

Joint Beamforming and Scheduling Design in Hierarchical Cognitive Radio Networks

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Abstract

Cognitive radio (CR) has been recently regarded as an important technique to improve spectrum utilization. In the thesis, we present a joint design approach to integrate power allocation, antenna beamforming and user scheduling to maximize the system sum rate of multiple users in the hierarchical CR networks with multi-carrier transmissions. Concurrent transmissions between CR users and licensed users are considered to improve the spectrum efficiency. The challenge of hierarchical CR networks is to manage mutual interference between the CR and licensed systems. We first apply the semi-definite relaxation (SDR) technique to transfer the original mixed integer non-linear problem (MINLP) to a convex problem. Then, a sum rate optimization algorithm is proposed to determine the optimal power allocation, antenna beamforming and user scheduling under the limitation of the interference to the licensed networks. The simulation results show that the proposed algorithm can improve the sum rate at least 20% compared to an exhaustive search user scheduling plus zero-forcing beamforming, while overcoming the primary system with sacrifice of 10% sum rate below the maximum sum rate. The proposed methodology provides many important insights into the system design and deployment principles for future hierarchical CR networks.

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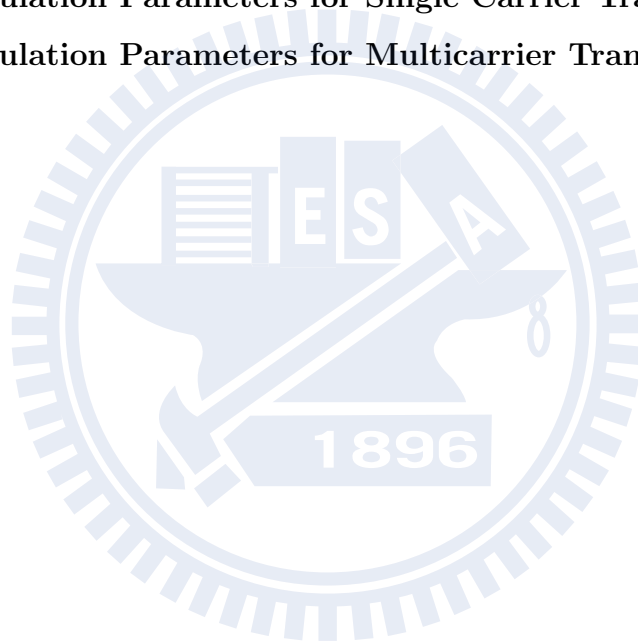
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CHAPTER 1

Introduction

Recently, the amount of systems and services grow rapidly in wireless communications. Most of frequency bands, which is suitable for wireless communications, is assigned by the Federal Communications Commission (FCC) in the United States. However, the measurement by FCC shows that the licensed band is not utilized most of time [1]. Thus, cognitive radio (CR) is proposed as an important technique to improve spectral efficiency of previous spectrum policy [2] [3], which allow the unlicensed users utilize the licensed band smartly without degrading performance quality of licensed users significantly. According to different side information requirements, CR networks can be categorized into three paradigms [4]: underlay, overlay and interweave. The underlay paradigm allows cognitive users to transmit concurrently with licensed users under the limitation of interference to licensed users [5–9]. The overlay paradigm requires the large amount of side information such as codebook or messages of licensed users to relay the messages of the licensed users and serve unlicensed users, simultaneously. The interweave paradigm can be referred as opportunistic communication. The CR users dynamic access the spectrum hole, which is the unused licensed band.

In this thesis, the hierarchical underlying CR networks are considered, where the unlicensed users reuse the same spectrum with licensed users, simultaneously. The major challenge in the hierarchical CR networks is to manage the interference

between licensed and unlicensed systems for the purpose of the sum rate maximization. Multiple-input and multiple-output (MIMO) is one of potential techniques for capacity enhancement and interference mitigation. The optimal coding scheme for MIMO system is dirty paper coding (DPC) [10], which has been shown to achieve Shannon capacity in downlink channel. However, DPC has very high complexity and difficult to implement. The linear beamforming is more practical scheme for MIMO system. In general, beamforming is utilized for sum rate maximization [11] and transmit power minimization [12], while satisfying the quality of service (QoS) requirement. The work in [11] developed a joint design of beamforming and power control to increase the system sum rate in the single cell deployment. The objective of [12] was to minimize the transmit power under the QoS constraint for each user, which is based on the uplink and downlink duality.

Scheduling is another potential technique to improve capacity by employing multi-user diversity. The joint design of antenna beamforming and user scheduling has been studied in [6,8,13–15]. In [6], a joint beamforming and scheduling design was proposed in the single cell deployment, which is based on zero-forcing and orthogonality between users. In [6,8], a user selection algorithm was proposed to mitigate the cross-tier interference between primary and secondary system, but the interference power constraint to primary system may not be satisfied simultaneously. In addition, scheduling can maximize the number of served users [14,15]. In [14], the joint design of transmit power minimization and utilization maximization was formulated as a mixed integer non-linear problem (MINLP), and an approximation of was made by semi-definite relaxation (SDR). The work in [15] considered the same objective as [14] in underlay CR system, and branch and bound (BnB) was utilized to solve the MINLP.

1.1 Problem and Solution

In this thesis, we consider hierarchical CR networks in which a multiuser broadcast secondary system is underlaid in a licensed system. The joint power allocation, beamforming weight and user scheduling design in the hierarchical CR networks is proposed to maximize the sum rate of the secondary system with the limitation of cross-tier interference and transmit power at secondary base station (BS). In [7], a joint design power allocation and beamforming has been studied in the same scenario for a fixed given users set. In this thesis, we relax the assumption to allow the number of secondary users being larger than transmit antennas, where the user scheduling must be required. The joint design of beamforming and scheduling is usually formulated as a MINLP such as [14, 15], which is very difficult to solve. The key contribution of this thesis is to transfer the original sum rate maximization problem, i.e. a MINLP, to a convex optimization problem by using SDR reformulation and introducing additional constraint on the intra-user interference power. In addition, our algorithm can strictly constrain the interference to the primary system below the threshold. The simulation results show that the system sum rate can be improved significantly if user scheduling is adopted. Furthermore, we show that our scheme can exploit the soft capacity gain from dynamic interference constraints.

1.2 Thesis Outline

The rest of this thesis is organized as follows. Chapter 2 introduces the background of cognitive radio and convex optimization. Some related works are also described in Chapter 2. Chapter 3 shows the system and signal model in the hierarchical CR networks with single and multicarrier transmissions. In Chapter 4, semi-definite relaxation approach and an iterative algorithm are proposed to determine the optimal

solution of the MINLP. Numerical results are shown in Chapter 5. Concluding remarks are given in Chapter 6.



CHAPTER 2

Background

2.1 Overview on Hierarchical Cognitive Radio System

In wireless communications, all users coexisting in the same frequency band would interfere with each other. Thus, the right of use spectrum in the United State is assigned by Federal Communication Commission (FCC). Most of spectrum bands, suitable for wireless communication, have already been licensed by FCC. As the requirement of wireless communication has grown very fast in recent years, the useful spectrum becomes insufficient. However, the traffic loads of most licensed bands are not very high. Recent measurements by FCC have shown that the licensed bands are unused for almost 90% of time. FCC considers to adopt the new technique to dynamically accessing the licensed spectrum when there are no licensed users access. That is the licensed band can be shared with non-licensed users in certain conditions.

Several techniques can share the licensed bands to non-licensed users, such as industrial, scientific and medical (ISM) band, sensor network and CR. In particular, CR was regarded as an important technique to improve the spectrum utilization. Indeed, there are still other ways for concurrent transmissions of licensed and unlicensed users without impacting the quality of licensed users. Thus, cognitive radio network requires interference mitigation techniques, and some information of coexist-

ing users, such as spectrum activity, channel condition, codebooks or message. Based on the available side information, there are three CR network paradigms [4]: underlay, overlay and interweave. We briefly describe three paradigms as follows:

1. **Underlay Paradigm:** The underlay paradigm allows communication by CR, which assumes that the cognitive transmitter knows the channel to non-cognitive receiver. In this paradigm, cognitive users are usually called *secondary users*, and non-cognitive users is called *primary users*. The concurrent transmission between cognitive and non-cognitive users occurs only if the interference power resulted from cognitive transmitter is below certain threshold. The interference constraint can be satisfied by several ways, such as multiple antennas beamforming, spread spectrum and, ultra-wideband (UWB). The channel between cognitive and non-cognitive transmitters can be approximated via reciprocal if the cognitive transmitter can overhear the transmission from the non-cognitive receivers. Since the interference constraint is usually quite strict, the coverage of cognitive systems are usually small, and the transmit power of cognitive transmitter is small, too.
2. **Overlay Paradigm:** In the overlay paradigm, the codebooks or messages of non-cognitive system are known at cognitive transmitter. If the cognitive system follows the uniform standard for communication such as Worldwide Interoperability for Microwave Access (WiMAX) and Long Term Evolution-Advanced (LTE-A), the codebook could be obtained. In addition, the messages of non-cognitive can decode at cognitive receiver by known codebook when the non-cognitive transmitter broadcast the messages. Then, cognitive transmitter can obtain the messages by feedback from cognitive receiver. Side information of non-cognitive users can be used in different ways to cancel or mitigate the interference at cognitive and non-cognitive receivers. On the one hand, the interference resulted from non-cognitive users can be cancelled at cognitive receiver. The cognitive transmitter

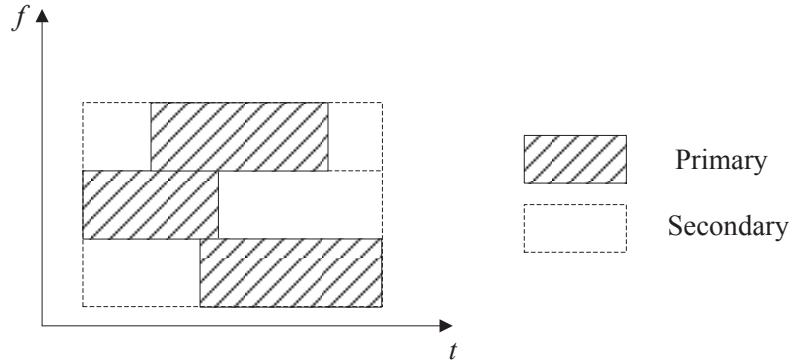


Figure 2.1: Example of opportunistic communications. Void area represent the spectrum, i.e. the licensed users do not utilize.

can assign part of power to transmit the message of non-cognitive users. By proper power allocation at cognitive transmitter, the performance quality of non-cognitive receiver may keep the same or improve, and cognitive users transmit their own message simultaneously. Therefore, the overlay paradigm can improve the spectrum efficiency significantly.

