Quasi–Two-Dimensional Simulation of Scour and Deposition in Alluvial Channels

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ABSTRACT: Most existing sediment routing models are one-dimensional models and hence they cannot be used to simulate lateral channel bed deformations. Conversely, a two-dimensional model is computationally time consuming, and is not suitable for long-term simulation. Using the stream tube concept, a one-dimensional model is extended to a quasi–two-dimensional model in this study. It is capable of simulating lateral variations of channel cross section through the adjustments of stream tube boundaries. The model has an option to treat the suspended load and bed load separately and hence is able to simulate the deposition behavior of the suspended sediment under a nonequilibrium process. The model also provides options to solve either de Saint Venant equation or energy equation and hence can be applied in both steady and unsteady flow conditions. An assessment of this model's performance has been conducted through a comparison to an analytical solution and a set of experimental data. The application of this model to the Keelung River and the Shiemen Reservoir in Taiwan also gave convincing results.

INTRODUCTION

Through the history of civilization, humankind has tended to concentrate its activity in river basins. This has led naturally to the imposition of man-made changes on rivers, through the construction and operation of dams and reservoirs for flood control, hydropower generation, river training, navigation, waste disposal in rivers, etc. Yet the response of rivers to such interventions often creates new problems, possibly diminishing the utility of the river as a resource and often negating the beneficial effects associated with the original plan.

The evolution of the river bed in alluvial channels has been studied by many researchers using analytical and numerical approaches. The analytical approach is insufficient for a natural river study. With rapid growth in computer technology, numerical models have become a popular means for studying mobile-bed hydraulics.

During the past decade, several numerical models have been developed. Most of the computer codes, such as HEC2SR (Simons & Li Assoc., Inc. 1980), FLUVIAL-12 (Chang and Hill 1976), HEC6 (Thomas and Prashum 1977), IALLUVIAL (Karim and Kennedy 1982), SEDICOUP (Holly and Rahuel 1990), BRALLUVIAL (Holly et al. 1985), CHARIMA (Holly et al. 1990), and ONED3X (Lai 1987), stayed with a one-dimensional approach. The BRALLUVIAL and CHARIMA codes were developed for simulating bed evolution of a branched and looped channel system. One-dimensional simulation of bed evolution in alluvial rivers is not sufficiently mature. Many key factors related to physical phenomena such as nonuniformity of the sediment and modeling techniques still need in-depth study and investigation.

In spite of the immaturity of model development, researchers still apply their efforts to extend mobile bed computational techniques to quasi-two-, two-, and even three-dimensional algorithms to meet the engineering need of solving real-life river

problems. Example models of this type are GSTARS (Molinas and Yang 1986), TABS2 (Thomas et al. 1984), and MOBED2 (Spasojevic 1988). Multidimensional codes are often developed for solving a particular or local problem. TABS2 is a horizontal two-dimensional uncoupled model for the mobile-bed channels. MOBED2 is an unsteady horizontal two-dimensional coupled model which solves the governing equations for the water and nonuniform sediment movements simultaneously in one time step. GSTARS is a quasisteady semi-two-dimensional model with the use of a stream tube concept to simulate the bed evolution process. However, in this model bed load and suspended load are not treated separately, hence it can not reflect the nonequilibrium deposition of the suspended sediment.

To remedy the shortcomings of GSTARS, a new model is developed in the present paper. This newly developed model, named USTARS, uses the stream tube concept and includes the capability of simulating the movement of suspended load and bed load, and their interactions. Hence, it is able to simulate deposition patterns of the suspended sediment in a nonequilibrium process. The model also provides options in choosing to solve either de Saint Venant equation or energy equation, and hence is applicable to both steady and unsteady flow conditions. The details of the mathematical basis and numerical techniques are described in the following sections. Experimental data and analytical solution are used to assess the model's capability. The model is applied to simulate the bed evolution of the Keelung River and Shiemen Reservoir in Taiwan to demonstrate its engineering applicability.

THEORETICAL BASIS AND GOVERNING EQUATIONS

Stream Tube

Stream tubes are imaginary tubes bounded by streamlines. A schematic diagram for the stream tube configuration is shown in Fig. 1. Since the velocity vectors are tangential to the streamlines, no convective exchange occurs across streamlines. In the present study, a one-dimensional hydrodynamic calculation is performed first to determine the hydraulic characteristics of the full-channel cross-section, and then the channel is divided into a certain number of subsections or stream tubes within a section based on the principle of equal conveyance. The mobile-bed computation is then performed in each tube to calculate the channel-bed evolutions, and hence it is able to reflect the lateral variations of channel cross-sections. In this model, the number of tubes can be assigned arbitrarily. It is expected that the use of more tubes would give a better

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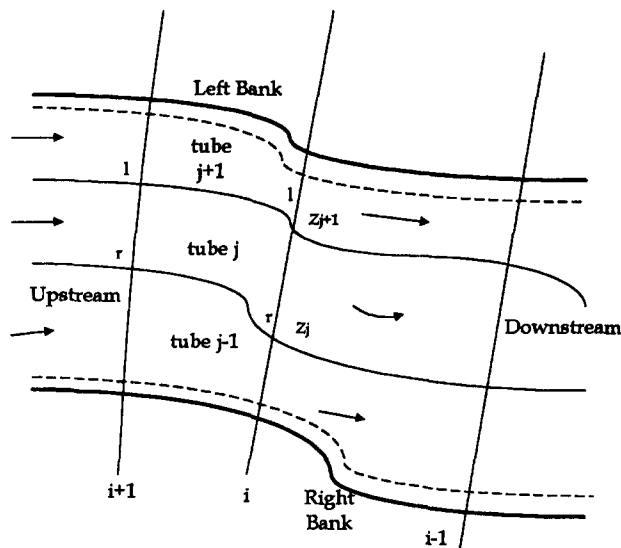
FIG. 1. Schematic Diagram of Stream Tube Configuration

two-dimensional interpretation, although the flow and sediment transport characteristics are still treated as one dimensional in each stream tube. The stream tube concept is applied in both steady and unsteady flow calculations, however, due to longitudinal variations of the discharge, the tube width will be different longitudinally in the unsteady flow computation. Cunge and Holly (1980) used the stream tube concept to simulate the pollutant transport along a channel. Molinas and Yang (1986) applied a similar concept for the development of a mobile bed model with total load transport. In the present paper, the stream tube concept is adopted to develop a mobile bed model with the capability of simulating bed load, suspended load, and their interactions. Governing equations and the computation procedures are described in the subsequent sections.

Equations for Hydraulic Routing

Two options, steady and unsteady, are available for hydraulic routing. Although an unsteady flow model is more general than a steady flow model, most engineers are more familiar with a HEC-6 type quasisteady model, hence both options are presented. The governing equations are listed as follows.

Unsteady Flow Computation

The de Saint Venant equations are used in the unsteady flow computation. These include a continuity equation and a one-dimensional momentum equation

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\alpha \frac{Q^2}{A}\right) + gA\left(\frac{\partial y}{\partial x}\right) + gAS_f - \frac{Q}{A}q = 0 \tag{2}$$

where A = channel cross-sectional area; Q = flow discharge; t = time; x = coordinate in flow direction; q = lateral inflow/outflow discharge per unit length; α = momentum correction coefficient; g = gravitational acceleration; y = water surface elevation; $S_f(=Q|Q|/K^2)$ = friction slope; $K[=(A/n)R^{2/3}]$ = channel conveyance; n = roughness coefficient of Manning's formula; and R = hydraulic radius.

Steady Flow Computation

The one-dimensional energy equation, which is given below, is solved in the steady flow computation

$$\frac{d}{dx}\left(y + \alpha \frac{Q^2}{2gA^2}\right) = -S_f \tag{3}$$

The lateral inflow/outflow discharge is included in Q, and hence Q will be different longitudinally if there exists lateral inflow or outflow. It can be seen more clearly in the difference form of this equation, which will appear in (17).

