

Adaptive controller with desired pole/zero assignment

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Abstract: An adaptive algorithm is presented to incorporate pole/zero assignment as a principle design criterion. As this algorithm does not require solving a diophantine equation at each sampling step, it possesses much computational advantage over the existing approaches of an adaptive pole-assignment control algorithm. This computational advantage is very important for the adaptive control system to the actual application. The internal stability requirement is achieved so that this algorithm can easily handle any minimum- or nonminimum-phase system. The desired zero assignment of the sensitivity function can make high-order reference signal tracking possible. Several simulations are given to illustrate that our approach is suitable for application and easy to implement.

1 Introduction

Recently, various kinds of adaptive control have been developed. Self-tuning regulators are an important class of adaptive controllers, they are easy to implement and process with a wide variety of characteristic unknown parameters, the presence of time delay, time-varying process dynamics [1-4]. Adaptive controllers based on pole assignment design have been discussed by several authors. The approaches taken by Wellstead *et al.* [5, 6] can be detuned to avoid the excessive control effort of minimum-variance controllers. Their methods can also be applied to nonminimum-phase systems with unknown or varying time delay. The adaptive pole assignment schemes developed by Vogel and Edgar [7], Allidina and Hughes [8] and Åström and Wittenmark [9] contain several desirable features, such as robustness as well as ease of set-point tracking. The approach of McDermott and Mellichamp [10] has discussed the additional characteristic of treating steady-state offset; this approach can also treat the nonminimum-phase systems.

Almost all the adaptive pole-assignment algorithms mentioned above need to solve a polynomial identity (diophantine equation) at each sampling step. It is time consuming and is an obvious obstacle to actual application. To avoid solving the diophantine equation, Elliot [11] has presented a direct adaptive control structure. With his method, the nonlinear problem of estimating $2m$ controller parameters should be replaced by a linear parameter estimation problem involving $4m$ parameters, where m is the order of minimum realisation of the plant transfer function. Leal and Landau [12] have proposed a quasi-adaptation algorithm which employs two different identification algorithms to avoid solving the diophantine equation: the first one identifies the plant's parameters and then provides the second one, which, in turn, identifies the controller parameters. But no matter what has been done by increasing the identified parameters or the times of identification, it is still time consuming.

In this paper, we introduce a more general and simpler algorithm to synthesise an adaptive controller with desired pole/zero assignment. This algorithm is computationally easier to implement than the approaches discussed above, because it does not require the solution of the diophantine equation nor the increase of the identified parameters or

the times of identification. This algorithm also contains several characteristics; the internal stability requirement is achieved so that this algorithm can easily handle any minimum- or nonminimum-phase system; the desired zero assignment of the sensitivity function can make high-order reference signal tracking possible [13-16]. Several simulated examples are given to show that our approach is suitable for application and easy to implement.

In Section 2, a design principle based on pole/zero assignment is elaborated so as to ensure the internal stability and reference signal tracking ability of the closed-loop systems. In Section 3, the adaptive algorithm with desired pole/zero assignment is derived. Several simulations are given in Section 4.

2 Control of known plant

2.1 Problem formulation

Consider the pole/zero assignment problem of the control system in Fig. 1.

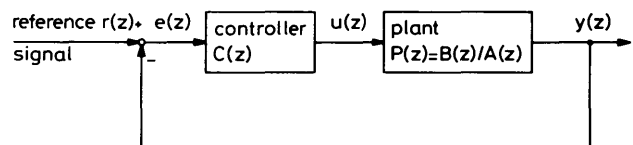


Fig. 1 Basic feedback system

Let the process be described as

$$A(z)y(z) = B(z)u(z) \quad (1)$$

where $y(z)$ and $u(z)$ denote the process output and input, respectively. $A(z)$ and $B(z)$ are relatively prime polynomials of degree n_a and n_b , respectively, and are given by

$$A(z) = z^{n_a} + \alpha_1 z^{n_a-1} + \dots + \alpha_{n_a} \quad (2)$$

$$B(z) = \beta_0 z^{n_b} + \beta_1 z^{n_b-1} + \dots + \beta_{n_b} \quad (3)$$

The plant is also described by the following rational form:

$$P(z) = \frac{B(z)}{A(z)} \quad (4)$$

Note that, if the system time delay is present, we are modelling it as a set of zero coefficients in the polynomial $B(z)$. In this case, the degree of $B(z)$ must be selected to be large enough so that the deadtime of the process can be represented by z^{-k} , while the k leading coefficients of $B(z)$ become zero [10].

