

國立交通大學
土木工程學系
碩士論文

利用三維彈性理論配合 Ritz 法分析具內部裂縫

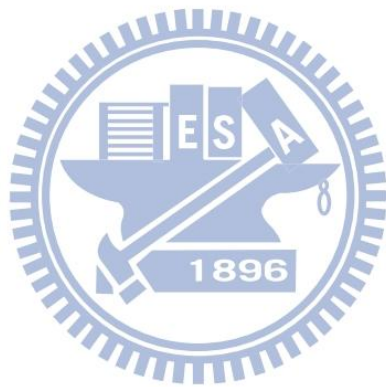
矩形功能梯度材料板之振動問題

Vibrations of Functionally Graded Material Rectangular Plates with
Internal Cracks Using Three-Dimensional Elasticity with the Ritz Method

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中華民國一百零一年一月



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碩士論文

A Thesis

Submitted to Department of Civil Engineering
College of Engineering

National Chiao Tung University

in partial Fulfillment of the Requirements

for the Degree of

Master

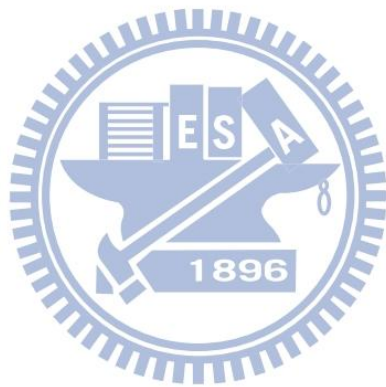
in

Civil Engineering

January 2012

Hsinchu, Taiwan, Republic of China

中華民國一百零一年一月



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摘要

本研究利用 3D 彈性理論配合 Ritz 法分析含內部裂縫矩形功能梯度板自由振動；提出能準確描述裂縫尖端奇異性並滿足跨越裂縫位移不連續行為之裂縫函數配合滿足邊界條件之多項式函數當允許函數。透過收斂性分析，並與文獻中具裂縫均質板的結果比較，驗證所發展電腦程式的正確性與所提解之準確性。本文所考慮的功能梯度板是由金屬鋁(Al)及陶瓷(Al_2O_3)組成，使用簡單的冪級數與體積分數來描述材料的性質。本文探討不同材料組成、不同邊界條件、不同板厚及不同裂縫長度、角度與位置對板的振動行為之影響，並繪有中平面及三維變形之模態圖；此些結果均首見於文獻。

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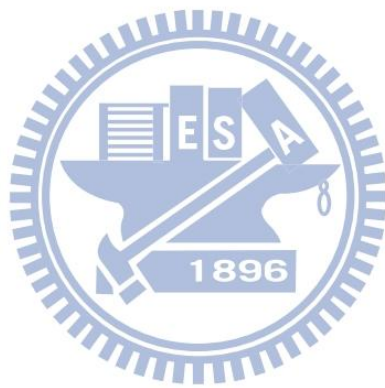
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Abstract

This study examines the three-dimensional (3-D) vibrations of functionally graded material (FGM) rectangular plates having internal cracks. We employ 3-D theory of elasticity and a variational Ritz methodology, and propose new hybrid series of mathematically complete orthogonal polynomials and crack functions as the assumed displacement fields to enhance the convergence modeling of the stress singular behavior of the crack terminus edge front in a rectangular FGM plate. The proposed admissible hybrid series properly describe the $\mathcal{O}(1/\sqrt{r})$ 3-D stress singularities at the terminus edge front of the crack, allowing for displacement discontinuities across the crack sufficient to explain the most general 3-D “mixed modes” of local crack-edge deformation and stress fields typically seen in fracture mechanics. The correctness and validity of the proposed approach are confirmed through comprehensive convergence studies and comparisons with published results for homogeneous and FGM rectangular plates with internal cracks based on various plate theories. The FGM plates under consideration are composed of aluminum and ceramic (Al/Al₂O₃). The locally effective material properties are estimated by a simple power law and the effects of the volume fraction on the frequencies are investigated. Frequency data,

mode shapes and nodal patterns for FGM rectangular plates with different boundary conditions and thickness-width ratios, and having internal cracks with varying crack length ratios, crack positions, and crack inclination angles are reported herein for the first time in the published literature.





誌謝

本論文得以順利完成，首先要感謝恩師 黃炯憲教授耐心與悉心的指導。研究期間，老師嚴謹且不厭其煩的糾正錯誤，使學生的專業知識有所增長；並不斷提供新設備與優質的研究環境，使得研究能順利完成。在車禍期間，您特地前來探望關心，著實令學生感到非常溫馨。在論文修改期間，您細心的研讀並提出建議，使學生的論文得以更加完整。

論文口試期間，承蒙交通大學土木工程學系 劉俊秀教授、鄭復平教授與 林昌佑教授於口試期間提供寶貴的意見，謹此表示最誠摯的感謝。

這份論文能夠順利完成，感謝研究室明儒學長、凱平學長的指導，在課業與研究上耐心且大方的指導與協助。尤其是明儒學長在論文撰寫與口試期間，幫忙訂正論文與簡報，並提供諸多寶貴之建議。同時也感謝威智學長、連杰學長、靖俞學長、政甯學長、志偉學長、政淵學長、榕師學長、宛臻學姊和嫻糧學姊，以及勇奇學長、穎泰學長、宗輝學長、辰彥學長、家宇學長。另外，感謝同窗中原、鈞誠與維莘，因為有你們的相互扶持，使我在生活與課業上獲益良多。也感謝同學進順、宣好、連峰、柏林、綸桓、孟軒、思伶、江祥，學弟妹承哲、裕鈞、旭進、芳琳，還有宣治、俊佐、晟佑和俊超一起在生活和研究中成長、幫助。還要感謝所有中原大學

的老師們、同學們，尤其是 陳遠亮教授多麼無私的將自己的時間全部花在學生的課業上，並啟發學生的思考力。當然還要感謝花蓮的朋友們與在新竹研究所期間認識的所有人們。

最重要的，我要感謝我的家人，爸爸、媽媽從小的栽培及付出，並且辛苦的工作，你們一路支持，讓我無後顧之憂。還有姊姊的鼓勵及家裡最可愛的 QQ。遠在美國的爺爺、奶奶、姑姑、姑丈、叔叔、表哥、堂弟、堂妹，還有花蓮的外公、外婆、大阿姨、舅舅、三阿姨、小阿姨、表哥、表姊、表弟妹們，真的非常感謝你們對我的支持與關心。

最後，我要感謝雅鈞，在這一段時間的陪伴、幫助，感恩妳的包容與忍讓，且不時給我鼓勵，帶給我很大的動力。同時對我的照顧並帶給我滿滿的歡樂，讓我這一路走來不孤單。

在這即將離開的時刻，這些日子滿滿的回憶湧上心頭，這些點點滴滴將陪伴我走向未來。凡走過必留下痕跡，凡努力必造就意義。

目錄

摘要	i
Abstract	ii
誌謝	v
目錄	vii
表目錄	ix
圖目錄	xxii
第一章 緒論	1
1.1 研究動機與方法	1
1.2 文獻回顧	3
1.3 內容概要	7
第二章 分析方法	9
2.1 功能梯度材料公式	9
2.2 3D 彈性理論之應變能與動能	10
2.3 利用 Ritz 法求解板之自然振動頻率	11

2.4 允許函數之建構.....	14
第三章 收斂性分析.....	18
3.1 厚度影響.....	20
3.2 裂縫長度影響.....	21
第四章 數值結果.....	24
4.1 裂縫對振動頻率之影響.....	24
4.2 不同理論所得結果比較.....	35
4.3 板之振動模態探討.....	36
第五章 結論與建議.....	39
5.1 結論.....	39
5.2 建議.....	40
參考文獻.....	42



表目錄

表 2.1 材料參數.....	49
表 3.1 具水平內部裂縫簡支方形均質薄板($a/b = 1.0, h/b = 0.01,$ $(x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.2, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析.....	50
表 3.2 具水平內部裂縫簡支之 Al/Al ₂ O ₃ FGM 方形中厚板($a/b = 1.0,$ $h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.2, \alpha = 0^\circ, m = 0.2$)之無因 次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析.....	55
表 3.3 具水平內部裂縫簡支方形均質中厚板($a/b = 1.0, h/b = 0.1,$ $(x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.2, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析.....	58
表 3.4 具水平內部裂縫簡支方形均質薄板($a/b = 1.0, h/b = 0.05,$ $(x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.3, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析.....	61
表 3.5 具水平內部裂縫簡支方形均質中厚板($a/b = 1.0, h/b = 0.1,$ $(x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.3, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析.....	66

表 3.6 具水平內部裂縫簡支之 Al/Al ₂ O ₃ FGM 方形中厚板($a/b = 1.0$, $h/b = 0.1$, $(x_0/a, y_0/b) = (0.5, 0.5)$, $d/a = 0.5$, $\alpha = 0^\circ$, $m=0.2$)之無因 次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析.....	69
表 3.7 具水平內部裂縫簡支之 Al/Al ₂ O ₃ FGM 方形中厚板($a/b = 1.0$, $h/b = 0.1$, $(x_0/a, y_0/b) = (0.5, 0.5)$, $d/a = 0.3$, $\alpha = 0^\circ$, $m=5$)之無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析.....	72
表 3.8 具水平內部裂縫簡支之 Al/Al ₂ O ₃ FGM 方形中厚板($a/b = 1.0$, $h/b = 0.1$, $(x_0/a, y_0/b) = (0.5, 0.5)$, $d/a = 0.5$, $\alpha = 0^\circ$, $m=5$)之無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析.....	75
表 3.9 具水平內部裂縫簡支方形均質中厚板($a/b = 1.0$, $h/b = 0.1$, $(x_0/a, y_0/b) = (0.5, 0.5)$, $d/a = 0.5$, $\alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析.....	78
表 3.10 具垂直內部裂縫懸臂之 Al/Al ₂ O ₃ FGM 方形中厚板($a/b = 1.0$, $h/b = 0.1$, $(x_0/a, y_0/b) = (0.5, 0.5)$, $d/b = 0.2$, $\alpha = 90^\circ$, $m=5$)之無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析.....	81
表 3.11 具垂直內部裂縫懸臂之 Al/Al ₂ O ₃ FGM 方形中厚板($a/b = 1.0$, $h/b = 0.1$, $(x_0/a, y_0/b) = (0.5, 0.5)$, $d/b = 0.6$, $\alpha = 90^\circ$, $m=5$)之無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析.....	84

表 3.12 具垂直內部裂縫懸臂方形均質中厚板($a/b = 1.0, h/b = 0.1,$ $(x_0/a, y_0/b) = (0.5, 0.5), d/b = 0.2, \alpha = 90^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析.....	87
表 3.13 具垂直內部裂縫懸臂方形均質中厚板($a/b = 1.0, h/b = 0.1,$ $(x_0/a, y_0/b) = (0.5, 0.5), d/b = 0.6, \alpha = 90^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析.....	90
表 4.1 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02, (x_0/a, y_0/b) = 0.5$)	93
表 4.2 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5)$)..	94
表 4.3 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2, (x_0/a, y_0/b) = (0.5,$ $0.5)$)	95
表 4.4 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02, (x_0/a, y_0/b) = (0.5,$ $0.5)$)	96
表 4.5 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5)$) ..	97

表 4.6 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$).....	98
表 4.7 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$ $0.75)$).....	99
表 4.8 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$ $0.75)$).....	100
表 4.9 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$ $0.75)$).....	101
表 4.10 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$ $0.75)$).....	102
表 4.11 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$ $0.75)$).....	103
表 4.12 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$) ...	104

表 4.13 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板無因次化頻

率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 105

表 4.14 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板無因次化

頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) ... 105

表 4.15 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板無因次化頻

率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) ... 106

表 4.16 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板無因次化頻

率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 106

表 4.17 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板無因次化

頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 107

表 4.18 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板無因次化頻

率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)107

表 4.19 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板無因次

化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$
 $0.5)$)..... 108

表 4.20 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板無因

次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$
 $0.5)$)..... 109

表 4.21 具不同水平內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	110
表 4.22 具不同水平內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	111
表 4.23 具不同水平內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	112
表 4.24 具不同水平內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	113
表 4.25 具不同水平內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.75)).....	114
表 4.26 具不同水平內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.75))	114
表 4.27 具不同水平內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次	

化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$
 0.75)) 115

表 4.28 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板無因次

化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$
 0.75)) 115

表 4.29 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板無因

次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$
 0.75))..... 116

表 4.30 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板無因次

化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$
 0.75)) 116

表 4.31 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板無因次

化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$
 0.5)) 117

表 4.32 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板無因

次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$
 0.5)) 117

表 4.33 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板無因次

化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$

0.5))	118
表 4.34 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	118
表 4.35 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5)).....	119
表 4.36 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	119
表 4.37 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.25,$ 0.5)).....	120
表 4.38 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.25,$ 0.5)).....	120
表 4.39 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.25,$ 0.5))	121

表 4.40 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.25,$ 0.5))	121
表 4.41 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因 次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.25,$ 0.5))	122
表 4.42 具不同垂直內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次 化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.25,$ 0.5))	122
表 4.43 具不同內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次化頻 率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	123
表 4.44 具不同內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 中厚板無因次化 頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5)).....	123
表 4.45 具不同內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 厚板無因次化頻 率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5,$ 0.5))	124
表 4.46 具不同內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 薄板無因次化頻	

率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 124

表 4.47 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板無因次化

頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 125

表 4.48 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板無因次化頻

率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 125

表 4.49 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 中厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 126

表 4.50 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 126

表 4.51 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 中厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 127

表 4.52 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a,$

$y_0/b) = (0.5, 0.5))$ 127

表 4.53 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 中厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.1$ 、
 $(x_0/a, y_0/b) = (0.25, 0.5))$ 128

表 4.54 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.2$ 、 $(x_0/a,$
 $y_0/b) = (0.25, 0.5))$ 128

表 4.55 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 中厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b=0.1$ 、 $(x_0/a,$
 $y_0/b) = (0.25, 0.5))$ 129

表 4.56 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

方形 FGM 厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b=0.2$ 、 $(x_0/a,$
 $y_0/b) = (0.25, 0.5))$ 129

表 4.57 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 中厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b=2$ 、
 $h/b=0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5))$ 130

表 4.58 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b=2$ 、

$h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$) 130

表 4.59 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 中厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b = 2$ 、

$h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)..... 131

表 4.60 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b = 2$ 、 $h/b = 0.2$ 、

$(x_0/a, y_0/b) = (0.5, 0.5)$)..... 131

表 4.61 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 中厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b = 2$ 、

$h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$) 132

表 4.62 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b = 2$ 、

$h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)..... 132

表 4.63 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 中厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b = 2$ 、

$h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$) 133

表 4.64 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃

矩形 FGM 厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b = 2$ 、 $h/b = 0.2$ 、

$(x_0/a, y_0/b) = (0.25, 0.5))$ 133

表 4.65 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形

FGM 中厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.1 \cdot (x_0/a,$
 $y_0/b) = (0.5, 0.5))$ 134

表 4.66 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形

FGM 厚板無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.2 \cdot (x_0/a, y_0/b)$
 $= (0.5, 0.5))$ 134

表 4.67 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形

FGM 中厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b=0.1 \cdot (x_0/a, y_0/b)$
 $= (0.5, 0.5))$ 135

表 4.68 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形

FGM 厚板無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b=0.2 \cdot (x_0/a, y_0/b) =$
 $(0.5, 0.5))$ 135

表 4.69 具不同水平內部裂縫簡支之方形均質板厚度效應及不同理

論比較($(x_0/a, y_0/b) = (0.5, 0.5))$ 136

圖目錄

圖 2.1 具內部裂縫矩形板示意圖	137
圖 2.2 具內部裂縫奇異點與連續線段示意圖	138
圖 4.1 Al/Al ₂ O ₃ 功能梯度材料參數 E(z)和ρ(z)沿厚度變化圖根據式 (2.1)	139
圖 4.2 具不同內部裂縫簡支之均質與 Al/Al ₂ O ₃ FGM 方形薄板 2D 模態圖(h/b=0.02)	140
圖 4.3 具不同水平內部裂縫簡支之方形均質中厚板模態圖	141
圖 4.4 具不同水平內部裂縫簡支之 Al/Al ₂ O ₃ 方形 FGM 中厚板模態 圖($\hat{m}=5$)	151
圖 4.5 具不同內部裂縫懸臂之方形均質中厚板模態圖(h/b=0.1) .	161
圖 4.6 具不同內部裂縫懸臂之 Al/Al ₂ O ₃ 方形 FGM 中厚板模態圖 ($\hat{m}=5, h/b=0.1$)	171
圖 4.7 具不同內部裂縫兩端固定端，兩端自由端之方形均質中厚板 模態圖(h/b=0.1)	181
圖 4.8 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al ₂ O ₃ 方形 FGM 中厚板模態圖($\hat{m}=5, h/b=0.1$)	189

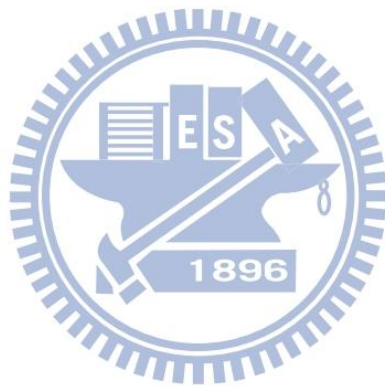
圖 4.9 具不同內部裂縫兩端固定端，兩端自由端之矩形均質中厚板

模態圖($a/b=2, h/b=0.1$) 197

圖 4.10 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形

FGM 中厚板模態圖($\hat{m}=5, a/b=2, h/b=0.1$) 201





第一章 緒論

1.1 研究動機與方法

功能梯度材料(functionally graded material, 簡稱 FGM)的概念是由日本的研究團隊在 1980 年代中期首先提出的。其為了滿足航太、國防工業、機械及土木工程等苛刻的需求，而發展出陶瓷-金屬功能梯度材料。FGM 是一種由不同材料(如陶瓷和金屬)依照不同比例組合而成的材料，結構和性能在材料厚度或長度方向連續變化的非均質複合材料，可分為金屬-合金，金屬-非金屬，非金屬-陶瓷、金屬-陶瓷、陶瓷-陶瓷等多種組合方式。其組成材料、含量、結構等參數將可依工程需要，而設計出具有高強度、剛度、耐熱性、耐磨性等的合適材料。其不僅強化了材料本身的強度、韌性及耐高溫性，亦使其內部界面消除，解決傳統複合材料容易在界面上發生應力集中而破壞的現象。1980 年代中期，日本學者新野正之(Niino Masayuki)、平井敏雄(Hirai Toshio)與渡邊龍三(Ryuzo Watanabe)首先提出功能梯度材料(Functionally Gradient Materials, 簡稱 FGM)的概念，並提出其製造理論與技術(Niino 和 Maeda, 1990)。

最初研究 FGM 的目的，是為了解決航太方面的問題，進而發展的金屬和超耐熱陶瓷結合的研究，由於 FGM 的特點，它很快的就被利用在其

他功能材料的構想和研究中，用途從航太工業擴大到核能、電子、汽車、化學、生物醫學科學領域，應用前景廣闊，為熱門研究方向。

經過近年的發展應用，由於 FGM 的概念，使其能為材料創造出某種特殊功能，且為材料的結構分析帶來了新的探討，進而引起物理、力學、生物與材料研究者的特別關注。研究 FGM 複合結構是航空業進一步發展的關鍵，特別是航太工業。此外 FGM 已漸漸擴大其應用範圍，並很可能首先應用於汽車工業。FGM 對於各種耐熱、結構承載上的充分應用提供理論依據和力學分析，有效的減輕導彈重量、火箭的自重、提高飛行器的耐熱極限及承載能力有者極重要的戰略意義，對國防事業有巨大的經濟價值。

由於 FGM 有極大的實用性，材料沿梯度變化的概念所蘊含的意義深遠，最近在生物力學方面，已經提出了胚胎中存在著決定發育的關鍵力學性質，其結構的組成在特定方向上也呈梯度分布，並建立了梯度結構理論，由此可見 FGM 已擴展至生物領域，相信 FGM 往後會有更多元的發展。

板殼為結構常見之元件，在應用上常面臨奇異點(singularity)之問題，通常發生於：(1)幾何形狀不連續(如裂縫尖銳切角或邊界條件)；(2)載重點處(集中載重或載重強度急遽改變)；(3)材料性質之陡變(如複合材料)。當

分析板殼元件含有奇異點時，須找到能正確描述奇異點特性之漸進解，方能得到準確之收斂數值解。

本研究即首先依 3D 彈性理論，並利用 Ritz 法將角函數(corner function) 帶入允許函數(admissible function)中，探討功能梯度板因裂縫所致之應力奇異行為；並分析含裂縫之功能梯度板自由振動。

1.2 文獻回顧

關於具有裂縫矩形板之研究，已有許多關於均質板由於形狀幾何與材料所引起應力奇異之文章，且大部分文獻探討靜態行為，僅有少數研究振動行為者，其中又以古典薄板理論或平面線彈性理論為主。

對於求解矩形薄板之振動問題，以下文獻分別利用不同方法建立所需之積分方程求解。Lynn 和 Kumbasar (1967)分析了具裂縫四邊簡支承矩形板振動問題，首先用 Green's 函數來表示板之位移分量，並進一步將欲求問題轉換成齊性 Fredholm 第一型積分方程(homogeneous Fredholm integral equations of the first kind)，再求解積分方程；Stahl 和 Keer (1972)利用對偶級數方程(dual series equations)得齊性 Fredholm 第二型積分方程(homogeneous Fredholm integral equations of the second kind) 來求解振動問題。Aggarwala 和 Ariel (1981)應用 Stahl 和 Keer (1972)之方法，求解簡支

方形板具有位於中心點十字型裂縫或兩組(水平與垂直)對稱於中心點之邊緣裂縫振動問題。Hirano 和 Okazaki (1980)亦針對一組對邊為簡支承之裂縫矩形板，利用 Levy 的解，將裂縫兩邊的不連續位移當作未知函數，並進一步以加權餘數法(weighted residual method)來滿足邊界條件。另外，Nezu (1982)則是修正 Lynn 和 Kumbasar (1967)之方法，以 Levy 解建立所須之 Green 函數。Solecki (1983)之作法類似 Hirano 和 Okazaki (1980)，不過使用四邊簡支承板之解析解，將描述裂縫處位移和轉角的不連續函數，進行 finite Fourier 轉換，求解裂縫板振動問題。

於數值方法中，最常利用有限元素法與 Ritz 法分析具裂縫矩形板之振動問題。Qian 等人(1991)發展了一有限元素的解法，對於含裂縫尖端的元素，經由對應力強度因子的積分，建構元素的勁度矩陣。Yuan 和 Dickinson (1992)將一矩形板分成幾個區塊，在區塊連接邊界是以加入來彈簧處理；因此，就可用 Ritz 法中的正規允許函數來求解，不必用特別的函數來描述裂縫。Krawczuk (1993)則提出類似 Qian 等人(1991)的解決方式，唯一不同的是對裂縫尖端元素勁度矩陣，採用封閉形式(closed form)的積分。

利用 Ritz 法分析板振動問題，主要條件為所使用允許函數之適當性。Yuan 和 Dickinson (1992)將矩形板分成數個區塊，並加置人工彈簧於各區塊連結之邊界上；因此，可用傳統的允許函數(regular admissible functions)

於各區域來求解，不必用特別的函數來描述裂縫。Liew 等人(1994)用類似於 Yuan 和 Dickinson (1992)之切割方法，且僅要求各區塊之允許函數於兩區塊交接處，以積分形式滿足允許函數及其一階微分之連續性。Khadem 和 Rezaee (2000)利用 Levy's solution 建立所謂修正比較函數(modified comparison functions)，作為 Ritz 法所中的允許函數(admissible functions)，分析具水平裂縫簡支承矩形板於不同裂縫長度、深度與位置時之振動。然而，因為 Khadem 和 Rezaee (2000)使用之允許函數較為特殊，其僅適用於處理至少一對邊是簡支承 (two opposite edges simply supported)之裂縫矩形板振動問題。Huang 和 Leissa (2009)，利用 Williams (1952)推導裂縫尖端之漸近解，提出了一組可準確描述邊緣裂縫奇異行為的允許函數，並將此允許函數用於求解不同邊界條件下(四邊簡支承與自由端)含邊緣裂縫薄板的振動問題。李昱成 (2009)使用 Ritz 法分析裂縫矩形板之自由振動，引入一組特殊函數，來描述裂縫尖端之奇異性並滿足跨裂縫之位移與轉角不連續行為。上述應用 Ritz 法求解具裂縫板振動之文獻回顧中，絕大部分均透過切割次區域之技巧處理(Yuan 和 Dickinson、Liew 等與 Lee 和 Lim)，先在各區域內選擇適當之允許函數，再利用各種類似連續條件，建立全域之允許函數。但這些文獻忽略了 Ritz 法頻率從上限收斂的特性，因為其允許函數在兩個次區域連接處並非處處連續，而且例如 Yuan 和 Dickinson

(1992)安裝人工彈簧於次區域之連接處，雖然強迫滿足了必要之連續條件，但彈簧的勁度也會影響數據正確性。

亦有少數文獻分析具裂縫厚板之振動：Lee 和 Lim (1993)根據 Reissner 定理，利用區域分解的方法決定 Ritz 法求解含水平中心裂縫簡支矩形 Mindlin 板之允許函數，並求解振動頻率。Bachene 等 (2009)討論在 Mindlin 板理論架構下利用擴展的有限元素法(extended finite element method (X-FEM))分析含水平裂縫矩形薄板(厚寬比 $h/b \leq 1/500$)的自然振動頻率。Hosseini-Hashemi 等人 (2010)利用 Mindlin 板理論，並建立封閉型式的特徵方程分析具裂縫之矩形板，並提高了自然振動頻率的計算速度。Zhou 等人(2011)利用移動式最小平方元素法(MLS- element method)分析含裂縫之 Mindlin 板振動問題。Huang 等人 (2011)使用 Ritz 法分析含裂縫矩形 Mindlin 板之自然振動頻率與模態，為了準確描述裂縫，提出一組新的允許函數，此允許函數包含能滿足邊界條件之多項式函數與能準確描述裂縫尖端之奇異性並滿足跨越裂縫位移與轉角不連續行為之裂縫函數。

大部分研究功能梯度板，是以探討熱相關的問題，僅少數文獻探討關於 FGM 板或複合材料板振動。基於古典板理論，Yang 和 Shen (2001)研究含平面應力之四邊固定 FGM 方形板之振動行為。He 等人 (2001)研究附有壓電感應器的 FGM 方形板之振動。Zhao 等人 (2009)採用一階剪力變形板

理論去分析 FGM 方形板在不同邊界條件下之振動。Reddy (2000) 導出一有限元素解來分析 FGM 板之振動行為。Ferreira 等人 (2006) 採用無網格葛勒金法去解決一般方程。Matsunaga (2008) 根據 2D 高階板理論，導出一系列的基礎動力方程來分析簡支方形 FGM 板。Qian 等人 (2004) 根據高階剪力變形板理論，利用無網格葛勒金法去解決一般方程，並研究 FGM 板之振動行為。Vel 和 Batra (2004) 提出 3D 解來分析簡支矩形 FGM 板之振動。Hosseini-Hashemi 等人 (2011) 利用 Mindlin 板理論分析圓形與環狀之壓電層與 FGM 板耦合面內與面外振動模態。張明儒 (2008) 推導功能梯度材料厚板的漸近解，並探討板幾何所致的應力奇異性；進而將該漸近解引入懸臂斜形板和具邊緣裂縫簡支承矩形板之振動分析中，並計算具邊緣裂縫矩形板的應力強度因子。王凱平 (2010) 利用 3D 彈性理論探討具邊緣裂縫 FGM 板振動。本研究將延續王凱平之研究，利用 3D 彈性理論分析具內部裂縫 FGM 板由不同邊界條件、不同厚度、不同裂縫位置、不同裂縫長度及角度之振動行為。

1.3 內容概要

本論文共分為五章，其內容如下：

第一章 說明本研究的動機與目的，並回顧相關文獻。

第二章 介紹本研究所使用之理論與方法，並引入允許函數。

第三章 以 3D 彈性理論為基礎，並利用 Ritz 法求解含內部裂縫矩形板之振動，驗證允許函數之適用性，並探討不同邊界條件、不同厚度、不同裂縫長度和不同材料特性參數對收斂性之影響。

第四章 以第三章所用之允許函數分析不同邊界條件、不同厚度、不同裂縫位置、不同裂縫長度及角度和不同材料特性等參數對振動特性之影響。

第五章 本研究之結論與建議。



第二章 分析方法

本章將介紹功能梯度板之材料性質，再以 3D 彈性理論為架構，配合 Ritz 法求解含裂縫之功能梯度板（參考圖 2.1、圖 2.2）振動。

2.1 功能梯度材料公式

FGM 材料為非均勻性。本文中所探討的功能梯度板是由金屬鋁(Al)跟陶瓷(Al_2O_3)組成，其材料性質如表 2.1 所示。假設功能梯度板的材料特性沿厚度方向變化，可表示為

$$P(z) = P_b + V(z)\Delta P \quad (2.1)$$

其中 $P(z)$ 為材料的有效性質， P_b 代表板底層 ($z = -h/2$) 的材料性質， ΔP 是 P_b 跟板頂層 ($z = h/2$) 材料性質的差異，

$$V(z) = \left(\frac{z}{h} + \frac{1}{2} \right)^{\hat{m}} \quad (2.2)$$

h 代表板厚； \hat{m} 是控制材料沿厚度 (z) 方向變化的參數。

2.2 3D 彈性理論之應變能與動能

利用 $x-y-z$ 直角座標系統表示 3D 彈性理論之應變能與動能。應變能以張量分量表示成：

$$U = \int \frac{1}{2} \sigma_{ij} \varepsilon_{ij} dV \quad (2.3)$$

$$\text{其中 } \sigma_{ij} = [\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_{yz}]^T \quad (2.4a)$$

$$\varepsilon_{ij} = [\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{zz} \quad 2\varepsilon_{xy} \quad 2\varepsilon_{xz} \quad 2\varepsilon_{yz}]^T \quad (2.4b)$$

彈性體之本構方程為



$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{yz} \end{Bmatrix} \quad (2.5)$$

其中 λ 與 μ 為拉梅常數(Lamé constants)；即

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} \quad (2.6)$$

當考慮 FGM 時，上式中之 μ 及 λ 均為 z 之函數；利用此本構方程，應變能密度(strain energy density)可表示成：

$$\frac{1}{2}\sigma_{ij}\varepsilon_{ij} = \frac{1}{2}(2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk})\varepsilon_{ij} \quad (2.7)$$

再依微小變形理論中，應變與位移的關係

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (2.8)$$

其中 u_i 代表 u_1, u_2, u_3 ，其為 x 、 y 與 z 方向的位移， $u_{i,j}$ 代表 i 方向的位移在 j 方向上的變化率， $i, j = 1, 2, 3$

將式(2.7)及式(2.8)代入式(2.3)得

$$U = \int \left[\frac{\mu(z)}{4}(u_{i,j} + u_{j,i})^2 + \frac{1}{2}\lambda(z)u_{i,i}u_{k,k} \right] dV \quad (2.9)$$

三維問題之動能可表示成

$$T = \frac{1}{2} \int_V \rho(z)(\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2) dV \quad (2.10)$$

其中 ρ 為單位體積重量

2.3 利用 Ritz 法求解板之自然振動頻率

假設板內某點位移為 $u_j (j=1, 2, 3)$ ，並令

$$u_j = \bar{U}_j e^{i\omega t} \quad (2.11)$$

式中 ω 為自然振動頻率， t 為時間。利用 Ritz 法求解板之自然振動頻率，定義能量函數為

$$\Pi = U_{\max} - T_{\max} \quad (2.12)$$

其中 Π 為結構總勢能， T_{\max} 為一振動週期內最大動能， U_{\max} 為一振動週期內最大應變能。

依式(2.9)及(2.10)可得最大應變能及動能分別可表示為

$$U_{\max} = \int \left[\frac{\mu(z)}{4} (\bar{U}_{i,j} + \bar{U}_{j,i})^2 + \frac{1}{2} \lambda(z) \bar{U}_{i,i} \bar{U}_{k,k} \right] dV \quad (2.13)$$

$$T_{\max} = \frac{\omega^2}{2} \int_V \rho(z) (\bar{U}_1^2 + \bar{U}_2^2 + \bar{U}_3^2) dV \quad (2.14)$$

將 \bar{U}_j 利用完備性之允許函數序列可展開得

$$\bar{U}_j = \sum_{i=1}^{\infty} A_{ij} \tilde{U}_{ij} \quad (2.15)$$

其中 \tilde{U}_{ij} 為滿足幾何邊界條件的允許函數。為使整體總勢能 Π 最小，即

$$\frac{\partial \Pi}{\partial A_{1j}} = \frac{\partial \Pi}{\partial A_{2j}} = \frac{\partial \Pi}{\partial A_{3j}} = 0 \quad (2.16)$$

上式經整理可簡化為特徵值問題，並求解自然振動頻率 ω ，以及振動模態。

整理後可得一組特徵方程式

$$\begin{bmatrix} K^{11} & K^{12} & K^{13} \\ K^{21} & K^{22} & K^{23} \\ K^{31} & K^{32} & K^{33} \end{bmatrix} \begin{Bmatrix} A_{1j} \\ A_{2j} \\ A_{3j} \end{Bmatrix} = \omega^2 \begin{bmatrix} M^{11} & 0 & 0 \\ 0 & M^{22} & 0 \\ 0 & 0 & M^{33} \end{bmatrix} \begin{Bmatrix} A_{1j} \\ A_{2j} \\ A_{3j} \end{Bmatrix} \quad (2.17)$$

勁度矩陣 $[K]$ 與質量矩陣 $[M]$ ，經推導得矩陣各元素表示式為：

$$K_{ij}^{11} = \int_V \left\{ \mu(z) [2\tilde{U}_{1i,1}\tilde{U}_{1j,1} + \tilde{U}_{1i,2}\tilde{U}_{1j,2} + \tilde{U}_{1i,3}\tilde{U}_{1j,3}] + \lambda(z)\tilde{U}_{1i,1}\tilde{U}_{1j,1} \right\} dV \quad (2.18a)$$

$$K_{ij}^{12} = \int_V \left\{ \mu(z)\tilde{U}_{1i,2}\tilde{U}_{2j,1} + \lambda(z)\tilde{U}_{1i,1}\tilde{U}_{2j,2} \right\} dV \quad (2.18b)$$

$$K_{ij}^{13} = \int_V \left\{ \mu(z)\tilde{U}_{1i,3}\tilde{U}_{3j,1} + \lambda(z)\tilde{U}_{1i,1}\tilde{U}_{3j,3} \right\} dV \quad (2.18c)$$

$$K_{ij}^{22} = \int_V \left\{ \mu(z) [\tilde{U}_{2i,1}\tilde{U}_{2j,1} + 2\tilde{U}_{2i,2}\tilde{U}_{2j,2} + \tilde{U}_{2i,3}\tilde{U}_{2j,3}] + \lambda(z)\tilde{U}_{2i,2}\tilde{U}_{2j,2} \right\} dV \quad (2.18d)$$

$$K_{ij}^{23} = \int_V \left\{ \mu(z)\tilde{U}_{2i,3}\tilde{U}_{3j,2} + \lambda(z)\tilde{U}_{2i,2}\tilde{U}_{3j,3} \right\} dV \quad (2.18e)$$

$$K_{ij}^{33} = \int_V \left\{ \mu(z) [\tilde{U}_{3i,1}\tilde{U}_{3j,1} + \tilde{U}_{3i,2}\tilde{U}_{3j,2} + 2\tilde{U}_{3i,3}\tilde{U}_{3j,3}] + \lambda(z)\tilde{U}_{3i,3}\tilde{U}_{3j,3} \right\} dV \quad (2.18f)$$

$$M_{ij}^{11} = \omega^2 \int_V \rho(z) \left\{ \tilde{U}_{1i}\tilde{U}_{1j} \right\} dV \quad (2.19a)$$

$$M_{ij}^{22} = \omega^2 \int_V \rho(z) \left\{ \tilde{U}_{2i}\tilde{U}_{2j} \right\} dV \quad (2.19b)$$

$$M_{ij}^{33} = \omega^2 \int_V \rho(z) \left\{ \tilde{U}_{3i}\tilde{U}_{3j} \right\} dV \quad (2.19c)$$

2.4 允許函數之建構

在 Ritz 法中所使用之允許函數必須滿足自然或幾何邊界條件。為詳細表示，以下以 \bar{U} 、 \bar{V} 、 \bar{W} 分別替代上節所示之 \bar{U}_1 、 \bar{U}_2 及 \bar{U}_3 。本文所使用之允許函數含完備的多項式函數集合 ($\bar{U}_p(x, y, z)$ 、 $\bar{V}_p(x, y, z)$ 、 $\bar{W}_p(x, y, z)$) 與能正確描述尖端處奇異應力行為及沿裂縫位移不連續現象之裂縫函數集 ($\bar{U}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ 、 $\bar{V}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ 、 $\bar{W}_c^*(r_1, r_2, \theta_1, \theta_2, z)$)，即令

$$\bar{U}(x, y, z) = \bar{U}_p(x, y, z) + \bar{U}_c^*(r_1, r_2, \theta_1, \theta_2, z) \quad (2.20a)$$

$$\bar{V}(x, y, z) = \bar{V}_p(x, y, z) + \bar{V}_c^*(r_1, r_2, \theta_1, \theta_2, z) \quad (2.20b)$$

$$\bar{W}(x, y, z) = \bar{W}_p(x, y, z) + \bar{W}_c^*(r_1, r_2, \theta_1, \theta_2, z) \quad (2.20c)$$

為延遲式(2.17)病態矩陣之出現，依 Bhat (1985)所提之正交多項式函數建構允許函數， \bar{U}_p 、 \bar{V}_p 、 \bar{W}_p 表示成：

$$\bar{U}_p(x, y, z) = \sum_{k=1}^{\hat{N}_z} \sum_{j=1}^J \sum_{i=1}^I a_{ijk} f_{ui}(x) g_{uj}(y) z^{k-1} \quad (2.21a)$$

$$\bar{V}_p(x, y, z) = \sum_{k=1}^{\hat{N}_z} \sum_{j=1}^J \sum_{i=1}^I b_{ijk} f_{vi}(x) g_{vj}(y) z^{k-1} \quad (2.21b)$$

$$\bar{W}_p(x, y, z) = \sum_{k=1}^{\hat{N}_z} \sum_{j=1}^J \sum_{i=1}^I c_{ijk} f_{wi}(x) g_{wj}(y) z^{k-1} \quad (2.21c)$$

其中， I 和 J 和 \hat{N}_z 為多項式之項數， a_{ijk} 、 b_{ijk} 、 c_{ijk} 為待定係數， $f_{ui}(x)$ 、 $f_{vi}(x)$ 、 $f_{wi}(x)$ 、 $g_{uj}(y)$ 、 $g_{vj}(y)$ 、 $g_{wj}(y)$ 是經由 Gram-Schmidt 的演算程序所獲得之 x 向及 y 向正交多項式函數，分別滿足 x 向及 y 向之幾何邊界條件。

以建構 $f_{ui}(x)$ 為例，考慮 $x=0$ 及 $x=a$ 處簡支承；

$$f_{u0}(x) = x(a-x) \quad (2.22a)$$

$$f_{u1}(x) = x(a-\beta_1)f_{u0}(x) \quad (2.22b)$$

$$f_{ui}(x) = (x-\beta_i)f_{ui-1}(x) - C_i f_{ui-2}(x) \quad (2.22c)$$

$$\text{其中 } \beta_i = \frac{\int_0^a x f_{ui-1}^2(x) dx}{\int_0^a f_{ui-1}^2(x) dx}, \quad C_i = \frac{\int_0^a x f_{ui-1}(x) f_{ui-2}(x) dx}{\int_0^a f_{ui-2}^2(x) dx},$$

$f_{ui}(x)$ 滿足 $\int_0^a f_{uk}(x) f_{ul}(x) dx = \begin{cases} 0, & uk \neq ul \\ a_{ukul}, & uk = ul \end{cases}$ 使用正交多項式可改善使用一般多項式造成病態矩陣之現象。

為描述內部裂縫尖端應力奇異性與裂縫處位移不連續現象，必需引入另一組函數集，以幫助 Ritz 法辨識裂縫之存在。取 $\bar{U}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ 、

$\bar{V}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ 與 $\bar{W}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ (其中 r_1, r_2, θ_1 及 θ_2 之定義，參看圖 2.1) 表示成：

$$\begin{aligned} \bar{U}_c^*(r_1, r_2, \theta_1, \theta_2, z) = & f_{Bu}(x, y) \sum_{k=1,2}^{\hat{N}_z} \left\{ \sum_{n=1}^{N_{1u-1}} \sum_{m=0}^n \bar{a}_{mnk}^{(A)} r_1^{\frac{2n-1}{2}} \cos \frac{2m+1}{2} \theta_1 + \sum_{n=1}^{N_{1u-2}} \sum_{m=0}^n \bar{a}_{mnk}^{(B)} r_2^{\frac{2n-1}{2}} \cos \frac{2m+1}{2} \theta_2 + \right. \\ & \left. r_2^\beta \sin^2 \frac{\theta_2}{2} \sum_{n=1}^{N_{2u-1}} \sum_{m=0}^n \tilde{a}_{mnk}^{(A)} r_1^{\frac{2n-1}{2}} \sin \frac{2m+1}{2} \theta_1 + r_1^\beta \sin^2 \frac{\theta_1}{2} \sum_{n=1}^{N_{2u-2}} \sum_{m=0}^n \tilde{a}_{mnk}^{(B)} r_2^{\frac{2n-1}{2}} \sin \frac{2m+1}{2} \theta_2 \right\} z^{k-1} \end{aligned} \quad (2.23a)$$

$$\begin{aligned} \bar{V}_c^*(r_1, r_2, \theta_1, \theta_2, z) = & f_{Bv}(x, y) \sum_{k=1,2}^{\hat{N}_z} \left\{ \sum_{n=1}^{N_{1v-1}} \sum_{m=0}^n \bar{b}_{mnk}^{(A)} r_1^{\frac{2n-1}{2}} \cos \frac{2m+1}{2} \theta_1 + \sum_{n=1}^{N_{1v-2}} \sum_{m=0}^n \bar{b}_{mnk}^{(B)} r_2^{\frac{2n-1}{2}} \cos \frac{2m+1}{2} \theta_2 + \right. \\ & \left. r_2^\beta \sin^2 \frac{\theta_2}{2} \sum_{n=1}^{N_{2v-1}} \sum_{m=0}^n \tilde{b}_{mnk}^{(A)} r_1^{\frac{2n-1}{2}} \sin \frac{2m+1}{2} \theta_1 + r_1^\beta \sin^2 \frac{\theta_1}{2} \sum_{n=1}^{N_{2v-2}} \sum_{m=0}^n \tilde{b}_{mnk}^{(B)} r_2^{\frac{2n-1}{2}} \sin \frac{2m+1}{2} \theta_2 \right\} z^{k-1} \end{aligned} \quad (2.23b)$$

$$\begin{aligned} \bar{W}_c^*(r_1, r_2, \theta_1, \theta_2, z) = f_{Bw}(x, y) \sum_{k=1,2}^{\hat{N}_z} \left\{ \sum_{n=1}^{N_{1w-1}} \sum_{m=0}^n \bar{c}_{mnk}^{(A)} r_1^{\frac{2n-1}{2}} \cos \frac{2m+1}{2} \theta_1 + \sum_{n=1}^{N_{1w-2}} \sum_{m=0}^n \bar{c}_{mnk}^{(B)} r_2^{\frac{2n-1}{2}} \cos \frac{2m+1}{2} \theta_2 + \right. \\ \left. r_2^\beta \sin^2 \frac{\theta_2}{2} \sum_{n=1}^{N_{2w-1}} \sum_{m=0}^n \tilde{c}_{mnk}^{(A)} r_1^{\frac{2n-1}{2}} \sin \frac{2m+1}{2} \theta_1 + r_1^\beta \sin^2 \frac{\theta_1}{2} \sum_{n=1}^{N_{2w-2}} \sum_{m=0}^n \tilde{c}_{mnk}^{(B)} r_2^{\frac{2n-1}{2}} \sin \frac{2m+1}{2} \theta_2 \right\} z^{k-1} \end{aligned} \quad (2.23c)$$

式 (2.23) 中，由於 $\sin\left(\frac{2m+1}{2}\theta_1\right)$ 在 $\theta_1 = \pm\pi$ 時並不連續，此特性恰可用於描述真實解跨越裂縫之位移不連續行為；然而此函數於 $\theta_2 = 0$ 時亦為不連續(即圖 2.2 中之虛線部分)，不符合真實物理現象，因此於 $\sin\left(\frac{2m+1}{2}\theta_1\right)$ 項乘上 $\sin^2\left(\frac{\theta_2}{2}\right)$ 以保證於 $\theta_2 = 0$ 時(圖 2.2 中 $\overline{2B}$ 線段)為連續的。其不僅可限制函數跨越裂縫之位移不連續，並維持函數原有之對稱與反對稱特性。另外，由於 $\sin^2\left(\frac{\theta_2}{2}\right)$ 函數之一階導函數於 $r_2 = 0$ 具 $\frac{1}{r_2}$ 之奇異性，因此另外乘上一函數 r_2^β ，其中取 $\beta \geq 0.5$ 以避免該一階導函數造成裂縫尖端處不適當之奇異性。同理，對 $\sin\left(\frac{2m+1}{2}\theta_2\right)$ 項做類似之處理。當取 $\beta > 1$ 時，則 $r_2^\beta \sin^2\left(\frac{\theta_2}{2}\right)$ 或 $r_1^\beta \sin^2\left(\frac{\theta_1}{2}\right)$ 之一階導函數不會於 $r_2 \rightarrow 0$ 或 $r_1 \rightarrow 0$ 處產生奇異性，故在本研究取 $\beta = 1.5$ ，讓裂縫尖端處之應力奇異行為主要由雙疊加符號內之函數來描述。

式 (2.23) 中函數 $f_{Bu}(x, y)$ 、 $f_{Bv}(x, y)$ 與 $f_{Bw}(x, y)$ ，為使 \bar{U}_c 、 \bar{V}_c 及 \bar{W}_c 滿足幾何邊界條件。若考慮簡支承，則可表示成

$$f_{Bu}(x, y) = y(BL - y) \quad (2.24a)$$

$$f_{Bv}(x, y) = x(AL - x) \quad (2.24b)$$

$$f_{Bw}(x,y) = xy(AL-x)(BL-x) \quad (2.24c)$$

以上所定義之 $\bar{U}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ 、 $\bar{V}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ 與 $\bar{W}_c^*(r_1, r_2, \theta_1, \theta_2, z)$ 皆以極座標 (可參考圖 2.1) 表示之，其中卡式座標 (x_i, y_i) 與極座標 (r_i, θ_i) 之轉換關係如下：

$$r_i = \sqrt{\bar{x}_i^2 + \bar{y}_i^2} \quad (2.25)$$

$$\theta_i = \tan^{-1} \frac{\bar{y}_i}{\bar{x}_i} \quad (2.26)$$

其中，

$$\bar{x}_i = (x - A_{xi}) \cos \alpha_i + (y - A_{yi}) \sin \alpha_i, \quad (2.27a)$$

$$\bar{y}_i = -(x - A_{xi}) \sin \alpha_i + (y - A_{yi}) \cos \alpha_i, \quad (2.27b)$$

(A_{xi}, A_{yi}) 為裂縫尖端之 (x, y) 座標，下標 i 為分別內部裂縫兩個奇異點， $i=1, 2$ ，其中 $\alpha_1 = \alpha + \pi$ ， $\alpha_2 = \alpha$ ， α 為裂縫與水平軸之夾角。

在以下分析中，式(2.23)取

$$N_{1u_1} = N_{1u_2} = N_{2u_1} = N_{2u_2} = N_{1v_1} = N_{1v_2} = N_{2v_1} = N_{2v_2} = N_{1w_1} = N_{1w_2} = N_{2w_1} = N_{2w_2} = \bar{N}$$

。

第三章 收斂性分析

本文利用 Ritz 法求得自然振動頻率，當使用足夠多項允許函數時，求得之解將準確從上限收斂至真解。於收斂性分析中，吾人將考慮簡支和懸臂，具不同裂縫長度、不同板厚之均質與 FGM 方形板，並與文獻值比較，以驗證解之正確性。

文中之數值解乃利用 Fortran 程式語言，並使用“IMSL”函式庫進行雙重積分與求解特徵值。為求高精度，以 128-bit 之變數撰寫程式，並使用 64 位元系統執行。本文所得之數值結果僅列出前五個模態並以無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 表示，其中下標 c 指的是陶瓷(ceramic)。波松比 (ν) 取為 0.3， a 為板長， b 為板寬， h 為板厚， d 為裂縫長度， $(x_0/a, y_0/b)$ 為裂縫在中平面之中心位置（幾何參數定義於圖 2.1 中），氧化鋁(Alumina, Al_2O_3) 的材料性質 $E_c = 380Gpa$ ， $\rho_c = 3800kg/m^3$ 。

表 3.1~表 3.13 為具內部裂縫方形板在不同邊界條件下之收斂性分析，各表中 (I, J) 為式(2.21)多項式函數之項數，而 \bar{N} (No. of Corner Functions) 為式(2.23)裂縫函數之項數數目， \hat{N}_z 為允許函數中 z 方向所使用之項數，如此所使用允許函數總項數為 $[(\bar{N} + 3) \times \bar{N} \times 2 + I \times J] \times 3 \times \hat{N}_z$ 。

表 3.1 所列之收斂性分析者為四邊簡支(SSSS)均質方形板($a/b=1$)，其厚寬比為($h/b=0.01$)，含位於 $(x_0/a, y_0/b) = (0.5, 0.5)$ 且長 $d/a=0.2$ 之水平內部

裂縫，考慮無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 。在各模態首列是未加入裂縫函數即($\bar{N}=0$)所得的無因次化頻率，單使用多項式函數並無法有效判別裂縫的存在與行為；將式(2.23)之裂縫函數加入允許函數時，隨著裂縫函數項數提高所得到的結果也逐漸與文獻值接近。Stahl 和 Keer 探討之問題是架構在薄板理論下，利用 Fredholm 積分，求解具裂縫簡支矩形薄板之自然振動頻率；與 3D 彈性理論之結果比較後，發現該文獻高估自然振動頻率，因薄板理論未考慮剪力變形。表 3.1 中所列五個模態皆為面外模態(out-of-plane mode)。 $\hat{N}_z=2$ 之結果較 $\hat{N}_z=3$ 或 4 明顯不精準。當 $I=J=7$ 、 $\bar{N}=6$ 及 $\hat{N}_z=3$ (總項數 1413)時，可達至少 3 位有效位數收斂。

表 3.2 為四邊簡支(SSSS)功能梯度方形板($a/b=1, \hat{m}=0.2$)，厚寬比($h/b=0.1$)，含位於 $(x_0/a, y_0/b)=(0.5, 0.5)$ 且長 $d/a=0.2$ 之水平內部裂縫之收斂性分析。此功能梯度材料由鋁和氧化鋁所組成，並依式(2.2)取控制材料特性參數 $\hat{m}=0.2$ 。當 $I=J=7$ 、 $\bar{N}=4$ 及 $\hat{N}_z=4$ (總項數 1260)時，所得之無因次化頻率可達至少 3 位有效位數之收斂值。

透過收斂性分析得以驗證允許函數之適用性及所發展程式的正確性。將進一步考慮不同邊界條件、不同厚度、不同裂縫長度和不同材料特性參數(\hat{m})，對於收斂性的影響。

3.1 厚度影響

表 3.3 與表 3.1 所考慮者相同，除了板厚從 $h/b=0.01$ 改為 $h/b=0.1$ 。表 3.1 當 $I=J=7$ 、 $\bar{N}=6$ 及 $\hat{N}_z=3$ (總項數 1413) 時，可達至少 3 位有效位數收斂。表 3.3 第一、二和三模態為面外模態(out-of-plane mode)，第四和五模態為面內模態(in-plane mode)，故四和五模態無法與文獻值相比較。當取 $I=J=6$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1392) 時，即可達至少 3 位有效位數之收斂值。一、二和三模態與 Stahl and Keer 的結果相比較，文獻值明顯高估自然振動頻率，因 $h/b=0.1$ 已非屬薄板；另外與 Huang (2011) 利用 Mindlin 板理論配合 Ritz 法所求得之自然振動頻率的結果相比，略大於 Mindlin 板理論之結果 0.4%。觀察發現，薄板需要使用較多的項數才能得到三位有效收斂。

表 3.4 所列收斂性分析者為四邊簡支(SSSS)均質方形板($a/b=1$)，其厚寬比為($h/b=0.05$)，含位於 $(x_0/a, y_0/b)=(0.5, 0.5)$ 且長 $d/a=0.3$ 之水平內部裂縫，考慮無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 。表 3.5 則為與表 3.4 所考慮者相同，除了板厚從 $h/b=0.05$ 改為 $h/b=0.1$ 。表 3.4 所列五個模態皆為面外模態(out-of-plane mode)。當取 $I=J=7$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1548) 時，可達至少 3 位有效位數之收斂值。與 Stahl and Keer 結果比較，文獻值仍高出 3D 彈性理論許多；而與 Huang(2011)的結果相比，略大於 Mindlin 板理論