Equations for Sediment Routing

Based on the results of the hydraulic routing, the channel is divided into a designated number of stream tubes, and then sediment routing is performed in every stream tube. The boundaries of the stream tubes are recomputed in every time step and to reflect the lateral movement of the sediment particles. Most of the sediment routing models adopt the total load equations to calculate the sediment transport capacity and thus can only be applied to the equilibrium conditions. To reflect the deposition characteristics of the suspended sediment in a nonequilibrium flow condition, such as reservoir, separate treatment of the suspended load and bed load is necessary. Both measures are available in this model and they are described separately in the following sections.

Total Load Equation Method

This method includes a sediment continuity equation and equations for calculating the total sediment transport rate. The sediment continuity equation is given as follows:

$$\frac{\partial Q_s}{\partial x} + (1 - p)\frac{\partial A_{dt}}{\partial t} = 0 \tag{4}$$

where Q_s = total sediment load; A_{dt} = amount of sediment deposition/scouring per unit length of stream tube; and p = channel bed porosity. Three formulas, namely Yang's (1973, 1984), Ackers and White (1973), and Engelund and Hansen (1967), are available to calculate the sediment discharge Q_s.

Separate Treatment Method

This method includes a sediment continuity equation, a sediment concentration convection-diffusion equation and a bed load equation. The Rouse number $W/\kappa u_*$, where W = fall velocity; κ = Karman's constant; and u_* = shear velocity, is used to distinguish between bed load and suspended load. Particle with $W/\kappa u_* > 5$ is treated as bed load and particle with $W/\kappa u_* \leqq 5$ is treated as suspended load. The sediment continuity equation is given as

$$(1 - p)\frac{\partial A_{dt}}{\partial t} + \frac{\partial}{\partial x}\sum_{k=1}^{N\,size}[q_t C_k] + \frac{\partial Q_b}{\partial x} = 0 \tag{5}$$

where Q_b = bed load transport rate in stream tube; q_t = flow discharge in stream tube; and C_k = depth-averaged concentration of suspended sediment of size fraction k in stream tube. The concentration C_k is calculated using the convection-diffusion equation shown as

$$\frac{\partial(C_k A_t)}{\partial t} + \frac{\partial}{\partial x}(C_k q_t) = \frac{\partial}{\partial x}\left(A_t k_x \frac{\partial C_k}{\partial x}\right) + S_k + \left.\left(hk_z \frac{\partial C_k}{\partial z}\right)\right|_r^t \tag{6}$$

where k_x and k_z = longitudinal and transverse dispersion coefficients; A_t = area across stream tube; h = flow depth; and S_k = source term of suspended sediment of size fraction k. A similar sediment continuity equation was proposed by Holly and Rahuel (1990). In their formulation the bed-sediment exchange is through the source term which will appear in (6). Both approaches were tested in the present study. Holly and Rahuel's equation tended to underestimate the scouring and

deposition quantities and (5) rendered a much better simulation and hence was chosen.

According to Van Rijn (1984) and Holly and Rahuel (1990), the source term S_k is the combination of deposition and resuspension, and can be expressed as

$$S_k = S_{ek} + S_{dk} = a - bC_k \qquad (7)$$

where S_{ek} and S_{dk} = quantities of sediment resuspension and deposition, respectively. The amount of sediment resuspension can be calculated by

$$S_{ek} = \rho B_t W_k \beta_k C_{ek} \qquad (8)$$

where B_t = width of stream tube; ρ = sediment-water mixture density; W_k = fall velocity of sediment of size fraction k; β_k = weight percentage of sediment of size fraction k; and C_{ek} = sediment concentration close to channel bed, which can be calculated by the equation proposed by Van Rijn (1984)

$$C_{ek} = 0.015 \frac{D_k}{a} \frac{T_k^{1.5}}{D_*^{0.3}} \qquad (9)$$

where D_k = particle diameter of size fraction k; D_* = particle parameter = $D_{50}\{[(s - 1)g]/v^2\}^{1/3}$; T_k = transport stage parameter = $[(u'_*)^2 - (u_{*cr})^2]/(u_{*cr})^2$; v = water kinematic viscosity; s = specific weight of sediment particle; u'_* = grain shear velocity = $g^{0.5}u/c'$; c' = Chezy coefficient related to grains = 18 $\log(12R_t/3D_{90})$; R_t = hydraulic radius in stream tube; and u_{*cr} = critical shear velocity.

The amount of sediment deposition can be calculated by using the following equation (Holly and Rahuel 1990):

$$S_{dk} = -\rho B_t W_k C_{dk} \qquad (10)$$

where C_{dk} = deposition concentration, which can be estimated by

$$C_{dk} = [3.25 + 0.55 \ln(W_k/\kappa u_*)]C_k \qquad (11)$$

with $\kappa = 0.4$.

The bed load transport rate Q_b can be calculated using the following equation:

$$Q_b = \int_l^r q_b \, dB \qquad (12)$$

where q_b = bed load discharge/unit width, which is calculated using Meyer-Peter and Müller formula, and r and l = right and left boundaries of stream tube.

Armoring Scheme

Most river beds consist of grains with a broad size fraction. If the flow over such a bed is depleted of sediment, fine particles are entrained more easily and the bed surface will become progressively coarser. Ultimately, an armor coat of large particles may form, and that stops further degradation. During the aggradation process, layers of sediment will be deposited on the bed surface and the bed surface will be progressively finer. To update the bed composition at every time step is necessary and crucial to a sediment routing model. Various techniques dealing with bed composition variation have been proposed. In the present study, the model adopts the conventional sorting and armoring techniques which were proposed by Bennet and Nordin (1977). In that model the bed is divided into several layers, and bed deposition accounting is accomplished through the use of two or three armor layers depending on whether scouring or deposition occurs at the cross section during the time step.

COMPUTATIONAL PROCEDURES

The simulation processes consist of three parts in every time step, i.e., flow computations, stream tube computations, and
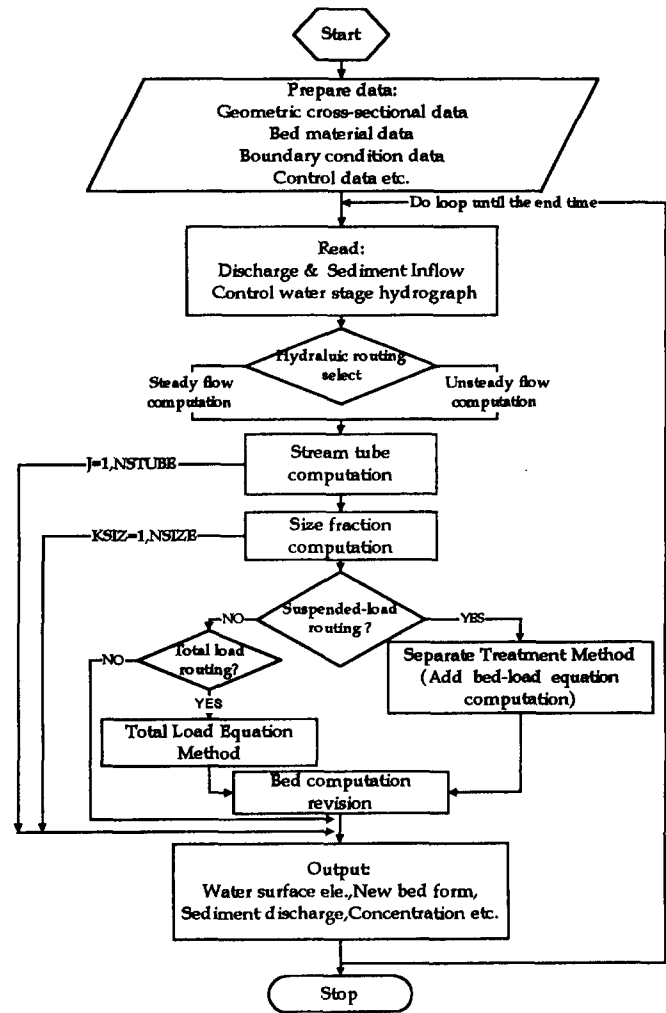
FIG. 2. Flowchart of Model

sediment routing. Flow computation is performed first to provide the basis for setting tube boundaries. Then, sediment routing is performed for each stream tube to calculate the amount of channel bed variations. The computational flow chart is shown in Fig. 2. These procedures are described sequentially in the following sections.

