CHAPTER 2
SYSTEM DYNAMIC MODELING

2.1 Introduction

In this chapter, the dynamic characters of automated manual transmission system are analyzed, and the dynamic models from system equations are created in Matlab® Simulink.

Since one of the objectives in this study is to assist Industrial Technology Research Institute (ITRI) to analyze and modify the prototype of a new clutch actuator, the dynamic analysis on clutch actuator is focused with more details.

There are two main methods for dynamic model creation in this chapter. With components that should be concerned with details or dynamic characters are commonplace, free-body analysis is applied. For example: all the structures in clutch actuator are analyzed with free body analysis. On the other hand, components that are more complicated and free body analyses are complex that are not worth for analyzing, or components that are not the emphasis of this study, are seen as black boxes, such as engine, gear box, clutch, etc.. In such black boxes, experiments data and curve fitting methods are used to create system equations that stand for dynamic characters of these components, and dynamic models are similarly created in Simulink® according to these system equations.

In the free body analyses, in order to simplify the complexity of the models, two main assumptions are supposed. First, all the components are seen as rigid bodies besides springs, since it is well approximated in most cases on vehicle [19]. Second, the gravity effects are ignored. These assumptions are hold in general cases. However, if the hypotheses are not held that errors caused by these assumptions are out of our capable range after a comparison with experiments data, the hypotheses should be modified in future works.

In this chapter, clutch actuator is specially analyzed in section 2.2 , and the powertrain of automated manual transmission is analyzed in section 2.3 , finally in section 2.4 , the system dynamic equations developed in section 2.2 and section 2.3 are built up into Matlab® Simulink models.

2.2 Dynamic Analysis of Clutch Actuator

The clutch actuator which is a prototype of Industrial Technology Research Institute is analyzed with dynamic theories in this section. At first, structures of the clutch actuator are introduced and decomposed into several parts. The analyses and system equation creation are progressed according to these parts.

2.2.1 Structure of Clutch Actuator

The structure of the clutch actuator is shown in Figure 2.2-1. The deeply blue part is a DC electrical motor with a worm gear shaft in red assembled on the armature. The worm gear shaft transmits power from electrical motor to worm gear, which is drawn in brown. The worm gear shaft and the worm gear work like a deceleration system and torque amplify system for the electrical motor. On the worm gear, a joint structure is used to connect the worm gear with a linkage structure drawn in black. The linkage is driven by the worm gear throw the joint structure. At the other end of the linkage is a ball and socket joint coupling with a lever in yellow used to drive the clutch with a lever ratio through the clutch lever in yellow. Near the ball and socket joint is a fix plate in green connected with the linkage through a circular furrow. On the left of the fix plate is a spring in blue, which is usually in compressed status to assist the movement of the fix plate to drive the linkage to disengage clutch. Therefore, the worm gear, linkage, and fix plate can work almost like a linkage-slider structure since the travel of the actuator output is very small.

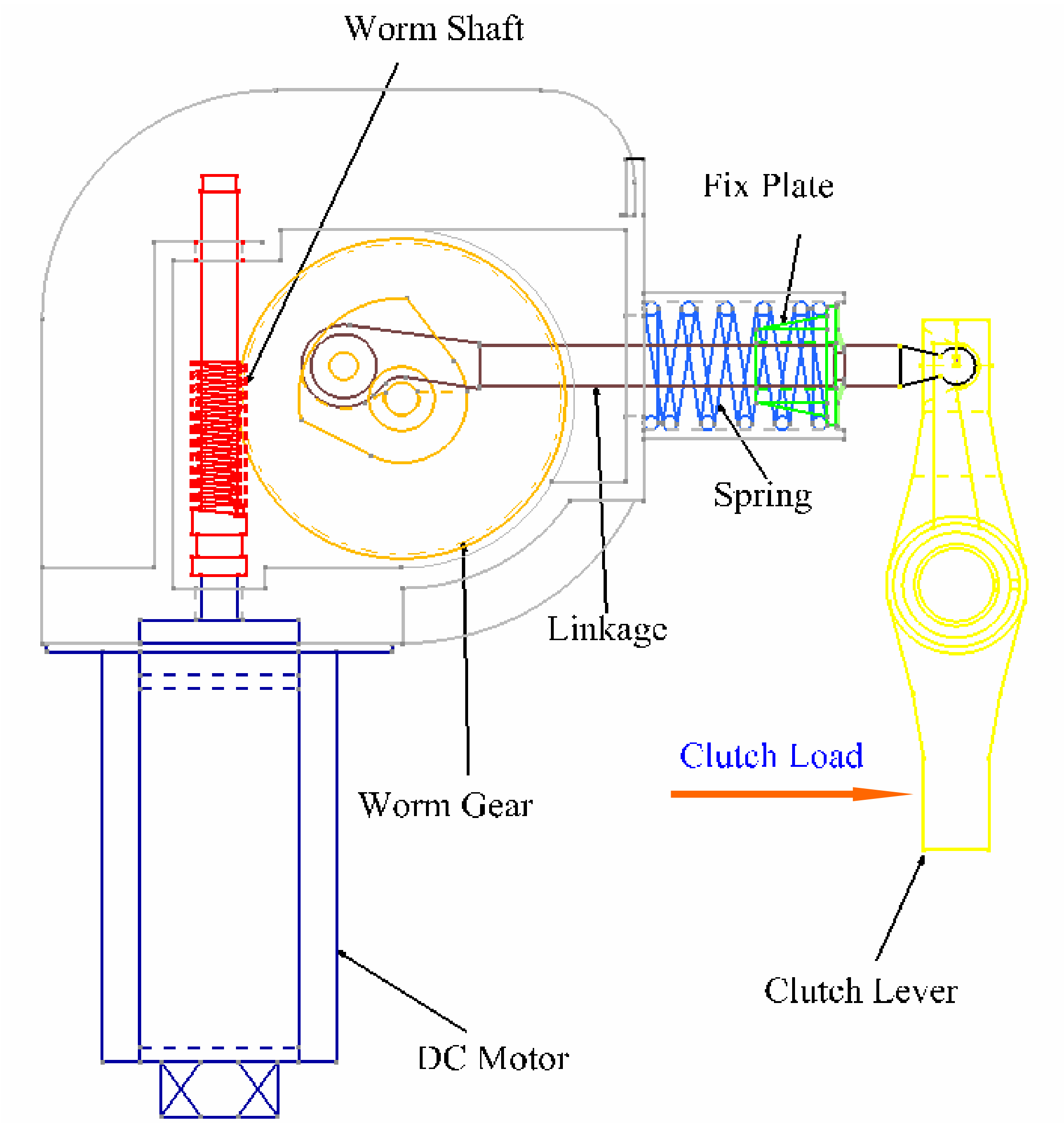

Figure 2.2-1 Clutch Actuator

The clutch actuator drives the clutch through the clutch lever as shown in Figure 2.2-2. Clutch actuator has the maximum movement of 8mm in the design. The lever has a length of 65mm. According to Figure 2.2-2, the rotation angle θ is no more than 0.1077 degree in general operation. Since θ is small, the travel of clutch actuator can be seen as the horizontal motion and the displacement is the same as the clutch travel according to the lever ratio of one in the prototype.

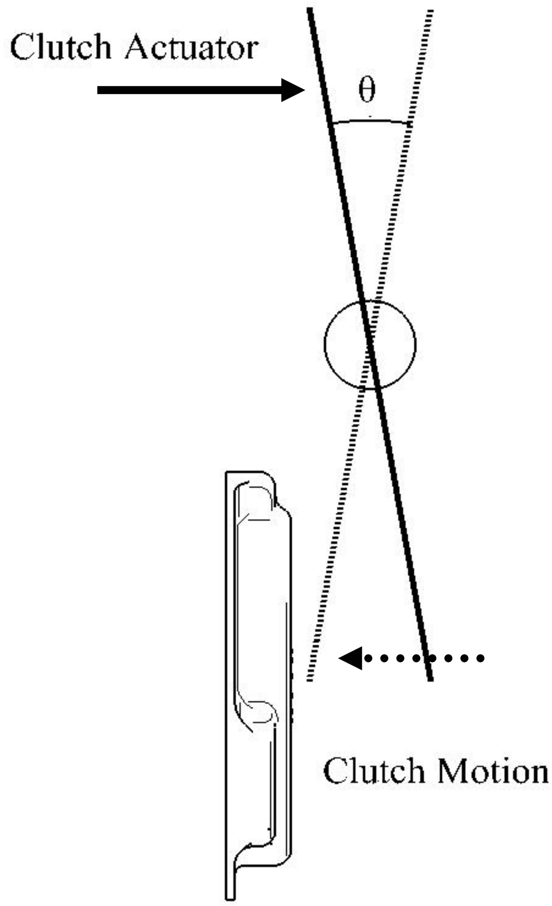

Figure 2.2-2 Clutch Lever

According to the structure, the clutch actuator is decomposed into to four subsystems: electrical motor, worm shaft, worm gear, and linkage system. In the following sections, dynamic analyses of each part are introduced according to these subsystems.

2.2.2 Electrical Motor

The electrical motor here is used to drive the clutch actuator on AMT vehicle, since most vehicles use direct current (DC) as their electric power, the model here should be created as a DC motor as used in the prototype. Figure 2.2-3 shows the common structure of a DC motor. In addition to housing and bearings, the nonturning part (stator) has magnets, which establish a magnetic field across the turning part (rotor). The brushes force current through the wire wound around the rotor. The (rotating) commutator causes the current always to be sent through the armature.

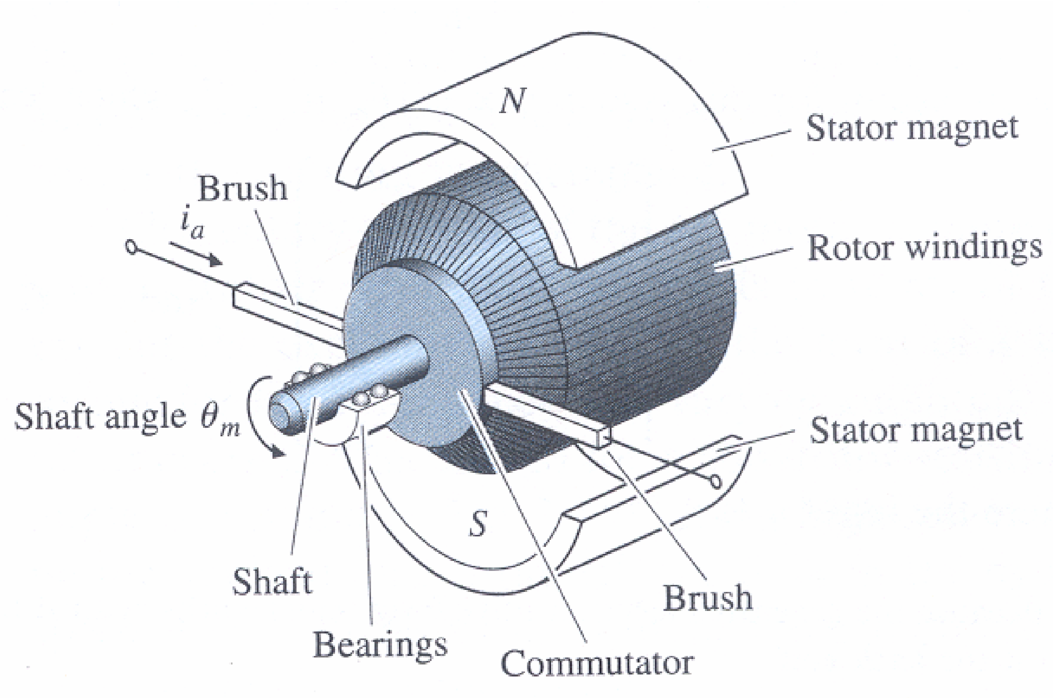

Figure 2.2-3 DC Motor [10]

In mechanical component, the torque developed by DC electrical motor is introduced by the force caused by magnetic flux. The relation of force and current is known as "law of motor", which can be expressed as [11]:

$$F = Bli \quad \text{Newton} \tag{2.2-1}$$

Where B is the magnetic field strength, l is the length of conductor, i is the current in conductor, and F is the force caused by current and magnetism. Hence the torque cause by F is:

$$T = Blir \quad \text{Newton} \cdot \text{meter} \tag{2.2-2}$$

Where T is the torque caused by current and magnetism.

Applying to free body diagram of motor rotor, as shown in Figure 2.2-4, the relation between current and torque can be obtained as shown in Eq.(2.2-3):

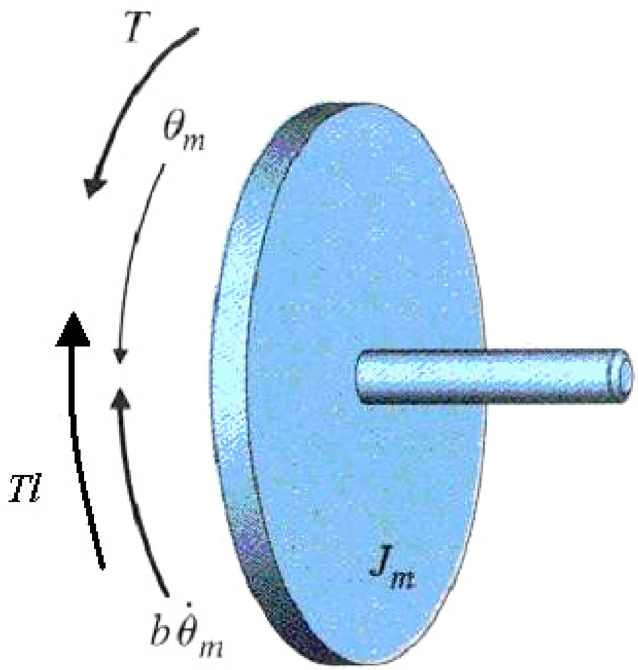

Figure 2.2-4 Free Body Diagram of Motor Rotor

$$J_m\ddot{\theta}_m + b\dot{\theta}_m + Tl = K_t i = T \qquad (2.2\text{-}3)$$

Were J_m is the inertia of rotor, b is the damping coefficient effects on the rotor, θ_m is the rotation angle of rotor, Tl is the extra loading torque on motor, and K_t is called torque constant, which can be expressed as:

$$K_t = Blr \qquad (2.2\text{-}4)$$

In electrical component, Figure 2.2-5 shows electrical circuit of the armature. According to Kirchooff Voltage Law (KVL) loop analysis, the relationship between current and voltage on DC motor can be stated as Eq.(2.2-5).

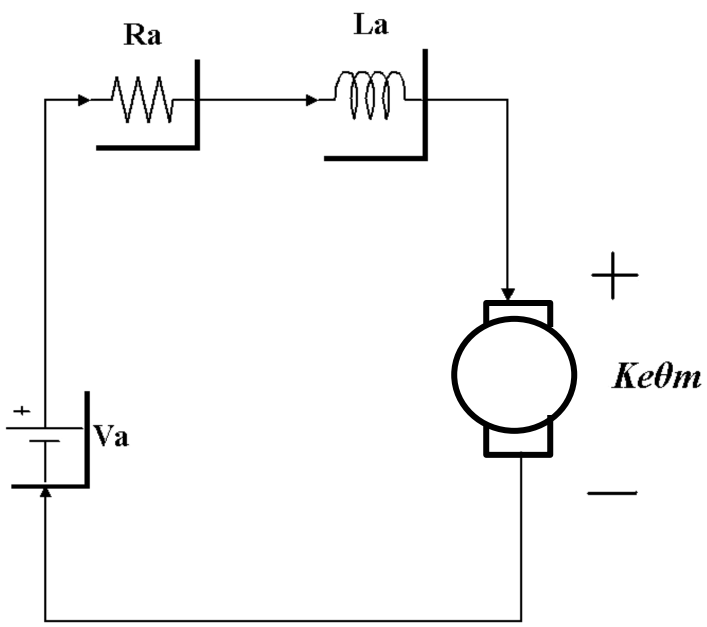

Figure 2.2-5 Electrical Circuit of DC Motor Armature

$$La\frac{di_a}{dt}+R_a i_a = v_a - K_e\dot{\theta}_m \qquad (2.2\text{-}5)$$

Where La, R_a, and v_a indicate inductance, resistance and source voltage of the electrical circuit in DC motor. K_e is called "EMF" (Electro Motive Force) constant which is decided by motor character. $K_e\dot{\theta}_m$ is the term of counter EMF introduced by armature rotation that causes motor working simultaneously like a generator generating voltage opposing the supplied voltage v_a.

According to Eq.(2.2-3) and Eq.(2.2-5), the system equations of the DC electrical motor on clutch actuator are obtained, which has an system input v_a, an system output θ_m, an outside loading Tl, and other system parameter terms.

2.2.3 Worm Shaft

The structure of worm shaft is shown in Figure 2.2-6. Worm shaft is connected with electrical motor armature at one end and transmits torque from motor to worm gear, as shown in Figure 2.2-7.

