

國立交通大學

電信工程研究所

碩士論文

多蜂巢多點協同下行傳輸網路下基於訊號
干擾雜音比限制之強韌性自動重傳放大後
中繼傳送設計

Robust CoMP AF Relaying for SINR
Constrained ARQ in Downlink Multi-Cell
Networks

研究生：郭俊義

指導教授：伍紹勳

中華民國 100 年 7 月 15 日

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Student : Chun-I Kuo

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國立交通大學
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摘 要

中繼器輔助下之多點協同波束成形技術被研究用來提供在下行多細胞網路中自動重傳以支持服務質量。考慮到多點協同傳輸之設計限制和可行性，兩種類型的強韌性多點協同放大後中繼傳送波束成形技術被提出用來在通道資訊不完美下維持網路中每位使用者之訊號干擾雜音比。模擬結果顯示被提出的低複雜度演算法可以提供接近最佳的效能，並且透過多點協同中繼傳送技術，大大提高靠近細胞邊界之使用者的下行吞吐量。此外，被提出的強韌性多點協同波束成形技術不僅可以減低系統傳送所需的率，即使用低複雜度的放大後中繼傳送技術也仍然能增加自動重傳支持服務質量的可能性。

Robust CoMP AF Relaying for SINR Constrained ARQ in Downlink Multi-Cell Networks

Student : Chun-I Kuo

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ABSTRACT

Relay-assisted coordinated multi-point (CoMP) beamforming (BF) methods are studied to support quality of service (QoS) in Automatic Retransmission reQuest (ARQ) for downlink multi-cell networks. Considering the design constraints and the feasibility of CoMP transmissions, two types of robust CoMP amplify-and-forward (AF) BF methods are proposed to maintain the signal to interference-plus-noise ratio for each subscriber station (SS) in the network subject to imperfect knowledge on the channel state information. Simulation results show that the proposed low-complexity algorithm can provide near optimal performance and the downlink throughput for SSs close to cell boundaries can be drastically improved via CoMP relaying. Besides, the proposed robust CoMP BF methods not only can reduce the system power consumption, but also can increase the feasibility to support QoS in ARQs even with the low-complexity AF relaying.

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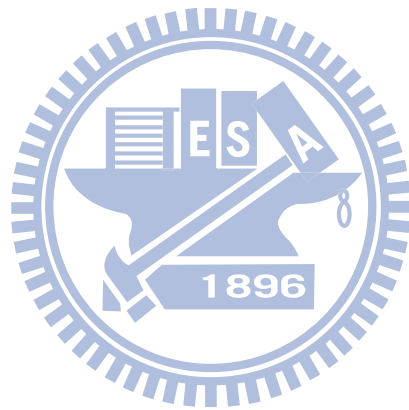
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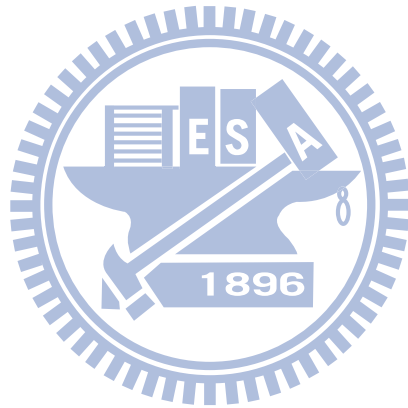
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Abstract

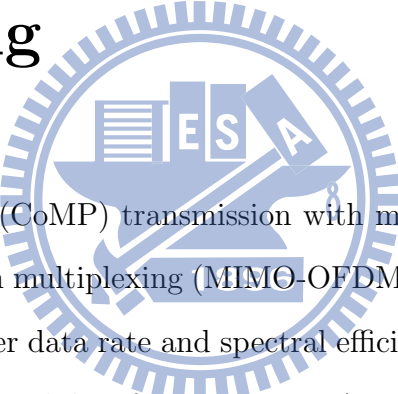
Relay-assisted coordinated multi-point (CoMP) beamforming (BF) methods are studied to support quality of service (QoS) in Automatic Retransmission reQuest (ARQ) for downlink multi-cell networks. Considering the design constraints and the feasibility of CoMP transmissions, two types of robust CoMP amplify-and-forward (AF) BF methods are proposed to maintain the signal to interference-plus-noise ratio for each subscriber station (SS) in the network subject to imperfect knowledge on the channel state information. Simulation results show that the proposed low-complexity algorithm can provide near optimal performance and the downlink throughput for SSs close to cell boundaries can be drastically improved via CoMP relaying. Besides, the proposed robust CoMP BF methods not only can reduce the system power consumption, but also can increase the feasibility to support QoS in ARQs even with the low-complexity AF relaying.

Chapter 1

Introduction and System

Architecture for CoMP AF

Beamforming



Coordinated multipoint (CoMP) transmission with multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) is one of the promising concepts to improve cell edge user data rate and spectral efficiency in Long Term Evolution (LTE) or Worldwide Interoperability for Microwave Access (WiMAX). In addition to CoMP transmissions, relaying has also been proposed to International Mobile Telecommunication (IMT)-Advanced to improve cell coverage and signal-to-interference plus noise ratio (SINR) of cell edge subscriber stations (SSs). Integrating the concept of CoMP transmissions and coordinated relaying, this work focuses on robust beamforming designs which concern about maintaining the quality of service (QoS) of each SS under the influences of channel quantization errors in a multi-cell wireless environments.

In general, coordinated BSs transmission can be categorized as inter-site CoMP and intra-site CoMP. Inter-site CoMP performs the cooperation among BSs in different cells, while intra-site CoMP only performs the cooperation among the sectors of one BS. Al-

though, compared to intra-site CoMP, inter-site CoMP would be much harder to realize due to existing challenges such as clustering, scheduling, synchronization, backhaul overhead, channel state information (CSI) feedback load and CSI estimation accuracy [1], the design of inter-site CoMP still attracts many research attention as it provides promising gains in system throughput [1]. To fully exploit the potential of inter-site CoMP processing, the coordinated BSs should jointly transmit to multiple SSs, where each SS is simultaneously served by coordinated BSs and therefore both data and channel state information (CSI) are exchanged between the BSs [1]. In this case the coordinated system actually becomes a single-cell multi-antenna system which is known as a virtual MIMO network, and existing downlink beamforming approach has been proposed in [2]. The main issue with [2] is that this scheme requires data sharing between the BSs, which is forbidden when limited backhaul is allowed. Motivated by this, [3] looked into coordinated beamforming (BF) schemes without data sharing among the BSs.

In spite of the potential of inter-site CoMP processing, the tradeoffs between the cost to provide CSI at the transmitter (CSIT), estimation error, and system resource allocated to training appears that the number of antennas that can be jointly coordinated (either on the same BS or across multiple BSs) is essentially limited [4]. In addition, to avoid executing the complex BF design for multiple BSs joint processing, limited MIMO network coordination is more practical and still yield a significant throughput gain, e.g., throughput increases to a factor of 2.5 by coordinating a cluster of seven cells [5]. For these purposes, several cells form a cluster, and a cluster control unit (CU) exists to collect CSIs, perform beamforming design and coordinate coherent transmissions in this system. Specifically, an adaptive clustering algorithm is required. Field trial results show that in spite of various aforementioned implementation constraints downlink CoMP transmission still be realized in real-world scenarios and that significant gain can still be achieved by forming small coordinated clusters in large-scale networks [1]. An extension and analysis of CoMP architecture in large multi-cell networks is also proposed and

investigated in [6].

Due to the essential constraints on CSI feedback, synchronization and the number of antennas that can be jointly coordinated, the CoMP architecture is often discussed for Orthogonal Frequency Division Multiple Access (OFDMA) system. A scheduling algorithm is required to jointly allocate the subcarriers for SSs in different cells of a CoMP system [7]. In LTE-Advanced context, subcarriers are allocated in blocks of adjacent subcarriers, which represent the Physical Resource Blocks (PRBs). Channel coherence bandwidth is assumed larger than the bandwidth of a PRB leading to flat fading over each PRB. Several PRBs exist in a coordinated system and each of them might be allocated to one or more SSs. This kind of resource allocation problem of spatially reusing PRBs within a CoMP region is investigated [7].

To perform coordinated BF in a CoMP system, SSs in a cluster need to accurately feed back their CSI to the CU; the CU exchange CSIs between BSs by backhaul. To yield a good tradeoff between the performance and overhead for CoMP systems, an efficient feedback method is proposed in [8], the results show that the reduction of the feedback load can be efficiently exploited to reduce the backhaul load through their proposed scheduling or BF scheme. In CoMP systems, synchronization is also a critical issue. Large distances between the base stations result in inevitable differences in time of arrival between the SSs' signals. This phenomenon further brings about inter-symbol interference in OFDM systems if the cyclic prefix length is exceeded. Moreover, inter-carrier interference happens because of carrier frequency offsets caused by imperfect oscillators. However, the gain of using CoMP is still demonstrated to be larger than the decline results from synchronization problems according to metrics that can be applied for an approximation of the impact of asynchronization in CoMP systems [9].

Despite the sophisticated MIMO techniques that might be used to suppress the multiple access interferences (MAI) in coordinated BSs transmission, SINR at cell boundaries is typically low arising from intrinsically weak received signal coming from the associated

BS, which make it difficult to maintain the quality of service (QoS) in data transmissions. To overcome the difficulties, yet to improve the throughput for SSs close to cell boundaries, RSs are deployed for automatic-repeat-request (ARQ) for SSs because of stronger average channel gain of RS-SS links. Furthermore, to achieve high spectral efficiency as coordinated BS systems, the cooperation among RSs in different cells are also considered. The coordinated RSs jointly retransmit to multiple SSs, where each SS is simultaneously served by coordinated RSs.

Generally speaking, in ARQ, the source retransmits if the destination fails to decode data and feeds back negative-acknowledgment. An evolutionary and useful ARQ scheme for cooperative relaying using a single RS is proposed in [10]. First, the BS transmits their data to the SS. At the same time, the relay overhears. Then, SSs broadcast a single bit to indicate successful direct transmission or not. If negative-acknowledgment is broadcasted, the relay retransmits. Otherwise, the relay just do nothing and the next transmission round begins. Another existing ARQ protocol is known as two-hop relaying. The BS first transmits their data to the RS and then the RS forwards the received signal to SS. At the end of transmission round, SSs decode only the signal received from the RS [11]. As for the types of relaying methods, two typical approaches are : decode and-forward (DF), which forwards only if successful decoding of data, and amplify-and-forward (AF), which simply amplifies the received signal without any decoding [10]. More detailed discussion about the architecture, feasibility and effectiveness of relaying techniques in LTE-advanced are presented in [12–14].

We have reviewed two coordinated BF schemes. One transforms interference from the other cells into useful signal by exchanging CSIs and data between transmitting nodes. The other one focuses on interference cancelation only by having global CSIs at the CU. Nonetheless, to fully exploit the potential of coordinated transmission, the CU requires to have the global CSI knowledge, which incurr overhead for channel estimation and feedback. Considering the difficulties for acquiring global CSI, coordinated beamforming

design considering partial CSIs at the CU is widely discussed. In the process of acquiring the CSI, the channels are first estimated by the SSs and then are quantized and fed back to the CU. Hence, the available CSIs at the CU are subject to uncertainties that mainly arise from estimation error and channel quantization. With a good estimator and small amount of bits reserved for quantized CSI feedback in the next-generation cellular wireless networks, quantization error is very likely a dominant source of error about the CSIs. To overcome this, we also consider a bounded uncertainty model in the CSIs at the CU. This model is a useful one for systems with low-rate feedback channel [15]. For this bounded channel uncertainty model, we design a BF that minimizes the transmitted power required to ensure that each SS's SINR requirement is satisfied for all channels within the specified uncertainty region. In this kind of worst-case analysis, which we discuss in this paper, the uncertainty possesses no statistical properties. Similar worst-case analysis considering bounded error model can also be founded in [16–19]. On the other hand, stochastic robustness analysis, in which the error model is assumed possessing statistical properties has also been investigated in [20, 21]. In contrast, a distributed beamforming design is carried out to avoid the overhead from another point of view based on a non-cooperative game theory proposed [22]. The main idea of non-cooperative game theory is that each SS may try to compete for the scarce resource and optimize its own performance based on its local CSIs.

Because of the limitation of the number of antennas that can be jointly coordinated, we assume each PRB of can only be assigned to one SS in a cell and served by its associated BS and RS via this PRB. With this assumption, we discuss our coordinated downlink beamforming design under a cooperation group composed of several cells at each of which a single BS and a amplify-and-forward (AF) RS are located. A two-cell cooperation group is illustrated in Fig. 1.1 and Fig. 1.2.

As for RSs retransmission, in spite of the stronger channel gain of RS-SS links resulting in each SS' strong desired received signal from their associated RSs, a critical issue is

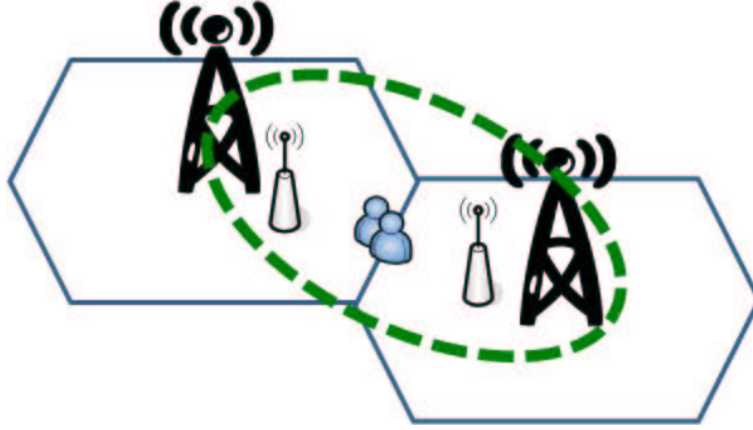


Figure 1.1: Topology of a two-cell cooperation group.

that simultaneous retransmission from different relays on the same sub-channel leads to cross-cell interference to SSs in the other cells. Fortunately, similar problems also exist in coordinated BS beamforming designs and two well-known coordinated BS beamforming schemes on the interference mitigation of BSs-SSs links have been surveyed in the previous introduction [2,3]. With the help of similar techniques, high-quality coordinated RSs retransmission is promising. However, different from BSs, resource allocated for exchanging information between RSs is commonly assumed to be extremely limited and hence virtual MIMO network is almost impossible in real-world. The other coordinated BF technique that only requires CSIs at the CU is more practical for coordinated RS BF. Even though exchanging data between BSs is more realistic, the same coordinated BF technique is preferable for coordinated BS BF as usual. Furthermore, to avoid QoS decline of each SS resulting from constrained cooperation between RSs, we discuss a scheme in which low-rate quantization for channel feedback is asked and no direct CSIs exchange between relays is allowed. We also investigate worst-case robust beamforming design which concerns about maintaining the QoS of each SS in the existence of channel quantization error as multi coordinated cells transmission and relaying are integrated. More detailed explanation and justification about the process of acquiring CSIs of the CU in this system is discussed in the next section.

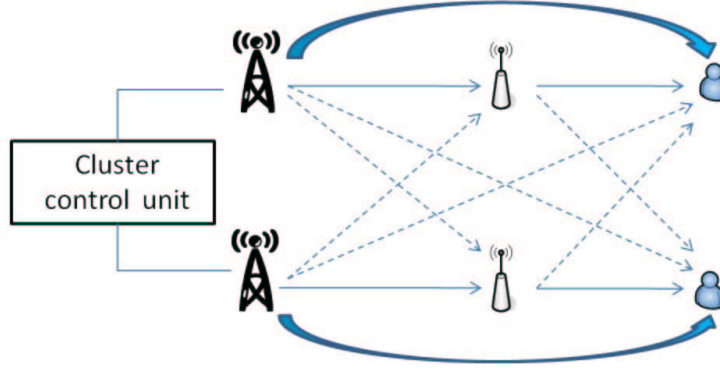


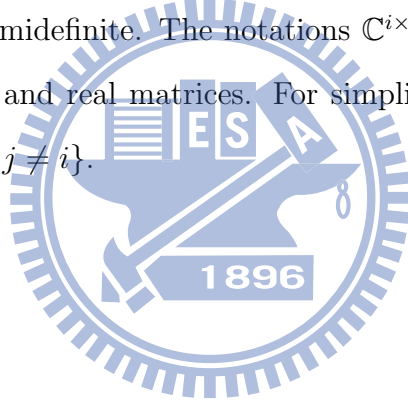
Figure 1.2: Another representation of a two-cell cooperation group. Solid lines and whereas dashed lines indicate useful transmission and interference respectively.

A similar integration of multi-cell coordinated transmission and relaying are also investigated [23]. However their interference mitigation schemes are quite different from that of our work. Following their illustrations, BSs-RSs links can be treated as broadcast channel or interference channel depending on the existence or absence of a backhaul connection between BSs. On the other hand, RSs-SSs links are regarded as an interference channel because of the absence of backhaul connection between RSs. The authors show that interference of broadcast channel and interference channel can be mitigated by using Dirty-Paper Coding [24] and Han-Kobayashi Coding [25] respectively.