3. **Interweave Paradigm:** The interweave paradigm can be referred as opportunistic communication, which is the original motivation of CR. The measurement by FCC shows that the licensed band is not utilized all the time. Therefore, the spectrum has many time and frequency holes in the licensed spectrum, which referred to spectrum hole as shown in Fig. 2.1. The cognitive users can improve the spectrum efficiency by opportunistic communication when the non-cognitive users do not utilize the licensed spectrum. In the interweave paradigm, one needs to know the spectrum activity information of non-cognitive users. The cognitive transmitter requires spectrum sensing technique to obtain this side information.

2.2 Introduction to Convex Optimization

A general optimization problem can be expressed as

$$\begin{aligned} \min f_0(\mathbf{x}) \\ \text{s.t. } f_i(\mathbf{x}) \leq b_i, i = 1, \dots, m \end{aligned} \tag{2.1}$$

where the vector $\mathbf{x} = [\mathbf{x}_1, \dots, \mathbf{x}_n]^T$ is the optimization variable. The function $f_0 : R^n \rightarrow R$ is the objective function. The function $f_i : R^n \rightarrow R, i = 1, \dots, m$ is the constraint function. The constant b_1, \dots, b_m is the bounds for the constraints. $\hat{\mathbf{x}}$ denotes the optimal solution of optimization problem(2.1). If the objective function and all constraints satisfy the following condition:

$$\begin{aligned} f_i(\alpha x + \beta y) = \alpha f_i(x) + \beta f_i(y), i = 1, \dots, m \\ \forall x, y \in R^n, \forall \alpha, \beta \in R \end{aligned} \tag{2.2}$$

the optimization problem is called linear programming. If objective or any function is not linear, the optimization problem is called non-linear programming. Furthermore, *convex optimization problems* mean that the objective and all constraints satisfy the following conditions:

$$\begin{aligned} f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y), i = 1, \dots, m \\ \alpha + \beta = 1 \end{aligned} \tag{2.3}$$

$$\forall x, y \in R^n, \forall \alpha, \beta \in R \text{ and } \alpha > 0, \beta > 0$$

In general, the non-linear programming is difficult to solve. There is no effective method to solve the general non-linear problem. However, some classes of non-linear optimization problems, such as least-squares problems and convex optimization problems, can be solved by efficient algorithms. There is in general no

analytical formula to solve the convex optimization problem. However, there is a very efficiency method, which is called the interior-point methods. The interior-point methods solve the problem in a short time even if the problem has thousands of variables and constraints. Although the convex optimization can be solved via interior method easily, the transformation from the general non-linear problems to convex problems are very difficult. In general, many skills and tricks such as variable transformation and function composition are required.

Even if a non-linear optimization problem can not be transferred to convex easily, several heuristic techniques based on convex optimization can help solving non-convex optimization problems. Furthermore, convex optimization can find the bounds of non-convex optimization problem by replacing or relaxing to the convex constraints from the original problem.

2.3 Literature Survey

In this section, we categorize the related work about the beamforming and/or scheduling design for the sum rate maximization and transmit power minimization in the hierarchical underlying CR systems. The works of [5] and [9] focused on the transmit power minimization in the hierarchical underlying CR systems. The work [5] proposed a joint beamforming and power control algorithm to minimize the transmit power of secondary system under the constraint that the QoS requirement must be satisfied. However, the channel information between primary BS and users is difficult to obtain at a secondary BS since the cooperation between the primary and secondary users are required. In [9], the interference power constraint of the primary system was satisfied instead of the QoS constraint of the primary users. The work [9] proposed two suboptimal algorithms based on the least square and admission control.

The works [6–8] focused on the sum rate maximization. The work [6] proposed

a suboptimal joint beamforming and scheduling algorithm based on zero forcing beamforming with equal power allocation and channel correlation between the primary and secondary users. However, the constraint of interference power to the primary system can not be satisfied in some cases. The work [7] develops a joint beamforming and power control iterative algorithm to sum rate maximization for fixed given serving set. However, the algorithm is only suitable in high SINR regime, which is not reasonable in interference-limited environment. A joint zero-forcing beamforming and user scheduling algorithm is proposed [8] to mitigate the cross-tier interference. The scheduling in [8] includes both the primary and secondary users, but it is not practical for the primary users scheduling in the underlay CR systems. We compare our work with above research in Table 2.1.

Based on the above discussions, we can summarize that the problem of joint power allocation, beamforming and scheduling design to maximize the sum rate of the secondary CR system under the interference constraint has not been investigated well.

Table 2.1: **Literature Survey**

	Power Control	Beamforming	Scheduling	Note
[5]	Joint design by weighted least square suboptimal		x	Manage interference by inactivating secondary users.
[6]	Equal power	Zero forcing	Orthogonality	Not guarantee the interference constraint
[7]	Joint design by convex optimization		x	Capacity approximation
[8]	x	Zero forcing	Orthogonality	Joint primary and secondary users scheduling.
Our works	Joint design by convex optimization			

CHAPTER 3

System Model and Problem Formulation

We consider a hierarchical underlying CR system, consisting of a primary system and secondary system. The primary system has a licensed spectrum. The secondary system is a multi-user broadcast system, which aims to provide services to secondary users under the condition that it can not interfere with the primary system. In the fourth generation (4G) of cellular wireless standards, the frequency division duplex (FDD) and time division duplex (TDD) are both considered. Therefore, what kind of duplexing modes of the primary and secondary is most suitable for hierarchical underlying CR system should be discussed. If the primary system is TDD, the secondary system may interfere primary downlink and uplink in one transmission time for both TDD and FDD secondary systems as shown in Fig. 3.1. The interference to primary system would hardly be managed. If the primary system is FDD, the secondary systems can transmit at the primary downlink or uplink spectrum. The secondary system utilizing the uplink spectrum of primary users is a better option for two reasons. First, the quality of service (QoS) requirement of uplink is usually less strict than downlink. Thus, it may endure larger interference from secondary systems. Second, in order to cancel the interference, the channel state information (CSI) must be known at the secondary BS for utilizing downlink spectrum. In general, the CSI to the primary BS is more easily obtained than CSI to primary users. Therefore, secondary system underlying the primary uplink spectrum is considered

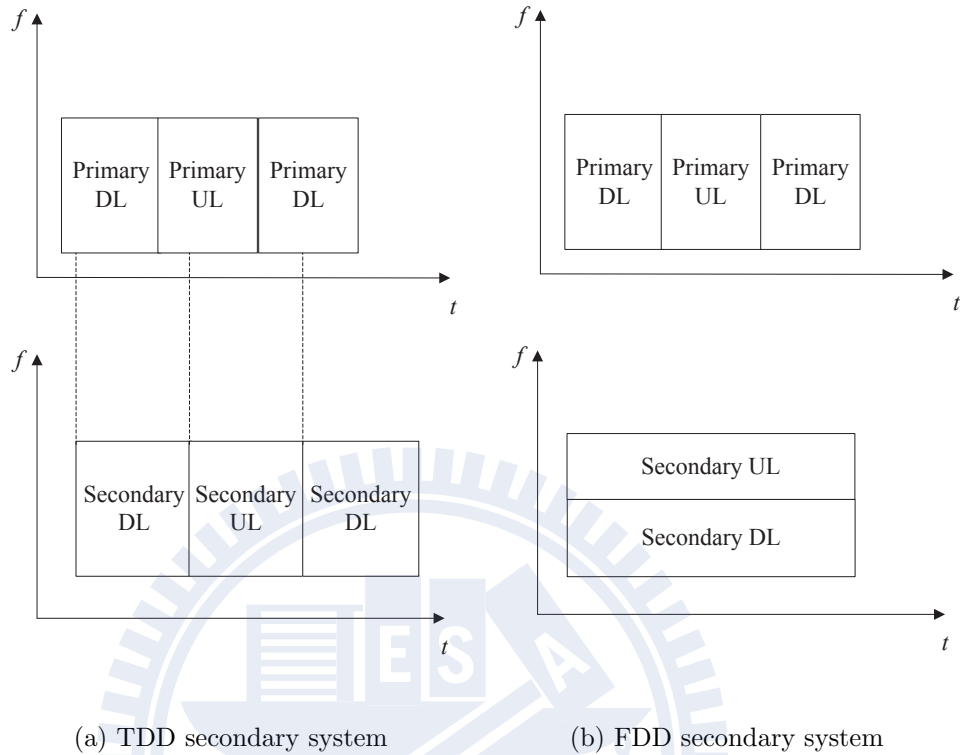
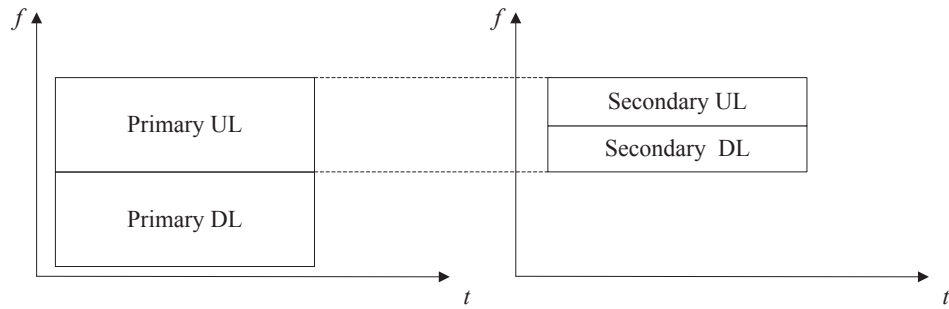


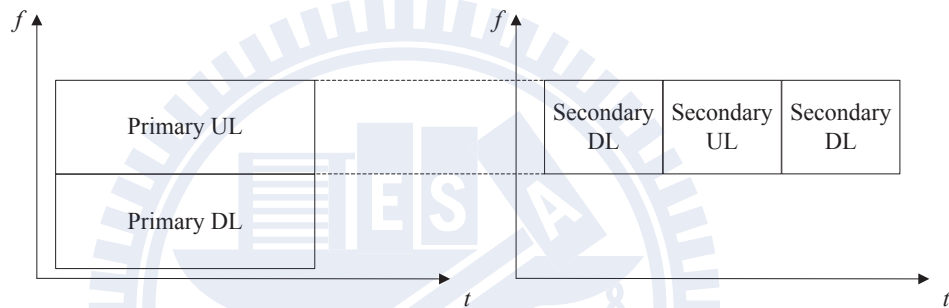
Figure 3.1: Spectrum usage of hierarchical CR system with FDD primary system and TDD/FDD secondary system.

as shown in Fig. 3.2. Since the CSI between the primary and the secondary users are also required, TDD secondary system shown in Fig. 3.2(b) is preferable for the purpose of feedback. To summarize, FDD primary system and TDD secondary system utilizing the primary uplink spectrum is considered in the thesis.