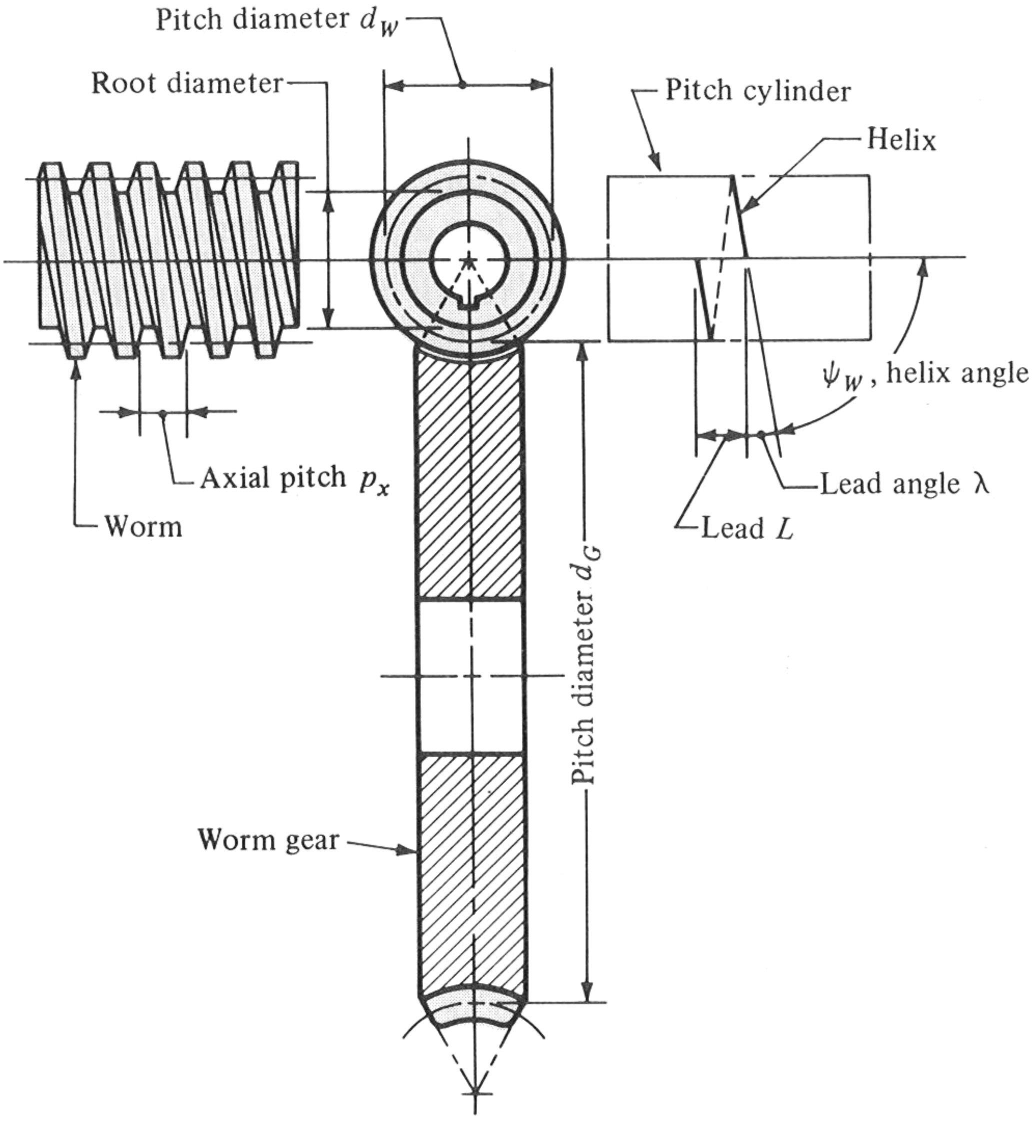

Figure 2.2-6 Structure of Worm Shaft [13]

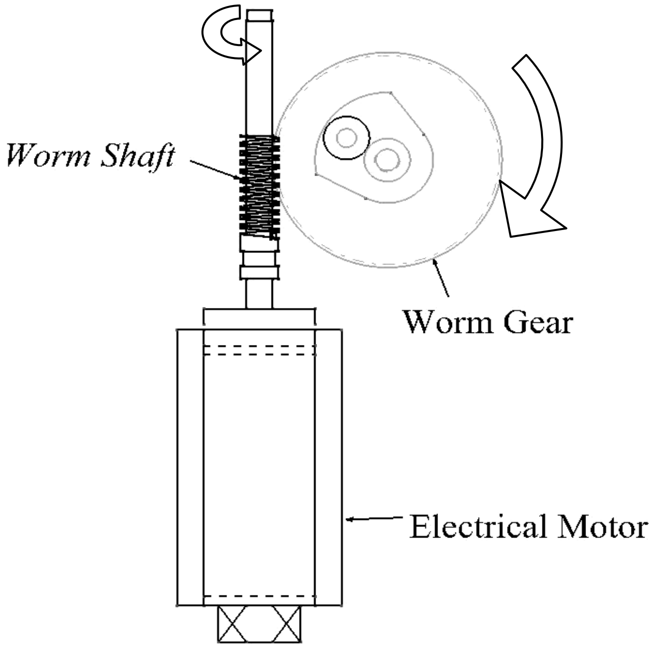

Figure 2.2-7 Worm Shaft Components

Figure 2.2-8 is free body diagram of the worm shaft coupling with a worm gear not shown. ϕ_n is pressure angle, and λ is lead angle of the worm shaft, which has a relation with pitch diameter d_w: $\tan\lambda = \frac{L}{\pi d_w}$, $L = p_x N_w$. Where p_x is axial pitch of worm shaft and N_w is thread number of the worm shaft [13]. The force exerted by the worm gear is W, and the torque transmitted from motor is T. W has three orthogonal components W^x, W^y, and W^z. From geometry of Figure 2.2-8, the three terms can be expressed as Eqs.(2.2-6).

$$
\begin{aligned}
W^x &= W\cos\phi_n \sin\lambda \\
W^y &= W\sin\phi_n \qquad\qquad (2.2\text{-}6) \\
W^z &= Wos\phi_n \cos\lambda
\end{aligned}
$$

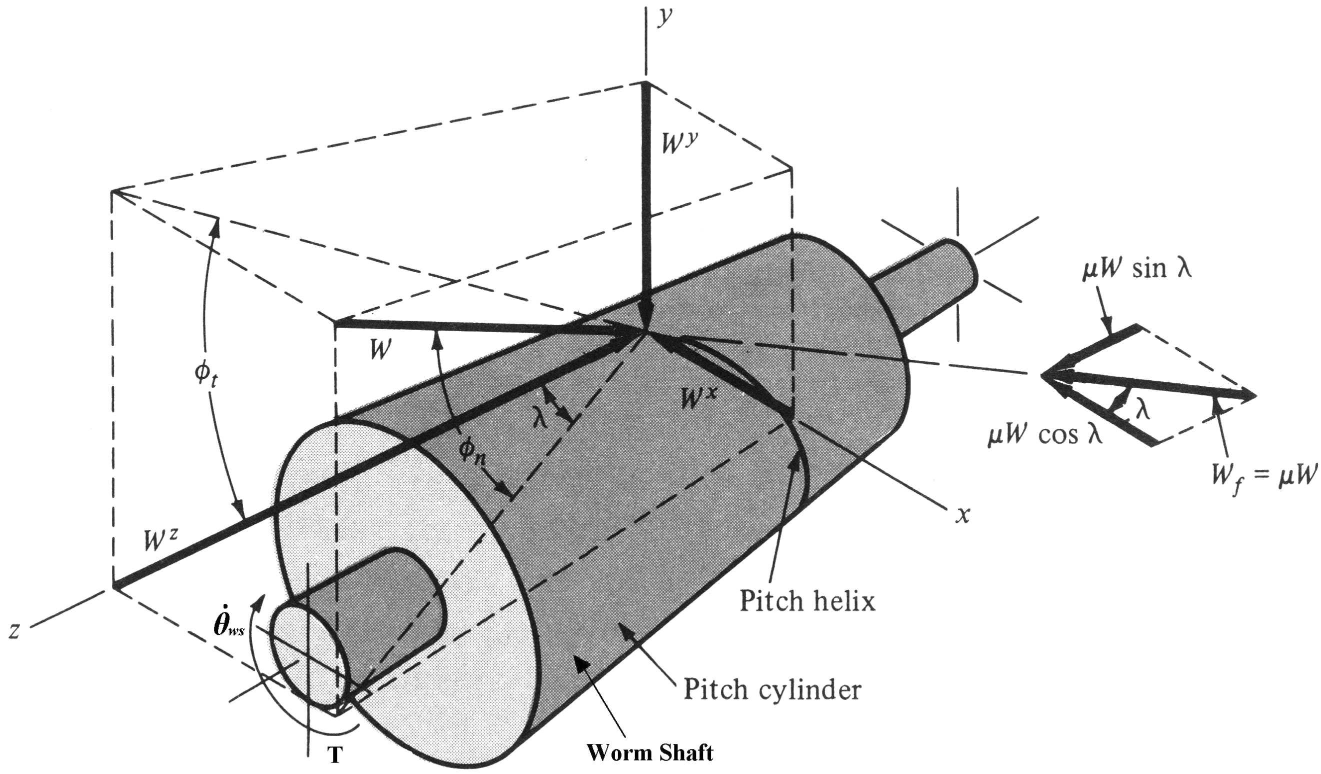

Figure 2.2-8 Free Body Diagram of Worm Shaft [13]

W^y is the separating, or radial, force for both worm shaft and worm gear, which is known as W_{Wr} and W_{Gr}. The axial force on worm shaft is W^z known as W_{Wa}, and tangential force W_{Gt} on worm gear, since the shaft angle here is $90°$. The tangential force which is a workable force to be transmitted on the worm shaft is W^z known as W_{Wt}, and on worm gear is an axial force known as W_{Ga}.

From Newton's third law, worm gear forces are opposite to the worm shaft forces, which can be summarized as Eqs.(2.2-7).

$$\begin{aligned} W_{Wt} &= -W_{Ga} = W^x \\ W_{Wr} &= -W_{Gr} = W^y \\ W_{Wa} &= -W_{Gt} = W^z \end{aligned} \qquad (2.2\text{-}7)$$

Since the relative motion between worm shaft and worm gear teeth are pure sliding, friction plays and important role in the transmission of worm gearing. By introducing a coefficient of friction μ, from Figure 2.2-8, the force W acting normal to the worm gear tooth produces a friction force $W_f = \mu W$, having a component $\mu W \cos\lambda$ in the negative x direction and another component $\mu W \sin\lambda$ in the positive z direction. Eqs.(2.2-6) therefore becomes Eqs.(2.2-8) which are considered with friction affection.

$$
\begin{aligned}
W^x &= W(\cos\phi_n \sin\lambda + \mu\cos\lambda) \\
W^y &= W\sin\phi_n \\
W^z &= (Wos\phi_n \cos\lambda - \mu\sin\lambda)
\end{aligned}
\qquad (2.2\text{-}8)
$$

From Eqs.(2.2-7) and Eqs.(2.2-8), the relation between pure tangential forces on worm shaft and worm gear, which is effective term on torque transmission, can be summarized as Eq.(2.2-9).

$$W_{wt} = W_{Gt}\frac{\cos\phi_n \sin\lambda}{\mu\sin\lambda - \cos\phi_n \cos\lambda} \qquad (2.2\text{-}9)$$

And the friction term affects on tangential direction can be expressed as:

$$W_{Wtf} = \mu W_{Gt}\frac{\cos\lambda}{\mu\sin\lambda - \cos\phi_n \cos\lambda} \qquad (2.2\text{-}10)$$

Were W_{Wtf} is the tangential force caused by friction between worm shaft tooth and worm gear tooth.

The terms of transmission torque and friction effect are separated here because of the convenience for further model creation, which in general can be expressed as [13]:

$$W_{wt} = W_{Gt}\frac{\cos\phi_n \sin\lambda + \mu\cos\lambda}{\mu\sin\lambda - \cos\phi_n \cos\lambda} \qquad (2.2\text{-}11)$$

For a simplified free body diagram as shown in Figure 2.2-9, the dynamic characters can be expressed as a system equation as shown in Eq.(2.2-12), which has an input torque T transmitted from electrical motor that the same with motor load Tl, an input loading torque from worm gear W_{Gt}, and an output of worm shaft rotation angle θ_{ws}.

$$T = Tl = W_{Wt} \, {d_W}/{2} + W_{Wtf} \, {d_W}/{2} + I_{WS} \, \ddot{\theta}_{WS} + C_{WS} \, \dot{\theta}_{WS} \qquad (2.2\text{-}12)$$

Where d_W is pitch diameter of the worm shaft, θ_{WS} is rotation angle of the worm shaft, I_{WS} is inertia moment of the worm shaft, C_{WS} is the damping coefficient of the worm shaft, and $a = \ddot{\theta}_{WS}$ as in Figure 2.2-9.

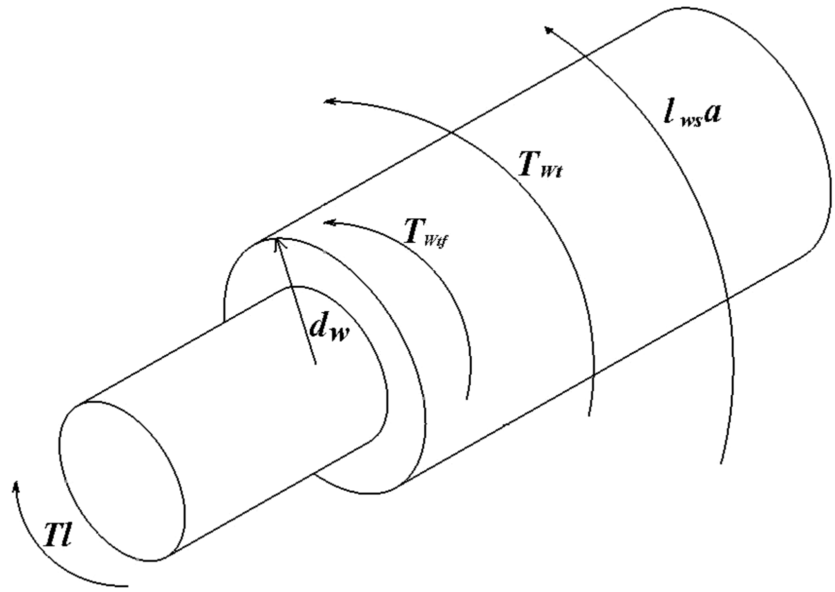

Figure 2.2-9 Simplified Free Body Diagram of Worm Shaft

2.2.4 Worm Gear

In the clutch actuator, worm gear couples with the worm shaft discussed in above subsection, which decelerates rotation speed and amplifies transmission torque of worm shaft. Besides, it also transforms rotation torque to a force which drives the linkage through a joint. As shown in Figure 2.2-10.

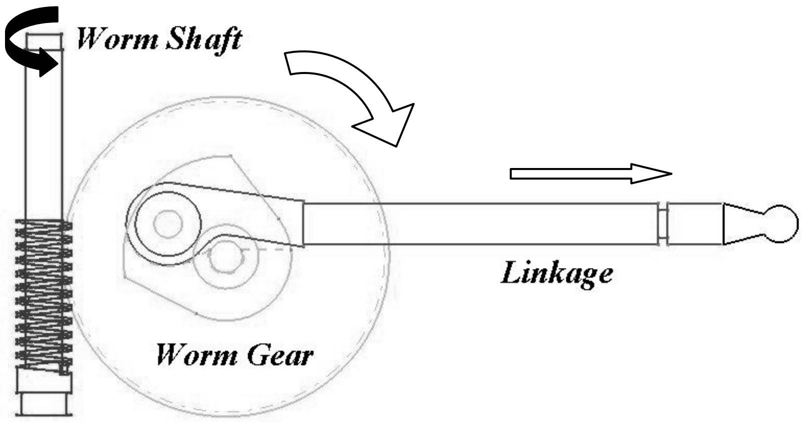

Figure 2.2-10 Worm Shaft Components

Free body diagram of worm gear is shown in Figure 2.2-11. Since structures here are supposed to be rigid bodies as mentioned before, free body analyses here are considered as a 2-D problem, where no deformation, which may change forces direction out of original plan, occurs. The moments on X and Y directions caused by the worm shaft and the uncommon plan force from linkage joint thus are assumed to be influence-less.