0.15%。表 3.5 第一、二和三模態為面外模態(out-of-plane mode)，第四和五模態為面內模態(in-plane mode)。當取 $I=J=7$ 、 $\bar{N}=4$ 及 $\hat{N}_z=4$ (總項數 1260)時,可達至少 3 位有效位數之收斂值。與 Stahl 和 Keer 的結果相比較,文獻值仍偏高;而與 Huang(2011)相比,本文略大於 Mindlin 板理論 0.5% 以下。觀察薄板仍需要使用較多的項數才能收斂。

3.2 裂縫長度影響

表 3.6 則為與表 3.2 所考慮者相同,除了裂縫長度從 $d/a=0.2$ 改為 $d/a=0.5$ 。表 3.2 中第四和五模態為面內模態(in-plane mode),當取 $I=J=6$ 、 $\bar{N}=4$ 及 $\hat{N}_z=4$ (總項數 1104)時,可達至少 3 位有效位數之收斂值。表 3.6 中第四和五模態亦為面內模態(in-plane mode),當取 $I=J=7$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1548)時,可達至少 3 位有效位數之收斂值。裂縫較短時,用比較少的允許函數就可收斂;裂縫較長時,則必須使用較多項數,尤其是裂縫函數 \bar{N} (crack function),方可使結果更精確。

表 3.7 所列收斂性分析者為四邊簡支(SSSS)功能梯度方形板($a/b=1$, $\hat{m}=5$),厚寬比($h/b=0.1$),含位於 $(x_0/a, y_0/b)=(0.5,0.5)$ 且長 $d/a=0.3$ 之水平內部裂縫,考慮無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 。表 3.8 則為與表 3.7 者相同,除了裂縫長度從 $d/a=0.3$ 改為 $d/a=0.5$ 。表 3.7 中第四和五模態為面內模態(in-plane mode),當取 $I=J=7$ 、 $\bar{N}=4$ 及 $\hat{N}_z=5$ (總項數 1575)時,可達至少 3

位有效位數之收斂值。表 3.8 中第四和五模態亦為面內模態(in-plane mode)，當取 $I=J=7$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1548)時，可達至少 3 位有效位數之收斂值。兩者比較，仍然為裂縫較短時用較少的允許函數就可收斂；裂縫較長時，則必須用較多項數。

表 3.9 則為與表 3.3 所考慮者相同，除了裂縫長度從 $d/a=0.2$ 改為 $d/a=0.5$ 。表 3.9 所列第四和五模態為面內模態(in-plane mode)。當取 $I=J=7$ 、 $\bar{N}=4$ 及 $\hat{N}_z=4$ (總項數 1260)時，可達至少 3 位有效位數之收斂值。兩者比較，裂縫較長時，必須用較多項數。

表 3.10 所列收斂性分析者為懸臂(CFFF)功能梯度方形板($a/b=1, \hat{m}=5$)，厚寬比($h/b=0.1$)，含位於 $(x_0/a, y_0/b)=(0.5, 0.5)$ 且長 $d/b=0.2$ 垂直(90°)之內部裂縫，考慮無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 。表 3.11 則為與表 3.10 所考慮者相同，除了裂縫長度從 $d/b=0.2$ 改為 $d/b=0.6$ 。表 3.10 中第四模態為面內模態(in-plane mode)，當取 $I=J=7$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1548)時，可達至少 3 位有效位數之收斂值。表 3.11 中第四模態為面內模態(in-plane mode)，當取 $I=J=7$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1548)時，可達至少 3 位有效位數之收斂值。此案例中裂縫長短結果比較，兩者收斂的允許函數項數相同；但一般而言，裂縫較長需要較多的項數，尤其是裂縫函數 \bar{N} (crack function)。

表 3.12 所列收斂性分析者為懸臂(CFFF)均質方形板($a/b=1$)，其厚寬比為($h/b=0.1$)，含位於 $(x_0/a, y_0/b)=(0.5,0.5)$ 且長 $d/b=0.2$ 垂直(90°)之內部裂縫，考慮無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 。表 3.13 則為與表 3.12 所考慮者相同，除了裂縫長度從 $d/b=0.2$ 改為 $d/b=0.6$ 。表 3.12 中第四模態為面內模態(in-plane mode)，當取 $I=J=7$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1548)時，可達至少 3 位有效位數之收斂值。表 3.13 中第四模態為面內模態(in-plane mode)，當取 $I=J=7$ 、 $\bar{N}=5$ 及 $\hat{N}_z=4$ (總項數 1548)時，可達至少 3 位有效位數之收斂值。兩者收斂的允許函數項數均相同。

在收斂性分析中，有許多 \hat{N}_z 差 1，而值接近收斂，其為將數值結果取四位有效，四捨五入後進位與捨去的差別(如表 3.1 中第一、二模態)。另外發現，面外模態(out-of-plane mode)可依 \hat{N}_z 觀察出收斂情形；而面內模態(in-plane mode)值不會依 \hat{N}_z 不同而改變，同時面內模態的頻率收斂變化也小於面外模態。由以上不同裂縫長度與角度、不同厚度及不同邊界條件的收斂性分析中，可發現本研究所提之允許函數在數值不穩定出現前，均可提供至少三位有效收斂之結果。

第四章 數值結果

經收斂性分析已驗證所使用的允許函數的正確性。本章將其應用於分析具有內部裂縫之矩形 FGM 板，探討在不同邊界條件和裂縫位置、角度及長度下，對振動行為的影響。允許函數之使用項數，乃根據前一章之收斂性分析，取法如下：當無裂縫時，取 $I=J=10$ 、 $\bar{N}=0$ (無裂縫函數)，當裂縫長度 d/a (或 d/b) ≤ 0.3 時，取 $I=J=8$ 、 $\bar{N}=5$ ，當裂縫長度 d/a (或 d/b) ≥ 0.3 時，取 $I=J=9$ 、 $\bar{N}=6$ ；若為薄板 ($h/b \leq 0.02$) 時，取 $\hat{N}_z=4$ ，若為厚板時，取 $\hat{N}_z=5$ 。表 4.1~表 4.68 中列出前五個模態之無因次化頻率值，根據第三章之收斂性分析可確認所得之解至少準確至三位有效。

4.1 裂縫對振動頻率之影響

因裂縫而導致之無因次化頻率折減量比，定義為：

$$\Delta\omega = \frac{\omega_{\text{無裂縫}} - \omega_{\text{裂縫}}}{\omega_{\text{無裂縫}}} \times 100\% ; \quad (4.1)$$

其中， $\omega_{\text{無裂縫}}$ 為完整板無因次化頻率值， $\omega_{\text{裂縫}}$ 為具裂縫板之無因次化頻率值。

4.1.1 四邊簡支(SSSS)板

表 4.1、表 4.2 和表 4.3 為具水平內部裂縫於 $(x_0/a, y_0/b)=(0.5,0.5)$ 處之簡支 Al/Al₂O₃ 方形 FGM 板，在不同板厚 $(h/b)=0.02、0.1、0.2$ ，不同裂縫長度 $(d/a)=0.1\sim 0.5$ ，不同材料特性參數 $(\hat{m})=0、0.2、1、5、10$ 下之無因次化自然振動頻率；而表 4.4、表 4.5 和表 4.6 分別為對應表 4.1、表 4.2 和表 4.3 之無因次化頻率折減量比 $\Delta\omega$ (參看式(4.1))。表 4.7、表 4.8 和表 4.9 為具水平內部裂縫於 $(x_0/a, y_0/b)=(0.5,0.75)$ 處之簡支 Al/Al₂O₃ 方形 FGM 板，在不同板厚 $(h/b)=0.02、0.1、0.2$ ，不同裂縫長度 $(d/a)=0.1\sim 0.5$ ，不同材料特性參數 $(\hat{m})=0、0.2、1、5、10$ 下之無因次化自然振動頻率；而表 4.10、表 4.11 和表 4.12 分別為對應表 4.7、表 4.8 和表 4.9 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.13、表 4.14 和表 4.15 為具 $\alpha=30^\circ$ 內部裂縫於 $(x_0/a, y_0/b)=(0.5,0.5)$ 處之簡支 Al/Al₂O₃ 方形 FGM 板，在不同板厚 $(h/b)=0.02、0.1、0.2$ ，不同裂縫長度 $(d/a)=0.1、0.3、0.5$ ，不同材料特性參數 $(\hat{m})=0、0.2、5$ 下之無因次化自然振動頻率；而表 4.16、表 4.17 和表 4.18 分別為對應表 4.13、表 4.14 和表 4.15 之無因次化頻率折減量比 $\Delta\omega$ 。觀察各表可發現

- (1) 考慮完整板(無裂縫者 $d/a=0$)中厚度為 $(h/b=0.1、0.2)$ 板之第四與第五模態頻率完全相同，厚度 $(h/b=0.02)$ 之第二與第三模態頻率

完全相同，此現象為幾何條件對稱具有的特徵值重根現象。

- (2) 當裂縫長度增加時，無因次化頻率隨之下降；此乃由於裂縫長度的增加，導致板勁度降低。觀察各案例 $d/a=0.1$ ，其頻率折減量比 $\Delta\omega$ 皆小於 1.2%；最大值 $\Delta\omega=1.17\%$ 發生在 $h/b=0.02, \hat{m}=10$ 的第五模態。當 $d/a=0.5$ 時，其頻率折減量比 $\Delta\omega$ 則可升至約 21%；最大值 $\Delta\omega=20.70\%$ 發生在 $h/b=0.1, \hat{m}=10$ 的第二模態。裂縫越短，勁度降低量越小，各模態頻率值降低量也越小。

- (3) 考慮不同厚度下裂縫位置改變對頻率之影響：

- (a) 當 $h/b=0.02$ 且裂縫長度固定時，水平裂縫位置從 $(x_0/a, y_0/b)=(0.5, 0.5)$ 改變至 $(x_0/a, y_0/b)=(0.5, 0.75)$ ，第一模態之頻率隨之增加，而第二模態頻率隨之減小；第三模態當 $d/a=0.1 \sim 0.3$ 時無明顯趨勢， $d/a=0.4 \sim 0.5$ 時頻率隨之增加；第四模態當 $d/a=0.1 \sim 0.3$ 時無明顯趨勢， $d/a=0.4 \sim 0.5$ 時頻率隨之減少；第五模態當 $d/a=0.1 \sim 0.3$ 時頻率隨之增加， $d/a=0.4 \sim 0.5$ 時則呈相反趨勢。

- (b) 當 $h/b=0.1$ 且裂縫長度固定時，水平裂縫位置從 $(x_0/a, y_0/b)=(0.5, 0.5)$ 改變至 $(x_0/a, y_0/b)=(0.5, 0.75)$ ，第一模態之頻率隨之增加；第二模態 $d/a=0.1 \sim 0.4$ 隨之減少， $d/a=0.5$ 呈相反

趨勢；第三模態無明顯趨勢；第四和五模態隨之減少。

- (c) 當 $h/b=0.2$ 且裂縫長度固定時，水平裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.5,0.75)$ ，第一模態之頻率隨之增加；第二模態 $d/a=0.1\sim 0.4$ 隨之減少， $d/a=0.5$ 呈相反趨勢；第三模態隨之減少；第四模態無明顯趨勢；第五模態 $d/a=0.1\sim 0.3$ 無明顯趨勢， $d/a=0.4\sim 0.5$ 時則隨之增加。

(4) 考慮不同厚度下裂縫角度改變對頻率之影響：

- (a) 當 $h/b=0.02$ 且裂縫位置 $(x_0/a, y_0/b)=(0.5,0.5)$ 時，角度由 $\alpha=0^\circ$ 改變至 $\alpha=30^\circ$ ，第一和二模態之頻率無一定趨勢，第三和四模態隨之減少，第五模態則隨之增加。

- (b) 當 $h/b=0.1$ 且裂縫位置 $(x_0/a, y_0/b)=(0.5,0.5)$ 時，角度由 $\alpha=0^\circ$ 改變至 $\alpha=30^\circ$ ，第一和三模態之頻率隨之減少，第二、四和五模態無一定趨勢。

- (c) 當 $h/b=0.2$ 且裂縫位置 $(x_0/a, y_0/b)=(0.5,0.5)$ 時，角度由 $\alpha=0^\circ$ 改變至 $\alpha=30^\circ$ ，第一模態之頻率隨之減少，第二和五模態 $d/a=0.1$ 及 $d/a=0.3$ 無一定趨勢， $d/a=0.5$ 隨之增加；而第三和四模態無一定之趨勢。

(5) 無因次化頻率隨材料特性參數(\hat{m})增加而下降，是因為 \hat{m} 增加所造成的勁度下降相較於質量減少來的快(如圖 4.1 所示)。

(6) 當厚板($h/b=0.1、0.2$)之第五模態主要以面內位移為主時(參看圖 4.3)，其無因次化頻率的變化對於裂縫長度之增加較不明顯，也就是其面內勁度較不受裂縫長度的影響(相較於面外者)。

(7) 厚度增加，其無因次化頻率將隨之下降，此因於無因次化頻率中含 $(\frac{1}{h})$ 之項。

4.1.2 懸臂(CFFF)板

表 4.19、表 4.20 和表 4.21 為具水平($\alpha=0^\circ$)內部裂縫於 $(x_0/a, y_0/b)=(0.5, 0.5)$ 處之懸臂 Al/Al₂O₃ 方形 FGM 板，在不同板厚(h/b) = 0.02、0.1、0.2，不同裂縫長度(d/a) = 0.1 ~ 0.5，不同材料特性參數(\hat{m}) = 0、0.2、1、5、10 下之無因次化自然振動頻率；而表 4.22、表 4.23 和表 4.24 分別為對應表 4.19、表 4.20 和表 4.21 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.25、表 4.26 和表 4.27 為具水平($\alpha=0^\circ$)內部裂縫於 $(x_0/a, y_0/b)=(0.5, 0.75)$ 處之懸臂 Al/Al₂O₃ 方形 FGM 板，在不同板厚(h/b) = 0.02、0.1、0.2，不同裂縫長度(d/a) = 0.1、0.3、0.5，不同材料特性參數(\hat{m}) = 0、0.2、5 下之無因次化自然振動頻率；而表 4.28、表 4.29 和表 4.30 分別為對應表 4.25、表 4.26 和表 4.27 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.31、表 4.32 和表 4.33 為具垂

直($\alpha = 90^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.5, 0.5)$ 處之懸臂 Al/Al₂O₃ 方形 FGM 板，在不同板厚 $(h/b) = 0.02、0.1、0.2$ ，不同裂縫長度 $(d/a) = 0.1、0.3、0.5$ ，不同材料特性參數 $(\hat{m}) = 0、0.2、5$ 下之無因次化自然振動頻率；而表 4.34、表 4.35 和表 4.36 分別為對應表 4.31、表 4.32 和表 4.33 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.37、表 4.38 和表 4.39 為具垂直($\alpha = 90^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.25, 0.5)$ 處之懸臂 Al/Al₂O₃ 方形 FGM 板，在不同板厚 $(h/b) = 0.02、0.1、0.2$ ，不同裂縫長度 $(d/a) = 0.1、0.3、0.5$ ，不同材料特性參數 $(\hat{m}) = 0、0.2、5$ 下之無因次化自然振動頻率；而表 4.40、表 4.41 和表 4.42 分別為對應表 4.37、表 4.38 和表 4.39 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.43、表 4.44 和表 4.45 為具($\alpha = 150^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.5, 0.5)$ 處之懸臂 Al/Al₂O₃ 方形 FGM 板，在不同板厚 $(h/b) = 0.02、0.1、0.2$ ，不同裂縫長度 $(d/a) = 0.1、0.3、0.5$ ，不同材料特性參數 $(\hat{m}) = 0、0.2、5$ 下之無因次化自然振動頻率；而表 4.46、表 4.47 和表 4.48 分別為對應表 4.43、表 4.44 和表 4.45 之無因次化頻率折減量比 $\Delta\omega$ 。觀察各表可發現裂縫對頻率之影響與前節所述者不同的是：

- (1) 觀察各案例 $d/a = 0.1$ ，其頻率折減量比 $\Delta\omega$ 大部分都小於 1.2%；且最大值 $\Delta\omega = 1.19\%$ 發生於 $h/b = 0.2, \alpha = 90^\circ, \hat{m} = 5, (x_0/a, y_0/b) = (0.25, 0.5)$ 的第一模態。當 $d/a = 0.5$ 時，其頻率折減量比 $\Delta\omega$ 則可升至約 20%；

最大值 $\Delta\omega=19.23\%$ 發生於 $h/b=0.2$, $\alpha=90^\circ$, $\hat{m}=5$,
 $(x_0/a, y_0/b)=(0.25,0.5)$ 的第一模態。

(2) 考慮不同厚度下裂縫位置改變對頻率之影響：

(a) 當 $h/b=0.02$ 且 $\alpha=0^\circ$ 時，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.5,0.75)$ ，第一至五模態之頻率皆隨之增加。當 $\alpha=90^\circ$ ，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.25,0.5)$ ，第一和二模態頻率隨之減小，第三、四和五模態則隨之增加。

(b) 當 $h/b=0.1$ 且 $\alpha=0^\circ$ 時，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.5,0.75)$ ，第一和三模態之頻率無明顯趨勢，第二、四和五模態隨之增加。當 $\alpha=90^\circ$ ，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.25,0.5)$ ，第一和四模態頻率隨之減小，第二、三模態則隨之增加，第五模態 $d/a=0.1$ 和 0.3 隨之增加， $d/a=0.5$ 則呈相反趨勢。

(c) 當 $h/b=0.2$ 且 $\alpha=0^\circ$ 時，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.5,0.75)$ ，第一模態之頻率無明顯趨勢，第二至五模態皆隨之增加。當 $\alpha=90^\circ$ ，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改

變至 $(x_0/a, y_0/b) = (0.25, 0.5)$ ，第一和三模態頻率隨之減小，第二、四模態則隨之增加，第五模態 $d/a = 0.1$ 和 0.3 隨之增加， $d/a = 0.5$ 則呈相反趨勢。

(3) 考慮不同厚度下裂縫角度改變對頻率之影響：

(a) 當 $h/b = 0.02$ 時，且裂縫位置 $(x_0/a, y_0/b) = (0.5, 0.5)$ 時，角度由 $\alpha = 0^\circ$ 改變至 $\alpha = 90^\circ$ ，第一和三模態之頻率隨之減少，第二、四和五模態則隨之增加。當角度由 $\alpha = 90^\circ$ 改變至 $\alpha = 150^\circ$ ，第一和三模態之頻率隨之增加，第二、四和五模態隨之減少。

(b) 當 $h/b = 0.1$ 時，且裂縫位置 $(x_0/a, y_0/b) = (0.5, 0.5)$ 時，角度由 $\alpha = 0^\circ$ 改變至 $\alpha = 90^\circ$ ，第一模態之頻率隨之減少，第二模態無一定趨勢，第三和五模態隨之增加，第四模態 $d/a = 0.1$ 和 0.3 隨之增加， $d/a = 0.5$ 則呈相反趨勢。當角度由 $\alpha = 90^\circ$ 改變至 $\alpha = 150^\circ$ ，第一和三模態之頻率隨之增加，第二和五模態隨之減少，第四模態則無一定趨勢。

(c) 當 $h/b = 0.2$ 時，且裂縫位置 $(x_0/a, y_0/b) = (0.5, 0.5)$ 時，角度由 $\alpha = 0^\circ$ 改變至 $\alpha = 90^\circ$ ，第一至四模態之頻率隨之減少，第五模態則隨之增加。當角度由 $\alpha = 90^\circ$ 改變至 $\alpha = 150^\circ$ ，第一和四模態之頻率

隨之增加，第二和五模態隨之減少，第三模態無一定趨勢。

4.1.3 兩端固定端，兩端自由端(CFCF)板

表 4.49 和表 4.50 為具垂直($\alpha = 90^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.5, 0.5)$ 處之兩端固定端，兩端自由端 Al/Al₂O₃ 方形 ($a/b = 1$) FGM 板，在不同板厚(h/b) = 0.1、0.2，不同裂縫長度(d/a) = 0.1、0.3、0.5，不同材料特性參數(\hat{m}) = 0、0.2、5 下之無因次化自然振動頻率；而表 4.51 和表 4.52 分別為對應表 4.49 和表 4.50 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.53 和表 4.54 為具垂直($\alpha = 90^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.25, 0.5)$ 處之兩端固定端，兩端自由端 Al/Al₂O₃ 方形 ($a/b = 1$) FGM 板，在不同板厚(h/b) = 0.1、0.2，不同裂縫長度(d/a) = 0.1、0.3、0.5，不同材料特性參數(\hat{m}) = 0、0.2、5 下之無因次化自然振動頻率；而表 4.55 和表 4.56 分別為對應表 4.53 和表 4.54 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.57 和表 4.58 為具垂直($\alpha = 90^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.5, 0.5)$ 處之兩端固定端，兩端自由端 Al/Al₂O₃ 矩形 ($a/b = 2$) FGM 板，在不同板厚(h/b) = 0.1、0.2，不同裂縫長度(d/a) = 0.1、0.3、0.5，不同材料特性參數(\hat{m}) = 0、0.2、5 下之無因次化自然振動頻率；而表 4.59 和表 4.60 分別為對應表 4.57 和表 4.58 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.61 和表 4.62 為具垂直($\alpha = 90^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.25, 0.5)$ 處之兩端固定端，兩端自由端 Al/Al₂O₃ 矩形 ($a/b = 2$) FGM 板，在不同板厚(h/b) = 0.1、

0.2，不同裂縫長度(d/a) = 0.1、0.3、0.5，不同材料特性參數(\hat{m}) = 0、0.2、5 下之無因次化自然振動頻率；而表 4.63 和表 4.64 分別為對應表 4.61 和表 4.62 之無因次化頻率折減量比 $\Delta\omega$ 。表 4.65 和表 4.66 為具($\alpha = 150^\circ$)內部裂縫於 $(x_0/a, y_0/b) = (0.5, 0.5)$ 處之兩端固定端 Al/Al₂O₃ 方形 ($a/b = 1$) FGM 板，在不同板厚(h/b) = 0.1、0.2，不同裂縫長度(d/a) = 0.1、0.3、0.5，不同材料特性參數(\hat{m}) = 0、0.2、5 下之無因次化自然振動頻率；而表 4.67 和表 4.68 分別為對應表 4.65 和表 4.66 之無因次化頻率折減量比 $\Delta\omega$ 。觀察各表可發現裂縫對頻率之影響：

- (1) 觀察各案例 $d/a = 0.1$ ，其頻率折減量比 $\Delta\omega$ 都小於 0.65%；且最大值 $\Delta\omega = 0.64\%$ 發生於方形 ($a/b = 1$)， $h/b = 0.1$ ， $\alpha = 90^\circ$ ， $\hat{m} = 5$ ， $(x_0/a, y_0/b) = (0.5, 0.5)$ 的第一模態。當 $d/a = 0.5$ 時，其頻率折減量比 $\Delta\omega$ 則可升至約 20%；最大值 $\Delta\omega = 19.11\%$ 發生於方形 ($a/b = 1$)， $h/b = 0.2$ ， $\alpha = 90^\circ$ ， $\hat{m} = 0$ ， $(x_0/a, y_0/b) = (0.5, 0.5)$ 的第五模態。

- (2) 考慮不同厚度下裂縫位置改變對頻率之影響：

- (a) 當 $h/b = 0.1$ 且為方型板 ($a/b = 1$) 時，裂縫位置從 $(x_0/a, y_0/b) = (0.5, 0.5)$ 改變至 $(x_0/a, y_0/b) = (0.25, 0.5)$ ，第一模態之頻率隨之增加，第二和五模態隨之減少，第三模態 $d/a = 0.1$ 和 0.3 隨之增加， $d/a = 0.5$ 則呈相反趨勢；第四模態 $d/a = 0.1$ 和 0.3 隨之減

少， $d/a=0.5$ 呈相反趨勢。當為矩型板 ($a/b=2$)，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.25,0.5)$ ，第一、四和五模態之頻率隨之增加，第二和三模態則隨之減少。

(b) 當 $h/b=0.2$ 且為方型板 ($a/b=1$) 時，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.25,0.5)$ ，第一模態之頻率隨之增加，第二和三模態隨之減少，第四模態 $d/a=0.1$ 隨之增加， $d/a=0.3$ 和 0.5 呈相反趨勢；第五模態 $d/a=0.1$ 和 0.3 隨之減少， $d/a=0.5$ 呈相反趨勢。當為矩型板 ($a/b=2$)，裂縫位置從 $(x_0/a, y_0/b)=(0.5,0.5)$ 改變至 $(x_0/a, y_0/b)=(0.25,0.5)$ ，第一和五模態之頻率隨之增加，第二、三和四模態則隨之減少。

(3) 考慮不同厚度下裂縫角度改變對頻率之影響

(a) 當 $h/b=0.1$ 且為方型板 ($a/b=1$) 時，裂縫位置 $(x_0/a, y_0/b)=(0.5,0.5)$ ，角度由 $\alpha=90^\circ$ 改變至 $\alpha=150^\circ$ ，第一和四模態之頻率隨之增加，第二、三和五模態則隨之減少。

(b) 當 $h/b=0.2$ 且為方型板 ($a/b=1$) 時，裂縫位置 $(x_0/a, y_0/b)=(0.5,0.5)$ ，角度由 $\alpha=90^\circ$ 改變至 $\alpha=150^\circ$ ，第一和五模態之頻率隨之增加，第二、三和四模態則隨之減少。

4.2 不同理論所得結果比較

厚度效應中，將允許函數用於分析不同厚度 ($h/b=0.01$ 、 0.05 、 0.1 及 0.2) 並包含位於 $(x_0/a, y_0/b)=(0.5,0.5)$ 之水平裂縫 ($d/a=0.3$ 及 0.5) SSSS 方形板，表 4.69 列出邊界條件為簡支之結果，其中 [] 之值為 Stahl 和 Keer (1972)，以古典薄板理論為架構，利用對偶級數方程得齊性 Fredholm 第二型積分方程求解所得結果；() 之值為李榕師 (2009)，利用 Ritz 法分析含裂縫矩形 Mindlin 板(一階剪力變型板理論，其中剪力修正係數取 $\kappa^2 = \pi^2/12$)；{ } 之值為本研究依 3D 彈性理論之結果(不含面內模態者)。

- (1) 當厚度 $h/b=0.01$ 時，相對於 3D 彈性理論之結果，古典板理論及一階剪力變型板理論之誤差分別小於 0.308% 與 0.154%，厚度增加為 $h/b=0.05$ 時誤差則分別介於 1.93% ~ 6.03% 與 0.018% ~ 0.407%；但如果厚度 $h/b=0.1$ ，其誤差則高達 5.11% ~ 17.30% 與 0.178% ~ 0.721%；當厚度 $h/b=0.2$ ，其誤差則高達 14.90% ~ 50.2% 與 0.453% ~ 1.50%。其中表 4.69 誤差最大值大部分發生在第二模態。古典板理論與 3D 彈性理論所得者之相對差異隨 h/b 增加而增加，同時相對差異大部分也隨 d/a 增加而增加。除了第五模態，當 $d/a=0.3$ 時，相對 3D 彈性理論誤差均小於 43.40%，且最大值發生在 $h/b=0.2$ 的第四模態，裂縫長度增加為 $d/a=0.5$ 時，相對 3D 彈性理論誤差均小於 48.60%，且

最大值發生在 $h/b=0.2$ 的第二模態。故古典板理論所得之結果會有高估的現象。

- (2) 當 $h/b \leq 0.1$ 時，Mindlin 板理論與 3D 彈性理論所得者之相對差異均小於 1%，誤差最大值大部分發生在第二和四模態，且誤差隨厚度增加而漸增。裂縫長度增加時，Mindlin 板理論與 3D 彈性理論所得者之相對差異也隨之增加，但較不明顯。當 $d/a=0.3$ 時，相對 3D 彈性理論誤差均小於 1.44%，最大值發生在 $h/b=0.2$ 的第四模態；當 $d/a=0.5$ 時，相對 3D 彈性理論誤差均小於 1.50%，最大值發生在 $h/b=0.2$ 的第四模態。因此由以上討論可知 Mindlin 板理論可用於分析板厚 $h/b \leq 0.2$ 者；但 Mindlin 板理論相較於 3D 彈性理論會稍有低估的現象。

4.3 板之振動模態探討

圖 4.2~4.10 為前述所分析具有裂縫之 FGM 板的部分 3D 振動模態圖與在中平面上之 2D 模態圖；圖中包含了各種裂縫開裂的形式。2D 圖中板內空白者乃為以面內位移為主之模態(in-plane mode)，其餘則為以面外位移為主之模態(out-of-plane mode)；面外模態為主之模態圖中，繪有面外位移等高線，虛線為 z 方向位移為零之節點線。

對於無裂縫之均質板而言，其縱向撓曲、側向撓曲、縱向延伸以及扭曲等模態，彼此間是不耦合的，並且形成相異的對稱模態。而對於具裂縫之 FGM 板，其各模態間相互耦合。

觀察無裂縫板之模態圖：

圖 4.2 為簡支(SSSS)方形之均質與 Al/Al₂O₃ FGM ($\hat{m}=5$)薄板 2D 振態圖，板厚為 $h/b=0.02$ ，其第一模態為無節點線之面外撓曲，第二和第三模態分別為具不同方向單一節點線之面外撓曲，第四模態為沿兩條相互垂直節點線之面外撓曲，第五模態為沿兩條相互平行節點線之面外撓曲。

圖 4.3 與圖 4.4 分別為簡支(SSSS)方形之均質板振態圖與 Al/Al₂O₃ FGM 板振態圖 ($\hat{m}=5$)，觀察板厚 $h/b=0.1$ ，其第一模態為無節點線之面外撓曲，第二和第三模態分別為具不同方向單一節點線之面外撓曲，第四和第五模態則為平面內的縱向延伸。板厚 $h/b=0.2$ ，其第一模態為無節點線之面外撓曲，第二和第三模態為平面內的縱向延伸，第四和第五模態則為具不同方向單一節點線之面外撓曲。

圖 4.5 與圖 4.6 分別為懸臂(CFFF)方形之均質板振態圖與 Al/Al₂O₃ FGM 板振態圖($\hat{m}=5$)。板厚 $h/b=0.1$ 之第一模態為一個無節點線之面外撓曲，第二模態為分別具不同方向單一節點線之面外扭曲，第三模態為沿單一節點線之面外撓曲，第四模態為平面內之延伸，第五模態為沿兩條對稱節點

線之面外撓曲。而板厚 $h/b=0.2$ 之第一、二和五模態與 $h/b=0.1$ 相同，第三模態為為平面內之延伸，第四模態為沿單一節點線之面外撓曲。

圖 4.7 與圖 4.8 為兩端固定端，兩端自由端(CFCF)方形之均質板振態圖與 Al/Al₂O₃ FGM 板振態圖($\hat{m}=5$)。板厚 $h/b=0.1$ 之第一模態為無節點線之面外撓曲，第二模態為單一節點線之面外扭曲，第三模態為兩條節點線之面外扭曲，第四模態為具不同方向單一節點線之面外撓曲，第五模態為沿兩條相互垂直之節點線之面外扭曲。板厚之第一和二模態同 $h/b=0.1$ ，第三模態為平面內之延伸，第四和五模態則同 $h/b=0.1$ 之第三和四模態。圖 4.9 與圖 4.10 同為兩端固定端，兩端自由端(CFCF)，形狀改為矩形($a/b=2$)之均質與 Al/Al₂O₃ FGM 板振態圖($\hat{m}=5$)，板厚 $h/b=0.1$ 之第一模態為無節點線之面外撓曲，第二模態為單一節點線之面外扭曲，第三模態為具不同方向單一節點線之面外撓曲，第四模態為沿兩條相互垂直之節點線之面外扭曲，第五模態為兩條節點線之面外扭曲。板厚 $h/b=0.2$ 之第一、二和三模態同 $h/b=0.1$ ，第四模態為平面內之延伸，第五模態同 $h/b=0.1$ 之第四模態。以上圖中可見均質板與 Al/Al₂O₃ FGM 板所呈現之振態圖相同，故模態圖並不受材料特性參數(\hat{m})所影響。

第五章 結論與建議

5.1 結論

本研究以 3D 彈性理論為架構，提出一組裂縫函數配合多項式函數組成 Ritz 法之允許函數，分析具內部裂縫矩形 FGM 板之自由振動。所提出之裂縫函數不僅能準確描述裂縫前端之應力奇異性，並符合裂縫處位移之不連續特性。依本研究的結果，吾人可得結論如下：

- (1) 在收斂性分析中，驗證了將裂縫函數引入允許函數，對於振動頻率之影響。只使用多項式函數無法解決裂縫問題；加入角函數不只加速解之收斂，並能了解每一振態於裂縫附近之應力奇異行為。當使用較多項之允許函數時，所得結果在數值不穩定出現前大部分皆為至少三位有效數字收斂。
- (2) 數值結果中，利用本文所提之函數求解內部裂縫之各種案例：簡支承、懸臂端及兩端固定端，兩端自由端三種邊界條件下，在不同裂縫位置（ $(x_0/a, y_0/b)=(0.5,0.5)$ 、 $(x_0/a, y_0/b)=(0.5,0.75)$ 或 $(x_0/a, y_0/b)=(0.25,0.5)$ ）、裂縫長度（ $d/b=0.1\sim 0.5$ ）、角度（ $\alpha=0^\circ$ 、 30° 、及 150° ）、板厚（ $h/b=0.02$ 、 0.1 和 0.2 ）與不同材料特性參數（ \hat{m} ）的振動分析；其中所有數值結果皆可達三位有效收斂，且繪有部分

- 3D 及 2D 之模態圖。另外，不論邊界條件或裂縫類型，裂縫長度增加將使得板的勁度下降，而頻率也跟著降低。當裂縫長度較短 ($d/a=0.1$) 時，前五個模態頻率值下降幅度皆小於 2%。
- (3) FGM 之材料特性參數 (\hat{m}) 影響 FGM 板沿 z 方向材料參數的分布狀況及板的勁度與質量。由數值分析的結果發現：當 FGM 材料為 Al/Al_2O_3 時， \hat{m} 越大將使無因次化頻率呈現遞減的趨勢。
- (4) 本研究所得之具內部裂縫均質板結果與依古典薄板理論所得者比較，古典薄板理論高估自然振動頻率。由於古典薄板理論不考慮剪力變形，致使當板厚(h/b)增加時，其所得頻率與 3D 彈性理論者相差亦增加；但此增加趨勢並未隨裂縫長度改變而明顯改變。
- (5) 本研究所得之具內部裂縫均質板結果與依 Mindlin 板理論所得者比較，Mindlin 板理論相較則稍有低估的現象，當 $h/b=0.2$ 時皆小於 1.5%，此低估現象可能由於剪力修正係數取值($\kappa^2 = \pi^2/12$)所造成。

5.2 建議

利用 Ritz 法分析具應力奇異點之功能梯度板振動，由多項式函數結合描述應力奇異性之角函數形成所需之允許函數，有效並精確的分析含裂縫板之問題。將來可直接應用本研究所提之函數於其他具應力奇異之問題。

亦可將本研究所提方法之精神應用於其他理論或求解不同材料(例：壓電材料)之問題。



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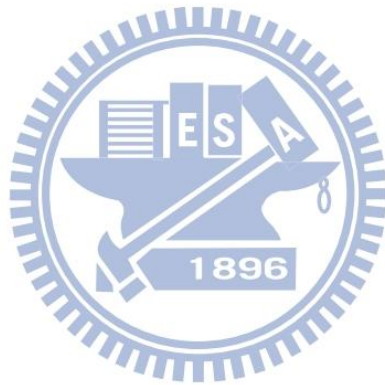


表 2.1 材料參數

Material	Properties		
	E(Gpa)	Poisson's ratio	$\rho(\text{kg/m}^3)$
Aluminum(Al)	70.0	0.3	2702
Alumina(Al_2O_3)	380	0.3	3800



表 3.1 具水平內部裂縫簡支方形均質薄板($a/b = 1.0, h/b = 0.01, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.2, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							Stahl and Keer (1972)
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
1	0	[7.024] (6.370) {6.370}	[6.609] (5.974) {5.974}	[6.609] (5.974) {5.974}	[6.609] (5.971) {5.971}	[6.609] (5.971) {5.971}	[6.609] (5.971) {5.971}	[6.609] (5.971) {5.971}	5.842*
	2	[6.741] (6.103) {6.103}	[6.596] (5.964) {5.964}	[6.592] (5.962) {5.961}	[6.589] (5.959) {5.959}	[6.587] (5.958) {5.957}	[6.586] (5.958) {5.957}	[6.584] (5.956) {5.955}	
	3	[6.582] (5.955) {5.954}	[6.579] (5.954) {5.953}	[6.576] (5.951) {5.950}	[6.574] (5.950) {5.949}	[6.570] (5.947) {5.947}	[6.569] (5.946) {5.946}	[6.566] (5.945) {5.944}	
	4	[6.530] (5.922) {5.921}	[6.528] (5.921) {5.920}	[6.511] (5.910) {5.910}	[6.501] (5.904) {5.903}	[6.497] (5.901) {5.900}	[6.491] (5.897) {5.897}	[6.489] (5.896) {5.895}	
	5	[6.442] (5.865) {5.865}	[6.440] (5.864) {5.864}	[6.430] (5.858) {5.857}	[6.428] (5.856) {5.855}	[6.425] (5.854) {5.854}	[6.422] (5.853) {5.852}	[6.421] (5.852) {5.851}	
	6	[6.418] (5.850) {5.849}	[6.418] (5.850) {5.849}	[6.408] (5.843) {5.842}	[6.408] (5.843) {5.842}	[6.405] (5.841) {5.840}	[6.405] (5.841) {5.840}	[6.404] (5.840) {5.839}	
	7	[6.410] (5.845) {5.844}	[6.408] (5.843) {5.842}	[6.403] (5.840) {5.839}	[6.403] (5.839) {5.838}	[6.401] (5.838) {5.837}	[6.401] (5.838) {5.837}	[6.400] (5.837) {5.836}	

Note: []: results from $\hat{N}_z = 2$; (): results from $\hat{N}_z = 3$; { }: results from $\hat{N}_z = 4$

表 3.1 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							Stahl and Keer (1972)
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
2	0	[19.73] (18.27) {18.27}	[19.64] (17.95) {17.95}	[16.56] (14.97) {14.97}	[16.56] (14.97) {14.97}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	14.88*
	2	[18.59] (17.06) {17.05}	[18.32] (16.73) {16.73}	[16.55] (14.96) {14.96}	[16.54] (14.95) {14.95}	[16.48] (14.89) {14.89}	[16.48] (14.89) {14.89}	[16.47] (14.89) {14.88}	
	3	[16.74] (15.17) {15.17}	[16.64] (15.05) {15.05}	[16.51] (14.93) {14.93}	[16.48] (14.89) {14.89}	[16.46] (14.88) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	
	4	[16.48] (14.90) {14.90}	[16.48] (14.89) {14.89}	[16.46] (14.88) {14.88}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	
	5	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.44] (14.87) {14.87}	[16.44] (14.87) {14.87}	
	6	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.45] (14.87) {14.87}	[16.44] (14.87) {14.87}	[16.44] (14.87) {14.87}	[16.44] (14.87) {14.86}	
	7	[16.45] (14.87) {14.87}	[16.44] (14.87) {14.87}	[16.44] (14.87) {14.87}	[16.44] (14.87) {14.87}	[16.44] (14.87) {14.86}	[16.44] (14.87) {14.86}	[16.44] (14.87) {14.86}	

Note: []: results from $\hat{N}_z = 2$; (): results from $\hat{N}_z = 3$; { }: results from $\hat{N}_z = 4$

表 3.1 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							Stahl and Keer (1972)
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
3	0	[19.73] (18.27) {18.27}	[19.64] (17.95) {17.95}	[16.56] (14.97) {14.97}	[16.56] (14.97) {14.97}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	14.93*
	2	[18.99] (17.36) {17.36}	[18.44] (16.83) {16.83}	[16.55] (14.96) {14.96}	[16.55] (14.96) {14.95}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	
	3	[16.82] (15.24) {15.23}	[16.65] (15.08) {15.07}	[16.53] (14.94) {14.94}	[16.53] (14.94) {14.94}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	
	4	[16.53] (14.94) {14.94}	[16.52] (14.93) {14.93}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	[16.51] (14.92) {14.92}	
	5	[16.52] (14.93) {14.93}	[16.51] (14.92) {14.92}	[16.50] (14.92) {14.92}	[16.50] (14.92) {14.92}	[16.50] (14.92) {14.92}	[16.50] (14.92) {14.91}	[16.50] (14.92) {14.91}	
	6	[16.50] (14.92) {14.91}	[16.50] (14.92) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	
	7	[16.50] (14.92) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	[16.50] (14.91) {14.91}	

Note: []: results from $\hat{N}_z = 2$; (): results from $\hat{N}_z = 3$; { }: results from $\hat{N}_z = 4$

表 3.1 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							Stahl and Keer (1972)
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
4	0	[30.96] (28.52) {28.51}	[30.93] (28.22) {28.22}	[26.48] (23.93) {23.93}	[26.48] (23.93) {23.93}	[26.41] (23.86) {23.86}	[26.41] (23.86) {23.86}	[26.41] (23.86) {23.86}	23.89*
	2	[30.64] (27.97) {27.96}	[28.53] (25.98) {25.97}	[26.46] (23.92) {23.91}	[26.45] (23.90) {23.90}	[26.40] (23.86) {23.86}	[26.40] (23.86) {23.85}	[26.40] (23.86) {23.85}	
	3	[27.22] (24.72) {24.71}	[26.62] (24.07) {24.07}	[26.44] (23.89) {23.89}	[26.42] (23.88) {23.88}	[26.40] (23.86) {23.85}	[26.40] (23.86) {23.85}	[26.40] (23.86) {23.85}	
	4	[26.43] (23.89) {23.89}	[26.42] (23.88) {23.87}	[26.41] (23.87) {23.87}	[26.40] (23.86) {23.86}	[26.40] (23.86) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	
	5	[26.41] (23.87) {23.86}	[26.40] (23.86) {23.85}	[26.40] (23.86) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	
	6	[26.40] (23.86) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	[26.39] (23.85) {23.85}	[26.39] (23.85) {23.85}	
	7	[26.40] (23.86) {23.85}	[26.40] (23.85) {23.85}	[26.40] (23.85) {23.85}	[26.39] (23.85) {23.85}	[26.39] (23.85) {23.85}	[26.39] (23.85) {23.85}	[26.39] (23.85) {23.85}	

Note: []: results from $\hat{N}_z = 2$; (): results from $\hat{N}_z = 3$; { }: results from $\hat{N}_z = 4$

表 3.1 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							Stahl and Keer (1972)
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
5	0	[194.8] (194.8) {194.8}	[46.60] (43.79) {43.76}	[46.60] (43.00) {43.00}	[33.47] (30.29) {30.26}	[33.47] (30.26) {30.26}	[33.02] (29.83) {29.82}	[33.02] (29.83) {29.82}	28.43*
	2	[45.66] (42.37) {42.34}	[44.60] (41.47) {41.45}	[43.12] (39.88) {39.86}	[33.28] (30.15) {30.14}	[33.22] (30.10) {30.09}	[32.77] (29.67) {29.66}	[32.74] (29.65) {29.64}	
	3	[37.09] (34.07) {34.06}	[34.40] (31.36) {31.35}	[32.99] (29.89) {29.88}	[32.91] (29.84) {29.83}	[32.73] (29.67) {29.66}	[32.59] (29.55) {29.54}	[32.57] (29.53) {29.52}	
	4	[32.35] (29.43) {29.42}	[32.34] (29.42) {29.41}	[31.95] (29.09) {29.09}	[31.86] (29.03) {29.02}	[31.82] (29.00) {28.99}	[31.79] (28.98) {28.97}	[31.77] (28.96) {28.96}	
	5	[31.33] (28.67) {28.66}	[31.28] (28.61) {28.60}	[31.21] (28.56) {28.55}	[31.18] (28.54) {28.53}	[31.16] (28.52) {28.51}	[31.13] (28.50) {28.49}	[31.12] (28.49) {28.48}	
	6	[31.09] (28.53) {28.52}	[31.09] (28.47) {28.46}	[31.00] (28.41) {28.40}	[31.00] (28.40) {28.39}	[30.98] (28.39) {28.37}	[30.97] (28.38) {28.37}	[30.96] (28.37) {28.36}	
	7	[31.02] (28.42) {28.41}	[30.99] (28.40) {28.39}	[30.95] (28.37) {28.35}	[30.95] (28.37) {28.35}	[30.93] (28.35) {28.34}	[30.93] (28.35) {28.34}	[30.92] (28.35) {28.33}	

Note: []: results from $\hat{N}_z = 2$; (): results from $\hat{N}_z = 3$; { }: results from $\hat{N}_z = 4$

表 3.2 具水平內部裂縫簡支之 Al/Al₂O₃ FGM 方形中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.2, \alpha = 0^\circ, m = 0.2$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) @ [] $\hat{N}_z = 3$; () $\hat{N}_z = 4$; { } $\hat{N}_z = 5$						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	(5.725)	(5.387)	(5.387)	(5.383)	(5.383)	(5.383)	(5.383)
		(5.707)	(5.371)	(5.371)	(5.371)	(5.371)	(5.371)	(5.371)
		{5.707}	{5.371}	{5.371}	{5.371}	{5.371}	{5.371}	{5.371}
	2	[5.303]	[5.255]	[5.250]	[5.244]	[5.242]	[5.241]	[5.240]
		(5.287)	(5.239)	(5.234)	(5.229)	(5.226)	(5.225)	(5.224)
		{5.286}	{5.239}	{5.234}	{5.229}	{5.226}	{5.225}	{5.224}
	3	[5.237]	[5.236]	[5.234]	[5.233]	[5.232]	[5.231]	[5.230]
		(5.221)	(5.220)	(5.218)	(5.217)	(5.215)	(5.215)	(5.213)
		{5.221}	{5.219}	{5.218}	{5.217}	{5.215}	{5.215}	{5.213}
	4	[5.224]	[5.224]	[5.221]	[5.220]	[5.218]	[5.218]	[5.216]
		(5.208)	(5.208)	(5.205)	(5.204)	(5.202)	(5.201)	(5.200)
		{5.208}	{5.208}	{5.205}	{5.204}	{5.202}	{5.201}	{5.200}
	5	[5.218]	[5.217]	[5.215]	[5.214]	[5.213]	[5.213]	[5.213]
		(5.202)	(5.200)	(5.199)	(5.198)	(5.197)	(5.197)	(5.197)
		{5.202}	{5.200}	{5.199}	{5.198}	{5.197}	{5.197}	{5.197}
2	0	(15.50)	(15.21)	(12.97)	(12.97)	(12.94)	(12.94)	(12.94)
		(15.37)	(15.08)	(12.90)	(12.90)	(12.86)	(12.86)	(12.86)
		{15.37}	{15.08}	{12.90}	{12.89}	{12.86}	{12.86}	{12.86}
	2	[13.16]	[13.00]	[12.83]	[12.81]	[12.80]	[12.80]	[12.80]
		(13.08)	(12.92)	(12.75)	(12.73)	(12.72)	(12.72)	(12.71)
		{13.08}	{12.92}	{12.75}	{12.73}	{12.72}	{12.71}	{12.71}
	3	[12.81]	[12.81]	[12.80]	[12.80]	[12.79]	[12.79]	[12.79]
		(12.73)	(12.73)	(12.72)	(12.71)	(12.71)	(12.71)	(12.71)
		{12.73}	{12.73}	{12.72}	{12.71}	{12.71}	{12.71}	{12.71}
	4	[12.80]	[12.79]	[12.79]	[12.79]	[12.79]	[12.79]	[12.79]
		(12.71)	(12.71)	(12.71)	(12.71)	(12.71)	(12.70)	(12.70)
		{12.71}	{12.71}	{12.71}	{12.71}	{12.70}	{12.70}	{12.70}
	5	[12.79]	[12.79]	[12.79]	[12.79]	[12.79]	[12.79]	[12.79]
		(12.71)	(12.71)	(12.71)	(12.70)	(12.70)	(12.70)	(12.70)
		{12.71}	{12.71}	{12.70}	{12.70}	{12.70}	{12.70}	{12.70}

表 3.2 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	(15.50)	(15.21)	(12.97)	(12.97)	(12.94)	(12.94)	(12.94)
		(15.37)	(15.08)	(12.90)	(12.90)	(12.86)	(12.86)	(12.86)
		{15.37}	{15.08}	{12.90}	{12.89}	{12.86}	{12.86}	{12.86}
	2	[13.57]	[13.48]	[12.96]	[12.95]	[12.93]	[12.93]	[12.93]
		(13.48)	(13.40)	(12.88)	(12.88)	(12.85)	(12.85)	(12.85)
	{13.48}	{13.39}	{12.88}	{12.88}	{12.85}	{12.85}	{12.85}	
3	[12.95]	[12.94]	[12.94]	[12.93]	[12.93]	[12.93]	[12.93]	
	(12.87)	(12.87)	(12.86)	(12.86)	(12.85)	(12.85)	(12.85)	
	{12.87}	{12.87}	{12.86}	{12.86}	{12.85}	{12.85}	{12.85}	
4	[12.93]	[12.93]	[12.93]	[12.93]	[12.93]	[12.93]	[12.93]	
	(12.86)	(12.85)	(12.85)	(12.85)	(12.85)	(12.85)	(12.85)	
	{12.86}	{12.85}	{12.85}	{12.85}	{12.85}	{12.85}	{12.85}	
5	[12.93]	[12.93]	[12.93]	[12.93]	[12.93]	[12.93]	[12.93]	
	(12.85)	(12.85)	(12.85)	(12.85)	(12.85)	(12.85)	(12.85)	
	{12.85}	{12.85}	{12.85}	{12.85}	{12.85}	{12.85}	{12.85}	
4	0	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
		{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}
	2	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	
3	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	
	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	
	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	
4	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	
	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	
	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	
5	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	
	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	
	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.2 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}
	2	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}
	3	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}
	4	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}
	5	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}	[18.56] (18.56) {18.56}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.3 具水平內部裂縫簡支方形均質中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.2, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) @ $[\] \hat{N}_z = 3; () \hat{N}_z = 4; \{ \} \hat{N}_z = 5$							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
1	0	[6.158] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	5.842* 5.577+
	2	[5.704] (5.684) {5.684}	[5.653] (5.633) {5.633}	[5.648] (5.628) {5.628}	[5.642] (5.622) {5.622}	[5.640] (5.620) {5.620}	[5.639] (5.619) {5.619}	[5.637] (5.617) {5.617}	
	3	[5.635] (5.614) {5.614}	[5.633] (5.613) {5.613}	[5.631] (5.611) {5.611}	[5.631] (5.610) {5.610}	[5.629] (5.608) {5.608}	[5.628] (5.608) {5.607}	[5.627] (5.606) {5.606}	
	4	[5.621] (5.601) {5.601}	[5.621] (5.600) {5.600}	[5.618] (5.598) {5.598}	[5.617] (5.597) {5.597}	[5.614] (5.594) {5.594}	[5.614] (5.594) {5.594}	[5.613] (5.592) {5.592}	
	5	[5.615] (5.594) {5.594}	[5.613] (5.593) {5.593}	[5.611] (5.591) {5.591}	[5.611] (5.590) {5.590}	[5.610] (5.589) {5.589}	[5.610] (5.589) {5.589}	[5.609] (5.589) {5.589}	
2	0	[16.64] (16.47) {16.47}	[16.32] (16.17) {16.17}	[13.94] (13.84) {13.84}	[13.94] (13.84) {13.84}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	14.88* 13.57+
	2	[14.13] (14.02) {14.02}	[13.96] (13.86) {13.86}	[13.78] (13.68) {13.68}	[13.76] (13.66) {13.66}	[13.75] (13.65) {13.65}	[13.75] (13.64) {13.64}	[13.74] (13.64) {13.64}	
	3	[13.76] (13.66) {13.66}	[13.76] (13.66) {13.66}	[13.75] (13.65) {13.65}	[13.74] (13.64) {13.64}	[13.74] (13.64) {13.64}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	
	4	[13.75] (13.64) {13.64}	[13.74] (13.64) {13.64}	[13.74] (13.64) {13.64}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	
	5	[13.74] (13.64) {13.64}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	[13.74] (13.63) {13.63}	