The hierarchical underlying CR system with FDD primary system and TDD secondary system is shown in Fig. 3.3. For simplification, assumes that only one primary utilizes the uplink band at a time. The primary BS, primary users and the secondary users are equipped with single antenna. The secondary BS are equipped with M . The secondary BS serve M secondary users at the most, which is selected



(a) FDD secondary system



(b) TDD secondary system

Figure 3.2: Spectrum usage of hierarchical CR system with TDD primary system and FDD/TDD secondary system.

from K secondary users, where $K > M$. Both frequency non-selective and frequency selective fading channel are considered in the next section.

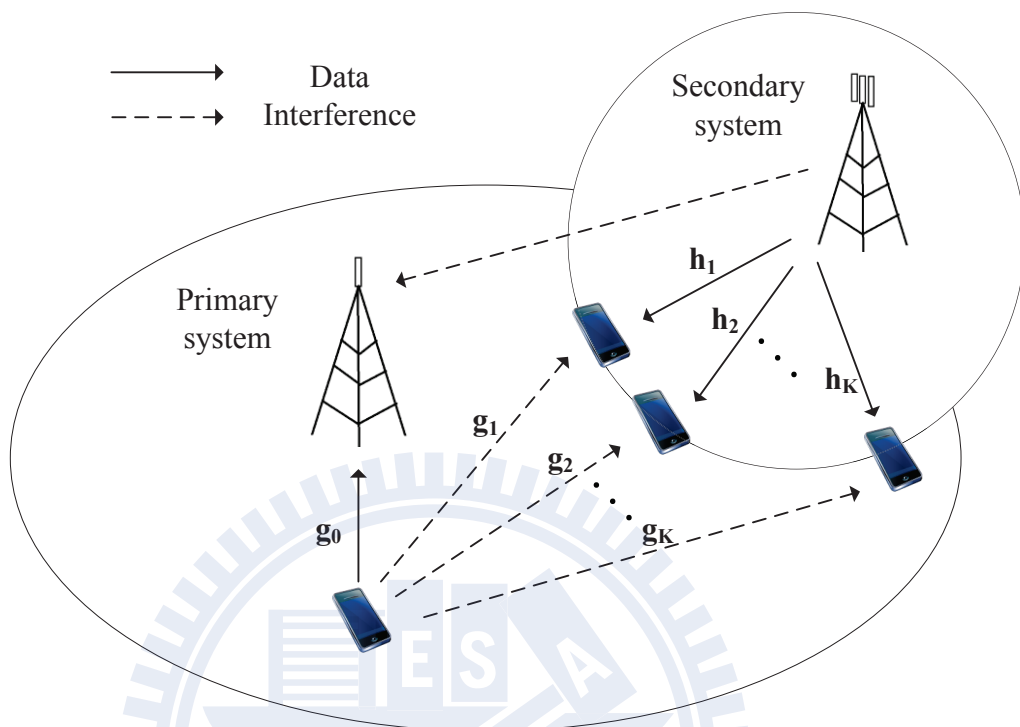


Figure 3.3: Hierarchical cognitive radio networks with a FDD primary system and a TDD secondary system, where the downlink spectrum of the secondary system utilizes the uplink spectrum of the primary system.

3.1 Single Carrier Hierarchical Cognitive Radio System

For a frequency non-selective channel, there is no inter-symbol interference (ISI). Thus, we may process the transmitted signal in the time domain, where s_k and x denote the transmitted signals from the secondary BS to the k^{th} secondary user and the primary user to primary BS, respectively. An $M \times 1$ vector \mathbf{w}_k represents beamforming weight for the k^{th} secondary user. S is the set of served users, where

$S \subseteq \{1, \dots, K\}$. The received signal of k^{th} secondary user is

$$r_k = \underbrace{(\mathbf{w}_k s_k)^\dagger \mathbf{h}_k}_{\text{desired signal}} + \underbrace{\left(\sum_{j \in S, j \neq k} \mathbf{w}_j s_j \right)^\dagger \mathbf{h}_k}_{\text{intra-users interference}} + \underbrace{\sqrt{Q} g_k x}_{\text{inter-cell interference}} + n_k, k \in S, \quad (3.1)$$

where \mathbf{h}_k denotes channel between M antennas of a secondary BS and the k^{th} secondary user, \sqrt{Q} denotes the transmit power of the primary users, g_k denotes channel between primary user and k^{th} secondary user, and n_k is a Gaussian noise for k^{th} secondary user with zero mean and variance σ_N^2 . We assume that the average power of signal $E[|s_k|^2]$ is normalized to one. Thus, the transmission power to the k^{th} users is $\|\mathbf{w}_k\|^2$. In addition, the received signal at the primary BS can be written as

$$r_0 = \sqrt{Q} g_0 x + \underbrace{\left(\sum_{j \in S} \mathbf{w}_j s_j \right)^\dagger \mathbf{h}_0}_{\text{inter-cell interference}} + n_0, \quad (3.2)$$

where n_0 is the noise at the primary. Here, \mathbf{h}_0 represents channel between all the served secondary users and the primary BS, g_0 represents channel between primary BS and primary user, and can be expressed as

$$g_k = \alpha_k a_k, k = 1, \dots, K \quad (3.3)$$

$$\mathbf{h}_k = \beta_k b_k \mathbf{v}_k, k = 0, \dots, K \quad (3.4)$$

where α_k and β_k are the long-term fading, which includes pathloss exponent of four, and log-normal shadow with standard deviation of 8 dB; a_k and b_k are short-term Rayleigh fading, Note that \mathbf{v}_k is the steering vector which represent the relative phase at each antenna

$$\mathbf{v}_k = [1, e^{j\phi_k}, \dots, e^{j(M-1)\phi_k}]^T \quad (3.5)$$

where ϕ_k depends on the carrier frequency and propagation direction of the plane wave.

For a hierarchical CR system, the interference to the primary system should be limited strictly. Therefore, we design beamforming weights to control the inter-cell interference (ICI) shown in (3.2) to be lower than a pre-defined threshold, while minimizing intra-use interference (IUI).

3.2 Multicarrier Hierarchical Cognitive Radio System

For the frequency selective channel, there are significant ISI due to multipath effects. The receive signals of the secondary users become

$$r_k = \underbrace{(\mathbf{w}_k s_k)^\dagger \mathbf{h}_{k,0}}_{\text{desired signal}} + \underbrace{\sum_{l=1}^{N_d} \sum_{j=1, j \neq k}^K (\mathbf{w}_j s_j)^\dagger \mathbf{h}_{k,l}}_{\text{intra-users and inter-symbol interference}} + \underbrace{\sqrt{Q} g_k x}_{\text{inter-cell interference}} + n_k, k \in S \quad (3.6)$$

where N_d is number of paths in the frequency-selective channel; $\mathbf{h}_{k,l}$ denotes channel between secondary BS and k^{th} secondary users in l^{th} delay tap. Here we assume that the channel from primary user is flat fading. $\mathbf{h}_{k,l}$ and g_k include the long-term fading as in the previous section. Although beamforming may cancel IUI and ICI, there still is significant ISI. Orthogonal frequency division multiplexing (OFDM) can be used to reduce ISI. We may process the transmitted signal in the frequency domain by fast Fourier transform (FFT). Then, the received signal of the k^{th} secondary user in the n_{sc}^{th} subcarrier can be expressed as

$$r_{k,n_{sc}} = \underbrace{(\tilde{\mathbf{w}}_{k,n_{sc}} \tilde{s}_{k,n_{sc}})^\dagger \tilde{\mathbf{h}}_{k,n_{sc}}}_{\text{desired signal}} + \underbrace{\sum_{j \in S_{n_{sc}}, j \neq k} (\tilde{\mathbf{w}}_{j,n_{sc}} \tilde{s}_{j,n_{sc}})^\dagger \tilde{\mathbf{h}}_{k,n_{sc}}}_{\text{intra-users interference}} \quad (3.7)$$

$$+ \underbrace{\sqrt{Q} \tilde{g}_k \tilde{x}}_{\text{inter-cell interference}} + \tilde{n}_{k,n_{sc}}, k \in S_{n_{sc}}$$

where $n_{sc} = [1, \dots, N_{sc}]$; N_{sc} is number of subcarriers; $\tilde{h}_{k,n_{sc}}$ denotes CSI for the k^{th} secondary users in the n_{sc} subcarriers; $S_{n_{sc}}$ is the set of secondary users selected in n_{sc}^{th} subcarrier. In addition, the received signal of the primary BS in the n_{sc}^{th} subcarrier can be expressed as

$$r_0 = \sqrt{Q}g_0x + \underbrace{\sum_{n_{sc}=1}^{N_{sc}} \sum_{j \in S_{n_{sc}}} (\mathbf{w}_{j,n_{sc}} \tilde{S}_{j,n_{sc}})^\dagger \mathbf{h}_{0,n_{sc}}}_{\text{inter-cell interference}} + n_0 \quad (3.8)$$

If there is no ISI, then we may design beamforming weights at each subcarrier to cancel the remaining interference.

3.3 Performance Metrics

3.3.1 System Sum Rate

We consider a multi-user broadcast system, and utilize Shannon capacity formula to model the system sum rate as follows:

$$C_{sum} = \sum_{k \in S} \log_2(1 + \gamma_k), \quad (3.9)$$

where S is the set of served users in each transmission; γ_k is the signal to interference and noise ratio (SINR) of the k^{th} user.

3.3.2 Fairness Index

Fairness is used to determine whether all the users share the resource fairly. There are several definitions of fairness. In this thesis, we utilize Jain's fairness index as follows:

$$F(x_1, \dots, x_K) = \frac{\left(\frac{1}{K} \sum_{i=1}^K x_i\right)^2}{\frac{1}{K} \sum_{i=1}^K x_i^2} = \frac{\left(\sum_{i=1}^K x_i\right)^2}{K \sum_{i=1}^K x_i^2}, \quad (3.10)$$

where x_i is the number of resource allocating to the i^{th} user; K is the number of users. The larger the fairness index F is, the more the fair the resource allocate. The range of F is between $\frac{1}{K}$ and 1. $\frac{1}{K}$ is the worst case, which represents all resources are allocated to one user. If fairness index is equal to one, it represents that all users are allocated the same amount of resource.