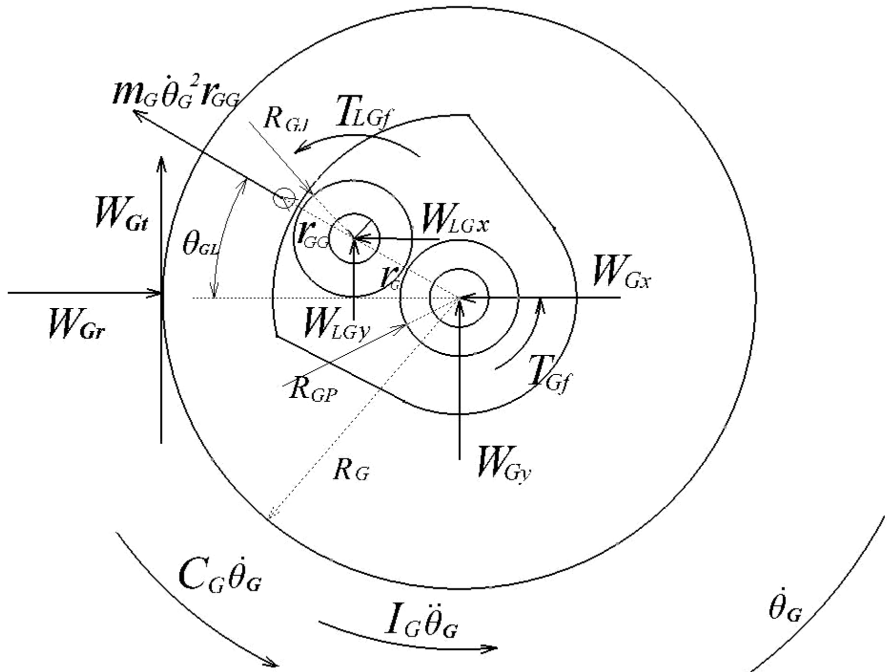

Figure 2.2-11 Free Body Diagram of Worm Gear

From Figure 2.2-11, considering the influences of inertia moment, damping, friction and forces that that act on the worm gear, the system dynamic equations according to the equilibrium of forces and moments are obtained as Eqs.(2.2-13).

$$\begin{aligned}
&\sum F_x = 0; \\
&W_{LGx} + W_{Gx} + m_G \dot{\theta}_G^{2} r_{GG} \cos\theta_{GL} = W_{Gr} \\
&\sum F_y = 0; \\
&W_{Gt} + W_{LGy} + W_{Gy} + m_G \dot{\theta}_G^{2} r_{GG} \sin\theta_{GL} = 0 \\
&\sum M = 0; \\
&W_{Gt} R_G + W_{LGy} r_G \cos\theta_{GL} = W_{LGx} r_G \sin\theta_{GL} + T_{LGf} + T_{Gf} + I_G \ddot{\theta}_G + C_G \dot{\theta}_G
\end{aligned} \qquad (2.2\text{-}13)$$

Where m_G is the mass, I_G is the inertia moment, C_G is the damping coefficient, θ_G is the rotation angle, and r_{GG} is the distance from mass center to axle of the worm gear. W_{LGx} and W_{LGy} are reacting forces from linkage in X and Y directions. W_{Gx} and W_{Gy} are forces on the axle of the worm gear in the directions of X and Y. W_{Gt} and W_{Gr} are tangential force and radial force on worm gear as mentioned in Eqs.(2.2-7). By rearrangement of Eqs.(2.2-7), Eqs.(2.2-8), and Eq.(2.2-11), W_{Gr} can be expressed by W_{Gt}, as shown in Eq.(2.2-14).

$$W_{Gr} = \frac{W_{Gt} \sin\phi_n}{\cos\phi_n \cos\lambda - \mu \sin\lambda} \qquad (2.2\text{-}14)$$

T_{Gf} and T_{LGf} are torques that caused by frictions on axle of worm gear and joint used to connect linkage.

Let the friction coefficient on axle of the worm gear be μ_{Ga}, on the joint be μ_{Gj}. The torques T_{Gf} and T_{LGf} caused by friction can be expressed as:

$$
\begin{aligned}
T_{Gf} &= \sqrt{{W_{Gx}}^2 + {W_{Gy}}^2}\,\mu_{Ga} R_{GP} \\
T_{LGf} &= \sqrt{{W_{LGx}}^2 + {W_{LGy}}^2}\,\mu_{Gj} R_{GJ}
\end{aligned}
\qquad (2.2\text{-}15)
$$

From Eqs.(2.2-13),Eq.(2.2-14) and Eqs.(2.2-15), the system dynamic equation of the worm gear can be rearranged as Eqs.(2.2-16). In which equation, outside influences are know as forces W_{LGx}, W_{LGy}, and W_{Gt}, which cause an system output of rotation angle known as θ_G. And parameters I_G, C_G, R_G, r_G, θ_{GL}, ϕ_n, λ, μ_{Ga}, μ_{Gj}, R_{GP}, r_{GG}, m_G and R_{GJ} are system characteristics defined by original design as shown in Figure 2.2-11.

$$
\begin{aligned}
& I_G \ddot{\theta}_G + C_G \dot{\theta}_G = (W_{Gt} R_G + W_{LGy} r_G \cos\theta_{GL} - W_{LGx} r_G \sin\theta_{GL}) - \\
& \mu_{Ga} R_{Gp} \sqrt{\left(\frac{W_{Gt}\sin\phi_n}{\cos\lambda\cos\phi_n - \mu\sin\lambda} - W_{LGx} - m_G \dot{\theta}_G^{\,2} r_{GG}\cos\theta_{GL}\right)^2 + (W_{Gt} + W_{LGy} + m_G \dot{\theta}_G^{\,2} r_{GG}\sin\theta_{GL})^2} - \\
& \mu_{Gj} R_{GJ} \sqrt{{W_{LGx}}^2 + {W_{LGy}}^2}
\end{aligned}
\qquad (2.2\text{-}16)
$$

On the other hand, there are lower and upper bounds in the rotation of the worm gear. The limits are caused by the design of mechanical interferences as shown in Figure 2.2-12 and Figure 2.2-13.

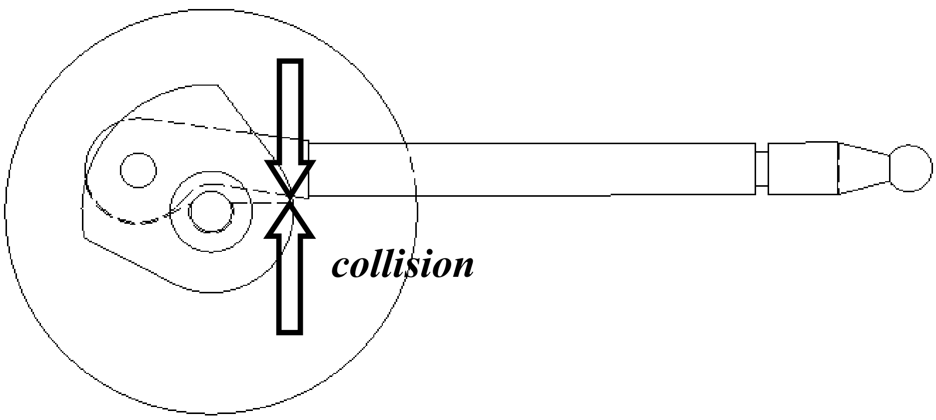

Figure 2.2-12 Lower End Collision

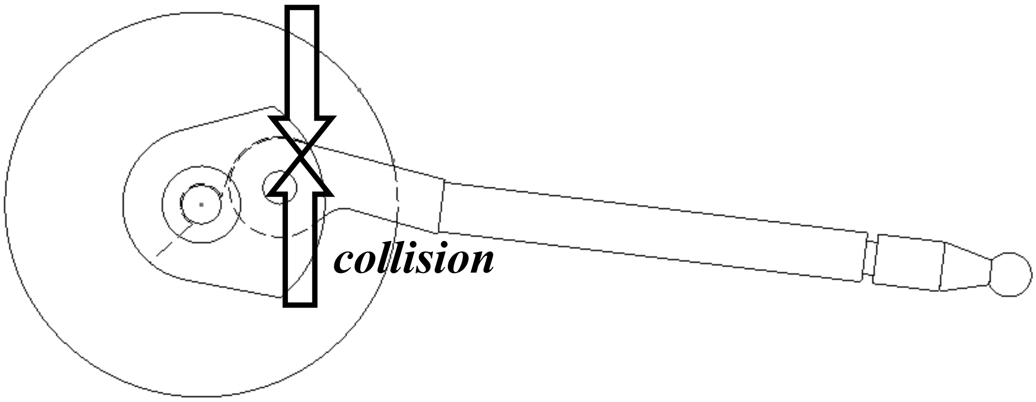

Figure 2.2-13 Upper End Collision

Since collision effect is not the emphases of the simulation, dynamic characters of collision here are seen as an impact to a hard spring as shown in Figure 2.2-14.

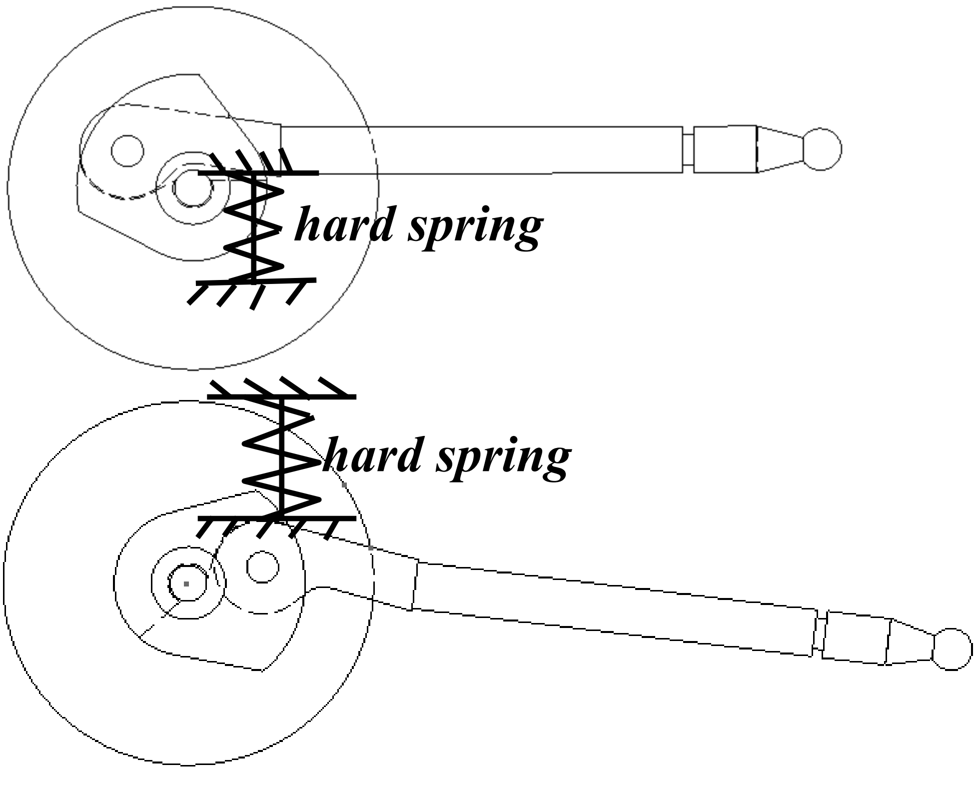

Figure 2.2-14 Model of Collision

Thus the system equation of Eqs.(2.2-16) is modified to be Eq.(2.2-17) while the upper or lower collision occurs.

$$I_G \ddot{\theta}_G + C_G \dot{\theta}_G + (K_c\theta_C + C_c\dot{\theta}_c) + \dot{\theta}_c) = W_{Gt}R_G + W_{LGy}r_G\cos\theta_{GL} - W_{LGx}r_G\sin\theta_{GL} -$$

$$\mu_{Ga}R_{Gp}\sqrt{\left(\frac{W_{Gt}\sin\phi_n}{\cos\lambda\cos\phi_n-\mu\sin\lambda} - W_{LGx} - m_G\dot{\theta}_G^{\,2} r_{GG}\cos\theta_{GL}\right)^2 + \left(W_{Gt}+W_{LGy}+m_G\dot{\theta}_G^{\,2} r_{GG}\sin\theta_{GL}\right)^2}$$

$$-\mu_{Gj}R_{GJ}\sqrt{W_{LGx}^{\,2}+W_{LGy}^{\,2}}$$

(2.2-17)

Where K_c and C_c are stiffness and damping coefficient of the virtual spring. θ_c is the angle degree that overtakes the lower or upper limit.

Also notes that the rotation angle between worm shaft and worm gear is $\theta_G = {}^{d_G\pi}\!/_{L}\,\theta_{WS}$ as shown in Figure 2.2-11.

2.2.5 Linkage System

Components of the linkage system of the clutch actuator are shown in Figure 2.2-15, which includes a linkage, a fix plate, and a spring.

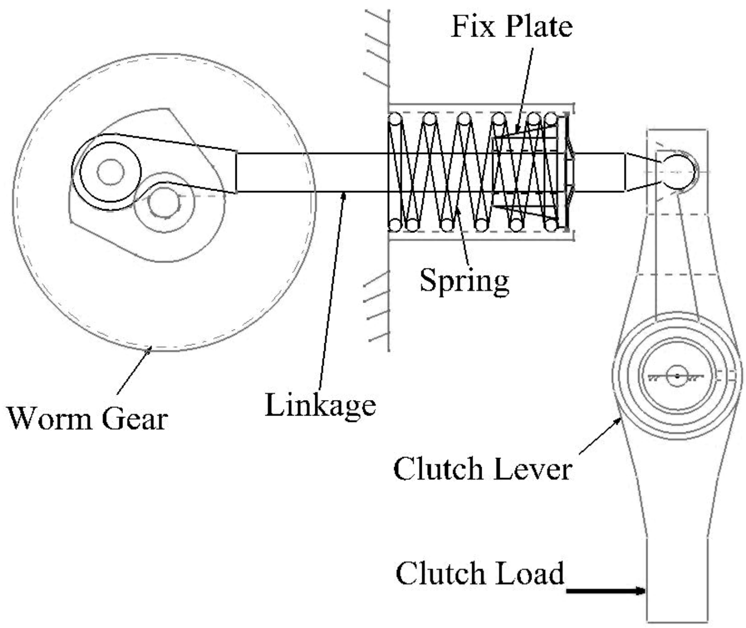

Figure 2.2-15 Linkage System Components

The linkage is driven by the worm gear discussed in previous subsection, by which produces a movement to drive the clutch lever through a ball and socket joint as shown. As mentioned in section 2.2.1 , the movement on the right end of the linkage, which is a ball and socket joint, can be seen as a motion in pure X direction.

The mutual motion from clutch actuator to clutch is shown in Figure 2.2-16, where structure of the clutch lever is simplified. The actuating force is transmitted from clutch actuator to the clutch lever, through a release bearing on the clutch, finally set clutch to a desired position.

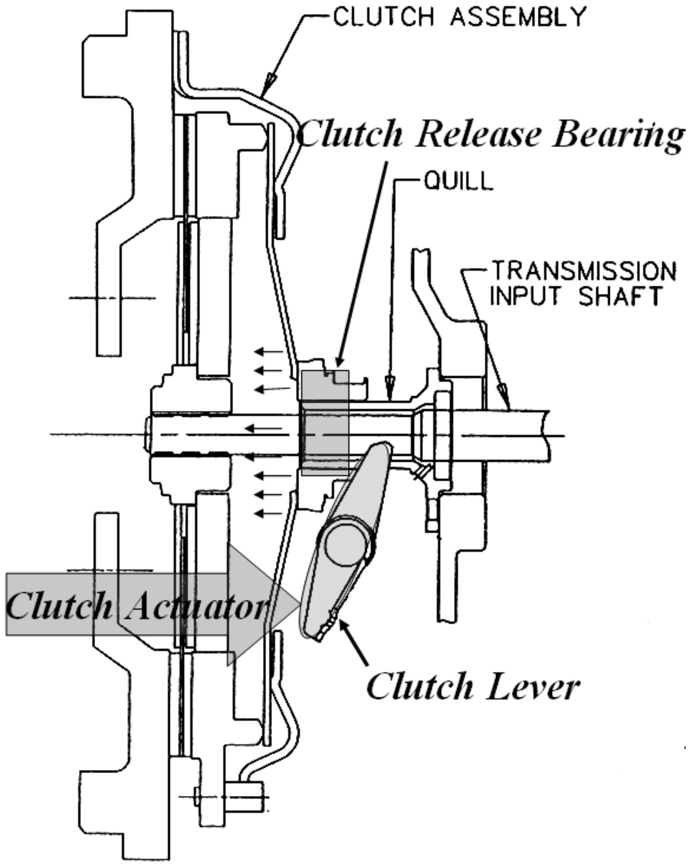

Figure 2.2-16 Structures from Clutch Actuator to Clutch

Figure 2.2-17 shows free body diagram of the linkage.

