action times and, hence, higher gains may be achieved at these wavelengths.

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CONVOLUTIONAL CODING FOR BPSK SYSTEM WITH ORTHOGONAL SOUNDING TONE ON FAST RAYLEIGH FADING CHANNEL

Indexing terms: Codes, Digital communication systems

For combating the fast Rayleigh fading usually encountered in mobile communication channels, convolutional coding and Viterbi decoding together with interleaving and the BPSK system using an orthogonal sounding tone are considered. The bit error probability performance of the system is analysed, and numerical results for the systems with rate-$\frac{1}{2}$ optimum codes of constraint lengths $K = 3$ to 7 are also provided.

Introduction: Digital communications over land mobile channels usually suffer from fast Rayleigh fading. The envelope of the mobile radio signal is Rayleigh-distributed and the phase of the received signal is uniformly distributed from 0 to 2π. Moreover, the time derivative of the random phase varies with time in a random manner (random FM), depending on both the vehicle speed and the carrier frequency. Phase decorrelation caused by random FM prohibits the use of a coherent receiver and creates a large error floor for a noncoherent receiver. To remove random FM, a BPSK system with an orthogonal sounding tone has been proposed by Yokoyama.[1] The performance analysis of such a system was further simplified by Davarian.[2] Analysis indicates that the bit error rate performance of the system is worse by 3 dB than that of the coherent BPSK in a slow and nonselective Rayleigh fading environment, when the power is equally divided between the BPSK signal and the sounding tone. Instead of the usually used diversity techniques, convolutional coding and Viterbi decoding with interleaving are considered here for further improvement of the system performance. To make interleaving practical even at low vehicle speed, an oscillating antenna technique[4] can be used. In the following, the bit error probability performance of the coded system is analysed under the assumption of full interleaving, and numerical results for the systems with rate-$\frac{1}{2}$ codes of constraint lengths $K = 3$ to 7 are also provided.

System description: A data sequence encoded by the rate-R convolutional encoder (Fig. 1) is mapped into a channel symbol sequence $x = \{x_i\}$, where x_i is either $+1$ or -1, corresponding to 1 or 0, respectively, of the encoder output bit. After full interleaving, the interleaved output sequence $\tilde{x}$ is transmitted to the discrete channel. Details of the discrete channel are depicted in Fig. 2. The modulator and the demodulator shown are those proposed by Yokoyama.[1] Channel fading imposes a random amplitude variation $a(t)$ and phase variation $\theta(t)$ on to the transmitted signal $s(t)$. In addition to being plagued by fading, $s(t)$ is also corrupted by the additive white Gaussian noise (AWGN) $n(t)$ with one-sided power spectral density N_0. The tone envelope extractor is used to extract the channel fading amplitude. The outputs of the discrete channel are de-interleaved. The Viterbi decoder with channel amplitude measurement performs maximum-likelihood (ML) decoding and recovers the transmitted data.

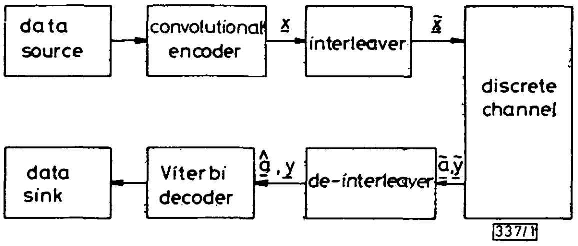

Fig. 1 *Block diagram of system under consideration*

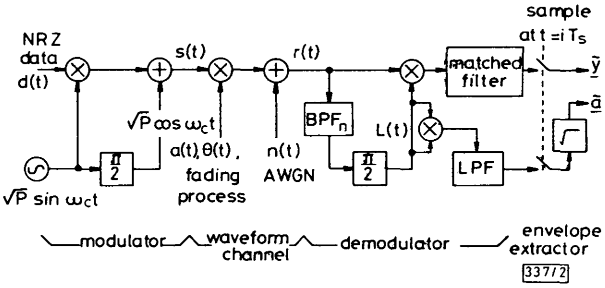

Fig. 2 *Details of discrete channel shown in Fig. 1*

Performance analysis: The received signal $r(t)$ can be expressed by

$$\begin{aligned} r(t) &= a(t)\sqrt{P}\, d(t) \sin\,[\omega_c t + \theta(t)] && \text{(faded BPSK)} \\ &\quad + a(t)\sqrt{P} \cos\,[\omega_c t + \theta(t)] && \text{(faded tone)} \\ &\quad + n(t) && (1) \end{aligned}$$

where P is the average transmitted power and $d(t)$ is the NRZ formated information-bearing waveform. To permit undistorted detection of the faded tone, the tone extraction filter BPF_n bandwidth W must be equal to $2f_D$, where f_D denotes the maximum Doppler shift. Since in practice f_D is smaller than the BPSK signalling rate $R_s = 1/T_s$, Yokoyama[1] has made the assumption that $W \ll R_s$. As a consequence of this assumption, the influence of noise on the extracted tone may be ignored and the extracted tone after a $\pi/2$ phase shift can be written as

$$L(t) = a(t)\sqrt{P} \sin\,[\omega_c t + \theta(t)] \qquad (2)$$

Hence the envelope extractor with output sequence $\tilde{\boldsymbol{a}} = \{\tilde{a}_i\}$, where $\tilde{a}_i = \surd(P/2)a_i$ and $a_i = a(iT_s)$, makes a perfect channel amplitude measurement. By employing Davarian's analysis result, the normalised matched filter output after de-interleaving is derived. It is given by