Hydraulic Routing

Unsteady Flow Computation

Eqs. (1) and (2) are transformed into difference equations using a Preissmann four point finite difference scheme. The difference equations are shown as

$$\frac{\phi}{\Delta t}(A_{i+1}^{n+1} - A_{i+1}^n) + \frac{(1 - \phi)}{\Delta t}(A_i^{n+1} - A_i^n) + \frac{\theta}{\Delta x}(Q_{i+1}^{n+1} - Q_i^{n+1})$$

$$+ \frac{(1 - \theta)}{\Delta x}(Q_{i+1}^n - Q_i^n) - q_i = 0 \qquad (13)$$

$$\frac{\phi}{\Delta t}(Q_{i+1}^{n+1} - Q_{i+1}^n) + \frac{1 - \phi}{\Delta x}(Q_i^{n+1} - Q_i^n)$$

$$+ 2\left\{ \alpha\theta\left[(1 - \phi)\frac{Q_i^{n+1}}{A_i^{n+1}} + \phi\frac{Q_{i+1}^{n+1}}{A_{i+1}^{n+1}} \right] \right.$$

$$\left. + \alpha(1 - \theta)\left[(1 - \phi)\frac{Q_i^n}{A_i^n} + \phi\frac{Q_{i+1}^n}{A_{i+1}^n} \right]\right\}$$

$$\cdot \left[\frac{\theta}{\Delta x}(Q_{i+1}^{n+1} - Q_i^{n+1}) + \frac{1 - \theta}{\Delta x}(Q_{i+1}^n - Q_i^n) \right]$$

$$- \alpha \left\{ \theta \left[(1 - \phi)\frac{Q_i^{n+1}}{A_i^{n+1}} + \phi\frac{Q_{i+1}^{n+1}}{A_{i+1}^{n+1}} \right]^2 \right.$$

$$\left. + (1 - \theta)\left[(1 - \phi)\frac{Q_i^n}{A_i^n} + \phi\frac{Q_{i+1}^n}{A_{i+1}^n} \right]^2 \right\}$$

$$\cdot \left[\frac{\theta}{\Delta x}(A_{i+1}^{n+1} - A_i^{n+1}) + \frac{1 - \theta}{\Delta x}(A_{i+1}^n - A_i^n) \right]$$

$$+ g\{\theta[(1 - \phi)A_i^{n+1} + \phi A_{i+1}^{n+1}] + (1 - \theta)[(1 - \phi)A_i^n + \phi A_{i+1}^n]\}$$

$$\cdot \left[\frac{\theta}{\Delta x}(y_{i+1}^{n+1} - y_i^{n+1}) + \frac{1 - \theta}{\Delta x}(y_{i+1}^n - y_i^n) \right]$$

$$+ g\{\theta[(1 - \phi)A_i^{n+1} + \phi A_{i+1}^{n+1}] + (1 - \theta)[(1 - \phi)A_i^n + \phi A_{i+1}^n]\}$$

$$\cdot \left\{ \theta \left[\beta\frac{Q_i^{n+1}|Q_i^{n+1}|}{(K_i^{n+1})^2} + (1 - \beta)\frac{Q_{i+1}^{n+1}|Q_{i+1}^{n+1}|}{(K_{i+1}^{n+1})^2} \right] \right.$$

$$\left. + (1 - \theta)\left[\beta\frac{Q_i^n|Q_i^n|}{(K_i^n)^2} + (1 - \beta)\frac{Q_{i+1}^n|Q_{i+1}^n|}{(K_{i+1}^n)^2} \right] \right\}$$

$$- \left\{ \theta \left[(1 - \phi)\frac{Q_i^{n+1}}{A_i^{n+1}} + \phi\frac{Q_{i+1}^{n+1}}{A_{i+1}^{n+1}} \right] \right.$$

$$\left. + (1 - \theta)\left[(1 - \phi)\frac{Q_i^n}{A_i^n} + \phi\frac{Q_{i+1}^n}{A_{i+1}^n} \right] \right\} q_i = 0 \qquad (14)$$

where i and $i + 1$ = downstream and upstream ends of simulated reach, respectively. The equations are then solved using a double sweep method to obtain the flow discharge and the water surface elevations.

Steady Flow Computation

The discretized energy equation, written for two adjacent point i and $i + 1$ along a channel is given as

$$y_i^{n+1} + \frac{\alpha_i}{2g}\left(\frac{Q_i^{n+1}}{A_i^{n+1}}\right)^2 = y_{i+1}^{n+1} + \frac{\alpha_{i+1}}{2g}\left(\frac{Q_{i+1}^{n+1}}{A_{i+1}^{n+1}}\right)^2 + (x_{i+1} - x_i)$$

$$\cdot [\beta_i S_{fi}^{n+1} + (1 - \beta_i)S_{fi+1}^{n+1}] \qquad (15)$$

To account for the lateral inflow/outflow, the relation of the discharges between two adjacent points is shown as

$$Q_i = Q_{i+1} + q_i(x_{i+1} - x_i) \qquad (16)$$

where q_i = lateral inflow/outflow discharge per unit length in section i.

This equation together with (13) are again solved using a double sweep method to obtain the discharge and the water surface elevation.

Stream Tube Computation

The algorithm for stream tube computation used for this model is the same as that of the GSTARS model developed by Molinas and Yang (1986). Following the initial flow computations at each computational point, stream tube locations across the channel satisfying equal conveyance requirements can be determined. To do this, the channel section at every computational point is first divided into ten subsections with equal width. The conveyance for each subsection is computed and summed to obtain the total conveyance for the section. Once the number of tubes is selected, the conveyance for each tube is obtained by dividing the section conveyance by the number of tubes. Then one determines the boundary for each tube based on the known conveyance of

the ten subsections. After the number and the location of stream tube are known, the sediment routing procedure is carried out for each tube along the channel in each time step. The transport processes in each tube are strictly one dimensional.

Sediment Routing

Total Load Equation Method

The difference equation of the sediment continuity equation for every size fraction, i.e., (4), is shown as

$$\Delta Z_{ki} = \frac{8\Delta t(Q_{ski-1} - Q_{ski})}{(1 - p)(2P_i + P_{i-1} + P_{i+1})(\Delta X_i + \Delta X_{i-1})} \qquad (17)$$

where ΔZ_{ki} = variation of bed elevation on section number i for size fraction k; and P_i = wetted parameter. The total variations of the channel bed elevations can be obtained by summarizing ΔZ_{ki} for every size fraction.

Separate Treatment Method

The difference equation for the sediment continuity equation, i.e., (5) is shown as

$$\Delta Z_{ki} = \frac{-4\Delta t}{(1 - p)(2P_i + P_{i+1} + P_{i-1})}$$

$$\cdot \left[\frac{\displaystyle\sum_{k=1}^{Nsize}[qC_k]_i - \sum_{k=1}^{Nsize}[qC_k]_{i-1} + Q_{bki} - Q_{bki-1}}{(\Delta X_i + \Delta X_{i-1})/2} \right] \qquad (18)$$

The concentration C_k is obtained by solving the convection-diffusion equation, i.e., (6). The split operator approach is used in solving this equation. The governing equation is separated into four portions, i.e., advection, longitudinal diffusion, transverse diffusion, and reaction. They are solved subsequently in one time step. The C_k and CX_k, $CX_k = \partial C_k/\partial x$, values obtained in the previous portion are served as the known values for the next portion. The computational techniques are described as the following: (To simplify the expression C is used to replace C_k from here on.)