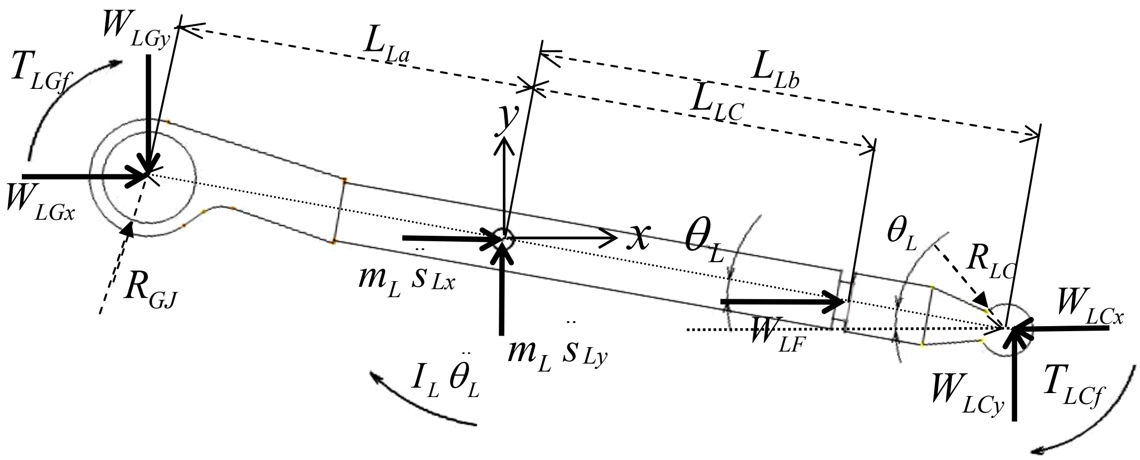

Figure 2.2-17 Free Body Diagram of the Linkage

From Figure 2.2-17, system equations according to equilibrium of forces and moments are shown in Eqs.(2.2-18).

$$
\begin{aligned}
&\sum F_x = 0; \\
&W_{LGx} + m_L \ddot{s}_{Lx} + W_{LF} = W_{LCx} \\
&\sum F_y = 0; \\
&W_{LGy} = m_L \ddot{s}_{Ly} + W_{LCy} \\
&\sum M_{xy} = 0; \\
&W_{LGx} L_{La} \sin\theta_L + W_{LCx} L_{Lb} \sin\theta_L + T_{LGf} + T_{LCf} + I_L \ddot{\theta}_L = W_{LGy} L_{La} \cos\theta_L + W_{LF} L_{LC} \sin\theta_L + W_{LCy} L_{Lb} \cos\theta_L
\end{aligned}
$$

(2.2-18)

Where the x-y axis system in Figure 2.2-17 is at mass center of the linkage. W_{LCx}, and W_{LCy} are forces that act on the ball and socket joint from the clutch lever. W_{LGx} and W_{LGy} are forces from worm gear as mentioned in Eq.(2.2-13). W_{LF} is force from the spring which assists motion of the linkage to disengage the clutch. S_{Lx} and S_{Ly} are displacements of mass center of the linkage in X and Y direction. θ_L is rotation angle of the linkage. I_L is inertia moment of the linkage in direction of Z. T_{LCf} and T_{LGf} are torques that caused by friction at two ends of the linkage. T_{LGf} has mentioned in Eqs.(2.2-15), and T_{LCf} can be expressed as Eq.(2.2-19), where μ_{LC} is friction coefficient on the ball and socket joint.

$$T_{LCf} = \mu_{LC} R_{LC} \sqrt{W_{LCx}^{\ 2} + W_{LCy}^{\ 2}} \tag{2.2-19}$$

From Figure 2.2-16, reacting force from clutch, transmitting through a release bearing and clutch lever, is know as W_{LCx}. To be simplification, the affection of masses on release bearing and clutch lever are simplified as m_{BL}. Thus, let the reacting force from clutch to be W_c, clutch lever ratio to be R_r, the relation between W_{LCx}, and W_c can be expressed as Eq.(2.2-20).

$$W_{LCx} = m_{BL} \ddot{s}_{LC} + C_{BL} \dot{s}_{LC} + R_r W_c \tag{2.2-20}$$

Where C_{BL} is the identity of damping coefficients from release bearing to ball and socket joint. S_{LC} is output of the clutch actuator, which is the same with motion of the ball and socket joint in X direction as mentioned in Figure 2.2-2. From Figure 2.2-18, S_{LC} can be expressed as Eq.(2.2-21), where θ_L is zero at initial condition according to original design of the prototype.

$$\begin{aligned} S_{LC} &= S_{Lx} + L_{Lb}(\cos\theta_L - 1) \\ S_{Ly} &= L_{Lb} \sin\theta_L \end{aligned} \tag{2.2-21}$$

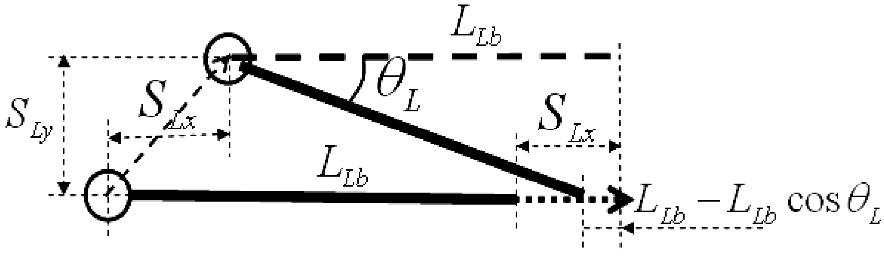

Figure 2.2-18 Linkage Arm Motion

Since rotation angle θ_L is always small during operation, the displacement of the furrow where W_{LF} acts can be seen as the displacement S_{LC}. From the basic dynamic model of spring, force from the spring W_{LF} can be expressed as:

$$-W_{LF} = M_{SP}\ddot{s}_{LC} + C_{SP}\dot{s}_{LC} + K_{SP}(s_{ins} - s_{LC}) \qquad (2.2\text{-}22)$$

where s_{ins} is pre-deformation of the spring, M_{SP} is identity mass of the spring and the fix plate, C_{SP} is damping coefficient of the spring, and K_{SP} is stiffness coefficient of the spring.

According to Eqs.(2.2-15), Eq.(2.2-19), Eq.(2.2-20), Eq.(2.2-21) and Eq.(2.2-22), Eqs.(2.2-18) can be modified into Eqs.(2.2-23), which are system equations of the linkage system..

$$\begin{aligned}
&\sum F_x = 0;\\
&W_{LGx} =\\
&m_{BL}(S_{Lx}+L_{Lb}(\cos\theta_L-1))'' + C_{BL}(S_{Lx}+L_{Lb}(\cos\theta_L-1))' + W_C - m_L\ddot{s}_{Lx}\\
&+(M_{SP}(S_{Lx}+L_{Lb}(\cos\theta_L-1))'' + C_{SP}(S_{Lx}+L_{Lb}(\cos\theta_L-1))' + K_{SP}(s_{ins}-s_{LC}))
\end{aligned}$$

$$\begin{aligned}
&\sum F_y = 0;\\
&W_{LGy} = m_L\ddot{s}_{Ly} + W_{LCy}
\end{aligned}$$

$$\begin{aligned}
&\sum M_{xy} = 0;\\
&W_{LGx}L_{La}\sin\theta_L + W_{LCx}L_{Lb}\sin\theta_L + \sqrt{W_{LGx}^{\;2}+W_{LGy}^{\;2}}\,\mu_{Gj}R_{GJ} + \mu_{LC}R_{LC}\sqrt{W_{LCx}^{\;2}+W_{LCy}^{\;2}} + I_L\ddot{\theta}_L =\\
&W_{LGy}L_{La}\cos\theta_L - (M_{SP}(S_{Lx}+L_{Lb}(\cos\theta_L-1))'' + C_{SP}(S_{Lx}+L_{Lb}(\cos\theta_L-1))' + K_{SP}(s_{ins}-s_{LC}))L_{LC}\sin\theta_L + W_{LCy}L_{Lb}\cos\theta_L
\end{aligned}$$

$$(2.2\text{-}23)$$

Besides the dynamic properties of the clutch actuator discussed in subsections 2.2.2 , 2.2.3 , 2.2.4 , and 2.2.5 , the relative motion from the electrical motor rotation angle θ_{WS} to output of the linkage tip D_{out}, from Figure 2.2-19, can be expressed as Eq.(2.2-24).

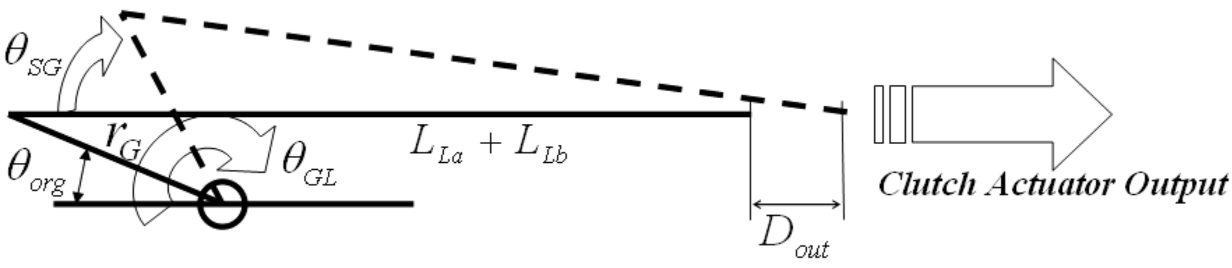

Figure 2.2-19 Motion Diagram of Clutch Actuator

$$D_{out} = S_{LC} = S_{Lx} - (L_{Lb} - L_{Lb}\cos\theta_L)$$
$$= (L_{La} + L_{Lb})\left(\sqrt{1 - \frac{{r_G}^2}{L_{La}+L_{Lb}}(\sin(\theta_{org} + \theta_{WS}/R_{SG}) - \sin\theta_{org})^2} - 1\right) + r_G(\cos\theta_{org} - \cos(\theta_{org} + \theta_{WS}/R_{SG}))$$

(2.2-24)

In Figure 2.2-19, $\theta_{GL} = \theta_{org} + \theta_{SG}$, where θ_{org} is an angle from joint to axle of worm gear, θ_{SG} is rotation angle of worm gear driven by worm shaft, which has a relation with worm shaft rotation angle: $\frac{\theta_G}{\theta_{WS}} = \frac{d_G}{d_W \tan\lambda}$.

2.3 Dynamic Analyses of AMT Transmission System

Dynamic analyses of AMT powertrain are introduced in this section. Not like clutch actuator on AMT which has just been developed in recent years; AMT powertrain is almost the same as MT powertrain which has been well developed with maturity for many years. Therefore, not like clutch actuator considered with detail on every element since it is a prototype needed to be modified, dynamic analyses of powertrain are more focus on the dynamic performance obtained from system overview and experiments data.

2.3.1 Structure of AMT Transmission System

Mechanical structure of AMT powertrain is shown in Figure 2.3-1.

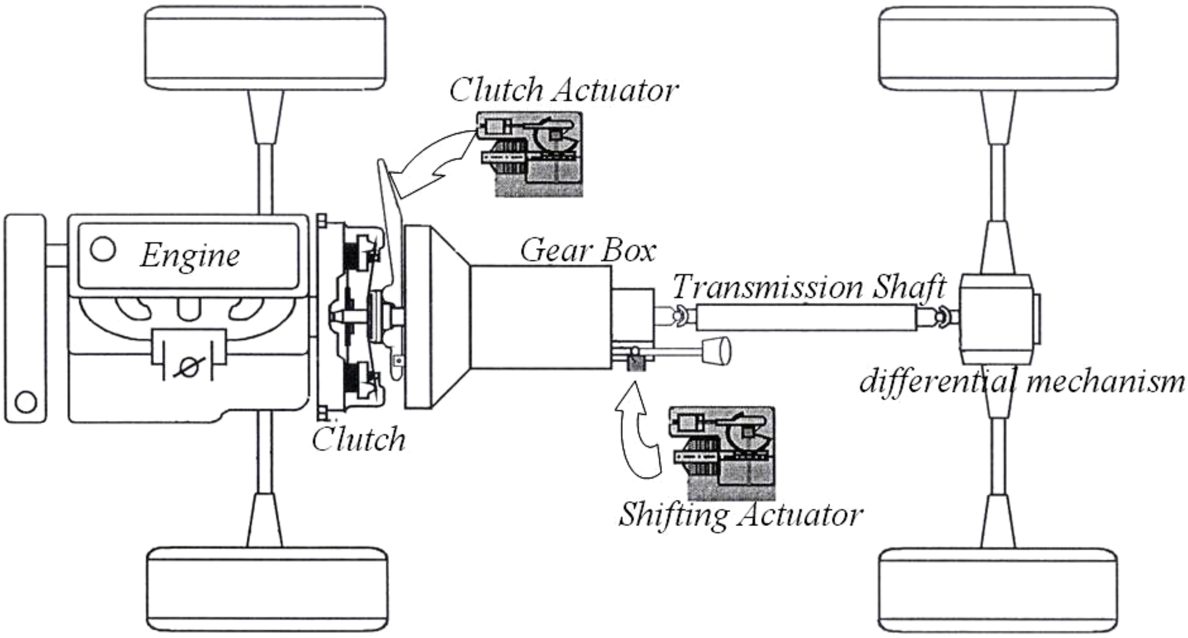

Figure 2.3-1 Structure of Automated Manual Transmission

The torque generated by engine is transmitted from engine shaft to clutch, where clutch couples output shaft of engine and input shaft of gear box. In the gear box, several gear ratios are available for input shaft to output shaft. During shifting process, synchronizer release the original engaged gear ratio, and then provides a torque to synchronize the next desired gear ratio to couple with output shaft. The desired transmission torque and rotation speed are transmitted from output shaft of gear box, throw a differential mechanism, to drive wheels.

A diagram of automated manual transmission powertrain is shown in Figure 2.3-2. System blocks of engine, clutch, gear box, and car loading are to be analyzed in the following subsections.

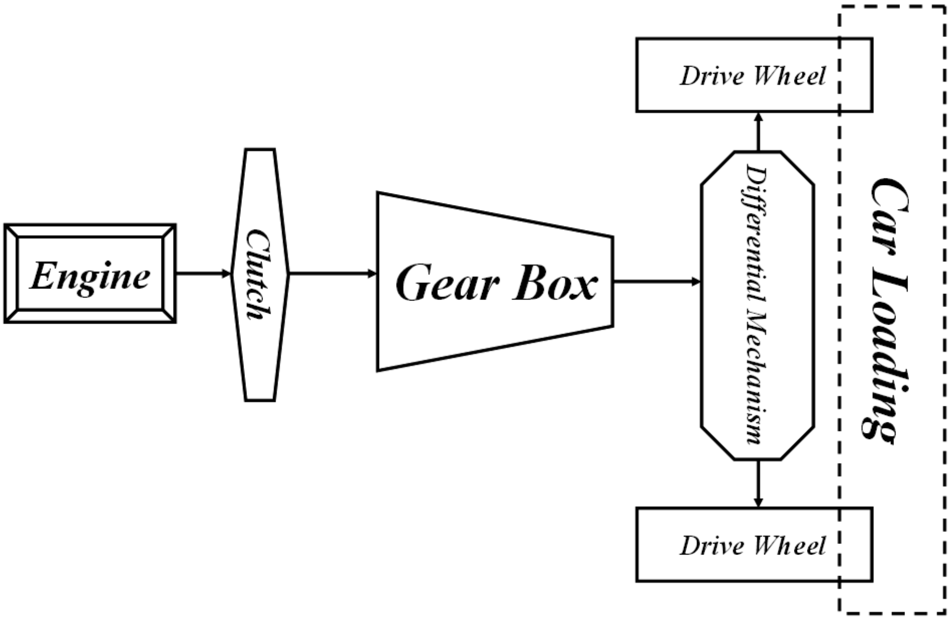

Figure 2.3-2 Automated Manual Transmission Diagram

2.3.2 Engine

Engine is power source of the transmission system, as shown in Figure 2.3-3.

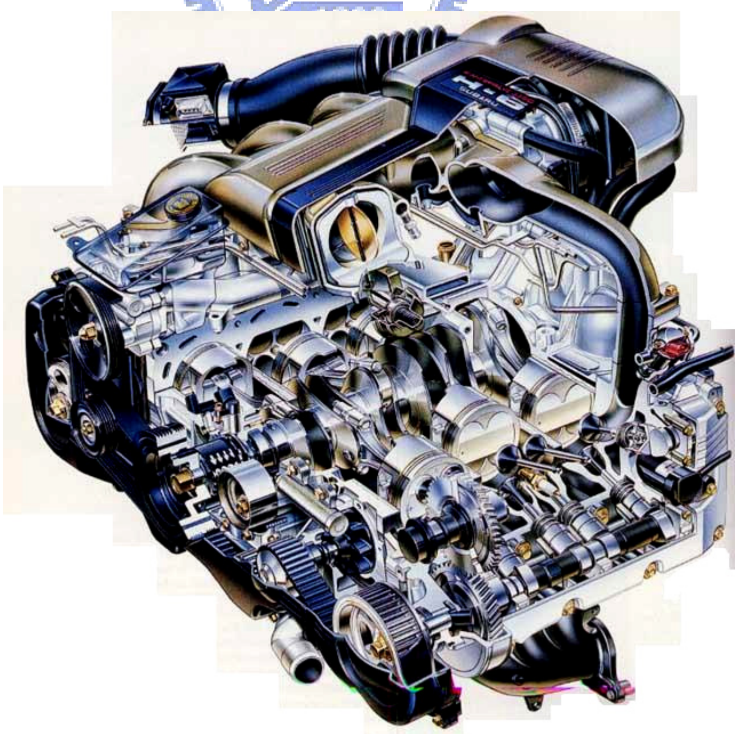

Figure 2.3-3 Vehicle Engine [14]

Since structure of engine is complex, which includes many mechanical and electrical components like piston, crank, cooling system, speed control, temperature control, firing control, etc. The analyses of engine with detail components are neither practical nor worthful here. Furthermore, study of engine is not the emphases of this study which focuses on AMT shifting and clutch control. Thus, dynamic model of engine is created into a black box, as mentioned in section 2.1 .

