

A Study of the Gait Control of a Quadruped Walking Vehicle

TSU-TIAN LEE AND CHING-LONG SHIH

Abstract—Some of the fundamental problems of the gait control of a quadruped walking vehicle are addressed. It is shown that the product of the duty factor and the stride length is equal to the length of the boundary of the reachable area of the leg. Furthermore, the mathematical expression representing the relationship between the stability margin, the stride length and duty factor are also formulated. These equations are expressed in terms of quantities that specify the configuration of the quadruped walking vehicle. Based on the derived results, a graphical approach to determine the required stride length and the duty factor that corresponding to a regular gait with a prescribed static stability margin is presented. This graphical approach is then adopted to determine the regular gait of a quadruped walking vehicle, and the results agree with the analytic approach.

I. INTRODUCTION

CURSORIAL animals are considered to have high off-road capability superior to that of the conventional wheeled or track vehicles. This fact has motivated considerable attempts to develop the multilegged walking vehicles. During the past several years, the generalized walking vehicle with multiple degrees-of-freedom was considered, and the computer-coordinated control of legs to complete the "walk" by suitably choosing the sequence for lifting legs was investigated. As a result, the first multilegged vehicle ever designed to walk autonomously under full computer control is a four-legged vehicle called "phoney pony" built by McGhee and Frank [1], [2]. Since then, various multilegged walking machines have been built. In particular, a one-legged hopping machine was built by Raibert [22]. This hopping machine must hop continually in order to maintain balance. A series of computer-controlled bipeds have been developed by Kato [29], Miura and Shimoyama [15], independently and individually. These bipeds could climb stairs under the control of an operator. A four-legged walking machine called PVII was built by Hirose and Umetani [10], [11]. The PVII can maintain a horizontal body orientation and has contact sensor on each foot to detect contact with obstacles in its path or with the ground. The OSU Hexapod, a six-legged walking machine, was built by McGhee *et al.* [5] at the Ohio State University in 1977. The OSU Hexapod, built primarily to study control algorithms for a walking machine, has been continuously improved by adding sensors systems and vision systems. A self-contained, terrian-

adaptive walking machine, known as adaptive suspension vehicle (ASV), is being developed at the Ohio State University.

In 1983, another six-legged vehicle was built by Raibert and Sutherland [24]. This machine was the first self-contained walking vehicle controlled by an onboard microcomputer.

Observing all these developed walking vehicles, we notice that, in general, the basic control problems that all the walking vehicles confront are 1) how to generate the trajectory and the average speed; 2) how to determine the best sequence for lifting and placing the feet; 3) what the suitable distance that each leg transfer in order to maintain the prescribed stability is; 4) how to control the body's inclination and height; and 5) how to develop a measurement system and information processing method to support the motion planning. The problem of choosing the best sequence for lifting and placing the feet of a walking vehicle is the gait-selection problem. Gait was originally studied by zoologists attempting to understand animal locomotion. It has not been studied systematically by researchers for the past twenty years. In the following, we shall briefly review previous works on gait study.

In 1968, McGhee [1] gave a strong mathematical base for the analysis of walking gaits. He defined the basic terminologies such as stride length, duty factor, phase ... etc. Furthermore, McGhee and Frank [2] used longitudinal stability margins to study statically stable walking gaits of a quadruped. They proved mathematically that there is a unique optimum gait which maximizes the longitudinal stability margin of a quadruped. The resultant gait stability margin is $\beta - 3/4$, where β is the duty factor or ratio of the contact phase time to the stride period.

Later, McGhee and Jain [3] defined another mathematical description of a gait, the "event sequence." They showed that the 5040 theoretically possible connected quadruped gaits can be reduced into 45 equivalence classes.

In 1973, Bessonov and Umnov [30] showed by numerical experimentation that a regular and symmetric hexapod gait with the following phase relationships maximizes the longitudinal stability margin of all periodic gaits of a hexapod:

$$\begin{aligned}\phi_3 &= \beta \\ \phi_5 &= 2\beta - 1, \quad \beta \geq 0.5\end{aligned}\quad (1)$$

where ϕ_n is the phase of leg n and β is the duty factor. Phase is measured as the ratio of the time footfall of leg n lags behind that of leg 1 to the stride period.

In 1974, Sun [31] developed a computer program to

Manuscript received September 6, 1985; revised January 14, 1986.

T. Lee is with the Institute of Control Engineering, National Chiao Tung University, Hsinchu, Taiwan.

C. Shih is with the Department of Electrical Engineering, The Ohio State University, Columbus, OH 43210, USA.

IEEE Log Number 8608267.

calculate the longitudinal stability margins of six-legged regular and symmetric gaits. From the results of his program, he also found that the gait defined by (1) maximizes the gait stability margin of hexapod gaits.

In 1979 a nonperiodic gait, termed *free gait*, was introduced by McGhee and Iswandhi [4]. Algorithms of free gait have been implemented into simulated coordination programs and vehicle motion displayed on graphic terminals. This preliminary study has shown the practicality of this algorithm for a walking vehicle.