Advection-Step. The advection portion of (6) can be written as

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = 0 \qquad (19)$$

where U = average velocity.

Using the Holly-Preissmann two-point four-order scheme, the difference equation of (19) can be obtained, when the Courant Number is less than 1

$$C_{i+1}^{n+1} = C_\xi^n = a_1C_i^n + a_2C_{i+1}^n + a_3CX_i^n + a_4CX_{i+1}^n \qquad (20)$$

where $a_1 = r'^2(3 - 2r')$; $a_2 = 1 - a_1$; $a_3 = r'^2(1 - r')\Delta x$; $a_4 = -r'(1 - r')^2\Delta x$; and $r' = (U_i^n\Delta t)/\Delta x$.

Differentiating (19) with respect to x, the difference equation can be obtained

$$CX_{i+1}^{n+1} = CX_\xi^n\frac{1 - \dfrac{\Delta t}{2}\dfrac{\partial U}{\partial x}\Big|_{\xi, t^n}}{1 + \dfrac{\Delta t}{2}\dfrac{\partial U}{\partial x}\Big|_{i+1, t^{n+1}}} \qquad (21)$$

where $CX_\xi^n = b_1C_i^n + b_2C_{i+1}^n + b_3CX_i^n + b_4CX_{i+1}^n$; $b_1 = [6r'(r' - 1)]/\Delta x$; $b_2 = -b_1$; $b_3 = r'(3r' - 2)$; and $b_4 = (r' - 1)(3r' - 1)$.

When Courant Number is greater than 1, the difference equation of (19) can be obtained

$$C_{i+1}^{n+1} = C_i^\tau = A_1 C_i^n + A_2 C_i^{n+1} + A_3 CX_i^n + A_4 CX_i^{n+1} \quad (22)$$

where $A_1 = s'^2(3 - 2s')$; $A_2 = 1 - A_1$; $A_3 = -U_i^n s'^2(1 - s')\Delta t$; $A_4 = U_i^{n+1} s'(1 - s')^2 \Delta t$; and $s' = (\Delta x/U_{i+1}^{n+1})/\Delta t$.

Differentiating (19) with respect to t, and then transforming CT to CX, the difference equation is obtained

$$CX_{i+1}^{n+1} = CX_i^\tau \frac{U_i^\tau + \dfrac{s'\Delta t}{2} \left.\dfrac{\partial U}{\partial t}\right|_{i,\tau}}{U_{i+1}^{n+1} - \dfrac{s'\Delta t}{2} \left.\dfrac{\partial U}{\partial t}\right|_{i+1,\tau^{n+1}}} \quad (23)$$

where $CX_i^\tau = B_1 C_i^n + B_2 C_i^{n+1} + B_3 CX_i^n + B_4 CX_i^{n+1}$; $B_1 = [-6s'(s' - 1)]/\Delta t U_i^\tau$; $B_2 = -B_1$; $B_3 = (U_i^n/U_i^\tau)s'(3s' - 2)$; and $B_4 = (U_i^{n+1}/U_i^\tau)(s' - 1)(3s' - 1)$.

Longitudinal Diffusion Step. The longitudinal diffusion portion of (6) can be written as

$$\frac{\partial C}{\partial t} - \frac{1}{A}\frac{\partial}{\partial x}\left(Ak_x \frac{\partial C}{\partial x}\right) = 0 \quad (24)$$

Using the Crank-Nicholson central difference method, (24) can be discretized as

$$C_i^{n+1} - C_i^n = f_1(C_{i+1}^{n+1} - C_i^{n+1}) - f_2(C_i^{n+1} - C_{i-1}^{n+1}) \quad (25)$$

where

$$f_1 = \frac{\Delta t}{\dfrac{1}{2}(x_{i+1} - x_{i-1})A_i} \frac{\left[\dfrac{1}{2}(A_i k_{xi} + A_{i+1}k_{xi+1})\right]}{(x_{i+1} - x_i)}$$

$$f_2 = \frac{\Delta t}{\dfrac{1}{2}(x_{i+1} - x_{i-1})A_i} \frac{\left[\dfrac{1}{2}(A_i k_{xi} + A_{i-1}k_{xi-1})\right]}{(x_i - x_{i-1})}$$

Differentiating (24) with respect to x, and then using the Tee scheme, the difference equation becomes

$$CX_i^{n+1} - CX_i^n = g_1 CX_{i+1}^{n+1} - (g_2 + g_3)CX_i^{n+1} + g_4 CX_{i-1}^{n+1} \quad (26)$$

where

$$g_1 = \frac{\Delta t}{\dfrac{1}{2}(A_{i+1} + A_i)} \frac{A_{i+1}k_{xi+1}}{\dfrac{1}{2}(x_{i+1} - x_{i-1}) \cdot (x_{i+1} - x_i)}$$

$$g_2 = \frac{\Delta t}{\dfrac{1}{2}(A_{i+1} + A_i)} \frac{A_i k_{xi}}{\dfrac{1}{2}(x_{i+1} - x_{i-1}) \cdot (x_{i+1} - x_i)}$$

$$g_3 = \frac{\Delta t}{\dfrac{1}{2}(A_{i-1} + A_i)} \frac{A_i k_{xi}}{\dfrac{1}{2}(x_{i+1} - x_{i-1}) \cdot (x_i - x_{i-1})}$$

$$g_4 = \frac{\Delta t}{\dfrac{1}{2}(A_{i-1} + A_i)} \frac{A_{i-1}k_{xi-1}}{\dfrac{1}{2}(x_{i+1} - x_{i-1}) \cdot (x_i - x_{i-1})}$$

The values of C and CX can be obtained by using Gaussian Elimination Method to solve the tri-diagonal matrix formed by (25) and (26).

Transverse-Diffusion Step. The transverse-diffusion portion of (6) can be written as

$$\frac{\partial C}{\partial t} = \frac{1}{A}\left.\left(hk_z \frac{\partial C}{\partial z}\right)\right|_r^l \quad (27)$$

Using the same method as the longitudinal diffusion step, (27) can be discretized as

$$C_{i,j}^{n+1} - C_{i,j}^n = r_1(C_{i,j+1}^{n+1} - C_{i,j}^{n+1}) - r_2(C_{i,j}^{n+1} - C_{i,j-1}^{n+1}) \quad (28)$$

where

$$r_1 = \frac{\Delta t}{\dfrac{1}{2}(A_{i,j+1} + A_{i,j})} \frac{\dfrac{1}{2}(h_{i,j}k_{zi,j} + h_{i,j+1}k_{zi,j+1})}{(z_{i,j+1} - z_{i,j})}$$

$$r_2 = \frac{\Delta t}{\dfrac{1}{2}(A_{i,j+1} + A_{i,j})} \frac{\dfrac{1}{2}(h_{i,j}k_{zi,j} + h_{i,j-1}k_{zi,j-1})}{(z_{i,j} - z_{i,j-1})}$$

Differentiating (27) with respect to x, and then using the Tee scheme, the difference equation is shown as

$$-r_1 \cdot CX_{i,j+1}^{n+1} + (r_1 + r_2 + 1)CX_{i,j}^{n+1} - r_2 CX_{i,j-1}^{n+1} = CX_{i,j}^n + q_1 + q_2 \quad (29)$$

where

$$q_1 = -\frac{(A_{i+1,j} - A_{i,j})}{\Delta x}\cdot\frac{1}{A_{i,j}}[r_1(C_{i,j+1}^{n+1} - C_{i,j}^{n+1}) - r_2(C_{i,j}^{n+1} - C_{i,j-1}^{n+1})]$$

$$q_2 = -\frac{h_{i+1,j}k_{zi+1,j} - h_{i,j}k_{zi,j}}{\Delta x}\frac{1}{(h_{i,j}k_{zi,j})}[r_1(C_{i,j+1}^{n+1} - C_{i,j}^{n+1}) - r_2(C_{i,j}^{n+1} - C_{i,j-1}^{n+1})]$$

The values of C and CX can be obtained by using Gaussian Elimination Method to solve the tri-diagonal matrix formed by (28) and (29).