The character of engine to be concerned is torque, which has correlations with engine speed, throttle position, engine temperature (TPS), and other operating conditions. Even engine temperature or other operation conditions are variables of different driving condition which can't be controlled in initial. Thus, control parameters of engine torque are chosen to be engine speed and throttle position. Engine speed influences torque be generated, and depends on the dynamic characters of loading condition. Throttle position controls engine power to be generated, which is controlled by accelerator dominated by driver and Transmission Control Unit (TCU) setup up in car.

Table 1.1-1 shows experiments data of engine torque measured by ITRI.

R.P.M / **TPS**	**400**	**800**	**1200**	**1600**	**2000**	**2400**	**3200**
10	56.22	57.21	51.23	49.48	40.95	29.27	20.00
20	57.88	63	63.03	67.24	66.65	59.46	48.58
30	57.92	63.01	63.12	74.43	76.73	72.36	73.10
40	58.13	65.12	68.22	76.47	78.86	76.10	82.63
50	58.92	65.84	73.04	80.97	80.26	79.67	87.64
60	59.01	67.1	73.11	80.46	80.66	82.50	89.14
70	59.21	67.81	73.23	81.13	80.74	84.23	89.85
80	59.23	68.1	76.11	80.91	85.62	85.54	90.21
90	63.13	69.38	76.23	81.06	85.95	86.48	90.32
100	67.32	71.98	78.32	81.37	85.59	91.85	97.1

3600	4000	4400	4800	5200	5600	6000
14.10	5.12	0.82	0.00	0.00	0.00	0.00
42.22	38.40	31.32	32.03	27.96	23.15	18.61
72.22	72.08	69.63	64.75	57.67	50.34	44.18
84.68	85.68	86.45	83.00	72.48	64.38	53.43
90.54	90.44	92.67	91.86	83.12	72.01	73.48
92.65	93.92	96.38	95.64	87.33	76.83	77.68
93.74	95.06	98.75	97.45	89.41	79.28	79.38
93.94	95.58	99.66	98.36	90.55	80.39	80.23
93.87	101.90	100.00	99.02	91.15	80.94	80.91
99.53	102.01	100.08	99.25	91.24	80.85	80.99

Table 2.3-1 Engine Torque Data (Nm)

The experiments data is from engine set up on the AMT prototype car with 4-cylinders, 1200 c.c., maximum power 70 h.p. at 6000 r.p.m., and maximum torque 102 Nm at 4000 r.p.m..

From Table 2.3-1, where first column is throttle position (TPS), first row is engine speed, and others are torques generated in different conditions. System dynamic model of engine, which has two input variables: throttle position (TPS) and engine speed and one output of engine torque, can be obtained from these data.

There are two methods to obtain a dynamic model of the engine, which can provides output with any input values. The first method is curve fitting. Curve fitting uses a predetermined system equation obtained from predicting or analyzing system dynamic characters. For example, system character equation of spring are analyzed to be: $F(x)_{pre} = mx'' + cx' + kx$, where (m,c,k) are unknown system parameters, x is system input of spring deflection, and $F(x)_{pre}$ is system output of exerted force. Using optimization methods to solve optimum problem where design variables are unknown system parameters (m,c,k) and cost function is to minimum square error between force from experiments data and force from predetermined system function $F(x)_{pre}$ at positions where experiments data exist. Thus, system equation $F(x)_{pre}$ can be obtained with optimized parameters (m,c,k). The second method is to create a look-up table [17]. The experiments data is firstly set up into look-up table, and then uses interpolation-extrapolation method [15] to compute unknown data at points where no experiments data exists. Thus system output is obtainable with any system inputs from look-up table.

Since curve fitting method requires a predetermined system equation according to system characters. Engine is a compound of many subsystems and hence system equation is complicated that not easy to be predicted or analyzed. Thus, look-up table method is used to create a black box of engine dynamic model.

From Table 2.3-1, using interpolation-extrapolation method to compute output torques where no experiment data is provided, an engine torque map according to engine speed and TPS is obtained as shown in Figure 2.3-4.

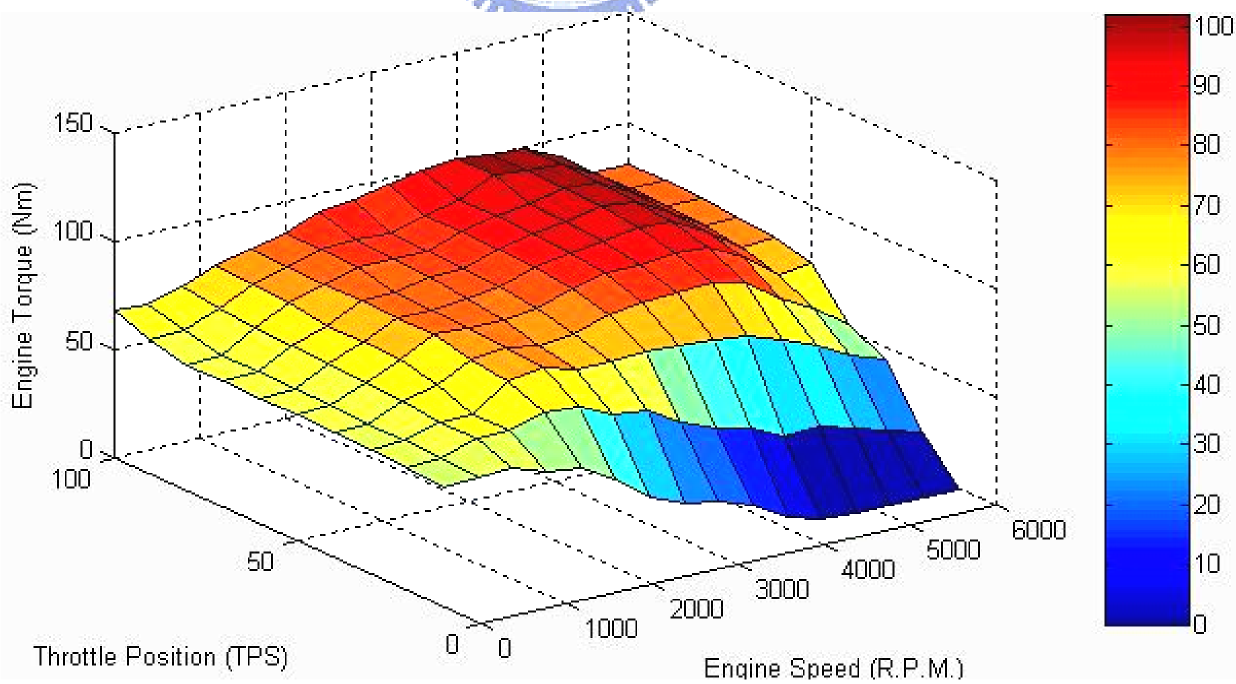

Figure 2.3-4 Map of Engine Torque

According to engine map shown in Figure 2.3-4, engine dynamic model is obtained from look-up table, which includes two inputs: throttle position and engine speed, and one output of engine torque.

2.3.3 Clutch

Clutch couples shafts of engine output and gear box input. The structure of clutch is shown in Figure 2.3-5.

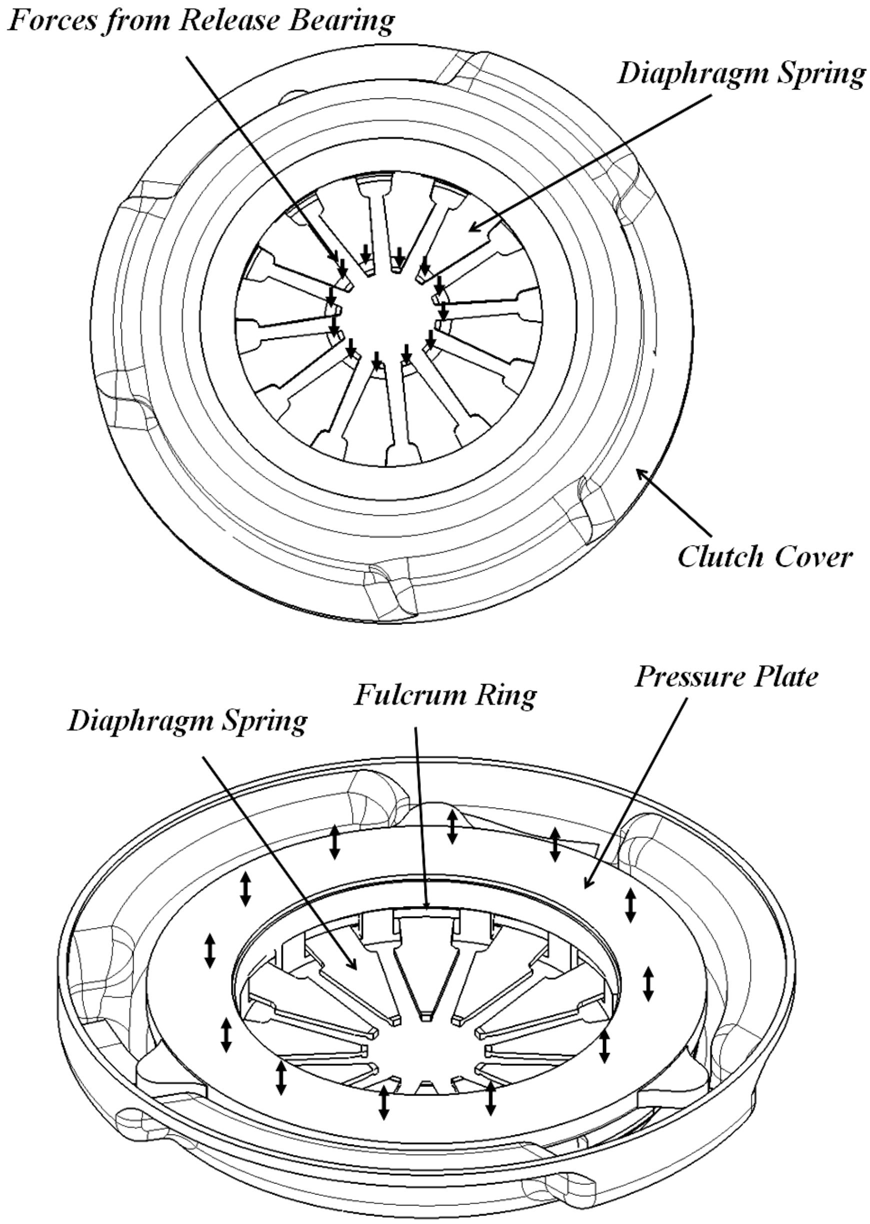

Figure 2.3-5 Structure of Clutch

The clutch cover (Figure 2.3-5) is assembled on a fly wheel which is connected with engine output shaft. The pressure plate couples with a clutch disc connected to gear box input shaft, as shown in Figure 2.3-6. Pressure plate and clutch disc is coupled by an initial threshold load from diaphragm spring after assembled. The couple force is controlled by position of a release bearing assembled beside the diaphragm spring as shown in Figure 2.2-16.

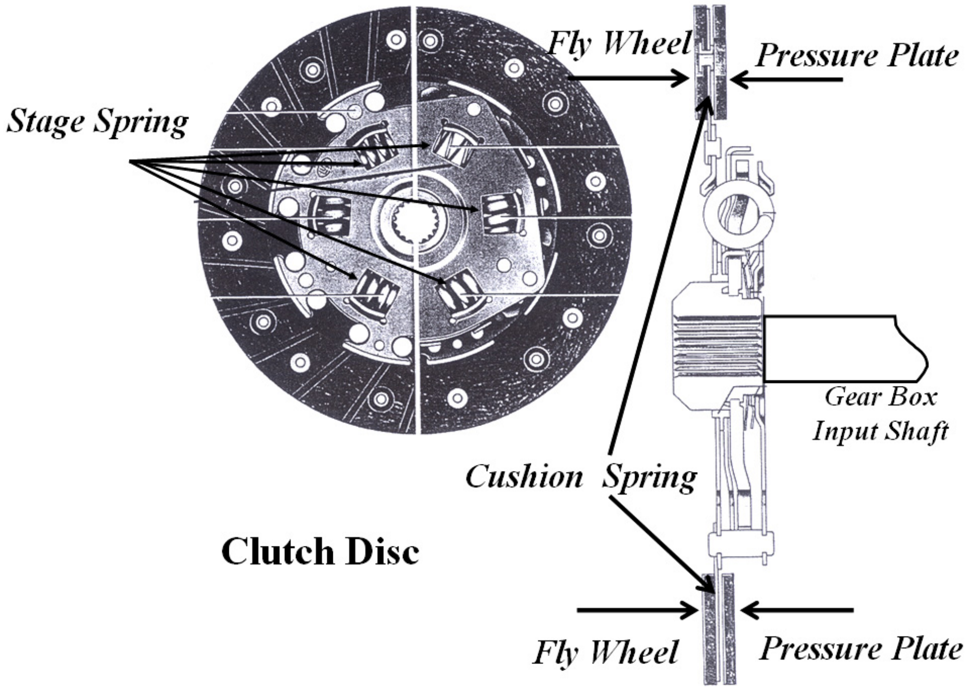

Figure 2.3-6 Structure of Clutch Disc [24]

Diaphragm spring is generally used in clutch since it has many advantages comparing to coil spring. However, the dynamic character of diaphragm spring is not like coil spring which has a linear relation between force and deflection as shown in Figure 2.3-7. The system equation of clutch should be created into black box since the dynamic characters is not easy to be expressed theoretically.

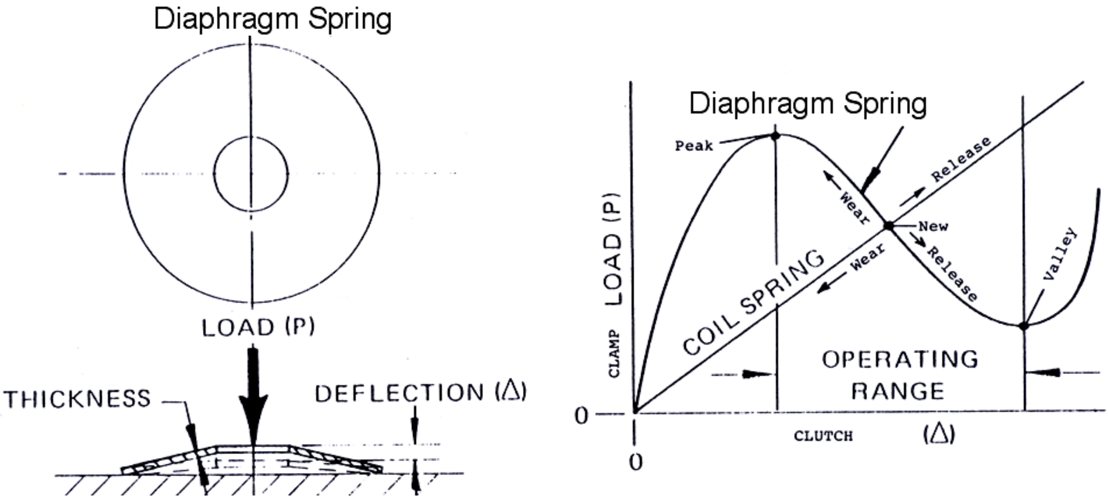

Figure 2.3-7 Character of Diaphragm Spring [24]

There are two inputs for clutch: engine torque and release bearing travel, and one output: torque transferable to input shaft of gear box.

Let engine torque be T_{eng}, engine speed be $\dot{\theta}_{eng}$, engine acceleration be $\ddot{\theta}_{eng}$, inertia moment of engine be I_{eng}, damping coefficient of engine be C_{eng}, torque from normal force between clutch disc and pressure plate be T_{cf}, output torque of clutch be T_{clu}, and rotation speed of clutch disc be $\dot{\theta}_{clu}$. From Figure 2.3-8, the relations between engine torque and torque out of clutch can be expressed as Eq.(2.3-1).

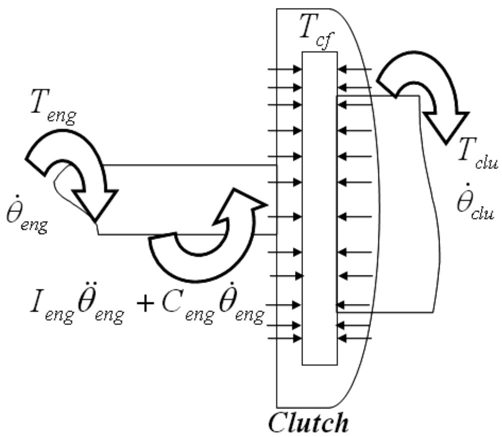

Figure 2.3-8 Free Body Diagram of Clutch

$$
\begin{aligned}
&if \quad T_{eng} - (I_{eng}\ddot{\theta}_{eng} + C_{eng}\dot{\theta}_{eng}) > T_{cf} \\
&\{T_{clu} = T_{cf}\} \\
&if \quad T_{eng} - (I_{eng}\ddot{\theta}_{eng} + C_{eng}\dot{\theta}_{eng}) \le T_{cf} \\
&\left\{\begin{array}{l}
if \quad \dot{\theta}_{eng} \ne \dot{\theta}_{clu} \\
\left[T_{clu} = T_{cf}\right] \\
if \quad \dot{\theta}_{eng} = \dot{\theta}_{clu} \\
\left[T_{clu} = T_{eng} - (I_{eng}\ddot{\theta}_{eng} + C_{eng}\dot{\theta}_{eng})\right]
\end{array}\right\}
\end{aligned}
\qquad (2.3\text{-}1)
$$

Considering torque T_{cf}, which is the torque able to be transferred from clutch caused by normal force N_{clu} between clutch disc and pressure plate. The relation between T_{cf} and N_{clu} can be expressed as Eq.(2.3-2) [24].

$$
T_{cf} = \frac{4}{3}\mu_{clutch}N_{clu}\left[\frac{(Ro^3 - Ri^3)}{(Ro^2 - Ri^2)}\right] \qquad (2.3\text{-}2)
$$

Where μ_{clutch} is friction coefficient between clutch disc and pressure plate, Ro is outer radius of pressure plate, and Ri is inner radius of pressure plate.