表 3.3 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
3	0	[16.64] (16.47) {16.47}	[16.32] (16.17) {16.17}	[13.94] (13.84) {13.84}	[13.94] (13.84) {13.84}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	14.93* 13.74+
	2	[14.57] (14.46) {14.45}	[14.48] (14.37) {14.37}	[13.92] (13.82) {13.82}	[13.92] (13.82) {13.82}	[13.89] (13.80) {13.80}	[13.89] (13.80) {13.80}	[13.89] (13.80) {13.80}	
	3	[13.91] (13.82) {13.81}	[13.91] (13.81) {13.81}	[13.90] (13.80) {13.80}	[13.90] (13.80) {13.80}	[13.89] (13.80) {13.80}	[13.89] (13.80) {13.80}	[13.89] (13.80) {13.80}	
	4	[13.90] (13.80) {13.80}	[13.89] (13.80) {13.80}	[13.89] (13.80) {13.80}	[13.89] (13.80) {13.79}	[13.89] (13.80) {13.79}	[13.89] (13.79) {13.79}	[13.89] (13.79) {13.79}	
	5	[13.89] (13.80) {13.80}	[13.89] (13.79) {13.79}	[13.89] (13.79) {13.79}	[13.89] (13.79) {13.79}	[13.89] (13.79) {13.79}	[13.89] (13.79) {13.79}	[13.89] (13.79) {13.79}	
4	0	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	-
	2	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	3	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	4	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	5	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.3 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
5	0	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	-
	2	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	3	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	4	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	5	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.4 具水平內部裂縫簡支方形均質薄板($a/b = 1.0, h/b = 0.05, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.3, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
1	0	[6.319]	[5.928]	[5.928]	[5.928]	[5.928]	[5.928]	[5.928]	5.698* 5.589+
		(6.313)	(5.922)	(5.922)	(5.922)	(5.922)	(5.922)	(5.922)	
		{6.313}	{5.922}	{5.922}	{5.922}	{5.922}	{5.922}	{5.922}	
	2	[5.814]	[5.769]	[5.753]	[5.747]	[5.740]	[5.738]	[5.732]	
		(5.806)	(5.761)	(5.745)	(5.738)	(5.732)	(5.730)	(5.723)	
		{5.806}	{5.761}	{5.745}	{5.738}	{5.731}	{5.730}	{5.723}	
3	[5.691]	[5.685]	[5.673]	[5.669]	[5.664]	[5.661]	[5.654]		
	(5.683)	(5.676)	(5.664)	(5.660)	(5.655)	(5.651)	(5.645)		
	{5.683}	{5.676}	{5.664}	{5.660}	{5.655}	{5.651}	{5.645}		
4	[5.624]	[5.619]	[5.613]	[5.612]	[5.609]	[5.608]	[5.606]		
	(5.615)	(5.610)	(5.604)	(5.603)	(5.600)	(5.599)	(5.598)		
	{5.615}	{5.610}	{5.604}	{5.603}	{5.600}	{5.599}	{5.598}		
5	[5.612]	[5.609]	[5.605]	[5.604]	[5.602]	[5.602]	[5.601]		
	(5.604)	(5.600)	(5.596)	(5.595)	(5.593)	(5.593)	(5.592)		
	{5.604}	{5.600}	{5.596}	{5.595}	{5.593}	{5.593}	{5.592}		
6	[5.609]	[5.605]	[5.601]	[5.601]	[5.599]	[5.599]	[5.599]		
	(5.600)	(5.596)	(5.592)	(5.592)	(5.591)	(5.591)	(5.590)		
	{5.600}	{5.596}	{5.592}	{5.592}	{5.591}	{5.591}	{5.590}		

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.4 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
2	0	[17.82] (17.77) {17.77}	[17.51] (17.45) {17.45}	[14.69] (14.66) {14.66}	[14.69] (14.66) {14.66}	[14.65] (14.62) {14.62}	[14.65] (14.62) {14.62}	[14.65] (14.62) {14.62}	14.67* 14.19+
	2	[15.45] (15.41) {15.41}	[15.10] (15.06) {15.06}	[14.38] (14.34) {14.34}	[14.34] (14.30) {14.30}	[14.31] (14.27) {14.27}	[14.29] (14.26) {14.26}	[14.29] (14.25) {14.25}	
	3	[14.33] (14.29) {14.29}	[14.31] (14.28) {14.28}	[14.30] (14.26) {14.26}	[14.28] (14.24) {14.24}	[14.28] (14.24) {14.24}	[14.27] (14.23) {14.23}	[14.27] (14.23) {14.23}	
	4	[14.30] (14.26) {14.26}	[14.28] (14.24) {14.24}	[14.28] (14.24) {14.24}	[14.27] (14.23) {14.23}	[14.26] (14.22) {14.22}	[14.26] (14.22) {14.22}	[14.26] (14.22) {14.21}	
	5	[14.28] (14.24) {14.24}	[14.27] (14.23) {14.23}	[14.27] (14.22) {14.22}	[14.26] (14.22) {14.22}	[14.26] (14.22) {14.22}	[14.25] (14.21) {14.21}	[14.25] (14.21) {14.21}	
	6	[14.26] (14.22) {14.22}	[14.26] (14.22) {14.22}	[14.26] (14.22) {14.22}	[14.25] (14.21) {14.21}	[14.25] (14.21) {14.21}	[14.25] (14.21) {14.21}	[14.25] (14.21) {14.21}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.4 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
3	0	[17.82] (17.77) {17.77}	[17.51] (17.45) {17.45}	[14.69] (14.66) {14.66}	[14.69] (14.66) {14.66}	[14.65] (14.62) {14.62}	[14.65] (14.62) {14.62}	[14.65] (14.62) {14.62}	14.90* 14.55+
	2	[15.96] (15.92) {15.92}	[15.73] (15.69) {15.69}	[14.66] (14.63) {14.63}	[14.65] (14.62) {14.62}	[14.62] (14.59) {14.59}	[14.62] (14.59) {14.59}	[14.62] (14.59) {14.59}	
	3	[14.68] (14.65) {14.65}	[14.65] (14.62) {14.62}	[14.63] (14.60) {14.60}	[14.62] (14.59) {14.59}	[14.62] (14.59) {14.59}	[14.62] (14.59) {14.59}	[14.62] (14.59) {14.59}	
	4	[14.65] (14.62) {14.62}	[14.61] (14.58) {14.58}	[14.61] (14.58) {14.58}	[14.61] (14.58) {14.58}	[14.61] (14.58) {14.58}	[14.61] (14.58) {14.58}	[14.61] (14.58) {14.58}	
	5	[14.65] (14.62) {14.62}	[14.61] (14.58) {14.58}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	
	6	[14.65] (14.62) {14.62}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	[14.60] (14.57) {14.57}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.4 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
4	0	[27.49] (27.37) {27.37}	[27.13] (27.01) {27.01}	[23.24] (23.17) {23.17}	[23.24] (23.17) {23.17}	[23.18] (23.11) {23.11}	[23.18] (23.11) {23.11}	[23.18] (23.11) {23.11}	23.87* 22.90+
	2	[25.08] (24.98) {24.98}	[24.20] (24.11) {24.11}	[23.13] (23.05) {23.05}	[23.11] (23.03) {23.03}	[23.07] (22.99) {22.99}	[23.06] (22.98) {22.98}	[23.06] (22.98) {22.98}	
	3	[23.28] (23.20) {23.20}	[23.14] (23.05) {23.05}	[23.09] (23.00) {23.00}	[23.06] (22.98) {22.98}	[23.05] (22.97) {22.97}	[23.04] (22.96) {22.96}	[23.04] (22.96) {22.96}	
	4	[23.09] (23.01) {23.01}	[23.06] (22.98) {22.98}	[23.05] (22.97) {22.97}	[23.04] (22.96) {22.96}	[23.04] (22.95) {22.95}	[23.03] (22.95) {22.95}	[23.03] (22.95) {22.95}	
	5	[23.07] (22.99) {22.99}	[23.04] (22.96) {22.96}	[23.04] (22.96) {22.96}	[23.03] (22.95) {22.95}	[23.03] (22.95) {22.95}	[23.03] (22.95) {22.95}	[23.03] (22.95) {22.94}	
	6	[23.05] (22.97) {22.97}	[23.04] (22.96) {22.95}	[23.04] (22.95) {22.95}	[23.03] (22.95) {22.95}	[23.03] (22.95) {22.95}	[23.03] (22.94) {22.94}	[23.03] (22.94) {22.94}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.4 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
5	0	[38.98] (38.98) {38.98}	[38.98] (38.98) {38.98}	[38.98] (38.98) {38.98}	[29.19] (29.07) {29.07}	[29.17] (29.06) {29.06}	[28.77] (28.66) {28.66}	[28.77] (28.66) {28.66}	27.12* 25.59+
	2	[34.83] (34.65) {34.65}	[32.89] (32.70) {32.70}	[29.77] (29.64) {29.64}	[27.36] (27.24) {27.24}	[27.23] (27.11) {27.11}	[26.95] (26.83) {26.83}	[26.89] (26.77) {26.77}	
	3	[27.24] (27.12) {27.12}	[26.90] (26.75) {26.75}	[26.52] (26.40) {26.40}	[26.44] (26.30) {26.30}	[26.31] (26.19) {26.19}	[26.25] (26.13) {26.13}	[26.20] (26.08) {26.08}	
	4	[26.07] (25.95) {25.94}	[25.98] (25.85) {25.85}	[25.85] (25.74) {25.74}	[25.85] (25.73) {25.73}	[25.82] (25.70) {25.70}	[25.81] (25.69) {25.69}	[25.80] (25.68) {25.68}	
	5	[25.86] (25.75) {25.75}	[25.82] (25.70) {25.70}	[25.79] (25.67) {25.67}	[25.78] (25.66) {25.66}	[25.76] (25.65) {25.65}	[25.76] (25.64) {25.64}	[25.75] (25.63) {25.63}	
	6	[25.83] (25.71) {25.71}	[25.78] (25.66) {25.66}	[25.75] (25.64) {25.64}	[25.75] (25.64) {25.64}	[25.74] (25.63) {25.63}	[25.74] (25.63) {25.62}	[25.74] (25.62) {25.62}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.5 具水平內部裂縫簡支方形均質中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.3, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [] $\hat{N}_z = 3$; () $\hat{N}_z = 4$; { } $\hat{N}_z = 5$							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
1	0	[6.158] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	5.698* 5.411+
	2	[5.552] (5.530) {5.530}	[5.531] (5.510) {5.510}	[5.522] (5.501) {5.501}	[5.520] (5.498) {5.498}	[5.515] (5.493) {5.493}	[5.514] (5.492) {5.492}	[5.509] (5.486) {5.486}	
	3	[5.488] (5.466) {5.466}	[5.487] (5.464) {5.464}	[5.477] (5.455) {5.455}	[5.475] (5.452) {5.452}	[5.470] (5.448) {5.448}	[5.469] (5.446) {5.446}	[5.465] (5.443) {5.443}	
	4	[5.457] (5.435) {5.435}	[5.453] (5.431) {5.431}	[5.449] (5.427) {5.427}	[5.449] (5.427) {5.427}	[5.446] (5.425) {5.425}	[5.446] (5.424) {5.424}	[5.445] (5.423) {5.423}	
	5	[5.450] (5.428) {5.428}	[5.447] (5.426) {5.426}	[5.445] (5.423) {5.423}	[5.444] (5.423) {5.423}	[5.443] (5.422) {5.422}	[5.443] (5.422) {5.422}	[5.443] (5.421) {5.421}	
2	0	[16.64] (16.47) {16.47}	[16.32] (16.17) {16.17}	[13.94] (13.84) {13.84}	[13.94] (13.84) {13.84}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	14.67* 13.15+
	2	[13.83] (13.73) {13.73}	[13.69] (13.58) {13.58}	[13.43] (13.32) {13.32}	[13.39] (13.28) {13.28}	[13.38] (13.26) {13.26}	[13.36] (13.25) {13.25}	[13.36] (13.25) {13.25}	
	3	[13.38] (13.27) {13.27}	[13.37] (13.26) {13.26}	[13.37] (13.25) {13.25}	[13.36] (13.24) {13.24}	[13.35] (13.24) {13.24}	[13.35] (13.23) {13.23}	[13.35] (13.23) {13.23}	
	4	[13.37] (13.26) {13.25}	[13.36] (13.24) {13.24}	[13.35] (13.24) {13.24}	[13.34] (13.23) {13.23}	[13.34] (13.22) {13.22}	[13.34] (13.22) {13.22}	[13.34] (13.22) {13.22}	
	5	[13.35] (13.24) {13.24}	[13.35] (13.23) {13.23}	[13.35] (13.23) {13.23}	[13.34] (13.22) {13.22}	[13.34] (13.22) {13.22}	[13.34] (13.22) {13.22}	[13.34] (13.22) {13.22}	

表 3.5 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
3	0	[16.64] (16.47) {16.47}	[16.32] (16.17) {16.17}	[13.94] (13.84) {13.84}	[13.94] (13.84) {13.84}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	14.90* 13.71+
	2	[14.80] (14.68) {14.68}	[14.71] (14.60) {14.59}	[13.89] (13.80) {13.80}	[13.88] (13.79) {13.79}	[13.87] (13.77) {13.77}	[13.87] (13.77) {13.77}	[13.87] (13.77) {13.77}	
	3	[13.89] (13.80) {13.80}	[13.88] (13.78) {13.78}	[13.87] (13.77) {13.77}	[13.86] (13.77) {13.77}	[13.86] (13.77) {13.76}	[13.86] (13.76) {13.76}	[13.86] (13.76) {13.76}	
	4	[13.88] (13.78) {13.78}	[13.86] (13.76) {13.76}	[13.86] (13.76) {13.76}	[13.86] (13.76) {13.76}	[13.86] (13.76) {13.76}	[13.86] (13.76) {13.76}	[13.85] (13.76) {13.76}	
	5	[13.88] (13.78) {13.78}	[13.85] (13.76) {13.76}	[13.85] (13.76) {13.76}	[13.85] (13.76) {13.76}	[13.85] (13.76) {13.76}	[13.85] (13.76) {13.76}	[13.85] (13.76) {13.76}	
4	0	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	-
	2	[19.47] (19.47) {19.47}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	
	3	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	
	4	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	
	5	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	[19.46] (19.46) {19.46}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.5 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
5	0	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	-
	2	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	3	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	4	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	5	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.6 具水平內部裂縫簡支之 Al/Al₂O₃ FGM 方形中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.5, \alpha = 0^\circ, m = 0.2$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) @ $[\] \hat{N}_z = 3; () \hat{N}_z = 4; \{ \} \hat{N}_z = 5$						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	(5.725)	(5.387)	(5.387)	(5.383)	(5.383)	(5.383)	(5.383)
		(5.707)	(5.371)	(5.371)	(5.371)	(5.371)	(5.371)	(5.371)
		{5.707}	{5.371}	{5.371}	{5.371}	{5.371}	{5.371}	{5.371}
	2	[4.901]	[4.890]	[4.872]	[4.871]	[4.853]	[4.851]	[4.839]
		(4.882)	(4.870)	(4.853)	(4.851)	(4.833)	(4.831)	(4.820)
		{4.882}	{4.870}	{4.853}	{4.851}	{4.833}	{4.831}	{4.820}
	3	[4.778]	[4.767]	[4.752]	[4.751]	[4.745]	[4.745]	[4.741]
		(4.760)	(4.747)	(4.734)	(4.733)	(4.727)	(4.727)	(4.724)
		{4.759}	{4.747}	{4.734}	{4.733}	{4.727}	{4.727}	{4.724}
	4	[4.750]	[4.740]	[4.736]	[4.735]	[4.734]	[4.733]	[4.733]
		(4.732)	(4.722)	(4.718)	(4.717)	(4.716)	(4.716)	(4.715)
		{4.732}	{4.722}	{4.718}	{4.717}	{4.716}	{4.716}	{4.715}
	5	[4.747]	[4.737]	[4.733]	[4.732]	[4.731]	[4.731]	[4.731]
		(4.729)	(4.719)	(4.715)	(4.715)	(4.714)	(4.714)	(4.713)
		{4.729}	{4.719}	{4.715}	{4.715}	{4.714}	{4.714}	{4.713}
2	0	(15.50)	(15.21)	(12.97)	(12.97)	(12.94)	(12.94)	(12.94)
		(15.37)	(15.08)	(12.90)	(12.90)	(12.86)	(12.86)	(12.86)
		{15.37}	{15.08}	{12.90}	{12.89}	{12.86}	{12.86}	{12.86}
	2	[11.26]	[10.92]	[10.68]	[10.64]	[10.61]	[10.57]	[10.56]
		(11.17)	(10.80)	(10.58)	(10.54)	(10.51)	(10.47)	(10.46)
		{11.17}	{10.80}	{10.58}	{10.54}	{10.50}	{10.47}	{10.46}
	3	[10.63]	[10.58]	[10.57]	[10.53]	[10.53]	[10.51]	[10.51]
		(10.53)	(10.47)	(10.46)	(10.42)	(10.42)	(10.40)	(10.40)
		{10.52}	{10.47}	{10.46}	{10.42}	{10.42}	{10.40}	{10.40}
	4	[10.56]	[10.52]	[10.51]	[10.50]	[10.50]	[10.50]	[10.49]
		(10.45)	(10.41)	(10.40)	(10.39)	(10.39)	(10.38)	(10.38)
		{10.45}	{10.41}	{10.40}	{10.39}	{10.39}	{10.38}	{10.38}
	5	[10.52]	[10.51]	[10.50]	[10.50]	[10.49]	[10.49]	[10.49]
		(10.41)	(10.40)	(10.39)	(10.38)	(10.38)	(10.37)	(10.37)
		{10.41}	{10.40}	{10.39}	{10.38}	{10.38}	{10.37}	{10.37}

表 3.6 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	[15.50]	[15.21]	[12.97]	[12.97]	[12.94]	[12.94]	[12.94]
		(15.37)	(15.08)	(12.90)	(12.90)	(12.86)	(12.86)	(12.86)
		{15.37}	{15.08}	{12.90}	{12.89}	{12.86}	{12.86}	{12.86}
	2	[13.27]	[13.13]	[12.81]	[12.79]	[12.78]	[12.77]	[12.77]
		(13.18)	(13.05)	(12.73)	(12.71)	(12.70)	(12.70)	(12.69)
4	3	[12.92]	[12.76]	[12.75]	[12.75]	[12.74]	[12.74]	[12.73]
		(12.84)	(12.68)	(12.67)	(12.67)	(12.66)	(12.66)	(12.65)
	{12.84}	{12.68}	{12.67}	{12.67}	{12.66}	{12.66}	{12.65}	
	4	[12.90]	[12.72]	[12.71]	[12.71]	[12.71]	[12.71]	[12.71]
		(12.82)	(12.65)	(12.63)	(12.63)	(12.63)	(12.63)	(12.63)
{12.82}	{12.64}	{12.63}	{12.63}	{12.63}	{12.63}	{12.63}		
5	[12.89]	[12.72]	[12.71]	[12.70]	[12.70]	[12.70]	[12.70]	
	(12.81)	(12.64)	(12.63)	(12.63)	(12.63)	(12.63)	(12.63)	
{12.81}	{12.64}	{12.63}	{12.63}	{12.63}	{12.63}	{12.63}		
4	0	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
		{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}
	2	[18.42]	[18.38]	[18.38]	[18.37]	[18.37]	[18.37]	[18.37]
		(18.42)	(18.38)	(18.38)	(18.37)	(18.37)	(18.37)	(18.37)
{18.42}	{18.38}	{18.38}	{18.37}	{18.37}	{18.37}	{18.37}		
3	[18.37]	[18.37]	[18.36]	[18.36]	[18.36]	[18.36]	[18.36]	
	(18.37)	(18.37)	(18.36)	(18.36)	(18.36)	(18.36)	(18.36)	
	{18.37}	{18.37}	{18.36}	{18.36}	{18.36}	{18.36}	{18.36}	
4	[18.36]	[18.36]	[18.36]	[18.36]	[18.36]	[18.36]	[18.36]	
	(18.36)	(18.36)	(18.36)	(18.36)	(18.36)	(18.36)	(18.36)	
{18.36}	{18.36}	{18.36}	{18.36}	{18.36}	{18.36}	{18.36}		
5	[18.36]	[18.36]	[18.36]	[18.36]	[18.36]	[18.36]	[18.36]	
	(18.36)	(18.36)	(18.36)	(18.36)	(18.36)	(18.36)	(18.36)	
{18.36}	{18.36}	{18.36}	{18.36}	{18.36}	{18.36}	{18.36}		

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.6 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
		{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}
	2	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
		{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}
	3	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
		{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}
	4	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
		{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}
	5	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]	[18.56]
		(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)	(18.56)
		{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}	{18.56}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.7 具水平內部裂縫簡支之 Al/Al₂O₃ FGM 方形中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.3, \alpha = 0^\circ, m=5$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [] $\hat{N}_z = 4$; () $\hat{N}_z = 5$; { } $\hat{N}_z = 6$						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	[4.005] (4.004) {4.003}	[3.773] (3.773) {3.772}	[3.773] (3.773) {3.772}	[3.773] (3.772) {3.772}	[3.773] (3.772) {3.772}	[3.773] (3.772) {3.772}	[3.773] (3.772) {3.772}
	2	[3.607] (3.607) {3.606}	[3.594] (3.594) {3.593}	[3.588] (3.588) {3.587}	[3.586] (3.586) {3.585}	[3.583] (3.582) {3.582}	[3.582] (3.581) {3.581}	[3.579] (3.578) {3.577}
	3	[3.566] (3.565) {3.564}	[3.564] (3.563) {3.563}	[3.558] (3.558) {3.557}	[3.557] (3.556) {3.555}	[3.554] (3.554) {3.553}	[3.553] (3.552) {3.552}	[3.551] (3.550) {3.550}
	4	[3.546] (3.545) {3.545}	[3.544] (3.543) {3.543}	[3.541] (3.541) {3.540}	[3.541] (3.541) {3.540}	[3.541] (3.538) {3.538}	[3.541] (3.538) {3.538}	[3.538] (3.538) {3.538}
	5	[3.542] (3.541) {3.541}	[3.540] (3.540) {3.539}	[3.539] (3.538) {3.538}	[3.538] (3.538) {3.537}	[3.538] (3.538) {3.537}	[3.538] (3.538) {3.537}	[3.538] (3.538) {3.537}
2	0	[10.61] (10.60) {10.60}	[10.41] (10.41) {10.40}	[8.958] (8.954) {8.952}	[8.957] (8.953) {8.951}	[8.933] (8.930) {8.928}	[8.933] (8.930) {8.928}	[8.933] (8.929) {8.927}
	2	[8.879] (8.875) {8.873}	[8.772] (8.768) {8.766}	[8.604] (8.600) {8.598}	[8.577] (8.573) {8.571}	[8.566] (8.562) {8.560}	[8.557] (8.553) {8.551}	[8.555] (8.551) {8.549}
	3	[8.573] (8.569) {8.567}	[8.563] (8.558) {8.556}	[8.558] (8.553) {8.551}	[8.549] (8.544) {8.542}	[8.546] (8.542) {8.540}	[8.542] (8.538) {8.536}	[8.540] (8.536) {8.534}
	4	[8.558] (8.554) {8.552}	[8.549] (8.544) {8.542}	[8.544] (8.540) {8.538}	[8.538] (8.533) {8.531}	[8.536] (8.532) {8.529}	[8.534] (8.529) {8.527}	[8.533] (8.529) {8.526}
	5	[8.545] (8.541) {8.538}	[8.541] (8.536) {8.534}	[8.537] (8.533) {8.530}	[8.535] (8.530) {8.528}	[8.534] (8.530) {8.527}	[8.533] (8.528) {8.525}	[8.532] (8.528) {8.525}

表 3.7 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	[10.61]	[10.41]	[8.958]	[8.957]	[8.933]	[8.933]	[8.933]
		(10.60)	(10.41)	(8.954)	(8.953)	(8.930)	(8.930)	(8.929)
		{10.60}	{10.40}	{8.952}	{8.951}	{8.928}	{8.928}	{8.927}
	2	[9.485]	[9.427]	[8.926]	[8.921]	[8.910]	[8.909]	[8.909]
		(9.480)	(9.422)	(8.922)	(8.917)	(8.906)	(8.906)	(8.905)
	{9.478}	{9.420}	{8.920}	{8.915}	{8.904}	{8.904}	{8.903}	
3	[8.927]	[8.915]	[8.912]	[8.908]	[8.907]	[8.906]	[8.906]	
	(8.923)	(8.911)	(8.908)	(8.904)	(8.903)	(8.903)	(8.902)	
	{8.921}	{8.909}	{8.906}	{8.902}	{8.901}	{8.901}	{8.900}	
4	[8.917]	[8.904]	[8.904]	[8.904]	[8.903]	[8.903]	[8.902]	
	(8.913)	(8.901)	(8.900)	(8.900)	(8.899)	(8.899)	(8.898)	
	{8.911}	{8.899}	{8.898}	{8.898}	{8.897}	{8.897}	{8.896}	
5	[8.915]	[8.902]	[8.902]	[8.901]	[8.901]	[8.901]	[8.901]	
	(8.911)	(8.898)	(8.898)	(8.898)	(8.897)	(8.897)	(8.897)	
	{8.909}	{8.896}	{8.896}	{8.896}	{8.896}	{8.895}	{8.895}	
4	0	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	2	[12.63]	[12.62]	[12.62]	[12.62]	[12.62]	[12.61]	[12.61]
		(12.63)	(12.62)	(12.62)	(12.62)	(12.62)	(12.61)	(12.61)
	{12.63}	{12.62}	{12.62}	{12.62}	{12.62}	{12.61}	{12.61}	
3	[12.62]	[12.62]	[12.62]	[12.61]	[12.61]	[12.61]	[12.61]	
	(12.62)	(12.62)	(12.62)	(12.61)	(12.61)	(12.61)	(12.61)	
	{12.62}	{12.62}	{12.62}	{12.61}	{12.61}	{12.61}	{12.61}	
4	[12.62]	[12.61]	[12.61]	[12.61]	[12.61]	[12.61]	[12.61]	
	(12.61)	(12.61)	(12.61)	(12.61)	(12.61)	(12.61)	(12.61)	
	{12.61}	{12.61}	{12.61}	{12.61}	{12.61}	{12.61}	{12.61}	
5	[12.61]	[12.61]	[12.61]	[12.61]	[12.61]	[12.61]	[12.61]	
	(12.61)	(12.61)	(12.61)	(12.61)	(12.61)	(12.61)	(12.61)	
	{12.61}	{12.61}	{12.61}	{12.61}	{12.61}	{12.61}	{12.61}	

Note: []: results from $\hat{N}_z = 4$; (): results from $\hat{N}_z = 5$; { }: results from $\hat{N}_z = 6$

表 3.7 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	2	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	3	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	4	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	5	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}

Note: []: results from $\hat{N}_z = 4$; (): results from $\hat{N}_z = 5$; { }: results from $\hat{N}_z = 6$

表 3.8 具水平內部裂縫簡支之 Al/Al₂O₃ FGM 方形中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.5, \alpha = 0^\circ, m=5$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [] $\hat{N}_z = 3$; () $\hat{N}_z = 4$; { } $\hat{N}_z = 5$						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	[4.005] (4.004) {4.003}	[3.773] (3.773) {3.772}	[3.773] (3.773) {3.772}	[3.773] (3.772) {3.772}	[3.773] (3.772) {3.772}	[3.773] (3.772) {3.772}	[3.773] (3.772) {3.772}
	2	[3.445] (3.418) {3.417}	[3.438] (3.410) {3.409}	[3.426] (3.399) {3.398}	[3.425] (3.396) {3.395}	[3.413] (3.385) {3.384}	[3.411] (3.383) {3.382}	[3.403] (3.376) {3.375}
	3	[3.363] (3.337) {3.336}	[3.355] (3.328) {3.327}	[3.346] (3.320) {3.319}	[3.345] (3.319) {3.318}	[3.341] (3.315) {3.315}	[3.341] (3.315) {3.314}	[3.338] (3.313) {3.312}
	4	[3.344] (3.318) {3.318}	[3.338] (3.312) {3.311}	[3.335] (3.309) {3.309}	[3.334] (3.309) {3.308}	[3.334] (3.308) {3.308}	[3.333] (3.308) {3.307}	[3.333] (3.308) {3.307}
	5	[3.342] (3.316) {3.316}	[3.335] (3.310) {3.309}	[3.333] (3.308) {3.307}	[3.333] (3.307) {3.307}	[3.332] (3.307) {3.306}	[3.332] (3.307) {3.306}	[3.332] (3.307) {3.306}
2	0	[10.61] (10.60) {10.60}	[10.41] (10.41) {10.40}	[8.958] (8.954) {8.952}	[8.957] (8.953) {8.951}	[8.933] (8.930) {8.928}	[8.933] (8.930) {8.928}	[8.933] (8.929) {8.927}
	2	[7.830] (7.696) {7.692}	[7.590] (7.425) {7.420}	[7.439] (7.289) {7.285}	[7.409] (7.260) {7.255}	[7.387] (7.238) {7.234}	[7.361] (7.211) {7.207}	[7.356] (7.206) {7.201}
	3	[7.403] (7.250) {7.246}	[7.366] (7.211) {7.207}	[7.356] (7.200) {7.196}	[7.332] (7.178) {7.174}	[7.329] (7.173) {7.169}	[7.319] (7.164) {7.160}	[7.318] (7.161) {7.157}
	4	[7.350] (7.196) {7.192}	[7.325] (7.171) {7.167}	[7.318] (7.158) {7.153}	[7.311] (7.151) {7.146}	[7.310] (7.148) {7.143}	[7.306] (7.145) {7.140}	[7.305] (7.142) {7.138}
	5	[7.322] (7.164) {7.160}	[7.316] (7.157) {7.153}	[7.312] (7.148) {7.144}	[7.306] (7.142) {7.137}	[7.305] (7.140) {7.135}	[7.302] (7.137) {7.133}	[7.302] (7.136) {7.132}

表 3.8 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	[10.61]	[10.41]	[8.958]	[8.957]	[8.933]	[8.933]	[8.933]
		(10.60)	(10.41)	(8.954)	(8.953)	(8.930)	(8.930)	(8.929)
		{10.60}	{10.40}	{8.952}	{8.951}	{8.928}	{8.928}	{8.927}
	2	[9.271]	[9.175]	[8.958]	[8.945]	[8.935]	[8.933]	[8.930]
		(9.134)	(9.047)	(8.838)	(8.826)	(8.817)	(8.815)	(8.812)
	{9.130}	{9.042}	{8.834}	{8.822}	{8.813}	{8.811}	{8.808}	
3	[9.029]	[8.926]	[8.918]	[8.916]	[8.908]	[8.907]	[8.905]	
	(8.906)	(8.806)	(8.799)	(8.796)	(8.789)	(8.788)	(8.786)	
	{8.902}	{8.802}	{8.795}	{8.792}	{8.785}	{8.784}	{8.782}	
4	[9.017]	[8.900]	[8.890]	[8.889]	[8.889]	[8.888]	[8.888]	
	(8.893)	(8.781)	(8.771)	(8.771)	(8.770)	(8.770)	(8.770)	
	{8.889}	{8.777}	{8.767}	{8.767}	{8.766}	{8.766}	{8.766}	
5	[9.011]	[8.896]	[8.887]	[8.887]	[8.886]	[8.886]	[8.886]	
	(8.887)	(8.777)	(8.769)	(8.769)	(8.768)	(8.768)	(8.768)	
	{8.882}	{8.774}	{8.765}	{8.765}	{8.765}	{8.765}	{8.764}	
4	0	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	2	[12.53]	[12.50]	[12.49]	[12.49]	[12.49]	[12.49]	[12.49]
		(12.52)	(12.49)	(12.49)	(12.48)	(12.48)	(12.48)	(12.48)
	{12.52}	{12.49}	{12.49}	{12.48}	{12.48}	{12.48}	{12.48}	
3	[12.49]	[12.49]	[12.48]	[12.48]	[12.48]	[12.48]	[12.48]	
	(12.48)	(12.48)	(12.48)	(12.47)	(12.47)	(12.47)	(12.47)	
	{12.48}	{12.48}	{12.48}	{12.47}	{12.47}	{12.47}	{12.47}	
4	[12.48]	[12.48]	[12.48]	[12.48]	[12.48]	[12.48]	[12.48]	
	(12.47)	(12.47)	(12.47)	(12.47)	(12.47)	(12.47)	(12.47)	
	{12.47}	{12.47}	{12.47}	{12.47}	{12.47}	{12.47}	{12.47}	
5	[12.48]	[12.48]	[12.48]	[12.48]	[12.48]	[12.48]	[12.48]	
	(12.47)	(12.47)	(12.47)	(12.47)	(12.47)	(12.47)	(12.47)	
	{12.47}	{12.47}	{12.47}	{12.47}	{12.47}	{12.47}	{12.47}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.8 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	2	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	3	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	4	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}
	5	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]	[12.64]
		(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)	(12.64)
		{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}	{12.64}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.9 具水平內部裂縫簡支方形均質中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/a = 0.5, \alpha = 0^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [] $\hat{N}_z = 3$; () $\hat{N}_z = 4$; { } $\hat{N}_z = 5$							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
1	0	[6.158] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	[5.795] (5.777) {5.777}	5.358* 5.060+
	2	[5.267] (5.244) {5.244}	[5.257] (5.232) {5.232}	[5.238] (5.214) {5.214}	[5.236] (5.212) {5.211}	[5.217] (5.193) {5.193}	[5.216] (5.191) {5.191}	[5.203] (5.179) {5.179}	
	3	[5.140] (5.117) {5.117}	[5.128] (5.104) {5.104}	[5.113] (5.090) {5.090}	[5.112] (5.089) {5.089}	[5.105] (5.083) {5.083}	[5.105] (5.082) {5.082}	[5.101] (5.079) {5.079}	
	4	[5.110] (5.088) {5.088}	[5.100] (5.078) {5.077}	[5.095] (5.073) {5.073}	[5.095] (5.073) {5.073}	[5.093] (5.071) {5.071}	[5.093] (5.071) {5.071}	[5.092] (5.070) {5.070}	
	5	[5.107] (5.085) {5.085}	[5.096] (5.074) {5.074}	[5.092] (5.070) {5.070}	[5.092] (5.070) {5.070}	[5.091] (5.069) {5.069}	[5.091] (5.069) {5.069}	[5.090] (5.069) {5.069}	
2	0	[16.64] (16.47) {16.47}	[16.32] (16.17) {16.17}	[13.94] (13.84) {13.84}	[13.94] (13.84) {13.84}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	13.02* 11.02+
	2	[12.05] (11.94) {11.93}	[11.70] (11.55) {11.55}	[11.45] (11.32) {11.32}	[11.41] (11.28) {11.28}	[11.37] (11.24) {11.24}	[11.33] (11.20) {11.20}	[11.33] (11.20) {11.19}	
	3	[11.40] (11.27) {11.27}	[11.34] (11.21) {11.21}	[11.33] (11.19) {11.19}	[11.29] (11.16) {11.15}	[11.29] (11.15) {11.15}	[11.27] (11.13) {11.13}	[11.27] (11.13) {11.13}	
	4	[11.32] (11.19) {11.19}	[11.28] (11.15) {11.15}	[11.27] (11.13) {11.13}	[11.26] (11.12) {11.12}	[11.26] (11.12) {11.12}	[11.25] (11.11) {11.11}	[11.25] (11.11) {11.11}	
	5	[11.28] (11.14) {11.14}	[11.27] (11.13) {11.13}	[11.26] (11.12) {11.12}	[11.25] (11.11) {11.11}	[11.25] (11.11) {11.11}	[11.25] (11.10) {11.10}	[11.25] (11.10) {11.10}	

表 3.9 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
3	0	[16.64] (16.47) {16.47}	[16.32] (16.17) {16.17}	[13.94] (13.84) {13.84}	[13.94] (13.84) {13.84}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	[13.90] (13.81) {13.81}	14.74* 13.50+
	2	[14.25] (14.14) {14.14}	[14.10] (14.00) {14.00}	[13.76] (13.66) {13.66}	[13.74] (13.64) {13.64}	[13.73] (13.63) {13.63}	[13.72] (13.63) {13.63}	[13.72] (13.62) {13.62}	
	3	[13.88] (13.77) {13.77}	[13.71] (13.61) {13.61}	[13.70] (13.60) {13.60}	[13.70] (13.60) {13.60}	[13.69] (13.59) {13.59}	[13.68] (13.58) {13.58}	[13.68] (13.58) {13.58}	
	4	[13.86] (13.75) {13.75}	[13.67] (13.57) {13.57}	[13.66] (13.56) {13.56}	[13.66] (13.56) {13.56}	[13.65] (13.56) {13.56}	[13.65] (13.56) {13.55}	[13.65] (13.55) {13.55}	
	5	[13.85] (13.74) {13.74}	[13.67] (13.57) {13.57}	[13.65] (13.55) {13.55}	[13.65] (13.55) {13.55}	[13.65] (13.55) {13.55}	[13.65] (13.55) {13.55}	[13.65] (13.55) {13.55}	
4	0	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	-
	2	[19.33] (19.33) {19.33}	[19.29] (19.29) {19.29}	[19.29] (19.29) {19.29}	[19.29] (19.29) {19.29}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	
	3	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	
	4	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.28] (19.28) {19.28}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	
	5	[19.28] (19.28) {19.28}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	[19.27] (19.27) {19.27}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.9 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]							文獻值
		3×3	4×4	5×5	6×6	7×7	8×8	9×9	
5	0	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	-
	2	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	3	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	4	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	
	5	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	[19.48] (19.48) {19.48}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

* : Stahl and Keer (1972)之結果

+ : Huang et al. (2011) 之結果

表 3.10 具垂直內部裂縫懸臂之 Al/Al₂O₃ FGM 方形中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/b = 0.2, \alpha = 90^\circ, m=5$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [] $\hat{N}_z = 3$; () $\hat{N}_z = 4$; { } $\hat{N}_z = 5$						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	[0.7076] (0.7063) {0.7062}	[0.6987] (0.6969) {0.6968}	[0.6915] (0.6900) {0.6900}	[0.6894] (0.6879) {0.6878}	[0.6878] (0.6863) {0.6862}	[0.6870] (0.6855) {0.6854}	[0.6864] (0.6849) {0.6848}
	2	[0.6915] (0.6897) {0.6897}	[0.6886] (0.6867) {0.6867}	[0.6842] (0.6826) {0.6826}	[0.6832] (0.6815) {0.6815}	[0.6818] (0.6801) {0.6800}	[0.6811] (0.6793) {0.6793}	[0.6805] (0.6788) {0.6787}
	3	[0.6853] (0.6836) {0.6836}	[0.6849] (0.6832) {0.6832}	[0.6821] (0.6804) {0.6803}	[0.6819] (0.6802) {0.6801}	[0.6806] (0.6789) {0.6788}	[0.6805] (0.6788) {0.6787}	[0.6799] (0.6782) {0.6781}
	4	[0.6826] (0.6809) {0.6808}	[0.6823] (0.6805) {0.6805}	[0.6808] (0.6791) {0.6791}	[0.6805] (0.6788) {0.6788}	[0.6800] (0.6782) {0.6782}	[0.6797] (0.6780) {0.6779}	[0.6793] (0.6776) {0.6775}
	5	[0.6804] (0.6787) {0.6786}	[0.6803] (0.6786) {0.6785}	[0.6799] (0.6782) {0.6781}	[0.6798] (0.6781) {0.6780}	[0.6793] (0.6776) {0.6775}	[0.6792] (0.6775) {0.6774}	[0.6789] (0.6772) {0.6771}
2	0	[1.719] (1.710) {1.709}	[1.646] (1.632) {1.632}	[1.639] (1.624) {1.624}	[1.620] (1.606) {1.605}	[1.618] (1.603) {1.603}	[1.612] (1.598) {1.597}	[1.611] (1.597) {1.596}
	2	[1.648] (1.634) {1.634}	[1.626] (1.610) {1.610}	[1.618] (1.602) {1.602}	[1.609] (1.594) {1.594}	[1.606] (1.590) {1.590}	[1.602] (1.587) {1.587}	[1.601] (1.586) {1.586}
	3	[1.622] (1.606) {1.606}	[1.615] (1.599) {1.599}	[1.612] (1.596) {1.596}	[1.605] (1.590) {1.589}	[1.603] (1.588) {1.588}	[1.601] (1.586) {1.586}	[1.600] (1.585) {1.585}
	4	[1.611] (1.595) {1.595}	[1.608] (1.592) {1.592}	[1.606] (1.590) {1.590}	[1.602] (1.587) {1.587}	[1.601] (1.586) {1.586}	[1.600] (1.585) {1.585}	[1.599] (1.585) {1.584}
	5	[1.605] (1.589) {1.589}	[1.603] (1.588) {1.587}	[1.602] (1.587) {1.586}	[1.600] (1.586) {1.585}	[1.600] (1.585) {1.585}	[1.599] (1.585) {1.584}	[1.599] (1.584) {1.584}

表 3.10 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	[4.501] (4.500) {4.500}	[4.119] (4.080) {4.079}	[4.059] (4.016) {4.014}	[4.031] (3.992) {3.990}	[4.019] (3.981) {3.979}	[4.015] (3.976) {3.975}	[4.011] (3.972) {3.971}
	2	[4.026] (3.983) {3.981}	[3.982] (3.940) {3.939}	[3.948] (3.904) {3.903}	[3.927] (3.886) {3.885}	[3.919] (3.880) {3.878}	[3.916] (3.876) {3.875}	[3.911] (3.871) {3.869}
	3	[3.944] (3.902) {3.901}	[3.937] (3.895) {3.894}	[3.921] (3.880) {3.879}	[3.918] (3.878) {3.877}	[3.909] (3.869) {3.868}	[3.908] (3.867) {3.866}	[3.905] (3.864) {3.863}
	4	[3.913] (3.873) {3.872}	[3.911] (3.871) {3.869}	[3.907] (3.867) {3.865}	[3.906] (3.865) {3.864}	[3.898] (3.858) {3.857}	[3.896] (3.856) {3.854}	[3.893] (3.853) {3.852}
	5	[3.902] (3.861) {3.860}	[3.900] (3.859) {3.858}	[3.894] (3.855) {3.853}	[3.893] (3.853) {3.852}	[3.890] (3.850) {3.849}	[3.890] (3.850) {3.848}	[3.887] (3.848) {3.846}
4	0	[5.174] (5.137) {5.136}	[4.336] (4.336) {4.336}	[4.314] (4.314) {4.314}	[4.300] (4.300) {4.300}	[4.295] (4.295) {4.295}	[4.291] (4.291) {4.291}	[4.289] (4.289) {4.289}
	2	[4.326] (4.326) {4.326}	[4.284] (4.284) {4.284}	[4.263] (4.263) {4.263}	[4.254] (4.253) {4.253}	[4.247] (4.247) {4.247}	[4.244] (4.243) {4.243}	[4.241] (4.240) {4.240}
	3	[4.276] (4.276) {4.276}	[4.267] (4.266) {4.266}	[4.256] (4.255) {4.255}	[4.248] (4.248) {4.247}	[4.244] (4.243) {4.243}	[4.241] (4.240) {4.240}	[4.240] (4.239) {4.239}
	4	[4.260] (4.259) {4.259}	[4.255] (4.255) {4.255}	[4.249] (4.249) {4.249}	[4.245] (4.245) {4.245}	[4.242] (4.242) {4.242}	[4.240] (4.240) {4.240}	[4.239] (4.238) {4.238}
	5	[4.251] (4.250) {4.250}	[4.248] (4.248) {4.248}	[4.246] (4.245) {4.245}	[4.243] (4.243) {4.243}	[4.241] (4.241) {4.241}	[4.240] (4.239) {4.239}	[4.239] (4.238) {4.238}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.10 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[6.377]	[5.793]	[5.139]	[5.112]	[5.077]	[5.074]	[5.070]
		(6.324)	(5.733)	(5.094)	(5.066)	(5.033)	(5.031)	(5.028)
		{6.322}	{5.731}	{5.093}	{5.064}	{5.032}	{5.030}	{5.026}
	2	[5.395]	[5.322]	[5.066]	[5.055]	[5.026]	[5.024]	[5.019]
		(5.352)	(5.272)	(5.020)	(5.009)	(4.983)	(4.981)	(4.977)
	{5.351}	{5.270}	{5.019}	{5.008}	{4.982}	{4.980}	{4.976}	
3	[5.065]	[5.059]	[5.030]	[5.028]	[5.020]	[5.019]	[5.017]	
	(5.022)	(5.016)	(4.987)	(4.985)	(4.978)	(4.977)	(4.975)	
	{5.021}	{5.015}	{4.986}	{4.984}	{4.977}	{4.975}	{4.973}	
4	[5.033]	[5.030]	[5.020]	[5.018]	[5.015]	[5.013]	[5.012]	
	(4.990)	(4.986)	(4.977)	(4.975)	(4.972)	(4.971)	(4.970)	
	{4.988}	{4.985}	{4.976}	{4.974}	{4.971}	{4.970}	{4.968}	
5	[5.017]	[5.016]	[5.014]	[5.013]	[5.010]	[5.010]	[5.009]	
	(4.975)	(4.974)	(4.971)	(4.971)	(4.968)	(4.968)	(4.967)	
	{4.973}	{4.972}	{4.970}	{4.969}	{4.967}	{4.967}	{4.966}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.11 具垂直內部裂縫懸臂之 Al/Al₂O₃ FGM 方形中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/b = 0.6, \alpha = 90^\circ, m=5$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [] $\hat{N}_z = 3$; () $\hat{N}_z = 4$; { } $\hat{N}_z = 5$						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	[0.7076] (0.7063) {0.7062}	[0.6987] (0.6969) {0.6968}	[0.6915] (0.6900) {0.6900}	[0.6894] (0.6879) {0.6878}	[0.6878] (0.6863) {0.6862}	[0.6870] (0.6855) {0.6854}	[0.6864] (0.6849) {0.6848}
	2	[0.6599] (0.6565) {0.6564}	[0.6523] (0.6489) {0.6488}	[0.6460] (0.6429) {0.6429}	[0.6441] (0.6411) {0.6410}	[0.6421] (0.6390) {0.6389}	[0.6411] (0.6379) {0.6378}	[0.6401] (0.6369) {0.6368}
	3	[0.6356] (0.6324) {0.6323}	[0.6322] (0.6291) {0.6291}	[0.6305] (0.6274) {0.6273}	[0.6294] (0.6263) {0.6263}	[0.6287] (0.6256) {0.6255}	[0.6274] (0.6245) {0.6244}	[0.6271] (0.6241) {0.6240}
	4	[0.6296] (0.6265) {0.6264}	[0.6279] (0.6250) {0.6249}	[0.6271] (0.6241) {0.6240}	[0.6268] (0.6238) {0.6237}	[0.6261] (0.6232) {0.6231}	[0.6258] (0.6229) {0.6228}	[0.6255] (0.6225) {0.6225}
	5	[0.6273] (0.6244) {0.6243}	[0.6264] (0.6235) {0.6234}	[0.6260] (0.6231) {0.6230}	[0.6258] (0.6229) {0.6229}	[0.6254] (0.6225) {0.6224}	[0.6253] (0.6224) {0.6223}	[0.6250] (0.6221) {0.6220}
2	0	[1.719] (1.710) {1.709}	[1.646] (1.632) {1.632}	[1.639] (1.624) {1.624}	[1.620] (1.606) {1.605}	[1.618] (1.603) {1.603}	[1.612] (1.598) {1.597}	[1.611] (1.597) {1.596}
	2	[1.637] (1.620) {1.619}	[1.605] (1.586) {1.586}	[1.598] (1.579) {1.578}	[1.584] (1.565) {1.564}	[1.581] (1.562) {1.561}	[1.576] (1.558) {1.557}	[1.575] (1.556) {1.556}
	3	[1.597] (1.578) {1.578}	[1.586] (1.566) {1.566}	[1.584] (1.564) {1.564}	[1.580] (1.560) {1.560}	[1.577] (1.558) {1.557}	[1.574] (1.555) {1.555}	[1.573] (1.554) {1.554}
	4	[1.584] (1.564) {1.564}	[1.580] (1.561) {1.561}	[1.577] (1.558) {1.557}	[1.574] (1.556) {1.555}	[1.573] (1.555) {1.554}	[1.572] (1.554) {1.553}	[1.571] (1.553) {1.553}
	5	[1.578] (1.559) {1.559}	[1.575] (1.556) {1.556}	[1.574] (1.555) {1.555}	[1.572] (1.554) {1.554}	[1.572] (1.554) {1.553}	[1.571] (1.553) {1.553}	[1.571] (1.553) {1.552}

表 3.11 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	[4.501]	[4.119]	[4.059]	[4.031]	[4.019]	[4.015]	[4.011]
		(4.500)	(4.080)	(4.016)	(3.992)	(3.981)	(3.976)	(3.972)
		{4.500}	{4.079}	{4.014}	{3.990}	{3.979}	{3.975}	{3.971}
	2	[3.601]	[3.564]	[3.526]	[3.502]	[3.479]	[3.465]	[3.453]
		(3.556)	(3.522)	(3.477)	(3.454)	(3.432)	(3.419)	(3.405)
4	3	[3.331]	[3.302]	[3.267]	[3.242]	[3.236]	[3.227]	[3.224]
		(3.285)	(3.256)	(3.225)	(3.202)	(3.196)	(3.187)	(3.184)
	4	[3.234]	[3.221]	[3.214]	[3.211]	[3.207]	[3.205]	[3.203]
		(3.194)	(3.181)	(3.175)	(3.172)	(3.168)	(3.166)	(3.164)
		{3.193}	{3.180}	{3.174}	{3.171}	{3.167}	{3.165}	{3.163}
5	[3.220]	[3.210]	[3.207]	[3.205]	[3.201]	[3.200]	[3.198]	
	(3.181)	(3.172)	(3.168)	(3.167)	(3.163)	(3.162)	(3.161)	
4	0	[5.174]	[4.336]	[4.314]	[4.300]	[4.295]	[4.291]	[4.289]
		(5.137)	(4.336)	(4.314)	(4.300)	(4.295)	(4.291)	(4.289)
		{5.136}	{4.336}	{4.314}	{4.300}	{4.295}	{4.291}	{4.289}
	2	[3.884]	[3.847]	[3.831]	[3.823]	[3.818]	[3.814]	[3.812]
		(3.884)	(3.847)	(3.831)	(3.823)	(3.818)	(3.814)	(3.812)
4	3	[3.847]	[3.831]	[3.822]	[3.816]	[3.814]	[3.811]	[3.810]
		(3.846)	(3.831)	(3.822)	(3.815)	(3.813)	(3.811)	(3.809)
	4	[3.846]	[3.831]	[3.822]	[3.815]	[3.813]	[3.810]	[3.809]
		[3.830]	[3.822]	[3.817]	[3.813]	[3.812]	[3.810]	[3.809]
		(3.829)	(3.821)	(3.817)	(3.813)	(3.811)	(3.809)	(3.808)
5	[3.829]	[3.821]	[3.817]	[3.813]	[3.811]	[3.809]	[3.808]	
	{3.820}	{3.814}	{3.813}	{3.811}	{3.810}	{3.808}	{3.807}	

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.11 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [®]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[6.377]	[5.793]	[5.139]	[5.112]	[5.077]	[5.074]	[5.070]
		(6.324)	(5.733)	(5.094)	(5.066)	(5.033)	(5.031)	(5.028)
		{6.322}	{5.731}	{5.093}	{5.064}	{5.032}	{5.030}	{5.026}
	2	[5.091]	[5.038]	[4.881]	[4.872]	[4.849]	[4.846]	[4.840]
		(5.041)	(4.986)	(4.839)	(4.829)	(4.806)	(4.803)	(4.797)
		{5.040}	{4.984}	{4.838}	{4.827}	{4.804}	{4.802}	{4.796}
3	[4.832]	[4.819]	[4.799]	[4.791]	[4.781]	[4.779]	[4.777]	
	(4.790)	(4.776)	(4.757)	(4.749)	(4.739)	(4.738)	(4.736)	
		{4.789}	{4.775}	{4.756}	{4.748}	{4.738}	{4.737}	{4.735}
4	[4.801]	[4.789]	[4.779]	[4.777]	[4.775]	[4.774]	[4.772]	
	(4.759)	(4.747)	(4.738)	(4.736)	(4.734)	(4.733)	(4.732)	
		{4.758}	{4.746}	{4.737}	{4.735}	{4.733}	{4.732}	{4.730}
5	[4.789]	[4.781]	[4.774]	[4.773]	[4.772]	[4.771]	[4.771]	
	(4.748)	(4.740)	(4.733)	(4.732)	(4.731)	(4.730)	(4.730)	
		{4.747}	{4.739}	{4.732}	{4.731}	{4.730}	{4.729}	{4.729}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.12 具垂直內部裂縫懸臂方形均質中厚板($a/b = 1.0, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5), d/b = 0.2, \alpha = 90^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$ 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [] $\hat{N}_z = 3$; () $\hat{N}_z = 4$; { } $\hat{N}_z = 5$						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	[1.076] (1.075) {1.075}	[1.063] (1.062) {1.062}	[1.052] (1.051) {1.051}	[1.049] (1.048) {1.047}	[1.046] (1.045) {1.045}	[1.045] (1.044) {1.044}	[1.044] (1.043) {1.043}
	2	[1.052] (1.051) {1.051}	[1.048] (1.046) {1.046}	[1.041] (1.040) {1.040}	[1.040] (1.038) {1.038}	[1.037] (1.036) {1.036}	[1.036] (1.035) {1.035}	[1.035] (1.034) {1.034}
	3	[1.043] (1.041) {1.041}	[1.042] (1.041) {1.041}	[1.038] (1.036) {1.036}	[1.038] (1.036) {1.036}	[1.036] (1.034) {1.034}	[1.035] (1.034) {1.034}	[1.035] (1.033) {1.033}
	4	[1.039] (1.037) {1.037}	[1.038] (1.037) {1.037}	[1.036] (1.034) {1.034}	[1.035] (1.034) {1.034}	[1.035] (1.033) {1.033}	[1.034] (1.033) {1.033}	[1.034] (1.032) {1.032}
	5	[1.035] (1.034) {1.034}	[1.035] (1.034) {1.034}	[1.035] (1.033) {1.033}	[1.034] (1.033) {1.033}	[1.034] (1.032) {1.032}	[1.033] (1.032) {1.032}	[1.033] (1.031) {1.031}
2	0	[2.624] (2.615) {2.615}	[2.515] (2.503) {2.503}	[2.504] (2.491) {2.491}	[2.476] (2.463) {2.463}	[2.473] (2.459) {2.459}	[2.463] (2.450) {2.450}	[2.461] (2.448) {2.448}
	2	[2.519] (2.506) {2.506}	[2.485] (2.471) {2.471}	[2.473] (2.459) {2.459}	[2.460] (2.446) {2.446}	[2.454] (2.440) {2.440}	[2.448] (2.435) {2.435}	[2.446] (2.433) {2.433}
	3	[2.479] (2.465) {2.465}	[2.468] (2.454) {2.454}	[2.464] (2.450) {2.450}	[2.452] (2.439) {2.439}	[2.450] (2.436) {2.436}	[2.446] (2.433) {2.433}	[2.445] (2.432) {2.432}
	4	[2.463] (2.448) {2.448}	[2.457] (2.444) {2.443}	[2.455] (2.441) {2.440}	[2.449] (2.435) {2.435}	[2.447] (2.433) {2.433}	[2.445] (2.431) {2.431}	[2.443] (2.430) {2.430}
	5	[2.452] (2.439) {2.438}	[2.449] (2.436) {2.435}	[2.448] (2.434) {2.434}	[2.446] (2.432) {2.432}	[2.445] (2.432) {2.432}	[2.444] (2.430) {2.430}	[2.443] (2.430) {2.430}