Despite the rich results about the worst-case analysis of amplify-and-forward relaying for bounded error model [26], [27], robust beamforming based on integration of coordinated BS transmission and RS retransmission has not been studied. We first formulate the robust BF designs considering herein as a Semidefinite Programming (SDP) based on the S-Procedure [28] and semi-definite relaxation (SDR) [29]. We then develop necessary conditions for the optimal beamforming for the robust BF problem and develop a suboptimal and nearly closed-form expression of the BFER based on a two-tier iterated QCQP. The SDR-based beamforming method is shown by simulations to achieve the near optimal performance. And the performance of the low-complexity suboptimal scheme based on the iterated QCQP is shown to be very close to that with the SDR

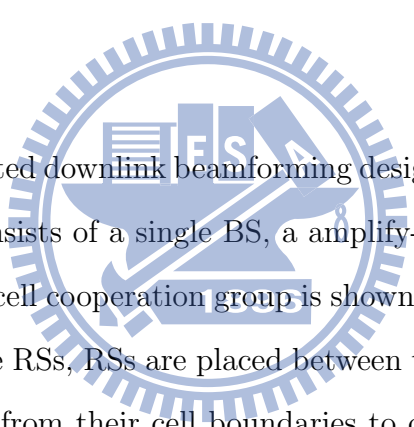
approach, and is more friendly for practical implementations. Simulation results also show that the downlink throughput for SSs close to cell boundaries of the multi-cell system can be drastically improved via CoMP AF relaying. The proposed robust CoMP AF BF methods is a promising scheme to compromise the two factors of performance, the system power consumption and the feasibility to support QoS in ARQs.

Notations: We follow the conventions to use lowercase letters, e.g., a , to represent scalars, boldfaced lowercase letters, e.g., \mathbf{a} , to represent vectors and uppercase letters, e.g., \mathbf{A} , to represent matrices. $\text{vec}(\cdot)$ stands for concatenating the columns of the matrix argument into a vector. $[\mathbf{a}]_i$ stands for i -th entry of vector \mathbf{a} . $[\mathbf{A}]_{i,j}$ stands for the (i,j) entry of matrix \mathbf{A} . $\|\cdot\|$ represents the vector Euclidean norm. The superscripts 'T', 'H' and '-1' represent transpose, Hermitian transpose, and inverse of matrices. $\mathbf{A} \succeq 0$ means that matrix \mathbf{A} is positive semidefinite. The notations $\mathbb{C}^{i \times j}$ and $\mathbb{R}^{i \times j}$ represent the sets of $i \times j$ -dimensional complex and real matrices. For simplicity, $\forall i \triangleq \{i = 1 \dots N\}$ and $\forall j \neq i \triangleq \{i, j = 1 \dots N \text{ but } j \neq i\}$.



Chapter 2

Transmission Protocol and Problem Setting for CoMP AF Beamforming



We discuss our coordinated downlink beamforming design under a cooperation group of N cells, each of which consists of a single BS, an amplify-and-forward (AF) RS and a SS. The topology of a three-cell cooperation group is shown in Fig. 2.1. To maintain the signal quality received at the RSs, RSs are placed between the BSs and the SSs, and are still in a fair distance away from their cell boundaries to overhear and forward signals to SSs near the cell boundaries. Consequently, the interferences to a RS from the BSs of its adjacent cells are very small. Next, we carefully show transmission mechanism, challenges and our solutions in this system.

The transmission scheme of a cooperation group is introduced below based on a two-cell cooperation group example as shown in Fig. 2.2, and this example can be generalized to N -cell cases. In the first transmission round of a cooperation group, the scheduler would first choose two SSs in different cells and then coordinate BSs in this group to transmit data to these SSs. At the end of transmission round, SSs would decode the packet and detect the error. If both the SSs detect errors, scheduler would coordinate

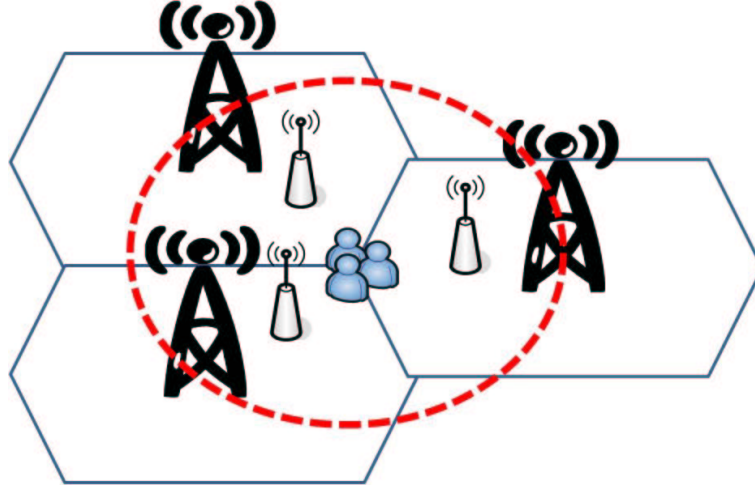


Figure 2.1: Topology of a three-cell cooperation group.

RSs to retransmit data to these SSs in the next transmission round. If both the SSs do not detect any error, scheduler would coordinate BSs to continue transmitting data to these SSs. However, if only one SS detect errors, scheduler would choose a new partner for them respectively to form a new direct transmission group and retransmission group.

To design CoMP BF, the CU requires having knowledge of global CSIs and informs transmitting nodes of their adequate beamforming matrices or vectors. Existing works have been carefully investigated the feasibility of coordinated BSs transmission [1]. However, there are few literatures discussing the design of coordinated transmission among RSs in different cells. Here, we propose a scheme for it. By retransmission of RSs, data is actually passed through BS-RS and RS-SS links. Hence, CSIs of those links are required for BF design. To acquire these CSIs, training signals are transmitted, estimated CSIs are fed back by RSs or SSs to their associated BSs and then RSs are informed of their own beamforming matrices by their own associated BSs. The flow chart is shown in Fig. 2.3.

To yield optimal performance, the CU and RSs require complete knowledge of CSIs and of their own BF matrices respectively. In this work, we assume RSs are able to have complete knowledge of their own BF matrices. However, due to the amount of bits

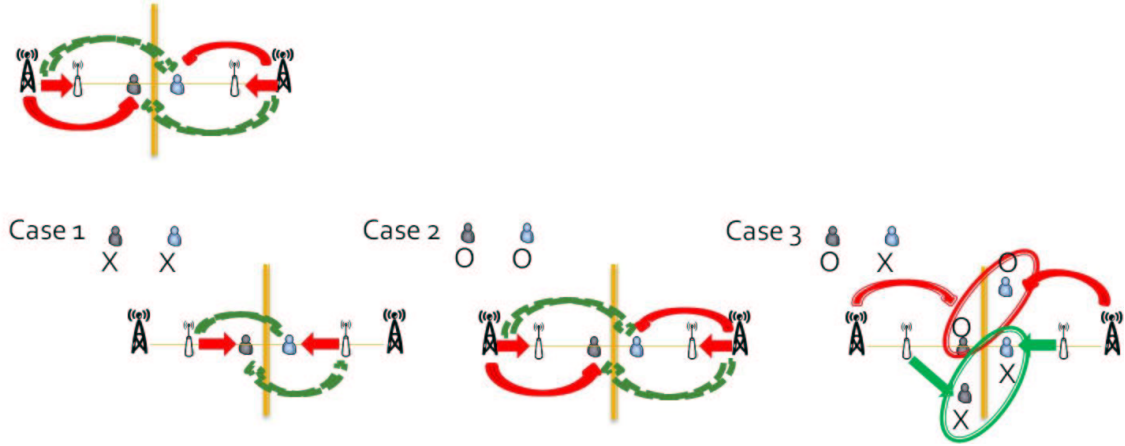


Figure 2.2: Three possible cases next to coordinated BSs transmission in a two-cell cooperation group. Solid lines and whereas dashed lines indicate useful transmission and interference respectively.

reserved for quantized CSI feedback is small in the standards for the next-generation wireless cellular networks, full CSI at the CU is impractical and hence we consider a uncertainty region which is useful and common to model quantization error. And we also design BF based on criterion of minimizes the transmitted power required to ensure that each user's QoS requirement is satisfied for all channels within this specified region. Besides, those trivial and weak interference links (i.e., links between BSs and their non-associated RSs) are neglected in our analysis. Similar relaxation is also common for CoMP system to yield a good tradeoff between performance and CSIs feedback overhead [8]. In addition, by neglecting those trivial interferences, optimal BF matrices for each RS are shown to possess a specified structure, which results in reducing computation effort of BF design at the CU and decreasing resource reserved for informing RSs of their own BF.

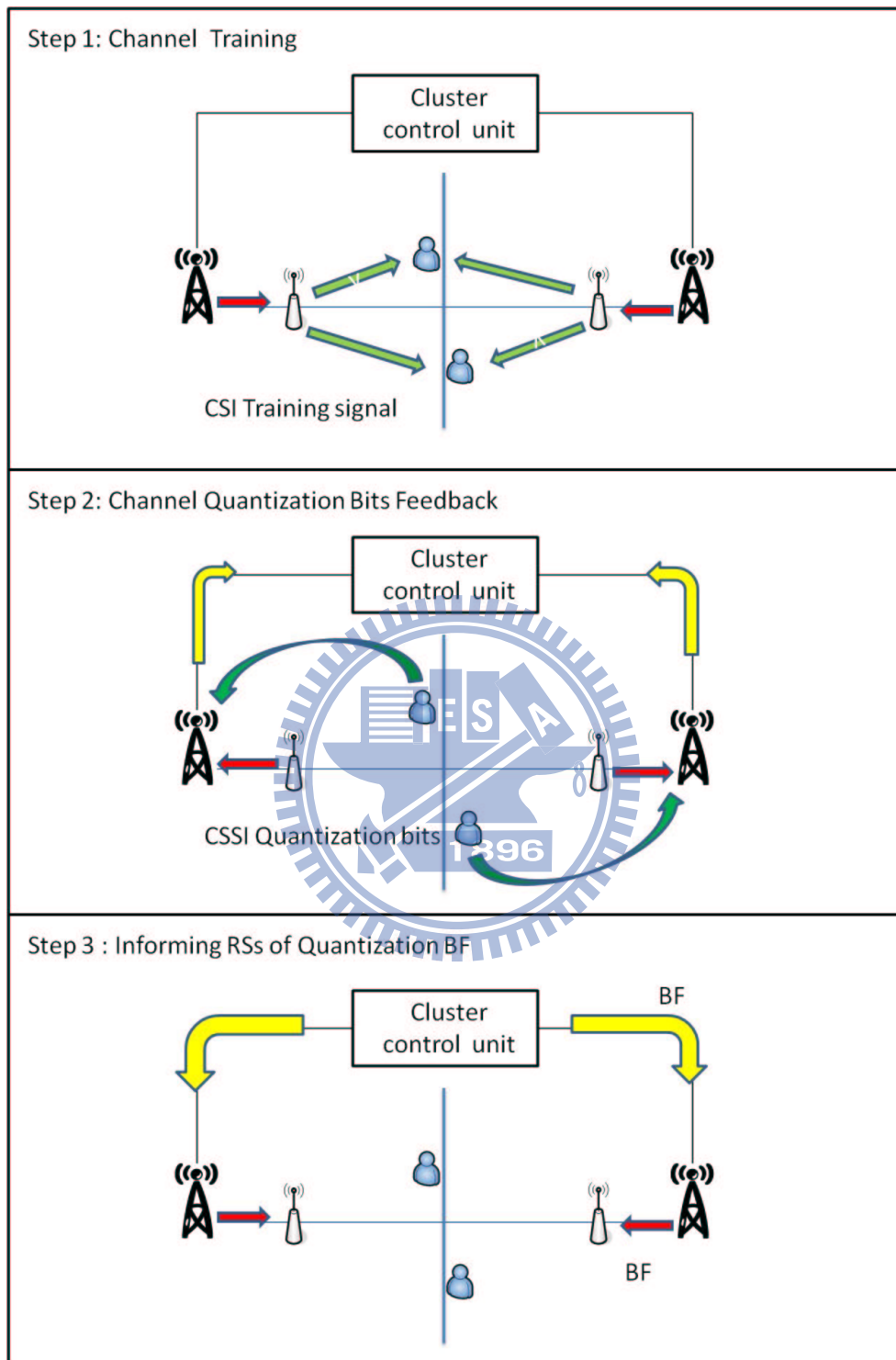


Figure 2.3: Flow chart of designing beamforming at cluster control unit for every relay station in a cooperation group.

Chapter 3

System Model for CoMP AF

Beamforming

Under the system architecture introduced previously, RSs of a cell receive signals only from their associated BS. In contrast, a SS_{*i*} close to the edge of its cell C_{*i*}, will receive its desired signal from BS_{*i*} or its associated RS_{*i*}, and meanwhile pick up interferences from BS_{*j*} or RS_{*j*}, $\forall j \neq i$. To suppress the interferences efficiently in coordinated transmissions without data sharing, each BS or RS must be at least equipped with N antennas (We assume just N antennas in this paper), while each SS is assumed having one antenna only. Besides, we assume perfect channel state information at the receiver (CSIR), while partial CSI at the CU caused by quantization error is considered. Also, we assume RSs have complete knowledge of their own BF matrices.

Here, we just illustrate system models for coordinated BSs transmission and coordinated RSs retransmission. Signals sent from BSs and received by SS_{*i*} can be modeled as

$$y_{s_i} = \sum_{j=1}^N \mathbf{h}_{b_j, s_i}^H \mathbf{w}_{b_j} x_j + n_{s_i} \quad (3.1)$$

where x_j stands for the data symbols of unit power sent for SS_{*j*} from BS_{*j*}, and $\mathbf{w}_{b_j} \in \mathbb{C}^{N \times 1}$ is the BF vectors for BS_{*j*}. The channel coefficients from the N transmit antennas

of BS_{*j*} to the antenna of SS_{*i*} are modeled as flat Rayleigh faded and are denoted by $\mathbf{h}_{b_j, s_i} \in \mathbb{C}^{N \times 1}$. Besides, the noise $n_{s_i} \in \mathbb{C}$ is zero-mean complex Gaussian distributed with its variance equal to σ^2 , denoted by $n_{s_i} \sim \mathcal{CN}(0, \sigma^2)$. The total average transmit power of the BSs is $P_b = \sum_{i=1}^N \|\mathbf{w}_{b_i}\|^2$. Given σ^2 , \mathbf{h}_{b_j, s_i} and $\mathbf{w}_{b_j}, \forall i, j$, the SINR observed at SS_{*i*} for signals sent from BSs can be expressed as

$$\text{SINR}_i^{(b)} = \frac{|\mathbf{h}_{b_i, s_i}^H \mathbf{w}_{b_i}|^2}{\sum_{j \neq i} |\mathbf{h}_{b_j, s_i}^H \mathbf{w}_{b_j}|^2 + \sigma^2} \quad (3.2)$$

On the other hand, the signal received at the relay RS_{*i*} is modeled as

$$\mathbf{y}_{r_i} = \mathbf{h}_{b_i, r_i} x_i + \mathbf{n}_{r_i} \quad (3.3)$$

where $\mathbf{h}_{b_i, r_i} = \mathbf{H}_{b_i, r_i} \mathbf{w}_{b_i}$.

Since the interferences from BS_{*j*}, $\forall j \neq i$, of adjacent cells are negligible in our system setting. The noise vector $\mathbf{n}_{r_i} \in \mathbb{C}^{N \times 1}$ is $\sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$, and the channel coefficient between each antenna pair from BS_{*i*} to RS_{*i*} is also modeled as flat Rayleigh faded. Thus, the channel coefficients can be expressed in a matrix form of $\mathbf{H}_{b_i, r_i} \in \mathbb{C}^{N \times N}$. However, $\mathbf{h}_{b_i, r_i} \in \mathbb{C}^{N \times 1}$ can be regarded as equivalent channel coefficients between BS_{*i*} and RS_{*i*}.

Consider simple ARQs with coordinated AF BF in our system architecture. The signals received at SS_{*i*} from RS_{*j*}, $\forall j$, in ARQ phases can be expressed as

$$\mathbf{y}_{r_i, s_i} = \mathbf{h}_{r_i, s_i}^H \mathbf{W}_{r_i} \mathbf{y}_{r_i} + \sum_{j \neq i} \mathbf{h}_{r_j, s_i}^H \mathbf{W}_{r_j} \mathbf{y}_{r_j} + n_{s_i} \quad (3.4)$$

where $\mathbf{W}_{r_i} \in \mathbb{C}^{N \times N}$ is the BF matrix employed by RS_{*i*} in C_{*i*} and $\mathbf{h}_{r_j, s_i} \in \mathbb{C}^{N \times 1}$ stands for the channel vector from RS_{*j*} to SS_{*i*}. Again all coefficients in \mathbf{h}_{r_j, s_i} are still modeled as flat Rayleigh faded. Given σ^2 and $\mathbf{h}_{b_i, r_i}, \forall i$, the total average transmit power of the

relays is defined as

$$P_r \triangleq \sum_{i=1}^N [\|\mathbf{W}_{r_i} \mathbf{h}_{b_i, r_i}\|^2 + \sigma^2 \text{trace}(\mathbf{W}_{r_i}^H \mathbf{W}_{r_i})] \quad (3.5)$$

The SINR observed at SS_i for signals sent from RS_i , $\forall i$, can thus be expressed as

$$\text{SINR}_i^{(r)} = \frac{|\mathbf{h}_{r_i, s_i}^H \mathbf{W}_{r_i} \mathbf{h}_{b_i, r_i}|^2}{\sum_{j \neq i} |\mathbf{h}_{r_j, s_i}^H \mathbf{W}_{r_j} \mathbf{h}_{b_j, r_j}|^2 + (\sum_j \|\mathbf{h}_{r_j, s_i}^H \mathbf{W}_{r_j}\|^2 + 1)\sigma^2} \quad (3.6)$$

According to the aforementioned system setting and signal model, we study in the sequel coordinated AF BF methods for CoMP ARQ in order to maintain the SINR for SS_i , $\forall i$, in the sub-network.