Reaction Step. The reaction portion of (6) is shown as

$$\frac{\partial C}{\partial t} = a_p - b_p C \quad (30)$$

where $a_p = a/A$ and $b_p = b/A$. There exists an analytical solution for (30), and shown as

$$C_i^{n+1} = (a_p/b_p) + (C_i^n - a_p/b_p)e^{-b_p \Delta t} \quad (31)$$

Differentiating (30) with respect to x and the difference expression form for CX_i^{n+1} can be obtained as

$$CX_i^{n+1} = \frac{1}{(1 + b_{pi}\Delta t)}\left[CX_i^n + \left(\frac{a_{pi} - a_{pi-1}}{\Delta x}\right)\Delta t \right.$$
$$\left. - \left(\frac{b_{pi} - b_{pi-1}}{\Delta x}\cdot C_i^{n+1}\right)\Delta t\right] \quad (32)$$

BOUNDARY CONDITIONS

For water flow computation, the imposition of a discharge hydrograph is usually needed at the upstream boundary, whereas a stage hydrograph or a rating curve is required at the downstream boundary. For sediment routing, both the inflowing suspended-load concentration and the bed-load discharge must be imposed at the upstream boundary. The longitudinal diffusion process needs one more boundary condition at the downstream end, for which one can apply a Neumann diffusion condition. Usually, a zero concentration gradient is used. At the bank boundary, a zero concentration gradient is used for the computation of the lateral diffusion process. At the internal boundary between neighboring tubes, zero discharge flux is assumed; hence there is no sediment convected across the tube. Since the tubes are divided with equal conveyance,

604 / JOURNAL OF HYDRAULIC ENGINEERING / JULY 1997

the water, suspended and bed load sediment inflow discharges should be assigned equally for all tubes at the upstream boundary.

MODEL PERFORMANCE ASSESSMENT

The numerical scheme for suspended load computation is first assessed by comparing it with an analytical solution. A sample of suspended sediment with Gaussian distribution is released at the inlet of a rectangular channel, and the model is used to predict the concentration 10 and 20 min later. A time interval equal to 20 s is selected to compute concentration in this case. The results are shown in Fig. 3, which indicate that the simulation results do not have any unexpected numerical oscillation or numerical diffusion problems. The algorithm performs well for the simple channel case.

A set of experimental data from Lee and Yu (1991) is used to further verify the suspended load computation procedure of this newly developed model. In a series of their experiments, an idealized backwater zone of a reservoir is formed in the flume. A sediment inflow with a certain concentration is released at the upstream end of the flume. Then the longitudinal and vertical variations of sediment concentrations and hydraulic characteristics were measured. Since the transverse variation of the flow pattern is small, only one stream tube is used in the simulation. The sediment concentration profile along the flume after a certain time period is shown in Fig. 4. The simulation results tend to overestimate in the upstream reach of the flume, but the overall agreement is quite satisfactory.
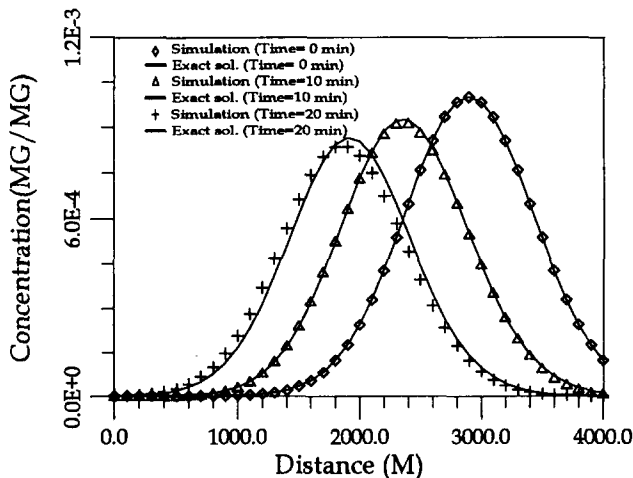

FIG. 3. Longitudinal Variations of Sediment Concentration, Compared with Analytical Solution
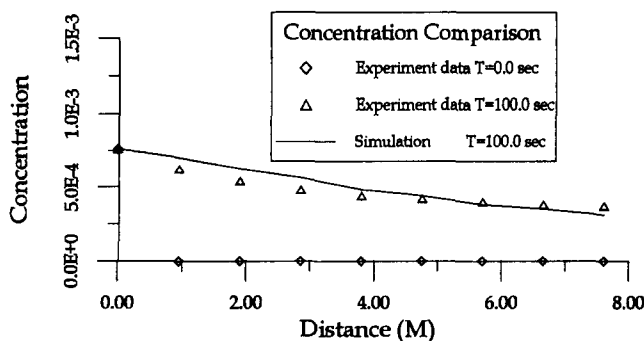

FIG. 4. Comparison of Simulated Sediment Concentration with Experimental Data

APPLICATION TO KEELUNG RIVER

The Keelung River, located in the Taipei area of Taiwan, is one of the main branches of the Tan-Hsui River system. The Taipei area through which the Tan-Hsui River passes has a population of about six million. Therefore, effective management and flood control of the Tan-Hsui River system is one of the most important tasks in Taiwan. In the past few decades, many construction projects and studies have been completed in an attempt to maintain river stability. However, due to variable river characteristics such as rapid slope variation, a wide range of bed material size distribution along the channel, complex river morphology, and the rapidly rising peak flood condition, any man-made activities can significantly affect the river behavior. Detailed analysis and simulation along the longitudinal and lateral variations of the Keelung River channel bed are needed, which renders the one-dimensional mobile bed model insufficient. Therefore, the stream tube mobile bed model with quasi−two-dimensional capability appears to be useful. The Keelung River study reach is selected to test the model's applicability.

Data Preparation

The study reach is about 23 km long and is in the estuarian area. The location map is shown in Fig. 5. Field data from 1989, including geometric cross-sectional data and bed material data, are used as the initial condition. The average slope of the study reach is about $1.24 * 10^{-4}$, and the average mean particle size is about 0.05 mm. Detailed cross-sectional data and the bed material composition for each computational point will not be given here for brevity.

The test simulation was performed using data between 1989 to 1993. Data from 1989 to 1990 were used to calibrate the model and data from 1990 to 1993 were used for verification. Past experience and field observation indicate that there is insignificant sediment transport for flow discharges less than 100 m^3/s. Therefore, only discharges greater than 100 m^3/s are selected as the input inflow conditions. The up-stream inflow suspended sediment concentrations versus the inflow water discharge rating curve obtained by the Taiwan Provincial Water Conservancy Bureau, $Q_s = 4.532Q^{1.13}$, are used for the upstream boundary condition. At the downstream boundary, the measured stage hydrographs were used as the downstream boundary conditions. The measured cross-sectional data in 1990 were used to compare with the simulation results as the basis for the parameter examination. A time interval of 1 h was used in the simulations.