In 1983, Tsai successfully implemented a follow-the-leader gait into a computer program on the OSU Hexapod [32]. In a follow-the-leader mode on each side of the body, the front leg steps on a selected foothold, and the middle leg steps in the footprints of the front leg. The rear leg steps in the footprints of the middle leg.

More recently, a simple analytical approach using the concept of leg local phase to calculate the stability margin of periodic gaits was developed by Song [33]. The continuous follow-the-leader gait proposed by Song [33] provides the vehicle with a means of walking in rough terrain at a reasonably high speed.

Thus far, most of gait studies are mainly aimed at forward straight-line locomotion with a notable exception of the paper by Hirose [14], in which the crab walk was defined and the standard gait for the crab walk of a quadruped walking vehicle was derived. Although the gait of quadruped walking vehicle is well studied by the pioneering work of McGhee [1], [2] and recently by Hirose [14], the following problems remain unanswered.

- 1) What is the mathematical relation between the stability margin and stride length?
- 2) Observing most of the periodic gaits, one notices that in general the duty factor decreases while the stride length increase, and vice versa. It is natural to ask, "is the product of the duty factor and the stride length equal to a constant?" If yes, does this constant have some physical meaning related to the configurations and structure of the quadruped walking vehicle?"
- 3) Is there any relationship between the static stability margin, the height, and the distance from foothold positions to the boundary of the reachable area?
- 4) What is the distance that the body should move during a complete leg cycle in order to maintain the prescribed static stability margin?

This paper studies these problems, and some solutions to these problems are presented. To be more precisely, it will be shown that the product of the duty factor and the stride length is equal to the length of boundary of the reachable area of the leg. Furthermore, the mathematical equations relating static stability margin, duty factor, and stride length are derived. It will also be shown that the derived results fit in with McGhee's results [2], [4] (see also Remarks 5-7). Based on the results, a graphical method to determine the regular gaits of a quadruped walking vehicle with specified static stability margin on even terrain or on constant slope terrain is presented. We show that once the figure that graphically shows the relations between

the stability margin, the stride length, and the duty factor has been constructed, various physically realizable gaits with specified static stability margin on terrains with different slope angle can be obtained from the same figure. This graphical method turns out to be very useful for the design and control of a quadruped walking vehicle.

II. DESCRIPTION OF A QUADRUPED WALKING VEHICLE

Fig. 1 shows a quadruped walking vehicle constructed at the National Chiao Tung University (NCTU). It is referred to as NCTU Quadruped-1. The NCTU Quadruped-1 has a total weight of 50 kg and leg length of 0.6 m. The detailed dimensions are shown in Fig. 2. Each of the four legs contains three dc servo motors. There are eight tactile sensors installed at the bottom as well as the side of each leg. A balanced sensor for posture detection of the body is placed in the middle of the quadruped vehicle.

Because the task of motion planning and execution involved computation that must be accomplished in realtime, work has been initiated to partition both tasks so that it may be executed on dual single board computer (SBC) 86/12. One SBC 86/12 is responsible for motion planning and gait control, the other SBC 86/12 controls 12 dc servo motors so that the resultant motions are as planned.

For the ease of presentation, the NCTU Quadruped-1 with its reachable region of the leg is modeled as shown in Fig. 3. The notations appeared on Fig. 3 will be defined in the next section.

III. GAIT CONTROL

In this section, we establish a general framework for gait control problems. Results will be generated that quantitatively characterize the stability margin for the regular gait of a quadruped. The following definitions are required for the developments to be followed [2].

A. Definition and Notation

Definition 1: The support state of a k -legged locomotion system is a binary row vector $Y(t)$, such that at any time $Y_i(t) = 1$ if leg i is in contact with the supporting surface; $Y_i(t) = 0$ otherwise.

Definition 2: The support pattern associate with a given support state is the convex hull of the point set in a horizontal plane which contains the area of the vertical projections of all supporting feet.

Definition 3: Consider any supporting state sequence with $q(t)$ the location of the vertical projection of the vehicle center of gravity. Then the support pattern is statically stable at time t if and only if $q(t)$ is contained in its interior.

Definition 4: The stride length of a gait with stability margin S is the distance $\lambda(s)$ by which the body of a locomotion machine is translated in order to provide stability margin S during a complete leg cycle.