Relation between normal force N_{clu}, release bearing motion D_{out} and release bearing loading W_c is emphasis of the black box.

The deformation of diaphragm spring can't base on Hook's law, since it has no linear relation between loading force and deformation. Moreover, using finite element method to find the relation between loading and deformation is not easy since most of finite element packages are linear which always simplify the transform term it use, for example: Catia. The best way to obtain data of black box is from experiments.

The real line in Figure 2.3-9 shows relation between release bearing travel and release bearing load from experiments [ITRI].

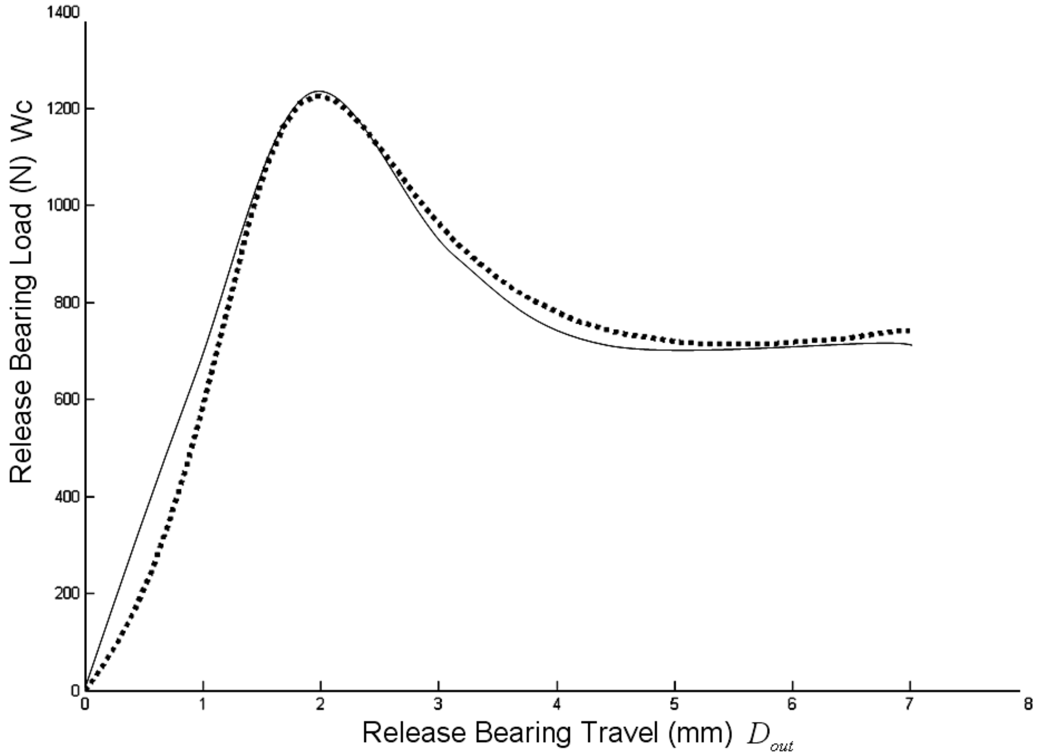

Figure 2.3-9 Release Bearing Travel vs. Load [ITRI]

Using curve fitting method, a fitted curve is obtained as shown in Eq.(2.3-3), and drawn in dash line in Figure 2.3-9.

$$W_c(D_{out}) = (74.578D_{out}{}^3 - 194.726D_{out}{}^2 + 1053.5D_{out})/(D_{out}{}^2 - 3.02D_{out} + 3.608)$$

(2.3-3)

Cushion spring plays an import rule in clutch engagement. As shown in Figure 2.3-10, the range of cushion spring deflection includes the entire zone of engage modulation. Thus, cushion spring deflection is equal to travel of pressure plate D_p before full disengagement, and normal force N_{clu} can be seen as deflection force of cushion spring. In conclusion, the normal force N_{clu} can be represented in terms of pressure plate travel D_p which has a relation with release bearing travel D_{out}.

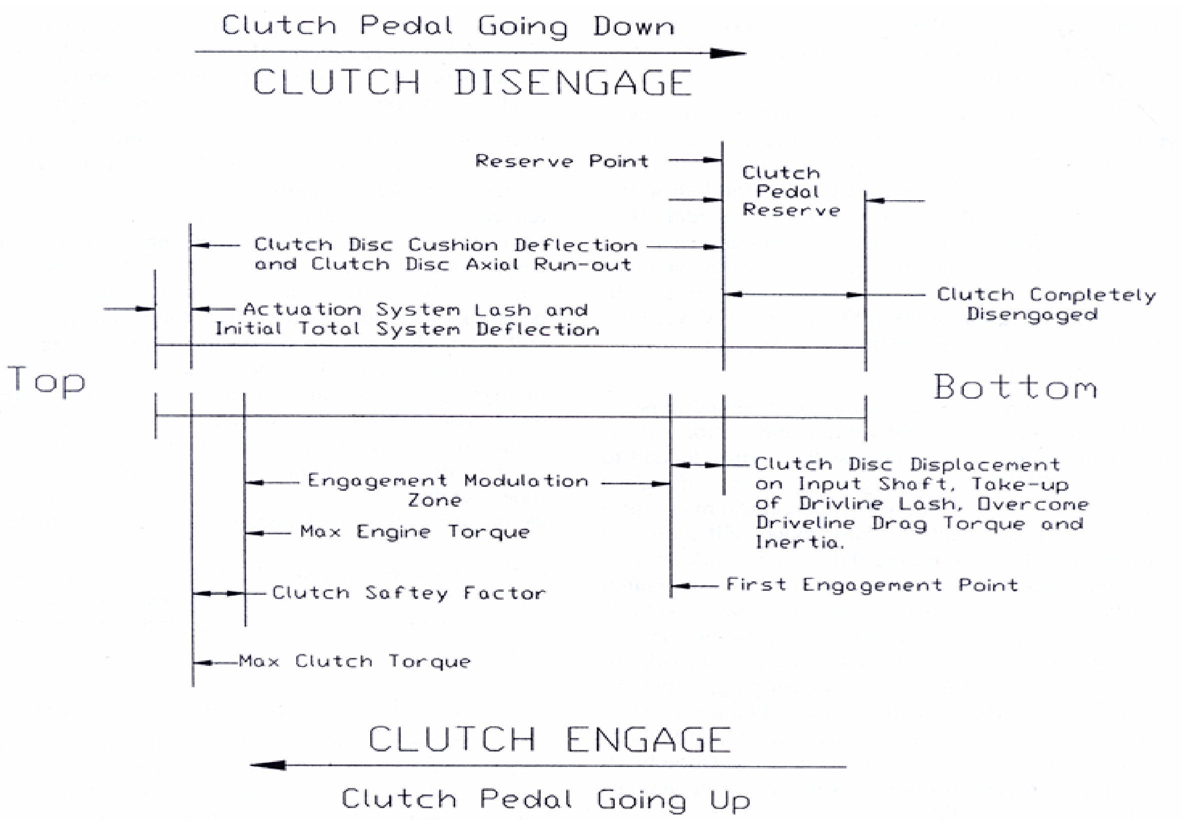

Figure 2.3-10 Clutch Pedal Travel Description [24]

The real line in Figure 2.3-11 shows the relation between release bearing travel and pressure plate travel from experiments data. From these data, a five degrees polynomial equation, which represents the relation between D_{out} and D_p, can be obtained using curve fitting method. The equation is shown in Eq.(2.3-4)

$$D_p(D_{out}) = 0.0002615D_{out}^{\ 5} - 0.005043D_{out}^{\ 4} + 0.02873D_{out}^{\ 3} - 0.02388D_{out}^{\ 2} + 0.004408D_{out} \tag{2.3-4}$$

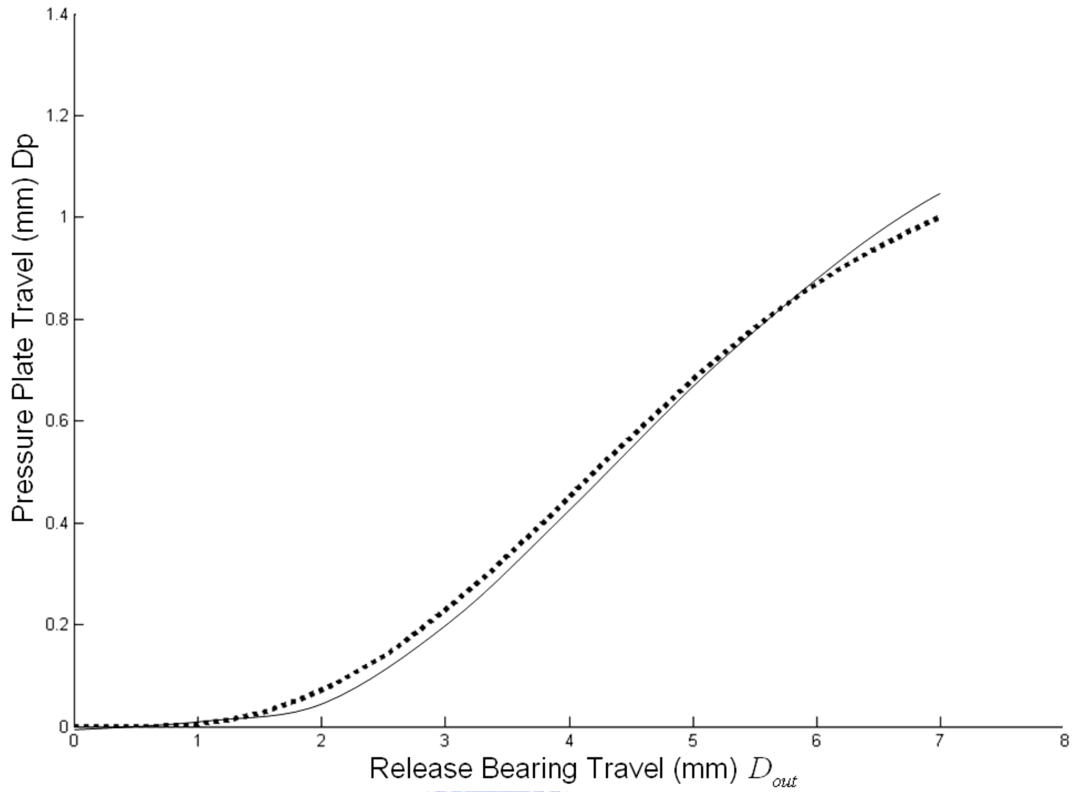

Figure 2.3-11 Relation between R.B. Travel and P.P. Travel [ITRI]

And then from Figure 2.3-12 the relation between deflection and load of cushion spring can be obtained as Eq.(2.3-5)

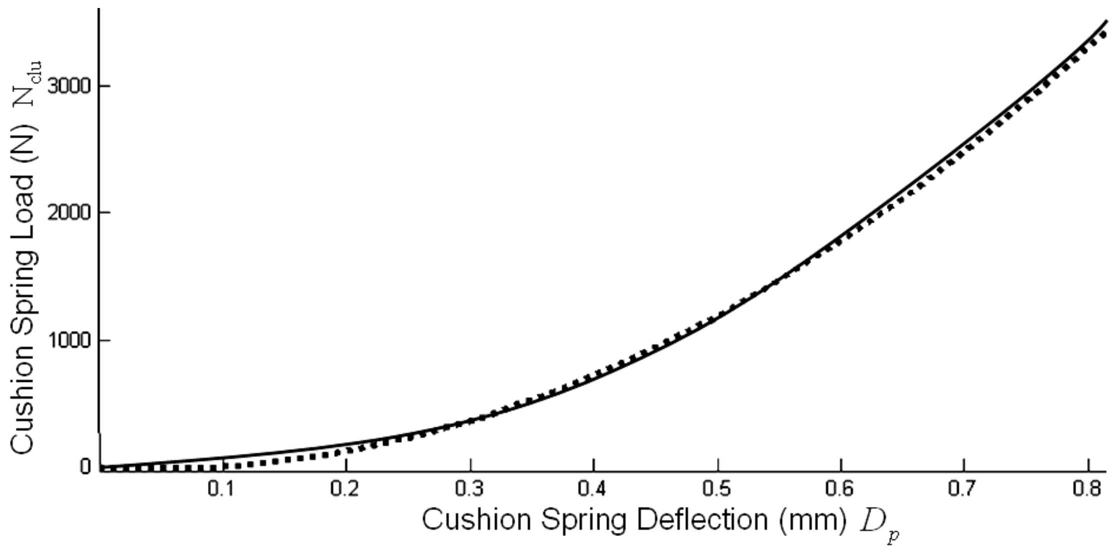

Figure 2.3-12 Cushion Spring Character [ITRI]

$$N_{clu}(D_p) = 5819 D_p^{\ 2} - 545 D_p \tag{2.3-5}$$

From Eq.(2.3-4) and Eq.(2.3-5), a function of $N_{clu}(D_{out})$ can be obtained.

To provide more comfort to driver and passenger, stage springs, which work like a torsional damper, always used in clutch as shown in Figure 2.3-6. The stage springs play important rules on clutch, it reduce gear rattle and boom noises transmitted into the passenger compartment [24].

Considering with stage springs, a free body diagram from output shaft of engine to input shaft of gear box is shown in Figure 2.3-13.

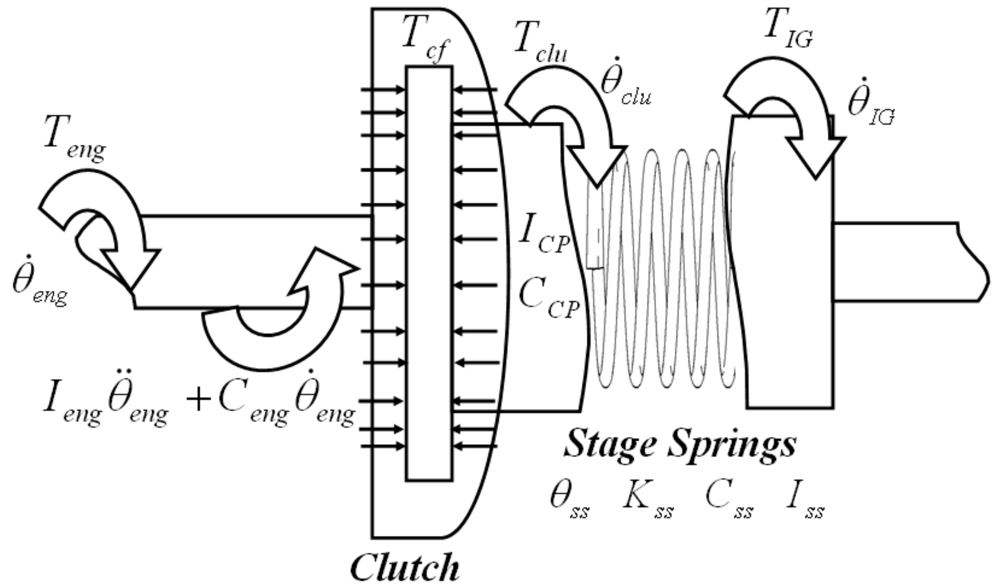

Figure 2.3-13 Free Body Diagram of Clutch

From Figure 2.3-13, and considering Eq.(2.3-3), Eq.(2.3-4), and Eq.(2.3-5), system equation of clutch can be expressed as Eqs.(2.3-6).

$$
\begin{aligned}
&if \quad T_{eng} - (I_{eng}\ddot{\theta}_{eng} + C_{eng}\dot{\theta}_{eng}) > T_{cf} \\
&\left\{ T_{IG} = T_{cf} - \left[(I_{CP} + \frac{1}{2} I_{ss})\ddot{\theta}_{clu} + (C_{CP} + C_{ss})\dot{\theta}_{clu} \right] \right\} \\
&if \quad T_{eng} - (I_{eng}\ddot{\theta}_{eng} + C_{eng}\dot{\theta}_{eng}) \le T_{cf} \\
&\left\{ \begin{array}{l}
if \quad \dot{\theta}_{eng} \ne \dot{\theta}_{clu} \\
\left[T_{IG} = T_{cf} - \left[(I_{CP} + \frac{1}{2} I_{ss})\ddot{\theta}_{clu} + (C_{CP} + C_{ss})\dot{\theta}_{clu} \right] \right] \\
if \quad \dot{\theta}_{eng} = \dot{\theta}_{clu} \\
\left[T_{IG} = T_{eng} - (I_{eng}\ddot{\theta}_{eng} + C_{eng}\dot{\theta}_{eng}) - \left[(I_{CP} + \frac{1}{2} I_{ss})\ddot{\theta}_{clu} + (C_{CP} + C_{ss})\dot{\theta}_{clu} \right] \right]
\end{array} \right\} \\
&\left\{ \begin{array}{l}
\theta_{IG} = \theta_{clu} - \theta_{ss}; \\
K_{ss}\theta_{ss} + C_{ss}\dot{\theta}_{ss} = T_{IG};
\end{array} \right.
\end{aligned}
\qquad (2.3\text{-}6)
$$

Where I_{CP} is inertia moment of clutch disc, I_{ss} is inertia moment of stage springs, C_{CP} is damping coefficient of clutch disc, C_{ss} is damping coefficient of stage springs, T_{IG} is output torque of clutch, θ_{IG} is rotation angle of output shaft of clutch, θ_{ss} is deformation angle of stage springs, and K_{ss} is stiffness coefficient of stage springs.