3.4 Problem Formulation

Based on the signal model described in Sections 3.1 and 3.2, the single carrier transmission can be regarded as a special case in multicarrier transmissions. Therefore, the following problems are formulated with multicarrier transmissions

The average SINR for the k^{th} user in the j^{th} subcarrier is given by

$$\gamma_{k,j} = \frac{\left| \mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j} \right|^2}{\sum_{i \in S_j, i \neq k} \left| \mathbf{w}_{i,j}^\dagger \mathbf{h}_{k,j} \right|^2 + B_{k,j}}, \quad (3.11)$$

where $\sum_{i \in S_j, i \neq k} \left| \mathbf{w}_{i,j}^\dagger \mathbf{h}_{k,j} \right|^2$ is the interference between secondary users with each other in j^{th} subcarrier. $\sum_{i \in S_j, i \neq k} \left| \mathbf{w}_{i,j}^\dagger \mathbf{h}_{k,j} \right|^2$ can be regarded as the intra-user interference. $Q|g_{k,j}|^2$ is the inter-cell interference resulted from the primary user, and σ_N^2 is the thermal noise. The inter-cell interference resulted from the primary user can not be controlled at the secondary BS. Noise can be combined with the inter-cell interference as $B_{k,j} = Q|g_{k,j}|^2 + \sigma_N^2, \forall k$, which is a constant to the secondary users in each transmission. Since S_j includes unknown users, we introduce a new variable \mathbf{b}_j , regarded to the scheduling parameter. If the i^{th} secondary is served in the j^{th} subcarrier, $b_{i,j}$ is equal to one; otherwise, $b_{i,j}$ is equal to zero. The relationship between S_j and \mathbf{b}_j can be written as

$$S_j = \{b_{i,j} | b_{i,j} = 1, i = 1, \dots, K\} \quad (3.12)$$

Then, the system sum rate can be formulated as follows:

$$C_{sum} = \max_{\mathbf{b}, \mathbf{W}_1, \dots, \mathbf{W}_{N_{sc}}} \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \log_2 \left(1 + \frac{b_{k,j} \left| \mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j} \right|^2}{\sum_{i \in S_j, i \neq k}^K b_{i,j} \left| \mathbf{w}_{i,j}^\dagger \mathbf{h}_{k,j} \right|^2 + B_{k,j}} \right), \quad (3.13)$$

where $\mathbf{W}_j = [\mathbf{w}_{1,j}, \dots, \mathbf{w}_{K,j}]$ is the beamforming weight matrix in the j^{th} subcarrier. The subcarriers assigned to k^{th} secondary user are $\{j | b_{k,j} = 1, j = 1, \dots, N_{sc}\}$, and the number of users served in the j^{th} subcarrier is $\sum_{i=1}^K b_{i,j}$, which must be smaller than or equal to M . Therefore, (3.13) can be regarded as the secondary system sum rate after user selection and subcarrier allocation. Our goal is to maximize the sum rate of the secondary system subject to three constraints:

1. The interference power to the primary BS is strictly limited to a pre-defined threshold.
2. The number of served users in each transmission must be smaller than M in each subcarrier.
3. The transmit power from secondary BS is limited.

Thus, the sum rate maximization problem can be formulated as follows:

$$\max_{\mathbf{b}, \mathbf{W}_1, \dots, \mathbf{W}_{N_{sc}}} \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \log_2 \left(1 + \frac{b_{k,j} \left| \mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j} \right|^2}{\sum_{i \in S_j, i \neq k}^K b_{i,j} \left| \mathbf{w}_{i,j}^\dagger \mathbf{h}_{k,j} \right|^2 + B_{k,j}} \right),$$

subject to

$$(C.1) \sum_{i=1}^K b_{i,j} \left| \mathbf{w}_{i,j}^\dagger \mathbf{h}_{0,j} \right|^2 \leq I_{\max}; \quad (3.14)$$

$$(C.2) \sum_{i=1}^K b_{i,j} \leq M, \forall j = 1, \dots, N_{sc} ;$$

$$(C.3) \sum_{j=1}^{N_{sc}} \sum_{i=1}^K b_{i,j} \|\mathbf{w}_{i,j}\|^2 \leq P_{max} ;$$

$$(C.4) b_{i,j} \in \{0, 1\}, \forall i = 1, \dots, K, j = 1, \dots, N_{sc} ,$$

where I_{max} and P_{max} are the interference power and transmission power constrain, respectively. In (C.2), one dimension of antenna domain should be used to cancel the interference to the primary BS. Thus, secondary BS can only serve $M - 1$ users in each transmission at most. However, if proper user set is selected or the interference constraint is not too strict, secondary BS may serve M users at a time. In the objective function of (3.14), we jointly design beamforming weight \mathbf{W} and scheduling parameter \mathbf{b} to maximize the sum rate of the secondary BS with transmit power constraint (C.3). Note that the transmit power to the i^{th} secondary user in the j^{th} subcarrier is $\|\mathbf{w}_{i,j}\|^2$. Therefore, (C.3) also implies that power allocation is not only to each secondary user but also for subcarriers. Hence, (3.14) can be regarded as a joint optimization problem for multi-user beamforming, power allocation and scheduling in a system with multiple subcarriers.

CHAPTER 4

Joint Power allocation, Beamforming and Scheduling Design in Hierarchical Cognitive Radio System with Multi-carrier Transmissions

In this chapter, we present a joint power allocation, beamforming and scheduling design to maximize the sum rate of the secondary BS in a hierarchical CR system. We consider the multicarrier transmissions. The sum rate maximization problem is transferred from a MINLP to a convex problem, and an iterative algorithm is proposed to determine the optimal power allocation, beamforming weight and user scheduling.

4.1 Convexity of Optimization Problem

The optimization problem (3.14) is non-trivial for following two reasons:

1. Two variables \mathbf{W} and \mathbf{b} are continuous and discrete, respectively. It is called mixed integer non-linear optimization problem (MINLP), which is hard to solve. Although branch and bound (BnB) is a known method to solve mixed integer (MI) problem, it still difficult to solve a non-linear problem (NLP). However, the maximization problem (3.14) just includes the multiplication of two variables.

2. The objective is neither concave nor convex function respect to \mathbf{W} Therefore, the maximization problem can not be solved by convex optimization even if we assume the optimal \mathbf{b} is known.

In this section, we attempt to simplify and reformulate the non-convex optimization problem (3.14) to a convex optimization problem.

First, check whether each variable and constrain are necessary. Then, two observations are made as follows:

1. For the sum-rate maximization purpose, non-served users should not be allocated power. Thus, the variable \mathbf{b} can be dropped, and we can utilize $\|\mathbf{w}_i\|^2$, which is transmit power allocated to i^{th} secondary user, to represent scheduling instead of b_i , where $b_i = 0$ and $b_i = 1$ corresponding to $\|\mathbf{w}_i\|^2 = 0$ and $\|\mathbf{w}_i\|^2 > 0$, respectively.
2. As shown in (C.2) of (3.14), the number of served users at a time must be not larger than M . Otherwise, the beamforming weight matrix \mathbf{W} can not null the inter-user interference. We can expect that the rank of optimal beamforming weight matrix $\hat{\mathbf{W}}$ must be smaller than or equal to M . Then, (C.2) in (3.14) can be dropped.

After the above simplifications, the optimization problem (3.14) can be reformulated as follows:

$$\max_{\mathbf{w}_1, \dots, \mathbf{w}_{N_{sc}}} \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \log_2 \left(1 + \frac{|\mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j}|^2}{\sum_{i=1, i \neq k}^K |\mathbf{w}_{i,j}^\dagger \mathbf{h}_{k,j}|^2 + B_{k,j}} \right),$$

subject to

$$(C.1) \sum_{i=1}^K \left| \mathbf{w}_{i,j}^\dagger \mathbf{h}_{0,j} \right|^2 \leq I_{\max} ; \quad (4.1)$$

$$(C.2) \sum_{j=1}^{N_{sc}} \sum_{i=1}^K \|\mathbf{w}_{i,j}\|^2 \leq P_{\max} .$$

Now, (C.1) and (C.2) are norm square and quadratic form, respectively. Both of them are convex functions. However, the objective function is logarithm of quadratic-over-quadratic, which is not a concave function. If capacity $\log_2(1 + \text{SINR})$ is approximated by $\log_2(\text{SINR})$, the problem can be solved by convex optimization with an iterative algorithm in [7]. However, this assumption does not make sense in our problem, since SINR of non-served users are zero. In the next section, the semi-definite relaxation approaches are applied to this problem.

4.2 Semi-Definite Relaxation Approach

By linear algebra, we know that $\text{Tr}(\mathbf{A}\mathbf{B}) = \text{Tr}(\mathbf{B}\mathbf{A})$ for arbitrary matrix \mathbf{A} and \mathbf{B} . The quadratic form can be expressed to the form of trace function as:

$$\begin{aligned} \left| \mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j} \right|^2 &= (\mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j})(\mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j})^* = \text{Tr}(\mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j} \mathbf{h}_{k,j}^\dagger \mathbf{w}_{k,j}) \\ &= \text{Tr}(\mathbf{w}_{k,j} \mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j} \mathbf{h}_{k,j}^\dagger) \end{aligned} \quad (4.2)$$

This equation is affine respect to matrix $\mathbf{w}_{k,j} \mathbf{w}_{k,j}^\dagger$. Thus, new variables are introduced

$$\mathbf{H}_{k,j} = \mathbf{h}_{k,j} \mathbf{h}_{k,j}^\dagger, \forall k \in [0, K], j \in [1, N_{sc}] \quad (4.3)$$

and

$$\mathbf{W}_{k,j} = \mathbf{w}_{k,j} \mathbf{w}_{k,j}^\dagger, \forall k \in [1, K], j \in [1, N_{sc}] \quad (4.4)$$

Then, (4.2) can be expressed as:

$$\text{Tr}(\mathbf{w}_{k,j} \mathbf{w}_{k,j}^\dagger \mathbf{h}_{k,j} \mathbf{h}_{k,j}^\dagger) = \text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{k,j}), \forall k \in [1, K], j \in [1, N_{sc}] \quad (4.5)$$