The reference signal is assumed to have the representation

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$$r(z) = \frac{N(z)}{M(z)} = \frac{N(z)}{M_-(z)M_+(z)} \quad (5)$$

where $M(z)$ and $N(z)$ are relatively prime polynomials, and the polynomials $M_-(z)$ and $M_+(z)$ denote the factors of the polynomial $M(z)$, where $M_-(z)$ and $M_+(z)$ have their zeros in $|z| < 1$ and $|z| \geq 1$, respectively.

Let us define the sensitivity function $S(z)$ as [17]

$$S(z) = (1 + P(z)C(z))^{-1} \quad (6)$$

where $C(z)$ denotes the controller. From Fig. 1, it is seen that the tracking error signal $e(z)$ is given as

$$e(z) = S(z)r(z) \quad (7)$$

To track a high-order reference signal $r(z)$ (for example, unit step or ramp signal etc.), the right-hand side of eqn. 7 must not have any pole in $|z| \geq 1$ [13–16], i.e. the sensitivity function $S(z)$ must have a sufficient number of zeros to cancel the poles of $r(z)$ in $|z| \geq 1$ [13–16]. Hence the zero assignment of $S(z)$ is very important in high-order reference-signal tracking problems. The importance of pole assignment has been shown in many previously published papers [10, 18, 19]. Our design objective is to synthesise a controller $C(z)$ such that the sensitivity function $S(z)$ has all desired poles and some desired zeros for the purpose of high-order reference signal tracking.

2.2 Pole/zero assignment with known plant

For the purposes of high-order reference signal tracking and desired pole assignment, and from eqns. 5 and 7, the sensitivity function must be of the following form:

$$S(z) = \frac{W(z)M_+(z)}{g(z)} \quad (8)$$

where $g(z)$ is a Hurwitz polynomial containing the desired poles in $|z| < 1$, and $W(z)$ is an undetermined monic polynomial which should be determined to satisfy the following internal stability constraint.

2.2.1 Definition: [17] The sensitivity function $S(z)$ is said to be internally stable (or realisable) if the closed loop of Fig. 1 is asymptotically stable for some choices of the controller $C(z)$, i.e. no pole/zero cancellation between $C(z)$ and $P(z)$ in $|z| \geq 1$.

2.2.2 Lemma 1: [17, 27] The sensitivity function $S(z) \neq 0$ is internally stable (or realisable) if, and only if, all the following conditions hold:

- $S(z)$ is analytic in $|z| \geq 1$
- every zero of the polynomial $A(z)$ in $|z| \geq 1$ is a zero of $S(z)$ of at least the same multiplicity
- every zero of the polynomial $B(z)$ in $|z| \geq 1$ is a zero of $1 - S(z)$ of at least the same multiplicity.

2.2.3 Remark: If $S(z)$ satisfies the requirements of internal stability, then we can directly obtain the controller $C(z) = (1 - S(z))/(P(z)S(z))$ without worrying about any unstable hidden mode [17, 15].