Parameter Examination

n Value

For the flow simulation, the energy loss coefficient is the key parameter for the model calibration. The measured water
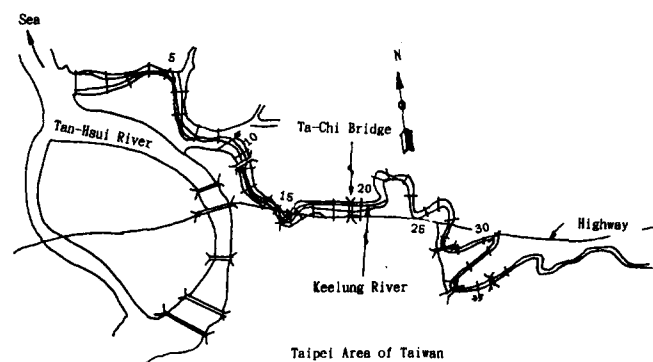

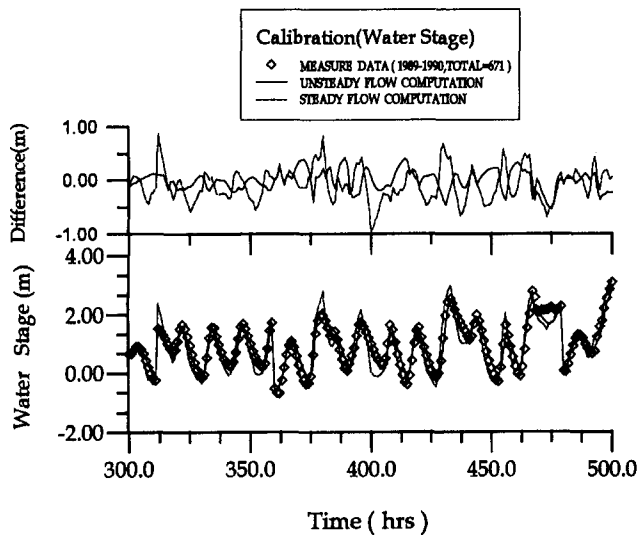**FIG. 5. Location Map of Keelung River**

FIG. 6. Comparisons of Simulated Stage Hydrograph with Field Data at Ta-Chi Bridge (1990)

stage data from the gauge station at Ta-Chi Bridge were used to calibrate the Manning's n value of the model. The simulation results are given in Fig. 6. The overall agreements are good. Both unsteady and steady flow computation options are applied, the steady flow model tends to overestimate the water surface elevations and the unsteady flow model slightly underestimates the water stages. Since the Keelung River is an estuarian river, the unsteady flow model is recommended.

Diffusion Coefficient

The diffusion coefficients in both longitudinal and lateral directions have to be examined for the simulation of suspended load transport. In practice, the diffusivity calibration may require setting up a sufficient number of stations inside the study reach for collecting enough suspended sediment concentration distribution data under various flow conditions. However, due to the lack of sufficient field data, no calibration was made to justify the values of longitudinal and transverse diffusivity. The simple relations $k_x = 5.93u_*h$ and $k_z = 0.23u_*h$ suggested by Elder (1959) were used in the present study.

Sensitivity Analyses

Due to lack of sediment transport data, sensitivity analyses were performed first to choose the total load equations. Three different formulas, namely, Yang's formula, Ackers-White formula and Engelund-Hansen formula, are available in the model. The results are shown in Fig. 7. It was found that Yang's formula rendered the best simulation, and hence it was chosen. Sensitivity analyses were also performed to investigate the influence of the stream tube numbers on the simulation results. Three different tube numbers, 1, 3, and 5, were investigated. It was found that using 3 and 5 tubes rendered the best simulation results and hence 5 tubes was chosen in the simulation. The simulated longitudinal bed profiles and cross-sectional channel bed profiles of section 30 are shown in Figs. 8 and 9, respectively. Investigations were also performed to study influences of using different sediment treatment measures, namely, total load equation method and separate treatment method. Since Keelung River is an estuarine river, the Elders (1959) relations are not applicable to calculate k_x and k_z, so that the total load equation method rendered the more accurate results and hence the total load equation method was chosen in the simulation. The comparisons are shown in Figs. 10 and 11, respectively.
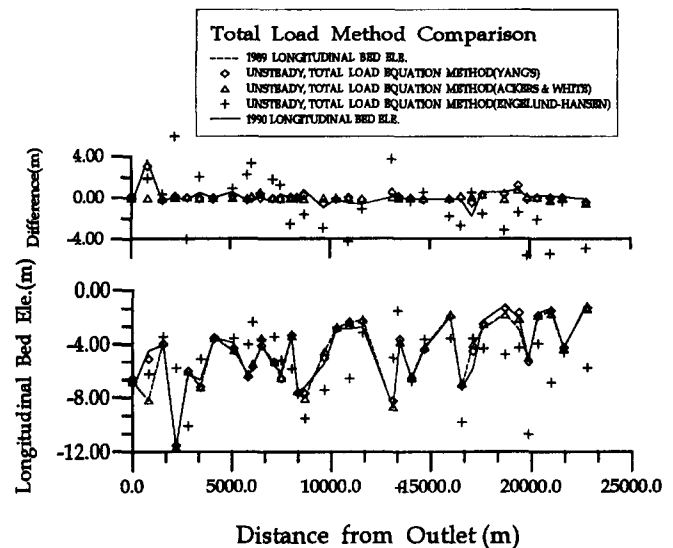
FIG. 7. Comparisons of Simulated Longitudinal-Bed Elevations with Field Data Using Different Total Load Equations
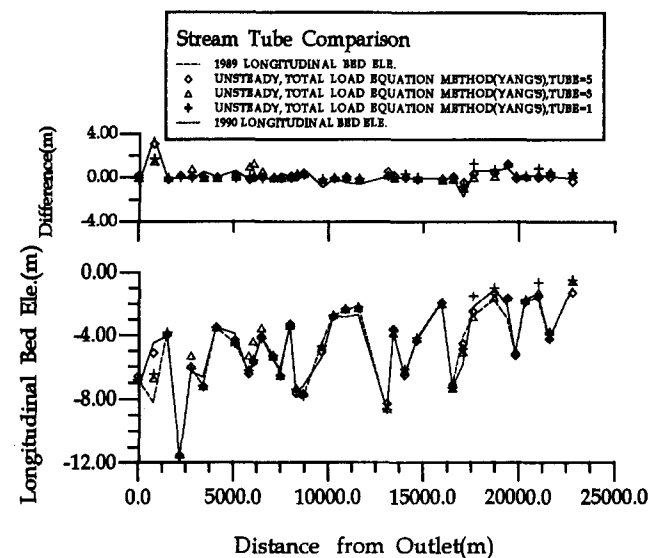

FIG. 8. Comparisons of Simulated Longitudinal-Bed Elevations with Field Data Using Different Stream Tube Numbers
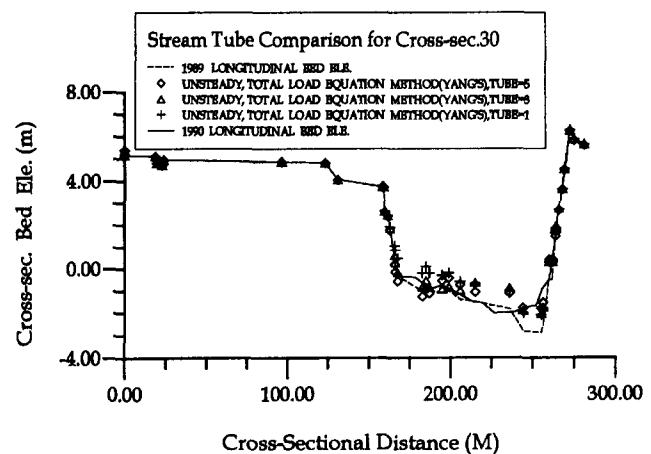