表 3.12 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	[6.927] (6.927) {6.927}	[6.309] (6.276) {6.276}	[6.222] (6.182) {6.182}	[6.173] (6.140) {6.140}	[6.158] (6.125) {6.125}	[6.149] (6.116) {6.116}	[6.143] (6.111) {6.110}
	2	[6.171] (6.134) {6.133}	[6.102] (6.065) {6.065}	[6.050] (6.012) {6.012}	[6.017] (5.982) {5.982}	[6.005] (5.971) {5.971}	[6.000] (5.966) {5.965}	[5.993] (5.958) {5.957}
	3	[6.044] (6.007) {6.007}	[6.033] (5.997) {5.996}	[6.008] (5.973) {5.973}	[6.004] (5.970) {5.969}	[5.990] (5.955) {5.955}	[5.988] (5.953) {5.953}	[5.983] (5.948) {5.947}
	4	[5.996] (5.961) {5.961}	[5.992] (5.958) {5.957}	[5.987] (5.951) {5.951}	[5.985] (5.949) {5.949}	[5.973] (5.938) {5.937}	[5.969] (5.934) {5.934}	[5.965] (5.930) {5.930}
	5	[5.978] (5.943) {5.942}	[5.976] (5.940) {5.940}	[5.967] (5.932) {5.932}	[5.964] (5.930) {5.929}	[5.960] (5.925) {5.925}	[5.959] (5.925) {5.924}	[5.955] (5.921) {5.920}
4	0	[7.941] (7.907) {7.907}	[6.676] (6.676) {6.676}	[6.642] (6.642) {6.642}	[6.621] (6.621) {6.621}	[6.612] (6.612) {6.612}	[6.606] (6.606) {6.606}	[6.604] (6.603) {6.603}
	2	[6.660] (6.660) {6.660}	[6.595] (6.595) {6.595}	[6.563] (6.563) {6.563}	[6.549] (6.548) {6.548}	[6.538] (6.538) {6.538}	[6.533] (6.533) {6.533}	[6.529] (6.529) {6.529}
	3	[6.583] (6.583) {6.583}	[6.568] (6.568) {6.568}	[6.551] (6.551) {6.551}	[6.540] (6.539) {6.539}	[6.533] (6.533) {6.533}	[6.529] (6.529) {6.529}	[6.527] (6.527) {6.526}
	4	[6.557] (6.557) {6.557}	[6.551] (6.551) {6.551}	[6.542] (6.542) {6.542}	[6.535] (6.535) {6.535}	[6.531] (6.531) {6.531}	[6.528] (6.528) {6.527}	[6.526] (6.526) {6.525}
	5	[6.544] (6.543) {6.543}	[6.540] (6.540) {6.540}	[6.536] (6.536) {6.536}	[6.533] (6.532) {6.532}	[6.530] (6.529) {6.529}	[6.527] (6.526) {6.526}	[6.526] (6.525) {6.525}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.12 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[9.793] (9.744) {9.744}	[8.894] (8.839) {8.839}	[7.880] (7.841) {7.841}	[7.838] (7.798) {7.798}	[7.780] (7.744) {7.744}	[7.777] (7.741) {7.741}	[7.770] (7.734) {7.733}
	2	[8.276] (8.238) {8.238}	[8.167] (8.121) {8.121}	[7.769] (7.728) {7.728}	[7.750] (7.710) {7.710}	[7.705] (7.667) {7.667}	[7.701] (7.663) {7.663}	[7.694] (7.657) {7.657}
	3	[7.764] (7.727) {7.727}	[7.756] (7.718) {7.717}	[7.712] (7.674) {7.673}	[7.709] (7.671) {7.670}	[7.696] (7.659) {7.658}	[7.694] (7.657) {7.656}	[7.690] (7.653) {7.653}
	4	[7.715] (7.677) {7.677}	[7.710] (7.672) {7.672}	[7.695] (7.657) {7.657}	[7.691] (7.654) {7.654}	[7.686] (7.650) {7.649}	[7.685] (7.648) {7.647}	[7.681] (7.645) {7.645}
	5	[7.690] (7.653) {7.653}	[7.688] (7.651) {7.651}	[7.685] (7.648) {7.648}	[7.684] (7.647) {7.647}	[7.679] (7.643) {7.643}	[7.679] (7.642) {7.642}	[7.677] (7.641) {7.641}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.13 具垂直內部裂縫懸臂方形均質中厚板($a/b = 1.0, h/b = 0.1,$
 $(x_0/a, y_0/b) = (0.5, 0.5), d/b = 0.6, \alpha = 90^\circ$)之無因次化頻率 $\omega(b^2/h)\sqrt{\rho/E}$
 收斂性分析

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@] [$\hat{N}_z = 3$; ($\hat{N}_z = 4$); { $\hat{N}_z = 5$ }						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
1	0	[1.076] (1.075) {1.075}	[1.063] (1.062) {1.062}	[1.052] (1.051) {1.051}	[1.049] (1.048) {1.047}	[1.046] (1.045) {1.045}	[1.045] (1.044) {1.044}	[1.044] (1.043) {1.043}
	2	[1.006] (1.002) {1.002}	[0.9940] (0.9909) {0.9908}	[0.9844] (0.9816) {0.9816}	[0.9815] (0.9788) {0.9788}	[0.9785] (0.9756) {0.9756}	[0.9769] (0.9740) {0.9739}	[0.9754] (0.9724) {0.9724}
	3	[0.9686] (0.9656) {0.9656}	[0.9633] (0.9604) {0.9604}	[0.9606] (0.9577) {0.9576}	[0.9588] (0.9560) {0.9559}	[0.9577] (0.9548) {0.9548}	[0.9557] (0.9530) {0.9530}	[0.9551] (0.9524) {0.9524}
	4	[0.9591] (0.9562) {0.9562}	[0.9565] (0.9538) {0.9538}	[0.9552] (0.9524) {0.9524}	[0.9547] (0.9520) {0.9519}	[0.9537] (0.9510) {0.9509}	[0.9531] (0.9505) {0.9504}	[0.9526] (0.9500) {0.9499}
	5	[0.9555] (0.9528) {0.9528}	[0.9541] (0.9515) {0.9514}	[0.9535] (0.9508) {0.9508}	[0.9532] (0.9506) {0.9505}	[0.9524] (0.9498) {0.9498}	[0.9524] (0.9497) {0.9497}	[0.9519] (0.9493) {0.9492}
2	0	[2.624] (2.615) {2.615}	[2.515] (2.503) {2.503}	[2.504] (2.491) {2.491}	[2.476] (2.463) {2.463}	[2.473] (2.459) {2.459}	[2.463] (2.450) {2.450}	[2.461] (2.448) {2.448}
	2	[2.504] (2.489) {2.489}	[2.455] (2.438) {2.438}	[2.445] (2.427) {2.427}	[2.424] (2.406) {2.406}	[2.419] (2.402) {2.402}	[2.411] (2.395) {2.395}	[2.409] (2.392) {2.392}
	3	[2.444] (2.427) {2.427}	[2.427] (2.409) {2.409}	[2.424] (2.406) {2.406}	[2.417] (2.400) {2.400}	[2.412] (2.395) {2.395}	[2.408] (2.391) {2.391}	[2.406] (2.389) {2.389}
	4	[2.424] (2.406) {2.406}	[2.418] (2.400) {2.400}	[2.412] (2.395) {2.395}	[2.408] (2.391) {2.391}	[2.407] (2.390) {2.390}	[2.405] (2.388) {2.388}	[2.404] (2.387) {2.387}
	5	[2.414] (2.397) {2.397}	[2.409] (2.392) {2.392}	[2.407] (2.391) {2.390}	[2.405] (2.389) {2.389}	[2.405] (2.388) {2.388}	[2.403] (2.387) {2.387}	[2.403] (2.387) {2.386}

表 3.13 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
3	0	[6.927] (6.927) {6.927}	[6.309] (6.276) {6.276}	[6.222] (6.182) {6.182}	[6.173] (6.140) {6.140}	[6.158] (6.125) {6.125}	[6.149] (6.116) {6.116}	[6.143] (6.111) {6.110}
	2	[5.530] (5.491) {5.490}	[5.472] (5.435) {5.435}	[5.416] (5.371) {5.371}	[5.377] (5.334) {5.334}	[5.343] (5.300) {5.300}	[5.321] (5.280) {5.280}	[5.303] (5.260) {5.259}
	3	[5.112] (5.070) {5.070}	[5.067] (5.025) {5.025}	[5.013] (4.975) {4.974}	[4.973] (4.936) {4.936}	[4.964] (4.928) {4.927}	[4.949] (4.913) {4.913}	[4.944] (4.908) {4.908}
	4	[4.960] (4.924) {4.924}	[4.939] (4.903) {4.903}	[4.929] (4.893) {4.893}	[4.924] (4.888) {4.888}	[4.918] (4.883) {4.883}	[4.914] (4.879) {4.879}	[4.910] (4.876) {4.875}
	5	[4.938] (4.903) {4.903}	[4.923] (4.888) {4.887}	[4.917] (4.882) {4.882}	[4.914] (4.879) {4.879}	[4.907] (4.873) {4.872}	[4.906] (4.872) {4.872}	[4.903] (4.870) {4.869}
4	0	[7.941] (7.907) {7.907}	[6.676] (6.676) {6.676}	[6.642] (6.642) {6.642}	[6.621] (6.621) {6.621}	[6.612] (6.612) {6.612}	[6.606] (6.606) {6.606}	[6.604] (6.603) {6.603}
	2	[5.977] (5.977) {5.977}	[5.921] (5.921) {5.921}	[5.896] (5.896) {5.896}	[5.884] (5.884) {5.884}	[5.876] (5.876) {5.876}	[5.871] (5.871) {5.870}	[5.868] (5.867) {5.867}
	3	[5.920] (5.920) {5.920}	[5.896] (5.896) {5.896}	[5.882] (5.882) {5.882}	[5.873] (5.873) {5.873}	[5.870] (5.869) {5.869}	[5.866] (5.865) {5.865}	[5.864] (5.863) {5.863}
	4	[5.894] (5.894) {5.894}	[5.882] (5.882) {5.882}	[5.875] (5.875) {5.875}	[5.869] (5.869) {5.869}	[5.867] (5.867) {5.866}	[5.864] (5.864) {5.863}	[5.863] (5.862) {5.862}
	5	[5.880] (5.880) {5.879}	[5.872] (5.871) {5.871}	[5.870] (5.869) {5.869}	[5.866] (5.866) {5.866}	[5.864] (5.864) {5.864}	[5.862] (5.862) {5.862}	[5.861] (5.861) {5.861}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 3.13 (續上頁)

Mode No.	Crack Functions (\bar{N})	Polynomial Solution Size ($I \times J$) [@]						
		3×3	4×4	5×5	6×6	7×7	8×8	9×9
5	0	[9.793]	[8.894]	[7.880]	[7.838]	[7.780]	[7.777]	[7.770]
		(9.744)	(8.839)	(7.841)	(7.798)	(7.744)	(7.741)	(7.734)
		{9.744}	{8.839}	{7.841}	{7.798}	{7.744}	{7.741}	{7.733}
	2	[7.813]	[7.732]	[7.486]	[7.472]	[7.436]	[7.432]	[7.422]
		(7.768)	(7.685)	(7.449)	(7.434)	(7.398)	(7.394)	(7.384)
		{7.768}	{7.685}	{7.448}	{7.433}	{7.397}	{7.393}	{7.384}
	3	[7.409]	[7.389]	[7.358]	[7.346]	[7.329]	[7.326]	[7.322]
		(7.372)	(7.352)	(7.321)	(7.308)	(7.293)	(7.289)	(7.286)
		{7.372}	{7.351}	{7.321}	{7.308}	{7.292}	{7.289}	{7.286}
	4	[7.361]	[7.342]	[7.327]	[7.323]	[7.320]	[7.319]	[7.315]
		(7.324)	(7.305)	(7.291)	(7.287)	(7.284)	(7.283)	(7.280)
		{7.324}	{7.305}	{7.291}	{7.287}	{7.284}	{7.282}	{7.279}
	5	[7.341]	[7.328]	[7.318]	[7.317]	[7.315]	[7.314]	[7.313]
		(7.305)	(7.292)	(7.282)	(7.281)	(7.279)	(7.278)	(7.277)
		{7.305}	{7.292}	{7.282}	{7.280}	{7.279}	{7.278}	{7.277}

Note: []: results from $\hat{N}_z = 3$; (): results from $\hat{N}_z = 4$; { }: results from $\hat{N}_z = 5$

表 4.1 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = 0.5$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	5.965	14.88	14.88	23.76	29.66
		0.1	5.939	14.88	14.88	23.74	29.36
		0.2	5.816	14.82	14.87	23.72	28.05
		0.3	5.665	14.58	14.84	23.68	26.71
		0.4	5.493	13.96	14.77	23.58	25.49
		0.5	5.318	12.82	14.66	23.28	24.52
	0.2	0	5.536	13.81	13.81	22.06	27.54
		0.1	5.513	13.81	13.81	22.04	27.27
		0.2	5.399	13.75	13.81	22.03	26.05
		0.3	5.259	13.53	13.78	21.99	24.80
		0.4	5.099	12.97	13.71	21.89	23.67
		0.5	4.937	11.91	13.61	21.62	22.77
	1	0	4.554	11.36	11.36	18.15	22.66
		0.1	4.536	11.36	11.36	18.14	22.45
		0.2	4.442	11.31	11.36	18.12	21.44
		0.3	4.330	11.14	11.33	18.10	20.44
		0.4	4.196	10.67	11.28	18.02	19.49
		0.5	4.063	9.806	11.19	17.79	18.74
	5	0	3.925	9.787	9.787	15.62	19.49
		0.1	3.906	9.785	9.786	15.61	19.27
		0.2	3.825	9.742	9.782	15.59	18.42
		0.3	3.725	9.581	9.760	15.56	17.53
		0.4	3.611	9.175	9.714	15.49	16.74
		0.5	3.496	8.419	9.638	15.30	16.10
	10	0	3.800	9.475	9.475	15.12	18.86
		0.1	3.781	9.472	9.474	15.10	18.64
		0.2	3.703	9.430	9.473	15.09	17.81
		0.3	3.606	9.273	9.448	15.06	16.96
		0.4	3.495	8.876	9.403	14.99	16.19
		0.5	3.386	8.149	9.331	14.80	15.59

表 4.2 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	5.777	13.81	13.81	19.48	19.48
		0.1	5.728	13.78	13.80	19.48	19.48
		0.2	5.588	13.63	13.79	19.48	19.48
		0.3	5.421	13.22	13.76	19.46	19.48
		0.4	5.241	12.38	13.68	19.40	19.48
		0.5	5.068	11.10	13.55	19.27	19.48
	0.2	0	5.371	12.86	12.86	18.56	18.56
		0.1	5.325	12.84	12.86	18.56	18.56
		0.2	5.196	12.70	12.85	18.56	18.56
		0.3	5.041	12.33	12.82	18.54	18.56
		0.4	4.873	11.55	12.74	18.48	18.56
		0.5	4.713	10.37	12.62	18.36	18.56
	1	0	4.426	10.63	10.63	16.20	16.20
		0.1	4.389	10.61	10.63	16.20	16.20
		0.2	4.283	10.50	10.62	16.20	16.20
		0.3	4.155	10.20	10.59	16.17	16.20
		0.4	4.018	9.579	10.53	16.07	16.12
		0.5	3.885	8.608	10.43	15.76	16.00
	5	0	3.772	8.929	8.929	12.64	12.64
		0.1	3.749	8.910	8.929	12.64	12.64
		0.2	3.647	8.806	8.922	12.63	12.64
		0.3	3.537	8.525	8.897	12.61	12.64
		0.4	3.419	7.963	8.845	12.57	12.64
		0.5	3.306	7.125	8.764	12.47	12.64
	10	0	3.642	8.588	8.588	11.52	11.52
		0.1	3.610	8.572	8.587	11.52	11.52
		0.2	3.520	8.461	8.580	11.52	11.52
		0.3	3.413	8.178	8.557	11.50	11.52
		0.4	3.299	7.622	8.507	11.47	11.52
		0.5	3.190	6.810	8.429	11.39	11.52

表 4.3 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	5.304	9.742	9.742	11.65	11.65
		0.1	5.258	9.742	9.742	11.60	11.64
		0.2	5.121	9.739	9.742	11.37	11.64
		0.3	4.959	9.728	9.742	10.84	11.60
		0.4	4.790	9.698	9.742	9.917	11.04
		0.5	4.633	8.763	9.638	9.742	9.974
	0.2	0	4.950	9.280	9.280	10.92	10.92
		0.1	4.907	9.280	9.280	10.88	10.92
		0.2	4.778	9.277	9.280	10.67	10.91
		0.3	4.627	9.266	9.280	10.18	10.88
		0.4	4.470	9.229	9.280	9.338	10.52
		0.5	4.322	8.245	9.183	9.280	9.500
	1	0	4.099	8.089	8.089	9.107	9.107
		0.1	4.063	8.088	8.089	9.077	9.107
		0.2	3.956	8.085	8.089	8.912	9.100
		0.3	3.830	8.068	8.089	8.528	9.074
		0.4	3.699	7.796	8.083	8.089	9.019
		0.5	3.575	6.915	8.014	8.089	8.277
	5	0	3.406	6.296	6.296	7.347	7.347
		0.1	3.377	6.296	6.296	7.317	7.346
		0.2	3.289	6.292	6.296	7.165	7.341
		0.3	3.185	6.274	6.296	6.823	7.322
		0.4	3.077	6.123	6.296	6.362	7.124
		0.5	2.976	5.462	6.257	6.296	6.444
	10	0	3.265	5.749	5.749	6.977	6.977
		0.1	3.238	5.749	5.749	6.946	6.976
		0.2	3.154	5.746	5.749	6.791	6.971
		0.3	3.056	5.734	5.749	6.445	6.954
		0.4	2.953	5.677	5.749	5.910	6.510
		0.5	2.856	5.149	5.713	5.749	5.886

表 4.4 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.44	0.00	0.00	0.08	1.01
		0.2	2.50	0.40	0.07	0.17	5.43
		0.3	5.03	2.02	0.27	0.34	9.95
		0.4	7.91	6.18	0.74	0.76	14.06
		0.5	10.85	13.84	1.48	2.02	17.33
	0.2	0.1	0.42	0.00	0.00	0.09	0.98
		0.2	2.47	0.43	0.00	0.14	5.41
		0.3	5.00	2.03	0.22	0.32	9.95
		0.4	7.89	6.08	0.72	0.77	14.05
		0.5	10.82	13.76	1.45	1.99	17.32
	1	0.1	0.40	0.00	0.00	0.06	0.93
		0.2	2.46	0.44	0.00	0.17	5.38
		0.3	4.92	1.94	0.26	0.28	9.80
		0.4	7.86	6.07	0.70	0.72	13.99
		0.5	10.78	13.68	1.50	1.98	17.30
	5	0.1	0.48	0.02	0.01	0.06	1.13
		0.2	2.55	0.46	0.05	0.19	5.49
		0.3	5.10	2.10	0.28	0.38	10.06
		0.4	8.00	6.25	0.75	0.83	14.11
		0.5	10.93	13.98	1.52	2.05	17.39
	10	0.1	0.50	0.03	0.01	0.13	1.17
		0.2	2.55	0.47	0.02	0.20	5.57
		0.3	5.11	2.13	0.28	0.40	10.07
		0.4	8.03	6.32	0.76	0.86	14.16
		0.5	10.89	13.99	1.52	2.12	17.34

表 4.5 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.85	0.22	0.07	0.00	0.00
		0.2	3.27	1.30	0.14	0.00	0.00
		0.3	6.16	4.27	0.36	0.10	0.00
		0.4	9.28	10.35	0.94	0.41	0.00
		0.5	12.27	19.62	1.88	1.08	0.00
	0.2	0.1	0.86	0.16	0.00	0.00	0.00
		0.2	3.26	1.24	0.08	0.00	0.00
		0.3	6.14	4.12	0.31	0.11	0.00
		0.4	9.27	10.19	0.93	0.43	0.00
		0.5	12.25	19.36	1.87	1.08	0.00
	1	0.1	0.84	0.19	0.00	0.00	0.00
		0.2	3.23	1.22	0.09	0.00	0.00
		0.3	6.12	4.05	0.38	0.19	0.00
		0.4	9.22	9.89	0.94	0.80	0.49
		0.5	12.22	19.02	1.88	2.72	1.23
	5	0.1	0.61	0.21	0.00	0.00	0.00
		0.2	3.31	1.38	0.08	0.08	0.00
		0.3	6.23	4.52	0.36	0.24	0.00
		0.4	9.36	10.82	0.94	0.55	0.00
		0.5	12.35	20.20	1.85	1.34	0.00
	10	0.1	0.88	0.19	0.01	0.00	0.00
		0.2	3.35	1.48	0.09	0.00	0.00
		0.3	6.29	4.77	0.36	0.17	0.00
		0.4	9.42	11.25	0.94	0.43	0.00
		0.5	12.41	20.70	1.85	1.13	0.00

表 4.6 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.87	0.00	0.00	0.43	0.09
		0.2	3.45	0.03	0.00	2.40	0.09
		0.3	6.50	0.14	0.00	6.95	0.43
		0.4	9.69	0.45	0.00	14.88	5.24
		0.5	12.65	10.05	1.07	16.38	14.39
	0.2	0.1	0.87	0.00	0.00	0.37	0.00
		0.2	3.47	0.03	0.00	2.29	0.09
		0.3	6.53	0.15	0.00	6.78	0.37
		0.4	9.70	0.55	0.00	14.49	3.66
		0.5	12.69	11.15	1.05	15.02	13.00
	1	0.1	0.88	0.01	0.00	0.33	0.00
		0.2	3.49	0.05	0.00	2.14	0.08
		0.3	6.56	0.26	0.00	6.36	0.36
		0.4	9.76	3.62	0.07	11.18	0.97
		0.5	12.78	14.51	0.93	11.18	9.11
	5	0.1	0.85	0.00	0.00	0.41	0.01
		0.2	3.44	0.06	0.00	2.48	0.08
		0.3	6.49	0.35	0.00	7.13	0.34
		0.4	9.66	2.75	0.00	13.41	3.04
		0.5	12.62	13.25	0.62	14.31	12.29
	10	0.1	0.83	0.00	0.00	0.44	0.01
		0.2	3.40	0.05	0.00	2.67	0.09
		0.3	6.40	0.26	0.00	7.63	0.33
		0.4	9.56	1.25	0.00	15.29	6.69
		0.5	12.53	10.44	0.63	17.60	15.64

表 4.7 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	5.965	14.88	14.88	23.76	29.66
		0.1	5.951	14.76	14.88	23.76	29.50
		0.2	5.888	14.28	14.87	23.73	28.81
		0.3	5.789	13.67	14.85	23.63	27.71
		0.4	5.646	13.01	14.81	23.39	25.38
		0.5	5.470	12.39	14.72	21.46	23.01
	0.2	0	5.536	13.81	13.81	22.06	27.54
		0.1	5.524	13.71	13.81	22.06	27.39
		0.2	5.465	13.26	13.80	22.03	26.75
		0.3	5.374	12.70	13.79	21.94	25.74
		0.4	5.241	12.08	13.74	21.72	23.58
		0.5	5.078	11.50	13.65	19.95	21.37
	1	0	4.554	11.36	11.36	18.15	22.66
		0.1	4.544	11.28	11.36	18.15	22.54
		0.2	4.496	10.91	11.36	18.13	22.02
		0.3	4.421	10.45	11.34	18.05	21.19
		0.4	4.312	9.940	11.31	17.87	19.42
		0.5	4.178	9.466	11.24	16.44	17.58
	5	0	3.925	9.787	9.787	15.62	19.49
		0.1	3.915	9.706	9.785	15.62	19.38
		0.2	3.873	9.384	9.779	15.60	18.92
		0.3	3.808	8.984	9.766	15.53	18.19
		0.4	3.713	8.542	9.735	15.37	16.64
		0.5	3.597	8.138	9.685	14.06	15.12
	10	0	3.800	9.475	9.475	15.12	18.86
		0.1	3.791	9.393	9.472	15.12	18.75
		0.2	3.750	9.081	9.467	15.10	18.31
		0.3	3.686	8.692	9.454	15.03	17.60
		0.4	3.594	8.265	9.423	14.87	16.08
		0.5	3.481	7.873	9.361	13.58	14.63

表 4.8 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	5.777	13.81	13.81	19.48	19.48
		0.1	5.746	13.58	13.79	19.43	19.48
		0.2	5.667	13.08	13.77	19.24	19.48
		0.3	5.547	12.48	13.74	18.92	19.46
		0.4	5.387	11.86	13.67	18.47	19.39
		0.5	5.199	11.28	13.57	16.95	17.60
	0.2	0	5.371	12.86	12.86	18.56	18.56
		0.1	5.342	12.65	12.85	18.51	18.56
		0.2	5.269	12.19	12.83	18.33	18.56
		0.3	5.158	11.62	12.80	18.03	18.53
		0.4	5.010	11.05	12.74	17.60	18.47
		0.5	4.836	10.51	12.64	15.88	17.08
	1	0	4.426	10.63	10.63	16.20	16.20
		0.1	4.403	10.46	10.62	16.15	16.20
		0.2	4.343	10.07	10.60	16.00	16.20
		0.3	4.252	9.603	10.58	15.73	16.17
		0.4	4.132	9.127	10.53	15.36	15.77
		0.5	3.988	8.678	10.45	13.21	14.91
	5	0	3.772	8.929	8.929	12.64	12.64
		0.1	3.752	8.786	8.922	12.60	12.64
		0.2	3.699	8.460	8.908	12.48	12.63
		0.3	3.619	8.068	8.883	12.27	12.61
		0.4	3.513	7.671	8.840	11.98	12.47
		0.5	3.389	7.291	8.771	10.74	11.63
10	0	3.642	8.588	8.588	11.52	11.52	
	0.1	3.622	8.449	8.581	11.49	11.52	
	0.2	3.570	8.137	8.566	11.38	11.52	
	0.3	3.492	7.762	8.542	11.19	11.50	
	0.4	3.389	7.382	8.500	10.93	11.44	
	0.5	3.268	7.017	8.433	10.23	10.60	

表 4.9 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	5.304	9.742	9.742	11.65	11.65
		0.1	5.272	9.714	9.742	11.47	11.64
		0.2	5.187	9.623	9.739	11.06	11.61
		0.3	5.059	9.465	9.728	10.55	11.58
		0.4	4.896	9.242	9.698	10.03	11.51
		0.5	4.710	8.969	9.483	9.632	10.28
	0.2	0	4.950	9.280	9.280	10.92	10.92
		0.1	4.920	9.254	9.280	10.75	10.91
		0.2	4.841	9.167	9.277	10.36	10.89
		0.3	4.723	9.017	9.267	9.880	10.86
		0.4	4.571	8.804	9.237	9.388	10.80
		0.5	4.397	8.544	8.877	9.175	9.795
	1	0	4.099	8.089	8.089	9.107	9.107
		0.1	4.075	8.066	8.088	8.965	9.100
		0.2	4.010	7.991	8.085	8.625	9.082
		0.3	3.912	7.859	8.072	8.213	9.053
		0.4	3.787	7.675	7.787	8.043	9.002
		0.5	3.644	7.343	7.448	7.976	8.559
	5	0	3.406	6.296	6.296	7.347	7.347
		0.1	3.386	6.278	6.296	7.241	7.341
		0.2	3.331	6.220	6.292	6.981	7.327
		0.3	3.247	6.119	6.279	6.656	7.303
		0.4	3.141	5.976	6.234	6.321	7.263
		0.5	3.020	5.800	5.883	6.211	6.714
	10	0	3.265	5.749	5.749	6.977	6.977
		0.1	3.246	5.733	5.749	6.881	6.972
		0.2	3.193	5.680	5.746	6.642	6.959
		0.3	3.112	5.586	5.736	6.342	6.936
		0.4	3.009	5.455	5.710	6.018	6.830
		0.5	2.893	5.295	5.572	5.720	6.145

表 4.10 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.23	0.81	0.00	0.00	0.54
		0.2	1.29	4.03	0.07	0.13	2.87
		0.3	2.95	8.13	0.20	0.55	6.57
		0.4	5.35	12.57	0.47	1.56	14.43
		0.5	8.30	16.73	1.08	9.68	22.42
	0.2	0.1	0.22	0.72	0.00	0.00	0.54
		0.2	1.28	3.98	0.07	0.14	2.87
		0.3	2.93	8.04	0.14	0.54	6.54
		0.4	5.33	12.53	0.51	1.54	14.38
		0.5	8.27	16.73	1.16	9.56	22.40
	1	0.1	0.22	0.70	0.00	0.00	0.53
		0.2	1.27	3.96	0.00	0.11	2.82
		0.3	2.92	8.01	0.18	0.55	6.49
		0.4	5.31	12.50	0.44	1.54	14.30
		0.5	8.26	16.67	1.06	9.42	22.42
	5	0.1	0.25	0.83	0.02	0.00	0.56
		0.2	1.32	4.12	0.08	0.13	2.92
		0.3	2.98	8.20	0.21	0.58	6.67
		0.4	5.40	12.72	0.53	1.60	14.62
		0.5	8.36	16.85	1.04	9.99	22.42
	10	0.1	0.24	0.87	0.03	0.00	0.58
		0.2	1.32	4.16	0.08	0.13	2.92
		0.3	3.00	8.26	0.22	0.60	6.68
		0.4	5.42	12.77	0.55	1.65	14.74
		0.5	8.39	16.91	1.20	10.19	22.43

表 4.11 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.54	1.67	0.14	0.26	0.00
		0.2	1.90	5.29	0.29	1.23	0.00
		0.3	3.98	9.63	0.51	2.87	0.10
		0.4	6.75	14.12	1.01	5.18	0.46
		0.5	10.01	18.32	1.74	12.99	9.65
	0.2	0.1	0.54	1.63	0.08	0.27	0.00
		0.2	1.90	5.21	0.23	1.24	0.00
		0.3	3.97	9.64	0.47	2.86	0.16
		0.4	6.72	14.07	0.93	5.17	0.48
		0.5	9.96	18.27	1.71	14.44	7.97
	1	0.1	0.52	1.60	0.09	0.31	0.00
		0.2	1.88	5.27	0.28	1.23	0.00
		0.3	3.93	9.66	0.47	2.90	0.19
		0.4	6.64	14.14	0.94	5.19	2.65
		0.5	9.90	18.36	1.69	18.46	7.96
	5	0.1	0.53	1.60	0.08	0.32	0.00
		0.2	1.94	5.25	0.24	1.27	0.08
		0.3	4.06	9.64	0.52	2.93	0.24
		0.4	6.87	14.09	1.00	5.22	1.34
		0.5	10.15	18.34	1.77	15.03	7.99
	10	0.1	0.55	1.62	0.08	0.26	0.00
		0.2	1.98	5.25	0.26	1.22	0.00
		0.3	4.12	9.62	0.54	2.86	0.17
		0.4	6.95	14.04	1.02	5.12	0.69
		0.5	10.27	18.29	1.80	11.20	7.99

表 4.12 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.60	0.29	0.00	1.55	0.09
		0.2	2.21	1.22	0.03	5.06	0.34
		0.3	4.62	2.84	0.14	9.44	0.60
		0.4	7.69	5.13	0.45	13.91	1.20
		0.5	11.20	7.93	2.66	17.32	11.76
	0.2	0.1	0.61	0.28	0.00	1.56	0.09
		0.2	2.20	1.22	0.03	5.13	0.27
		0.3	4.59	2.83	0.14	9.52	0.55
		0.4	7.66	5.13	0.46	14.03	1.10
		0.5	11.17	7.93	4.34	15.98	10.30
	1	0.1	0.59	0.28	0.01	1.56	0.08
		0.2	2.17	1.21	0.05	5.29	0.27
		0.3	4.56	2.84	0.21	9.82	0.59
		0.4	7.61	5.12	3.73	11.68	1.15
		0.5	11.10	9.22	7.92	12.42	6.02
	5	0.1	0.59	0.29	0.00	1.44	0.08
		0.2	2.20	1.21	0.06	4.98	0.27
		0.3	4.67	2.81	0.27	9.41	0.60
		0.4	7.78	5.08	0.98	13.96	1.14
		0.5	11.33	7.88	6.56	15.46	8.62
	10	0.1	0.58	0.28	0.00	1.38	0.07
		0.2	2.21	1.20	0.05	4.80	0.26
		0.3	4.69	2.84	0.23	9.10	0.59
		0.4	7.84	5.11	0.68	13.75	2.11
		0.5	11.39	7.90	3.08	18.02	11.92

表 4.13 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
30°	0	0	5.965	14.88	14.88	23.76	29.66
		0.1	5.939	14.88	14.88	23.73	29.40
		0.3	5.668	14.57	14.83	23.31	27.44
		0.5	5.281	12.81	14.58	22.25	26.43
	0.2	0	5.536	13.81	13.81	22.06	27.54
		0.1	5.513	13.81	13.81	22.03	27.30
		0.3	5.262	13.52	13.77	21.64	25.48
		0.5	4.903	11.89	13.53	20.66	24.54
	5	0	3.925	9.787	9.787	15.62	19.49
		0.1	3.907	9.783	9.786	15.60	19.31
		0.3	3.727	9.577	9.754	15.32	18.02
		0.5	3.471	8.411	9.585	14.62	17.36

表 4.14 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
30°	0	0	5.777	13.81	13.81	19.48	19.48
		0.1	5.722	13.78	13.80	19.48	19.48
		0.3	5.412	13.21	13.74	19.45	19.48
		0.5	5.017	11.10	13.45	19.26	19.48
	0.2	0	5.371	12.86	12.86	18.56	18.56
		0.1	5.320	12.84	12.86	18.56	18.56
		0.3	5.033	12.32	12.80	18.53	18.56
		0.5	4.665	10.37	12.53	18.35	18.39
	5	0	3.772	8.929	8.929	12.64	12.64
		0.1	3.736	8.910	8.929	12.64	12.64
		0.3	3.531	8.525	8.888	12.62	12.63
		0.5	3.272	7.132	8.700	12.48	12.61

表 4.15 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
30°	0	0	5.304	9.742	9.742	11.65	11.65
		0.1	5.251	9.742	9.742	11.59	11.64
		0.3	4.951	9.727	9.742	10.84	11.59
		0.5	4.584	8.782	9.630	9.742	10.37
	0.2	0	4.950	9.280	9.280	10.92	10.92
		0.1	4.901	9.280	9.280	10.87	10.92
		0.3	4.619	9.266	9.280	10.18	10.87
		0.5	4.276	8.262	9.175	9.282	9.874
	5	0	3.406	6.296	6.296	7.347	7.347
		0.1	3.373	6.296	6.296	7.313	7.346
		0.3	3.180	6.279	6.291	6.827	7.315
		0.5	2.945	5.475	6.230	6.319	6.693

表 4.16 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
30°	0	0.1	0.44	0.00	0.00	0.13	0.88
		0.3	4.98	2.08	0.34	1.89	7.48
		0.5	11.47	13.91	2.02	6.36	10.89
	0.2	0.1	0.42	0.00	0.00	0.14	0.87
		0.3	4.95	2.10	0.29	1.90	7.48
		0.5	11.43	13.90	2.03	6.35	10.89
	5	0.1	0.46	0.04	0.01	0.13	0.92
		0.3	5.04	2.15	0.34	1.92	7.54
		0.5	11.57	14.06	2.06	6.40	10.93

表 4.17 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
30°	0	0.1	0.95	0.22	0.07	0.00	0.00
		0.3	6.32	4.34	0.51	0.15	0.00
		0.5	13.16	19.62	2.61	1.13	0.00
	0.2	0.1	0.95	0.16	0.00	0.00	0.00
		0.3	6.29	4.20	0.47	0.16	0.00
		0.5	13.14	19.36	2.57	1.13	0.92
	5	0.1	0.95	0.21	0.00	0.00	0.00
		0.3	6.39	4.52	0.46	0.16	0.08
		0.5	13.26	20.13	2.56	1.27	0.24

表 4.18 具不同內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
30°	0	0.1	1.00	0.00	0.00	0.52	0.09
		0.3	6.66	0.15	0.00	6.95	0.52
		0.5	13.57	9.85	1.15	16.38	10.99
	0.2	0.1	0.99	0.00	0.00	0.46	0.00
		0.3	6.69	0.15	0.00	6.78	0.46
		0.5	13.62	10.97	1.13	15.00	9.58
	5	0.1	0.97	0.00	0.00	0.46	0.01
		0.3	6.64	0.27	0.08	7.08	0.44
		0.5	13.53	13.04	1.05	13.99	8.90

表 4.19 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	1.053	2.568	6.443	8.206	9.333
		0.1	1.051	2.562	6.430	8.139	9.315
		0.2	1.051	2.560	6.434	7.986	9.306
		0.3	1.051	2.558	6.434	7.756	9.273
		0.4	1.050	2.556	6.432	7.472	9.190
		0.5	1.049	2.554	6.427	7.157	9.023
	0.2	0	0.9774	2.384	5.981	7.618	8.665
		0.1	0.9759	2.378	5.972	7.556	8.648
		0.2	0.9758	2.377	5.972	7.414	8.640
		0.3	0.9755	2.375	5.972	7.202	8.609
		0.4	0.9747	2.373	5.971	6.938	8.533
		0.5	0.9738	2.372	5.966	6.646	8.378
	1	0	0.8039	1.961	4.920	6.266	7.129
		0.1	0.8027	1.957	4.913	6.217	7.116
		0.2	0.8025	1.956	4.912	6.100	7.109
		0.3	0.8023	1.954	4.913	5.926	7.084
		0.4	0.8017	1.953	4.911	5.710	7.021
		0.5	0.8009	1.952	4.908	5.470	6.894
	5	0	0.6931	1.689	4.238	5.397	6.134
		0.1	0.6922	1.685	4.233	5.352	6.122
		0.2	0.6919	1.683	4.232	5.249	6.117
		0.3	0.6917	1.681	4.232	5.097	6.094
		0.4	0.6911	1.680	4.231	4.909	6.040
		0.5	0.6905	1.679	4.227	4.701	5.930
10	0	0.6711	1.634	4.104	5.225	5.937	
	0.1	0.6703	1.630	4.098	5.181	5.925	
	0.2	0.6700	1.629	4.097	5.080	5.919	
	0.3	0.6698	1.627	4.097	4.932	5.898	
	0.4	0.6693	1.626	4.096	4.750	5.845	
	0.5	0.6686	1.624	4.093	4.548	5.738	

表 4.20 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	1.042	2.446	6.107	6.601	7.732
		0.1	1.041	2.439	6.103	6.582	7.649
		0.2	1.041	2.429	6.102	6.524	7.459
		0.3	1.041	2.419	6.102	6.426	7.198
		0.4	1.040	2.409	6.100	6.291	6.894
		0.5	1.039	2.399	6.088	6.117	6.576
	0.2	0	0.9678	2.275	5.682	6.289	7.193
		0.1	0.9675	2.269	5.682	6.273	7.120
		0.2	0.9673	2.260	5.682	6.218	6.945
		0.3	0.9667	2.251	5.680	6.125	6.703
		0.4	0.9658	2.242	5.676	5.993	6.417
		0.5	0.9648	2.233	5.665	5.828	6.122
	1	0	0.7964	1.876	4.685	5.491	5.926
		0.1	0.7962	1.871	4.685	5.477	5.867
		0.2	0.7960	1.864	4.685	5.429	5.725
		0.3	0.7956	1.857	4.683	5.348	5.528
		0.4	0.7948	1.849	4.680	5.233	5.293
		0.5	0.7940	1.842	4.671	5.051	5.088
	5	0	0.6844	1.595	3.969	4.287	5.026
		0.1	0.6843	1.591	3.969	4.276	4.973
		0.2	0.6841	1.584	3.969	4.239	4.847
		0.3	0.6837	1.577	3.967	4.175	4.674
		0.4	0.6831	1.569	3.964	4.086	4.470
		0.5	0.6824	1.562	3.956	3.973	4.262
	10	0	0.6622	1.539	3.827	3.908	4.851
		0.1	0.6620	1.535	3.827	3.898	4.800
		0.2	0.6618	1.528	3.827	3.864	4.676
		0.3	0.6615	1.520	3.806	3.826	4.507
		0.4	0.6609	1.513	3.725	3.823	4.310
		0.5	0.6602	1.505	3.622	3.815	4.107

表 4.21 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	1.016	2.233	3.306	5.361	6.828
		0.1	1.016	2.227	3.298	5.361	6.750
		0.2	1.016	2.212	3.269	5.361	6.553
		0.3	1.016	2.195	3.221	5.359	6.285
		0.4	1.015	2.178	3.152	5.356	5.980
		0.5	1.014	2.160	3.065	5.339	5.671
	0.2	0	0.9448	2.084	3.150	5.012	6.372
		0.1	0.9447	2.077	3.142	5.012	6.300
		0.2	0.9445	2.064	3.115	5.012	6.117
		0.3	0.9441	2.049	3.069	5.010	5.869
		0.4	0.9436	2.033	3.004	5.007	5.586
		0.5	0.9426	2.017	2.920	4.990	5.300
	1	0	0.7785	1.723	2.754	4.150	5.258
		0.1	0.7784	1.718	2.747	4.150	5.198
		0.2	0.7783	1.707	2.723	4.149	5.049
		0.3	0.7779	1.695	2.683	4.148	4.845
		0.4	0.7775	1.683	2.626	4.144	4.613
		0.5	0.7766	1.670	2.553	4.128	4.381
	5	0	0.6637	1.432	2.154	3.396	4.347
		0.1	0.6637	1.427	2.148	3.396	4.297
		0.2	0.6635	1.417	2.130	3.395	4.168
		0.3	0.6633	1.406	2.098	3.394	3.992
		0.4	0.6629	1.393	2.054	3.392	3.790
		0.5	0.6622	1.381	1.998	3.380	3.586
	10	0	0.6408	1.374	1.962	3.249	4.180
		0.1	0.6407	1.370	1.957	3.249	4.132
		0.2	0.6406	1.360	1.940	3.249	4.007
		0.3	0.6403	1.349	1.911	3.248	3.836
		0.4	0.6400	1.336	1.871	3.246	3.640
		0.5	0.6394	1.324	1.819	3.237	3.440

表 4.22 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.19	0.23	0.20	0.82	0.19
		0.2	0.19	0.31	0.14	2.68	0.29
		0.3	0.19	0.39	0.14	5.48	0.64
		0.4	0.28	0.47	0.17	8.94	1.53
		0.5	0.38	0.55	0.25	12.78	3.32
	0.2	0.1	0.15	0.25	0.15	0.81	0.20
		0.2	0.16	0.29	0.15	2.68	0.29
		0.3	0.19	0.38	0.15	5.46	0.65
		0.4	0.28	0.46	0.17	8.93	1.52
		0.5	0.37	0.50	0.25	12.76	3.31
	1	0.1	0.15	0.20	0.14	0.78	0.18
		0.2	0.17	0.25	0.16	2.65	0.28
		0.3	0.20	0.36	0.14	5.43	0.63
		0.4	0.27	0.41	0.18	8.87	1.51
		0.5	0.37	0.46	0.24	12.70	3.30
	5	0.1	0.13	0.24	0.12	0.83	0.20
		0.2	0.17	0.36	0.14	2.74	0.28
		0.3	0.20	0.47	0.14	5.56	0.65
		0.4	0.29	0.53	0.17	9.04	1.53
		0.5	0.38	0.59	0.26	12.90	3.33
	10	0.1	0.12	0.24	0.15	0.84	0.20
		0.2	0.16	0.31	0.17	2.78	0.30
		0.3	0.19	0.43	0.17	5.61	0.66
		0.4	0.27	0.49	0.19	9.09	1.55
		0.5	0.37	0.61	0.27	12.96	3.35

表 4.23 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.10	0.29	0.07	0.29	1.07
		0.2	0.10	0.70	0.08	1.17	3.53
		0.3	0.10	1.10	0.08	2.65	6.91
		0.4	0.19	1.51	0.11	4.70	10.84
		0.5	0.29	1.92	0.31	7.33	14.95
	0.2	0.1	0.03	0.26	0.00	0.25	1.01
		0.2	0.05	0.66	0.00	1.13	3.45
		0.3	0.11	1.05	0.04	2.61	6.81
		0.4	0.21	1.45	0.11	4.71	10.79
		0.5	0.31	1.85	0.30	7.33	14.89
	1	0.1	0.03	0.27	0.00	0.25	1.00
		0.2	0.05	0.64	0.00	1.13	3.39
		0.3	0.10	1.01	0.04	2.60	6.72
		0.4	0.20	1.44	0.11	4.70	10.68
		0.5	0.30	1.81	0.30	8.01	14.14
	5	0.1	0.01	0.25	0.00	0.26	1.05
		0.2	0.04	0.69	0.00	1.12	3.56
		0.3	0.10	1.13	0.05	2.61	7.00
		0.4	0.19	1.63	0.13	4.69	11.06
		0.5	0.29	2.07	0.33	7.32	15.20
	10	0.1	0.03	0.26	0.00	0.26	1.05
		0.2	0.06	0.71	0.00	1.13	3.61
		0.3	0.11	1.23	0.55	2.10	7.09
		0.4	0.20	1.69	2.67	2.18	11.15
		0.5	0.30	2.21	5.36	2.38	15.34

表 4.24 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.00	0.27	0.24	0.00	1.14
		0.2	0.00	0.94	1.12	0.00	4.03
		0.3	0.00	1.70	2.57	0.04	7.95
		0.4	0.10	2.46	4.66	0.09	12.42
		0.5	0.20	3.27	7.29	0.41	16.94
	0.2	0.1	0.01	0.34	0.25	0.00	1.13
		0.2	0.03	0.96	1.11	0.00	4.00
		0.3	0.07	1.68	2.57	0.04	7.89
		0.4	0.13	2.45	4.63	0.10	12.34
		0.5	0.23	3.21	7.30	0.44	16.82
	1	0.1	0.01	0.29	0.25	0.00	1.14
		0.2	0.03	0.93	1.13	0.02	3.97
		0.3	0.08	1.63	2.58	0.05	7.85
		0.4	0.13	2.32	4.65	0.14	12.27
		0.5	0.24	3.08	7.30	0.53	16.68
	5	0.1	0.00	0.35	0.28	0.00	1.15
		0.2	0.03	1.05	1.11	0.03	4.12
		0.3	0.06	1.82	2.60	0.06	8.17
		0.4	0.12	2.72	4.64	0.12	12.81
		0.5	0.23	3.56	7.24	0.47	17.51
	10	0.1	0.02	0.29	0.25	0.00	1.15
		0.2	0.03	1.02	1.12	0.00	4.14
		0.3	0.08	1.82	2.60	0.03	8.23
		0.4	0.12	2.77	4.64	0.09	12.92
		0.5	0.22	3.64	7.29	0.37	17.70

表 4.25 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	1.053	2.568	6.443	8.206	9.333
		0.1	1.053	2.568	6.443	8.194	9.333
		0.3	1.052	2.563	6.438	8.048	9.279
		0.5	1.051	2.557	6.430	7.689	9.091
	0.2	0	0.9774	2.384	5.981	7.618	8.665
		0.1	0.9773	2.384	5.981	7.607	8.665
		0.3	0.9766	2.379	5.976	7.472	8.615
		0.5	0.9754	2.374	5.969	7.140	8.440
	5	0	0.6931	1.689	4.238	5.397	6.134
		0.1	0.6930	1.688	4.238	5.389	6.134
		0.3	0.6925	1.685	4.235	5.291	6.099
		0.5	0.6916	1.681	4.230	5.052	5.974

表 4.26 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	1.042	2.446	6.107	6.601	7.732
		0.1	1.042	2.442	6.107	6.593	7.703
		0.3	1.041	2.422	6.102	6.493	7.487
		0.5	1.040	2.403	6.091	6.274	7.032
	0.2	0	0.9678	2.275	5.682	6.289	7.193
		0.1	0.9678	2.271	5.682	6.281	7.166
		0.3	0.9671	2.253	5.677	6.186	6.968
		0.5	0.9660	2.236	5.667	5.978	6.549
	5	0	0.6844	1.595	3.969	4.287	5.026
		0.1	0.6844	1.592	3.969	4.282	5.006
		0.3	0.6840	1.578	3.965	4.217	4.860
		0.5	0.6832	1.565	3.958	4.075	4.554

表 4.27 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0	1.016	2.233	3.306	5.361	6.828
		0.1	1.016	2.228	3.302	5.361	6.794
		0.3	1.016	2.198	3.252	5.357	6.537
		0.5	1.015	2.166	3.143	5.345	6.020
	0.2	0	0.9448	2.084	3.150	5.012	6.372
		0.1	0.9448	2.078	3.146	5.012	6.341
		0.3	0.9444	2.051	3.099	5.008	6.105
		0.5	0.9435	2.022	2.995	4.996	5.629
	5	0	0.6637	1.432	2.154	3.396	4.347
		0.1	0.6637	1.428	2.151	3.396	4.325
		0.3	0.6634	1.408	2.119	3.393	4.153
		0.5	0.6629	1.385	2.048	3.385	3.804

表 4.28 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.00	0.00	0.00	0.15	0.00
		0.3	0.09	0.19	0.08	1.93	0.58
		0.5	0.19	0.43	0.20	6.30	2.59
	0.2	0.1	0.01	0.00	0.00	0.14	0.00
		0.3	0.08	0.21	0.08	1.92	0.58
		0.5	0.20	0.42	0.20	6.27	2.60
	5	0.1	0.01	0.06	0.00	0.15	0.00
		0.3	0.09	0.24	0.07	1.96	0.57
		0.5	0.22	0.47	0.19	6.39	2.61

表 4.29 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.00	0.16	0.00	0.12	0.38
		0.3	0.10	0.98	0.08	1.64	3.17
		0.5	0.19	1.76	0.26	4.95	9.05
	0.2	0.1	0.00	0.18	0.00	0.13	0.38
		0.3	0.07	0.97	0.09	1.64	3.13
		0.5	0.19	1.71	0.26	4.95	8.95
	5	0.1	0.00	0.19	0.00	0.12	0.40
		0.3	0.06	1.07	0.10	1.63	3.30
		0.5	0.18	1.88	0.28	4.95	9.39

表 4.30 具不同水平內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.75)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
0°	0	0.1	0.00	0.22	0.12	0.00	0.50
		0.3	0.00	1.57	1.63	0.07	4.26
		0.5	0.10	3.00	4.93	0.30	11.83
	0.2	0.1	0.00	0.29	0.13	0.00	0.49
		0.3	0.04	1.58	1.62	0.08	4.19
		0.5	0.14	2.98	4.92	0.32	11.66
	5	0.1	0.00	0.28	0.14	0.00	0.51
		0.3	0.05	1.68	1.62	0.09	4.46
		0.5	0.12	3.28	4.92	0.32	12.49

表 4.31 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.053	2.568	6.443	8.206	9.333
		0.1	1.052	2.566	6.414	8.189	9.330
		0.3	1.037	2.563	6.132	8.063	9.316
		0.5	1.004	2.558	5.605	7.888	9.218
	0.2	0	0.9774	2.384	5.981	7.618	8.665
		0.1	0.9760	2.382	5.954	7.602	8.662
		0.3	0.9627	2.379	5.693	7.486	8.649
		0.5	0.9319	2.375	5.205	7.323	8.559
	5	0	0.6931	1.689	4.238	5.397	6.134
		0.1	0.6920	1.687	4.219	5.385	6.132
		0.3	0.6824	1.685	4.030	5.301	6.123
		0.5	0.6603	1.681	3.682	5.186	6.058