Let us factorise the denominator $A(z)$ and numerator $B(z)$ of the plant $P(z)$ as follows:

$$A(z) = A_+(z)A_-(z) \quad (9)$$

$$B(z) = B_+(z)B_-(z) \quad (10)$$

where the polynomials $A_+(z)$ and $B_+(z)$ have all their zeros in $|z| \geq 1$ while the polynomials $A_-(z)$ and $B_-(z)$ have all their zeros in $|z| < 1$. Let us denote

$$B_+(z) = \prod_{i=1}^n (z - q_i)^{m_i} \quad (11)$$

where n is the number of distinct zeros q_i of $B(z)$ in $|z| \geq 1$, and m_i is the multiplicity of q_i . From the condition (b) of lemma 1, in order to let the desired $S(z)$ satisfy the requirement of internal stability, the numerator of the sensitivity function in eqn. 8 must contain $A_+(z)$, i.e. the sensitivity function must be of the following form:

$$S(z) = \frac{W(z)M_+(z)}{g(z)} = \frac{l(z)A_+(z)M_+(z)}{g(z)} \quad (12)$$

where $g(z)$ and $M_+(z)$ contain the desired poles and zeros, respectively, and $l(z)$ is a polynomial to be determined by the condition (c) of lemma 1. (Note that if some of the zeros of $A_+(z)$ are equal to the zeros of $M_+(z)$, then the least common multiplier of $A_+(z)$ and $M_+(z)$ is chosen to be a factor of the numerator of the sensitivity function $S(z)$ [20].)

From eqn. 12, we obtain

$$1 - S(z) = \frac{g(z) - l(z)A_+(z)M_+(z)}{g(z)} \quad (13)$$

To satisfy the condition (c) of the internal stability of lemma 1, the numerator $g(z) - l(z)A_+(z)M_+(z)$ of $1 - S(z)$ must contain $B_+(z)$, i.e. it must be of the following form:

$$h(z) = g(z) - l(z)A_+(z)M_+(z) = B_+(z)F(z) \quad (14)$$

i.e.

$$h(q_i) = g(q_i) - l(q_i)A_+(q_i)M_+(q_i) = 0 \quad i = 1, 2, \dots, n \quad (15)$$

where n is the number of distinct zeros q_i of $B(z)$ in $|z| \geq 1$. And then

$$l(q_i) = \frac{g(q_i)}{A_+(q_i)M_+(q_i)} \quad i = 1, 2, \dots, n \quad (16)$$

Let

$$l(z) = z^n + l_1 z^{n-1} + \dots + l_{n-1} z + l_n \quad (17)$$

We obtain

$$\begin{bmatrix} q_1^{n-1} & \dots & q_1 & 1 \\ q_2^{n-1} & \dots & q_2 & 1 \\ \vdots & & \vdots & \vdots \\ q_n^{n-1} & \dots & q_n & 1 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix} = \begin{bmatrix} \frac{g(q_1)}{A_+(q_1)M_+(q_1)} - q_1^n \\ \frac{g(q_2)}{A_+(q_2)M_+(q_2)} - q_2^n \\ \vdots \\ \frac{g(q_n)}{A_+(q_n)M_+(q_n)} - q_n^n \end{bmatrix} \quad (18)$$

By solving the n simultaneous equations in eqn. 18, the polynomial $l(z)$ can be determined. And then the sensitivity function $S(z)$ in eqn. 12 is internally stable.

Remarks:

(i) If $B_+(z)$ has coincident zeros in $|z| \geq 1$, instead of solving eqn. 18, we can compare the coefficients between both sides of eqn. 14, and then a set of $n + n_f$ simultaneous equations must be solved to determine the polynomials $l(z)$ and $F(z)$, where n and n_f are the number of undetermined coefficients of polynomials $l(z)$ and $F(z)$, respectively. In general, the assumed order of the plant to be identified in the adaptive system will not be too high, so that the number of the zeros of the plant in $|z| \geq 1$ is few, i.e. the number of simultaneous equations should be solved is few. Especially, in the identification model, because the

disturbance is introduced, it is almost impossible to have coincident zeros of plant in $|z| \geq 1$. If $B_+(z)$ has nearly coincident zeros in $|z| \geq 1$, we can follow the algorithms described by Golub and Van Loan (Algorithms 5.6-1 and 5.6-2 of Reference 28) to solve eqn. 18.