As mentioned previously, to avoid heavy overhead caused by data sharing, we note that the data x_i for SS_i are still sent through their associated BS_i or RS_i inside the sub-network only. Given σ and \mathbf{h}_{b_j, s_i} , $\forall i, j$, the BF, \mathbf{w}_{b_i} , can be jointly designed to meet a target SINR, γ_0 , for each SS_i , which is known as SINR constrained power minimization problem of base stations (SPMP-BS).

SPMP-BS

$$\min_{\{\mathbf{w}_{b_i}\}_{i=1}^N} P_b \quad \text{s.t.} \quad \text{SINR}_i^{(b)} \geq \gamma_0, \forall i$$

Furthermore, given σ^2 , \mathbf{h}_{b_i, r_i} , and \mathbf{h}_{r_j, s_i} , $\forall i, j$, the beamformers \mathbf{W}_{r_i} for RS_i can be jointly designed to meet the target SINR γ_0 as well, according to SINR constrained power minimization problem of relay stations (SPMP-RS).

SPMP-RS

$$\min_{\{\mathbf{w}_{r_i}\}_{i=1}^N} P_r \quad \text{s.t.} \quad \text{SINR}_i^{(r)} \geq \gamma_0, \forall i$$

The design methodology outlined in SPMP-BS and SPMP-RS requires the full CSI for \mathbf{h}_{b_j, s_i} , \mathbf{h}_{r_j, s_i} and \mathbf{h}_{b_j, s_i} , $\forall i, j$.

To cope with the influences of channel quantization error, we study robust BF designs to maintain the SINR for each SS. Considering the uncertainties of \mathbf{h}_{b_j, s_i} , \mathbf{h}_{r_j, s_i} and \mathbf{h}_{b_j, s_i}

in our designs, they are modeled as

$$\begin{aligned}
\mathbf{h}_{b_j, s_i} &\triangleq \bar{\mathbf{h}}_{b_j, s_i} + \mathbf{e}_{b_j, s_i} \quad \forall i, j \\
\mathbf{h}_{r_j, s_i} &\triangleq \bar{\mathbf{h}}_{r_j, s_i} + \mathbf{e}_{r_j, s_i} \quad \forall i, j \\
\mathbf{h}_{b_i, r_i} &\triangleq \bar{\mathbf{h}}_{b_i, r_i} + \mathbf{e}_{b_i, r_i} \quad \forall i
\end{aligned} \tag{3.7}$$

where $\bar{\mathbf{h}}_{b_j, s_i}$, $\bar{\mathbf{h}}_{r_j, s_i}$ and $\bar{\mathbf{h}}_{b_i, r_i}$ are quantized channel available at the CU, and \mathbf{e}_{b_j, s_i} and $\mathbf{e}_{r_j, s_i} \in \mathbb{C}^{N \times 1}$ and $\mathbf{e}_{b_i, r_i} \in \mathbb{C}^{N \times 1}$ are their associated unknown uncertainty vectors or matrices. According to the uncertainty models, the design criterion for \mathbf{w}_{b_i} is modified into robust SINR constrained power minimization problem of base stations (RSPMP-BS)

RSPMP-BS

$$\begin{aligned}
\min_{\{\mathbf{w}_{b_i}\}_{i=1}^N} P_b \quad \text{s.t.} \quad & \text{SINR}_i^{(b)} \geq \gamma_0, \quad \forall i \\
& \forall \|\mathbf{e}_{b_j, s_i}\| \leq \epsilon_{b, s}, \quad \forall i, j
\end{aligned}$$

where the $\text{SINR}_i^{(b)}$ still has the form of (3.2) while \mathbf{h}_{b_j, s_i} in (3.2) is replaced by $\bar{\mathbf{h}}_{b_j, s_i} + \mathbf{e}_{b_j, s_i}$. This optimization criterion guarantees that the SINR is satisfied for all \mathbf{e}_{b_j, s_i} , $\forall i, j$, as long as $\|\mathbf{e}_{b_j, s_i}\| \leq \epsilon_{b, s}$.

Considering channel quantization error, \mathbf{e}_{b_i, r_i} are actually influence the total transmit power of relays, P_r , and make it fluctuate even BF matrix \mathbf{W}_{r_i} are fixed. Hence, one kind of reasonable cost function is defined as minimizing the maximal transmit power. Hence, we reconsider a robust design criterion for \mathbf{W}_{r_i} called as general robust SINR constrained power minimization problem of relay stations (GRSPMP-RS)

GRSPMP-RS

$$\begin{aligned}
\min_{\{\mathbf{w}_{r_i}\}_{i=1}^N} \max_{\{\mathbf{e}_{b_i, r_i}\}_{i=1}^N} P_r \quad \text{s.t.} \quad & \text{SINR}_i^{(r)} \geq \gamma_0, \quad \forall i \\
& \forall \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r, s}, \quad \forall i, j \\
& \forall \|\mathbf{e}_{b_i, r_i}\| \leq \epsilon_{b, r}, \quad \forall i
\end{aligned}$$

where P_r and $\text{SINR}_i^{(r)}$ still have the form of (3.5) and (3.6) while \mathbf{h}_{r_j, s_i} and \mathbf{h}_{b_i, r_i} in (3.6) are replaced by $\bar{\mathbf{h}}_{r_j, s_i} + \mathbf{e}_{r_j, s_i}$ and $\bar{\mathbf{h}}_{b_i, r_i} + \mathbf{e}_{b_i, r_i}$ respectively.

A relaxed version of GRSPMP-RS named as robust SINR constrained power minimization problem of relay stations (RSPMP-RS) is also considered in this paper.

RSPMP-RS

$$\begin{aligned} \min_{\{\mathbf{w}_{r_i}\}_{i=1}^N} P_r \quad \text{s.t.} \quad & \text{SINR}_i^{(r)} \geq \gamma_0, \quad \forall i \\ & \forall \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r, s}, \quad \forall i, j \end{aligned}$$

where P_r and $\text{SINR}_i^{(r)}$ still have the form of (3.5) and (3.6) while \mathbf{h}_{r_j, s_i} in (3.6) are replaced by $\bar{\mathbf{h}}_{r_j, s_i} + \mathbf{e}_{r_j, s_i}$.

The signal models and design criteria introduced above allow us to construct the beamformers for coordinated BSs transmissions and for coordinated RSs retransmissions. In the next section, we first introduce our CoMP AF BF algorithms according to SPMP-RS. The robust AF BF algorithms are provided in Section 5 and 6.



Chapter 4

SINR Constrained CoMP AF Beamforming

In this chapter, we first assume no quantization error (i.e., unlimited rate for channel feedback) and therefore solve problem SPMP-RS. However, this optimization problem includes so many (N^3) primal variables that it seems requiring very huge computation effort at the CU. Besides, N-by-N BF matrices for RSs lead to require high downlink transmission resource reserved for informing RSs of their own BF.

Fortunately, with help of the following lemma, optimal BF matrix for each RS are shown to be rank one at this case. The rank one BF matrix for a certain RS can be decomposed as a column vector and a row vector. This row vector is actually composed of equivalent CSIs between its associated BS and itself. Hence, these CSIs may be estimated by itself and only a vector rather than a matrix needs to be transmitted by its associated BS, which lead to decrease resource reserved for informing RSs of their own BF. Furthermore, with knowledge of the form of a good BF, we may also reduce computation effort of BF design at the CU, which results from fewer optimization variables should be considered.

Besides, in addition to formulating SPMP-RS into a standard second order cone

programming (SOCP), to develop an approach which is more friendly for practical implementations and to gain more insight into the problem, we then provide optimality conditions in the following proposition, and suggest a simple fixed point iteration for finding the variables that satisfy them.

Lemma 1. *Optimal CoMP AF beamforming matrix of SPMP-RS $\mathbf{W}_{r_i}^*$, $\forall i$, are necessary rank one and always can be decomposed as below*

$$\mathbf{W}_{r_i}^* = \mathbf{w}_i \mathbf{h}_{b_i, r_i}^H \quad (4.1)$$

where $\mathbf{w}_i \in \mathbb{C}^{N \times 1}$.

Proof. From Lagrangian zero derivative condition which is actually necessary for optimality, \mathbf{W}_{r_i} can be just represented as the form of (4.1) by some mathematical manipulations. The details of derivations are summarized in APPENDIX A. \square

Based on Lemma 1, CoMP AF beamforming matrix \mathbf{W}_{r_i} of SPMP-RS can be assumed as $\mathbf{w}_{r_i} \frac{\mathbf{h}_{b_i, r_i}^H}{\|\mathbf{h}_{b_i, r_i}\|}$, where $\mathbf{w}_{r_i} \in \mathbb{C}^{N \times 1}$, without generality, and hence can be easily reformulated into an equivalent problem called transformed SINR constrained power minimization problem of relay stations (TSPMP-RS)

TSPMP-RS

$$\min_{p_0, \{\mathbf{w}_{r_i}\}_{i=1}^N} p_0 \quad \text{s.t.} \quad \sum_{i=1}^N c_i \|\mathbf{w}_{r_i}\|^2 \leq p_0,$$

$$\frac{1}{\gamma_0} \mathbf{w}_{r_i}^H \mathbf{A}_i \mathbf{w}_{r_i} \geq \sum_{j=1, j \neq i}^N \mathbf{w}_{r_j}^H \mathbf{B}_{j,i} \mathbf{w}_{r_j} + \sigma^2, \quad \forall i$$

where $\mathbf{A}_i \triangleq d_i (\mathbf{h}_{r_i, s_i} \mathbf{h}_{r_i, s_i}^H)$, $\mathbf{B}_{j,i} \triangleq c_j (\mathbf{h}_{r_j, s_i} \mathbf{h}_{r_j, s_i}^H)$ for $j \neq i$, $c_i = (\|\mathbf{h}_{b_i, r_i}\|^2 + \sigma^2)$ and $d_i \triangleq (\|\mathbf{h}_{b_i, r_i}\|^2 - \gamma_0 \sigma^2)$.

Clearly, both $\mathbf{B}_{j,i}$ are positive semi-definite (*p.s.d.*), $j \neq i \forall i, j$, denoted by $\mathbf{B}_{j,i} \succeq 0$. Thus, there exist $\mathbf{R}_i \in \mathbb{C}^{N^2 \times N^2}$ such that $\sum_{j=1, j \neq i}^N \mathbf{w}_{r_j}^H \mathbf{B}_{j,i} \mathbf{w}_{r_j} \triangleq \|\mathbf{R}_i^H \mathbf{w}\|^2$, where $\mathbf{w}^H = (\mathbf{w}_{r_1}^H, \dots, \mathbf{w}_{r_N}^H)$. Moreover, supposed that $\{\mathbf{w}_{r_1}^*, \dots, \mathbf{w}_{r_N}^*\}$ be a set of optimal BF vectors, it can be easily verified from (3.6) that $e^{j\theta_i} \mathbf{w}_{r_i}$ is also an optimal BF vector, for

arbitrary $\theta_i, \forall i$. Thus, without the loss of generality (*w.l.o.g.*), one can assume that $(\mathbf{e}_i \otimes \sqrt{d_i} \mathbf{h}_{r_i, s_i}^H) \mathbf{w} \geq 0$, where \mathbf{e}_i is a N-dimensional unit vector with one for the i -th entry and zeros elsewhere. As a result, the SINR constraint in TSPMP-RS can be further reformulated as

$$\underline{\mathbf{a}}_i(\mathbf{w}) \triangleq (a_i(\mathbf{w}) \quad \mathbf{a}_i(\mathbf{w})^T)^T \triangleq \left(\frac{1}{\sqrt{\gamma_0}} (\mathbf{e}_i \otimes \sqrt{d_i} \mathbf{h}_{r_i, s_i}^H) \mathbf{w} \quad (\mathbf{R}_i^H \mathbf{w})^T \quad \sigma \right)^T \succeq_K 0 \quad (4.2)$$

where the notation $\underline{\mathbf{v}} \succeq_K 0$, $\underline{\mathbf{v}}^H \triangleq [v, \mathbf{v}^H]$ denotes a generalized inequality: $v \geq \|\mathbf{V}\|$. Similarly, having $p \triangleq \sqrt{p_0}$, the power constraint in TSPMP-RS can also be reformulated as $\underline{\mathbf{p}}(p, \mathbf{w}) \triangleq (p \quad (\mathbf{S}^H \mathbf{w})^T)^T \succeq_K 0$, where $\mathbf{S} \in \mathbb{C}^{N^2 \times N^2}$. Consequently, TSPMP-RS can be cast into the form of the standard second order cone programming (SOCP) [30] which can be solved by available optimization solvers, *e.g.* CVX [31]. However, the general-purpose solvers are typically computationally inefficient for a specific optimization problem like TSPMP-RS. To reduce the complexity for CoMP AF BF, we propose a nearly closed-form method for solving TSPMP-RS, which is summarized in the following proposition.

Proposition 1. *Consider TSPMP-RS. If there exist $\lambda_i > 0, \forall i$, that satisfy*

$$\lambda_i = \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i, j} \right)^{-1} \mathbf{h}_{r_i, s_i}} \quad (4.3)$$

then the minimum transmit power P_r is equal to $\sum_{i=1}^N \lambda_i \sigma^2$.

Define $\delta_i \triangleq \sigma \sqrt{\sum_{j=1}^N [\mathbf{F}^{-1}]_{i, j}}$ in which the entry of the $N \times N$ matrix \mathbf{F} are given by $[\mathbf{F}]_{i, i} \triangleq (\frac{\lambda_i d_i}{r_0})^2 \mathbf{h}_{r_i, s_i}^H (c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i, j})^{-1} \frac{A_i}{r_0} (c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i, j})^{-1} \mathbf{h}_{r_i, s_i}$; otherwise, for $i \neq j$, $[\mathbf{F}]_{i, j} \triangleq -(\frac{\lambda_j d_j}{r_0})^2 \mathbf{h}_{r_j, u_j}^H (c_j \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i, j})^{-1} \mathbf{B}_{j, i} (c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i, j})^{-1} \mathbf{h}_{r_j, u_j}$.

The optimal $\mathbf{w}_{r_i}, \forall i$, are given by

$$\mathbf{w}_{r_i} = \frac{\lambda_i d_i \delta_i}{\gamma_0} \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i, j} \right)^{-1} \mathbf{h}_{r_i, s_i}. \quad (4.4)$$

Proof. As the statements above, TSPMP-RS actually can be represented as

$$\min_{\mathbf{w}} \|\mathbf{S}^H \mathbf{w}\|^2 \quad \text{s.t.} \quad \|\mathbf{R}_i^H \mathbf{w}\|^2 - \left| \frac{1}{\sqrt{\gamma_0}} (\mathbf{e}_i \otimes \sqrt{d_i} \mathbf{h}_{r_i, s_i}^H) \mathbf{w} \right|^2 + \sigma^2 \leq 0 \quad (4.5)$$

In [30], the authors has shown that Karush-Kuhn-Tucker (KKT) conditions are just sufficient and necessary for optimality considering those problems with the same form as (4.5). Hence, CoMP AF BF provided in this proposition are just derived from KKT conditions and must be optimal. The other details are summarized in APPENDIX C. \square

To efficiently find feasible λ_i satisfying (4.3), fixed-point iterations are done as below and can be shown to converge given any initial $\lambda_i^{(0)} > 0$ in APPENDIX B.

$$\lambda_i^{(n+1)} = f_i(\Lambda^{(n)}) \triangleq \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j^{(n)} \mathbf{B}_{i, j} \right)^{-1} \mathbf{h}_{r_i, s_i}} \quad (4.6)$$

where $\Lambda = [\lambda_1^{(n)}, \dots, \lambda_N^{(n)}]$.

Proposition 1 provides a simple method to design the BF matrices for RSs to meet the same target SINR, γ_0 , for each SS in the subnetwork with CoMP AF relaying. On the other hand, individual weighting factors can be used to scale \mathbf{A}_i in TSPMP-RS for SSs to achieve different target SINRs.

Chapter 5

Robust SINR Constrained CoMP

AF Beamforming

According to CSIs feedback flow chart illustrated in chapter 2, CSIs of BS-RS and RS-SS links are fed back by BS-RS and BS-SS reverse links respectively . Comparing to channel feedback by BS-SS reverse links, BS-RS reverse links usually possess larger channel gain and feedback-link capacity, because RSs are much nearer to BSs than SSs. At this case, more bits are allowed to quantize CSIs of BS-RS links and then CSIs of these links at the CU are within a much smaller specified uncertainty region.