Then, (4.1) can be transformed to the following equivalent problem:

$$\max_{\mathbf{w}_{1,1}, \dots, \mathbf{w}_{1,N_{sc}}, \mathbf{w}_{2,1}, \dots, \mathbf{w}_{K,N_{sc}}} \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \log_2 \left[1 + \frac{\text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{k,j})}{\sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{i,j} \mathbf{H}_{k,j}) + B_{k,j}} \right],$$

subject to

$$(C.1) \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{0,j}) \leq I_{\max}; \quad (4.6)$$

$$(C.2) \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j}) \leq P_{\max};$$

$$(C.3) \mathbf{W}_{k,j} \succeq 0, \forall k \in [1, K], j \in [1, N_{sc}];$$

$$(C.4) \text{Rank}(\mathbf{W}_{k,j}) = 1, \forall k \in [1, K], j \in [1, N_{sc}],$$

where $\mathbf{W}_{i,j}$ denotes the beamforming weight for the i^{th} secondary user in the j^{th} subcarrier. The problem is a semi-definite programming since all variables are rank one hermitian symmetric positive semi-definite matrix. Note that the rank constrain (C.4) is not convex. Hence, we relax (C.4) as well to obtain the following problem:

$$\max_{\mathbf{w}_{1,1}, \dots, \mathbf{w}_{1,N_{sc}}, \mathbf{w}_{2,1}, \dots, \mathbf{w}_{K,N_{sc}}} \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \log_2 \left[\frac{\text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{k,j}) + \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{i,j} \mathbf{H}_{k,j}) + B_{k,j}}{\sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{i,j} \mathbf{H}_{k,j}) + B_{k,j}} \right]$$

subject to

$$(C.1) \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{0,j}) \leq I_{\max}; \quad (4.7)$$

$$(C.2) \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j}) \leq P_{\max};$$

$$(C.3) \mathbf{W}_{k,j} \succeq 0, \forall k \in [1, K], j \in [1, N_{sc}].$$

Now, all constraints are convex, but the objective function is still non-concave. The objective function is the composition of logarithm and linear fractional function, which does not guarantee a concave function with respect to $\mathbf{W}_{k,j}, \forall k, j$. In order to transform the objective function to a concave function, we introduce additional variable

$$\mathbf{\Omega} = \begin{bmatrix} \Omega_{1,1} & \cdots & \Omega_{1,N_{sc}} \\ \vdots & \ddots & \vdots \\ \Omega_{K,1} & \cdots & \Omega_{K,N_{sc}} \end{bmatrix}, \quad (4.8)$$

where $\mathbf{\Omega}$ is the upper bound of intra-user interference power among all secondary users. Then, the optimization problem can be reformulated as:

$$\begin{aligned} & \max_{\mathbf{W}_{1,1}, \dots, \mathbf{W}_{1,N_{sc}}, \mathbf{W}_{2,1}, \dots, \mathbf{W}_{K,N_{sc}}} \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \log_2 \left[\frac{\text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{k,j}) + \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{i,j} \mathbf{H}_{k,j}) + B_{k,j}}{\sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{i,j} \mathbf{H}_{k,j}) + B_{k,j}} \right], \\ & \text{subject to} \\ & (C.1) \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{0,j}) \leq I_{\max}; \\ & (C.2) \sum_{j=1}^{N_{sc}} \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j}) \leq P_{\max}; \\ & (C.3) \mathbf{W}_{k,j} \succeq 0, \forall k \in [1, K], j \in [1, N_{sc}]; \\ & (C.4) \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{k,j}) \leq \Omega_{k,j}, \forall j = 1, \dots, N_{sc}, \end{aligned} \quad (4.9)$$

where $\Omega_{k,j}, \forall k, j$ are variables with non-negative values bounding the intra-user interference power to the k^{th} secondary user. In addition, (4.9) can be simplified to

the single carrier case as:

$$\max_{\mathbf{w}_{1,1}, \dots, \mathbf{w}_K} \sum_{k=1}^K \log_2 \left[\frac{\text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_k) + \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_i \mathbf{H}_k) + B_k}{\sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_i \mathbf{H}_k) + B_k} \right],$$

subject to

$$(C.1) \sum_{k=1}^K \text{Tr}(\mathbf{W}_k \mathbf{H}_0) \leq I_{\max}; \quad (4.10)$$

$$(C.2) \sum_{k=1}^K \text{Tr}(\mathbf{W}_k) \leq P_{\max};$$

$$(C.3) \mathbf{W}_k \succeq 0, \forall k \in [1, K];$$

$$(C.4) \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_i \mathbf{H}_k) \leq \Omega_k.$$

For any fixed $\mathbf{\Omega}$, (4.9) is a concave maximization problem, denoted by $R_1(\mathbf{\Omega})$.

The optimal solution $[\hat{\mathbf{W}}_1, \dots, \hat{\mathbf{W}}_K]$ of the convex problem (4.9) can be solved by toolbox CVX [16]. Then, one need to check the rank of the solution $\hat{\mathbf{W}} = [\hat{\mathbf{W}}_1, \dots, \hat{\mathbf{W}}_K]$. If all of them are one, it means that we do not relax any constraints. $\hat{\mathbf{W}}$ will also be the optimal solution of the original problem (4.1), if the optimal $\hat{\mathbf{\Omega}}$ is known. In addition, the optimal user scheduling and subcarrier assignment are:

$$\hat{S}_j = \{k | \text{Rank}(\mathbf{W}_k) = 1, k = 1, \dots, K\}, \forall j = 1, \dots, N_{sc} \quad (4.11)$$

Then, the globally optimal beamforming weight $\hat{\mathbf{W}}_{k,j}, \forall k, j$ of (4.10) can be converted into a feasible solution by $\hat{\mathbf{W}}_{k,j} = \hat{\mathbf{w}}_{k,j} \hat{\mathbf{w}}_{k,j}^\dagger, \forall k \in \hat{S}_j, j = [1, \dots, N_{sc}]$. If the rank of any $\mathbf{W}_{k,j}, \forall k, j$ is larger than one, we must extract a feasible vector $\mathbf{w}_{k,j}$ from $\mathbf{W}_{k,j}$, which is usually not an optimal solution. Fortunately, the rank of $\hat{\mathbf{W}}_{k,j}, \forall k, j$ is always smaller than or equal to one in our simulation experience.

4.3 Iterative Sum-Rate Maximization Algorithm

Although the optimal $\hat{\Omega}$ can be obtained by the exhaustive search, we still prefer to develop an efficient algorithm to obtain $\hat{\Omega}$ to reduce search space. In (C.4) of (4.9), the power allocation is across the subcarrier. Thus, the optimal beamforming weight in each subcarrier is correlated with others. A simplified formulation is first considered as:

$$\max_{\mathbf{W}_{1,1}, \dots, \mathbf{W}_{1,N_{sc}}, \mathbf{W}_{2,1}, \dots, \mathbf{W}_{K,N_{sc}}} \sum_{k=1}^K \log_2 \left[\frac{\text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{k,j}) + \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{i,j} \mathbf{H}_{k,j}) + B_{k,j}}{\sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{i,j} \mathbf{H}_{k,j}) + B_{k,j}} \right],$$

subject to

$$(C.1) \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{0,j}) \leq I_{\max}; \quad (4.12)$$

$$(C.2) \sum_{k=1}^K \text{Tr}(\mathbf{W}_{k,j}) \leq \frac{P_{\max}}{N_{sc}}, \forall j \in [1, N_{sc}];$$

$$(C.3) \mathbf{W}_{k,j} \succeq 0, \forall k \in [1, K], j \in [1, N_{sc}];$$

$$(C.4) \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_{k,j} \mathbf{H}_{k,j}) \leq \Omega_{k,j}, \forall j = 1, \dots, N_{sc},$$

where (C.2) represents equal power allocation over all subcarriers. Then, the power allocation, beamforming and scheduling in each subcarrier is independent with other subcarriers. That is, $\Omega_j = [\Omega_{1,j}, \dots, \Omega_{K,j}]^T$ can change without impacting other subcarrier. Therefore, an iterative algorithm can be developed for the single carrier transmission, and may apply to multicarrier transmission by utilizing to each subcarrier.

The following formulations are expressed as single carrier transmission. First, we define:

$$\begin{aligned}
R_2(\mathbf{W}, \boldsymbol{\Omega}) &= \sum_{k=1}^K \{ \log_2 [\text{Tr}(\mathbf{W}_k \mathbf{H}_k) + \sum_{i=1, i \neq k}^K \text{Tr}(\mathbf{W}_i \mathbf{H}_k) + B_k] \\
&\quad - \log_2 [\Omega_k + B_k] \} \\
&= g(\mathbf{W}) + f(\boldsymbol{\Omega}),
\end{aligned} \tag{4.13}$$

where

$$\begin{aligned}
g(\bar{\mathbf{W}}) &= \log_2 [\text{Tr}(\bar{\mathbf{W}}_k \mathbf{H}_k) + \sum_{i=1, i \neq k}^K \text{Tr}(\bar{\mathbf{W}}_i \mathbf{H}_k) + B_k] \\
f(\boldsymbol{\Omega}) &= -\log_2 (\Omega_k + B_k).
\end{aligned}$$

Then, the optimization problem is formulated about $\boldsymbol{\Omega}$ as follows:

$$R_1(\boldsymbol{\Omega}) = \max_{\mathbf{W} \in \Theta(\mathbf{W})} R_2(\mathbf{W}, \boldsymbol{\Omega}) \tag{4.14}$$

where $\Theta(\mathbf{W})$ is the feasible set with respect to (4.9). Note that the size of set Θ depends on the value of $\boldsymbol{\Omega}$. In (C.4) of (4.9), if $\boldsymbol{\Omega}$ becomes smaller, the set of available beamforming weight \mathbf{W} would become smaller. Consider two vector $\boldsymbol{\Omega}^{(1)} = [\Omega_1^{(1)}, \dots, \Omega_K^{(1)}]$ and $\boldsymbol{\Omega}^{(2)} = [\Omega_1^{(2)}, \dots, \Omega_K^{(2)}]$, corresponding to two non-empty set $\Theta^{(1)}$ and $\Theta^{(2)}$, respectively. Assume that $\boldsymbol{\Omega}^{(2)} \preceq \boldsymbol{\Omega}^{(1)}$, i.e., $\Omega_k^{(2)} \leq \Omega_k^{(1)}, \forall k$. The following observations can be made:

1. $f(\boldsymbol{\Omega}^{(2)}) \geq f(\boldsymbol{\Omega}^{(1)})$.
2. $\max_{\mathbf{W} \in \Theta^{(2)}} g(\mathbf{W}) \leq \max_{\mathbf{W} \in \Theta^{(1)}} g(\mathbf{W})$, since $\Theta^{(2)}$ is a subset of $\Theta^{(1)}$.

The first observation shows that the system sum rate can be improved by decreasing $\boldsymbol{\Omega}$, but the second observation seems to reveal opposite trend as $\boldsymbol{\Omega}$ decrease. Therefore, we take a simple example to investigate the relationship between sum rate and $\boldsymbol{\Omega}$.