(ii) In order to satisfy the requirement of causality, the sensitivity function $S(z)$ must be proper, i.e. the following inequality must be satisfied

$$\deg(g(z)) \geq \deg(l(z)) + \deg(A_+(z)) + \deg(M_+(z)) \quad (19)$$

where $\deg(\cdot)$ denotes the degree of a polynomial. And as

$$\deg(l(z)) = \deg(B_+(z)) \quad (20)$$

we obtain

$$\deg(g(z)) \geq \deg(A_+(z)) + \deg(B_+(z)) + \deg(M_+(z)) \quad (21)$$

where $\deg(A_+(z))$ and $\deg(B_+(z))$ denote the numbers of poles and zeros of the plant in $|z| \geq 1$, respectively, and $\deg(M_+(z))$ denotes the number of poles of the reference signal $r(z)$ in $|z| \geq 1$. Thus the number of assigned poles, i.e. $\deg(g(z))$ must be chosen to be large enough to satisfy the inequality eqn. 21.

From the above analysis, to satisfy the requirement of internal stability, the sensitivity function $S(z)$ with desired poles and zeros must be of the form in eqn. 12. And then we can obtain the corresponding controller $C(z)$ as

$$\begin{aligned} C(z) &= \frac{1 - S(z)}{P(z)S(z)} \quad (\text{from eqn. 6}) \\ &= \frac{A(z)(g(z) - l(z)A_+(z)M_+(z))}{B(z)l(z)A_+(z)M_+(z)} \\ &\quad (\text{from eqns. 4, 12 and 13}) \\ &= \frac{A_-(z)F(z)}{B_-(z)l(z)M_+(z)} \quad (\text{from eqns. 9, 10 and 14}) \quad (22) \end{aligned}$$

i.e. if we synthesise the controller $C(z)$ as in eqn. 22, then the sensitivity function $S(z)$ must contain $g(z)$, with desired poles, and $M_+(z)$, with desired zeros, and there is no pole/zero cancellation between $C(z)$ and $P(z)$ in $|z| \geq 1$, or we can obtain the corresponding control law

$$B_-(z)l(z)M_+(z)u(z) = A_-(z)F(z)e(z) \quad (23)$$

Remarks:

(i) If the plant is free of zeros in $|z| \geq 1$, then $l(z) = 1$, and we don't need to solve $l(z)$ from any equation

(ii) If the plant is free of poles and zeros in $|z| \geq 1$, then all the poles and zeros of $S(z)$ can be arbitrarily assigned without worrying about the problem of internal stability.

As an example in the following, we follow the design procedure mentioned here to present the design algorithm proposed. For a given nonminimum-phase system

$$P(z) = \frac{z + 1.1}{z(z - 1.2)} \quad (24)$$

how do we synthesise a controller $C(z)$ for three poles of $S(z)$ at $z = -0.3, -0.3$ and 0.5 , and one zero at $z = 1$, to be able to track a unit step signal? To satisfy condition (b) of the internal stability of lemma 1 and the reference signal tracking requirement, and as there is only one zero of the plant in $|z| \geq 1$, $S(z)$ must be of the following form:

$$S(z) = \frac{(z + l_1)(z - 1.2)(z - 1)}{(z + 0.3)^2(z - 0.5)} \quad (\text{from eqn. 12}) \quad (25)$$

From eqn. 14, to satisfy condition (c) of the internal stability of lemma 1, we have

$$\begin{aligned} h(z) &= (z + 0.3)^2(z - 0.5) - (z + l_1)(z - 1.2)(z - 1) \\ &= (z + 1.1)F(z) \quad (26) \end{aligned}$$

The term $F(z)$ should be determined as long as $l(z) = (z + l_1)$ is determined. And, from eqn. 15, we obtain

$$\begin{aligned} h(-1.1) &= (-1.1 + 0.3)^2(-1.1 - 0.5) \\ &\quad - (-1.1 + l_1)(-1.1 - 1.2)(-1.1 - 1) = 0 \quad (27) \end{aligned}$$

i.e.