表 4.32 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.042	2.446	6.107	6.601	7.732
		0.1	1.039	2.440	6.056	6.584	7.704
		0.3	1.020	2.420	5.727	6.427	7.567
		0.5	0.980	2.398	5.185	6.096	7.398
	0.2	0	0.9678	2.275	5.682	6.289	7.193
		0.1	0.9651	2.270	5.635	6.273	7.167
		0.3	0.9473	2.251	5.330	6.123	7.039
		0.5	0.9100	2.231	4.826	5.808	6.882
	5	0	0.6844	1.595	3.969	4.287	5.026
		0.1	0.6825	1.591	3.935	4.276	5.007
		0.3	0.6694	1.577	3.719	4.174	4.918
		0.5	0.6423	1.561	3.365	3.960	4.808

表 4.33 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.016	2.233	3.306	5.361	6.828
		0.1	1.013	2.227	3.298	5.315	6.805
		0.3	0.9897	2.195	3.220	5.010	6.686
		0.5	0.9431	2.156	3.055	4.529	6.162
	0.2	0	0.9448	2.084	3.150	5.012	6.372
		0.1	0.9417	2.078	3.142	4.969	6.350
		0.3	0.9203	2.048	3.068	4.684	6.239
		0.5	0.8774	2.013	2.911	4.233	5.872
	5	0	0.6637	1.432	2.154	3.396	4.347
		0.1	0.6615	1.427	2.149	3.367	4.333
		0.3	0.6458	1.405	2.098	3.172	4.259
		0.5	0.6145	1.378	1.991	2.862	4.032

表 4.34 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.09	0.08	0.45	0.21	0.03
		0.3	1.52	0.19	4.83	1.74	0.18
		0.5	4.65	0.39	13.01	3.88	1.23
	0.2	0.1	0.14	0.08	0.45	0.21	0.03
		0.3	1.50	0.21	4.82	1.73	0.18
		0.5	4.66	0.38	12.97	3.87	1.22
	5	0.1	0.16	0.12	0.45	0.22	0.03
		0.3	1.54	0.24	4.91	1.78	0.18
		0.5	4.73	0.47	13.12	3.91	1.24

表 4.35 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.29	0.25	0.84	0.26	0.36
		0.3	2.11	1.06	6.22	2.64	2.13
		0.5	5.95	1.96	15.10	7.65	4.32
	0.2	0.1	0.28	0.22	0.83	0.25	0.36
		0.3	2.12	1.05	6.20	2.64	2.14
		0.5	5.97	1.93	15.07	7.65	4.32
	5	0.1	0.28	0.25	0.86	0.26	0.38
		0.3	2.19	1.13	6.30	2.64	2.15
		0.5	6.15	2.13	15.22	7.63	4.34

表 4.36 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.30	0.27	0.24	0.86	0.34
		0.3	2.59	1.70	2.60	6.55	2.08
		0.5	7.18	3.45	7.59	15.52	9.75
	0.2	0.1	0.33	0.29	0.25	0.86	0.35
		0.3	2.59	1.73	2.60	6.54	2.09
		0.5	7.13	3.41	7.59	15.54	7.85
	5	0.1	0.33	0.35	0.23	0.85	0.32
		0.3	2.70	1.89	2.60	6.60	2.02
		0.5	7.41	3.77	7.57	15.72	7.25

表 4.37 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.053	2.568	6.443	8.206	9.333
		0.1	1.048	2.567	6.440	8.204	9.331
		0.3	0.9953	2.562	6.407	8.146	9.319
		0.5	0.9039	2.552	6.252	7.892	9.289
	0.2	0	0.9774	2.384	5.981	7.618	8.665
		0.1	0.9731	2.383	5.978	7.615	8.663
		0.3	0.9240	2.378	5.948	7.562	8.652
		0.5	0.8393	2.370	5.805	7.327	8.624
	5	0	0.6931	1.689	4.238	5.397	6.134
		0.1	0.6898	1.688	4.236	5.395	6.133
		0.3	0.6545	1.684	4.214	5.356	6.125
		0.5	0.5940	1.678	4.110	5.188	6.104

表 4.38 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.042	2.446	6.107	6.601	7.732
		0.1	1.031	2.442	6.101	6.580	7.726
		0.3	0.9660	2.425	6.032	6.384	7.634
		0.5	0.8639	2.401	5.754	5.965	7.360
	0.2	0	0.9678	2.275	5.682	6.289	7.193
		0.1	0.9578	2.271	5.677	6.269	7.187
		0.3	0.8988	2.256	5.619	6.083	7.104
		0.5	0.8030	2.234	5.361	5.683	6.848
	5	0	0.6844	1.595	3.969	4.287	5.026
		0.1	0.6771	1.592	3.965	4.273	5.021
		0.3	0.6334	1.580	3.916	4.146	4.959
		0.5	0.5656	1.563	3.725	3.875	4.781

表 4.39 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.016	2.233	3.306	5.361	6.828
		0.1	1.004	2.229	3.296	5.352	6.817
		0.3	0.9312	2.203	3.199	5.238	6.704
		0.5	0.8234	2.166	2.990	4.862	5.933
	0.2	0	0.9448	2.084	3.150	5.012	6.372
		0.1	0.9337	2.079	3.140	5.004	6.363
		0.3	0.8661	2.056	3.048	4.901	6.258
		0.5	0.7663	2.022	2.849	4.557	5.655
	5	0	0.6637	1.432	2.154	3.396	4.347
		0.1	0.6558	1.428	2.147	3.389	4.340
		0.3	0.6073	1.411	2.084	3.308	4.267
		0.5	0.5361	1.384	1.950	3.051	3.890

表 4.40 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.47	0.04	0.05	0.02	0.02
		0.3	5.48	0.23	0.56	0.73	0.15
		0.5	14.16	0.62	2.96	3.83	0.47
	0.2	0.1	0.44	0.04	0.05	0.04	0.02
		0.3	5.46	0.25	0.55	0.74	0.15
		0.5	14.13	0.59	2.94	3.82	0.47
	5	0.1	0.48	0.06	0.05	0.04	0.02
		0.3	5.57	0.30	0.57	0.76	0.15
		0.5	14.30	0.65	3.02	3.87	0.49

表 4.41 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	1.06	0.16	0.10	0.32	0.08
		0.3	7.29	0.86	1.23	3.29	1.27
		0.5	17.09	1.84	5.78	9.63	4.81
	0.2	0.1	1.03	0.18	0.09	0.32	0.08
		0.3	7.13	0.84	1.11	3.28	1.24
		0.5	17.03	1.80	5.65	9.64	4.80
	5	0.1	1.07	0.19	0.10	0.33	0.10
		0.3	7.45	0.94	1.34	3.29	1.33
		0.5	17.36	2.01	6.15	9.61	4.87

表 4.42 具不同垂直內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	1.18	0.18	0.30	0.17	0.16
		0.3	8.35	1.34	3.24	2.29	1.82
		0.5	18.96	3.00	9.56	9.31	13.11
	0.2	0.1	1.17	0.24	0.32	0.16	0.14
		0.3	8.33	1.34	3.24	2.21	1.79
		0.5	18.89	2.98	9.56	9.08	11.25
	5	0.1	1.19	0.28	0.32	0.21	0.16
		0.3	8.50	1.47	3.25	2.59	1.84
		0.5	19.23	3.35	9.47	10.16	10.51

表 4.43 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
150°	0	0	1.053	2.568	6.443	8.206	9.333
		0.1	1.053	2.567	6.442	8.179	9.333
		0.3	1.050	2.540	6.417	7.880	9.307
		0.5	1.042	2.487	6.373	7.421	9.170
	0.2	0	0.9774	2.384	5.981	7.618	8.665
		0.1	0.9773	2.383	5.979	7.592	8.664
		0.3	0.9746	2.358	5.957	7.317	8.641
		0.5	0.9672	2.309	5.916	6.891	8.514
	5	0	0.6931	1.689	4.238	5.397	6.134
		0.1	0.6930	1.688	4.237	5.378	6.134
		0.3	0.6910	1.670	4.220	5.179	6.117
		0.5	0.6857	1.634	4.191	4.875	6.027

表 4.44 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
150°	0	0	1.042	2.446	6.107	6.601	7.732
		0.1	1.042	2.441	6.097	6.586	7.670
		0.3	1.037	2.403	6.049	6.426	7.302
		0.5	1.027	2.341	5.991	6.092	6.812
	0.2	0	0.9678	2.275	5.682	6.289	7.193
		0.1	0.9672	2.270	5.673	6.274	7.135
		0.3	0.9628	2.236	5.629	6.122	6.795
		0.5	0.9536	2.178	5.576	5.804	6.342
	5	0	0.6844	1.595	3.969	4.287	5.026
		0.1	0.6840	1.592	3.962	4.277	4.984
		0.3	0.6808	1.566	3.930	4.173	4.739
		0.5	0.6740	1.525	3.890	3.957	4.415

表 4.45 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
 無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
150°	0	0	1.016	2.233	3.306	5.361	6.828
		0.1	1.016	2.227	3.298	5.351	6.766
		0.3	1.010	2.185	3.219	5.295	6.385
		0.5	0.997	2.117	3.052	5.228	5.880
	0.2	0	0.9448	2.084	3.150	5.012	6.372
		0.1	0.9441	2.078	3.143	5.003	6.315
		0.3	0.9387	2.039	3.067	4.951	5.961
		0.5	0.9275	1.976	2.908	4.890	5.494
	5	0	0.6637	1.432	2.154	3.396	4.347
		0.1	0.6632	1.427	2.149	3.389	4.307
		0.3	0.6592	1.400	2.097	3.352	4.057
		0.5	0.6509	1.354	1.989	3.307	3.721

表 4.46 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 薄板
 無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.02$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
150°	0	0.1	0.00	0.04	0.02	0.33	0.00
		0.3	0.28	1.09	0.40	3.97	0.28
		0.5	1.04	3.15	1.09	9.57	1.75
	0.2	0.1	0.01	0.04	0.03	0.34	0.01
		0.3	0.29	1.09	0.40	3.95	0.28
		0.5	1.04	3.15	1.09	9.54	1.74
	5	0.1	0.01	0.06	0.02	0.35	0.00
		0.3	0.30	1.12	0.42	4.04	0.28
		0.5	1.07	3.26	1.11	9.67	1.74

表 4.47 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
150°	0	0.1	0.00	0.20	0.16	0.23	0.80
		0.3	0.48	1.76	0.95	2.65	5.56
		0.5	1.44	4.29	1.90	7.71	11.90
	0.2	0.1	0.06	0.22	0.16	0.24	0.81
		0.3	0.52	1.71	0.93	2.66	5.53
		0.5	1.47	4.26	1.87	7.71	11.83
	5	0.1	0.06	0.19	0.18	0.23	0.84
		0.3	0.53	1.82	0.98	2.66	5.71
		0.5	1.52	4.39	1.99	7.70	12.16

表 4.48 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/a	Mode				
			1	2	3	4	5
150°	0	0.1	0.00	0.27	0.24	0.19	0.91
		0.3	0.59	2.15	2.63	1.23	6.49
		0.5	1.87	5.19	7.68	2.48	13.88
	0.2	0.1	0.07	0.29	0.22	0.18	0.89
		0.3	0.65	2.16	2.63	1.22	6.45
		0.5	1.83	5.18	7.68	2.43	13.78
	5	0.1	0.08	0.35	0.23	0.21	0.92
		0.3	0.68	2.23	2.65	1.30	6.67
		0.5	1.93	5.45	7.66	2.62	14.40

表 4.49 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.1$ 、 $(x_0/a, y_0/b)=(0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	6.294	7.321	11.73	16.10	17.42
		0.1	6.255	7.321	11.66	16.09	17.41
		0.3	5.987	7.309	11.34	15.68	17.33
		0.5	5.595	7.231	11.13	13.32	17.13
	0.2	0	5.864	6.825	10.94	15.05	16.29
		0.1	5.827	6.825	10.88	15.04	16.28
		0.3	5.578	6.814	10.58	14.67	16.21
		0.5	5.212	6.741	10.37	12.47	16.02
	5	0	4.079	4.728	7.546	10.29	11.12
		0.1	4.053	4.728	7.506	10.29	11.11
		0.3	3.881	4.720	7.301	10.01	11.06
		0.5	3.630	4.670	7.163	8.489	10.93

表 4.50 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.2$ 、 $(x_0/a, y_0/b)=(0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	5.382	6.089	8.893	9.536	12.28
		0.1	5.351	6.089	8.891	9.492	12.26
		0.3	5.148	6.080	8.891	9.263	11.84
		0.5	4.857	6.024	8.873	9.105	9.933
	0.2	0	5.050	5.715	8.473	8.940	11.58
		0.1	5.020	5.713	8.471	8.897	11.56
		0.3	4.827	5.707	8.470	8.679	11.18
		0.5	4.550	5.654	8.452	8.530	9.374
	5	0	3.400	3.820	5.774	5.976	7.609
		0.1	3.382	3.820	5.774	5.950	7.598
		0.3	3.259	3.815	5.773	5.813	7.348
		0.5	3.082	3.782	5.715	5.773	6.173

表 4.51 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.62	0.00	0.60	0.06	0.06
		0.3	4.88	0.16	3.32	2.61	0.52
		0.5	11.11	1.23	5.12	17.27	1.66
	0.2	0.1	0.63	0.00	0.55	0.07	0.06
		0.3	4.88	0.16	3.29	2.52	0.49
		0.5	11.12	1.23	5.21	17.14	1.66
	5	0.1	0.64	0.00	0.53	0.00	0.09
		0.3	4.85	0.17	3.25	2.72	0.54
		0.5	11.01	1.23	5.08	17.50	1.71

表 4.52 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.58	0.00	0.02	0.46	0.16
		0.3	4.35	0.15	0.02	2.86	3.58
		0.5	9.75	1.07	0.22	4.52	19.11
	0.2	0.1	0.59	0.03	0.02	0.48	0.17
		0.3	4.42	0.14	0.04	2.92	3.45
		0.5	9.90	1.07	0.25	4.59	19.05
	5	0.1	0.53	0.00	0.00	0.44	0.14
		0.3	4.15	0.13	0.02	2.73	3.43
		0.5	9.35	0.99	1.02	3.40	18.87

表 4.53 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.1$ 、 $(x_0/a, y_0/b)=(0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	6.294	7.321	11.73	16.10	17.42
		0.1	6.291	7.316	11.71	16.01	17.42
		0.3	6.205	7.284	11.44	15.35	17.19
		0.5	5.897	7.227	10.69	14.46	16.23
	0.2	0	5.864	6.825	10.94	15.05	16.29
		0.1	5.861	6.821	10.92	14.96	16.29
		0.3	5.783	6.791	10.67	14.35	16.25
		0.5	5.499	6.739	9.976	13.51	15.46
	5	0	4.079	4.728	7.546	10.29	11.12
		0.1	4.077	4.725	7.534	10.23	11.12
		0.3	4.019	4.703	7.355	9.817	11.09
		0.5	3.811	4.664	6.879	9.258	10.52

表 4.54 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.2$ 、 $(x_0/a, y_0/b)=(0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	5.382	6.089	8.893	9.536	12.28
		0.1	5.374	6.084	8.865	9.513	12.22
		0.3	5.261	6.050	8.605	9.254	11.78
		0.5	4.922	5.984	8.125	8.678	11.22
	0.2	0	5.050	5.715	8.473	8.940	11.58
		0.1	5.042	5.711	8.446	8.919	11.52
		0.3	4.938	5.679	8.199	8.677	11.10
		0.5	4.623	5.618	7.741	8.135	10.57
	5	0	3.400	3.820	5.774	5.976	7.609
		0.1	3.395	3.817	5.756	5.962	7.570
		0.3	3.321	3.795	5.591	5.798	7.296
		0.5	3.103	3.751	5.284	5.439	6.949

表 4.55 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b=0.1$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.05	0.07	0.17	0.56	0.00
		0.3	1.41	0.51	2.47	4.66	1.32
		0.5	6.31	1.28	8.87	10.19	6.83
	0.2	0.1	0.05	0.06	0.18	0.60	0.00
		0.3	1.38	0.50	2.47	4.65	0.25
		0.5	6.22	1.26	8.81	10.23	5.10
	5	0.1	0.05	0.06	0.16	0.58	0.00
		0.3	1.47	0.53	2.53	4.60	0.27
		0.5	6.57	1.35	8.84	10.03	5.40

表 4.56 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b=0.2$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.15	0.08	0.31	0.24	0.49
		0.3	2.25	0.64	3.24	2.96	4.07
		0.5	8.55	1.72	8.64	9.00	8.63
	0.2	0.1	0.16	0.07	0.32	0.23	0.52
		0.3	2.22	0.63	3.23	2.94	4.15
		0.5	8.46	1.70	8.64	9.00	8.72
	5	0.1	0.15	0.08	0.31	0.23	0.51
		0.3	2.32	0.65	3.17	2.98	4.11
		0.5	8.74	1.81	8.49	8.99	8.67

表 4.57 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 中厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b=2$ 、 $h/b=0.1$ 、 $(x_0/a, y_0/b)=(0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.643	2.594	4.429	5.852	7.791
		0.1	1.637	2.594	4.429	5.848	7.782
		0.3	1.596	2.593	4.414	5.820	7.664
		0.5	1.528	2.585	4.358	5.787	7.395
	0.2	0	1.526	2.413	4.120	5.447	7.247
		0.1	1.521	2.413	4.119	5.443	7.238
		0.3	1.483	2.412	4.106	5.418	7.130
		0.5	1.420	2.404	4.055	5.389	6.882
	5	0	1.076	1.691	2.887	3.801	5.065
		0.1	1.072	1.691	2.887	3.798	5.058
		0.3	1.045	1.691	2.876	3.778	4.981
		0.5	1.000	1.685	2.838	3.754	4.801

表 4.58 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b=2$ 、 $h/b=0.2$ 、 $(x_0/a, y_0/b)=(0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.562	2.357	3.971	3.996	5.121
		0.1	1.556	2.357	3.971	3.995	5.115
		0.3	1.514	2.355	3.965	3.972	5.076
		0.5	1.447	2.348	3.898	3.963	5.025
	0.2	0	1.455	2.200	3.735	3.784	4.791
		0.1	1.450	2.200	3.735	3.784	4.785
		0.3	1.411	2.199	3.714	3.781	4.749
		0.5	1.348	2.192	3.645	3.776	4.703
	5	0	1.012	1.509	2.555	2.589	3.251
		0.1	1.009	1.508	2.554	2.589	3.247
		0.3	0.9814	1.508	2.539	2.586	3.220
		0.5	0.9380	1.503	2.489	2.584	3.184

表 4.59 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b=2$ 、 $h/b=0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.37	0.00	0.00	0.07	0.12
		0.3	2.86	0.04	0.34	0.55	1.63
		0.5	7.00	0.35	1.60	1.11	5.08
	0.2	0.1	0.33	0.00	0.02	0.07	0.12
		0.3	2.82	0.04	0.34	0.53	1.61
		0.5	6.95	0.37	1.58	1.06	5.04
	5	0.1	0.37	0.00	0.00	0.08	0.14
		0.3	2.88	0.00	0.38	0.61	1.66
		0.5	7.06	0.35	1.70	1.24	5.21

表 4.60 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b=2$ 、 $h/b=0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.38	0.00	0.00	0.03	0.12
		0.3	3.07	0.08	0.15	0.60	0.88
		0.5	7.36	0.38	1.84	0.83	1.87
	0.2	0.1	0.34	0.00	0.00	0.00	0.13
		0.3	3.02	0.05	0.56	0.08	0.88
		0.5	7.35	0.36	2.41	0.21	1.84
	5	0.1	0.30	0.07	0.04	0.00	0.12
		0.3	3.02	0.07	0.63	0.12	0.95
		0.5	7.31	0.40	2.58	0.19	2.06

表 4.61 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 中厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b=2$ 、 $h/b=0.1$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.643	2.594	4.429	5.852	7.791
		0.1	1.642	2.593	4.414	5.852	7.787
		0.3	1.637	2.581	4.312	5.845	7.726
		0.5	1.625	2.569	4.136	5.815	7.523
	0.2	0	1.526	2.413	4.120	5.447	7.247
		0.1	1.526	2.411	4.106	5.447	7.243
		0.3	1.521	2.401	4.011	5.441	7.187
		0.5	1.510	2.390	3.848	5.414	7.068
	5	0	1.076	1.691	2.887	3.801	5.065
		0.1	1.076	1.690	2.877	3.801	5.062
		0.3	1.072	1.682	2.809	3.796	5.021
		0.5	1.064	1.673	2.694	3.777	4.888

表 4.62 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($a/b=2$ 、 $h/b=0.2$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0	1.562	2.357	3.971	3.996	5.121
		0.1	1.562	2.354	3.965	3.982	5.121
		0.3	1.555	2.339	3.884	3.898	5.115
		0.5	1.537	2.319	3.722	3.766	5.090
	0.2	0	1.455	2.200	3.735	3.784	4.791
		0.1	1.455	2.197	3.722	3.778	4.791
		0.3	1.448	2.183	3.631	3.714	4.786
		0.5	1.432	2.165	3.479	3.588	4.762
	5	0	1.012	1.509	2.555	2.589	3.251
		0.1	1.012	1.507	2.546	2.585	3.251
		0.3	1.007	1.496	2.483	2.541	3.248
		0.5	0.995	1.483	2.380	2.456	3.232

表 4.63 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b=2$ 、 $h/b=0.1$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.06	0.04	0.34	0.00	0.05
		0.3	0.37	0.50	2.64	0.12	0.83
		0.5	1.10	0.96	6.62	0.63	3.44
	0.2	0.1	0.00	0.08	0.34	0.00	0.06
		0.3	0.33	0.50	2.65	0.11	0.83
		0.5	1.05	0.95	6.60	0.61	2.47
	5	0.1	0.00	0.06	0.35	0.00	0.06
		0.3	0.37	0.53	2.70	0.13	0.87
		0.5	1.12	1.06	6.69	0.63	3.49

表 4.64 具不同垂直內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($a/b=2$ 、 $h/b=0.2$ 、 $(x_0/a, y_0/b) = (0.25, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
90°	0	0.1	0.00	0.13	0.15	0.35	0.00
		0.3	0.45	0.76	2.19	2.45	0.12
		0.5	1.60	1.61	6.27	5.76	0.61
	0.2	0.1	0.00	0.14	0.35	0.16	0.00
		0.3	0.48	0.77	2.78	1.85	0.10
		0.5	1.58	1.59	6.85	5.18	0.61
	5	0.1	0.00	0.13	0.35	0.15	0.00
		0.3	0.49	0.86	2.82	1.85	0.09
		0.5	1.68	1.72	6.85	5.14	0.58

表 4.65 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.1$ 、 $(x_0/a, y_0/b)=(0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
150°	0	0	6.294	7.321	11.73	16.10	17.42
		0.1	6.284	7.321	11.60	16.10	17.41
		0.3	6.224	7.297	10.87	15.94	17.32
		0.5	6.156	7.178	10.09	14.20	16.56
	0.2	0	5.864	6.825	10.94	15.05	16.29
		0.1	5.854	6.825	10.82	15.05	16.28
		0.3	5.798	6.803	10.14	14.90	16.19
		0.5	5.735	6.693	9.417	13.29	15.48
	5	0	4.079	4.728	7.546	10.29	11.12
		0.1	4.072	4.727	7.461	10.29	11.11
		0.3	4.033	4.712	6.990	10.19	11.05
		0.5	3.989	4.635	6.484	9.038	10.53

表 4.66 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率 $\omega(b^2/h)\sqrt{\rho_c/E_c}$ ($h/b=0.2$ 、 $(x_0/a, y_0/b)=(0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
150°	0	0	5.382	6.089	8.893	9.536	12.28
		0.1	5.374	6.089	8.888	9.423	12.28
		0.3	5.325	6.063	8.785	8.881	12.14
		0.5	5.268	5.965	8.105	8.781	10.59
	0.2	0	5.050	5.715	8.473	8.940	11.58
		0.1	5.041	5.715	8.467	8.834	11.58
		0.3	4.995	5.691	8.237	8.461	11.45
		0.5	4.941	5.600	7.599	8.361	9.988
	5	0	3.400	3.820	5.774	5.976	7.609
		0.1	3.395	3.820	5.774	5.905	7.607
		0.3	3.365	3.805	5.490	5.752	7.531
		0.5	3.330	3.748	5.035	5.602	6.704

表 4.67 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 中厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.1$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
150°	0	0.1	0.16	0.00	1.11	0.00	0.06
		0.3	1.11	0.33	7.33	0.99	0.57
		0.5	2.19	1.95	13.98	11.80	4.94
	0.2	0.1	0.17	0.00	1.10	0.00	0.06
		0.3	1.13	0.32	7.31	1.00	0.61
		0.5	2.20	1.93	13.92	11.69	4.97
	5	0.1	0.17	0.02	1.13	0.00	0.09
		0.3	1.13	0.34	7.37	0.97	0.63
		0.5	2.21	1.97	14.07	12.17	5.31

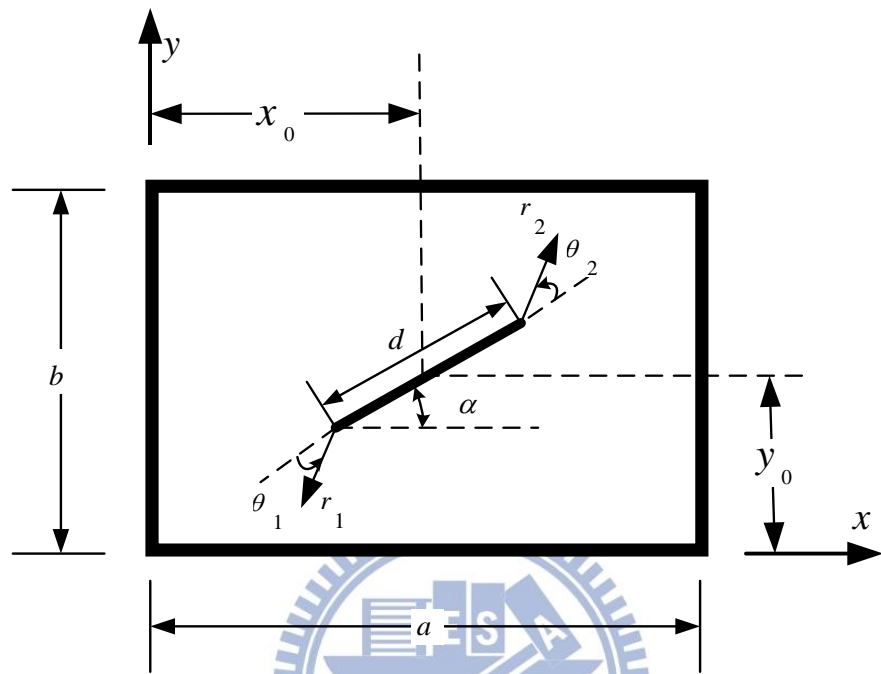
表 4.68 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 厚板
無因次化頻率折減量 $\Delta\omega(\%)$ ($h/b = 0.2$ 、 $(x_0/a, y_0/b) = (0.5, 0.5)$)

α	\hat{m}	d/b	Mode				
			1	2	3	4	5
150°	0	0.1	0.15	0.00	0.06	1.18	0.00
		0.3	1.06	0.43	1.21	6.87	1.14
		0.5	2.12	2.04	8.86	7.92	13.76
	0.2	0.1	0.18	0.00	0.07	1.19	0.00
		0.3	1.09	0.42	2.79	5.36	1.12
		0.5	2.16	2.01	10.32	6.48	13.75
	5	0.1	0.15	0.00	0.00	1.19	0.03
		0.3	1.03	0.39	4.92	3.75	1.03
		0.5	2.06	1.88	12.80	6.26	11.89

表 4.69 具不同水平內部裂縫簡支之方形均質板厚度效應及不同理論比較
 ($x_0/a, y_0/b$) = (0.5, 0.5)

d/a	h/b	Mode				
		1	2	3	4	5
0.3	0.01	[5.698]	[14.67]	[14.90]	[23.87]	[27.12]
		(5.701)	(14.65)	(14.89)	(23.81)	(27.09)
		{5.701}	{14.65}	{14.89}	{23.82}	{27.11}
	0.05	[5.698]	[14.67]	[14.90]	[23.87]	[27.12]
(5.589)		(14.19)	(14.55)	(22.90)	(25.59)	
{5.590}		{14.21}	{14.57}	{22.94}	{25.62}	
0.1	[5.698]	[14.67]	[14.90]	[23.87]	[27.12]	
	(5.411)	(13.15)	(13.71)	(20.85)	(23.00)	
0.2	0.2	[5.698]	[14.67]	[14.90]	[23.87]	[27.12]
		(4.933)	(10.69)	(11.48)	(16.40)	(17.83)
		{4.960}	{10.84}	{11.61}	{16.64}	{18.06}
	0.5	[5.358]	[13.02]	[14.74]	[23.52]	[24.86]
(5.347)		(12.96)	(14.71)	(23.46)	(24.76)	
{5.353}		{12.98}	{14.72}	{23.46}	{24.79}	
0.05	[5.358]	[13.02]	[14.74]	[23.52]	[24.86]	
	(5.232)	(12.23)	(14.36)	(22.40)	(23.56)	
	{5.238}	{12.28}	{14.37}	{22.44}	{23.60}	
0.1	[5.358]	[13.02]	[14.74]	[23.52]	[24.86]	
	(5.060)	(11.02)	(13.50)	(20.22)	(21.33)	
0.2	[5.358]	[13.02]	[14.74]	[23.52]	[24.86]	
	(4.612)	(8.640)	(11.32)	(15.73)	(16.71)	
	{4.633}	{8.764}	{11.43}	{15.97}	{16.89}	

Note: []:Stahl and Keer; (): Mindlin plate theory; { }: present 3-D elasticity-based solution



(a)板上視圖



(b)板側視圖

圖 2.1 具內部裂縫矩形板示意圖

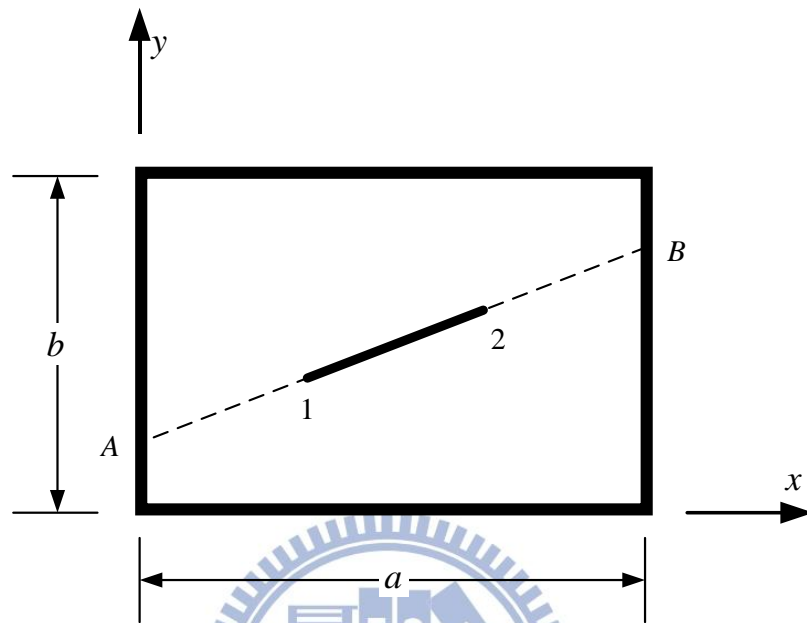


圖 2.2 具內部裂縫奇異點與連續線段示意圖

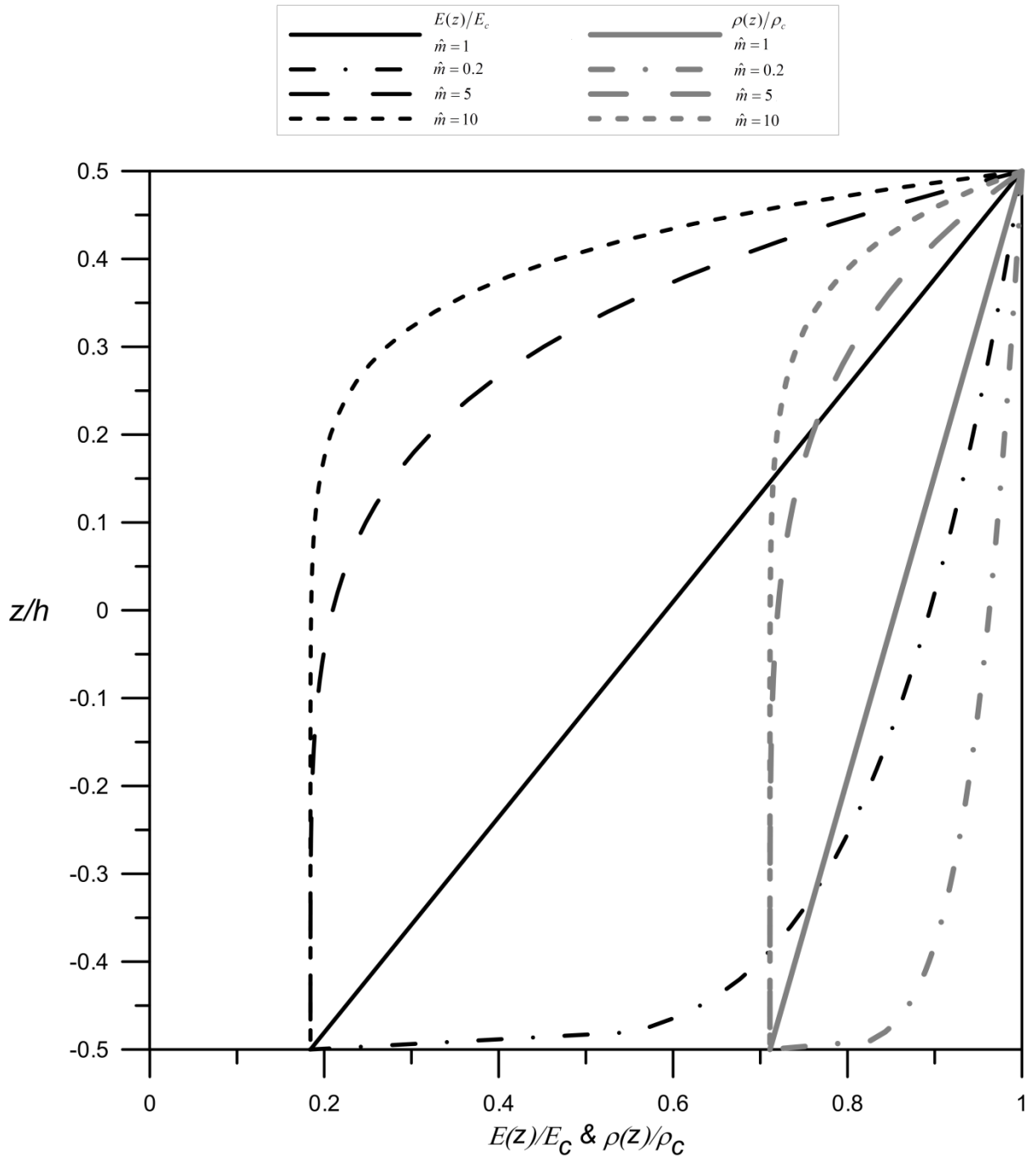


圖 4.1 Al/Al₂O₃ 功能梯度材料參數 $E(z)$ 和 $\rho(z)$ 沿厚度變化圖根據式(2.1)

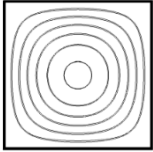
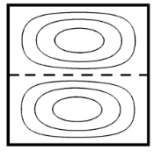
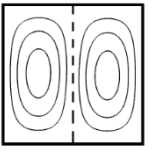
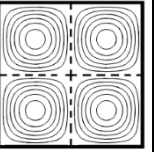
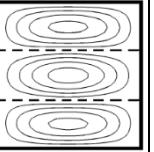
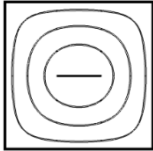
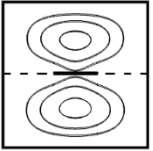
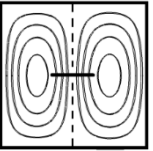
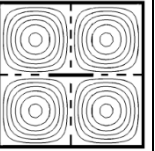
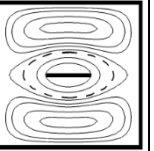
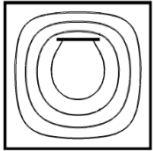
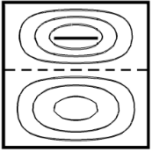
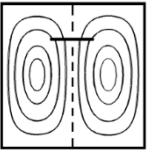
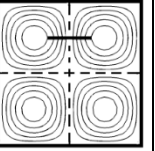
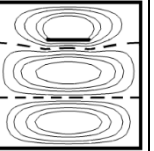
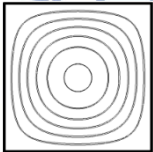
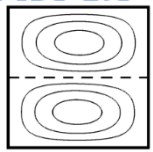
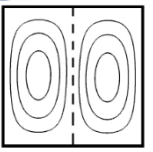
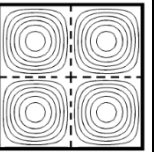
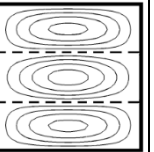
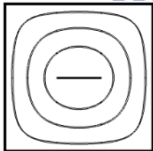
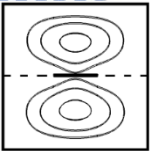
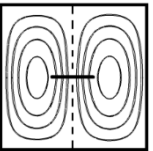
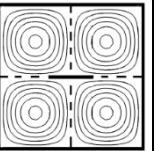
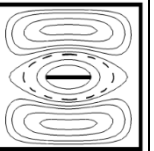
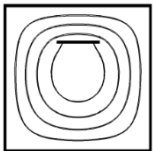
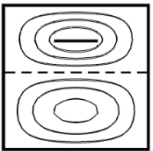
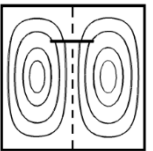
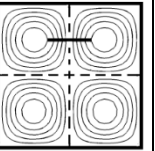
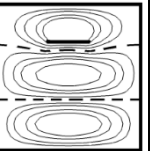
α	\hat{m}	$(x_0/a, y_0/b)$	d/a	Mode Number					
				1	2	3	4	5	
0°	0		0	 (5.965)	 (14.88)	 (14.88)	 (23.76)	 (29.66)	
		(0.5,0.5)	0.3	 (5.665)	 (14.58)	 (14.84)	 (23.68)	 (26.71)	
		(0.5,0.75)	0.3	 (5.789)	 (13.67)	 (14.85)	 (23.63)	 (27.71)	
	5			0	 (3.925)	 (9.787)	 (9.787)	 (15.62)	 (19.49)
		(0.5,0.5)	0.3	0.3	 (3.725)	 (9.581)	 (9.760)	 (15.56)	 (17.53)
		(0.5,0.75)	0.3	0.3	 (3.808)	 (8.984)	 (9.766)	 (15.53)	 (18.19)

圖 4.2 具不同內部裂縫簡支之均質與 $\text{Al}/\text{Al}_2\text{O}_3$ FGM 方形薄板 2D 模態圖
($h/b=0.02$)

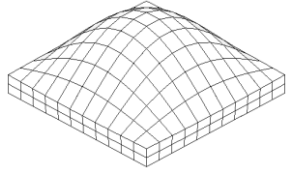
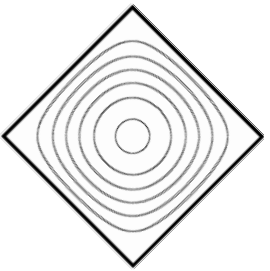
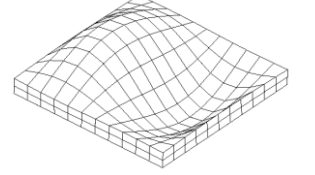
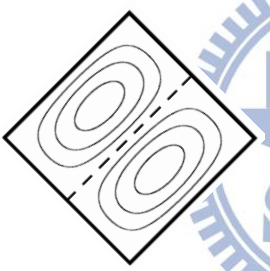
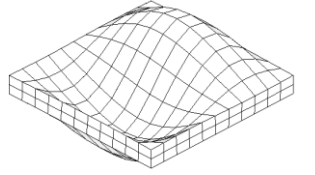

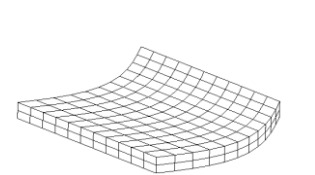
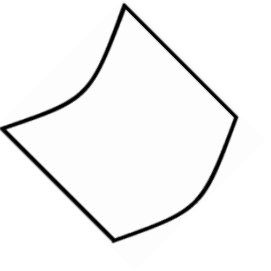
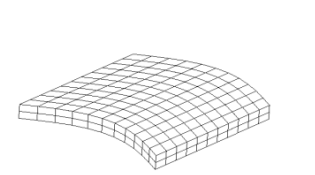
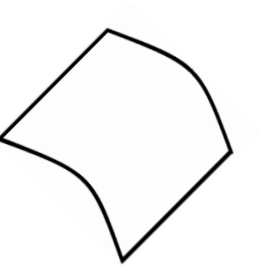
h/b	d/a	Mode				
		1	2	3	4	5
0.1	0	 (5.777) 	 (13.81) 	 (13.81) 	 (19.48) 	 (19.48) 

圖 4.3 具不同水平內部裂縫簡支之方形均質中厚板模態圖

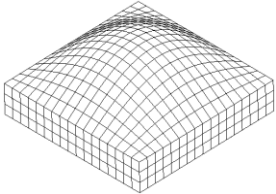
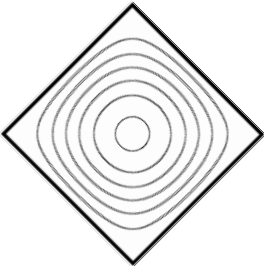
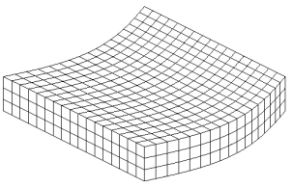
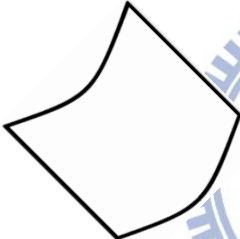
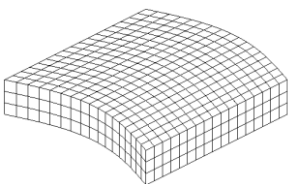

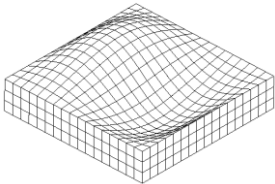
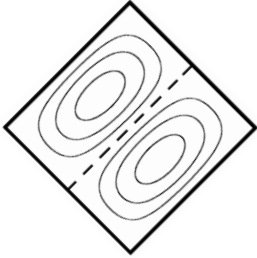
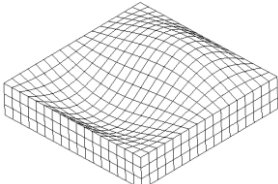
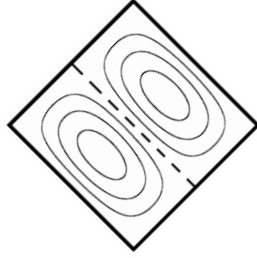
h/b	d/a	Mode				
		1	2	3	4	5
0.2	0	 (5.304) 	 (9.742) 	 (9.742) 	 (11.65) 	 (11.65) 

圖 4.3 (續上頁)

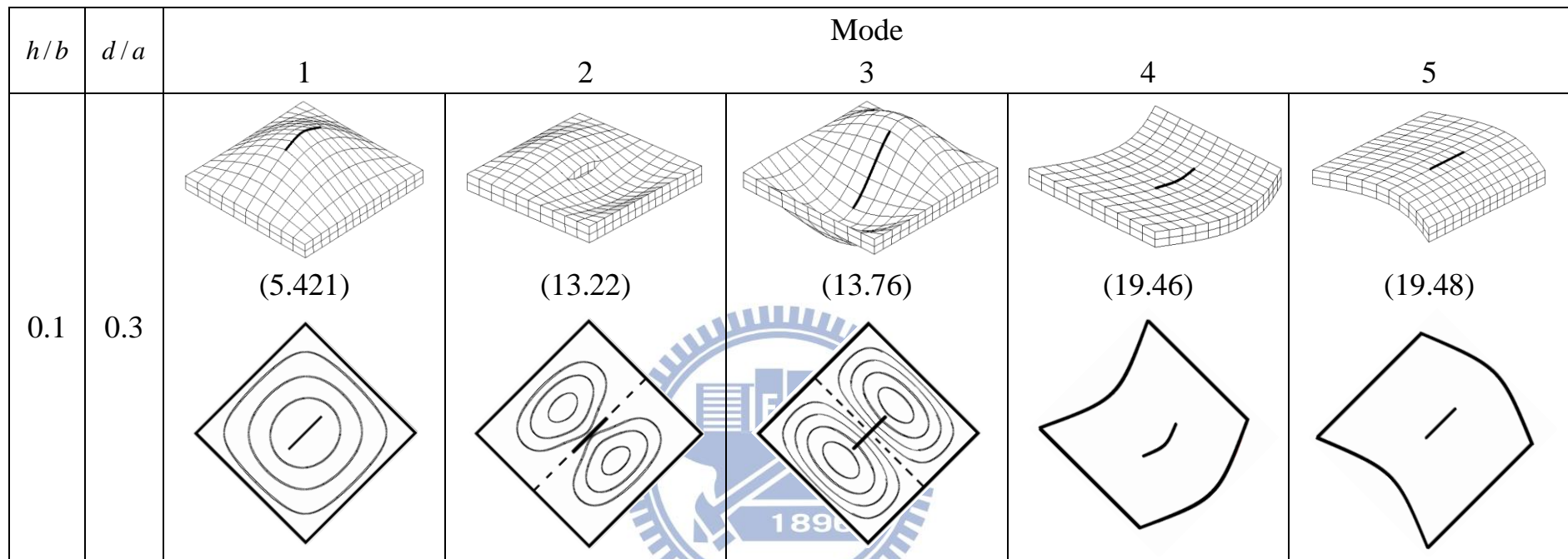


圖 4.3 (續上頁) $((x_0/a, y_0/b) = (0.5, 0.5))$

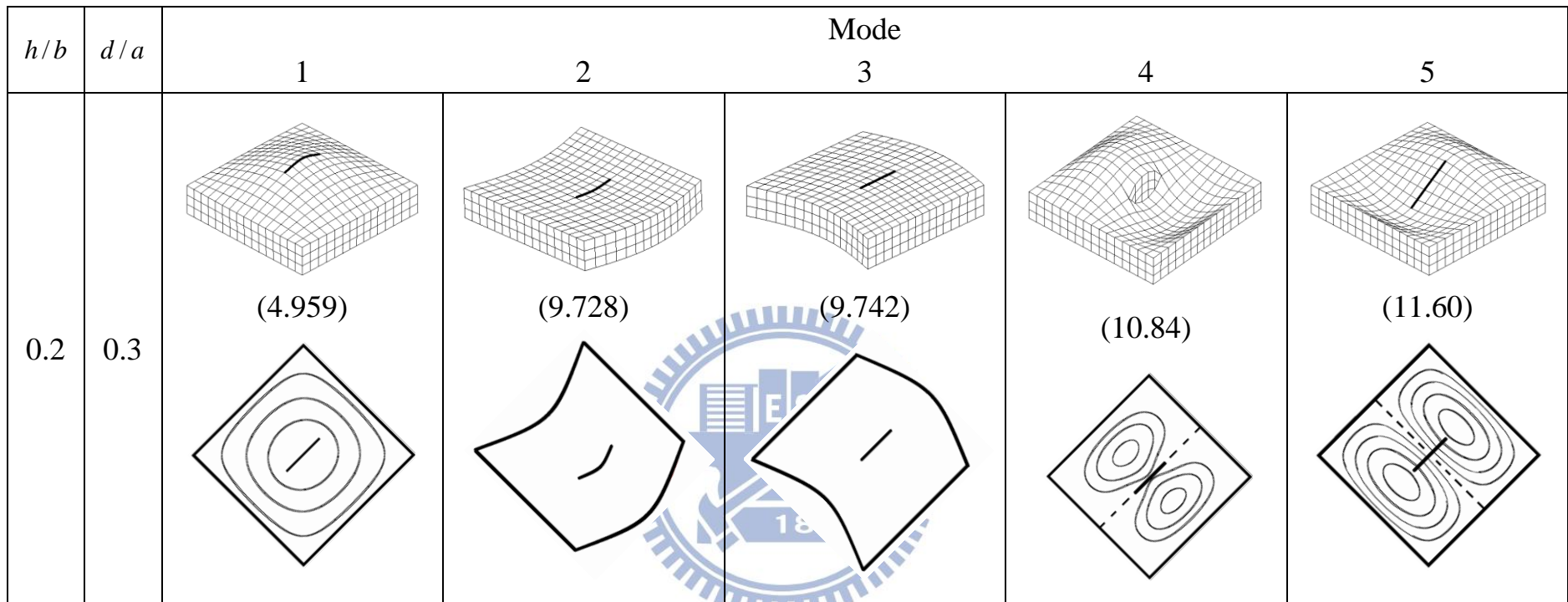


圖 4.3 (續上頁)

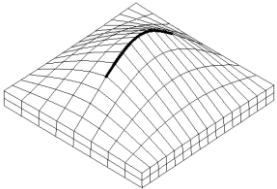
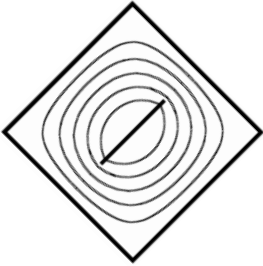
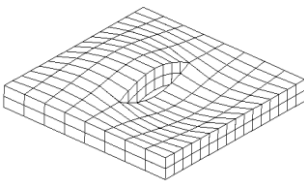
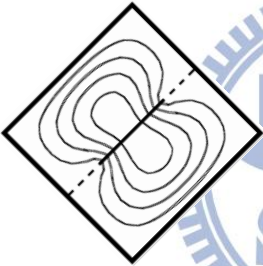
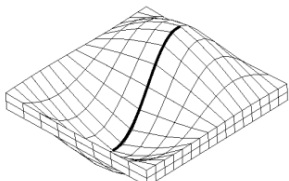

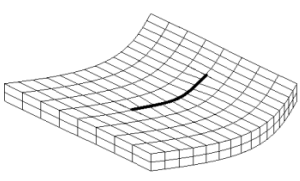
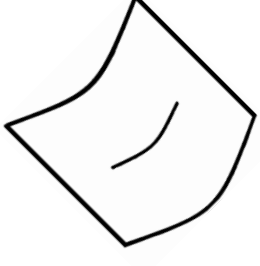
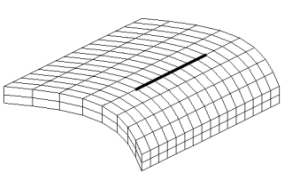
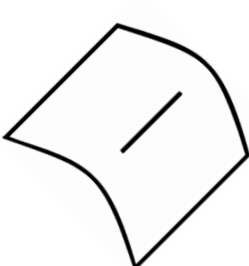
h/b	d/a	Mode				
		1	2	3	4	5
0.1	0.5	 (5.068) 	 (11.10) 	 (13.55) 	 (19.27) 	 (19.48) 

圖 4.3 (續上頁)

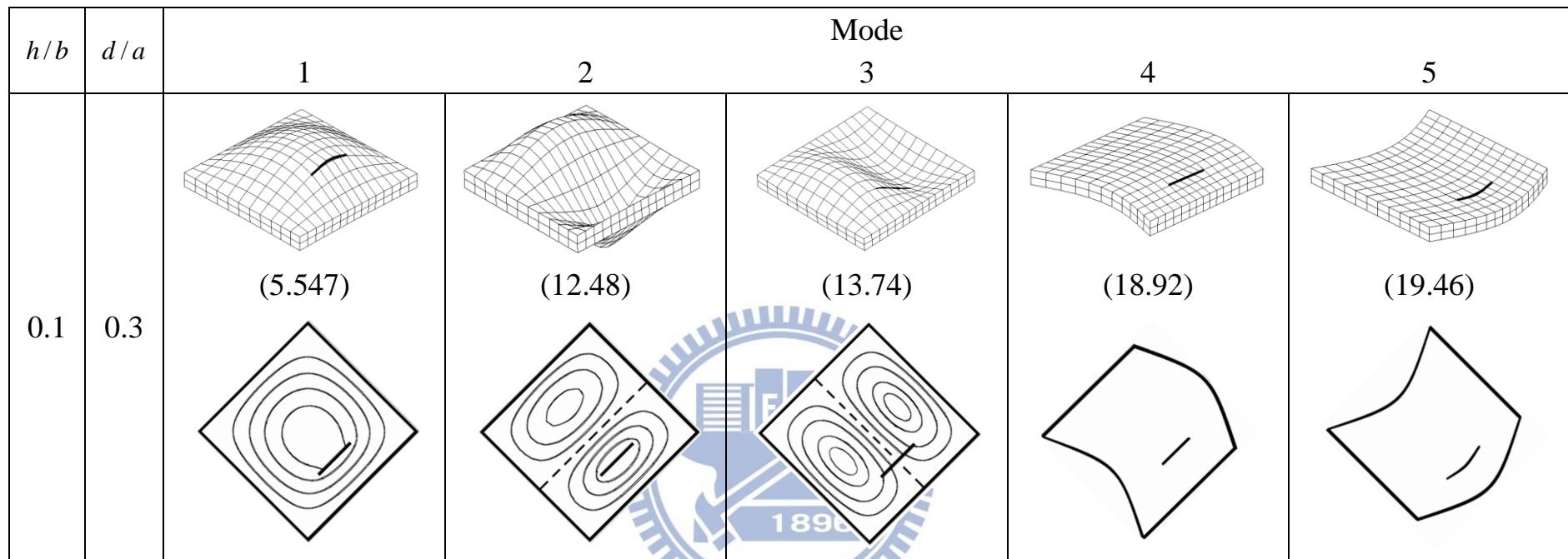


圖 4.3 (續上頁) ($((x_0/a, y_0/b) = (0.5, 0.75))$)