$$y_i = -\left(\frac{E_s}{N_0}\right)\hat{a}_i x_i + N_i \tag{3}$$

where $E_s = PT_s$ is the energy per channel symbol, $\{N_i\}$ is an identical independent (IID) sequence of Gaussian random variates with zero mean and unit variance, and $\{\hat{a}_i\}$, the de-interleaved version of $\{a_i\}$, is an IID sequence of random variates with density function given by

$$f(\hat{a}_i) = 2\hat{a}_i e^{-a_i^2} \qquad \hat{a}_i \geq 0 \tag{4}$$

The decoding rule for the Viterbi decoder with perfect channel amplitude measurement is as follows: the transmitted sequence $\boldsymbol{x} = \{x_i\}$ is selected and all the other possible sequences $\boldsymbol{x}' = \{x'_i\}$ are rejected if $\sum_i \hat{a}_i y_i x_i > \sum_i \hat{a}_i y_i x'_i$. Based on such a decoding rule and with the decision variable specified by eqn. 3, the bit error probability upper bound of the coded system can be derived in a similar manner as that presented by Dunham and Tzou.[5] For a rate-$R = b/n$ convolutional code, the bit error probability P_b is bounded by

$$P_b \leq \frac{1}{b} P_d Z^{-d} \left.\frac{\partial T(D, I)}{\partial I}\right|_{I=1,\, D=Z} \tag{5}$$

where $T(D, I)$ and d are the conventional augmented generating function and the minimum free distance of the code employed, respectively. The quantity Z is given by

$$Z = \frac{1}{1 + \dfrac{RE_b}{2N_0}} \tag{6}$$

and the term P_d is given by

$$P_d = E_A\left\{Q\left[\sqrt{\left(\frac{RE_b}{N_0}\sum_{i=1}^{d}\hat{a}_i^2\right)}\right]\right\} \tag{7}$$

Here

$$Q(z) = \frac{1}{\surd(2\pi)}\int_z^\infty e^{-w^2/2}\,dw \tag{8}$$

There is a closed-form expression for eqn. 7, which can be expressed as[6]

$$P_d = \left(\frac{1-u}{2}\right)^d \sum_{j=0}^{d-1}\binom{d-1+j}{j}\left(\frac{1+u}{2}\right)^j \tag{9}$$

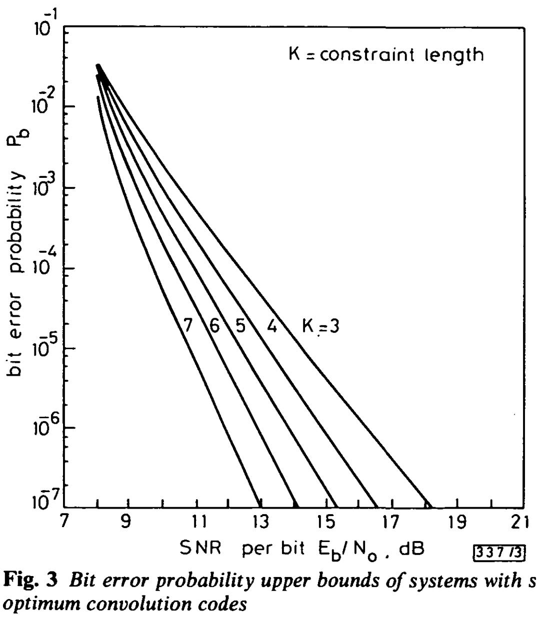

Fig. 3 *Bit error probability upper bounds of systems with selected rate-$\frac{1}{2}$ optimum convolution codes*

where

$$u = \sqrt{\left(\frac{RE_b/N_0}{2 + RE_b/N_0}\right)} \tag{10}$$

Note that $RE_b = E_s$, where E_b is the energy per source bit.

The performance bound (eqn. 5) is evaluated numerically for the systems with selected rate-$\frac{1}{2}$ optimum convolutional codes of constraint lengths $K = 3$ to 7. The results obtained are plotted in Fig. 3.

Discussion: We have derived the bit error probability performance bound for the coded system by ignoring the influence of channel noise on the extracted tone, and equally dividing the power between the BPSK signal and the sounding tone. However, such power splitting is usually not optimum. Davarian[2] has shown that, when the R_s/W ratio is taken into consideration and the noise influence on the extracted tone is not ignored, the error performance of the uncoded system can be improved by optimising the fraction of power consumed by the sounding tone. Hence the BER performance of the coded system may also be improved.

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IMPURITY ANALYSIS OF HEAVY METAL FLUORIDE GLASS FIBRES USING SELECTIVELY EXCITED PHOTOLUMINESCENCE

Indexing terms: Optical fibres, Photoluminescence, Optical measurement

A technique based on selectively excited fibre impurity luminescence (SEFIL) has been developed and successfully applied to the quantitative analysis of Pr^{3+}, Nd^{3+} and Fe^{3+} in heavy metal fluoride glass optical fibres. With fibre standards, SEFIL provides accurate analysis at impurity levels well below the requirements for long-distance fibre applications.

Heavy metal fluoride (HMF) glasses have significant technological potential for long-distance, repeaterless fibre-optic applications because their theoretical minimum loss is 10–100 times lower than that of silica. To approach these values requires that the concentrations of many transition metals and rare earths be reduced to 1 part in 10^9 or below.[1,2] Although still far from achieving these levels of purity, efforts at purifying HMF glass starting materials are often handicapped by the lack of adequate materials analysis. Loss spectrum measurements are one of the more sensitive impurity analysis