According to [20], transmission efficiency of total powertrain can be simplified to the efficiency of engine output torque. Thus, the term T_{eng} in Eq.(2.3-6) is assigned to be $\eta_{eng}T_{engR}$. Where η_{eng} is transmission efficiency of total powertrain, and T_{engR} is torque generated by engine as shown in engine map of Figure 2.3-4.

2.3.4 Gear Box

The analyses of gear box in this subsection include components of powertrain from clutch to driving wheels. As shown in the shadow parts of Figure 2.3-14.

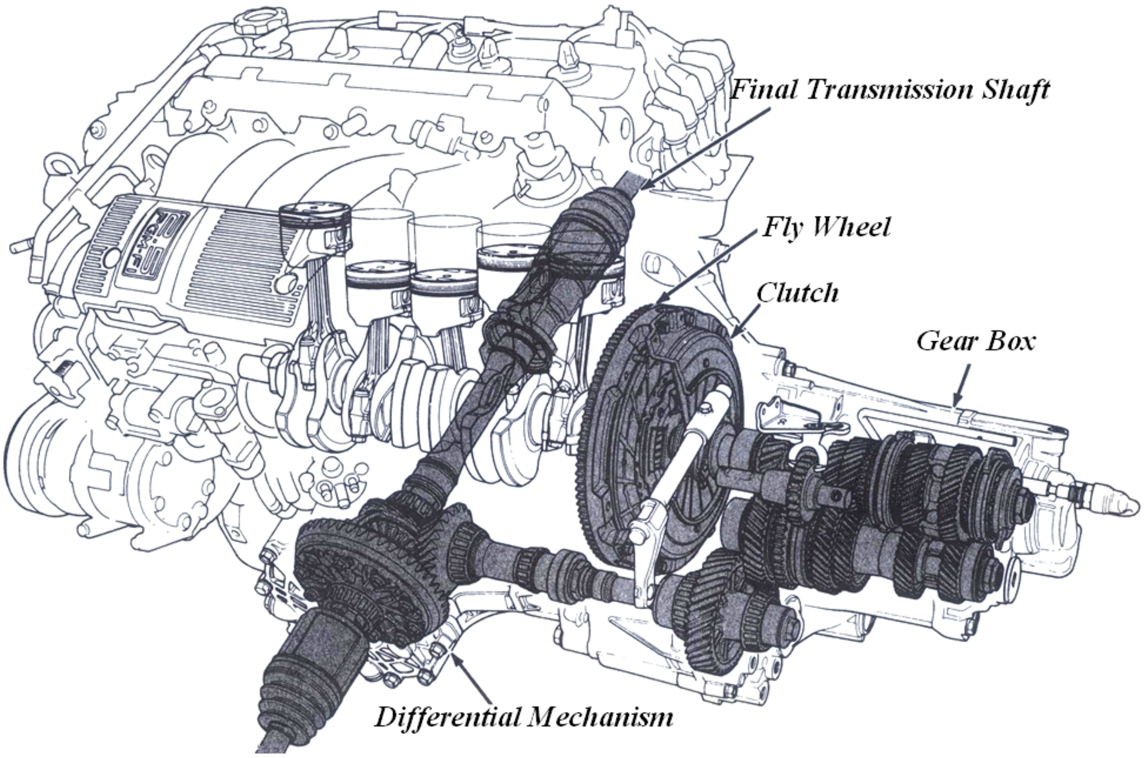

Figure 2.3-14 Final Transmission Structures [18]

Figure 2.3-14 shows final components of powertrain within a front-engine front-drive (FF) car, which is the type used in AMT prototype car. Torque is transmitted from clutch mentioned before into gear box, after a transformation by gears, transmitting the desired torque to output shaft. Finally, through a differential mechanism which only works while turning, the power is transmitted to the driving wheels to move a car. A simplified final powertrain diagram is shown in Figure 2.3-15.

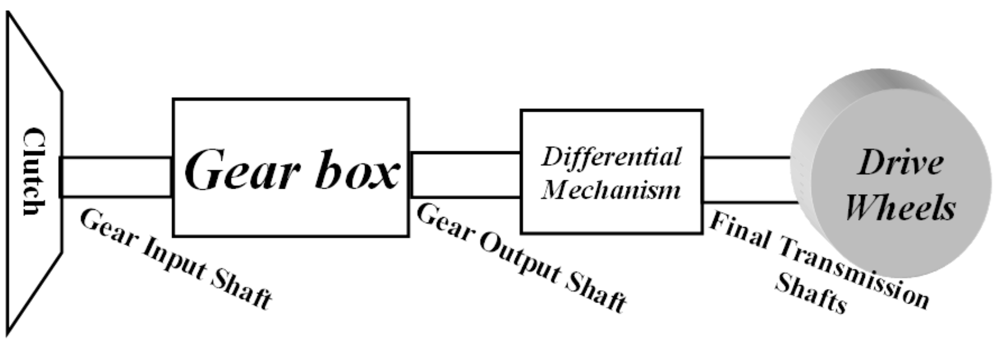

Figure 2.3-15 Simplified Final Powertrain Diagram

From Figure 2.3-15, the transmission components from clutch to drive wheels, which are objectives of this subsection, can be divided into two components: gear box and differential mechanism.

The inner structure of gear box is shown in Figure 2.3-16.

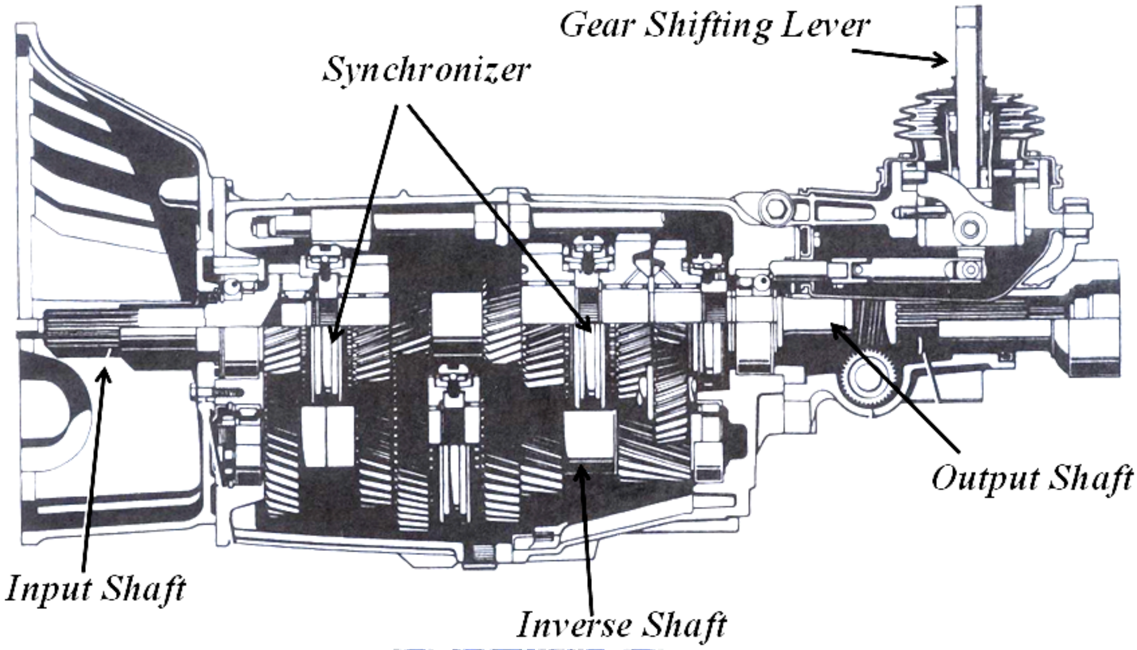

Figure 2.3-16 Inner Structure of Gear Box [18]

The torque transmitted from input shaft is firstly transmitted to inverse shaft by a couple of gears, and then the inverse shaft couples with output shaft by one of the gears ratios. Synchronizers are set between output shaft and gears on output shaft, which are used to engage gear with desired gear ratio with output shaft. A process of synchronizer engaging a gear is shown in Figure 2.3-17.

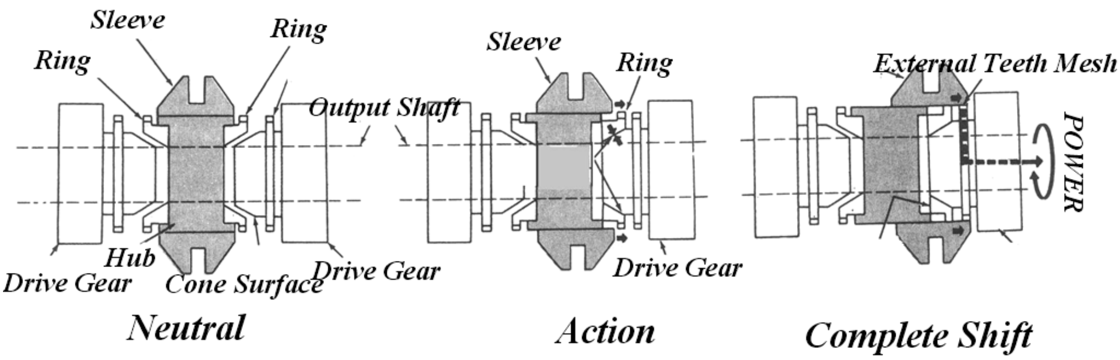

Figure 2.3-17 Synchronizer Action [18]

As shown in Figure 2.3-17. To shift the transmission into gear, the synchronizer sleeve is moved toward that gear. The sleeve slides on the hub splines and carries the three key with it. The keys butt against the synchronizer ring and push it toward the gear. This beings the cone surface in the ring into contact with the cone surface on the gear. Friction between the synchronizer ring and the gear brings the two into synchronous rotation, which rotate at the same speed. As shown in the right figure of Figure 2.3-17, when the external teeth on the synchronizer ring and the gear rotate at the same speed, the sleeve slides over them. This locks the gear to the shaft and completes the shift. Power flows from the gear, through the synchronizer sleeve and hub, to the shaft [18]. The same reverse process from right to left of Figure 2.3-17 is done to disengage a gear while shifting starts.

By the use of synchronizer, different transmission gear ratios in gear box can be shifted by disengage the original gear ratio and engage to the desire gear ratio, as shown in Figure 2.3-18. Note that synchronizer can be worked only after clutch disengages the transmission from engine which leads to a free rotation of input shaft.

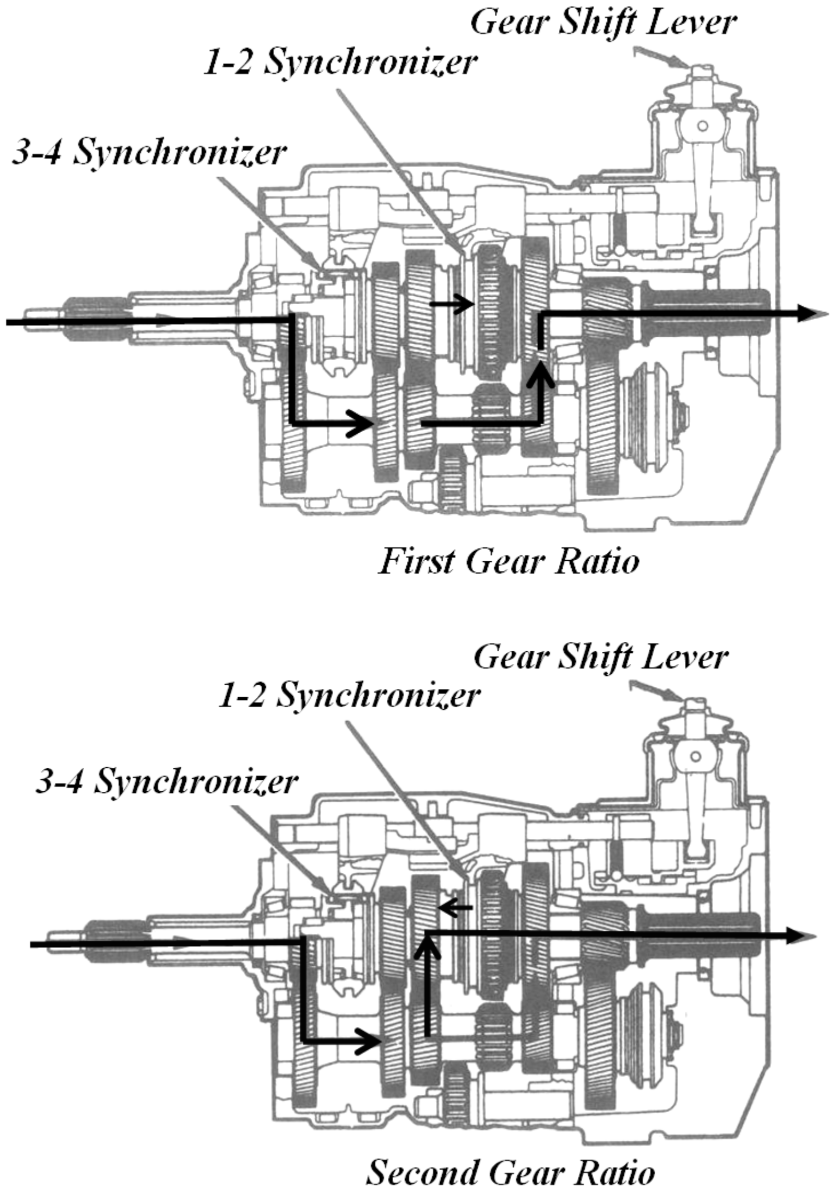

Figure 2.3-18 Power Flow of Gear Ratio 1&2 [18]

The first step to analyze dynamic characters of the gear box is to consider with a general condition while no shifting process is proceeded. A free body diagram of such condition is shown in Figure 2.3-19.

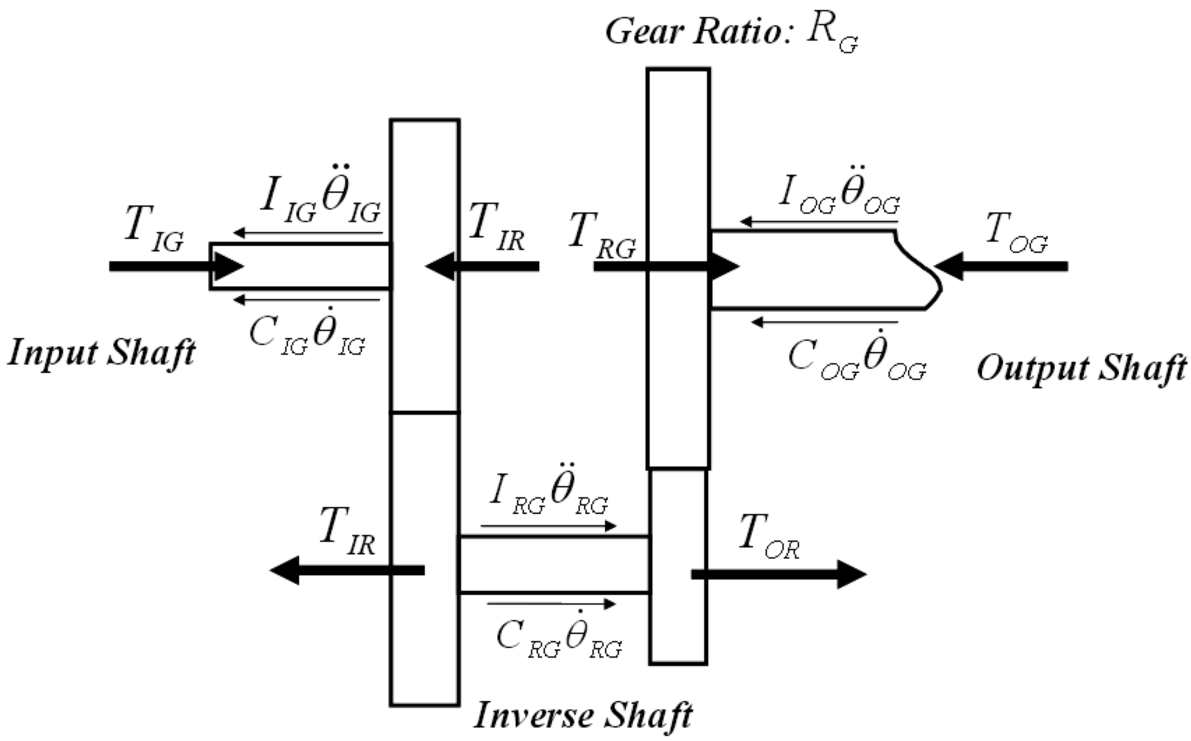

Figure 2.3-19 Free Body Diagram of Gear Box

Where the equilibrium equations can be expressed as Eqs.(2.3-7)

$$\begin{aligned} T_{IG} &= T_{IR} + I_{IG}\ddot{\theta}_{IG} + C_{IG}\dot{\theta}_{IG} \\ T_{IR} &= T_{OR} + I_{RG}\ddot{\theta}_{RG} + C_{RG}\dot{\theta}_{RG} \\ T_{RG} &= T_{OG} + I_{OG}\ddot{\theta}_{OG} + C_{OG}\dot{\theta}_{OG} \end{aligned} \quad (2.3\text{-}7)$$

T_{IG} is the torque transmitted from clutch, T_{IR} is the torque transmitted from input shaft to inverse shaft where the gear ratio between these two shaft is usually one, T_{OR} is the reacting torque from output shaft to the inverse shaft, T_{RG} is the torque from inverse shaft that acts on output shaft, T_{OG} is the torque transmitted to differential mechanism by output shaft; I_{IG}, I_{RG}, I_{OG} are inertia moments of input shaft, inverse shaft, and output shaft; C_{IG}, C_{RG}, C_{OG} are damping coefficients on input shaft, inverse shaft, and output shaft; θ_{IG}, θ_{RG}, θ_{OG} are rotation angle of input shaft, inverse shaft, and output shaft.