$$l_1 = \frac{(-1.1 + 0.3)^2(-1.1 - 0.5)}{(-1.1 - 1.2)(-1.1 - 1)} + 1.1 = 0.888 \quad (28)$$

and then, from eqn. 26, we obtain

$$F(z) = 1.412(z - 0.715) \quad (29)$$

From eqn. 22, we obtain the corresponding controller

$$C(z) = \frac{1.412z(z - 0.715)}{(z + 0.888)(z - 1)} \quad (30)$$

2.2.4 Discussion: The pole assignment algorithms introduced in References 5, 7, 8, 9 and 10 need to solve the following diophantine equation [21]:

$$A'(z)R(z) + B'(z)T(z) = G(z) \quad (31)$$

where $A'(z)$ and $B'(z)$ are the denominator and numerator polynomials of the plant, respectively, and $G(z)$ is the desired closed-loop polynomial with desired poles. If $A'(z)$ and $B'(z)$ are relatively prime and $r = \max(\deg(A'(z)), \deg(B'(z)))$, any polynomial $G(z)$ of degree $2r - 1$ can be obtained from eqn. 31 for unique polynomials $R(z)$ and $T(z)$ of degree $r - 1$ (Theorem 5.3.1 in Reference 21). In the preceding example, if we want to assign the desired poles $G(z) = (z + 0.3)^2(z - 0.5)$, by applying the diophantine eqn. 31, we have to solve the following equations for $R(z)$ and $T(z)$, where $R(z) = r_0z + r_1$ and $T(z) = t_0z + t_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.2 & 1 & 1 & 0 \\ 0 & -1.2 & 1.1 & 1 \\ 0 & 0 & 0 & 1.1 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ r_0 \\ t_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.1 \\ -0.21 \\ -0.045 \end{bmatrix} \quad (32)$$

In our method, the pole/zero assignment of the above example only needs to solve eqn. 28. But the pole assignment by the diophantine algorithm needs to solve four simultaneous equations in eqn. 32.

2.2.5 Comments:

(a) For $2r - 1$ pole assignments, the diophantine algorithm needs to solve $2r$ simultaneous equations [21], our algorithm only needs to solve n simultaneous equations, where n is the number of zeros of the plant in $|z| \geq 1$. In practical problems $2r \gg n$.

(b) The algorithms produced by solving the diophantine equation [5, 7-10] cannot solve the zero assignment of the sensitivity function to track the reference signal, but our algorithm can treat this problem very easily.

Because our method takes much computational advantage over the approaches by solving diophantine equation, this method is especially suitable for the adaptive control system.

In the adaptive case, as the plant is unknown, we need parameter identification at every sampling step to estimate $A(z)$ and $B(z)$ and compute $l(z)$ and $F(z)$ from eqns. 18 and

14, and then substitute these values into control law in eqn. 23 to generate the control signal $u(z)$. There are many methods for recursive parameter estimation. Least square scheme, which is one of the simplest recursive estimation schemes [9, 19, 21] can be applied to achieve the parameter estimation.

3 Adaptive control with desired pole/zero assignment

From the above analysis, adaptive control with desired pole/zero assignment will be described by the following algorithm:

Step 1: Calculate the estimations $\hat{A}(z)$ and $\hat{B}(z)$ of polynomials $A(z)$ and $B(z)$ by the recursive least-squares scheme, and perform the factorisation $\hat{A}(z) = \hat{A}_+(z)\hat{A}_-(z)$ and $\hat{B}(z) = \hat{B}_+(z)\hat{B}_-(z)$.

Step 2: Solve the coefficients of $l(z)$ from eqn. 16 or eqn. 18, with $A_+(q_i)$ replaced by $\hat{A}_+(q_i)$, $i = 1, 2, \dots, n$. And then, from the following equation,

$$g(z) - l(z)\hat{A}_+(z)M_+(z) = \hat{B}_+(z)F(z) \quad (33)$$

we can obtain the polynomial $F(z)$.