Hence, in this chapter, we first only consider dominant source lead to performance loss. That is, we consider CSI quantization error of RS-SS links (i.e., limited feedback of reverse BS-SS links) but no CSI quantization error of BS-RS links (i.e., unlimited feedback of BS-RS reverse links). Hence, a robust coordinated AF RSs downlink BF design known as RSPMP-RS is addressed, and CSI quantization error of all links would be further considered in the next chapter.

As illustrations of the previous chapter, a similar lemma is proposed and suggests that optimal BF matrices for each RS is rank one and possess a specified structure,

which may reduce computation effort of BF design at the CU and decrease resource reserved for informing RSs of their own BF.

In addition to formulating RSPMP-RS into a Semidefinite Programming (SDP) based on the S-Procedure and semi-definite relaxation (SDR) [28], to develop an approach which is more friendly for practical implementations and to gain more insight into the problem, we then provide necessary conditions for optimality in the following proposition, and suggest a simple two-tier fixed point iteration (as shown in the following algorithm) based on iterated quadratic constraints quadratic programming (QCQP) concept for finding the variables that satisfy them.

The SDR-based beamforming method is shown by simulations to achieve the nearly optimal performance. And the performance of the low-complexity suboptimal scheme based on the iterated QCQP is shown to be very close to that with the SDR approach, and is more friendly for practical implementations.

Lemma 2. *Optimal robust CoMP AF beamforming matrix for RSPMP-RS $\mathbf{W}_{r_i}^*$, $\forall i$, are necessary rank one and always can be decomposed as below*

$$\mathbf{W}_{r_i}^* = \mathbf{w}_i \mathbf{h}_{b_i, r_i}^H \quad (5.1)$$

where $\mathbf{w}_i \in \mathbb{C}^{N \times 1}$.

Proof. The idea of proof is just similar to Lemma 1's, and the details of derivations are also summarized in APPENDIX A. □

Based on Lemma 2, robust CoMP AF beamforming matrix \mathbf{W}_{r_i} can also be assumed as an equivalent problem called as $\mathbf{w}_{r_i} \frac{\mathbf{h}_{b_i, r_i}^H}{\|\mathbf{h}_{b_i, r_i}\|}$, where $\mathbf{w}_{r_i} \in \mathbb{C}^{N \times 1}$, without generality. RSPMP-RS can be then reformulated as transformed robust SINR constrained power minimization problem of relay stations (TRSPMP-RS)

$$\begin{array}{c}
\text{TRSPMP-RS} \\
\min_{\{\mathbf{w}_{r_i}\}_{i=1}^N} \sum_{i=1}^N c_i \|\mathbf{w}_{r_i}\|^2 \quad \text{s.t.} \quad \text{SINR}_i^{(r)} \geq \gamma_0, \forall i \\
\forall \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r, s}, \quad \forall i, j
\end{array}$$

And the $\text{SINR}_i^{(r)}$ can also be rewritten as

$$\text{where } \text{SINR}_i^{(r)} = \frac{\|\mathbf{h}_{b_i, r_i}\|^2 \mathbf{D}_{i,i}}{\sum_{j \neq i} \|\mathbf{h}_{b_j, r_j}\|^2 \mathbf{D}_{j,i} + (\sum_j \mathbf{D}_{j,i} + 1) \sigma^2} \quad (5.2)$$

where $\mathbf{D}_{j,i} \triangleq |(\bar{\mathbf{h}}_{r_j, s_i} + \mathbf{e}_{r_j, s_i})^H \mathbf{w}_{r_j}|^2$.

The problem above ensures that every SS will be served with an SINR at least better than the target SINR, γ_0 , for all channel uncertainties, \mathbf{e}_{r_j, s_i} , whose $\|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r, s}$. This, in fact, will impose an infinite number of non-convex constraints in this problem. Although the non-convex constraints can be transformed into convex ones with a method similar to what employed in Chapter 4, the approach used in Proposition 1 is still not capable of dealing with an infinite number of convex constraints. Hence, to solve TRSPMP-RS, the first challenge is to transform the infinite constraints into finite ones. We, in the sequel, present two methods to overcome the difficulty, which lead to two different approaches for solving TRSPMP-RS.

5.1 RSPMP-RS based on the S-Procedure and SDR

Following a procedure similar to what presented in [32–34], the infinite number of constraints in TRSPMP-RS can be transformed into a finite number of linear matrix inequalities (LMIs), using the S-procedure [28]:

Proposition 2. (*S-procedure*) Let $\mathbf{A}, \mathbf{C} \in \mathbb{C}^{N \times N}$, $\mathbf{b}, \mathbf{v} \in \mathbb{C}^{N \times 1}$, and $c \in \mathbb{R}$. The

inequalities $\mathbf{v}^H \mathbf{A} \mathbf{v} + \mathbf{b}^H \mathbf{v} + \mathbf{v}^H \mathbf{b} + c \geq 0$, $\forall \mathbf{v}^H \mathbf{C} \mathbf{v} \leq 1$ hold if and only if $\exists \lambda \geq 0$ such that

$$\begin{pmatrix} \mathbf{A} + \lambda \mathbf{C} & \mathbf{b} \\ \mathbf{b}^H & c - \lambda \end{pmatrix} \succeq 0. \quad (5.3)$$

Then, we introduce auxiliary variables $t_{j,i}$, $\forall i, j$, to help us reformulate TRSPMP-RS before we can apply S-Procedure. The resultant reformulation is given by

$$\begin{aligned} & \min_{\{t_{j,i} \geq 0\}_{i,j=1}^N, \{\mathbf{w}_{r_i}\}_{i=1}^N} \sum_{i=1}^N c_i \|\mathbf{w}_{r_i}\|^2 \\ \text{s.t.} \quad & \frac{d_i}{\gamma_0} \mathbf{D}_{i,i} \geq t_{i,i}, \quad c_j \mathbf{D}_{j,i} \leq t_{j,i}, \quad j \neq i, \quad \forall i, j \\ & \sum_{j=1, j \neq i}^N t_{j,i} + \sigma^2 \leq t_{i,i}, \quad \forall i \\ & \forall \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r,s}, \quad \forall i, j \end{aligned} \quad (5.4)$$

Based on this reformulation, we want to recast TRSPMP-RS into a semi-definite programming (SDP). We present the process step by step as follows.

1. Reformulate $\mathbf{D}_{j,i}$ into quadratic functions of \mathbf{e}_{r_j, s_i} :

$$\begin{aligned} c_j \mathbf{D}_{j,i} & \triangleq c_j |(\bar{\mathbf{h}}_{r_j, s_i} + \mathbf{e}_{r_j, s_i})^H \mathbf{w}_{r_j}|^2 \\ & = \mathbf{e}_{r_j, s_i}^H \mathbb{W}_{r_j} \mathbf{e}_{r_j, s_i} + \mathbf{e}_{r_j, s_i}^H \mathbb{W}_{r_j} \bar{\mathbf{h}}_{r_j, s_i} + \bar{\mathbf{h}}_{r_j, s_i}^H \mathbb{W}_{r_j} \mathbf{e}_{r_j, s_i} + \bar{\mathbf{h}}_{r_j, s_i}^H \mathbb{W}_{r_j} \bar{\mathbf{h}}_{r_j, s_i} \end{aligned}$$

where $\mathbb{W}_{r_j} \triangleq c_j \mathbf{w}_{r_j} \mathbf{w}_{r_j}^H$

2. Transform the infinite inequalities in (5.4) into LMIs via the S-Procedure:

According to Proposition 2, $c_j \mathbf{D}_{j,i} \leq t_{j,i}$ holds for all $\|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r,u}$, if and only if there exists a number $\lambda_{j,i} \geq 0$ such that the matrix (5.3) is *p.s.d.* if we have $\mathbf{v} \triangleq \mathbf{e}_{r_j, s_i}$, $\mathbf{A} \triangleq -\mathbb{W}_{r_j}$, $\mathbf{b} \triangleq -\mathbb{W}_{r_j} \bar{\mathbf{h}}_{r_j, s_i}$, $\mathbf{C} \triangleq \frac{1}{\epsilon_{r,s}^2} \mathbf{I}$, $c \triangleq t_{j,i} - \bar{\mathbf{h}}_{r_j, s_i}^H \mathbb{W}_{r_j} \bar{\mathbf{h}}_{r_j, s_i}$ and $\lambda \triangleq \lambda_{j,i}$. Following the similar steps, the constraints, $\frac{d_i}{\gamma_0} \mathbf{D}_{i,i} \geq t_{i,i}$, $\forall \|\mathbf{e}_{r_i, s_i}\| \leq \epsilon_{r,u}$, $\mathbf{e}_{r_i, s_i} \in \mathbb{C}^{N \times 1}$ can be transformed into LMIs with (5.3) as well.

3. Relax the rank-one constraint on \mathbb{W}_{r_j} , $\forall i$:

By doing this, the LMIs obtained above for $\frac{d_i}{\gamma_0} \mathbf{D}_{i,i}$ and $c_j \mathbf{D}_{j,i} \leq t_{j,i}$, $j \neq i$ can be expressed in convex forms. In other words, (5.4) can be relaxed and expressed in a SDP of the form

$$\begin{aligned}
& \min_{\mathbb{W}_{r_j}, t_{j,i} \geq 0, \lambda_{j,i} \geq 0} \sum_{i=1}^N \text{tr}(\mathbb{W}_{r_i}) && \text{s.t.} \\
& \frac{d_i}{c_i \gamma_0} \bar{\mathbf{H}}_{r_i, s_i}^H \mathbb{W}_{r_i} \bar{\mathbf{H}}_{r_i, s_i} + \text{Blkdiag} \left\{ \frac{\lambda_{i,i}}{c_{r,s}^2} \mathbf{I}_N, -(t_{i,i} + \lambda_{i,i}) \right\} \succeq 0, && \forall i \\
& \text{Blkdiag} \left\{ \frac{\lambda_{j,i}}{c_{r,s}^2} \mathbf{I}_N, t_{j,i} - \lambda_{j,i} \right\} - \bar{\mathbf{H}}_{r_j, s_i}^H \mathbb{W}_{r_j} \bar{\mathbf{H}}_{r_j, s_i} \succeq 0, && \forall j \neq i \\
& \sum_{j=1, j \neq i}^N t_{j,i} + \sigma^2 \leq t_{i,i}, \quad \mathbb{W}_{r_i} \succeq 0 && \forall i
\end{aligned} \tag{5.5}$$

where $\bar{\mathbf{H}}_{r_j, s_i} \triangleq (\mathbf{I}_N \bar{\mathbf{h}}_{r_j, s_i})$, $\forall i, j$ and the function $\text{Blkdiag}\{\mathbf{A}, \mathbf{B}\}$ puts its square matrix arguments into a block diagonal one.

The above SDR problem can be solved with the available optimization solvers such as SeDuMe or CVX [31, 35]. In fact, the beamformers obtained with the SDR approach are optimal if they satisfy the rank-one condition. Although the resultant \mathbb{W}_{r_i} , are not guaranteed to be rank-one $\forall i$, by inspection from our simulations, we notice that the solutions always satisfy the rank-one condition. Similar observations have been previously reported in [32–34]. Thus, the above SDR approach (5.5) can be practically considered as an nearly optimal solver for RSPMP-RS.

5.2 RSPMP-RS based on Iterated QCQP

The SDR-based approach presented in the previous section is elegant and provides a practically nearly optimal solver. However, the approach relies on a numerical toolbox which is not always available and less friendly for practical implementations. Motivated by the results in [36, 37], among others, we present in the sequel a more intuitive and commonly used approach to relieve the infinite number of constraints in TRSPMP-RS, making use of the triangular and the Cauchy-Schwarz inequalities. Nevertheless,

to resolve the conservativeness often seen in this approach, we introduce a number of auxiliary variables to ensure the tightness in the process of constraint relaxations, which together with a new formulation for programming leads to a nearly closed-form suboptimal solver. The suboptimal solver presents a performance very close to the one obtained with the SDR-approach in (5.5), yet is more computationally efficient and friendly for practical implementations. Moreover, it provides us an insight into the structure of the robust CoMP AF beamformers. Details of the derivations are presented below and in the Appendix C.

Recall TRSPMP-RS. To relieve the infinite number of constraints in programming, we need to bound the effects of uncertainties in $\mathbf{D}_{i,i}$, and $\mathbf{D}_{j,i}$ of (5.2) for all possible \mathbf{e}_{r_j,s_i} that satisfy $\|\mathbf{e}_{r_j,s_i}\| \leq \epsilon_{r,s}$, $\forall i, j$. Observing the form of $\mathbf{D}_{j,i}$ in (5.2), \mathbf{e}_{r_j,s_i} are embedded in $|\mathbf{e}_{j,i}^H \mathbf{w}_{r_i}|$ which can be upper bounded with the Cauchy-Schwarz inequality as follows

$$|\mathbf{e}_{r_j,s_i}^H \mathbf{w}_{r_i}| \leq \|\mathbf{e}_{r_j,s_i}\| \|\mathbf{w}_{r_i}\| \leq \epsilon_{r,s} \|\mathbf{w}_{r_i}\| \quad \forall i, j. \quad (5.6)$$

This upper bound is in fact tight for TRSPMP-RS as there exist \mathbf{e}_{r_j,s_i} that just make the inequalities also satisfy equality. Substituting this upper bound into $\mathbf{D}_{i,i}$ followed by some mathematical manipulations, we obtain

$$\begin{aligned} \mathbf{D}_{i,i} &= |(\bar{\mathbf{h}}_{r_i,s_i} + \mathbf{e}_{r_i,s_i})^H \mathbf{w}_{r_i}|^2 \geq (|\bar{\mathbf{h}}_{r_i,s_i}^H \mathbf{w}_{r_i}| - |\mathbf{e}_{r_i,s_i}^H \mathbf{w}_{r_i}|)^2 \\ &\geq |\bar{\mathbf{h}}_{r_i,s_i}^H \mathbf{w}_{r_i}|^2 - 2\epsilon_{r,s} |\bar{\mathbf{h}}_{r_i,s_i}^H \mathbf{w}_{r_i}| \|\tilde{\mathbf{w}}_{r_i}\| + \epsilon_{r,s}^2 \|\mathbf{w}_{r_i}\|^2 \end{aligned}$$

if $|\bar{\mathbf{h}}_{r_i,s_i}^H \mathbf{w}_{r_i}| \geq \epsilon_{r,s} \|\mathbf{w}_{r_i}\|$. This lower bound can further be expressed in a quadratic form if we define some parameters $v_{i,i}$, $\forall i$, such that $|\bar{\mathbf{h}}_{r_i,s_i}^H \mathbf{w}_{r_i}| \leq v_{i,i} \|\bar{\mathbf{h}}_{r_i,s_i}\| \|\mathbf{w}_{r_i}\|$, which is

$$\mathbf{D}_{i,i} \geq \mathbf{w}_{r_i}^H \left[\bar{\mathbf{h}}_{r_i,s_i} \bar{\mathbf{h}}_{r_i,s_i}^H + (\epsilon_{r,s}^2 - 2\epsilon_{r,s} v_{i,i} \|\bar{\mathbf{h}}_{r_i,s_i}\|) \mathbf{I}_N \right] \mathbf{w}_{r_i}. \quad (5.7)$$

To meet $|\bar{\mathbf{h}}_{r_i,s_i}^H \mathbf{w}_{r_i}| \geq \epsilon_{r,s} \|\mathbf{w}_{r_i}\|$, the parameter $v_{i,i}$ must also satisfy $v_{i,i} \geq \frac{\epsilon_{r,s}}{\|\bar{\mathbf{h}}_{r_i,s_i}\|}$.

Moreover, define $v_{j,i}$, $\forall j \neq i$, such that $|\bar{\mathbf{h}}_{r_j,s_i}^H \mathbf{w}_{r_j}| \leq v_{j,i} \|\bar{\mathbf{h}}_{r_j,s_i}\| \|\mathbf{w}_{r_j}\|$. By a similar

procedure, we have

$$\mathbf{D}_{j,i} = |(\bar{\mathbf{h}}_{r_j,s_i} + \mathbf{e}_{r_j,s_i})^H \mathbf{w}_{r_j}|^2 \leq (|\bar{\mathbf{h}}_{r_j,s_i} \mathbf{w}_{r_j}| + \epsilon_{r,s} \|\mathbf{w}_{r_j}\|)^2 \quad (5.8)$$

$$\leq \mathbf{w}_{r_j}^H \left[\bar{\mathbf{h}}_{r_j,s_i} \bar{\mathbf{h}}_{r_j,s_i}^H + (\epsilon_{r,s}^2 + 2\epsilon_{r,u} v_{j,i} \|\bar{\mathbf{h}}_{r_j,s_i}\|) \mathbf{I}_N \right] \mathbf{w}_{r_j}. \quad (5.9)$$

Based on (5.7) and (5.9), TRSPMP-RS can be reformulated as

$$\begin{aligned} & \min_{\mathbf{w}_{r_i}, v_{j,i}} \sum_{i=1}^N c_i \|\mathbf{w}_{r_i}\|^2 \\ \text{s.t.} \quad & \frac{1}{\gamma_0} \mathbf{w}_{r_i}^H \hat{\mathbf{A}}_i \mathbf{w}_{r_i} \geq \sum_{j=1, j \neq i}^N \mathbf{w}_{r_j}^H \hat{\mathbf{B}}_{j,i} \mathbf{w}_{r_j} + \sigma^2, \quad \forall i \\ & |\bar{\mathbf{h}}_{r_j,s_i}^H \mathbf{w}_{r_j}| \leq v_{j,i} \|\bar{\mathbf{h}}_{r_j,s_i}\| \|\mathbf{w}_{r_j}\|, \quad \forall i, j \\ & v_{i,i} \geq \frac{\epsilon_{r,s}}{\|\bar{\mathbf{h}}_{r_i,s_i}\|}, \quad v_{j,i} \geq 0, \quad \forall i, j \end{aligned} \quad (5.10)$$

where $\hat{\mathbf{A}}_i \triangleq d_i \left\{ \bar{\mathbf{h}}_{r_i,s_i} \bar{\mathbf{h}}_{r_i,s_i}^H + (\epsilon_{r,s}^2 - 2\epsilon_{r,u} v_{i,i} \|\bar{\mathbf{h}}_{r_i,s_i}\|) \mathbf{I}_N \right\}$, $d_i \triangleq (\|\mathbf{h}_{b_i,r_i}\|^2 - \gamma_0 \sigma^2)$ and $\hat{\mathbf{B}}_{j,i} \triangleq c_j \left\{ \bar{\mathbf{h}}_{r_j,s_i} \bar{\mathbf{h}}_{r_j,s_i}^H + (\epsilon_{r,s}^2 + 2\epsilon_{r,u} v_{j,i} \|\bar{\mathbf{h}}_{r_j,s_i}\|) \mathbf{I}_N \right\}$ for $j \neq i$.