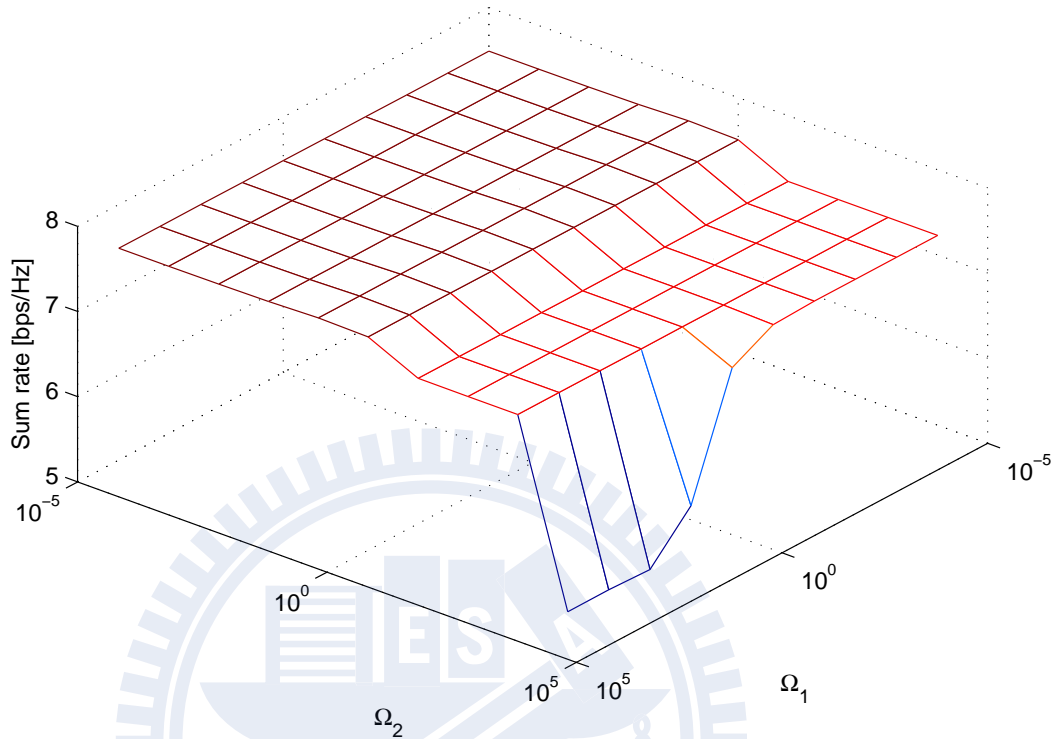


Figure 4.1: Sum-rate versus $\mathbf{\Omega}$ with three transmit antennas and two secondary users

Assumes that the number of transmit antennas is three, and there are two secondary users. As shown in Fig. 4.1, the secondary system sum rate increases significantly by decreasing $\mathbf{\Omega} = [\Omega_1, \Omega_2]$ when both Ω_1 and Ω_2 are large. However, the optimal value does not occurs at the boundary, which can be seen more clearly in Fig. 4.2 and Fig4.3. If $\mathbf{\Omega}$ decreases without restrictions, the sum rate can always be not improved. Note that if $\mathbf{\Omega}$ is set to be zero, the solution will become zero-forcing beamforming, which is usually not an optimal solution. To summarize, the system sum rate can be improved by decreasing the intra-user interference power constraint for those users in serving set $\Omega_k, k \in S$, but Ω_k can not be too small.

Therefore, an iterative algorithm can be developed to maximize the sum rate

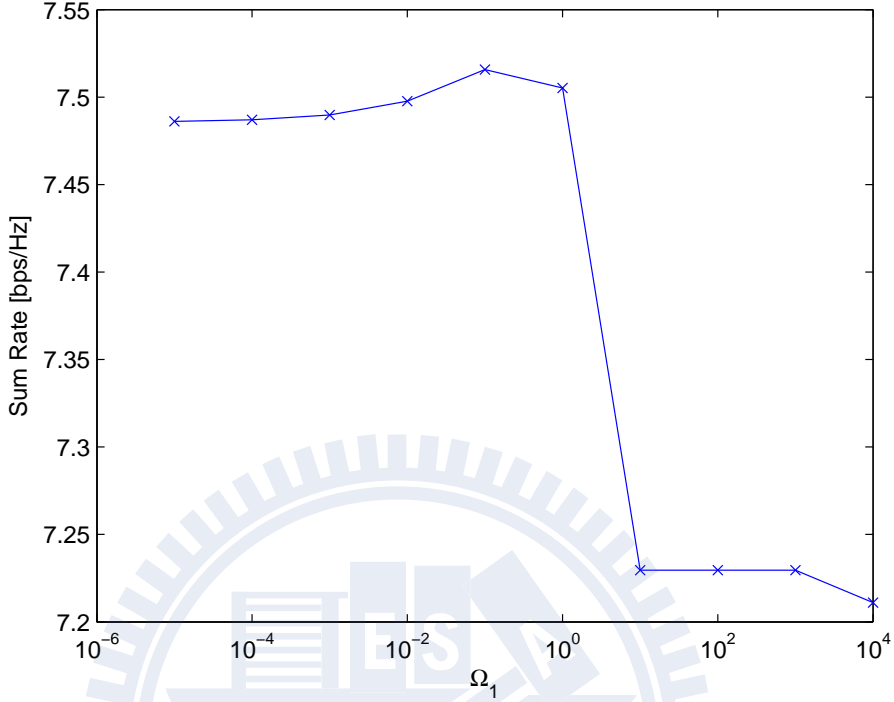


Figure 4.2: Sum-rate versus Ω with three transmit antennas and two secondary users, and fixed Ω_2 .

of the secondary system. The entire procedures of the iterative algorithm are shown in Fig. 4.4. First, we initialize $i = 1$ and $k = 1$, which corresponds iterative loop and user index, respectively. In order to not restrict the intra-user interference to those non-served users, the feasible set $\Omega^{(0)}$ is set to a sufficiently large non-negative real value. Then, we solve (4.10), and check whether initial $\Omega^{(0)}$ is feasible. Second, we choose a constant step size δ , where $0 < \delta < 1$, and update $\Omega^{(i-1)}$ to $\Omega^{(i)}$ by

$$\Omega^{(i)} = \Omega^{(i-1)} - (1 - \delta)\Omega_k^{(i)} \mathbf{e}_k . \quad (4.15)$$

where \mathbf{e}_k is the k^{th} column of a $K \times K$ identity matrix \mathbf{I}_K . Then, we solve (4.10) for new $\Omega^{(i)}$ and check whether the system sum rate improvement $R_1(\Omega^{(i)}) - R_1(\Omega^{(i-1)})$

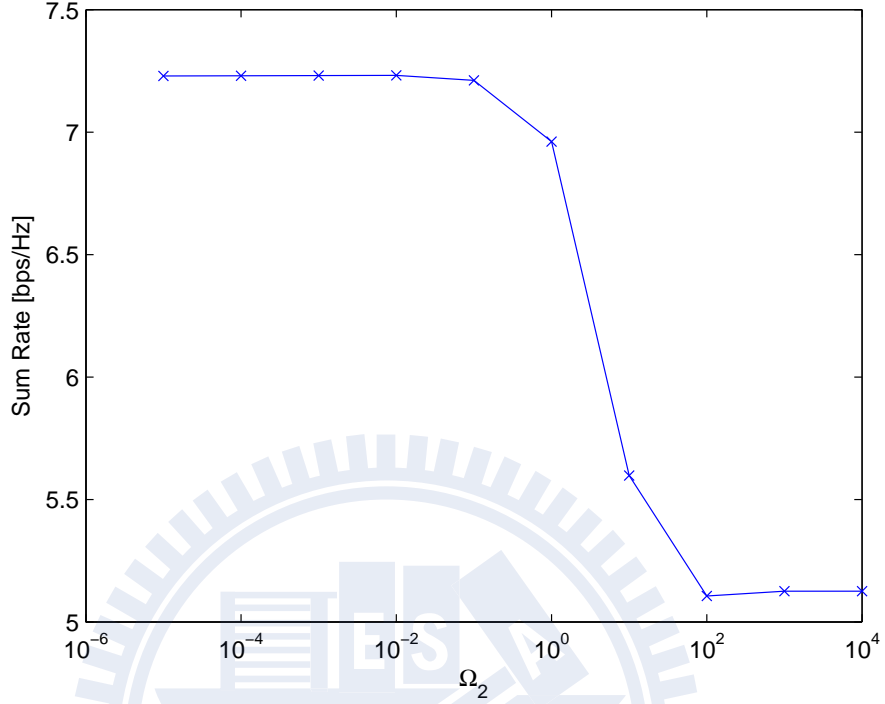


Figure 4.3: Sum-rate versus Ω with three transmit antennas and two secondary users, and fixed Ω_1 .

is larger than a pre-defined threshold $\Delta_r \geq 0$, which is a small constant value. If so, set $i = i + 1$ and continue to decrease Ω_k until the capacity improve insignificantly. Otherwise, set $\Omega^{(i)} = \Omega^{(i-1)}$, $R_1(\Omega^{(i)}) = R_1(\Omega^{(i-1)})$ and $\hat{\mathbf{W}}^{(i)} = \hat{\mathbf{W}}^{(i-1)}$, and decrease the intra-user interference to the next secondary user until $k = K$. Now, we have the optimal beamforming $\hat{\mathbf{W}} = [\hat{\mathbf{W}}_1, \dots, \hat{\mathbf{W}}_K]$, which includes rank one and zero matrix. The serving set \hat{S} includes the users with rank 1 solution, where the size of S is smaller than or equal to than the number of transmit antennas M . Although an iterative algorithm is utilized to find the optimal Ω , there is still an error compared with the optimal solution. The margin of this error depends on the step size δ and the number of secondary users. This error may increase as the number of secondary

users K increase. Therefore, the users, who are not in the serving set S , can be dropped and set $K = |S|$, where $|\cdot|$ denotes the size of a set. Again, we run the iterative algorithm for users in S . Note that it may still have a zero matrix solution for some users, which means the number of served users may be smaller than $|S|$. In the next chapter, numerical results with the single and multiple carrier transmissions are shown. The results of multicarrier transmissions are obtained by executing the iterative algorithm shown in Fig. 4.4 for each subcarrier.