Step 3: Substitute the obtained $l(z)$ and $F(z)$ into eqn. 23, and we can obtain the adaptive control law as in eqn. 23, with $A_-(z)$ and $B_-(z)$ replaced by $\hat{A}_-(z)$ and $\hat{B}_-(z)$, respectively.

By performing step 1 to step 3 at each sampling interval, the sensitivity function $S(z)$ will achieve the desired pole/zero assignment as $\hat{A}(z)$ and $\hat{B}(z)$ approach to $A(z)$ and $B(z)$, respectively.

Remarks:

(a) In general, the assumed order of the plant to be identified will not be too high, so that, in step 1, we can apply the existing formula to factorise $\hat{A}(z)$ and $\hat{B}(z)$ when the polynomials $\hat{A}(z)$ and $\hat{B}(z)$ are quadratic, cubic or quartic [13–15, 22]. When the polynomials $\hat{A}(z)$ and $\hat{B}(z)$ are of higher degree than quartic, the numerical method should be applied to solve the roots of $\hat{A}(z)$ and $\hat{B}(z)$.

(b) In step 2, when determining the polynomial $F(z)$, we can compare the coefficients between both sides of eqn. 33. As a result, the coefficients of $F(z)$ can be described as functions of the coefficients of $l(s)$ a priori, and then, as long as the coefficients of $l(z)$ are solved, the polynomial $F(z)$ should be determined immediately.

Because our adaptive pole/zero-assignment control algorithm is extendable to the nonminimum-phase systems, the possibility of pole/zero cancellation between $\hat{A}(z)$ and $\hat{B}(z)$ in $|z| \geq 1$ must be avoided. So that the convergence analysis of this adaptive pole/zero-assignment algorithm depends on how to show that all limit points of the parameter estimator correspond to the model (i.e. $\hat{A}(z)$ and $\hat{B}(z)$) are relatively prime. Several papers have been written to deal with the convergence problem of adaptive pole assignment to ensure that $\hat{A}(z)$ and $\hat{B}(z)$ are relatively prime [21, 33, 24]. Goodwin and Sin [21] have introduced a convergent algorithm of the adaptive pole assignment. However, the result is local in nature. Goodwin *et al.* [23] have suggested an alternative strategy which leads to global convergence of the adaptive pole-assignment algorithm. This strategy requires an addition of a persistently exciting external input to ensure that the parameters converge to their true values and to avoid pole/zero cancellation. Recently, Anderson and Johnstone [24] presented a detailed analysis of globally convergent algorithm for achieving a prescribed set of closed-loop poles without any

pole/zero cancellation. In their algorithm, persistency of excitation of an external input is also required.

4 Simulation

A demonstration of how the adaptive controller with pole/zero assignment works will now be given in terms of simulated examples:

Example 1: A nonminimum-phase system is considered with the following system description

$$P(z) = \frac{z + 2}{(z - 0.5)(z - 0.7)} \quad (34)$$

An adaptive control design with the following objectives is considered:

(i) two poles of the transfer function at $z = 0.2$ for robustness [19]

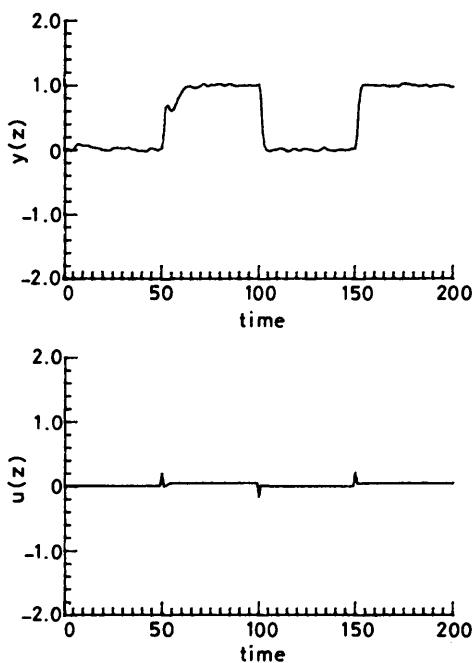


Fig. 2 Output response $y(z)$ and control signal $u(z)$ of the adaptive pole/zero assignment design in example 1