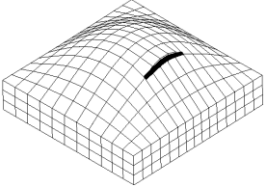
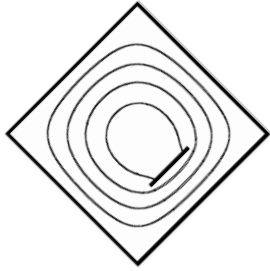
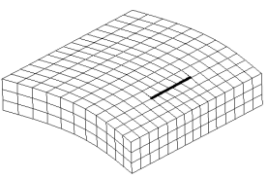
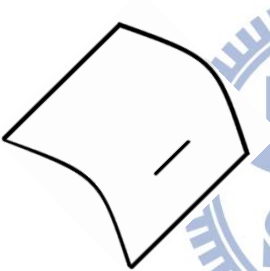
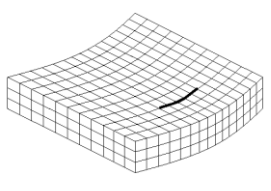

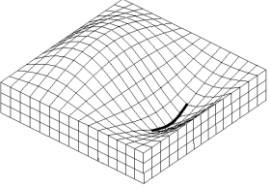
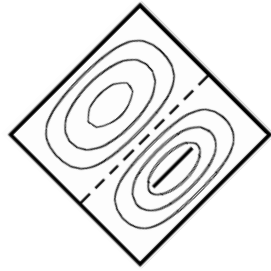
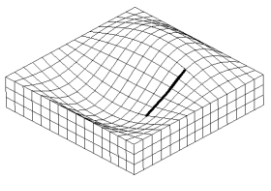
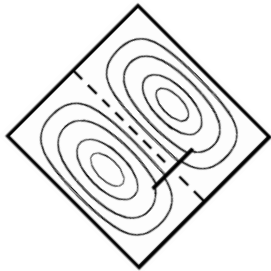
h/b	d/a	Mode				
		1	2	3	4	5
0.2	0.3	 (5.059) 	 (9.465) 	 (9.728) 	 (10.55) 	 (11.58) 

圖 4.3 (續上頁)

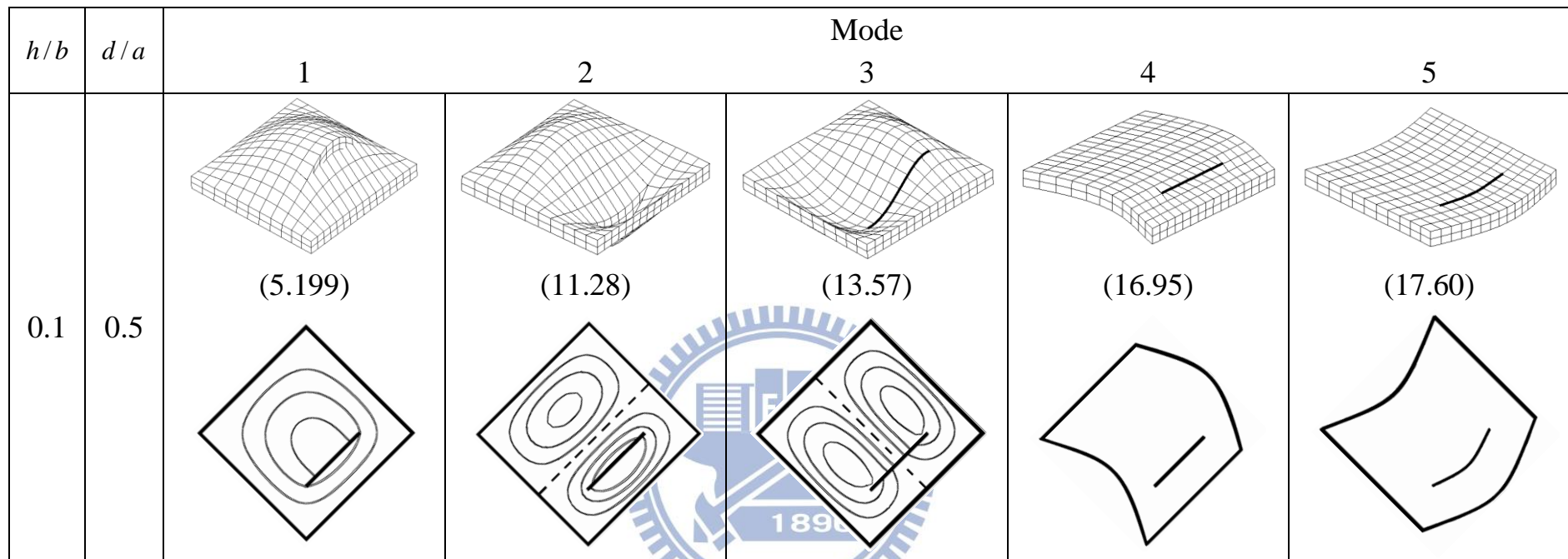


圖 4.3 (續上頁)

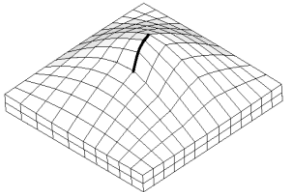
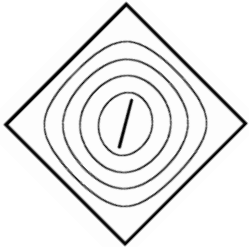
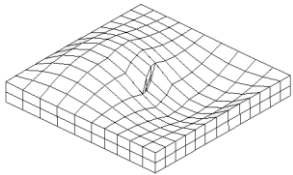

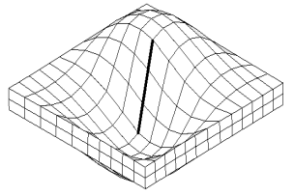

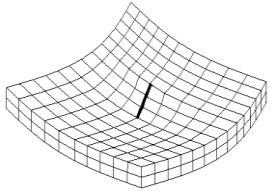
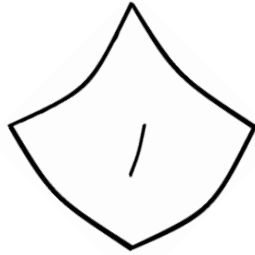
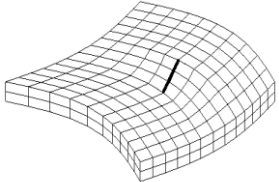

h/b	α	d/a	Mode				
			1	2	3	4	5
0.1	30°	0.3	 (5.412) 	 (13.21) 	 (13.74) 	 (19.45) 	 (19.48) 

圖 4.3 (續上頁) ($(x_0/a, y_0/b) = (0.5, 0.5)$)

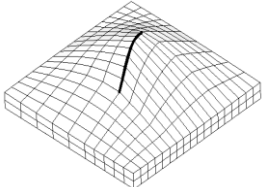
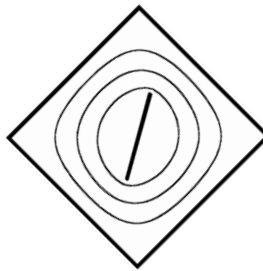
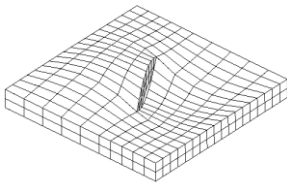
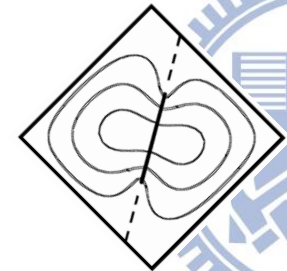
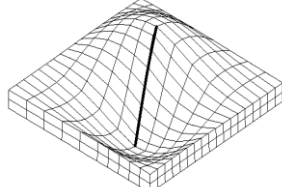

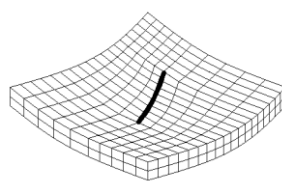
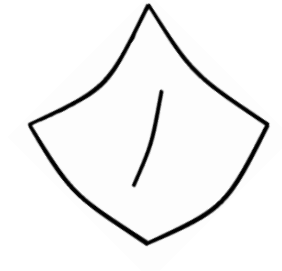
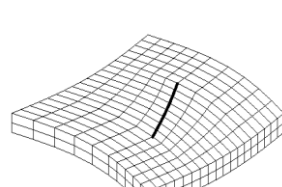
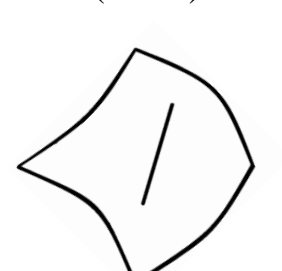
h/b	α	d/a	Mode				
			1	2	3	4	5
0.1	30°	0.5	 (5.017) 	 (11.10) 	 (13.45) 	 (19.26) 	 (19.48) 

圖 4.3 (續上頁)

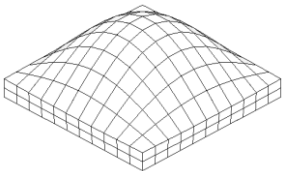
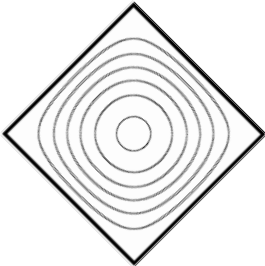
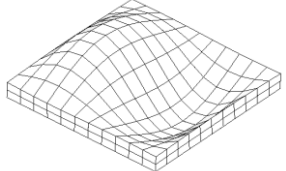

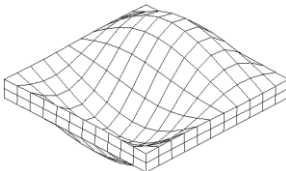

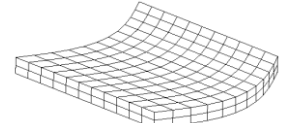
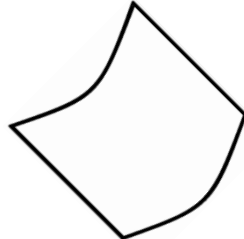
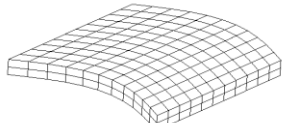
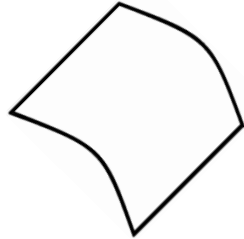
h/b	d/a	Mode				
		1	2	3	4	5
0.1	0	 (3.772) 	 (8.929) 	 (8.929) 	 (12.64) 	 (12.64) 

圖 4.4 具不同水平內部裂縫簡支之 Al/Al₂O₃ 方形 FGM 中厚板模態圖($\hat{m}=5$)

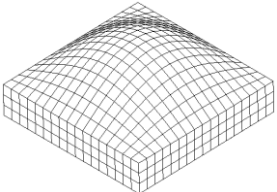
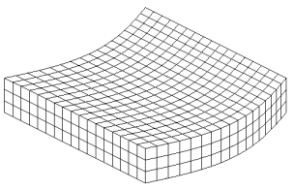
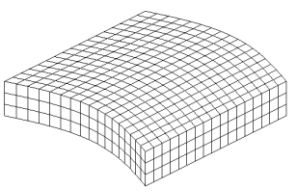
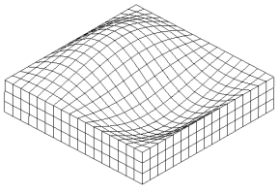
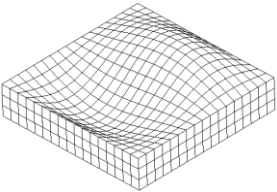
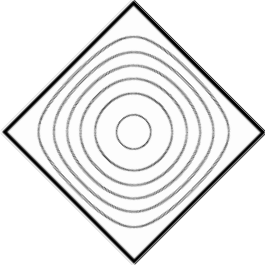

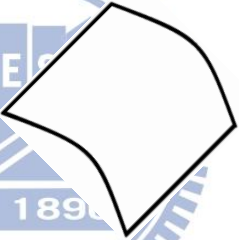
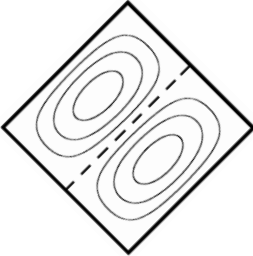
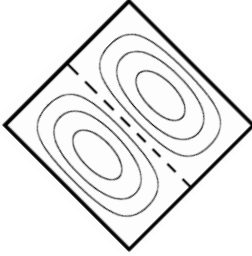
h/b	d/a	Mode				
		1	2	3	4	5
0.2	0	 (3.406)	 (6.296)	 (6.296)	 (7.347)	 (7.347)
						

圖 4.4 (續上頁)

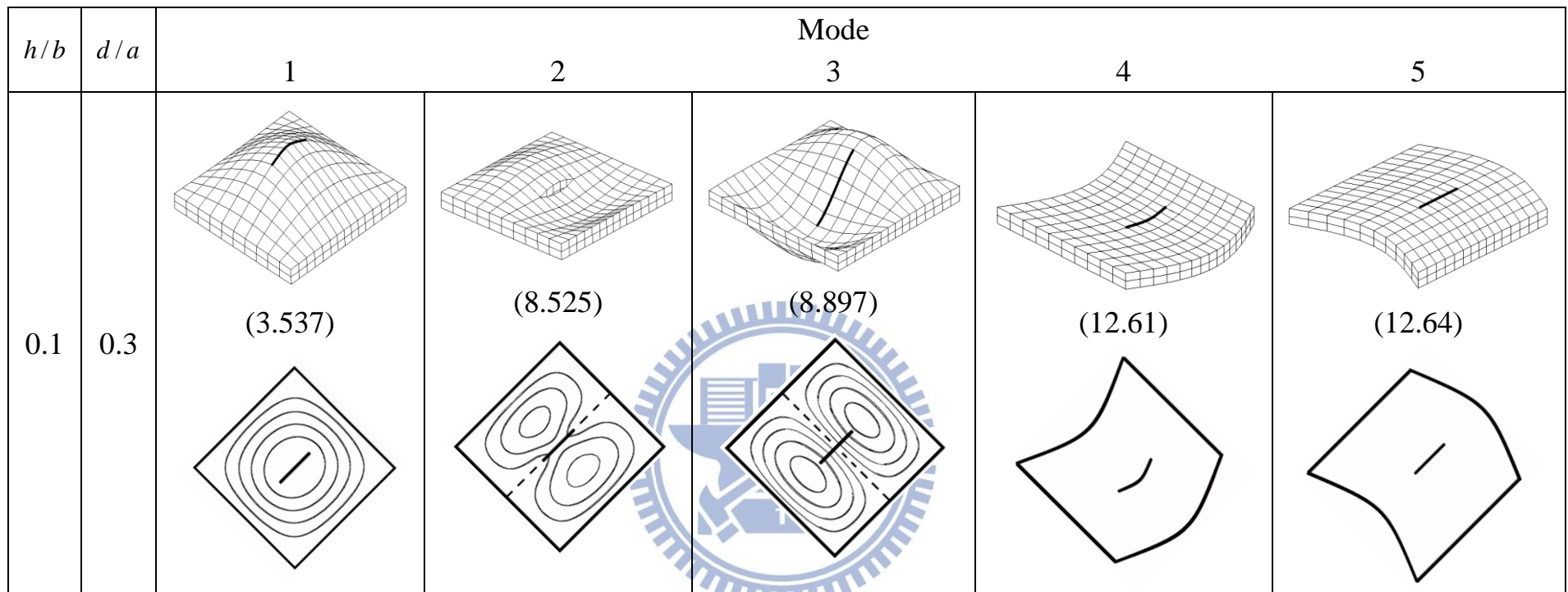


圖 4.4 (續上頁) $((x_0/a, y_0/b) = (0.5, 0.5))$

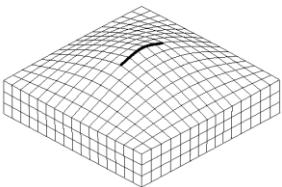
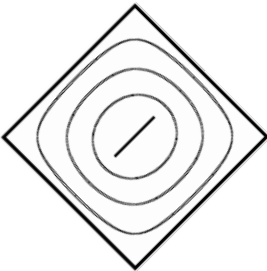
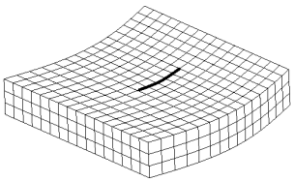
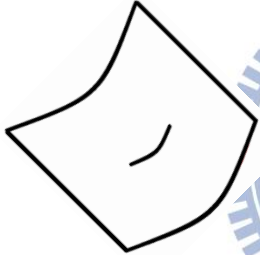
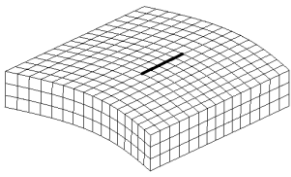
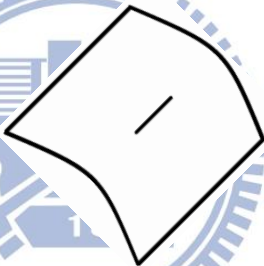
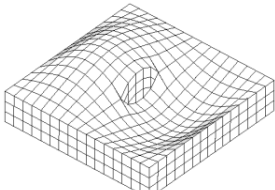
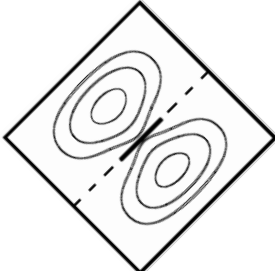
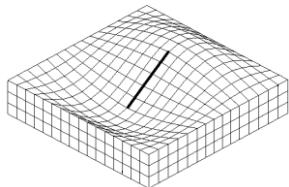
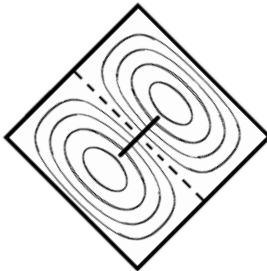
h/b	d/a	Mode				
		1	2	3	4	5
0.2	0.3	 (3.185) 	 (6.274) 	 (6.296) 	 (6.823) 	 (7.322) 

圖 4.4 (續上頁)

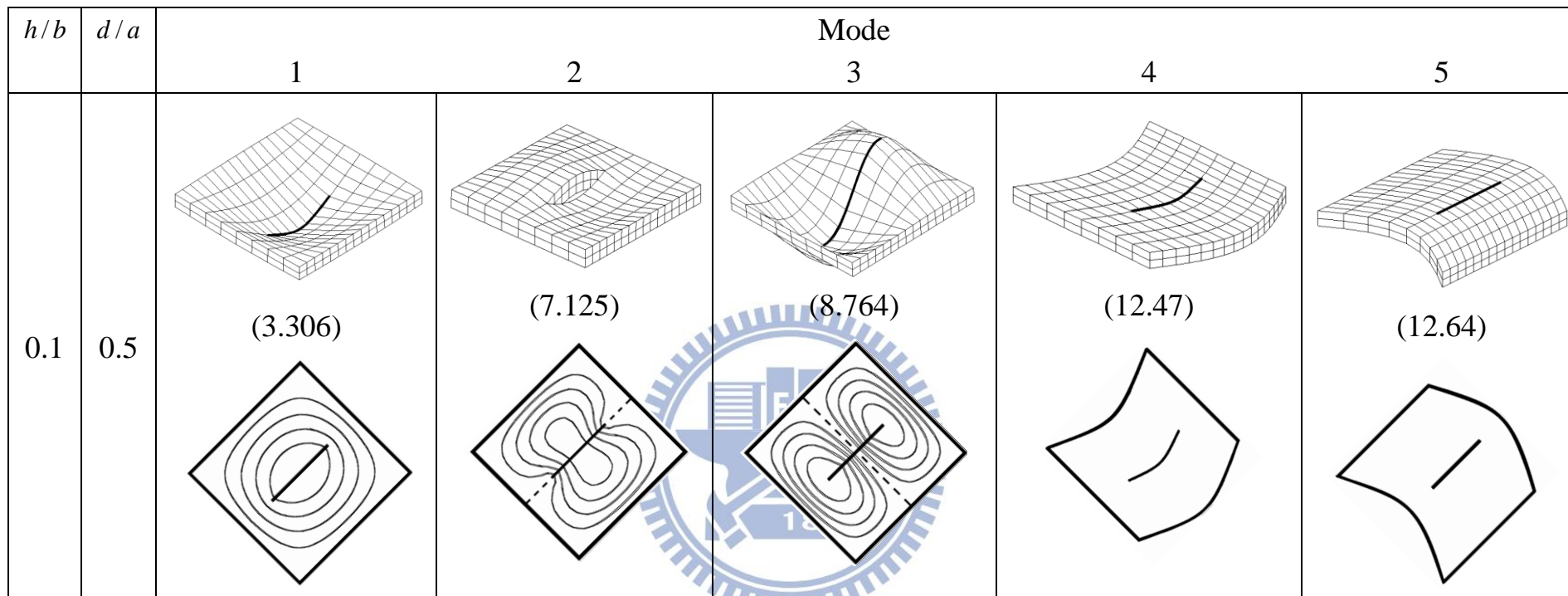


圖 4.4 (續上頁)

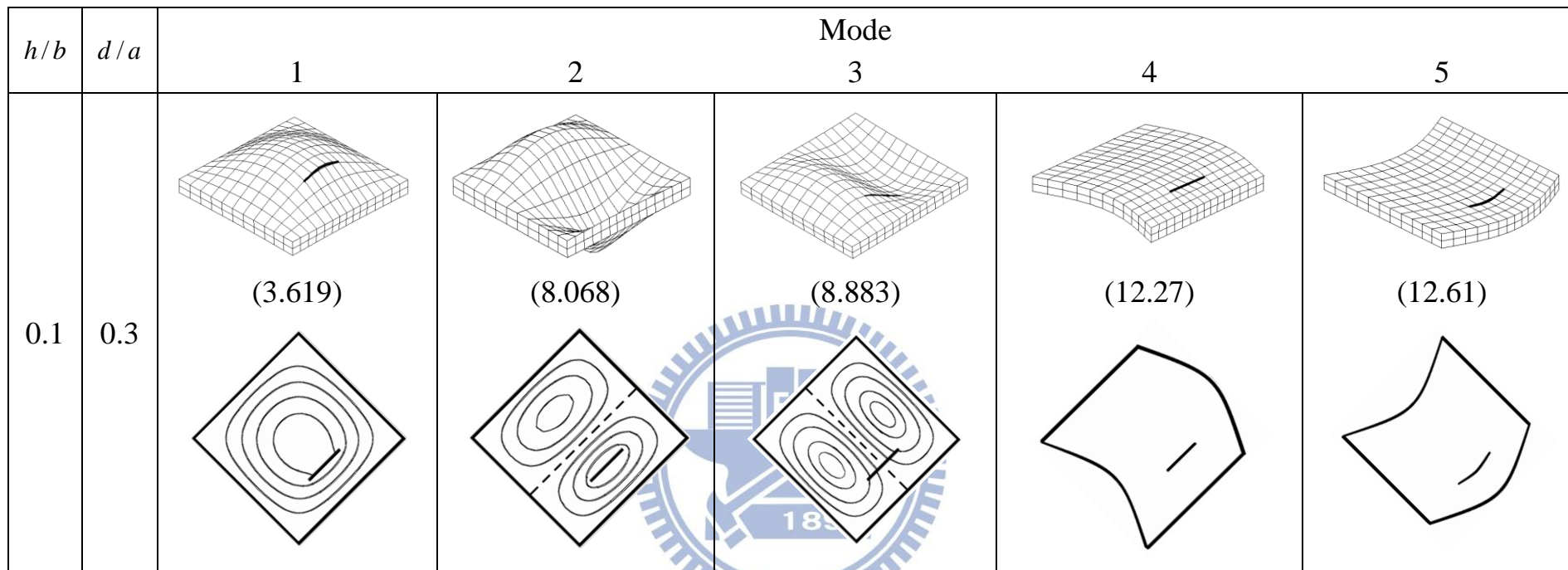


圖 4.4 (續上頁) ($(x_0/a, y_0/b) = (0.5, 0.75)$)

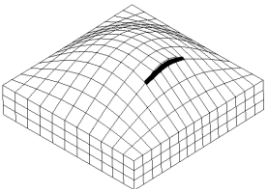
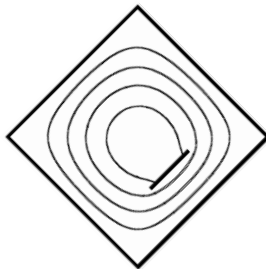
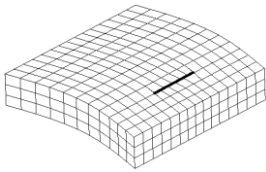
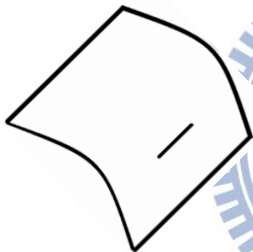
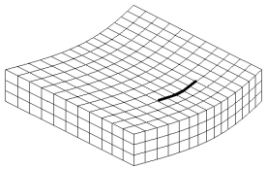
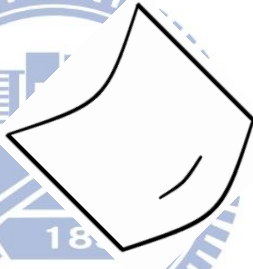
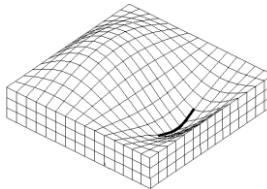
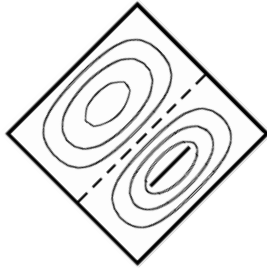
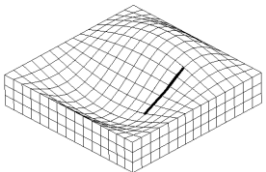
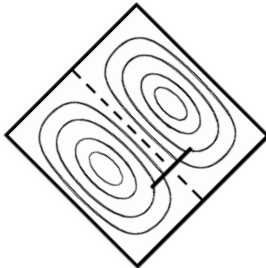
h/b	d/a	Mode				
		1	2	3	4	5
0.2	0.3	 (3.247) 	 (6.119) 	 (6.279) 	 (6.656) 	 (7.303) 

圖 4.4 (續上頁)

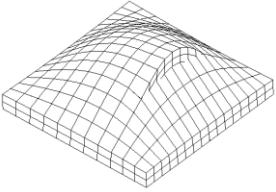
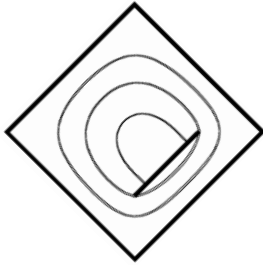
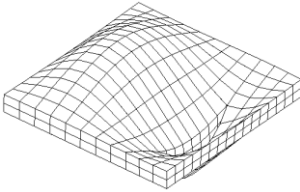

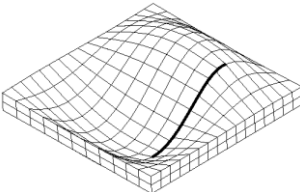

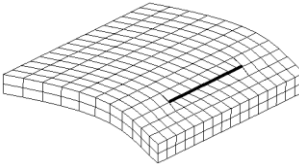
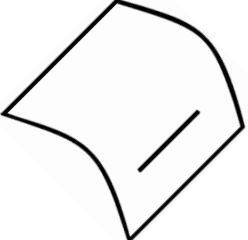
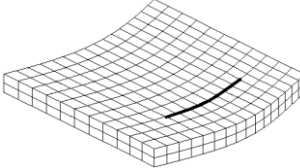
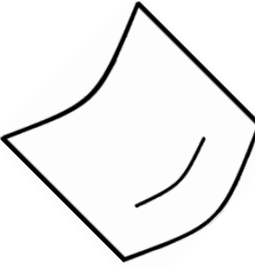
h/b	d/a	Mode				
		1	2	3	4	5
0.1	0.5	 (3.389) 	 (7.291) 	 (8.771) 	 (10.74) 	 (11.63) 

圖 4.4(續上頁)

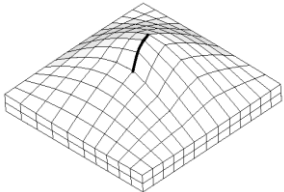
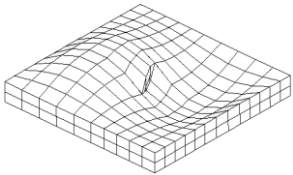
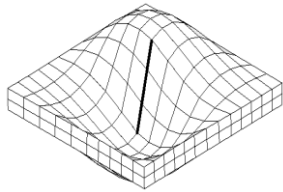
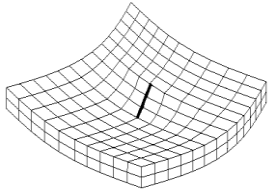
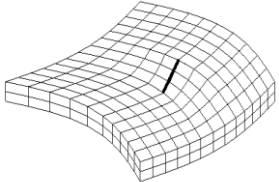
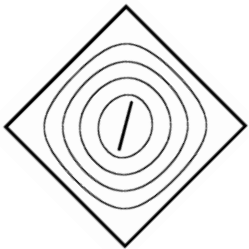


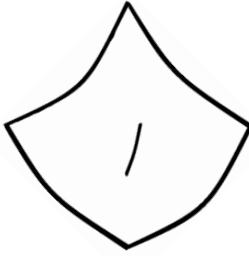

h/b	α	d/a	Mode				
			1	2	3	4	5
0.1	30°	0.3	 (3.531)	 (8.525)	 (8.888)	 (12.62)	 (12.63)
							

圖 4.4 (續上頁) $((x_0/a, y_0/b) = (0.5, 0.5))$

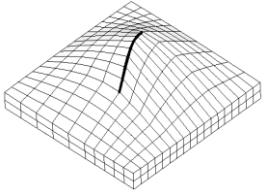
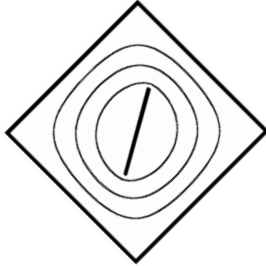
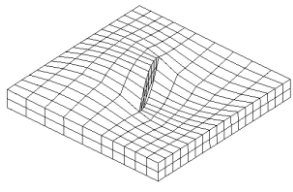
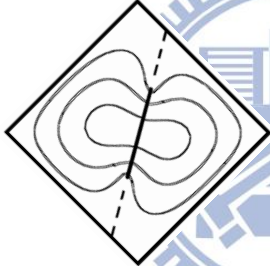
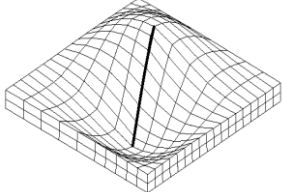

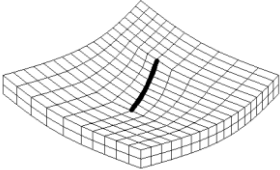
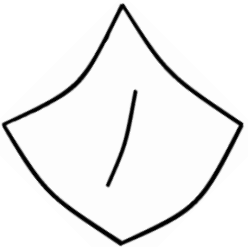
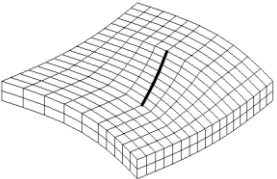
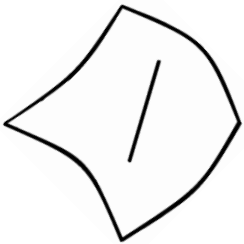
h/b	α	d/a	Mode				
			1	2	3	4	5
0.1	30°	0.5	 (3.272) 	 (7.132) 	 (8.700) 	 (12.48) 	 (12.61) 

圖 4.4 (續上頁)

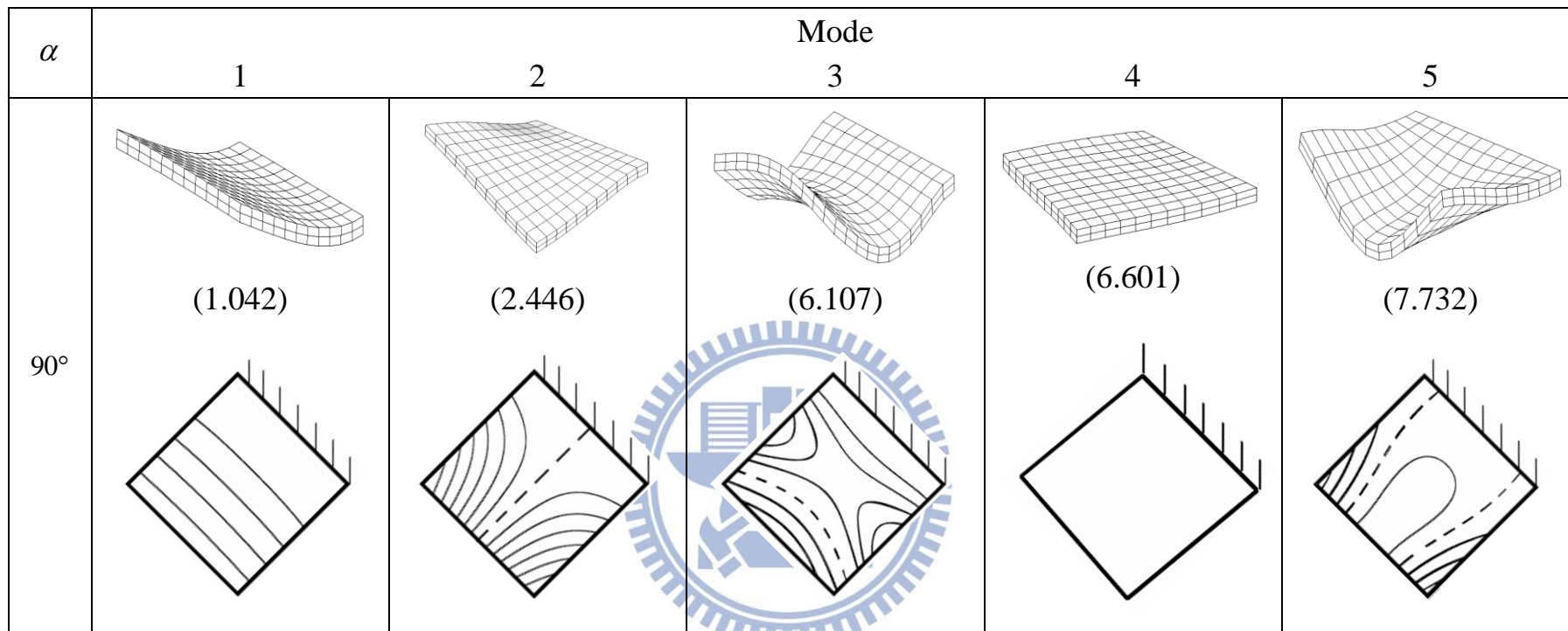


圖 4.5 具不同內部裂縫懸臂之方形均質中厚板模態圖($h/b=0.1$)

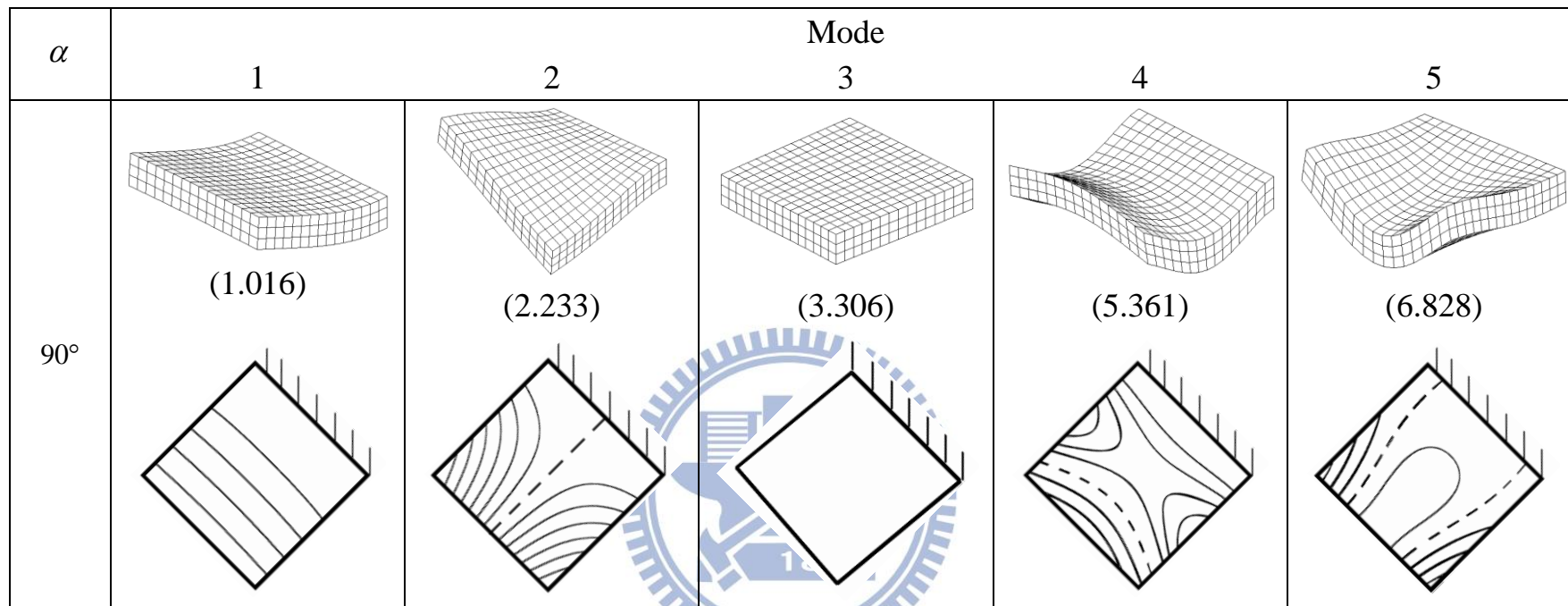


圖 4.5 (續上頁) ($h/b=0.2$)

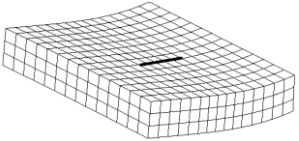
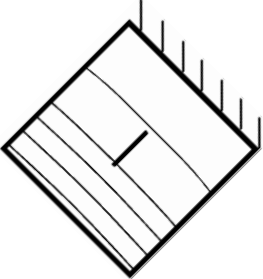
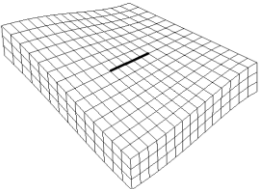
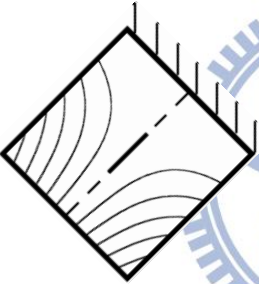
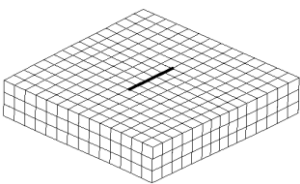
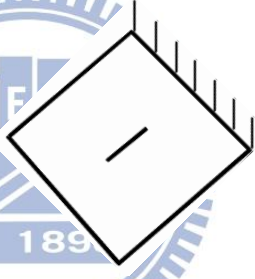
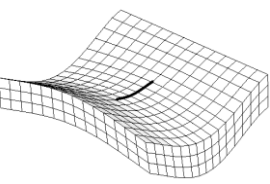
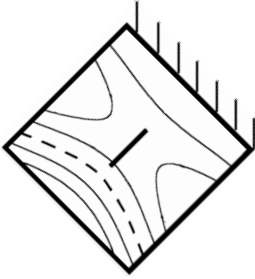
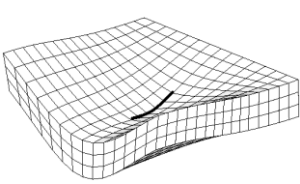
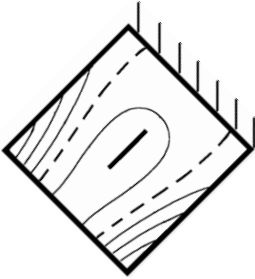
α	d/b	Mode				
		1	2	3	4	5
0°	0.3	 (1.016) 	 (2.195) 	 (3.221) 	 (5.359) 	 (6.285) 

圖 4.5 (續上頁) ($h/b=0.2$, $(x_0/a, y_0/b)=(0.5,0.5)$)

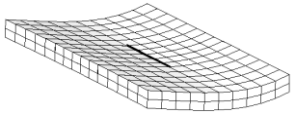
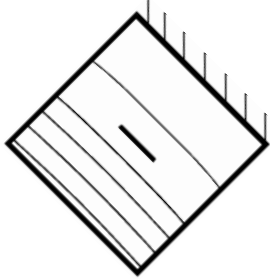
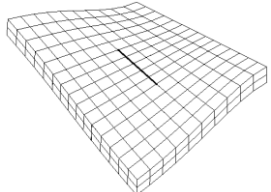
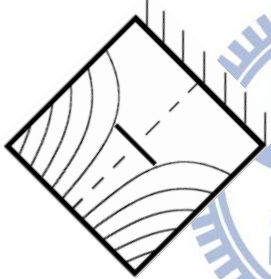
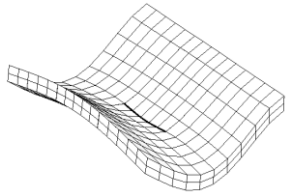
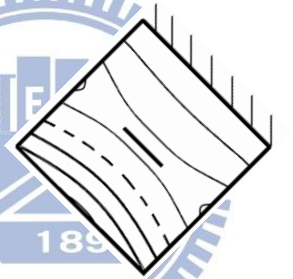
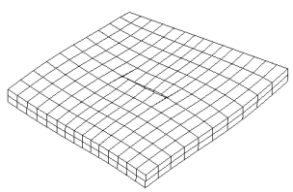
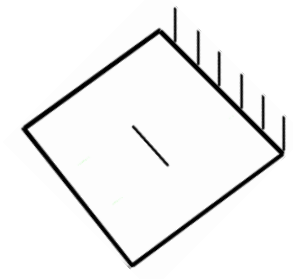
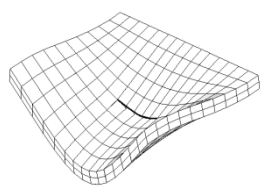
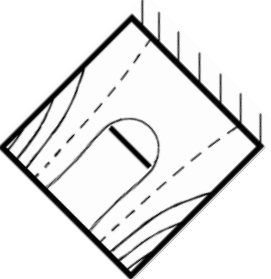
α	d/b	Mode				
		1	2	3	4	5
90°	0.3	 (1.020) 	 (2.420) 	 (5.727) 	 (6.427) 	 (7.567) 

圖 4.5 (續上頁) ($h/b=0.1, (x_0/a, y_0/b)=(0.5,0.5)$)

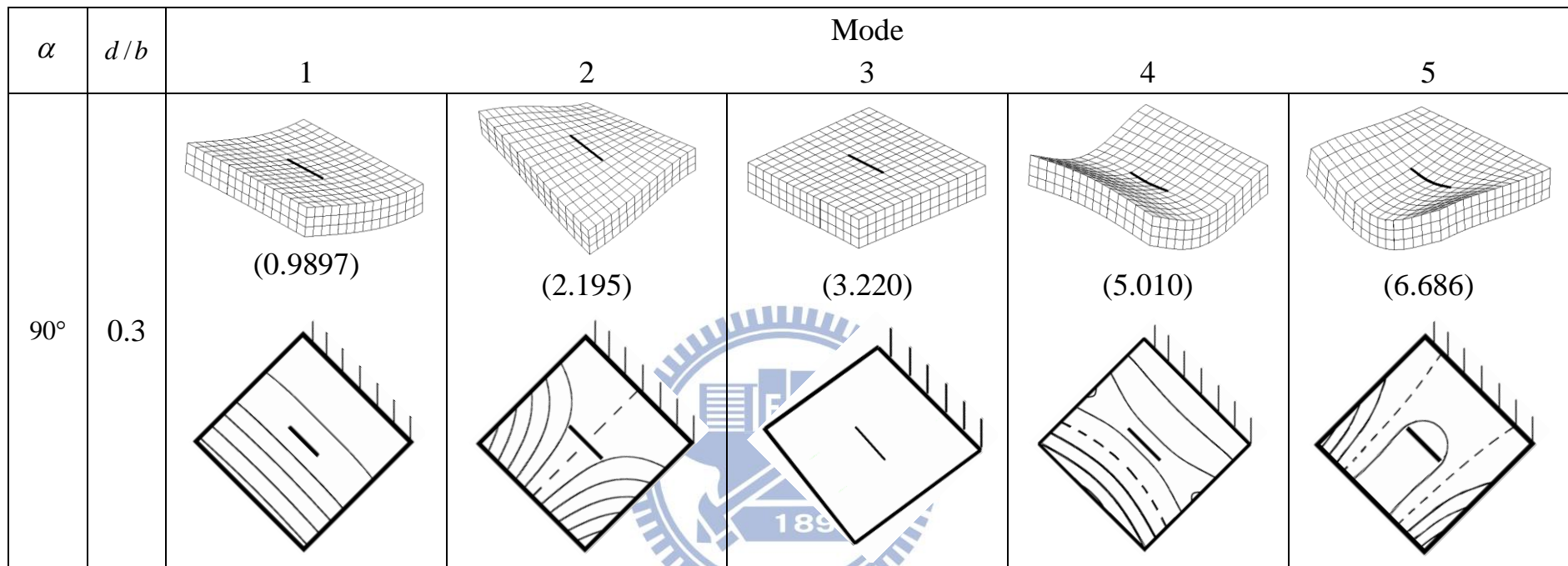


圖 4.5 (續上頁) ($h/b=0.2, (x_0/a, y_0/b)=(0.5,0.5)$)

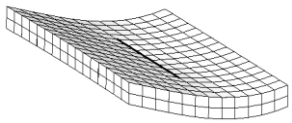
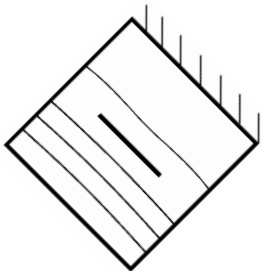
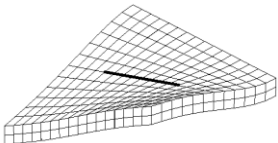
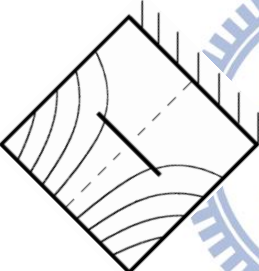
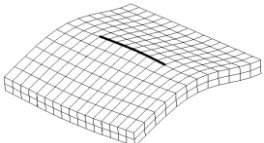
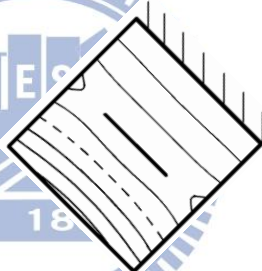
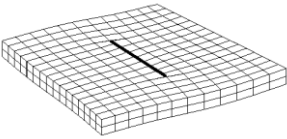
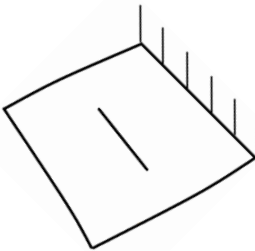
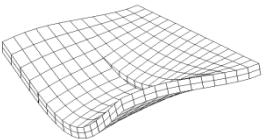
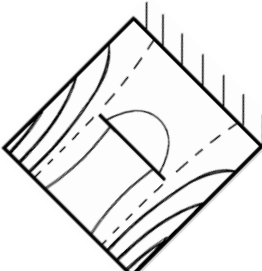
α	d/b	Mode				
		1	2	3	4	5
90°	0.5	 (0.9800) 	 (2.398) 	 (5.185) 	 (6.096) 	 (7.398) 

圖 4.5 (續上頁) ($h/b=0.1$, $(x_0/a, y_0/b)=(0.5,0.5)$)

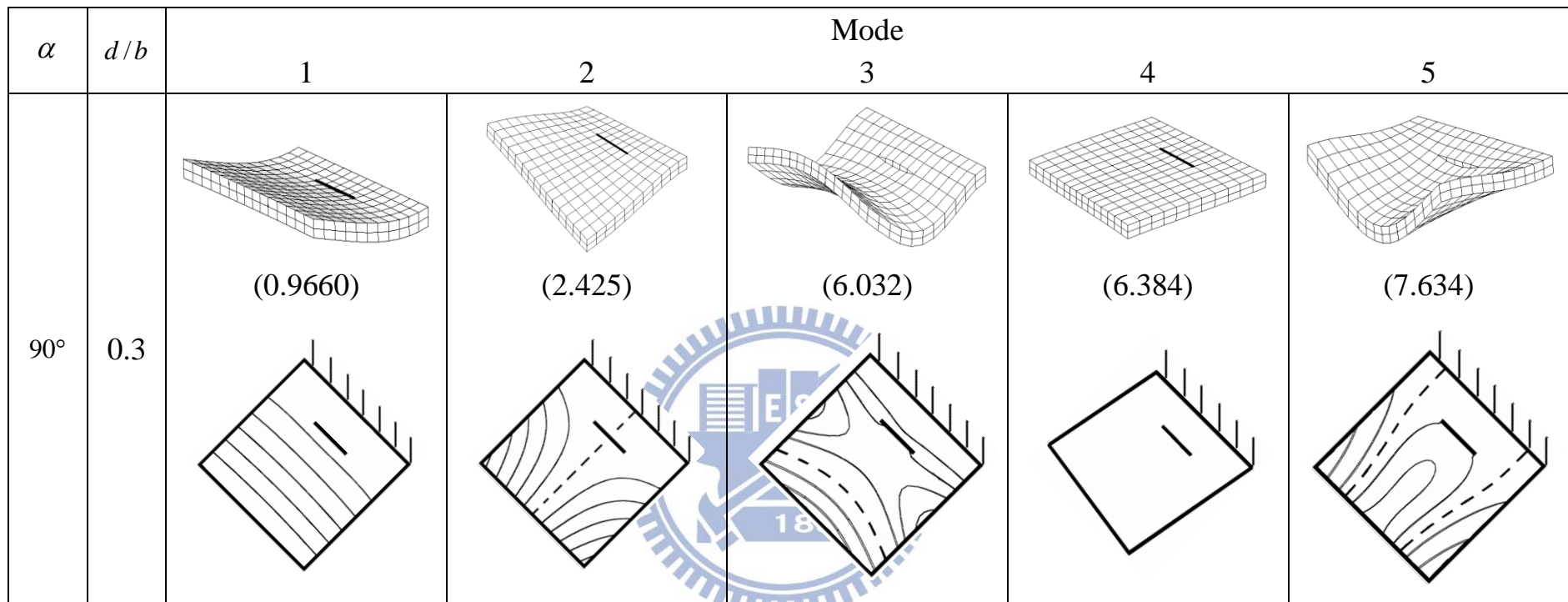


圖 4.5 (續上頁) ($h/b=0.1$, $(x_0/a, y_0/b)=(0.25,0.5)$)