The infinite number of constraints in TRSPMP-RS have been relieved and reformulated into finite ones in (5.10). And the tightness of the new constraints can be preserved if one can find the optimal $v_{i,i}^*$, $v_{j,i}^*$ and $\mathbf{w}_{r_j}^*$ that satisfy the equalities of $|\bar{\mathbf{h}}_{r_i,s_i}^H \mathbf{w}_{r_i}| = v_{i,i} \|\bar{\mathbf{h}}_{r_i,s_i}\| \|\mathbf{w}_{r_i}\|$ and $v_{j,i} \|\bar{\mathbf{h}}_{r_j,s_i}\| \|\mathbf{w}_{r_j}\| = |\bar{\mathbf{h}}_{r_j,s_i}^H \mathbf{w}_{r_j}|$. Although, the formulation of (5.10) is not in convex form, given $v_{i,i}$ and $v_{j,i}$, the programming can be viewed as a form of the quadratic constrained quadratic programming (QCQP) model in [30], or vice versus, which will provide an optimal $\mathbf{w}_{r_i}^*$ given the optimal set of parameters: $v_{i,i}^*$ and $v_{j,i}^*$. We next present an iterative framework to find the parameter set and \mathbf{w}_{r_i} according to the KKT conditions of (5.10). Given that the constraints of (5.10) are in general not convex, the iterative method only guarantees that the solutions satisfy the necessary conditions for (5.10). Even though we do not obtain an algorithm to find the optimal solutions, simulation results show that the resultant beamformers perform almost the same as do the ones obtained with the SDR-based approach.

Proposition 3. Consider the KKT conditions of (5.10). Define the dual variables, λ_i and $\tau_{i,j}$, $\forall i, j$. If there exist positive λ_i , $\forall i$, that satisfy

$$\lambda_i = \frac{\gamma_0}{d_i \bar{\mathbf{h}}_{r_i, s_i}^H \Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i}} \quad (5.11)$$

and $\tau_{i,j}$, $\forall i, j$, that satisfy

$$\tau_{i,i} = \frac{\epsilon_{r,s} d_i \lambda_i}{\gamma_0 v_{i,i} \|\bar{\mathbf{h}}_{r_i, s_i}\|} \quad \tau_{j,i} = \frac{\epsilon_{r,s} c_j \lambda_i}{v_{j,i} \|\bar{\mathbf{h}}_{r_j, s_i}\|} \quad \forall j \neq i \quad (5.12)$$

then the zero derivative conditions of KKT hold true.

where $\Phi_{i,i} \triangleq \Psi_i + \frac{\lambda_i d_i}{\gamma_0} (\bar{\mathbf{h}}_{r_i, s_i} \bar{\mathbf{h}}_{r_i, s_i}^H)$, $\Psi_i \triangleq c_i \mathbf{I}_N - \frac{\lambda_i}{\gamma_0} (\hat{\mathbf{A}}_i) + \sum_{m=1, m \neq i}^N \lambda_m \hat{\mathbf{B}}_{i,m} - \sum_{m=1}^N \tau_{i,m} \mathbf{G}_{i,m}$, $\mathbf{G}_{j,i} \triangleq v_{j,i}^2 \|\bar{\mathbf{h}}_{r_j, s_i}\|^2 \mathbf{I}_N - \bar{\mathbf{h}}_{r_j, s_i} \bar{\mathbf{h}}_{r_j, s_i}^H \quad \forall i, j$.

On the other hand, the complementary slackness conditions hold true if there exist feasible primal variables $v_{j,i} \quad \forall i, j$, that satisfy

$$v_{i,i} = \frac{\gamma_0}{\lambda_i d_i \|\bar{\mathbf{h}}_{r_i, s_i}\| \|\Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i}\|} \quad \forall i \quad v_{j,i} = \frac{1}{\lambda_i c_j \|\bar{\mathbf{h}}_{r_j, s_i}\| \|\Phi_{j,i}^{-1} \bar{\mathbf{h}}_{r_j, s_i}\|} \quad \forall j \neq i \quad (5.13)$$

and feasible primal variables \mathbf{w}_{r_i} , $\forall i$, such that \mathbf{W}_{r_i} consuming transmit power P_r equal to $\sum_{i=1}^N \lambda_i \sigma^2$ and satisfy

$$\mathbf{W}_{r_i} = \frac{\lambda_i d_i \delta_i}{\gamma_0} \Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i} \mathbf{h}_{b_i, r_i}^H \quad (5.14)$$

where $\Phi_{j,i} \triangleq \Psi_j - \lambda_i c_j \bar{\mathbf{h}}_{r_j, s_i} \bar{\mathbf{h}}_{r_j, s_i}^H$, $\forall j \neq i$.

$$\delta_i \triangleq \sigma \sqrt{\sum_{j=1}^N [\hat{\mathbf{F}}^{-1}]_{i,j}} \quad \forall i \quad (5.15)$$

where $[\hat{\mathbf{F}}]_{i,i} = (\frac{\lambda_i d_i}{r_0})^2 \bar{\mathbf{h}}_{r_i, s_i}^H \Phi_{i,i}^{-1} (\frac{\hat{\mathbf{A}}_i}{r_0}) \Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i}$ for the diagonal entries; otherwise, $\forall i \neq j$, $[\hat{\mathbf{F}}]_{i,j} = -(\frac{\lambda_j d_j}{r_0})^2 \bar{\mathbf{h}}_{r_j, s_j}^H \Phi_{j,j}^{-1} \hat{\mathbf{B}}_{j,i} \Phi_{j,j}^{-1} \bar{\mathbf{h}}_{r_j, s_j}$.

The details of the proof are summarized in the Appendix C. The proposition provides a simple strategy for finding the primal and dual variables that satisfy the KKT conditions. Given a feasible SINR requirement γ_0 and channel uncertainty radius $\epsilon_{r,s}$, all one has to do is to find the pairs of primal and dual variables satisfying all the equations in Proposition 3. The structure of the equations motivates us to find the solutions by fixed point iterations as (4.6). The entire procedure is summarized in Algorithm 1.



Algorithm 1 The procedure for finding the robust beamformers with Proposition 3

- 1: Initialize $n = 0$
- 2: Set $v_{j,i}^{(0)}$ to 0.001 $\forall j \neq i$
- 3: Set $v_{i,i}^{(0)}$ to one $\forall i$
- 4: **while** $|v_{j,i}^{(n+1)} - v_{j,i}^{(n)}| > 0 \forall i, j$ **do**
- 5: Initialize $m = 0$ and set $\lambda_i(0)$ to zero
- 6: **while** $|\lambda_i(m+1) - \lambda_i(m)| > 0 \forall i$ **do**
- 7: $\Phi_{i,i}(m)$ are yielded by replacing λ_i , $v_{j,i}$ and $\tau_{j,i}$ in $\Phi_{i,i}$ with $\lambda_i(m)$, $v_{j,i}^{(n)}$ and $\tau_{j,i}(m)$ respectively.

$$\lambda_i(m+1) = \frac{\gamma_0}{d_i \bar{\mathbf{h}}_{r_i, s_i}^H (\Phi_{i,i}(m))^{-1} \bar{\mathbf{h}}_{r_i, s_i}} \quad \forall i$$

$$\tau_{i,i}(m+1) = \frac{\epsilon_{r,s} d_i \lambda_i(m)}{\gamma_0 v_{i,i}^{(n)} \|\bar{\mathbf{h}}_{r_i, s_i}\|} \quad \forall i \quad \tau_{j,i}(m+1) = \frac{\epsilon_{r,s} c_j \lambda_i(m)}{v_{j,i}^{(n)} \|\bar{\mathbf{h}}_{r_j, s_i}\|} \quad \forall j \neq i$$

- 8: Set $m=m+1$
- 9: **end while**
- 10: $\lambda_i^{(n)} = \lambda_i(\infty)$ and $\tau_{j,i}^{(n)} = \tau_{j,i}(\infty) \forall i, j$
- 11: $\Phi_{j,i}^{(n)}$ are yielded by replacing λ_i , $v_{j,i}$ and $\tau_{j,i}$ in $\Phi_{j,i}$ with $\lambda_i^{(n)}$, $v_{j,i}^{(n)}$ and $\tau_{j,i}^{(n)}$ respectively.

$$v_{i,i}^{(n+1)} = \frac{\gamma_0}{\lambda_i^{(n)} d_i \|\bar{\mathbf{h}}_{r_i, s_i}\| \|(\Phi_{i,i}^{(n)})^{-1} \bar{\mathbf{h}}_{r_i, s_i}\|} \quad \forall i \quad (5.16)$$

$$v_{j,i}^{(n+1)} = \frac{1}{\lambda_i^{(n)} c_j \|\bar{\mathbf{h}}_{r_j, s_i}\| \|(\Phi_{j,i}^{(n)})^{-1} \bar{\mathbf{h}}_{r_j, s_i}\|} \quad \forall j \neq i \quad (5.17)$$

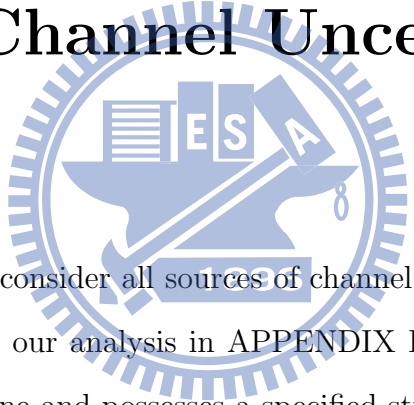
- 12: Set $n=n+1$
- 13: **end while**
- 14: $\Phi_{i,i}^{(\infty)}$ are yielded by replacing λ_i , $v_{j,i}$ and $\tau_{j,i}$ in $\Phi_{i,i}$ with $\lambda_i^{(\infty)}$, $v_{j,i}^{(\infty)}$ and $\tau_{j,i}^{(\infty)}$ respectively.
- 15: **if** The iterations above converge **then**
- 16: The robust AF BF of RSPMP-RS are given by

$$\mathbf{W}_{r_i} = \frac{\lambda_i^{(\infty)} d_i \delta_i}{\gamma_0} (\Phi_{i,i}^{(\infty)})^{-1} \bar{\mathbf{h}}_{r_i, s_i} \mathbf{h}_{b_i, r_i}^H \quad (5.18)$$

- 17: **else**
 - 18: This pair, $\{\gamma_0, \epsilon\}$, is infeasible for the given channel realization.
 - 19: **end if**
-

Chapter 6

Robust SINR Constrained CoMP AF Beamforming Accounting for All Sources of Channel Uncertainties



In this chapter, we fully consider all sources of channel uncertainties due to quantization error. Unfortunately, our analysis in APPENDIX D shows that optimal BF at this case is necessary rank one and possesses a specified structure as the previous case, which lead to increase computation effort of BF design at the CU and resource reserved for informing RSs of their own BF. To yield a good tradeoff between overhead and performance, we then develop a suboptimal but low-complexity BF design approach for this case. Moreover, from the analysis in APPENDIX D, this approach is shown to be a good approximation as CSIs of BS-RS links at the CU are within a small uncertainty region.

In this low-complexity approach, the robust BF of RS_{*i*}, W_{r_i} , is still defined as $\mathbf{w}_{r_i} \frac{\bar{\mathbf{h}}_{b_i, r_i}^H}{\|\bar{\mathbf{h}}_{b_i, r_i}\|}$, where $\mathbf{w}_{r_i} \in \mathbb{C}^{N \times 1}$. The CU decomposes the designed BF matrix for a certain RS as a column vector and a row vector, and then it only need to informs this

RS of the column vector part. The other part is just quantized channel of BS-RS links which generated at RS and is sure known by itself.

As the illustration above, $\text{SINR}_i^{(r)}$ can be rewritten as

$$\text{SINR}_i^{(r)} = \frac{\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| + \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right|^2 \mathbf{D}_{i,i}}{\sum_{j \neq i} \left\| \bar{\mathbf{h}}_{b_j, r_j} \right\| + \mathbf{p}_{j,j}^H \mathbf{e}_{b_j, r_j} \right|^2 \mathbf{D}_{j,i} + \left(\sum_j \mathbf{D}_{j,i} + 1 \right) \sigma^2} \quad (6.1)$$

where $\mathbf{p}_{i,i} \triangleq \frac{\bar{\mathbf{h}}_{b_i, r_i}}{\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\|}$, $\forall i$.

Observing (6.1), \mathbf{e}_{b_i, r_i} are embedded in $\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| + \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right|^2$, and hence, to yield worst case SINR of (6.1), the lower and upper bounding of $\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| + \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right|^2$ are both derived below and substituted into desired and interference signal respectively.

Cauchy-Schwarz inequality implies

$$\left| \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right| \leq \left\| \mathbf{e}_{b_i, r_i} \right\| \left\| \mathbf{p}_{i,i} \right\| \leq \epsilon_{b,r} \left\| \mathbf{p}_{i,i} \right\| \quad \forall i. \quad (6.2)$$

Substituting this upper bound into $\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| + \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right|^2$ followed by some mathematical manipulations, we obtain

$$\begin{aligned} \left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| + \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right|^2 &\geq \left(\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| - \left| \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right| \right)^2 \\ &\geq \left(\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| - d \epsilon_{b,r} \left\| \mathbf{p}_{i,i} \right\| \right)^2 \end{aligned} \quad (6.3)$$

if $\left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| \geq \epsilon_{b,r} \left\| \mathbf{p}_{i,i} \right\|$.

$$\begin{aligned} \left\| \bar{\mathbf{h}}_{b_i, r_i} \right\| + \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} \right|^2 &\leq \left(\left\| \bar{\mathbf{h}}_{b_j, r_j} \right\| + \left| \mathbf{p}_{j,j}^H \mathbf{e}_{b_j, r_j} \right| \right)^2 \\ &\leq \left(\left\| \bar{\mathbf{h}}_{b_j, r_j} \right\| + \epsilon_{b,r} \left\| \mathbf{p}_{j,j} \right\| \right)^2 \end{aligned} \quad (6.4)$$

On the other hand, \mathbf{e}_{b_i, r_i} are embedded in and cause the total transmit power of relays P_r fluctuating even beamforming matrix \mathbf{W}_{r_i} are fixed. Hence, one kind of reasonable

cost function is defined as minimizing the maximal transmit power. Given a set of available channel realizations and beamforming matrix, P_r can be upper bounded from (6.2) and (6.4) as below

$$\begin{aligned} P_r &= \sum_{i=1}^N \left(\|\bar{\mathbf{h}}_{b_i, r_i}\| + \mathbf{p}_{i,i}^H \mathbf{e}_{b_i, r_i} + \sigma^2 \right) \|\mathbf{w}_{r_i}\|^2 \\ &\leq \sum_{i=1}^N \left((\|\bar{\mathbf{h}}_{b_j, r_j} \mathbf{w}_{b_j}\| + \epsilon_{b,r} \|\mathbf{p}_{j,j}\|)^2 + \sigma^2 \right) \|\mathbf{w}_{r_i}\|^2 \end{aligned} \quad (6.5)$$

Finally, GRSPMP-RS can be reformulated as a problem below similar to TRSPMP-RS and can be efficiently solved by the proposed approaches in Section 5.

$$\begin{aligned} \min_{\{\mathbf{w}_{r_i}\}_{i=1}^N} \quad & \sum_{i=1}^N \hat{c}_i \|\mathbf{w}_{r_i}\|^2 \quad \text{s.t.} \quad \text{SINR}_i^{(r, \text{worst})} \geq \gamma_0, \quad \forall i \\ & \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r,s}, \quad \forall i, j \end{aligned} \quad (6.6)$$

where $\hat{c}_i \triangleq (\|\bar{\mathbf{h}}_{b_j, r_j}\| + \epsilon_{b,r} \|\mathbf{p}_{j,j}\|)^2 + \sigma^2$

And the $\text{SINR}_i^{(r, \text{worst})}$ can be written as

$$\text{SINR}_i^{(r, \text{worst})} = \frac{(\|\bar{\mathbf{h}}_{b_i, r_i}\| - \epsilon_{b,r} \|\mathbf{p}_{i,i}\|)^2 \mathbf{D}_{i,i}}{\sum_{j \neq i} (\|\bar{\mathbf{h}}_{b_j, r_j}\| + \epsilon_{b,r} \|\mathbf{p}_{j,j}\|)^2 \mathbf{D}_{j,i} + (\sum_j \mathbf{D}_{j,i} + 1) \sigma^2} \quad (6.7)$$

Chapter 7

Simulation Studies

We demonstrate throughput enhancement via simulations with the proposed CoMP AF BF methods in a multi-cell network. The effectiveness of the proposed robust CoMP AF BF algorithms are compared with the non-robust ones from the perspectives of power consumption and the feasibility of the beamformers. A system topology for a subnetwork of $N = 3$ is illustrated in Fig. 7.1. To model and investigate the effects of pathloss, the SSs are set to move altogether along the directions to their corresponding BSs in simulations. As shown in Fig. 7.1, the distance between BS and SS is defined as a unit distance, when SS is located at cell boundary corner. The distance from the SSs to the corner is denoted by ℓ_0 (unit distances). The channel variance $\sigma_{i,j}^2$ between a transmit node i and a receive node j is assumed as $\ell_{i,j}^{-n}$, where n is order of path loss and $\ell_{i,j}$ (unit distances) denote the distance between the nodes i and j . Besides, for the conciseness of performance comparisons, all channels are considered to be block-faded. Namely, the channel coefficients remain unchanged within the period of a packet interval, and change randomly from packet to packet. A packet consists of 100 QPSK symbols.