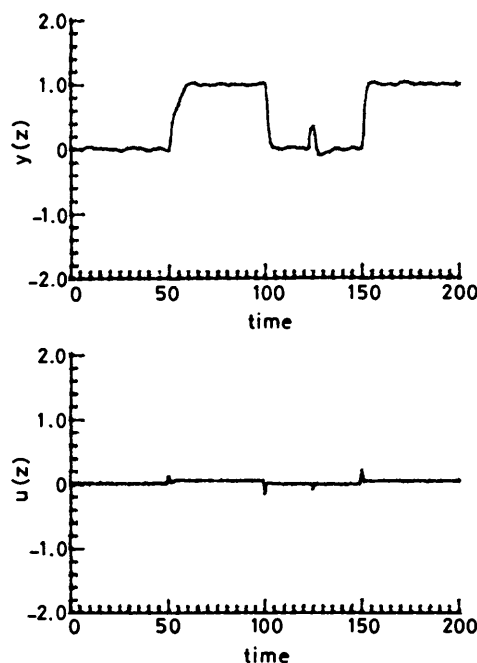


Fig. 3 As in Fig. 2 but with a load disturbance

(ii) one zero of the sensitivity function at $z = 1$ for tracking square-wave reference signal.

Following the proposed design algorithm, the behaviour of this system is shown in Fig. 2. It is seen that the behaviour of the closed-loop system is good enough in the second transient, because the parameters have converged after the first transient. Fig. 3 shows the behaviour when the output load is changed. It is seen that this adaptive pole/zero-assignment control algorithm is quite robust in its response to the load disturbance.

Example 2: The adaptive control with nonminimum-phase system (eqn. 24) discussed in Section 2.2 is simulated in this example. Fig. 4 shows the behaviour of this system

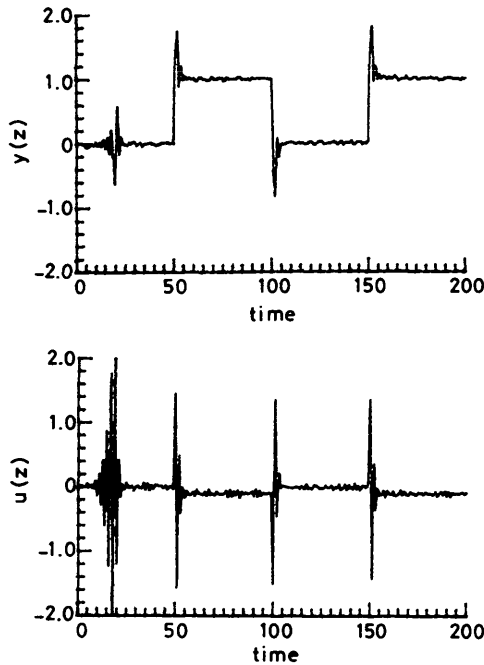


Fig. 4 Output response $y(z)$ and control signal $u(z)$ of the adaptive pole/zero assignment design in example 2

for tracking a square-wave reference signal. Again the behaviour of the closed-loop system is good enough in the second transient, because the parameters have converged after the first transient.

Example 3: A frequent problem encountered during machining systems is deterioration of system stability, which is caused by changes in the process parameters such as depth in cut. In order to solve this problem, the proposed adaptive pole/zero-assignment algorithm is applied. This machining system is modelled as in Fig. 5 [25]:

- F_r = cutting force
- v_f = feed rate
- n = spindle speed
- f = feed
- a = cutting depth
- k_s = specific cutting force
- h = sampling period

the value of the parameters of this system are chosen as

- $\xi = 0.7$
- $W_n = 60$ rad/s
- $n = 900$ rev
- $p = 0.7$
- $k_s = 933$
- $f = 0.197$ mm/rev
- $V_f = 2.96$ mm/s
- $h = 0.01$ s