Let transmission gear ratio in gear box be R_G, and gear ratio between input shaft and inverse shaft be one, the relationships between θ_{IG}, θ_{RG}, θ_{OG} and T_{IR}, T_{OR}, T_{RG} can be expressed as:

$$\begin{aligned}
&\theta_{IG} = \theta_{RG}; \dot{\theta}_{IG} = \dot{\theta}_{RG}; \ddot{\theta}_{IG} = \ddot{\theta}_{RG} \\
&\theta_{OG} = \theta_{RG}/R_G; \dot{\theta}_{OG} = \dot{\theta}_{RG}/R_G; \ddot{\theta}_{OG} = \ddot{\theta}_{RG}/R_G \\
&T_{IR} = T_{IR} \\
&T_{RG} = R_G T_{OR}
\end{aligned} \tag{2.3-8}$$

From Eq.(2.3-8), Eq.(2.3-7) can be expressed as a simplified system equation:

$$T_{IG} = (I_{IG} + I_{RG} + I_{OG}/R_G{}^2)\ddot{\theta}_{IG} + (C_{IG} + C_{RG} + C_{OG}/R_G{}^2)\dot{\theta}_{IG} + T_{OG}/R_G \tag{2.3-9}$$

Te second step is to consider the situation when shifting process is proceeded. Synchronizer exerts a torque T_{syn} on output shaft. The magnitude of T_{syn} is assumed to be constant here, since the torque depends on the shifting speed and rotating speed that is not easy to be determined theoretically, and the exact T_{syn}, which is not the emphases of this study, also plays no important rule on the shifting process. The direction of T_{syn} depends on shifting directions of up-shift or down-shift. When up-shift is executed, rotation speed of gear to be synchronized is slower than output shaft. Thus, synchronizer exerts a positive T_{syn} against the rotation direction to speed up the gear. In the same way, a negative T_{syn} is exerted while down-shift is executed. A free body diagram of up-shift process is shown in Figure 2.3-20.

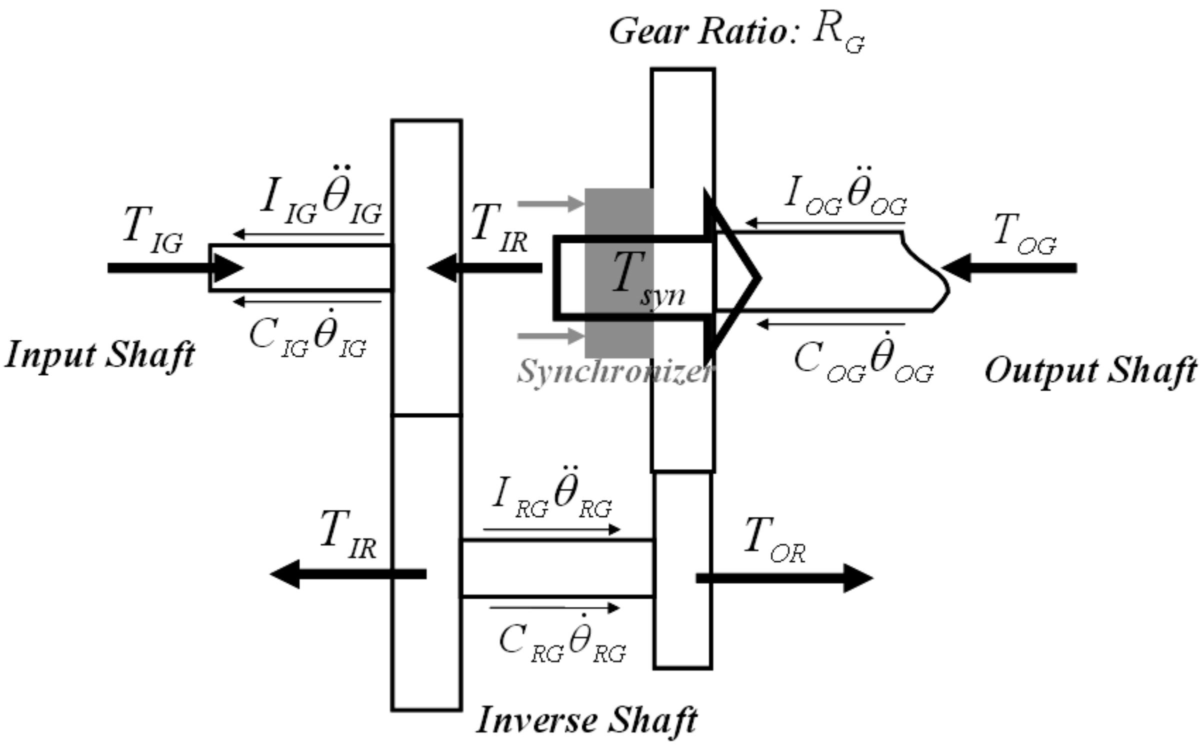

Figure 2.3-20 Free Body Diagram of Shifting Process

The system dynamic equations according to Figure 2.3-20 can be expressed as:

$$\begin{aligned} T_{IG} &= (I_{IG} + I_{RG})\ddot{\theta}_{IG} + (C_{IG} + C_{RG})\dot{\theta}_{IG} + T_{syn}/R_G \\ T_{syn} &= T_{OG} + I_{OG}\ddot{\theta}_{OG} + C_{OG}\dot{\theta}_{OG} \end{aligned} \qquad (2.3\text{-}10)$$

Considering with differential mechanism, it is only used to provide different speed upon the two drive wheels while vehicle is turning, such affection doesn't matter on transmission in general condition of straight driving which is the case in this study. However, differential mechanism still provides a final gear ratio that should be considered. Thus, it is simplified to a couple of gears with gear ratio R_{DM} .

The free body diagram of such simplified differential mechanism is shown in Figure 2.3-21, where the input shaft is output shaft of gear box, and the output shaft connects to drive wheels.

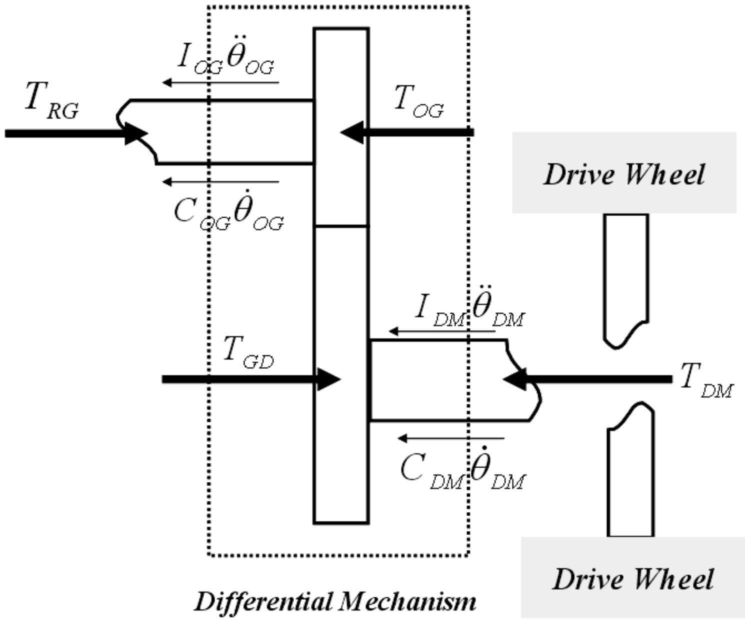

Figure 2.3-21 Free Body Diagram of Differential Mechanism

System dynamic equation of differential mechanism according to Figure 2.3-21 can be expressed as:

$$T_{GD} = I_{DM}\ddot{\theta}_{DM} + C_{DM}\dot{\theta}_{DM} + T_{DM} \tag{2.3-11}$$

Where T_{GD} is the torque transmitted from output shaft of gear box, I_{DM} is the identity inertia moment of shafts from differential mechanism to output shafts on drive wheels, C_{DM} is the damping coefficient of identity shaft, θ_{DM} is the rotation angle of output shaft which equals to rotation angle of drive wheels, and T_{DM} is the torque exerted on drive wheels.

Let the final gear ratio in differential mechanism be R_{DM}, the following relations are obtained:

$$\begin{aligned} &\theta_{DM} = {\theta_{OG}}/{R_{DM}}\,;\ddot{\theta}_{DM} = {\ddot{\theta}_{OG}}/{R_{DM}}\,;\dot{\theta}_{DM} = {\dot{\theta}_{OG}}/{R_{DM}} \\ &T_{GD} = R_{DM}T_{OG} \end{aligned} \tag{2.3-12}$$

From Eqs.(2.3-9), Eqs.(2.3-11), and Eqs.(2.3-12), the system dynamic equation of final powertrain from output shaft on clutch to output shaft on drive wheels can be expressed as Eq.(2.3-13), which is in condition when no shifting process is proceeded.

$$T_{IG} = (I_{IG} + I_{RG} + I_{OG}/{R_G}^2 + I_{DM}/{R_G}^2{R_{DM}}^2)\ddot{\theta}_{IG} + (C_{IG} + C_{RG} + C_{OG}/{R_G}^2 + C_{DM}/{R_G}^2{R_{DM}}^2)\dot{\theta}_{IG} + T_{DM}/R_G R_{DM} \tag{2.3-13}$$

The system dynamic equations while shifting process is proceeded, from Eqs.(2.3-10), Eqs.(2.3-11), and Eqs.(2.3-12), can be expressed as Eq.(2.3-14), which is an up-shift process.

$$\begin{aligned} T_{IG} &= (I_{IG} + I_{RG})\ddot{\theta}_{IG} + (C_{IG} + C_{RG})\dot{\theta}_{IG} + T_{syn}/R_G \\ T_{syn} - T_{DM}/R_{DM} &= (I_{OG} + I_{DM}/{R_{DM}}^2)\ddot{\theta}_{OG} + (C_{OG} + C_{DM}/{R_{DM}}^2)\dot{\theta}_{OG} \end{aligned} \tag{2.3-14}$$

From θ_{DM}, car speed v_{car} can be obtained as $v_{car} = \theta_{DM} r_{wh}$, where r_{wh} is radius of wheels.

In shifting process, system dynamic equation transfers from Eq.(2.3-13) to Eqs.(2.3-14) where synchronizer is actuated to disengage the original gear and to engage the new gear of the next desired transmission ratio. After shifting process is completed, where rotation speed is the same between the new gear and the output shaft of the gear box as in the condition of: $\dot{\theta}_{RG} = \dot{\theta}_{OG}/R_{G(NEW)}$, system equation transfers from Eqs.(2.3-14) to Eq.(2.3-13) .

2.3.5 Car Loading

There are two general forms to express the car loading on transmission system: full car equation and components equation. Full car equation concerns the relation between car loading and square of car speed, using experiments data to mesh the equation in the form of $F_x = f_1 v^2{}_{car} + f_0$, where F_x is the force of car loading, f_1 and f_0 are coefficients to be determined. Components equation concerns affections from rolling resistance, aerodynamic, gravity, and other outside loadings on a car, since it is an expression with part modules which is easier to be modified. Components equation is used in this study and is introduced in the following essay.

Forces that act on a car are shown in Figure 2.3-22.

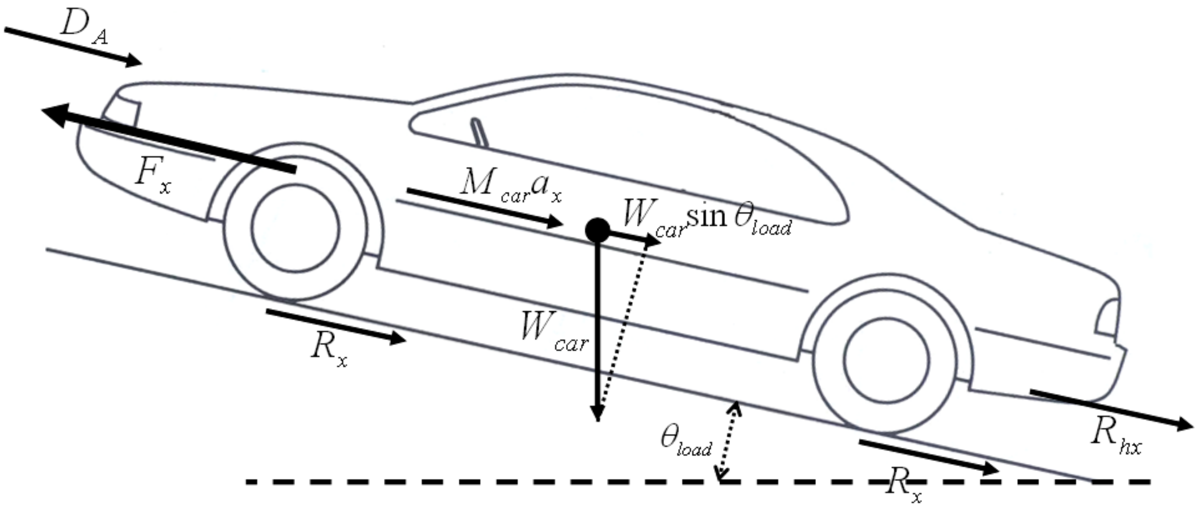

Figure 2.3-22 Forces Acting on a Vehicle

From Figure 2.3-22, loading condition of a car can be expressed as Eq.(2.3-15) [20], which is in the form of components equation.

$$F_x = M_{car}a_x + R_x + D_A + R_{hx} + W_{car}\sin\theta_{load} \qquad (2.3\text{-}15)$$

Where M_{car} is mass of the car, a_x is longitudinal acceleration of the car, R_x is rolling resistance forces on the four wheels, D_A is aerodynamic drag force, R_{hx} is hitch forces, W_{car} is the force according to the gravity that acts on a car, which is known as gM_{car} where g is the gravity, θ_{load} is gradient angle of the road, and F_x is the force from drive wheels transmitted from engine.

In general, rolling resistance forces R_x on wheels can be expressed as:

$$R_x = f_r W_{car} \qquad (2.3\text{-}16)$$

Where f_r is called rolling resistance coefficient.

The aerodynamic drag force D_A can be expressed as:

$$D_A = \frac{1}{2}\rho v^2{}_{car} C_D A = \frac{1}{2}\rho(\dot{\theta}_{DM} r_{wh})^2 C_D A \qquad (2.3\text{-}17)$$

Where ρ is air density, r_{wh} is radius of wheels, and v_{car} is speed of the car which equals to $\dot{\theta}_{DM} r_{wh}$ if the wind speed is zero, otherwise, it should be the relative speed between car and wind. C_D is aerodynamic drag coefficient, which is determined empirically from car, and A is frontal area of the car.

According to Eq.(2.3-13) where T_{DM} is the torque that acts on wheels, Eq.(2.3-15) can be expressed as:

$$T_{DM} = I_{wh}\ddot{\theta}_{DM} + C_{wh}\dot{\theta}_{DM} + (M_{car}\ddot{\theta}_{DM} r_{wh} + R_x + D_A + R_{hx} + W_{car}\sin\theta_{load})r_{wh} \qquad (2.3\text{-}18)$$

Where I_{wh} is the identity inertia moment of the four wheels and C_{wh} is the identity damping coefficient on wheels.

2.4 Dynamic Model Creation

Dynamic models according to system dynamic equations discussed in previous subsections are created into computer program in this section.

There are many commercial packages available to create dynamic models and do dynamic analysis, for example: Adams®, Working Model®, Visual Nastrain®, etc. However, the difficulty in engineering is not just the analyses of dynamic or static problems, but the problems to combine dynamic characters to control design, since the final purpose of engineering is to create a machine that is workable as expectation. Packets like Adams®, Working Model®, Visual Nastrain®, etc. are convenient for dynamic analysis. However, it is