For conciseness of presentation, the maximum number of ARQ rounds is limited to one in simulations as the improvement over two is not significant enough. In con-

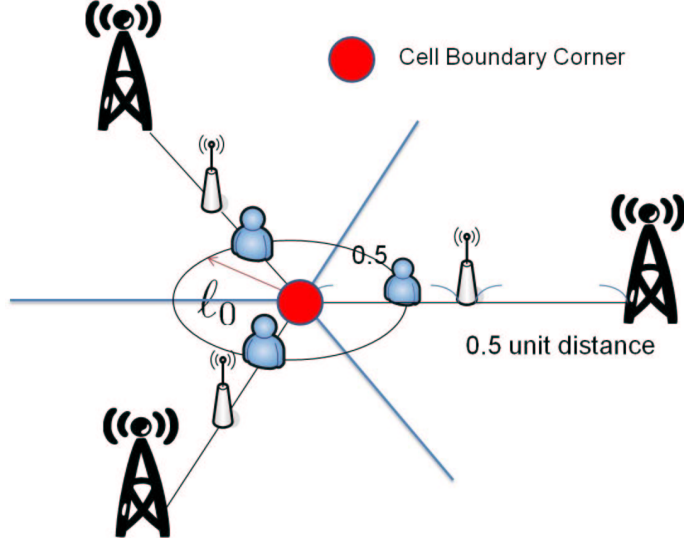


Figure 7.1: An illustration for a subnetwork of 3 SSs.

trast, for CoMP ARQs without the assistance of relays, the BSs perform the CoMP retransmissions by themselves.

To compare the throughput performance subject to a total transmit power P_0 for all the active terminals, the design criterion for CoMP AF BF follows

$$\max_{\mathbf{w}_{r_1}, \dots, \mathbf{w}_{r_N}} \min_i \text{SINR}_i^{(r)}, \quad \text{s.t.} \quad P_r \leq P_0, \quad \forall i \quad (7.1)$$

According to [30], problems of maximizing the minimum SINR such as (7.1) are inverse problems of power minimization problems such as SPMP-RS, by the strict monotonicity argument of the optimal solution of power minimization problems. The optimal objective value of (7.1) can be efficiently found via the 1-dimensional bisection search [30]. Moreover, CoMP BS BF also follows the same criterion, max-min SINR subject to power constraints.

In our previous analysis, we just do not count in the effect of links between BSs and their non-associated RSs. To investigate the rationality of this relaxation, we just test max-min SINR CoMP AF BF under two types of simulation environment at one of which these effects are relaxed and at the other the effect of all links exists. As shown in Fig.

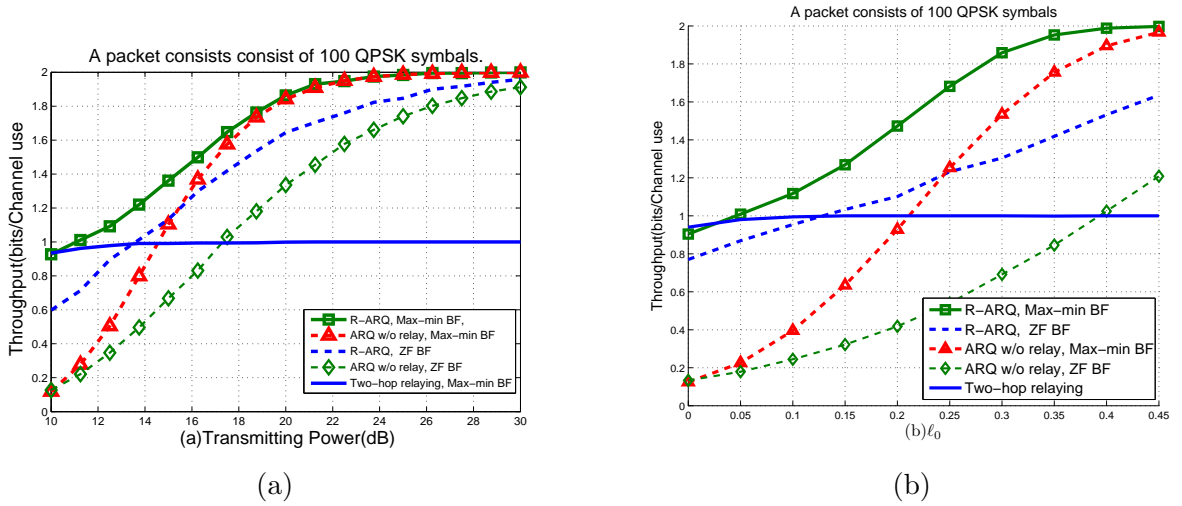


Figure 7.3: Simulated throughput when $N=3$: (a) $\ell_0 = 0$ (b) $P_0 = 10$ dB.

We demonstrate next the effectiveness of the robust CoMP AF BF designs. And the results are averaged over 1000 channel realizations. The SSs are evaluated at the cell edge, *i.e.*, $\ell_0 = 0$. In addition, to assess the feasibility of CoMP AF relaying, a BF scheme is considered infeasible for a certain channel realization if it can not ensure each SSs' SINR requirement for any CSIs error within uncertainty region given 60 dB total transmission power budget; otherwise, it is called a feasible one.

On the other hand, to demonstrate the effectiveness of the robust BFs, non-robust CoMP BFs are also proposed for comparison. Unlike robust BF, non-robust ones just neglect the existence of channel quantization error or only consider the existence of channel quantization error of some links. For clarity, considering $\epsilon_{r,s} > 0$ and $\epsilon_{b,r} = 0$, non-robust AF BFs are designed based on SPMP-RS rather than RSPMP-RS. In addition, considering $\epsilon_{r,s} > 0$ and $\epsilon_{b,r} > 0$, non-robust AF BFs are designed based on SPMP-RS or RSPMP-RS rather than GRSPMP-RS. However, these non-robust BFs are very likely fail to assure QoS and hence are then scaled up according to the procedure of tuning transmit power, which is summarized at Algorithm 2, for fair comparison of feasibility rate with the robust ones.

In Fig. 7.4 and 7.5, we implement non-robust CoMP AF BF based on SPMP-RS

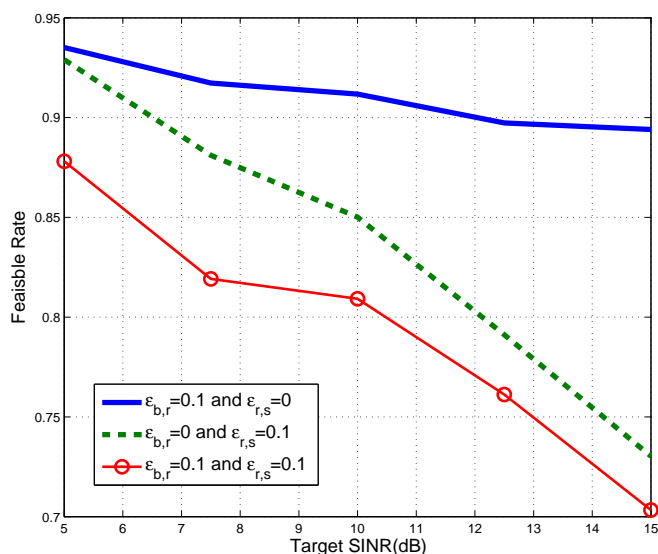


Figure 7.4: Feasible rates versus the target SINR when $N=2$.

under three types of partial CSI model. The results show that the existence of channel uncertainty between RSs and SSs has limited influence on feasible rate at high target SINR, which may show that error of RS-SS links may just dominant source lead to performance loss at high target SINR.

Fig. 7.6 presents the average power consumptions and feasible rates versus the target SINR γ_0 subject to no CSI quantization error of BS-RS links. The results for robust CoMP AF BF are obtained based on the SDR-based approach in Section 5.1. Subplot (a) compares the power consumptions using CoMP AF RS BF and CoMP BS BF with and without the robust criterion. On the other hand, subplot (b) shows the feasible rates of these BF schemes.

Without RSs, BSs are also able to retransmit data. However, apparently, relaying not only can reduce the system power consumption, but also can increase the feasibility to support QoS in ARQs even with the low-complexity AF relaying. Besides, the proposed robust CoMP BF scheme is more power efficient and feasible to support QoS subject to uncertainties in CSI in a multi-cell environment. The advantages of the robust CoMP

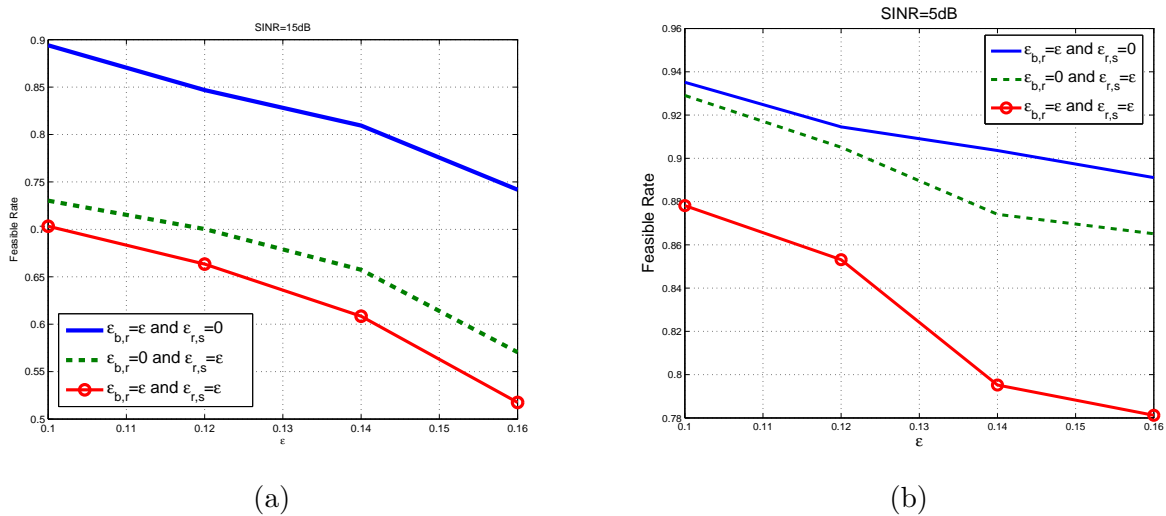


Figure 7.5: Feasible rates versus the upper limit, ϵ , on channel uncertainties when $N=2$: (a) $\gamma_0 = 15\text{dB}$ (b) $\gamma_0 = 5\text{dB}$.

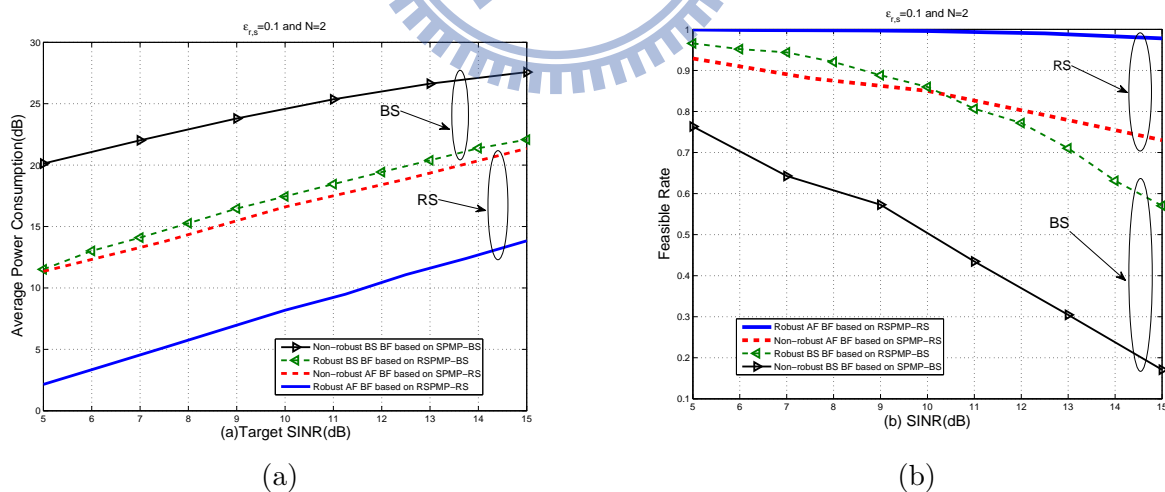


Figure 7.6: Subplot (a) shows the average power consumptions versus the target SINR, γ_0 , and subplot (b) presents the feasible rates versus γ_0 , both at $\epsilon=0.1$.

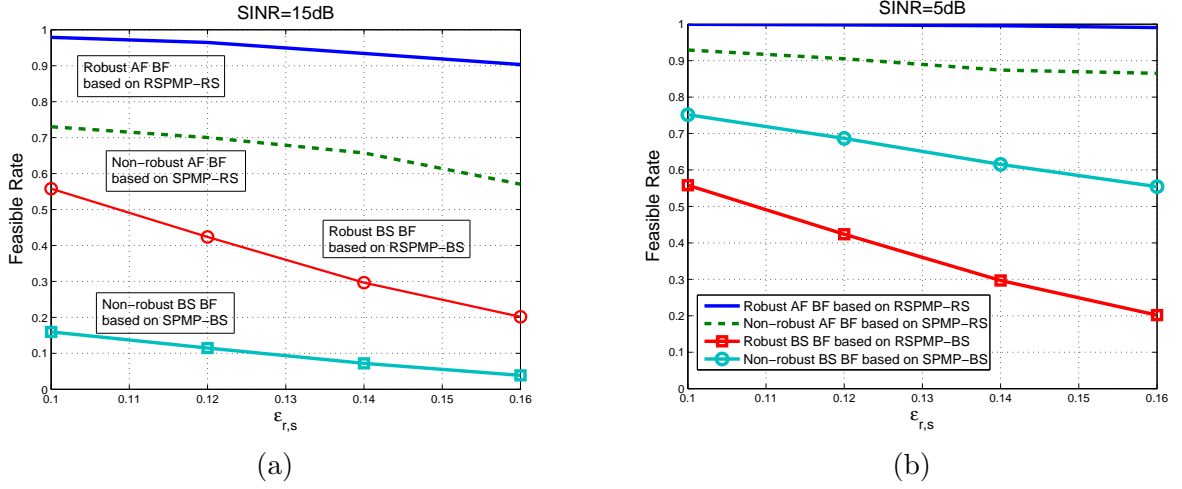


Figure 7.7: Feasible rates versus the upper limit, $\epsilon_{r,s}$, on channel uncertainties when subplot (a) $\gamma_0=15\text{dB}$, and subplot (b) $\gamma_0=5\text{dB}$

BF lie in its two figures that the robust scheme first finds the channel uncertainty which yields the worst SINR, and then minimizing the total transmit power subject to a SINR constraint under the specific channel uncertainty. In contrast, a beamformer designed without this robust mechanism would rather blindly increase its transmit power in order to meet the target SINR for all possible uncertainties, which often ends up enhancing the interferences rather than increasing the desired signal power, leading to a higher power consumption and a lower feasible rate. Fig. 7.7 compares the feasible rates for various schemes versus the upper limit, $\epsilon_{r,s}$, on the 2-norms of the channel uncertainties. Clearly, the robust CoMP BF schemes outperform the non-robust ones in all cases of $\epsilon_{r,s}$.