By employing the z -transformation [Table 3.1 in Reference 19], Fig. 5 can be represented as in Fig. 6; where a is the

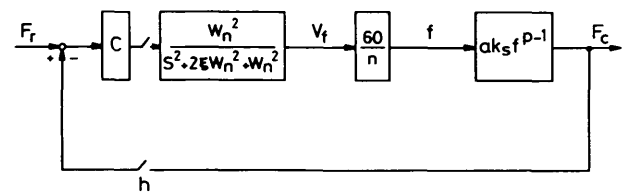


Fig. 5 Machine system

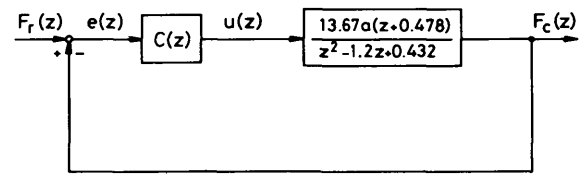


Fig. 6 Equivalent system of Fig. 5

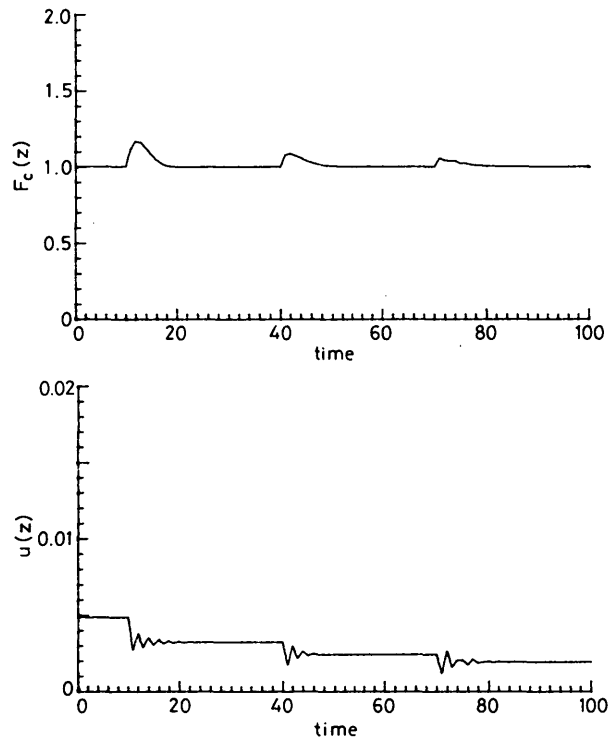


Fig. 7 Output response $F_c(z)$ and control signal $u(z)$ of the adaptive pole/zero assignment design in example 3
 $F_c(z)$ is normalised

cutting depth. By applying the adaptive pole/zero-assignment algorithm proposed in this paper, the simulated result is shown in Fig. 7 with one pole of the transfer function at $z = 0.5$ and one zero of the sensitivity function at $z = 1$. The data were taken after the parameters had converged. The cutting depth $a = 2$ mm before $t = 10$, $a = 3$ mm between $t = 10$ and $t = 40$, $a = 4$ mm between $t = 40$ and $t = 70$ and $a = 5$ mm after $t = 70$. The result shows that our adaptive pole/zero-assignment algorithm can quickly converge the cutting force to the desired steady-state value after the cutting depth is changed.

5 Conclusion

The adaptive control algorithm described in this paper is computationally easier to implement than the existing approaches of adaptive pole-assignment control algorithms, because the algorithm does not require solving a diophantine equation at each sampling step. This compu-

tational advantage is very important for the adaptive control system to the actual application. The internal stability requirement is achieved so that this algorithm can easily handle any minimum- or nonminimum-phase system. As well as the pole assignment, because the zeros of a system play an important role in the interaction between the closed-loop systems and their external environments, the zero assignment of control systems becomes very important. The desired zero assignment of the sensitivity function can make high-order reference-signal tracking possible. These characteristics are demonstrated by several simulations. This same approach can be extended to multivariable systems, and this is, presently, the subject of continuing research.

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