FIG. 9. Comparisons of Simulated Cross Sectional-Bed Elevations with Field Data Using Different Stream Tube Numbers (Section 30)
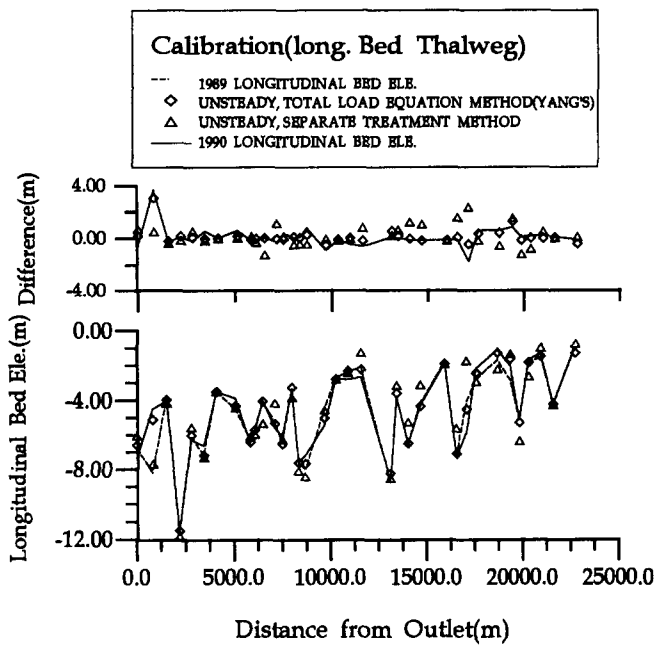
FIG. 10. Comparisons of Simulated Longitudinal-Bed Elevations with Field Data Using Different Sediment Routing Measures (1990)
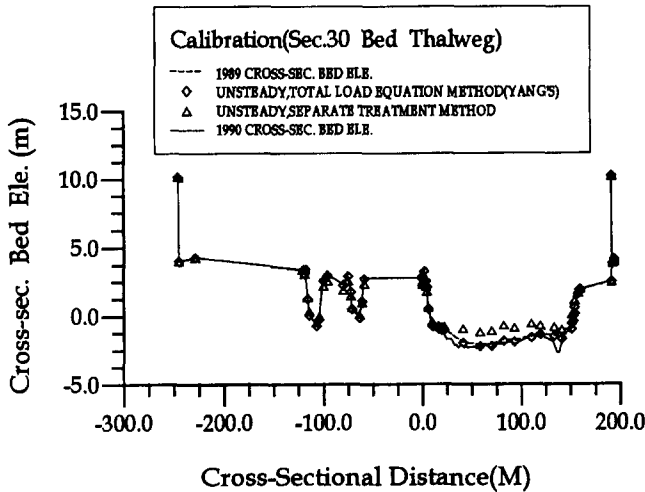

FIG. 11. Comparisons of Simulated Cross Sectional-Bed Elevations with Field Data Using Different Sediment Routing Measures (1990, Ta-Chi Bridge)
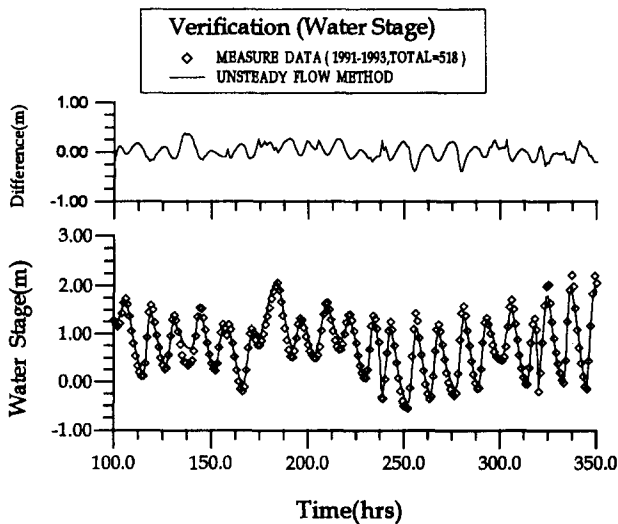

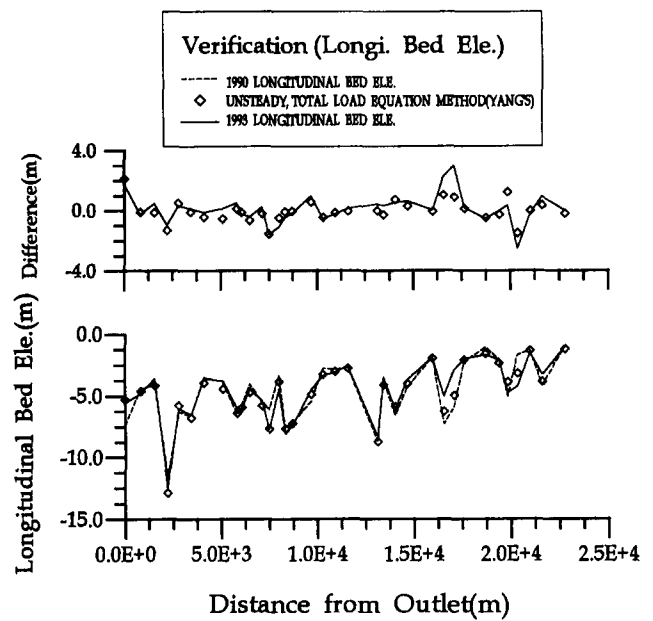**FIG. 12. Verifications of Stage Hydrograph (1993, Ta-Chi Bridge)**

FIG. 13. Verifications of Simulated Longitudinal-Bed Profiles (1990–1993)
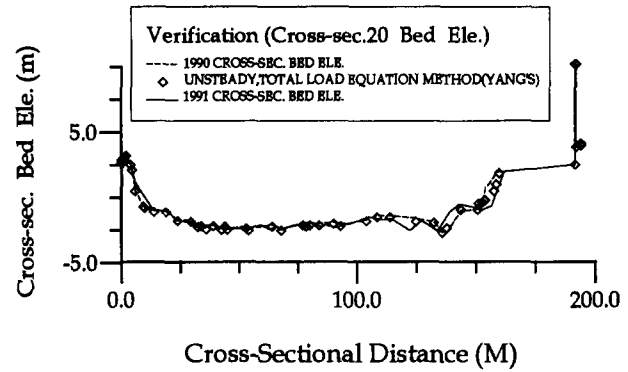

FIG. 14. Verifications of Simulated Cross Sectional-Bed Profiles (1990–1993, Ta-Chi Bridge)

Verification

Using the parameters, tube number, sediment transport formula and sediment treatment measures determined in the previous analyses, the model is applied to simulate the channel bed evolution of the Keelung River from 1990 to 1993. The comparisons of the longitudinal water surface elevation, longitudinal bed profiles and cross-sectional bed profiles of Ta-Chi Bridge are shown in Figs. 12–14, respectively. Fig. 12 shows that using the calibrated Manning's roughness, the model can simulate the temporal variations of the water surface elevations accurately. The simulated longitudinal and transverse bed profiles are shown in Figs. 13 and 14, respectively. The overall accuracy is acceptable. Five tubes were used in the simulation. The transverse variations of the bed profiles can be reflected by the stream tube concept and the accuracy is satisfactory.