The feasible rates and power consumptions between the two types of robust CoMP BF scheme based on SDR and iterated QCQP concepts are compared in Fig. 7.8 for $\epsilon_{r,s} = 0.1$. For QCQP-based scheme, the optimization process is considered non-convergent once the number of iterations is greater than 100. As can be seen from the subplot (a) that the feasible rates of QCQP-based scheme in Section 5.2 are only slightly less than that of the SDR-based scheme. Besides, simulations in subplot (b) also show that

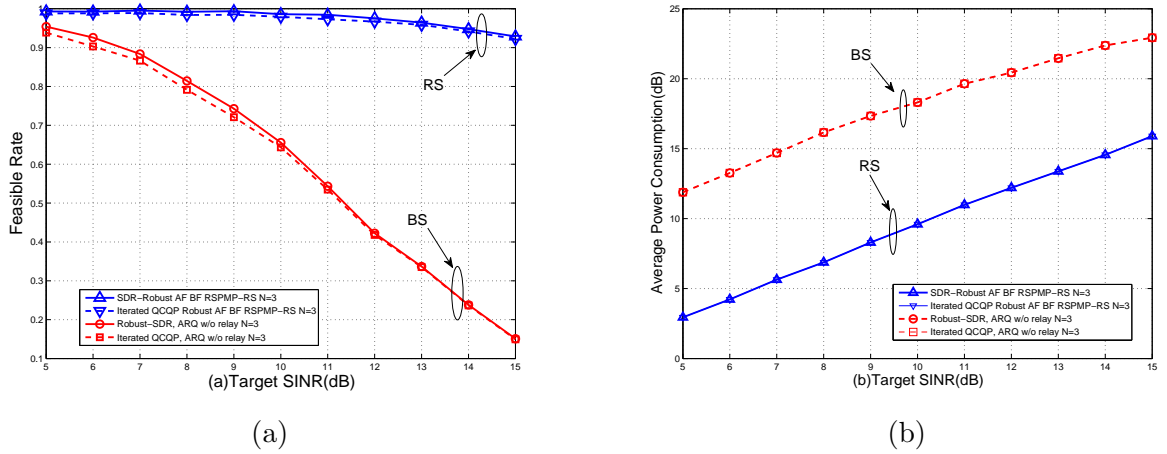


Figure 7.8: Feasible rates and power consumption versus the target SINR, γ_0 , when $\epsilon_{r,s}=0.1$.

the power consumptions of these two schemes are indistinguishable once QCQP-based scheme is feasible. In fact, the beamformers obtained with the SDR approach are optimal if they satisfy the rank-one condition. And by inspection from simulations, the CVX solver [31] seems always to yield the rank-one solutions. Similar observations have been previously reported in [32–34] as well. This indicates that Algorithm 1 is an efficient yet low-complexity sub-optimal solver.

In Fig. 7.9, we presents the average power consumptions and feasible rates versus the target SINR γ_0 subject to accounting for all sources of channel uncertainties. And the performance of non-robust CoMP AF BF based on SPMP-RS, RSPMP-RS and robust CoMP AF BF based on GRSPMP-RS are compared. Clearly, compared to CoMP BSs, relaying still can reduce the system power consumption and can increase the feasibility to support QoS in ARQs, even accounting for all sources of channel uncertainties. Besides, the proposed robust CoMP BF schemes in Chapter 6 apparently outperform the non-robust ones in all cases. Besides, the performance gain between robust one and non-robust one based on SPMP-RS seems to decrease at high SINR, which results from channel uncertainty between BSs and RSs become trivial at high target SINR as shown in the previous discussion and Fig. 7.4.

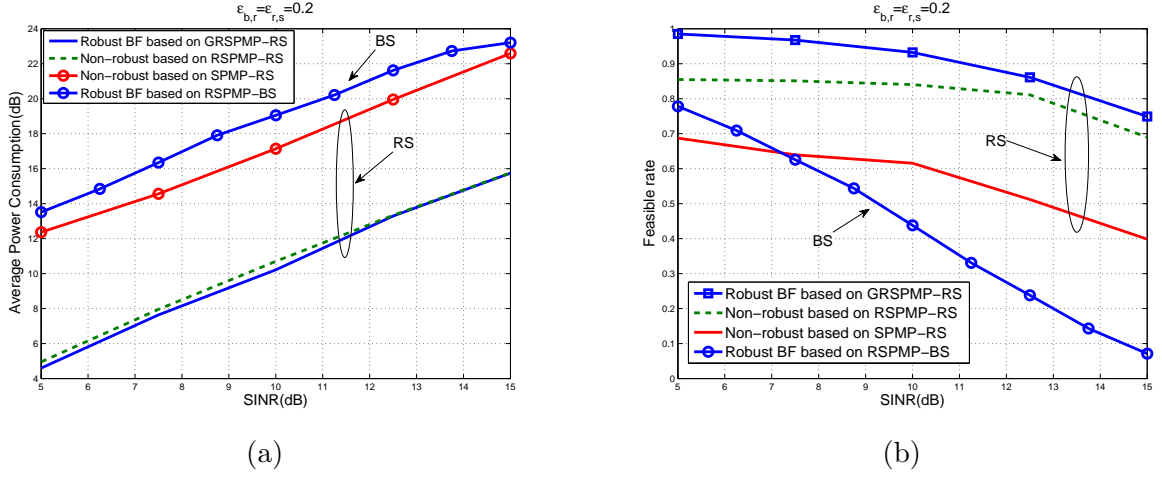


Figure 7.9: Subplot (a) shows the average power consumptions versus the target SINR, γ_0 , and subplot (b) presents the feasible rates versus γ_0 , both at $\epsilon_{b,r}$ and $\epsilon_{r,s}=0.2$.

Algorithm 2 The procedure of tuning power for the non-robust BF

- 1: Given power budget P_b and a BF set $\{W_{r_i}\}_{i=1}^N$ corresponding to total transmit power P_i
 - 2: $W'_{r_i} = \sqrt{\frac{P_b}{P_i}} W_{r_i} \quad \forall i$
 - 3: **if** $\{W'_{r_i}\}_{i=1}^N$ fulfill required SINR for 30000 sets of channel uncertainties **then**
 - 4: Non-robust BF is feasible for the given channel realization.
 - 5: $P_{max} = P_b$
 - 6: $P_{min} = 0$
 - 7: **while** $|P_{max} - P_{min}| > 0$ **do**
 - 8: $P = \frac{P_{max} + P_{min}}{2}$
 - 9: $W'_{r_i} = \sqrt{\frac{P}{P_i}} W_{r_i} \quad \forall i$
 - 10: **if** $\{W'_{r_i}\}_{i=1}^N$ fulfill required SINR for 30000 sets of channel uncertainties **then**
 - 11: $P_{max} = P$
 - 12: **else**
 - 13: $P_{min} = P$
 - 14: **end if**
 - 15: **end while**
 - 16: **else**
 - 17: Non-robust BF is infeasible for the given channel realization.
 - 18: **end if**
-

Chapter 8

Conclusions

Two types of robust CoMP AF BF methods were proposed to support QoS in ARQs for downlink multi-cell networks. The proposed CoMP BF methods do not require data exchanges between different cells of the network, and are able to maintain QoS in ARQs subject to imperfect CSI feedbacks. The SDR-based BF method was shown by simulations to achieve the optimal performance. And the performance of the low-complexity suboptimal scheme based on the iterated QCQP was shown to be very close to that with the SDR approach, and is more friendly for practical implementations. Simulation results show that the downlink throughput for users close to cell boundaries of the multi-cell system can be drastically improved via CoMP AF relaying. Besides, the proposed robust CoMP BF methods not only can reduce the system power consumption, but also can increase the feasibility to support QoS in ARQs even with the low-complexity AF relaying.

Chapter 9

APPENDIX

9.1 APPENDIX A

For simplicity, we only present the proof of Lemma 2, and Lemma 1 is its special case.

To present our robust CoMP AF BF algorithms designed according to RSPMP-RS, we first define a number of notations for matrices to be used frequently in the sequel. Specifically, we have $\mathbf{Q}_i \triangleq \mathbf{h}_{b_i, r_i} \mathbf{h}_{b_i, r_i}^H$, $\mathbf{m}_{j, i} \triangleq ([\mathbf{h}_{b_j, r_j}]_1 \mathbf{h}_{r_j, s_i}^H, \dots, [\mathbf{h}_{b_j, r_j}]_N \mathbf{h}_{r_j, s_i}^H)^H$ and $\tilde{\mathbf{w}}_{r_i} \triangleq \text{vec}(\mathbf{W}_{r_i})$. Following these definitions, the power consumption, P_r , used for the robust CoMP AF BF in RSPMP-RS can be expressed as

$$P_r \triangleq \sum_{i=1}^N [\|\mathbf{W}_{r_i} \mathbf{h}_{b_i, r_i}\|^2 + \sigma^2 \text{trace}(\mathbf{W}_{r_i}^H \mathbf{W}_{r_i})] = \sum_{i=1}^N \tilde{\mathbf{w}}_{r_i}^H [\sigma^2 \mathbf{I}_{N^2} + (\mathbf{Q}_i \otimes \mathbf{I}_N)^*] \tilde{\mathbf{w}}_{r_i} \quad (9.1)$$

where \otimes stands for the Kronecker product. And the SINR in (3.6) can be rewritten as

$$\text{SINR}_i^{(r)} = \frac{\tilde{\mathbf{w}}_{r_i}^H (\mathbf{m}_{i, i} \mathbf{m}_{i, i}^H) \tilde{\mathbf{w}}_{r_i}}{\sum_{j \neq i} \tilde{\mathbf{w}}_{r_j}^H (\mathbf{m}_{j, i} \mathbf{m}_{j, i}^H) \tilde{\mathbf{w}}_{r_j} + \sum_j \tilde{\mathbf{w}}_{r_j}^H (\sigma^2 \mathbf{I}_N \otimes (\mathbf{h}_{r_j, s_i} \mathbf{h}_{r_j, s_i}^H)) \tilde{\mathbf{w}}_{r_j}}. \quad (9.2)$$

Based on (9.1) and (9.2), the design criterion RSPMP-RS can be easily reformulated

into

$$\begin{aligned} \min_{\{\tilde{\mathbf{w}}_{r_i}\}_{i=1}^N} \sum_{j=1}^N \tilde{\mathbf{w}}_{r_j}^H \mathbf{C}_j \tilde{\mathbf{w}}_{r_j} \quad \text{s.t.} \quad & \frac{1}{\gamma_0} \tilde{\mathbf{w}}_{r_i}^H \mathbf{A}_i \tilde{\mathbf{w}}_{r_i} \geq \sum_{j=1}^N \tilde{\mathbf{w}}_{r_j}^H \mathbf{B}_{j,i} \tilde{\mathbf{w}}_{r_j} + \sigma^2, \quad \forall i \\ & \forall \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r, s}, \quad \mathbf{e}_{r_j, s_i} \in \mathbb{C}^{N \times 1}, \quad \forall i, j \end{aligned} \quad (9.3)$$

where $\mathbf{A}_i \triangleq \mathbf{m}_{i,i} \mathbf{m}_{i,i}^H$, $\mathbf{B}_{i,i} \triangleq \sigma^2 \mathbf{I}_N \otimes (\mathbf{h}_{r_i, s_i} \mathbf{h}_{r_i, s_i}^H)$ and $\mathbf{B}_{j,i} \triangleq \sigma^2 \mathbf{I}_N \otimes (\mathbf{h}_{r_j, s_i} \mathbf{h}_{r_j, s_i}^H) + \mathbf{m}_{j,i} \mathbf{m}_{j,i}^H$, $j \neq i$. Besides, $\mathbf{C}_i \triangleq \sigma^2 \mathbf{I}_{N^2} + (\mathbf{Q}_i \otimes \mathbf{I}_N)^*$.

Lagrangian associated with (9.3) is given by

$$L = \sum_{j=1}^N \tilde{\mathbf{w}}_{r_j}^H \mathbf{C}_j \tilde{\mathbf{w}}_{r_j} - \sum_{\mathcal{H}} \sum_{i=1}^N \lambda_{i,k} \left(\frac{1}{\gamma_0} \tilde{\mathbf{w}}_{r_m}^H \mathbf{A}_i \tilde{\mathbf{w}}_{r_i} - \sum_{j=1}^N \tilde{\mathbf{w}}_{r_j}^H \mathbf{B}_{j,i} \tilde{\mathbf{w}}_{r_j} - \sigma^2 \right) \quad (9.4)$$

where $\mathcal{H} = \left\{ \{\mathbf{e}_{r_j, s_i}\}_{i,j=1}^N \mid \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r, s}, \forall i, j \right\}$

Lagrangian zero derivative condition of (9.3) imply that,

$$\left\{ \mathbf{C}_i + \sum_{\mathcal{H}} \sum_{m=1}^N \lambda_{m,k} \mathbf{B}_{i,m} \right\} \tilde{\mathbf{w}}_{r_i} = \sum_{\mathcal{H}} \left(\frac{\lambda_{i,k}}{r_0} \right) \mathbf{m}_{i,i} \mathbf{m}_{i,i}^H \tilde{\mathbf{w}}_{r_i} \quad (9.5)$$

Reordering the $N^2 \times 1$ vectors of above equations and yielding $N \times N$ matrices below

$$\sigma^2 \boldsymbol{\Omega}_i \mathbf{W}_{r_i} = \left\{ \left(\sum_{\mathcal{H}} \frac{\lambda_{i,k} \mathbf{m}_{i,i}^H \tilde{\mathbf{w}}_{r_i}}{r_0} \mathbf{h}_{r_i, s_i} \right) - \boldsymbol{\Omega}'_i \mathbf{W}_{r_i} \mathbf{h}_{b_j, r_j} \right\} (\mathbf{h}_{b_j, r_j})^H \quad (9.6)$$

where $\boldsymbol{\Omega}_i \triangleq \mathbf{I} + \sum_{\mathcal{H}} \sum_{m=1}^N \lambda_{m,k} \mathbf{h}_{r_i, s_m} \mathbf{h}_{r_i, s_m}^H$ and $\boldsymbol{\Omega}'_i \triangleq \mathbf{I} + \sum_{\mathcal{H}} \sum_{m=1, m \neq i}^N \lambda_{m,k} \mathbf{h}_{r_i, s_m} \mathbf{h}_{r_i, s_m}^H$

It can be easily verified $\boldsymbol{\Omega}_i, \forall i$ are full rank and invertible, because the lagrangian variables $\lambda_{m,k}$ are positive. Hence the beamforming matrix \mathbf{W}_{r_i} can be represented as below

$$\mathbf{W}_{r_i} = (\sigma^2 \boldsymbol{\Omega}_i)^{-1} \left\{ \left(\sum_{\mathcal{H}} \frac{\lambda_{i,k} \mathbf{m}_{i,i}^H \tilde{\mathbf{w}}_{r_i}}{r_0} \mathbf{h}_{r_i, s_i} \right) - \boldsymbol{\Omega}'_i \mathbf{W}_{r_i} \mathbf{h}_{b_j, r_j} \right\} \mathbf{h}_{b_j, r_j}^H \quad (9.7)$$

9.2 APPENDIX B

The convergence proofs rely on the following lemma [30]:

Proposition 4. *If $\mathbf{E} \succeq 0$, $\mathbf{F} \succeq 0$ and \mathbf{c} is in the range of E , then*

$$\frac{1}{\mathbf{c}^H (\mathbf{E} + \mathbf{F})^{-1} \mathbf{c}} \geq \frac{1}{\mathbf{c}^H \mathbf{E}^{-1} \mathbf{c}} \quad (9.8)$$

with equality if and only if $\mathbf{F}(\mathbf{E} + \mathbf{F})^{-1} \mathbf{c} = 0$.

This proof is based on the standard function approach introduced in [38], which can be summarized as bellow. Consider the fixed point iteration

$$\lambda_i^{(n+1)} = f_i(\Lambda^{(n)}) \quad \forall i = 1, \dots, N \quad (9.9)$$

If the functions $f_i(\Lambda)$ obey the following properties, then the iteration has a fixed point, it's unique and for any initial $\Lambda^{(0)}$ will converge to it.

(1) *Positivity* - If $\lambda_i > 0 \forall i$, then $f_j(\Lambda) > 0 \forall j$.

$$f_i(\Lambda) = \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i,j} \right)^{-1} \mathbf{h}_{r_i, s_i}} \geq \frac{r_0}{d_i \mathbf{h}_{r_i, s_i}^H (c_i \mathbf{I}_N)^{-1} \mathbf{h}_{r_i, s_i}} > 0 \quad (9.10)$$

where we have used the *Proposition 4* with $\mathbf{E} = c_i \mathbf{I}_N$, $\mathbf{F} = \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i,j}$.

(2) *Monotonicity* - If $\lambda_i \geq \lambda'_i \forall i$, then $f_j(\Lambda) \geq f_j(\Lambda') \forall j$.

$$\begin{aligned} f_i(\Lambda) &= \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i,j} \right)^{-1} \mathbf{h}_{r_i, s_i}} \\ &= \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda'_j \mathbf{B}_{i,j} + \sum_{j=1, j \neq i}^N (\lambda_j - \lambda'_j) \mathbf{B}_{i,j} \right)^{-1} \mathbf{h}_{r_i, s_i}} \end{aligned} \quad (9.11)$$

$$\geq \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda'_j \mathbf{B}_{i,j} \right)^{-1} \mathbf{h}_{r_i, s_i}} = f_i(\Lambda') \quad (9.12)$$

where we have used the *Proposition 4* with $\mathbf{E} = c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j' \mathbf{B}_{i,j}$, $\mathbf{F} = \sum_{j=1, j \neq i}^N (\lambda_j - \lambda_j') \mathbf{B}_{i,j}$.