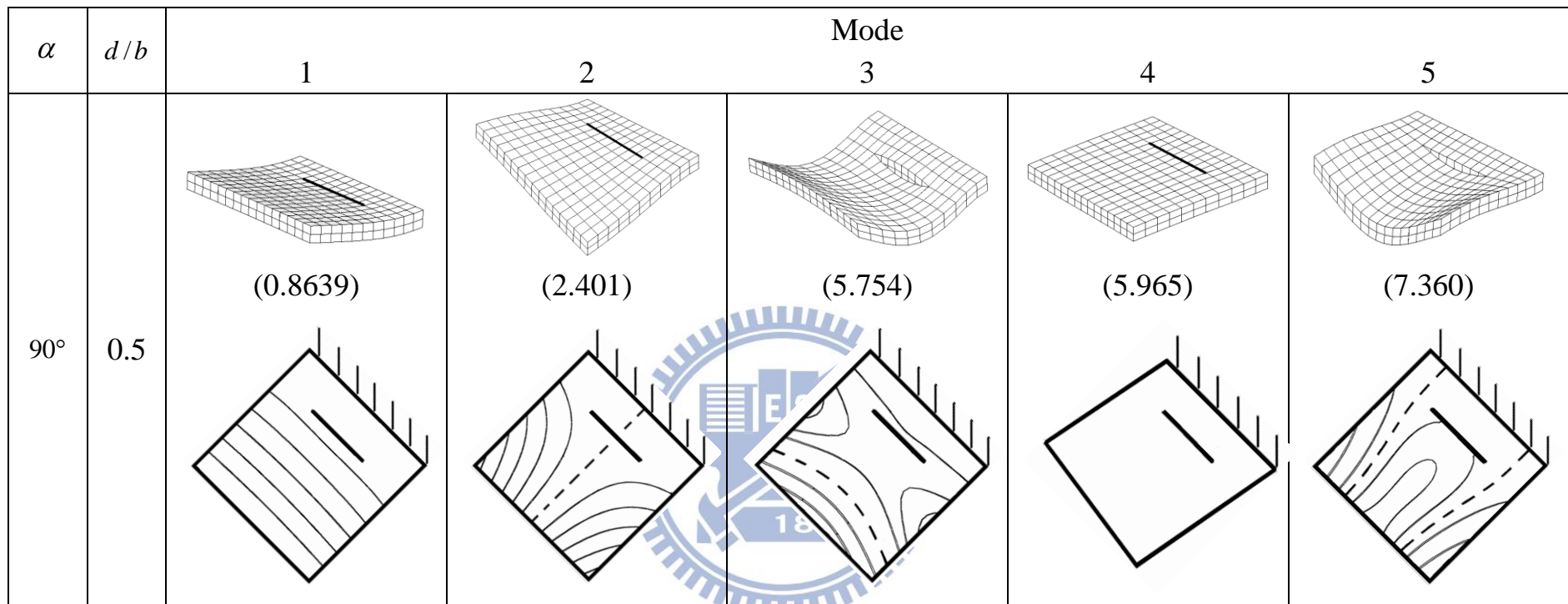


圖 4.5 (續上頁) ($h/b=0.1$, $(x_0/a, y_0/b)=(0.25,0.5)$)

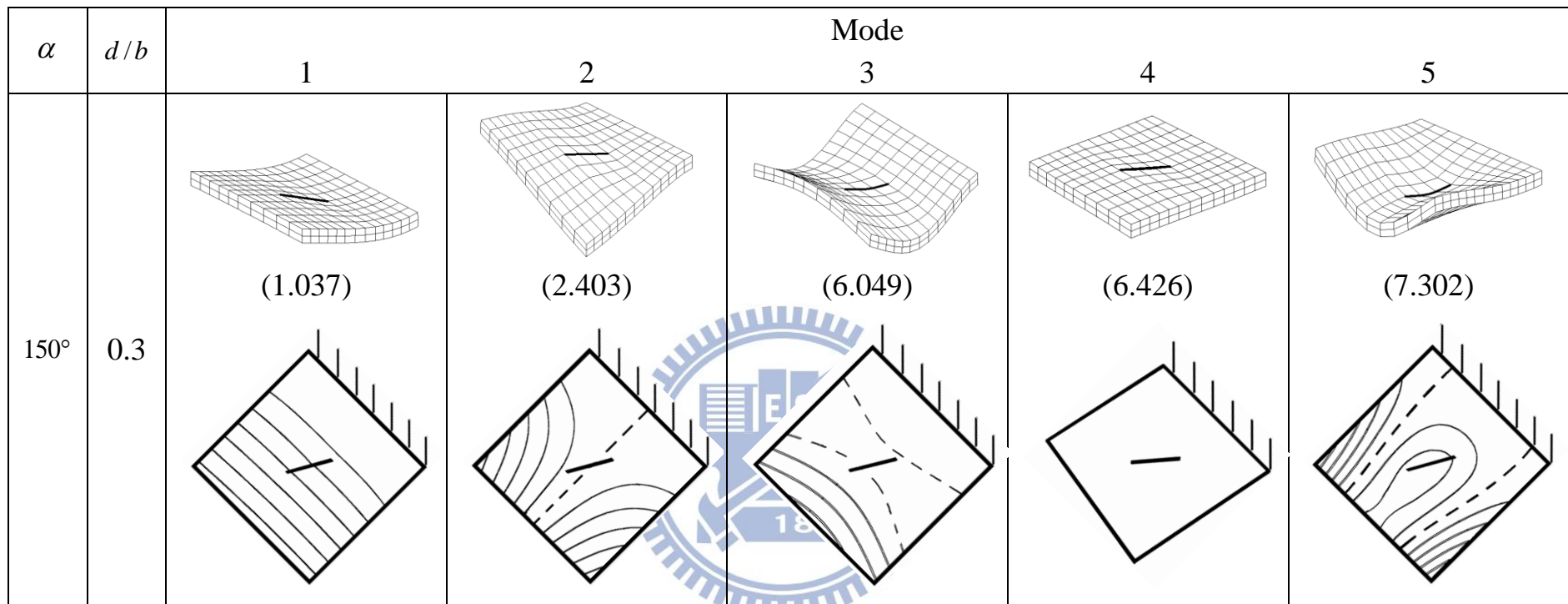


圖 4.5 (續上頁) ($h/b=0.1$, $(x_0/a, y_0/b)=(0.5,0.5)$)

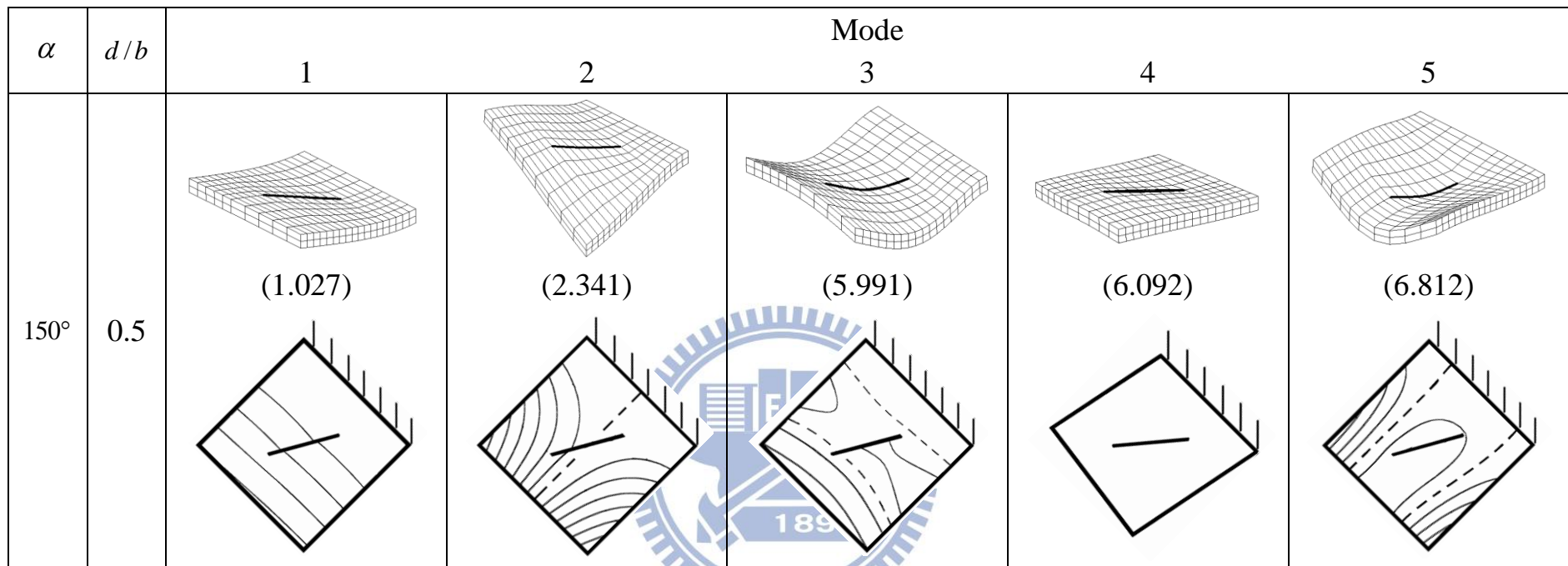


圖 4.5 (續上頁) ($h/b=0.1, (x_0/a, y_0/b)=(0.5,0.5)$)

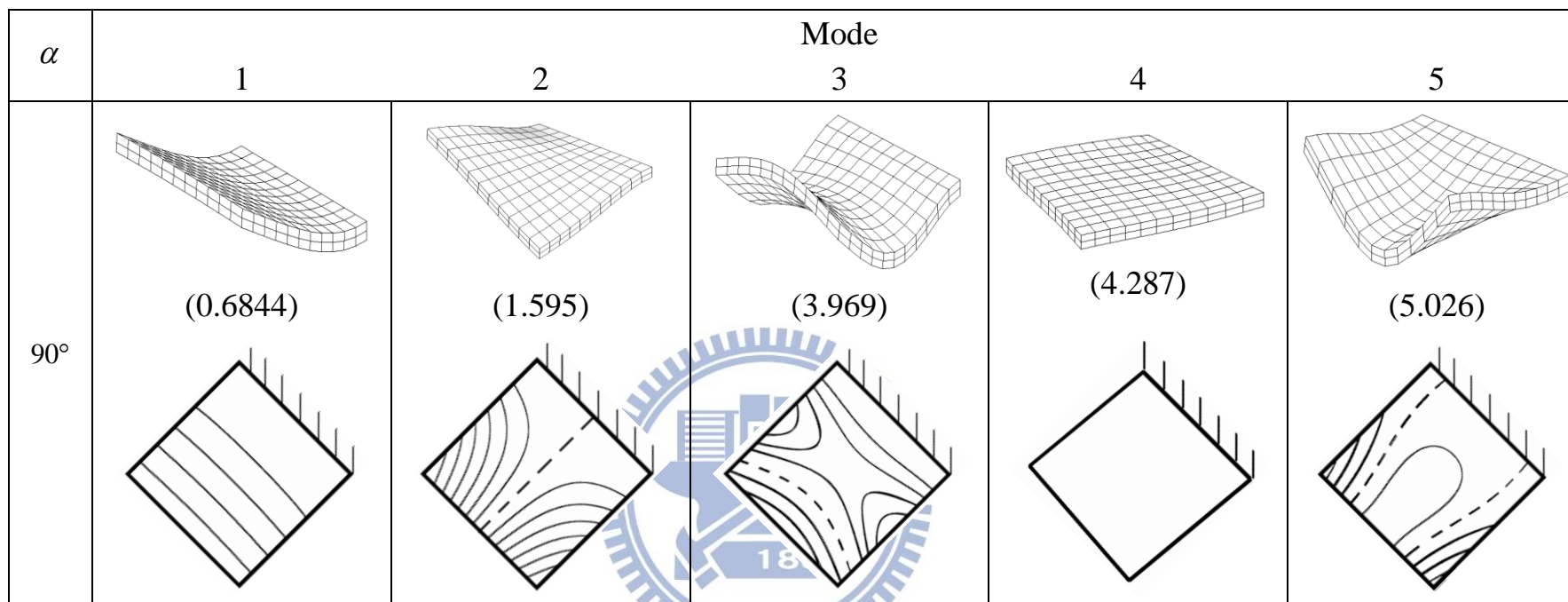


圖 4.6 具不同內部裂縫懸臂之 Al/Al₂O₃ 方形 FGM 中厚板模態圖($\hat{m}=5, h/b=0.1$)

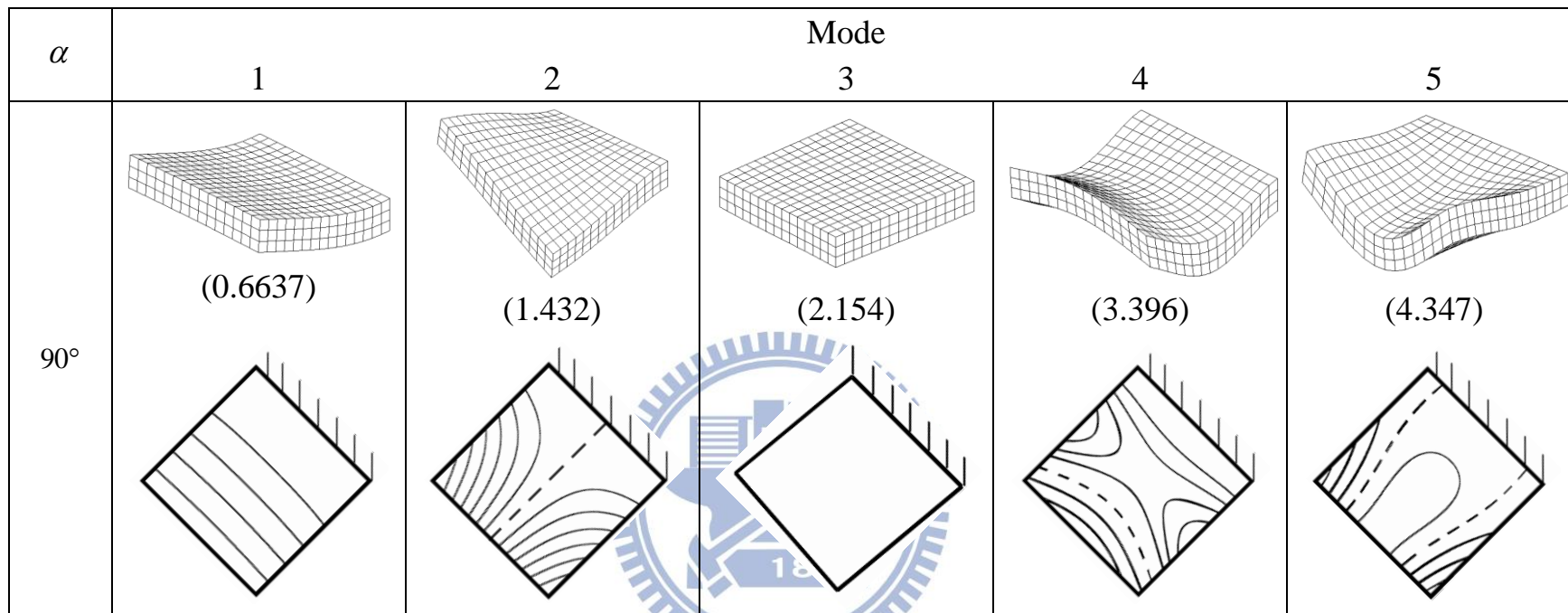


圖 4.6 (續上頁) ($\hat{m}=5, h/b=0.2$)

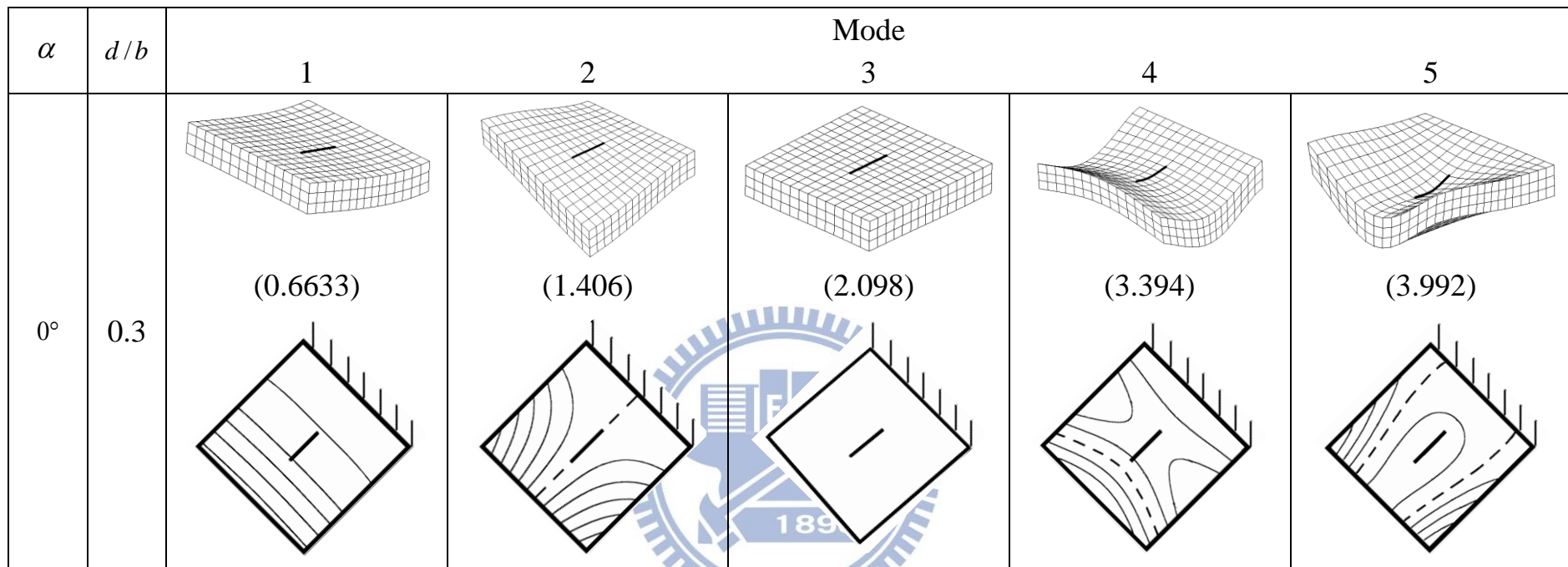


圖 4.6 (續上頁) ($\hat{m}=5, h/b=0.2, (x_0/a, y_0/b)=(0.5,0.5)$)

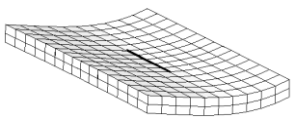
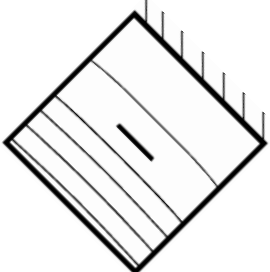
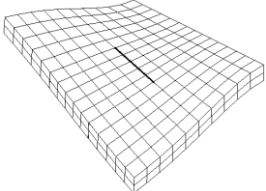
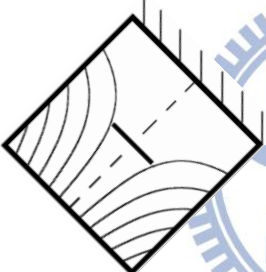
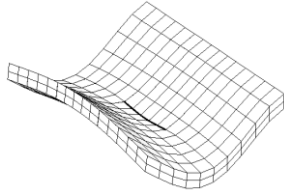
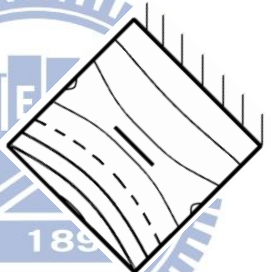
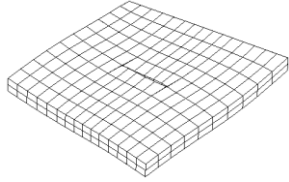
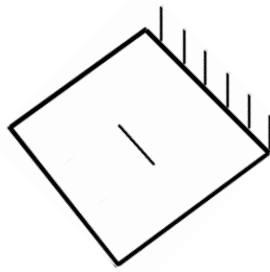
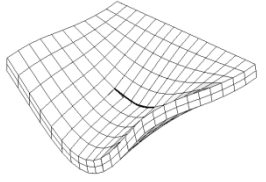
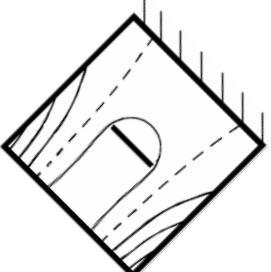
α	d/b	Mode				
		1	2	3	4	5
90°	0.3	 (0.6694) 	 (1.577) 	 (3.719) 	 (4.174) 	 (4.918) 

圖 4.6 (續上頁) ($\hat{m}=5, h/b=0.1, (x_0/a, y_0/b)=(0.5,0.5)$)

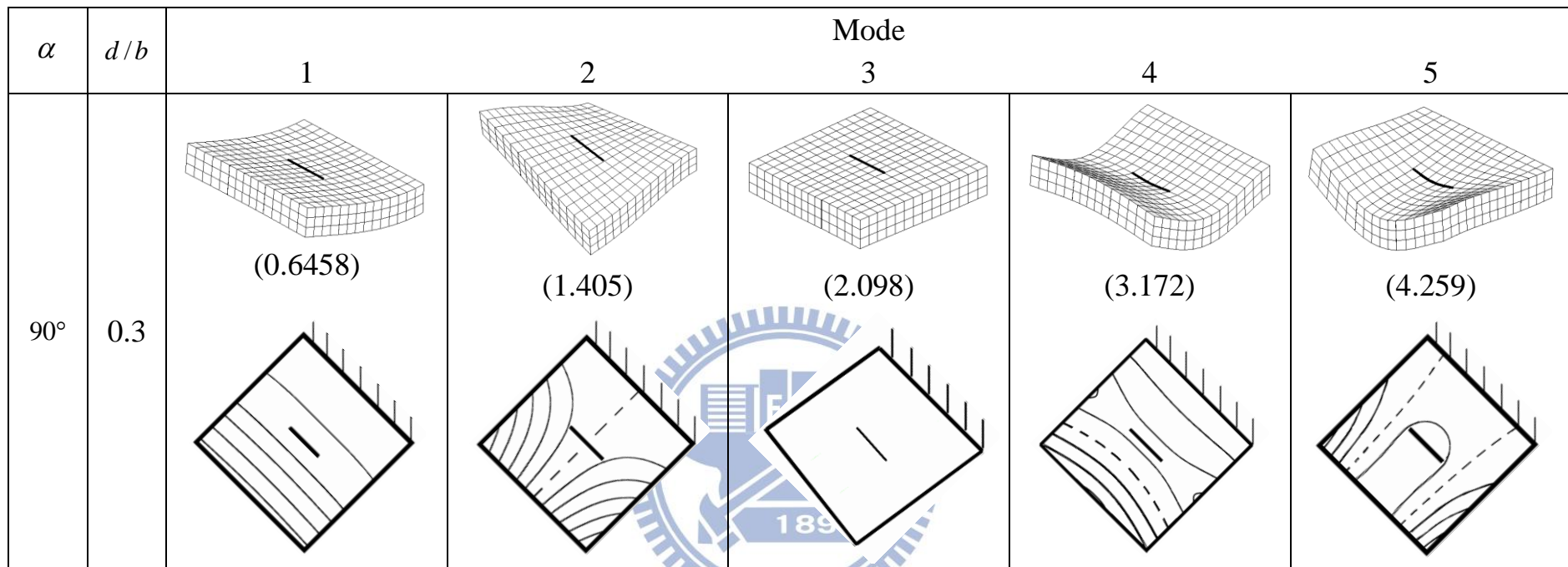


圖 4.6 (續上頁) ($\hat{m}=5, h/b=0.2, (x_0/a, y_0/b)=(0.5,0.5)$)

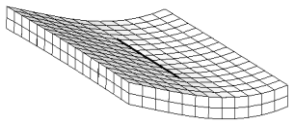
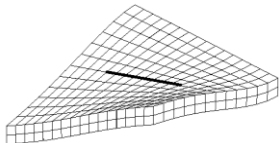
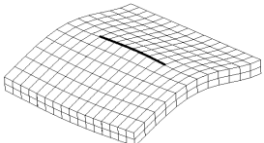
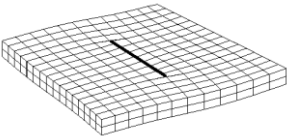
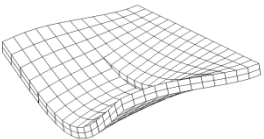
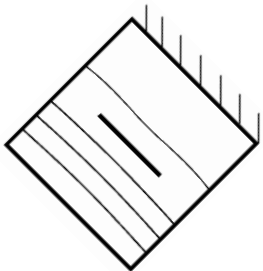
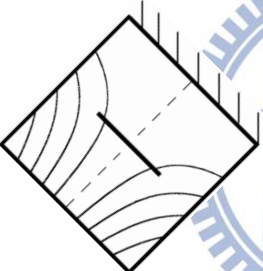
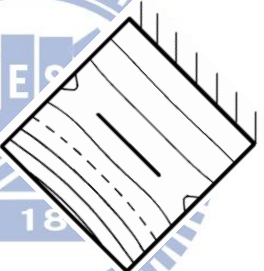
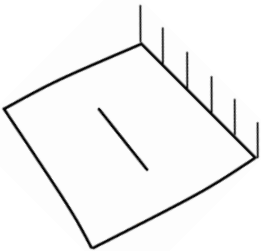
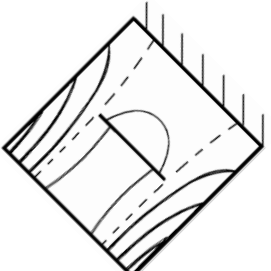
α	d/b	Mode				
		1	2	3	4	5
90°	0.5	 (0.6423)	 (1.561)	 (3.365)	 (3.960)	 (4.808)
						

圖 4.6 (續上頁) ($\hat{m} = 5, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5)$)

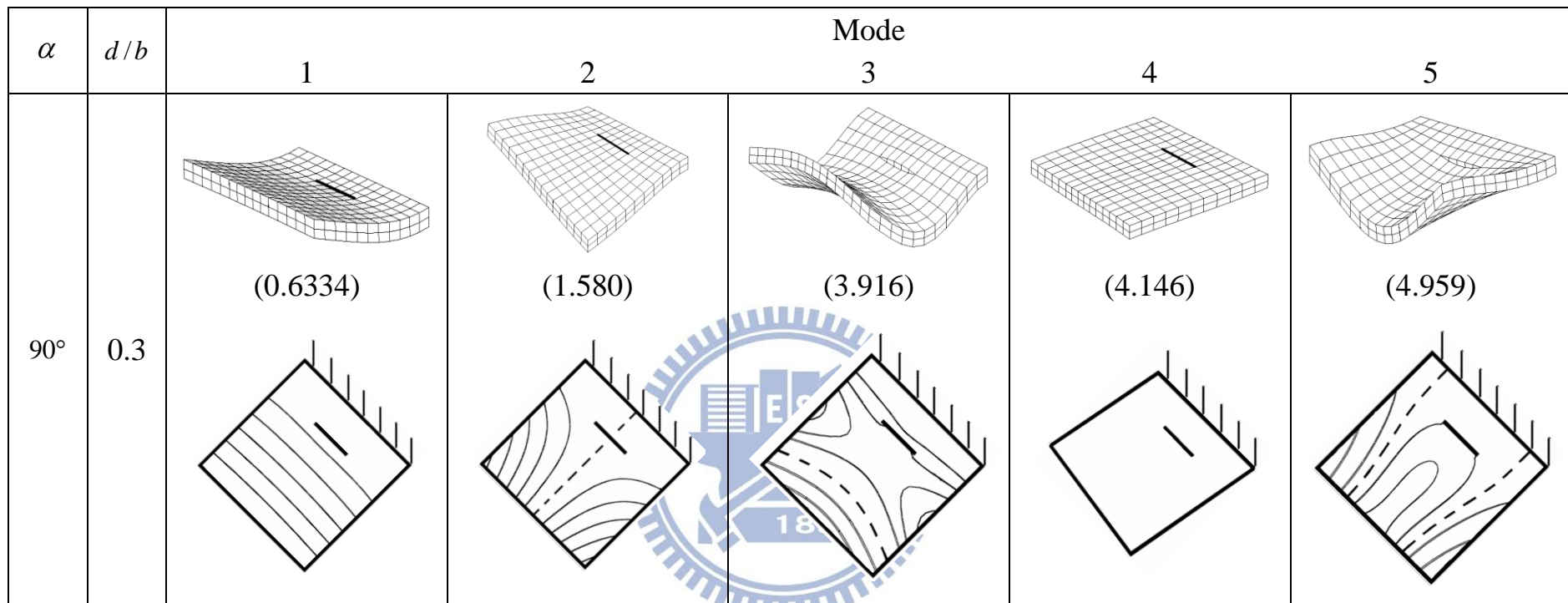


圖 4.6 (續上頁) ($\hat{m} = 5, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.25)$)

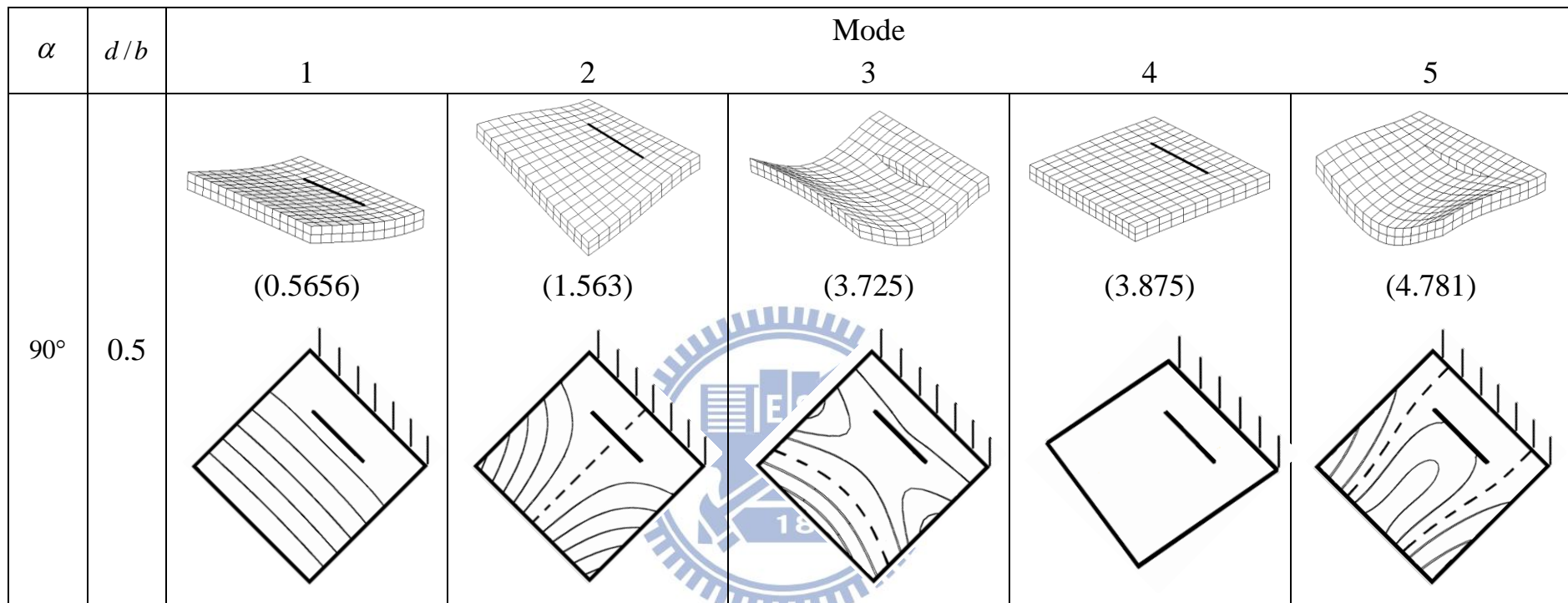


圖 4.6 (續上頁) ($\hat{m} = 5, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.25)$)

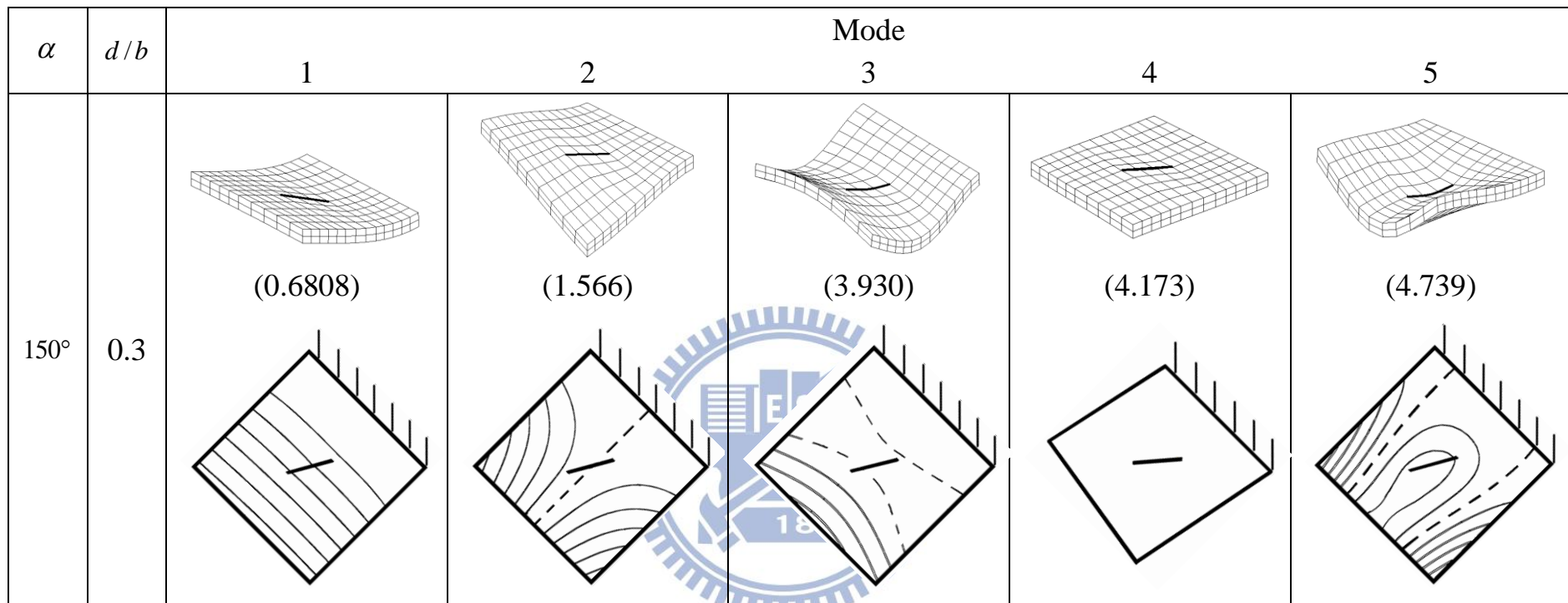


圖 4.6 (續上頁) ($\hat{m} = 5, h/b = 0.1, (x_0/a, y_0/b) = (0.5, 0.5)$)

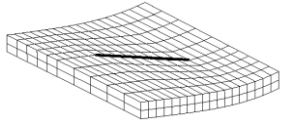
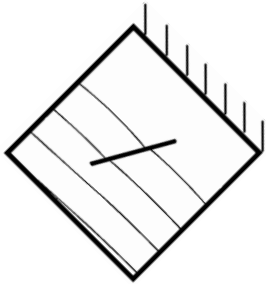
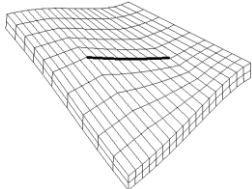
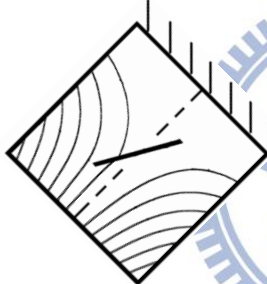
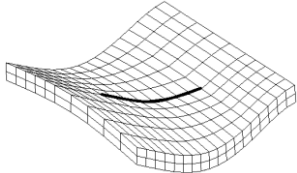

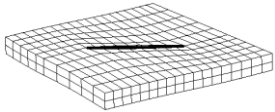
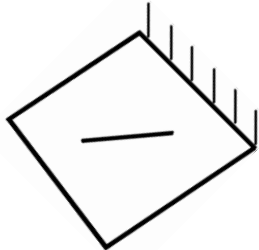
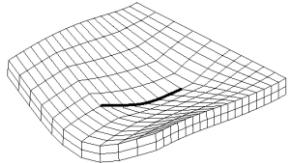
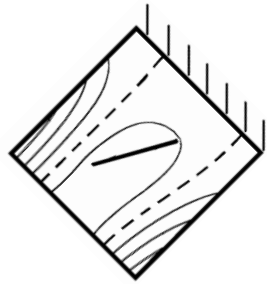
α	d/b	Mode				
		1	2	3	4	5
150°	0.5	 (0.6740) 	 (1.525) 	 (3.890) 	 (3.957) 	 (4.415) 

圖 4.6 (續上頁) ($\hat{m}=5, h/b=0.1, (x_0/a, y_0/b)=(0.5,0.5)$)

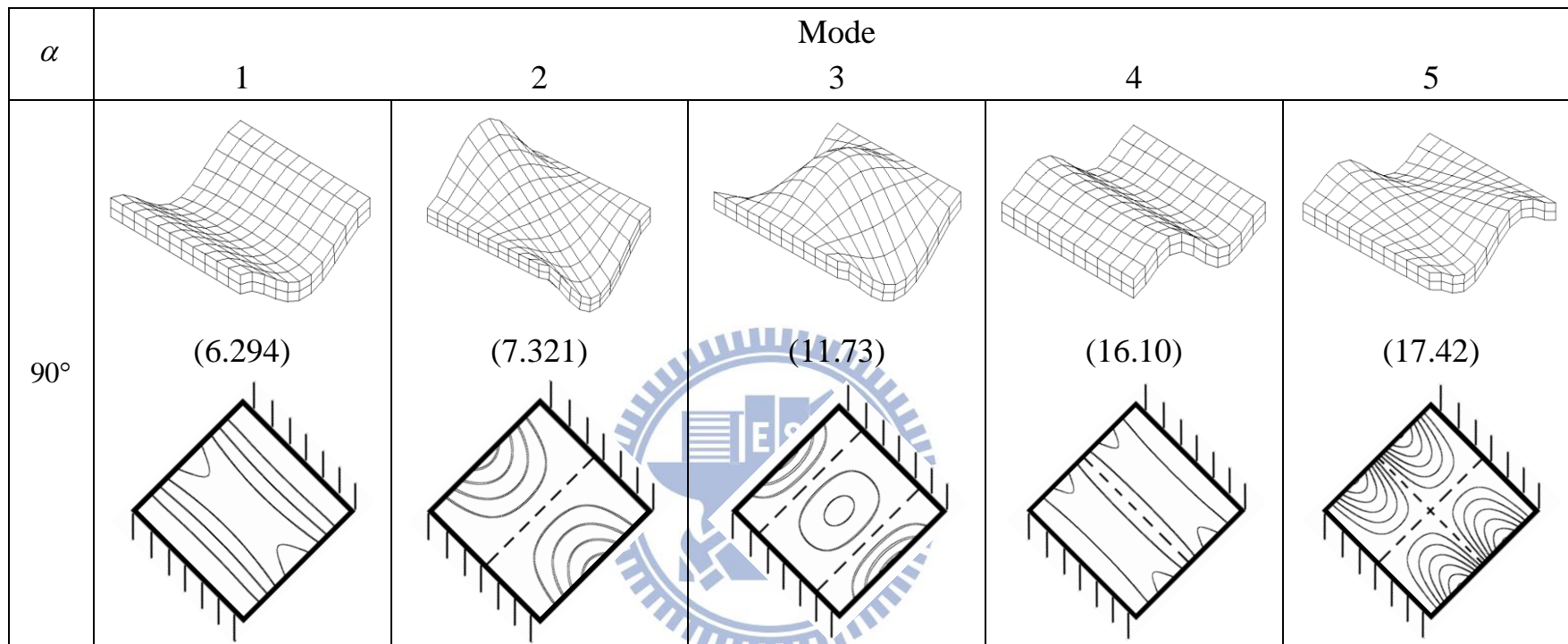


圖 4.7 具不同內部裂縫兩端固定端，兩端自由端之方形均質中厚板模態圖($h/b=0.1$)

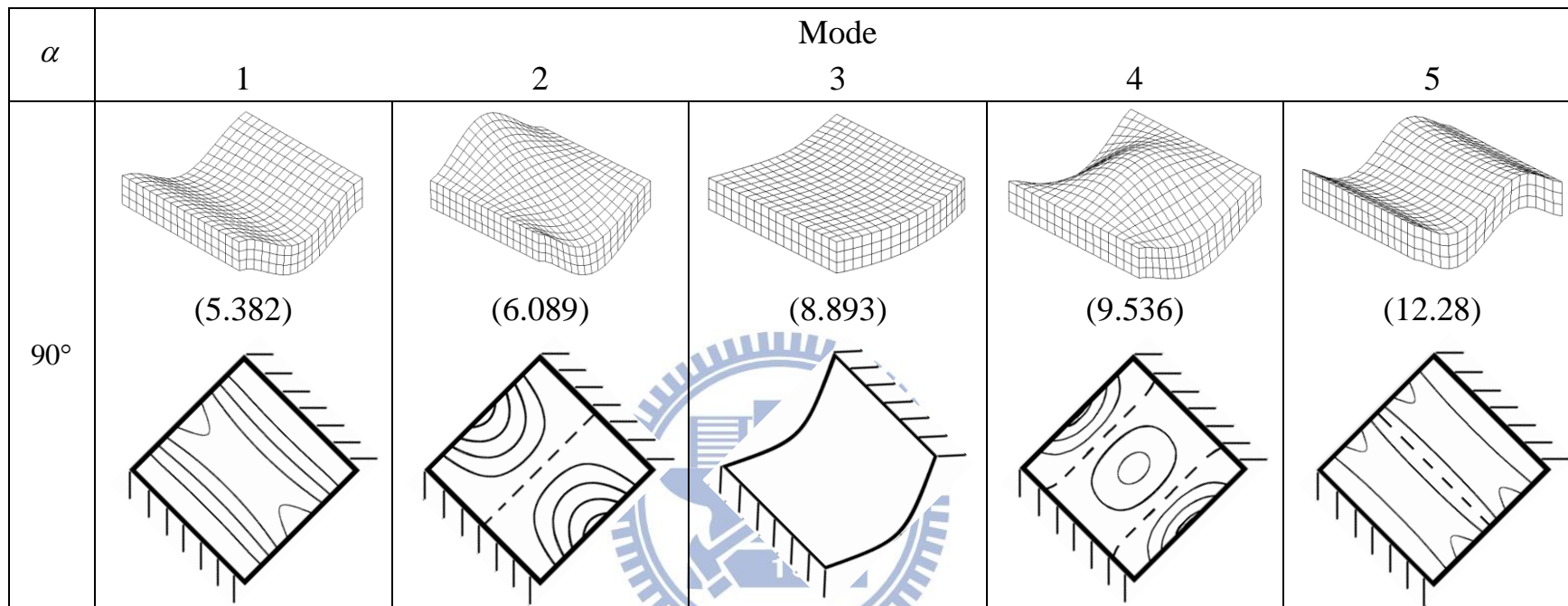


圖 4.7 (續上頁) ($h/b=0.2$)

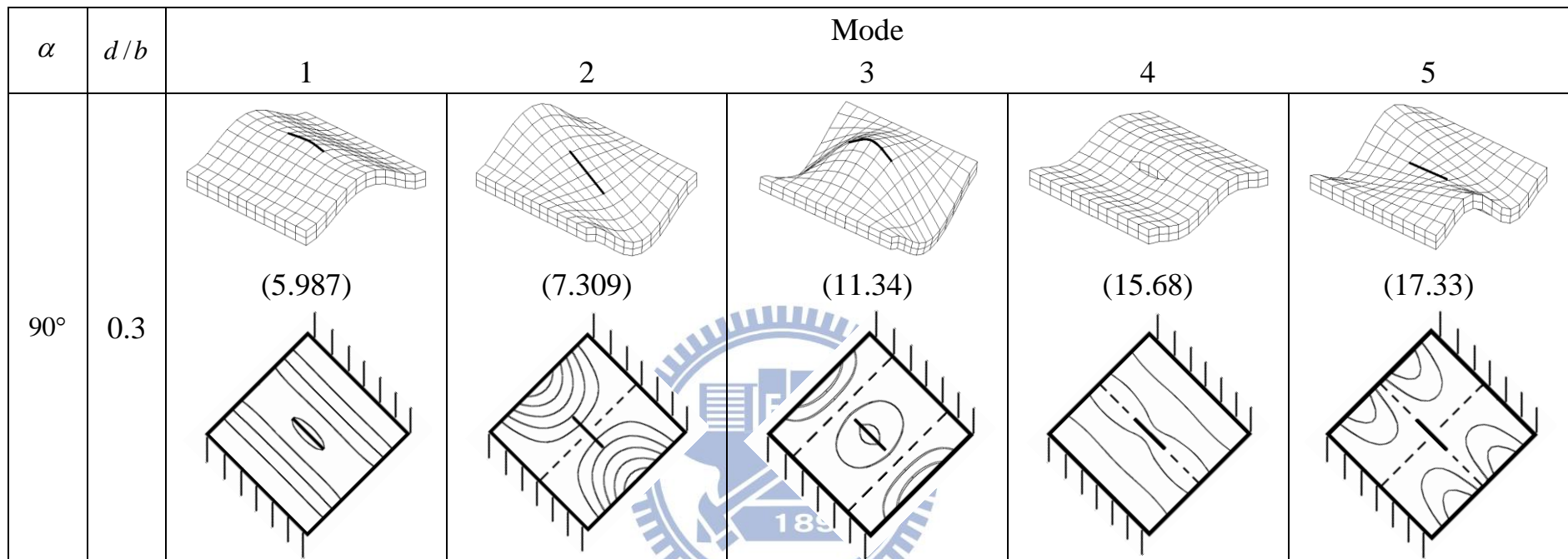


圖 4.7 (續上頁) ($h/b=0.1, (x_0/a, y_0/b)=(0.5,0.5)$)

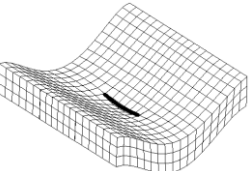
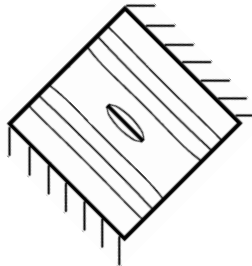
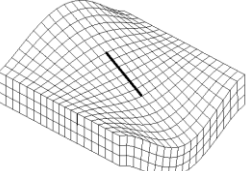

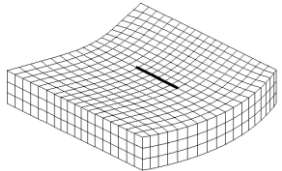

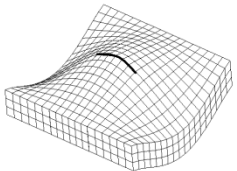
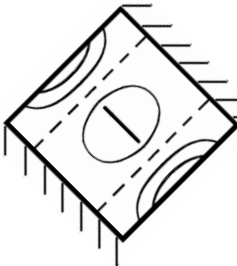
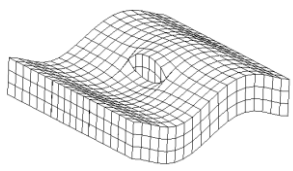
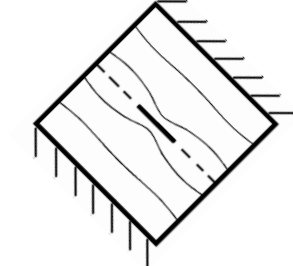
α	d/b	Mode				
		1	2	3	4	5
90°	0.3	 (5.148) 	 (6.080) 	 (8.891) 	 (9.263) 	 (11.84) 

圖 4.7 (續上頁) ($h/b=0.2$, $(x_0/a, y_0/b)=(0.5,0.5)$)

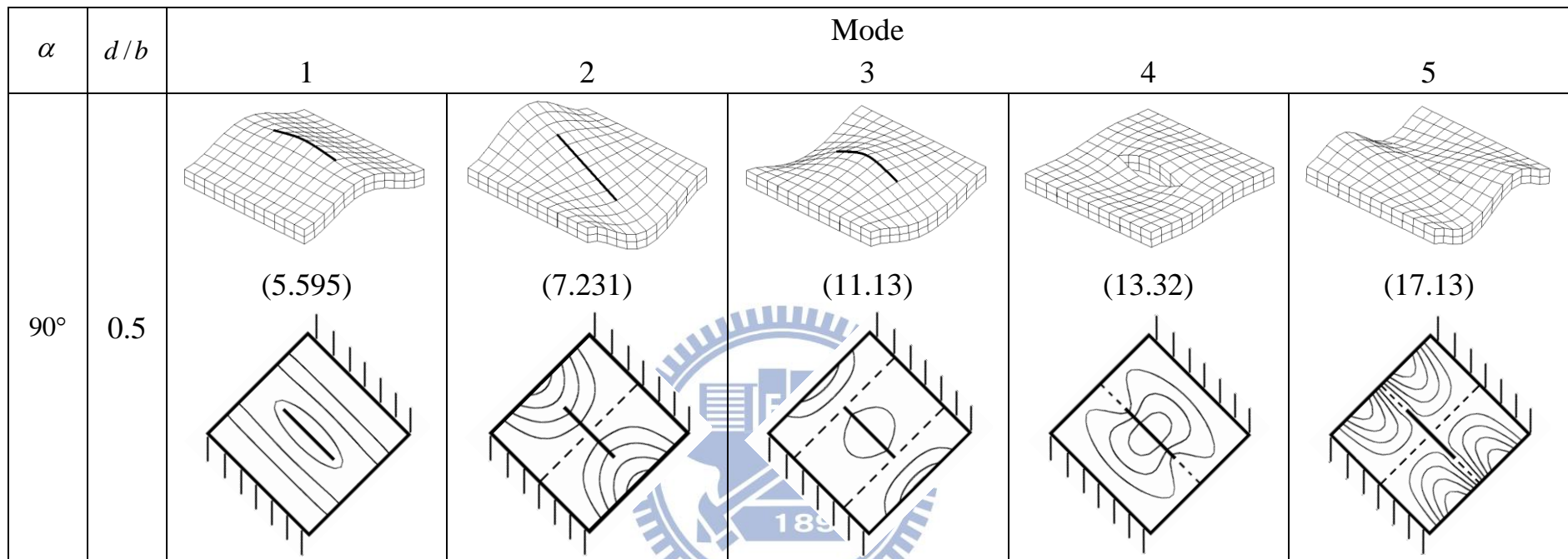


圖 4.7 (續上頁) ($h/b=0.1, (x_0/a, y_0/b)=(0.5,0.5)$)

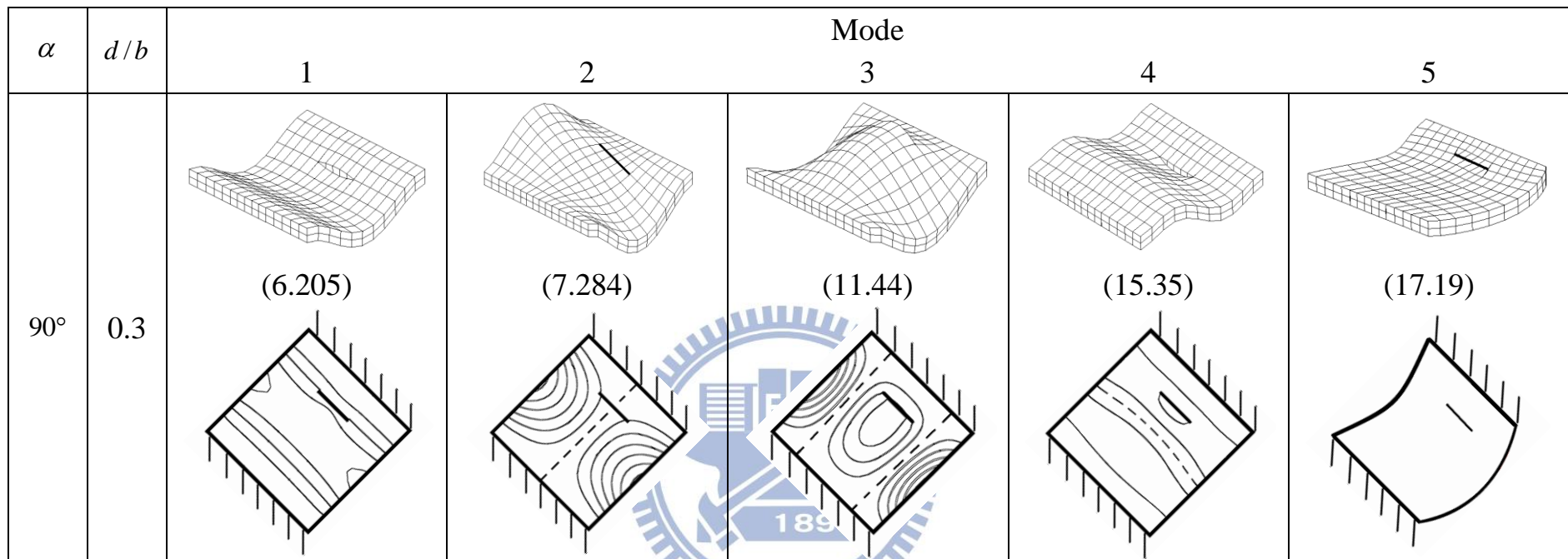


圖 4.7 (續上頁) ($h/b=0.1$, $(x_0/a, y_0/b)=(0.25,0.5)$)