APPLICATION TO SHIEMEN RESERVOIR

To test the applicabilities of the model in the reservoir and to show the advantages of the separate treatment method, the model is used to simulate the variations of the bed profiles of Shiemen Reservoir, a major reservoir located upstream of Tan-Hsui River. The location map of the reservoir is shown in Fig. 15. The simulation period is from 1981 to 1982. The measured
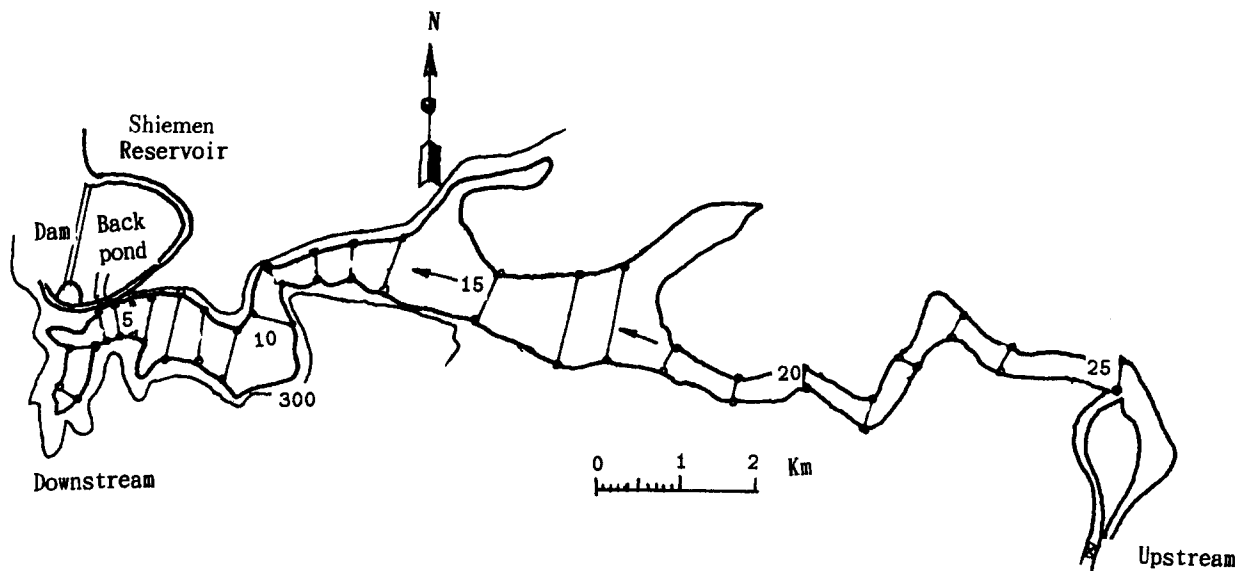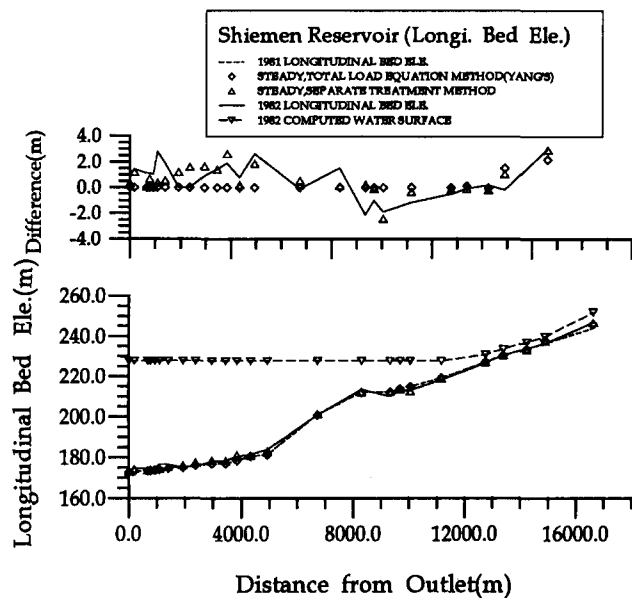
FIG. 15. Location Map of Shiemen Reservoir

FIG. 16. Comparisons of Simulated Longitudinal-Bed Elevations with Field Data Using Different Sediment Routing Measures for Shiemen Reservoir
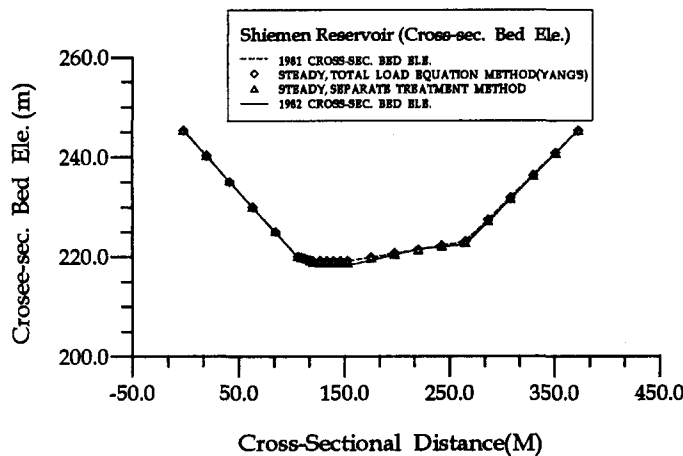

FIG. 17. Comparisons of Simulated Transverse-Bed Profiles with Field Data Using Different Sediment Routing Measures for Shiemen Reservoir (Section 20)

flow discharge versus sediment inflow discharge rating curve was used as the upstream boundary conditions and the measured water surface elevations at the dam were used as the downstream boundary conditions. The steady flow computation scheme was chosen, and both the total load equation method and separate treatment method were adopted in the simulation. The Manning's roughness coefficients were calibrated using the measured water surface elevations, the tube number chosen was 5 and Yang's formula was adopted to calculate the sediment transport rates. Using a time interval of 1 hour, the simulated longitudinal and transverse bed profiles are shown in Figs. 16 and 17, respectively. Fig. 16 shows that using the total load equation method, sediment deposition concentrates in the upstream reach, and the separate sediment treatment method is able to reflect the propagation behavior of the suspended sediment in the reservoir and renders a more accurate simulation.

CONCLUSIONS

A semi–two-dimensional sediment routing model is developed in the present study. This model integrates the stream tube flow model with a state-of-the-art sediment routing algorithm which is capable of simulating suspended and bed load separately. It is capable of simulating the deposition behaviors of the suspended sediment during a nonequilibrium process for both steady and unsteady flow conditions. The model's performance has been assessed through a comparison with an analytical solution and experimental data set. The assessment indicates that the model functions well.

The model's applicability has been demonstrated through an application to the Keelung River and Shiemen Reservoir in Taiwan. Convincing results from the simulations are obtained. A more complex and sophisticated calibration process for a large number of parameters may be needed in future studies.

ACKNOWLEDGMENTS

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APPENDIX II.   NOTATION

The following symbols are used in this paper:

A = channel cross sectional area;

A_{dt} = amount of sediment scouring/deposition per unit length of stream tube;

A_t = area across stream tube;

B_t = width of stream tube;

C_{dk} = deposition concentration;

C_{ek} = sediment concentration close to channel bed;

C_k = depth-averaged concentration of suspended sediment of size fraction k in stream tube;

c' = Chezy coefficient related to grains = $18 \log(12R_t/3D_{90})$;

D_k = particle diameter of size fraction k;

D_* = particle parameter = $D_{50}\{[(s-1)g]/\nu^2\}^{1/3}$;

g = gravitational acceleration;

h = flow depth;

K = channel conveyance;

k_x = longitudinal dispersion coefficients;

k_z = transverse dispersion coefficients;

l = left boundaries of stream tube;

n = roughness coefficient of Manning's formula;

P_i = wetted parameter;

p = channel bed porosity;

Q = flow discharge;

Q_b = bed load transport rate in stream tube;

Q_s = total sediment load;

q = lateral inflow/outflow discharge per unit length;

q_b = bed load discharge/unit width;

q_i = lateral inflow/outflow discharge per unit length in section i;

q_t = flow discharge in stream tube;

R = hydraulic radius;

R_t = hydraulic radius in stream tube;

r = right boundaries of stream tube;

S_f = energy slope;

S_k = source term of suspended sediment of size fraction k;

s = specific weight of sediment particle;

T_k = transport stage parameter = $[(u_*')^2 - (u_{*cr})^2]/(u_{*cr})^2$;

t = time;

U = average velocity;

u_{*cr} = critical shear velocity;

u_*' = grain shear velocity = $g^{0.5}u/c'$;

W = fall velocity of sediment particle;

W_k = fall velocity of sediment of size fraction k;

x = coordinate in flow direction;

y = water surface elevation;

z = coordinate in transverse direction;

α = momentum correction coefficient;

β_k = weight percentage of sediment of size fraction k;

ΔZ_i = variation of bed elevation for every size fraction;

κ = Karman's constant;

ν = water kinematic viscosity; and

ρ = sediment-water mixture density.