Sum rate v.s. user rate constraint; 8 DNs, 4 RNs and 32 subcarriers.

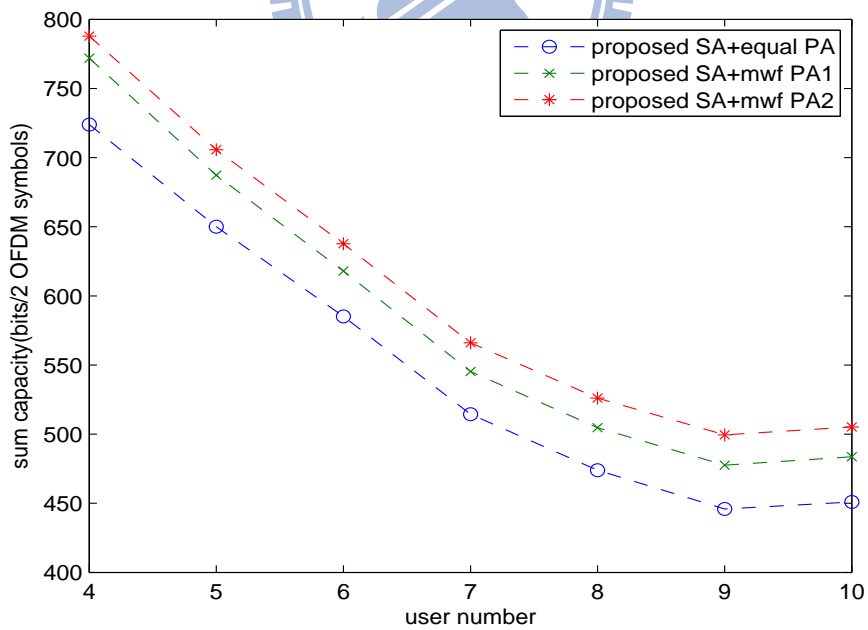


Figure 4.22: Sum rate v.s. user number; 4 relay nodes and 32 subcarriers and the minimum user rate requirement is 30 bits/2 OFDM symbols.

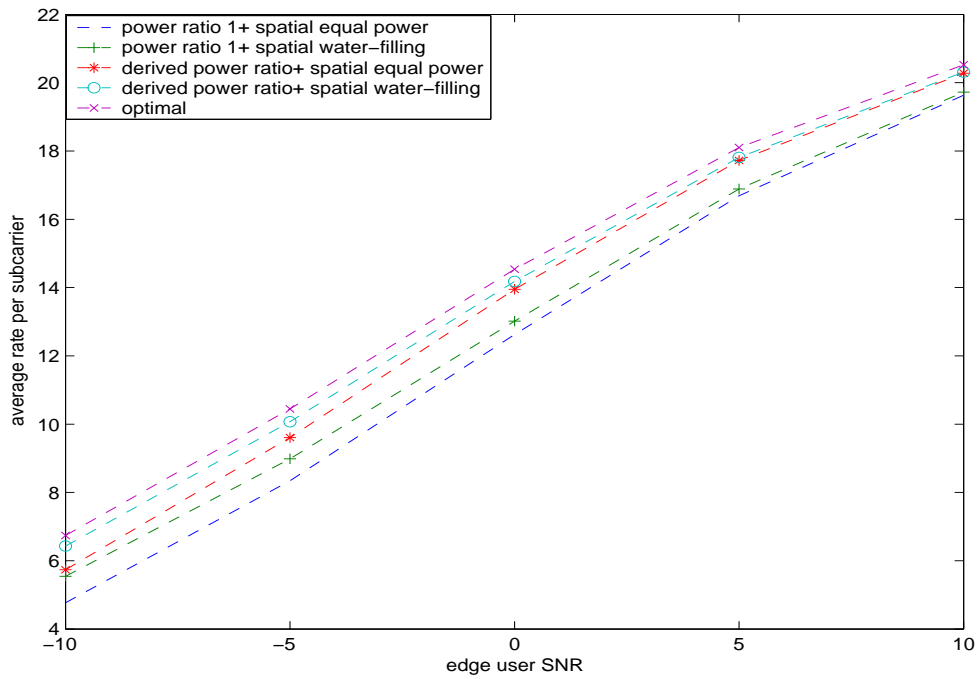


Figure 4.23: Rate per subcarrier v.s. edge user SNR; 1 DNs, 4 RNs and 1 subcarriers.