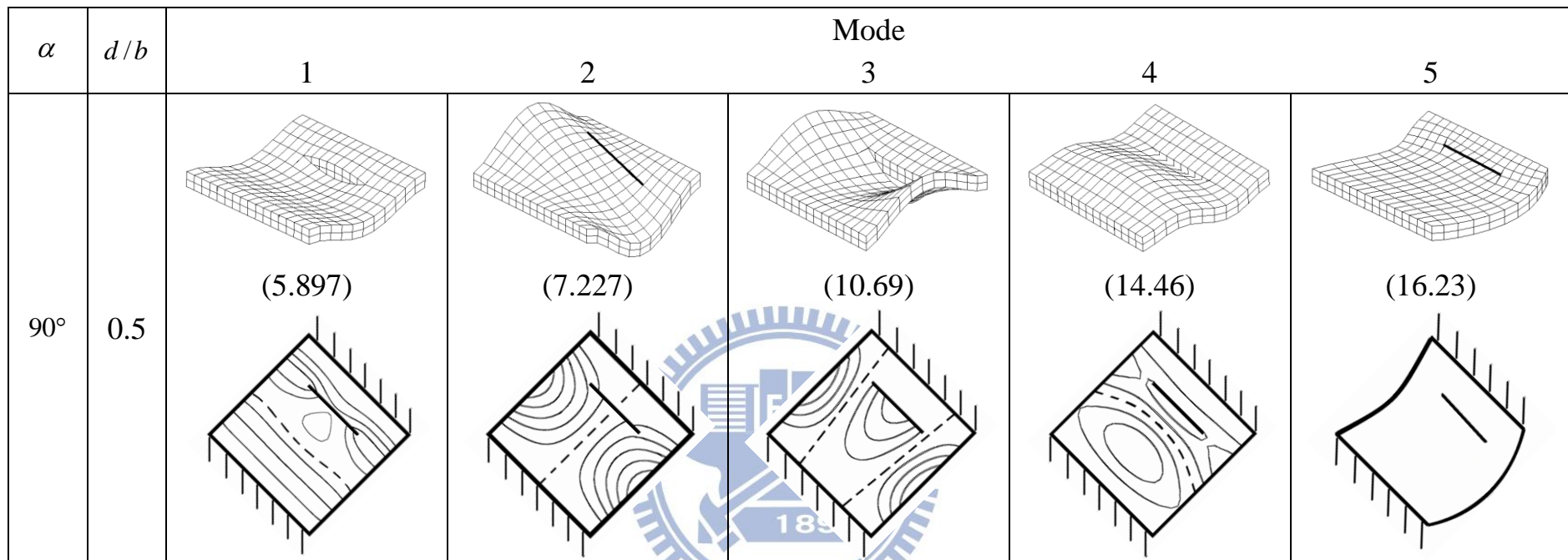


圖 4.7 (續上頁) ($h/b=0.1$, $(x_0/a, y_0/b)=(0.25,0.5)$)

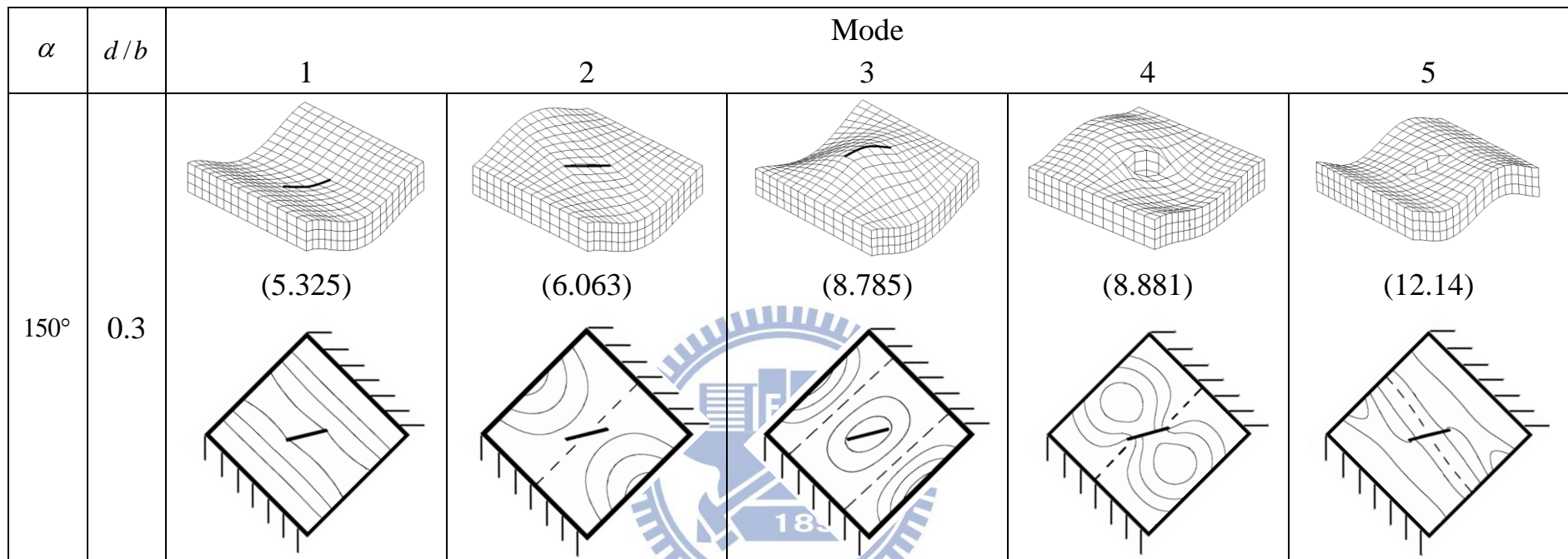


圖 4.7 (續上頁) ($h/b=0.2$, $(x_0/a, y_0/b)=(0.25,0.5)$)

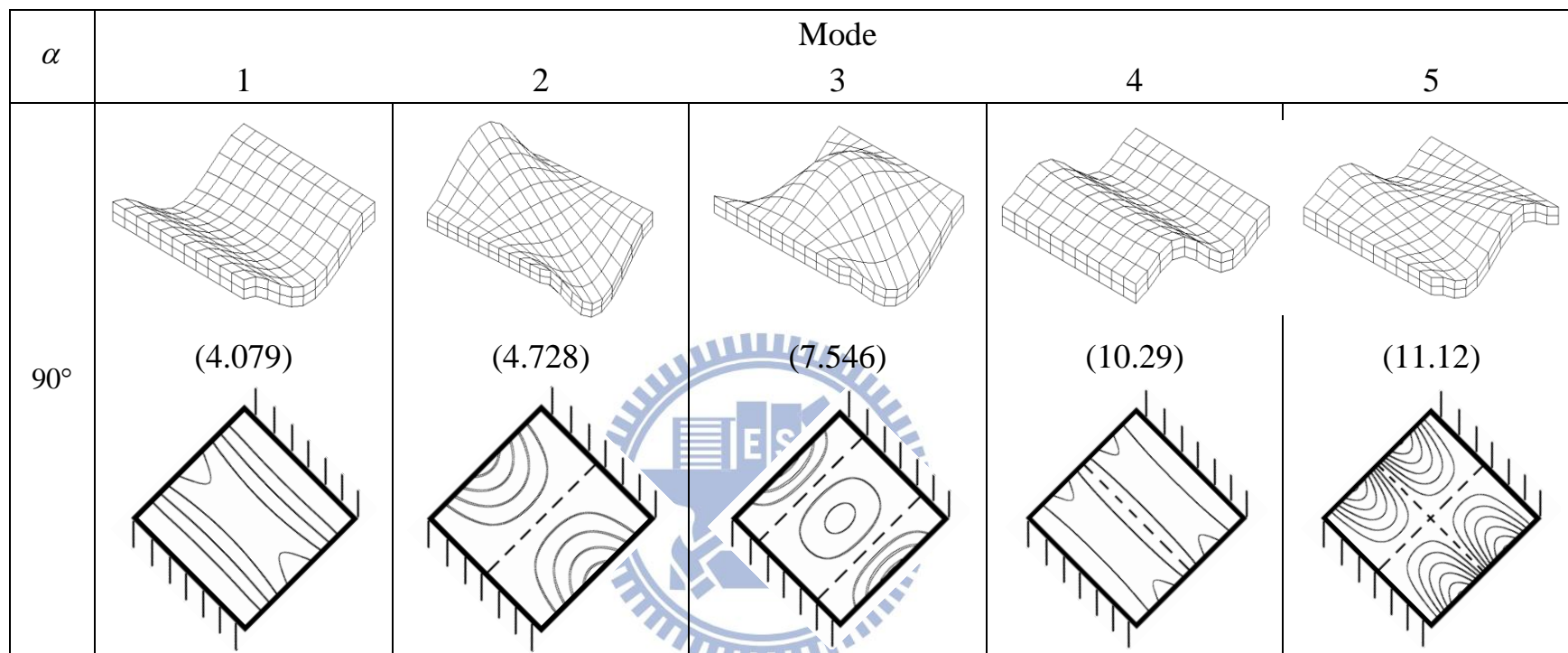


圖 4.8 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 方形 FGM 中厚板模態圖($\hat{m}=5, h/b=0.1$.)

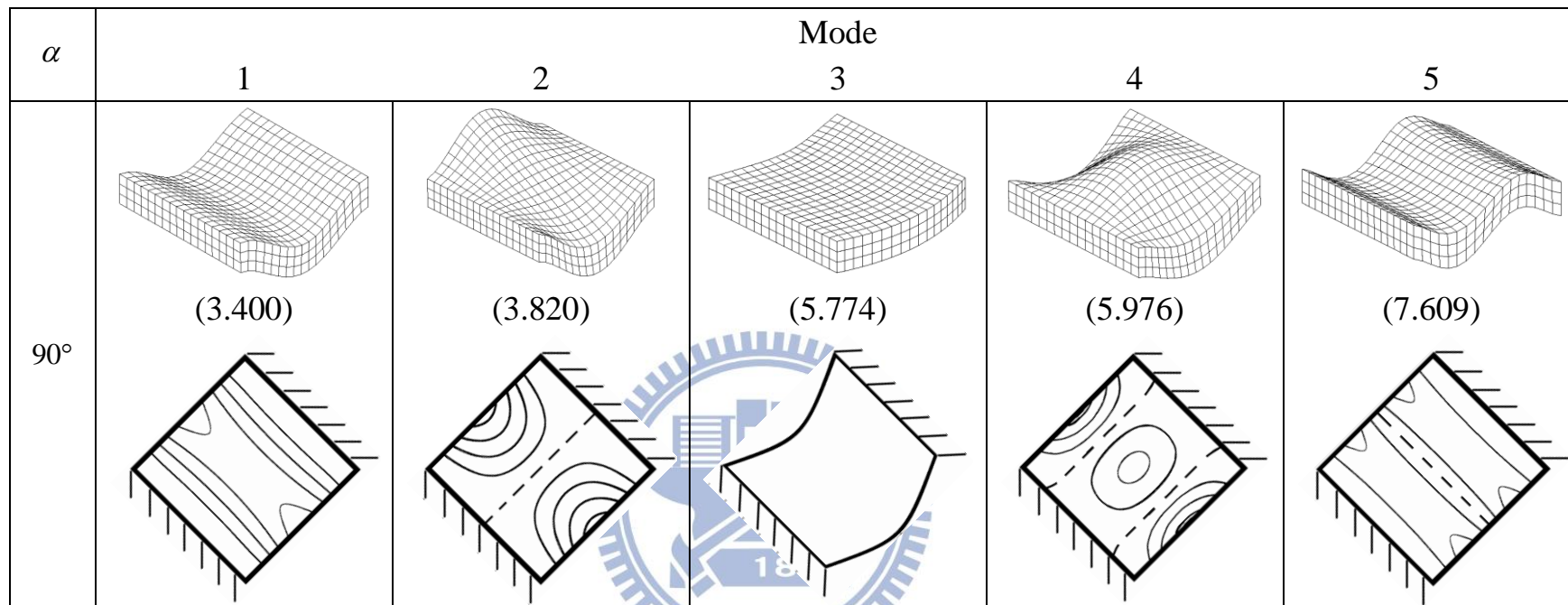


圖 4.8 (續上頁) ($\hat{m}=5, h/b=0.2$)

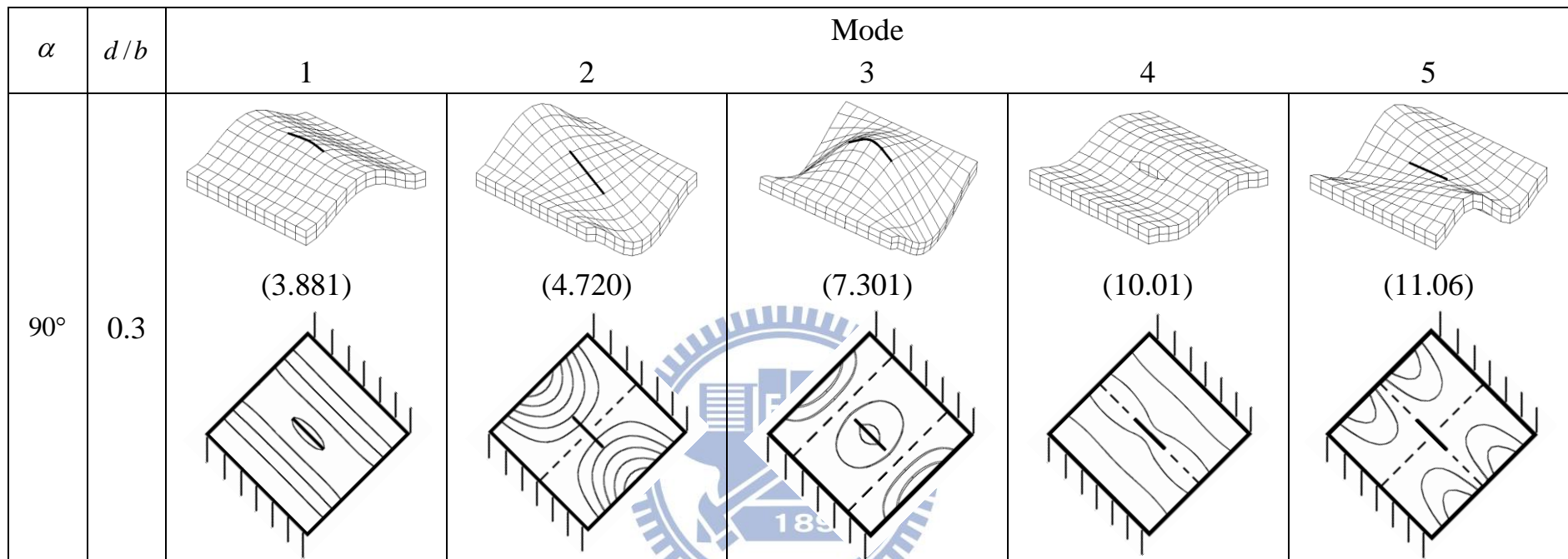


圖 4.8 (續上頁)($h/b=0.1, (x_0/a, y_0/b)=(0.5,0.5)$)

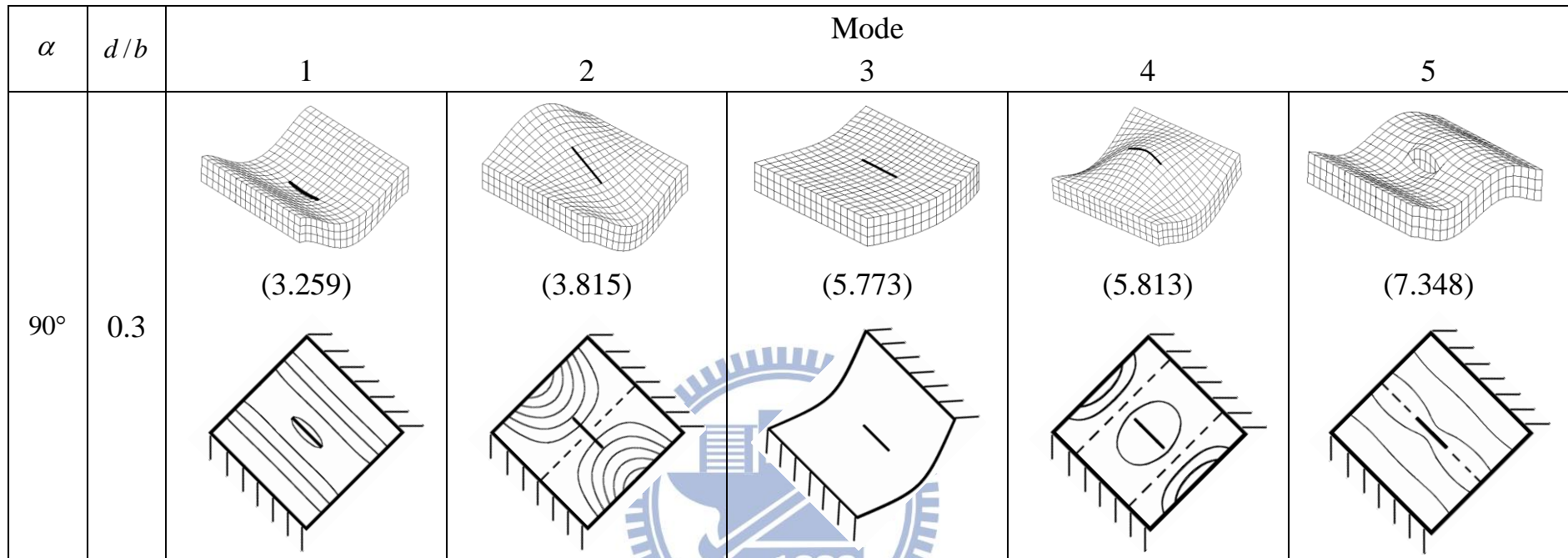


圖 4.8 (續上頁)($\hat{m} = 5, h/b = 0.2, (x_0/a, y_0/b) = (0.5, 0.5)$)

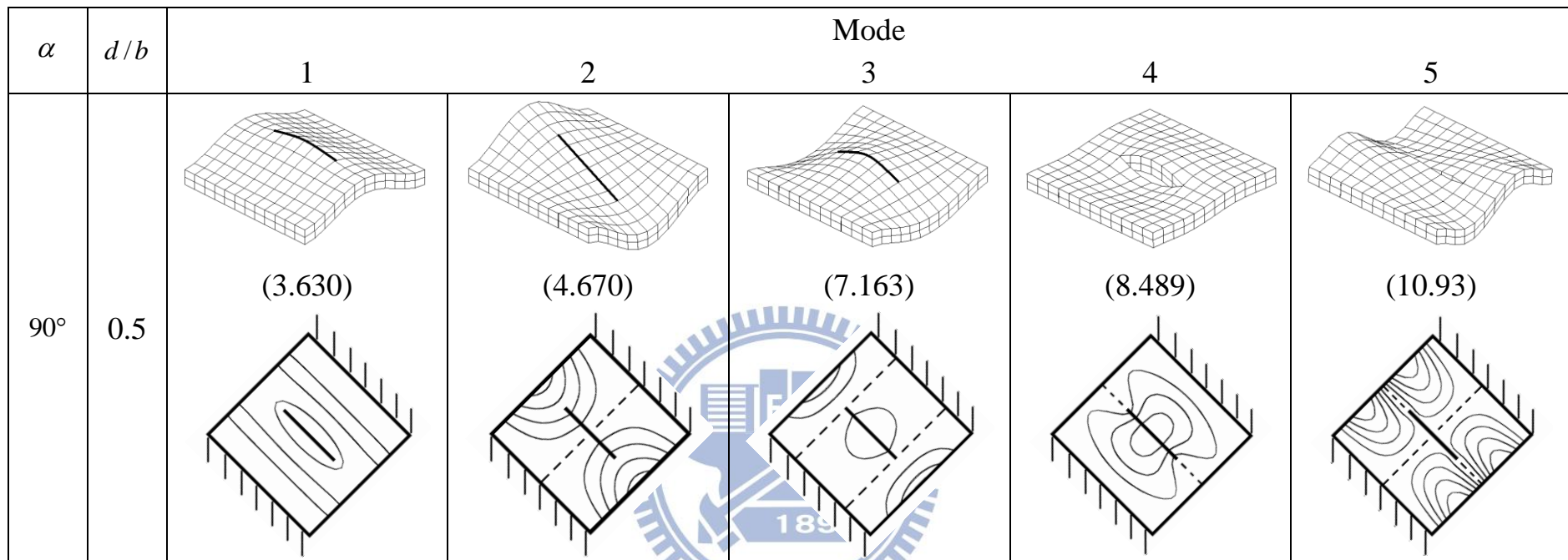


圖 4.8 (續上頁)

$$(\hat{m} = 5, h/b=0.1, (x_0/a, y_0/b) = (0.5, 0.5))$$

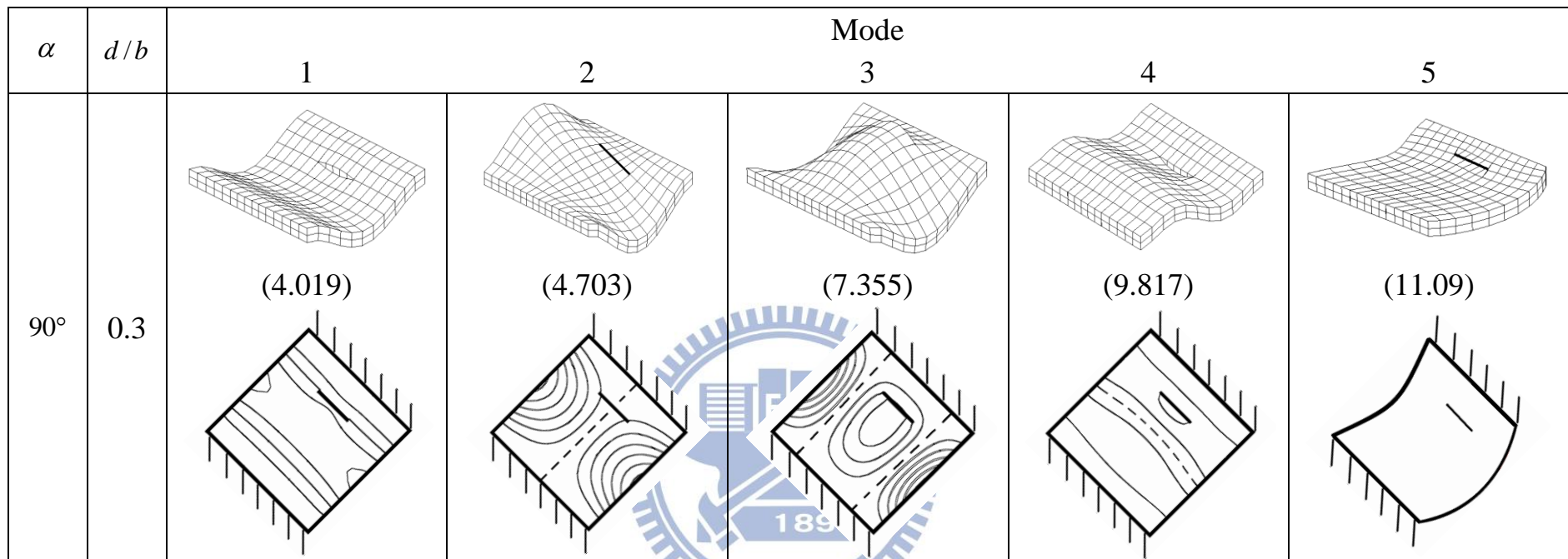


圖 4.8 (續上頁) ($\hat{m} = 5, h/b = 0.1, (x_0/a, y_0/b) = (0.25, 0.5)$)

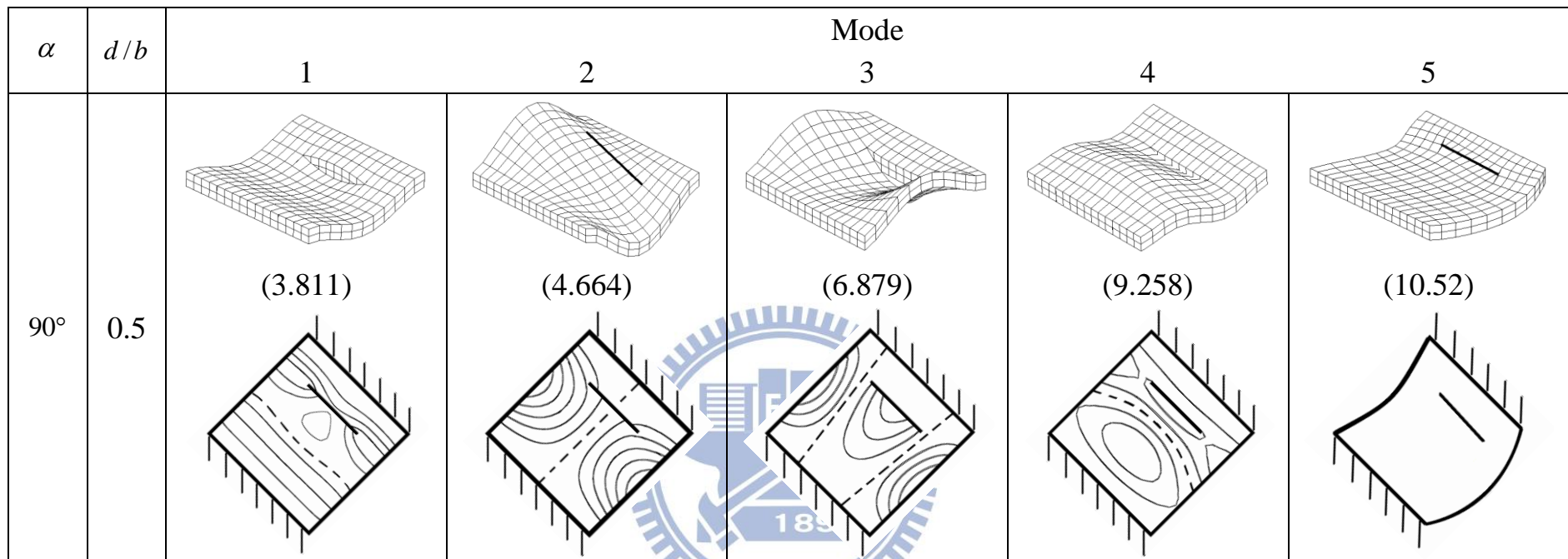


圖 4.8 (續上頁) ($\hat{m} = 5, h/b = 0.1, (x_0/a, y_0/b) = (0.25, 0.5)$)

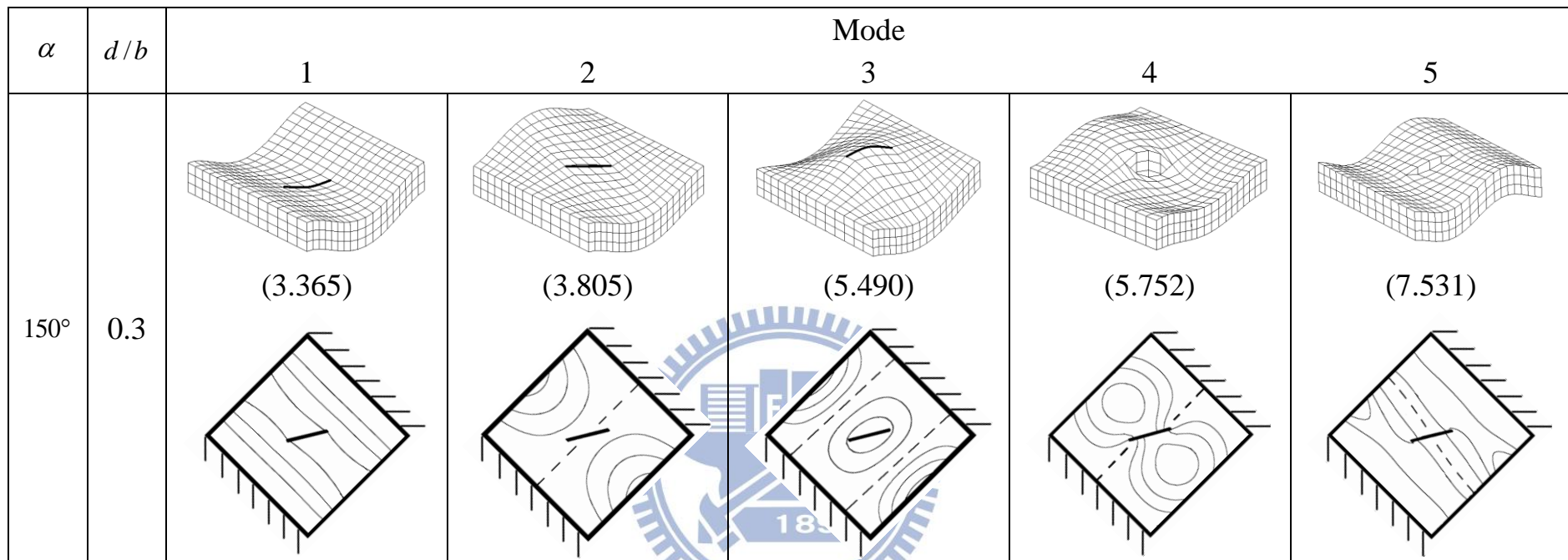


圖 4.8 (續上頁) ($\hat{m} = 5, h/b = 0.2, (x_0/a, y_0/b) = (0.25, 0.5)$)

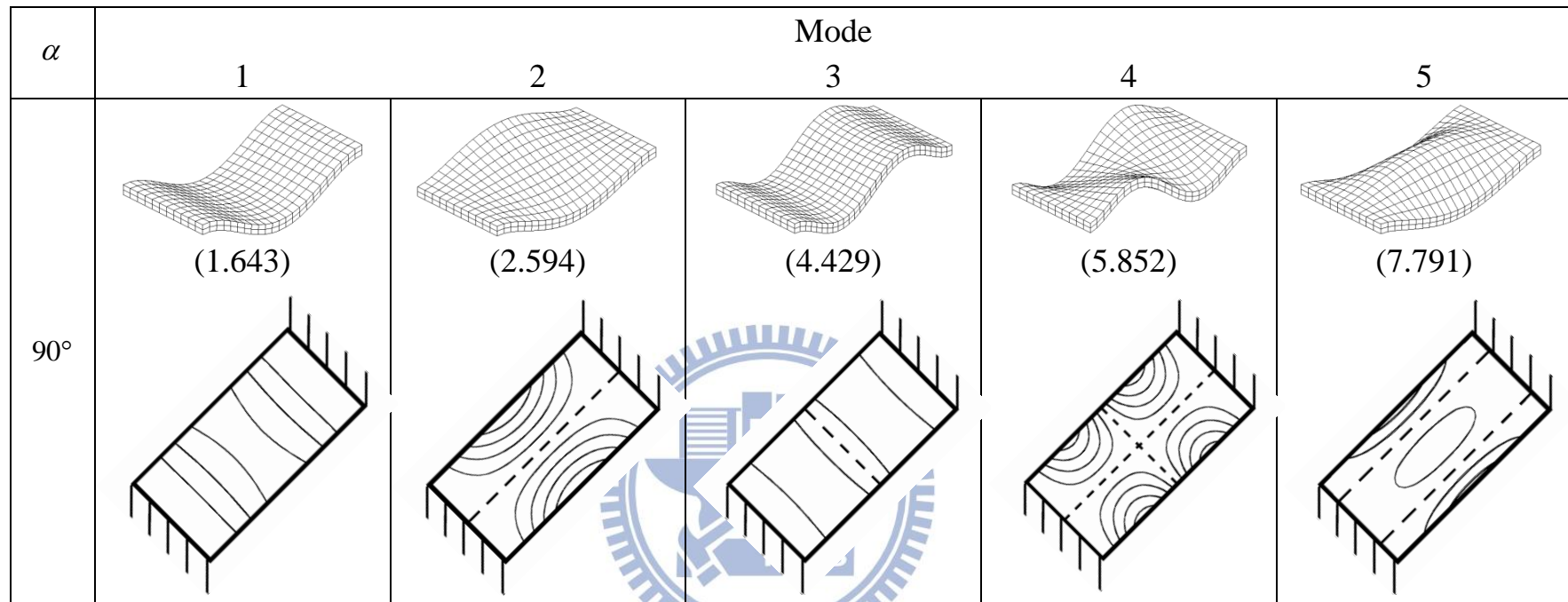


圖 4.9 具不同內部裂縫兩端固定端，兩端自由端之矩形均質中厚板模態圖($a/b=2, h/b=0.1$)

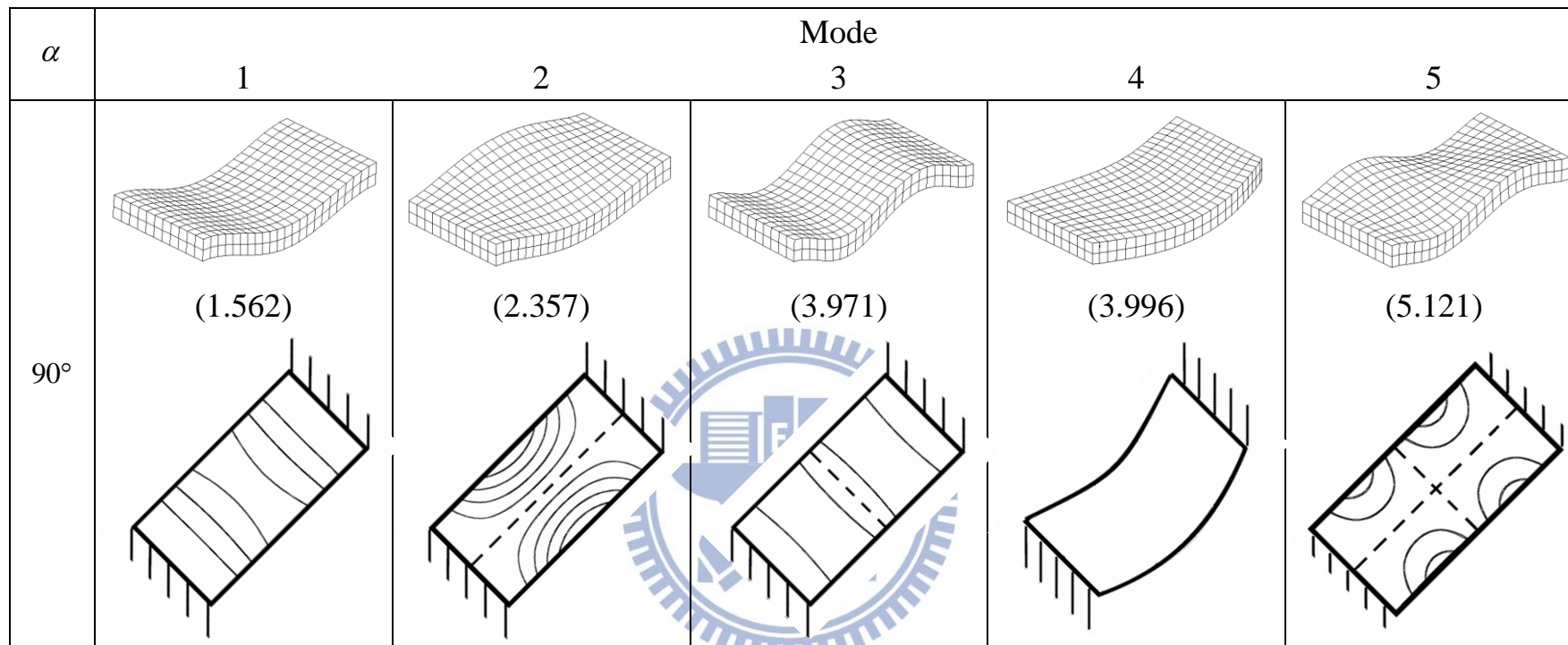


圖 4.9 (續上頁) ($a/b=2, h/b=0.2$)

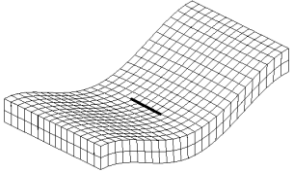
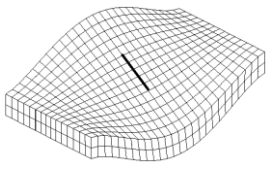
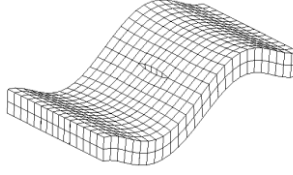
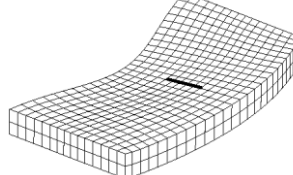
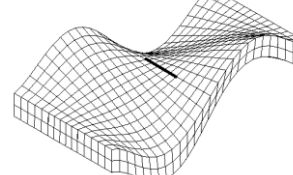
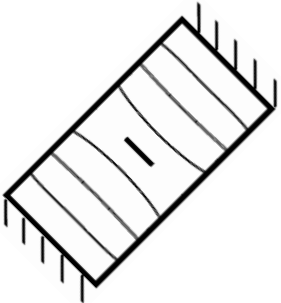
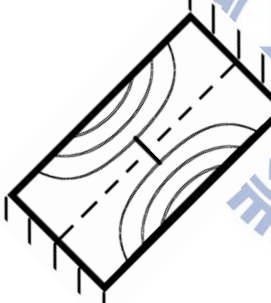
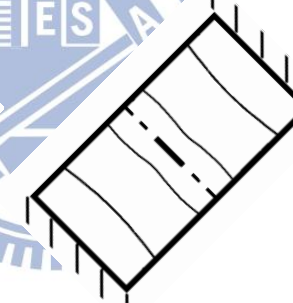
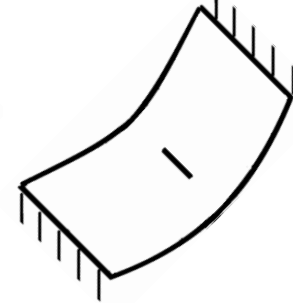
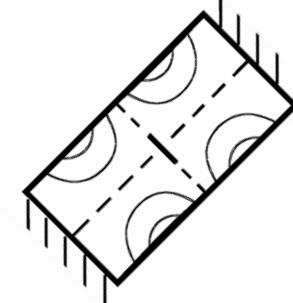
α	d/b	Mode				
		1	2	3	4	5
90°	0.3	 (1.514)	 (2.355)	 (3.965)	 (3.972)	 (5.076)
						

圖 4.9 (續上頁) ($a/b=2, h/b=0.2, (x_0/a, y_0/b)=(0.5,0.5)$)

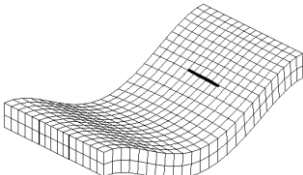
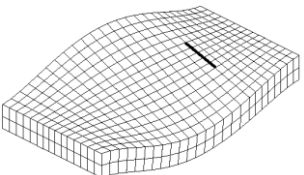
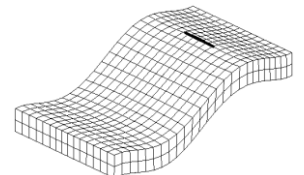
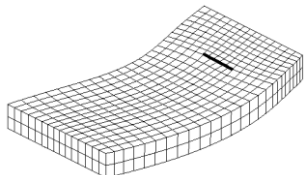
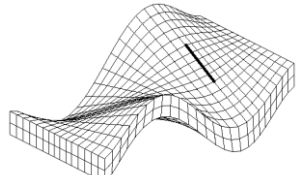
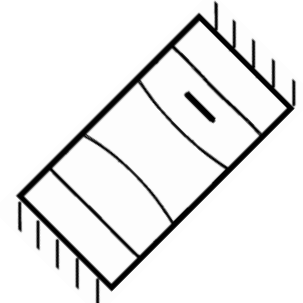

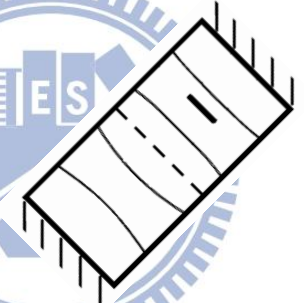
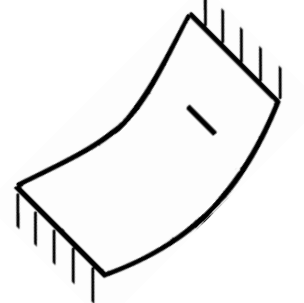
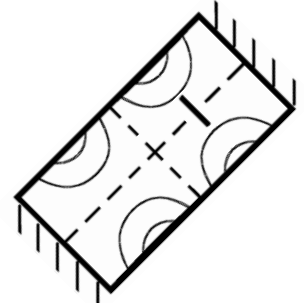
α	d/b	Mode				
		1	2	3	4	5
90°	0.3	 (1.555)	 (2.339)	 (3.884)	 (3.898)	 (5.115)
						

圖 4.9 (續上頁) ($a/b=2, h/b=0.2, (x_0/a, y_0/b)=(0.25,0.5)$)

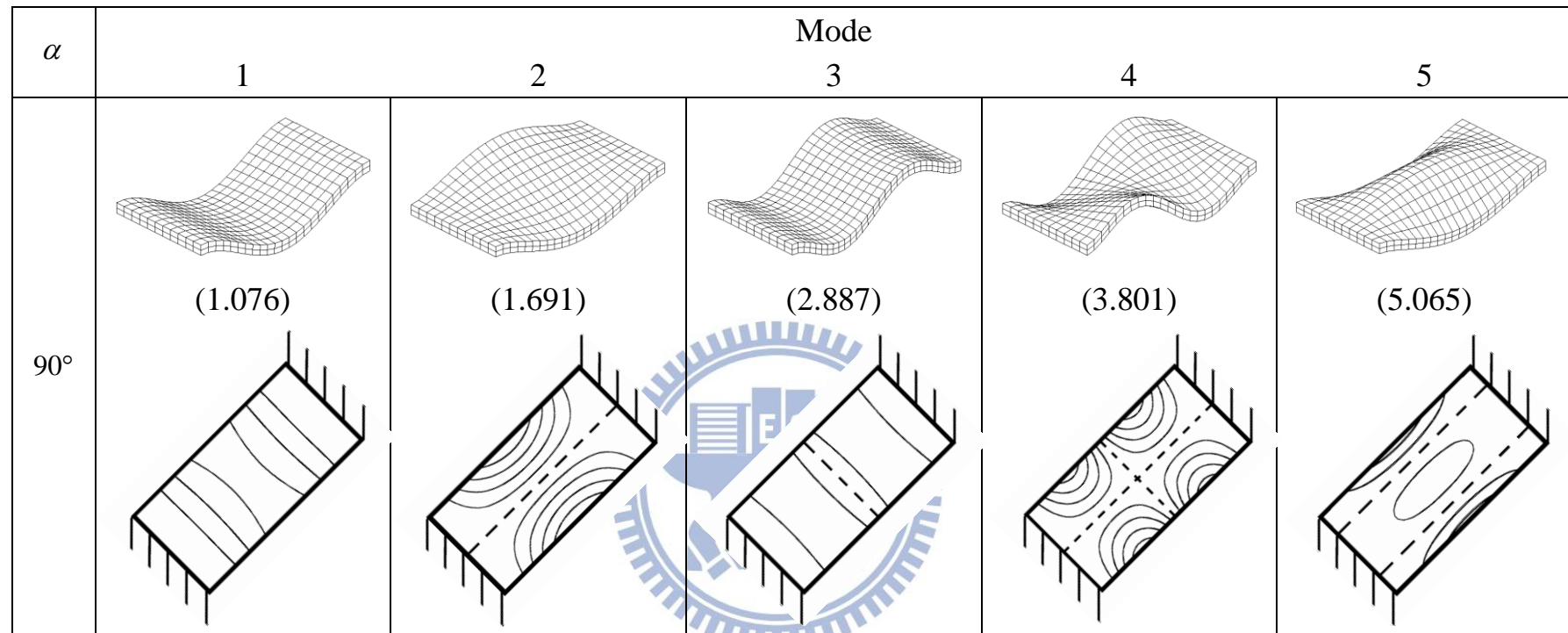


圖 4.10 具不同內部裂縫兩端固定端，兩端自由端之 Al/Al₂O₃ 矩形 FGM 中厚板模態圖($\hat{m}=5, a/b=2, h/b=0.1$)

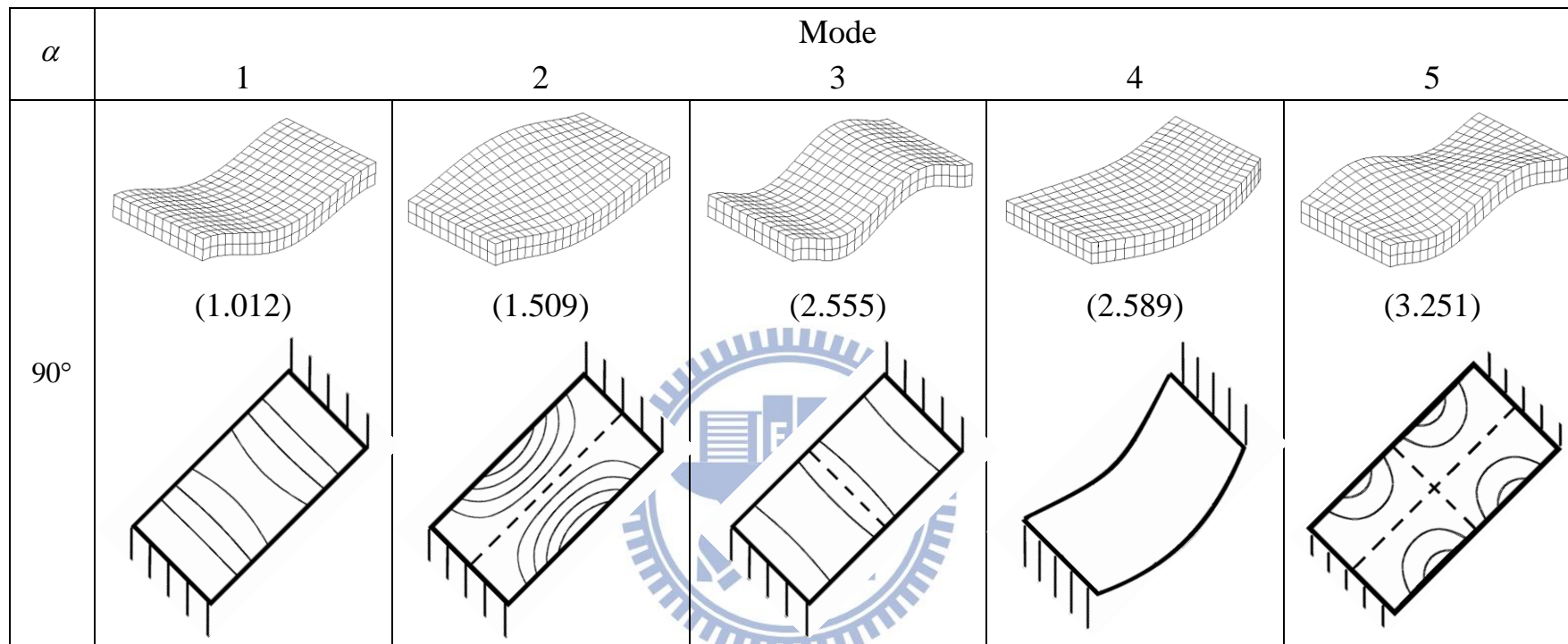


圖 4.10 (續上頁) ($\hat{m}=5, a/b=2, h/b=0.2$)

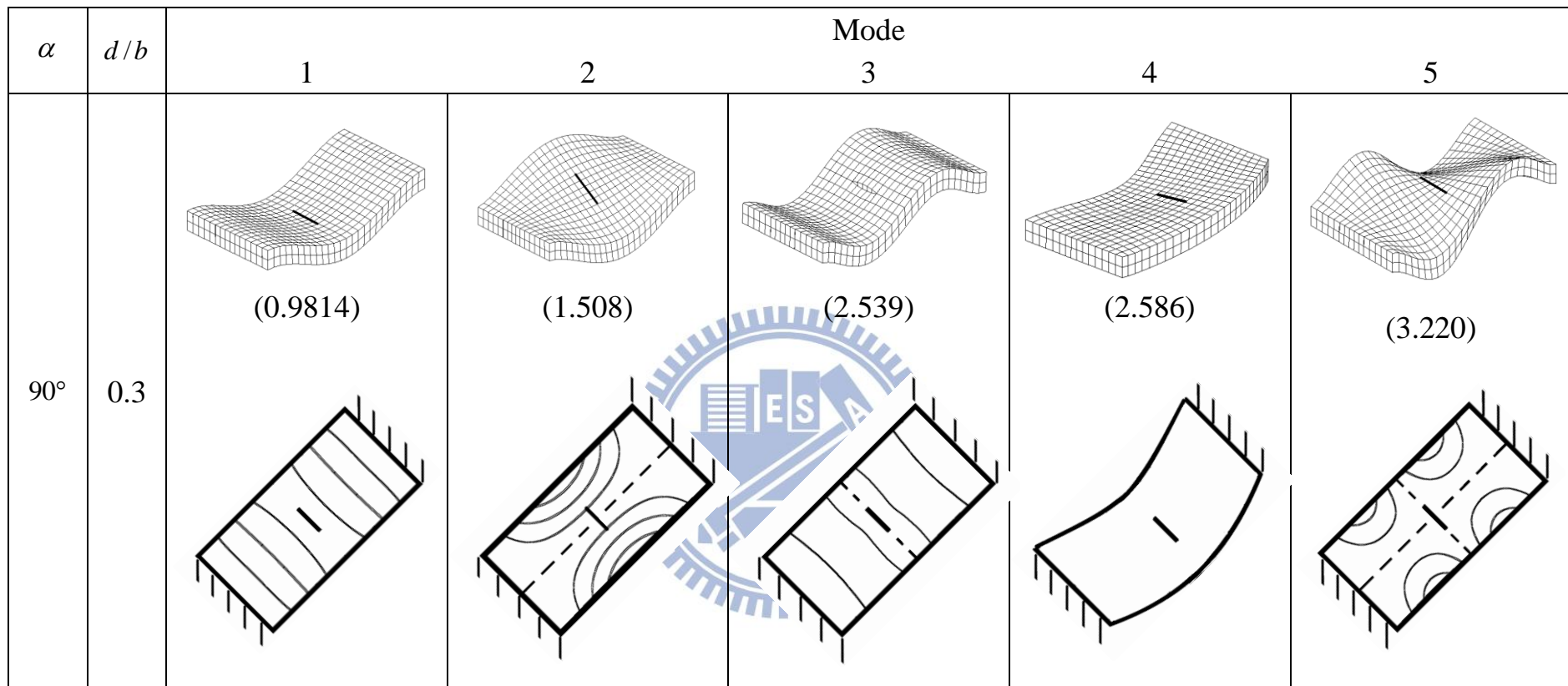


圖 4.10 (續上頁) ($\hat{m} = 5, a/b=2, h/b=0.2, (x_0/a, y_0/b)=(0.5,0.5)$)

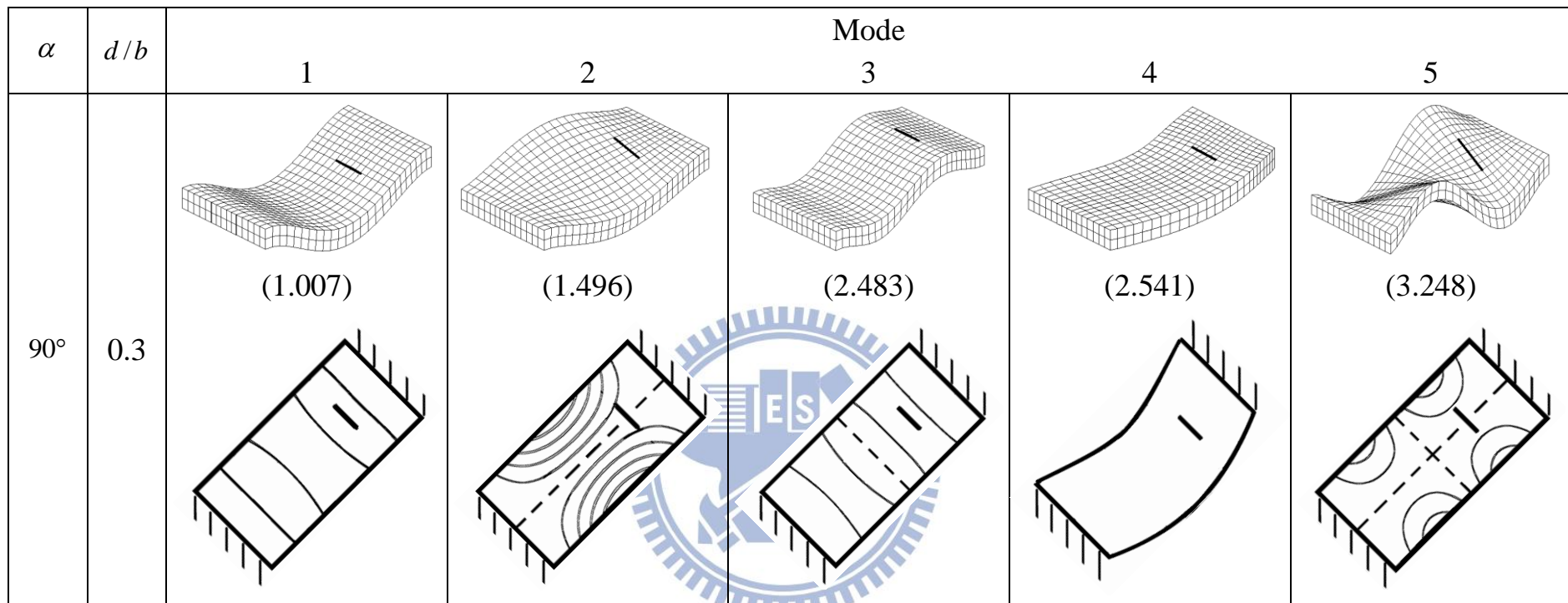


圖 4.10 (續上頁)($\hat{m} = 5, a/b=2, h/b=0.2, (x_0/a, y_0/b)=(0.25,0.5)$)