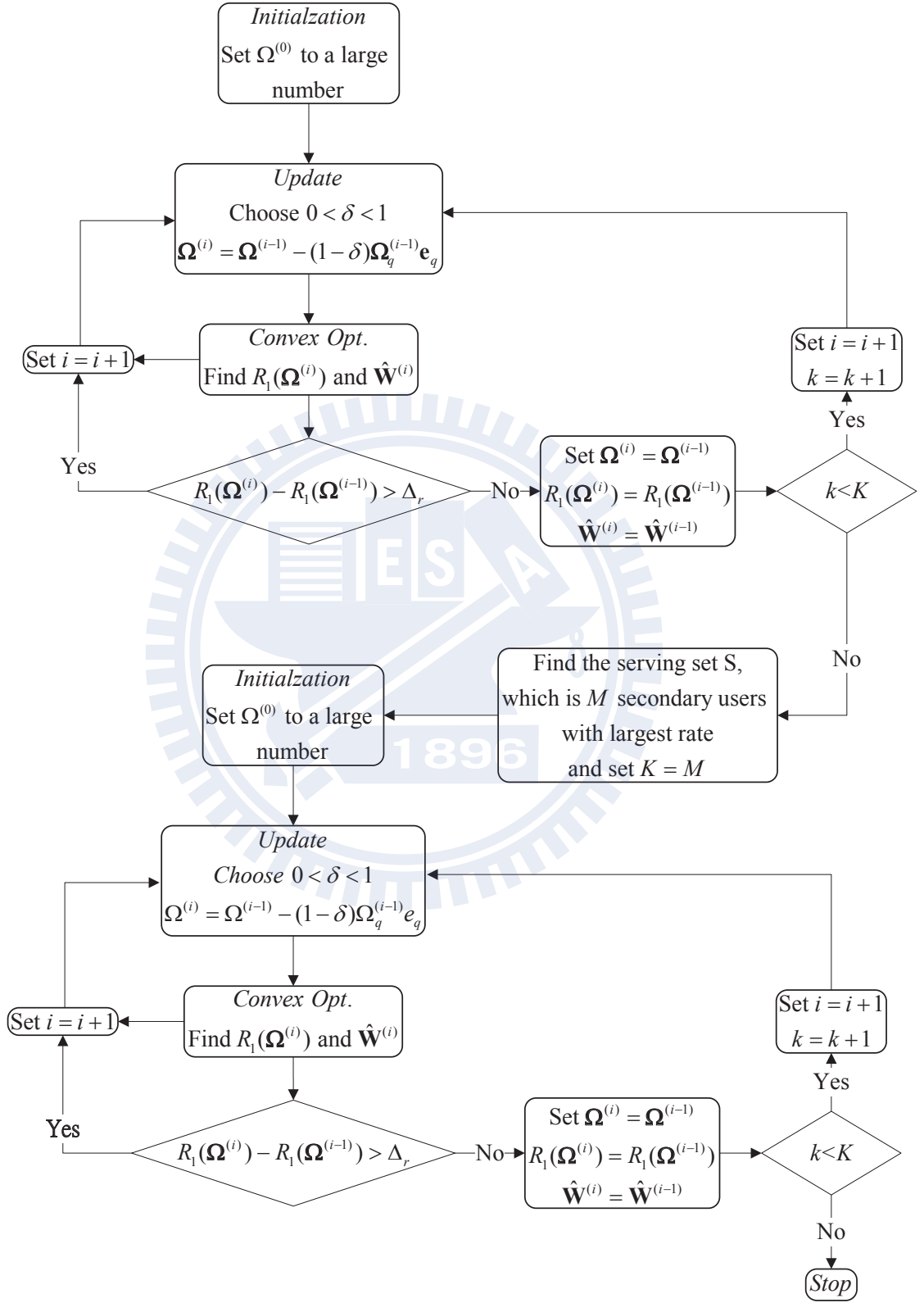


Figure 4.4: Procedures for the sum rate optimization algorithm.

CHAPTER 5

Numerical Results and Discussion

5.1 Single Carrier Case

5.1.1 Simulation Assumptions

In this subsection, we describe the simulation environments for evaluating the performance of the proposed iterative algorithm. The distribution of the primary and secondary systems are shown in Fig. 5.1. Assume that the primary and secondary BS are located at (0 m, 300 m) and (400 m, 0 m), respectively. The radius of the secondary system is 300 m. In a primary user with fixed location (0 m, 200 m), K secondary users are uniformly distributed at the secondary cell edge. The secondary BS equipped with M transmit antennas which are equal spacing with half a wavelength. Assume that the CSI is perfectly known at the secondary BS. The transmit power of the primary user and secondary BS are 20 dBm and 26 dBm, respectively. The noise power is -104 dBm. The pathloss exponent is four and the standard deviation of shadowing is 8 dB in both primary and secondary system. Simulation parameters are listed in Table 5.1.

There are two schemes for comparison: the first scheme is zero forcing beamforming between primary BS and $M-1$ secondary users with the exhausted scheduling and the optimal power allocation, which is regarded as a lower bound. Another scheme is singular value decomposition (SVD) to the secondary users, and does not control

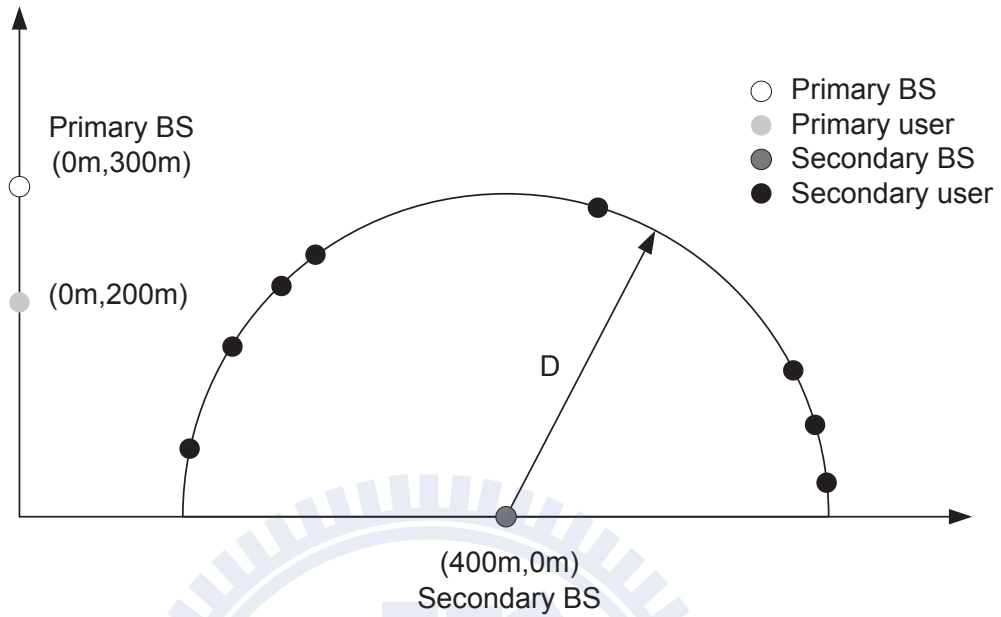


Figure 5.1: Distribution of primary and secondary systems.

the interference to the primary BS, but the secondary users are still interfered by the primary user. The SVD includes the transmitter and receiver beamforming, and is the optimal beamforming for the downlink MIMO system. The receiver beamforming in the hierarchical network architecture is completed by the cooperation between secondary users. However, secondary users do not cooperate with each others in the proposed algorithm. Thus, the SVD scheme can be regards as an upper bound.

5.1.2 Effects of Number of Transmit Antennas

Fig. 5.2 shows that the sum rate versus the number of the secondary BS transmit antennas for three beamforming and scheduling schemes. We can observe that the proposed scheduling scheme has significant improvement compared with the zero-forcing scheme. The gain comes from allowing some interference power to the primary

Table 5.1: **Simulation Parameters for Single Carrier Transmissions**

Position of primary BS	(0 m, 300 m)
Position of primary user	(0 m, 200 m)
Position of secondary BS	(400 m, 0 m)
Position of secondary users	Uniformly distribution at cell edge
Cell radius	300 m
Antenna spacing	equal spacing with $\frac{\lambda}{2}$
Transmit power of primary user	20 dBm
Transmit power of secondary BS	26 dBm
Noise power	-104 dBm
Pathloss exponent	4
Standard deviation of shadowing	8 dB
Channel type	Rayleigh fading

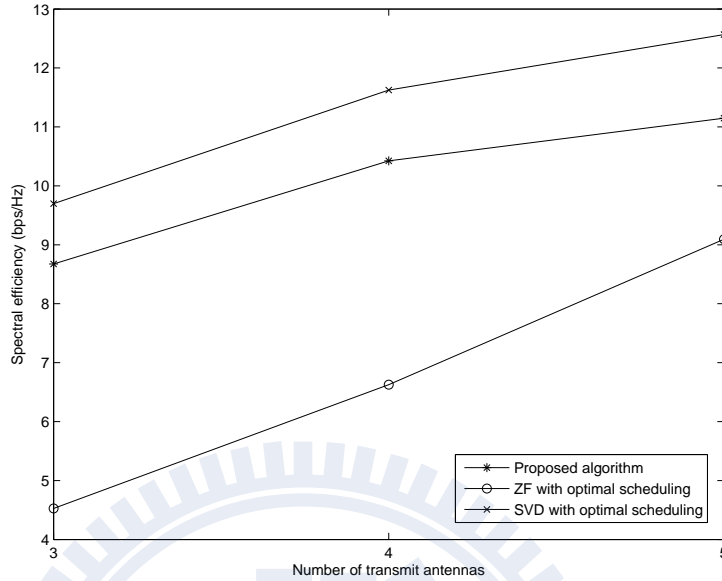


Figure 5.2: Sum rate for various numbers of transmit antennas, where the number of the secondary users is 10, and $I_{max} = 10$ dBm.

BS in our algorithm. However, interference power to the primary BS is zero. In addition, the proposed algorithm may serve M users in each transmission, but the zero-forcing scheme served $M - 1$ users fixedly. Shannon capacity formula is logarithm function, and the low SINR region can be regarded as the linear region. Thus, one more user can be served may lead to significant improvement.

5.1.3 Effects of Maximal Allowable Interference to Macro Cell

Fig. 5.3 shows that the sum rate versus the maximal allowable interference to primary BS I_{max} for the three considered beamforming and scheduling schemes. The SVD scheme does not consider the interference to primary BS, and the zero-forcing

always forces the interference to zero. Thus, the sum rate of these two schemes will not change as I_{max} changes. However, our algorithm increase significantly as I_{max} increases. It means that our proposed scheme is very flexible for different interference constraints I_{max} . As I_{max} increases, the sum rate of the proposed algorithm will gradually approach to the SVD scheme. Nevertheless, they will not have the same performance even if I_{max} goes infinity because of the received beamforming of SVD scheme. As I_{max} decreases, the sum rate of the proposed algorithm will gradually approach to the zero-forcing scheme. In fact, the proposed algorithm and the zero-forcing scheme will have the same performance as I_{max} approach to the negative infinity. However, the proposed algorithm can improve capacity significantly when I_{max} is large. Note that I_{max} will affect the coverage and transmit power of the secondary BS. Thus, the adaption of I_{max} is very important for the underlying CR system.