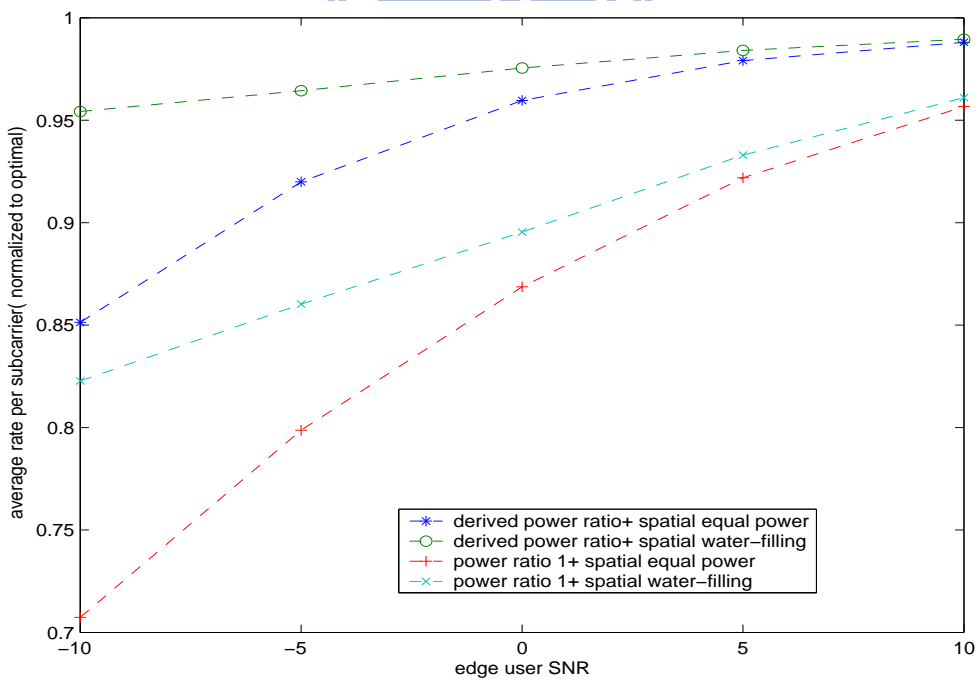


Figure 4.24: Rate per subcarrier(normalized to optimal) v.s. edge user SNR; 1 DNs, 4 RNs and 1 subcarriers.



Given  $P_T, \rho_{(n,k,m)}$ , for all  $k, n, m$   
 Set  $U_0 = \left\{ n \mid \sum_{k=1}^K \rho_{n,k,0} = 1 \right\}$  and  $U_1 = \left\{ n \mid \sum_{k=1}^K \sum_{m=1}^M \rho_{n,k,m} = 1 \right\}$   
 Set  $\kappa(n) = N_t, \kappa_s(n) = N_{t1}, \kappa_r(n) = N_{t2}$  for all  $n$   
 while  $\left( \left| \sum_{n=1}^N \sum_{k=1}^K \sum_{m=0}^M \rho_{(k,n,m)} \sum_{i=1}^{N_t} P_i(k, m, n) - P_T \right| \geq \epsilon \right)$   
     modify  $\lambda$  and calculate  $P(k, n)$   
     for  $(n \in U_0)$   
         calculate  $\mu_n$  by (4.32) and  $p_i(k, n)$  by (4.31)  
          $U_n = \{ i \mid p_i(n, k) > 0 \}$   
         while  $(|U_n| \neq \kappa(n))$   
             modify to the correct  $\kappa(n)$   
         end  
     end  
     for  $(n \in U_1)$   
         while  $(|R_s(k, m, n) - R_r(k, m, n)| \geq \epsilon)$   
             modify  $\lambda_{s,n}$  and calculate  $P_s(k, m, n)$   
             calculate  $\mu_{s,n}$  by (4.17) and  $p_{s,i}(k, m, n)$  by (4.15)  
              $U_{s,n} = \{ i \mid p_{s,i}(n, k) > 0 \}$   
             while  $(|U_{s,n}| \neq \kappa_s(n))$   
                 modify to the correct  $\kappa_s(n)$   
             end  
              $\lambda_{r,n} = 1 - \lambda_{s,n}$  and calculate  $P_r(k, m, n)$   
             calculate  $\mu_{r,n}$  by (4.18) and  $p_{r,i}(k, m, n)$  by (4.16)  
              $U_{r,n} = \{ i \mid p_{r,i}(n, k) > 0 \}$   
             while  $(|U_{r,n}| \neq \kappa_r(n))$   
                 modify to the correct  $\kappa_r(n)$   
             end  
         end  
     end  
 end

Table 4.1: Proposed optimal power allocation.

# Chapter 5

## Conclusion

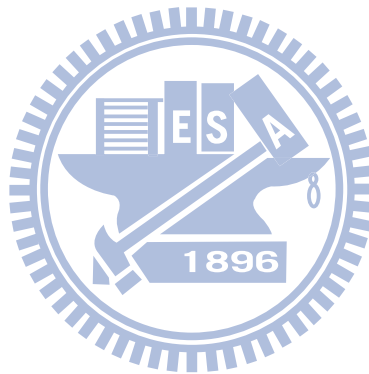
In this thesis, we present RA and relay selection algorithms with fairness (users' minimum rate requirements) consideration for a relay-aided OFDMA downlink network in the presence of CSI uncertainty for both SISO and MIMO scenarios. A tight CLB for a link with imperfect CSI channel is used as the basic performance measure. We derive the corresponding optimal source/relay power ratio for SISO networks and a near optimal source/relay power ratio for MIMO networks when the source-relay and relay-destination links are of the same rank. Based on these power ratios, we propose a practical low-complexity suboptimal subcarrier assignment algorithm which maximizes the sum of CLBs while satisfying the fairness requirement. Given a subcarrier assignment, we propose a near optimal power allocation for the SISO case. For the MIMO case, we obtain the optimal and a near optimal power allocation schemes. We provide numerical evidence to demonstrate that our algorithms suffer only minor sum rate performance degradation against a greedy approach that does not take into account the fairness constraints. It is also verified that our algorithms are capable of meeting all the fairness requirements with high probabilities.

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