(3) *Scalability* - If $\alpha > 1$, then $\alpha f_i(\Lambda) > f_i(\alpha \Lambda) \forall i$.

$$\alpha f_i(\Lambda) = \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(\alpha c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \alpha \lambda_j \mathbf{B}_{i,j} \right)^{-1} \mathbf{h}_{r_i, s_i}} \quad (9.13)$$

$$= \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \alpha \lambda_j \mathbf{B}_{i,j} + (\alpha - 1) c_i \mathbf{I}_N \right)^{-1} \mathbf{h}_{r_i, s_i}} \quad (9.14)$$

$$\geq \frac{\gamma_0}{d_i \mathbf{h}_{r_i, s_i}^H \left(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \alpha \lambda_j \mathbf{B}_{i,j} \right)^{-1} \mathbf{h}_{r_i, s_i}} = f_i(\alpha \Lambda) \quad (9.15)$$

where we have used the *Proposition 4* with $\mathbf{E} = c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \alpha \lambda_j \mathbf{B}_{i,j}$, $\mathbf{F} = (\alpha - 1) c_i \mathbf{I}_N$. The the inequality above holds with equality if and only if $\mathbf{F}(\mathbf{E} + \mathbf{F})^{-1} \mathbf{c} = 0$. This situation implies $(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i,j})^{-1} \mathbf{h}_{r_i, s_i} = 0$. Multiplying \mathbf{h}_{r_i, s_i}^H both sides further implies $\mathbf{h}_{r_i, s_i}^H (c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i,j})^{-1} \mathbf{h}_{r_i, s_i} = 0$ and contradicts the positivity property of the matrix $(c_i \mathbf{I}_N + \sum_{j=1, j \neq i}^N \lambda_j \mathbf{B}_{i,j})$.

9.3 APPENDIX C

We only present the proof of Proposition 3, and Proposition 1 is its special case.

Due to the fact that the equalities $v_{i,i} = \frac{\epsilon_{r,s}}{\|\mathbf{h}_{i,i}\|}$ result in trivial solutions, hence the Lagrangian associated with (5.10) is given by

$$\begin{aligned} L &= \sum_{m=1}^N c_m \|\mathbf{w}_{r_m}\|^2 - \sum_{m=1}^N \lambda_m \left\{ \frac{1}{\gamma_0} \mathbf{w}_{r_m}^H \hat{\mathbf{A}}_m \mathbf{w}_{r_m} - \sum_{n=1, n \neq m}^N \mathbf{w}_{r_n}^H \hat{\mathbf{B}}_{n,m} \mathbf{w}_{r_n} - \sigma^2 \right\} \\ &- \sum_{m=1}^N \sum_{n=1}^N \tau_{n,m} \mathbf{w}_{r_n}^H \mathbf{G}_{n,m} \mathbf{w}_{r_n} \end{aligned} \quad (9.16)$$

The KKT conditions are stated as follow :

1) *Feasibility* :

$$\begin{aligned}
\frac{1}{\gamma_0} \mathbf{w}_{r_i}^H \hat{\mathbf{A}}_i \mathbf{w}_{r_i} &\geq \sum_{j=1, j \neq i}^N \mathbf{w}_{r_j}^H \hat{\mathbf{B}}_{j,i} \mathbf{w}_{r_j} + \sigma^2 \\
\tilde{\mathbf{w}}_{r_j}^H \mathbf{G}_{j,i} \tilde{\mathbf{w}}_{r_j} &\geq 0 \\
\lambda_i, \tau_{i,j} &\geq 0, \forall i, j \\
v_{i,i} &\geq \frac{\epsilon_{r,s}}{\|\bar{\mathbf{h}}_{r_i, s_i}\|}, v_{j,i} \geq 0, \forall i, j
\end{aligned} \tag{9.17}$$

2) *Complementary Slackness* :

$$\frac{1}{\gamma_0} \mathbf{w}_{r_i}^H \hat{\mathbf{A}}_i \mathbf{w}_{r_i} = \sum_{j=1, j \neq i}^N \mathbf{w}_{r_j}^H \hat{\mathbf{B}}_{j,i} \mathbf{w}_{r_j} + \sigma^2 \tag{9.18}$$

$$|\bar{\mathbf{h}}_{r_j, s_i}^H \mathbf{w}_{r_j}| = v_{j,i} \|\bar{\mathbf{h}}_{r_j, s_i}\| \|\mathbf{w}_{r_j}\|, \forall i, j. \tag{9.19}$$

3) *Zero derivative* :

Based on (9.16), the expressions in (5.11) and (5.12) can be easily obtained by $\frac{dL}{dw_{j,i}} = 0 \forall i, j$.

Besides, via $\frac{dL}{d\mathbf{w}_{r_i}} = \mathbf{0}$, we have the condition that $\Psi_i \mathbf{w}_{r_i} = \mathbf{0}$ where Ψ_i is defined in the Proposition. The condition is equivalent to $(\Psi_i + \frac{\lambda_i d_i}{\gamma_0} \bar{\mathbf{h}}_{r_i, s_i} \bar{\mathbf{h}}_{r_i, s_i}^H) \mathbf{w}_{r_i} = (\frac{\lambda_i d_i}{\gamma_0} \bar{\mathbf{h}}_{r_i, s_i} \bar{\mathbf{h}}_{r_i, s_i}^H) \mathbf{w}_{r_i}$, which results in

$$\mathbf{w}_{r_i} = \left(\frac{\lambda_i d_i}{\gamma_0} \right) \left(\bar{\mathbf{h}}_{r_i, s_i}^H \mathbf{w}_{r_i} \right) \Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i} \tag{9.20}$$

Replace \mathbf{w}_{r_i} in the complementary conditions (9.18) with (9.20)

$$\left(\frac{\lambda_i d_i}{r_0} \right)^2 \bar{\mathbf{h}}_{r_i, s_i}^H \Phi_{i,i}^{-1} \left(\frac{\hat{\mathbf{A}}_i}{r_0} \right) \Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i} |\delta_i|^2 - \sum_{j \neq i} \left(\frac{\lambda_j d_j}{r_0} \right)^2 \bar{\mathbf{h}}_{r_j, u_j}^H \Phi_{j,j}^{-1} \hat{\mathbf{B}}_{j,i} \Phi_{j,j}^{-1} \bar{\mathbf{h}}_{r_j, u_j} |\delta_j|^2 = \sigma^2 \quad \forall i \tag{9.21}$$

where $\delta_i = \bar{\mathbf{h}}_{r_i, s_i}^H \mathbf{w}_{r_i}$.

The equations above can be rewritten in matrix form

$$\begin{pmatrix} |\delta_1|^2 \\ \vdots \\ |\delta_N|^2 \end{pmatrix} = \mathbf{F}^{-1} \begin{pmatrix} \sigma^2 \\ \vdots \\ \sigma^2 \end{pmatrix} \quad (9.22)$$

where $[\mathbf{F}]_{i,i} = (\frac{\lambda_i d_i}{r_0})^2 \bar{\mathbf{h}}_{r_i, s_i}^H \Phi_{i,i}^{-1} (\frac{\hat{\mathbf{A}}_i}{r_0}) \Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i}$ for the diagonal entries; otherwise, for $i \neq j$, $[\mathbf{F}]_{i,j} = -(\frac{\lambda_j d_j}{r_0})^2 \bar{\mathbf{h}}_{r_j, u_j}^H \Phi_{j,j}^{-1} \hat{\mathbf{B}}_{j,i} \Phi_{j,j}^{-1} \bar{\mathbf{h}}_{r_j, u_j}$.

Moreover, assuming $\{\mathbf{w}_{r_1}^* \dots \mathbf{w}_{r_N}^*\}$ as an optimal beamforming vector pair, it can be easily observed from (5.10) that $e^{j\theta_i} \mathbf{w}_{r_i} \forall i$ is also an optimal beamforming vector, where $\theta_i \forall i$ is an arbitrary phase. Hence, we may assume $\delta_i \geq 0$ without generality and $\bar{\mathbf{h}}_{r_i, s_i}^H \mathbf{w}_{r_i}$ in (9.20) is easily yielded from (9.22).

Following the procedure above, we may observe that the beamforming matrix can be yielded as long as λ_i and $v_{j,i}$ are known. Multiplying the both sides of (9.20) by $\bar{\mathbf{h}}_{r_i, s_i}^H$ leads to $\lambda_i = \frac{\gamma_0}{d_i \bar{\mathbf{h}}_{r_i, s_i}^H \Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i}}$ if $\bar{\mathbf{h}}_{r_i, s_i}^H \mathbf{w}_{r_i} \neq 0, \forall i$.

With the above results, we know that $\|\mathbf{w}_{r_i}\|^2 = (\frac{\lambda_i d_i}{\gamma_0})^2 \bar{\mathbf{h}}_{r_i, s_i}^H \Phi_{i,i}^{-2} \bar{\mathbf{h}}_{r_i, s_i} |\bar{\mathbf{h}}_{r_i, s_i}^H \mathbf{w}_{r_i}|^2$, and at the same time, to satisfy the complementary slackness condition $|\bar{\mathbf{h}}_{r_i, s_i}^H \mathbf{w}_{r_i}|^2 = v_{i,i}^2 \|\bar{\mathbf{h}}_{r_i, s_i}\|^2 \|\mathbf{w}_{r_i}\|^2$, we have $v_{i,i} = \frac{\gamma_0}{\lambda_i d_i \|\bar{\mathbf{h}}_{r_i, s_i}\| \|\Phi_{i,i}^{-1} \bar{\mathbf{h}}_{r_i, s_i}\|}, \forall i$.

Similarly, by subtracting $\lambda_j c_i \bar{\mathbf{h}}_{r_i, u_j} \bar{\mathbf{h}}_{r_i, u_j}^H \mathbf{w}_{r_i}$ on the both sides of $\Psi_i \mathbf{w}_{r_i} = 0$, we have another equation $(\Psi_i - \lambda_j c_i \bar{\mathbf{h}}_{r_i, u_j} \bar{\mathbf{h}}_{r_i, u_j}^H) \mathbf{w}_{r_i} = -\lambda_j c_i \bar{\mathbf{h}}_{r_i, u_j} \bar{\mathbf{h}}_{r_i, u_j}^H \mathbf{w}_{r_i}$, which leads to $\mathbf{w}_{r_i} = -(\lambda_j c_i) \Phi_{i,j}^{-1} \bar{\mathbf{h}}_{r_i, u_j} \bar{\mathbf{h}}_{r_i, u_j}^H \mathbf{w}_{r_i}$. With this result, we have $\|\mathbf{w}_{r_i}\|^2 = (\lambda_j c_i)^2 \bar{\mathbf{h}}_{r_i, u_j}^H \Phi_{i,j}^{-2} \bar{\mathbf{h}}_{r_i, u_j} |\bar{\mathbf{h}}_{r_i, u_j}^H \mathbf{w}_{r_i}|^2$, and meanwhile, to satisfy the condition $|\bar{\mathbf{h}}_{r_i, u_j}^H \mathbf{w}_{r_i}|^2 = v_{i,j}^2 \|\bar{\mathbf{h}}_{r_i, u_j}\|^2 \|\mathbf{w}_{r_i}\|^2$ for the complementary slackness, we obtain $v_{i,j} = \frac{1}{\lambda_j c_i \|\bar{\mathbf{h}}_{r_i, u_j}\| \|\Phi_{i,j}^{-1} \bar{\mathbf{h}}_{r_i, u_j}\|}, j \neq i, \forall i, j$.

The amount of transmit power is derived below.

$$\begin{aligned}
P_r &= \sum_{m=1}^N c_m \|\mathbf{w}_{r_m}\|^2 \\
&\stackrel{(a)}{=} \sum_{i=1}^N \mathbf{w}_{r_i}^H \left(\frac{\lambda_i}{r_0} \hat{\mathbf{A}}_i - \sum_{m=1, m \neq i}^N \lambda_m \hat{\mathbf{B}}_{i,m} \right) \mathbf{w}_{r_i} + \sum_{i=1}^N \mathbf{w}_{r_i}^H \left(\sum_{m=1}^N \tau_{i,m} \mathbf{G}_{i,m} \right) \mathbf{w}_{r_i} \\
&\stackrel{(b)}{=} \sum_{i=1}^N \frac{\lambda_i}{r_0} \mathbf{w}_{r_i}^H \hat{\mathbf{A}}_i \mathbf{w}_{r_i} - \sum_{i=1}^N \lambda_i \sum_{m=1, m \neq i}^N \mathbf{w}_{r_m}^H \hat{\mathbf{B}}_{m,i} \mathbf{w}_{r_m} \\
&\stackrel{(c)}{=} \sum_{i=1}^N \lambda_i \left(\frac{1}{r_0} \mathbf{w}_{r_i}^H \hat{\mathbf{A}}_i \mathbf{w}_{r_i} - \sum_{m=1, m \neq i}^N \mathbf{w}_{r_m}^H \hat{\mathbf{B}}_{m,i} \mathbf{w}_{r_m} \right) = \sum_{i=1}^N \lambda_i \sigma^2 \tag{9.23}
\end{aligned}$$

where the equality (a) results from zero derivative condition and (b), (c) both results from complementary slackness condition.

9.4 APPENDIX D

Considering RSPMP-RS with nonzero $\epsilon_{b,r}$ and $\epsilon_{r,s}$, an equivalent one is written below

$$\begin{aligned}
\min_{p_r, \{\mathbf{w}_{r_i}\}_{i=1}^N} p_r \quad \text{s.t.} \quad & P_r \leq p_r \\
& \text{SINR}_i^{(r)} \geq \gamma_0, \quad \forall i \\
& \forall \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r,s}, \quad \forall i, j \\
& \forall \|\mathbf{e}_{b_i, r_i}\| \leq \epsilon_{b,r}, \quad \forall i
\end{aligned} \tag{9.24}$$

And Lagrangian associated with (9.24) is given by

$$L = p_r - \sum_{\mathcal{R}} \left\{ \tau_k \left(p - \sum_{j=1}^N \tilde{\mathbf{w}}_{r_j}^H \mathbf{C}_j \tilde{\mathbf{w}}_{r_j} \right) + \sum_{i=1}^N \lambda_{i,k} \left(\frac{1}{\gamma_0} \tilde{\mathbf{w}}_{r_m}^H \mathbf{A}_i \tilde{\mathbf{w}}_{r_i} - \sum_{j=1}^N \tilde{\mathbf{w}}_{r_j}^H \mathbf{B}_{j,i} \tilde{\mathbf{w}}_{r_j} - \sigma^2 \right) \right\} \tag{9.25}$$

where $\mathcal{R} = \left\{ \left(\{\mathbf{e}_{r_j, s_i}\}_{i,j=1}^N, \{\mathbf{e}_{b_i, r_i}\}_{i=1}^N \right) \mid \|\mathbf{e}_{r_j, s_i}\| \leq \epsilon_{r,s}, \|\mathbf{e}_{b_i, r_i}\| \leq \epsilon_{b,r} \quad \forall i, j \right\}$

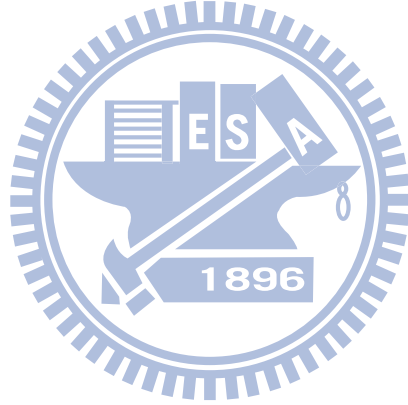
Following similar procedure as APPENDIX A, Lagrangian zero derivative condition

of (9.24) imply that

$$\mathbf{W}_{r_i} = \mathbf{\Pi}_i \left(\sum_{\mathcal{R}} \Delta_i (\bar{\mathbf{h}}_{b_i, r_i}^H + \mathbf{e}_{b_i, r_i})^H \right)^{\epsilon_{b_i, r_i} \rightarrow 0} \approx \mathbf{\Pi}_i \left(\sum_{\mathcal{R}} \Delta_i \right) \bar{\mathbf{h}}_{b_i, r_i}^H \quad (9.26)$$

where $\mathbf{\Pi}_i \triangleq \left(\sigma^2 \mathbf{I} + \sigma^2 \sum_{\mathcal{R}} \sum_{m=1}^N \lambda_{m,k} \mathbf{h}_{r_i, u_m} \mathbf{h}_{r_i, u_m}^H \right)^{-1}$, and

$$\Delta_i \triangleq \left(\frac{\lambda_{i,k} \mathbf{m}_{i,i}^H \tilde{\mathbf{w}}_{r_i}}{r_0} \mathbf{m}_{i,i} - \sum_{m=1, m \neq i}^N \lambda_{m,k} \mathbf{h}_{r_i, u_m} \mathbf{h}_{r_i, u_m}^H \mathbf{W}_{r_i} \mathbf{h}_{b_i, r_i} - \mathbf{W}_{r_i} \mathbf{h}_{b_i, r_i} \right)$$



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