5.1.4 Effects of Number of Cognitive Radio Users

Fig. 5.4 shows the sum rate versus the number of the secondary users for three beamforming and scheduling schemes. The slope of the proposed algorithm is same as the SVD scheme, and larger than zero-forcing scheme. Thus, the user diversity of the proposed algorithm is same as the exhausted scheduling.

5.2 Multicarrier Case

5.2.1 Simulation Assumptions

In this subsection, we describe the simulation environment for evaluating the performance of the proposed iterative algorithm. The distribution of primary and secondary systems are shown in Fig. 5.5. Assumes that the primary and secondary BS are lo-

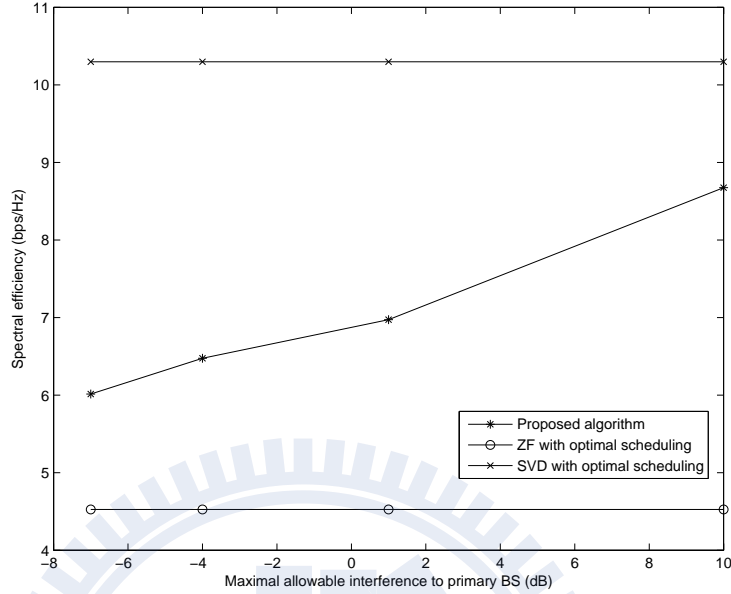


Figure 5.3: Sum rate for various numbers of maximal allowable interference to primary BS, where the number of transmit antennas is three, and the number of the secondary users is 10.

cated at (0 m, 300 m) and (400 m, 0 m), respectively. The radius of the secondary system is 300 m. There is a primary user with a fixed location (0 m, 200 m), and K secondary users are located fixedly at the secondary cell edge with equal separation. The secondary BS equipped with M transmit antennas which are equal spacing with half a wavelength. Assume that the CSI is perfectly known at secondary BS. The transmit power of the primary user is 20 dBm. The transmit power of secondary BS in each subcarrier is 20 dBm. The noise power is -110 dBm in each subcarrier. The pathloss exponent is four and the standard deviation of shadowing is 8 dB in both primary and secondary systems. The frequency-selective channel is i.i.d. Rayleigh fading in each sub-carrier. The summary of simulation parameters are shown in

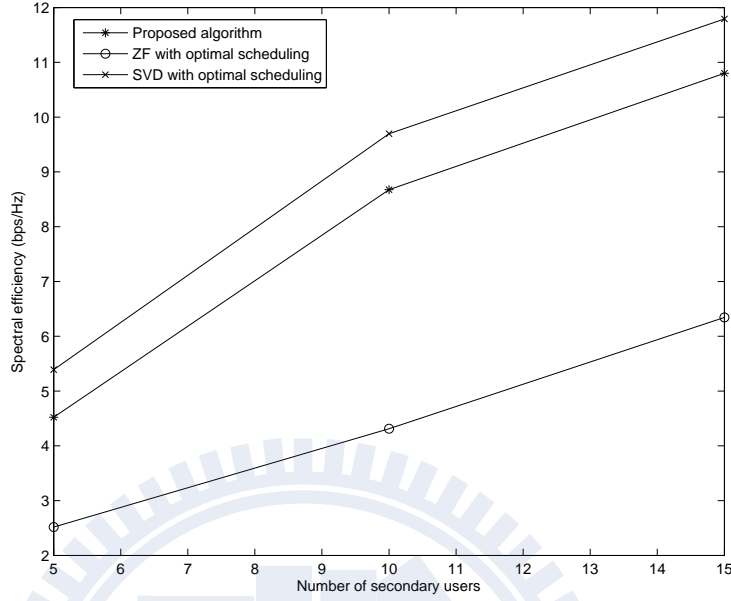


Figure 5.4: Sum rate for various numbers of secondary users, where the number of transmit antennas is three , and $I_{max} = 10$ dBm.

Table 5.2.

5.2.2 Fairness Compared with Single Carrier Transmissions

Fig. 5.6 shows the fairness index versus the number of subcarrier. Although the fairness is poor in single channel transmission, the fairness index improve significantly as the number of subcarriers increase. The fairness index is 0.93 for 8 subcarriers. Thus, the secondary BS can still serve users fairly on multicarrier transmissions even if the proposed algorithm is in order to maximize system sum rate.

Table 5.2: Simulation Parameters for Multicarrier Transmissions

Position of primary BS	(0 m, 300 m)
Position of primary user	(0 m, 200 m)
Position of secondary BS	(400 m, 0 m)
Position of secondary users	Fixed location at cell edge with equal separation.
Cell radius	300 m
Antenna spacing	equal spacing with $\frac{\lambda}{2}$
Transmit power of primary user	20 dBm
Transmit power of secondary BS	20 dBm in each subcarrier
Noise power	-110 dBm in each subcarrier.
Pathloss exponent	4
Standard deviation of shadowing	8 dB
Channel type	i.i.d. Rayleigh fading in each sub-carrier.

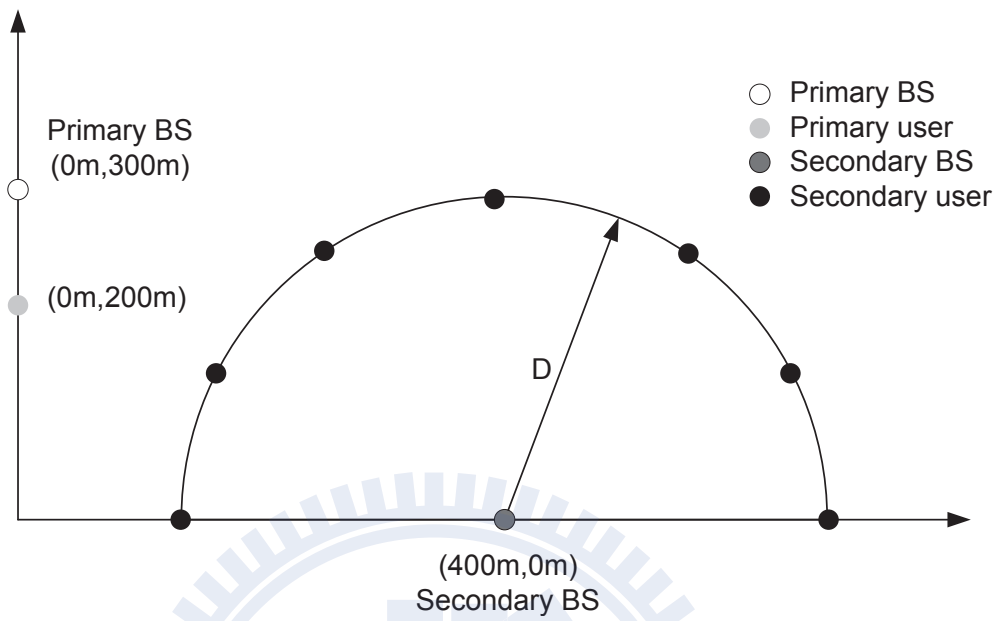


Figure 5.5: Distribution of primary and secondary systems.

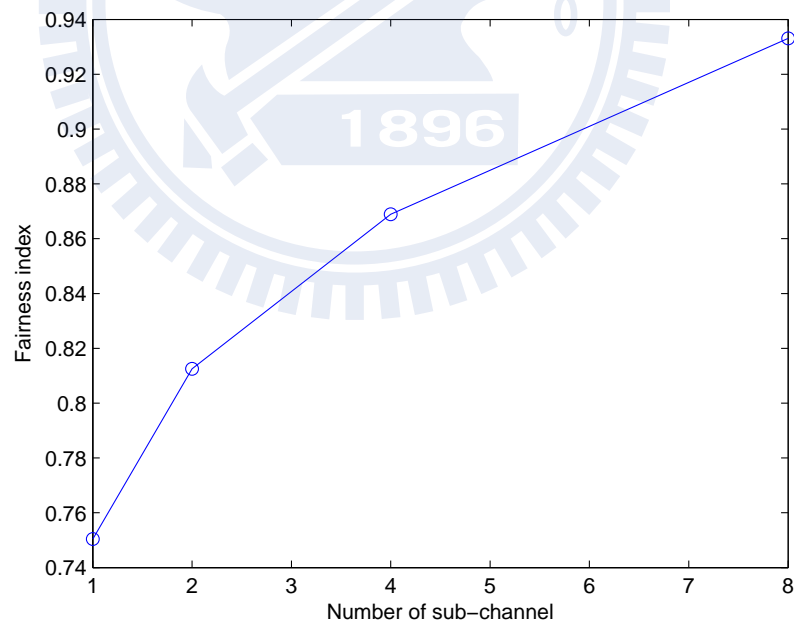


Figure 5.6: Fairness index for different number of transmit antennas.

CHAPTER 6

Conclusions

In this thesis, we developed a joint power allocation, transmitted beamforming and user scheduling design for the hierarchical cognitive radio networks in both frequency selective and frequency non-selective fading channel. SDR is applied to transfer the MINLP joint design problem to a convex problem. The numerical results show the performance for different numbers of antennas, secondary users and maximal allowable interference power to the primary system. The improvement of the sum rate saturates when a large number of transmit antennas, even the scheduling is considered jointly. The proposed algorithm is very flexible for different interference power constraints. The sum rate of the secondary system improve significantly as the interference power constraint becomes less strict, which is contributed by joint beamforming and scheduling design. The user diversity of the proposed algorithm is the same as the exhausted scheduling. Therefore, the scheduling of the proposed algorithm is approaching to the optimal value.

Our proposed algorithm can also be applied to the multicarrier transmissions. The proposed algorithm is designed for the sum rate maximization. Although, the fairness of the proposed is not the main design goal, users can be served quite fairly as the proposed algorithm is applied to the multicarrier transmissions.

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