Definition 5: The duty factor of a gait with stability margin S , denoted by $\beta_i(s)$, is the fraction of time that leg i is in contact with supporting surface during one complete cycle of locomotion in which the quadruped has static stability margin

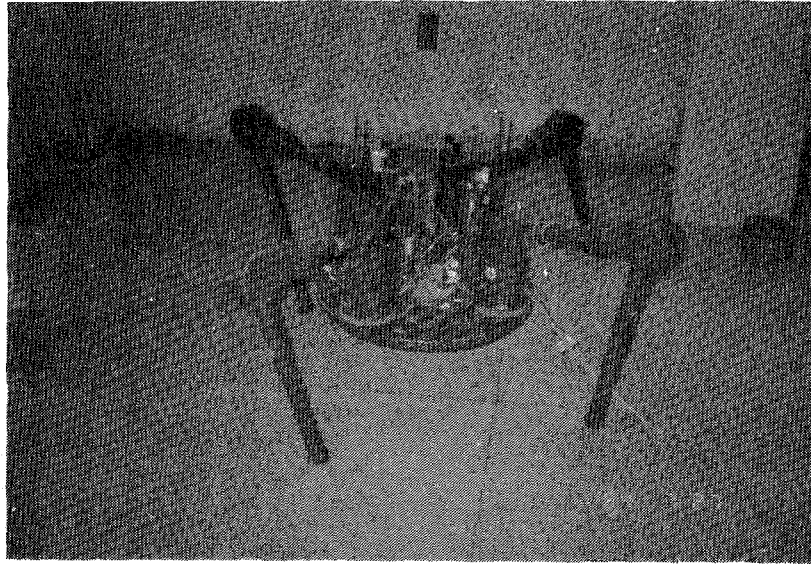


Fig. 1. Quadruped walking vehicle, NCTU Quadruped-1.

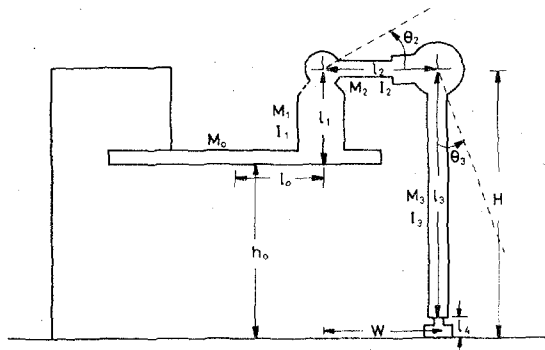


Fig. 2. Configuration and dimensions of NCTU Quadruped-1.

$$M_0 = 10 \text{ kg}, M_1 = 6.5 \text{ kg}$$

$$M_2 = 1.35 \text{ kg} \quad M_3 = 0.41 \text{ kg}$$

and

$$l_0 = 0.2475 \text{ m}, l_1 = 0.235 \text{ m}, l_2 = 0.40 \text{ m}, l_3 = 0.60 \text{ m}, l_4 = 0.053 \text{ m}.$$

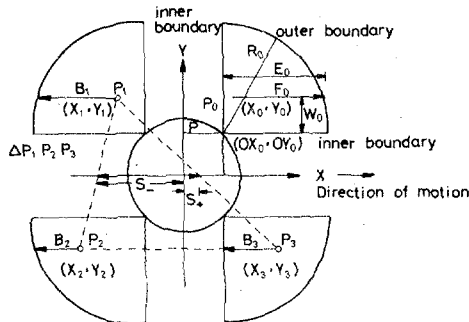


Fig. 3. Reachable region of quadruped walking robot in horizontal plane.

S . Note that a gait is regular if the duty factor of every leg is the same as that of every other leg.

In order to exploit the mathematical expression of the properties of the quadruped walking vehicle, the following notations, which appears in Fig. 3, are defined as following:

Definition 6: The symbol R_i is the radius of reachable sector of leg i in the horizontal plane.

Definition 7: The symbol $P_i(Q_i)$ is the distance on the x -axis (y -axis) between $q(t)$ and the origin of the sector i .

Definition 8: The symbol E_i is the distance between the boundaries of reachable area of leg i in the direction of motion.

Definition 9: The symbol W_i is the distance from foothold i to reachable inner boundary of leg i in the vertical motion direction.

Definition 10: Let $S_+(t)(S_-(t))$ be defined as the length of a vector pointing in the direction of motion (opposite direction of motion) to the intersection with the boundary of support pattern (see Fig. 3).

Definition 11: Let $B_i(t)(F_i(t))$ be defined as the length of a vector opposite (toward) motion from the current foothold i to an intersection with the reachable area boundary i .

Definition 12: The stability margin S is defined as the shortest distance from $q(t)$ to the nearest boundary along the direction of motion during one complete cycle of locomotion.

Definition 13: $A(S, \theta)$ is defined as the distance that the body moves, in order to provide a stability margin S on a constant slope plane with slope angle θ , while a leg is in the transfer state and the other three legs are in the supporting state. When $\theta = 0$, i.e., on the even terrain, the notation $A(s) = A(S, 0)$ is used. Furthermore, $A(0)$ denotes the distance that the body moves with zero stability margin while a leg is in the transfer state.

For the ease of presentation in the following, we assume that the quadruped walking vehicle moves either in $\pm x$ direction or in $\pm y$ direction only, and the quadruped has a symmetric structure ($P = Q, R_i = R, E_i = E$, and $W_i = W$) and regular gait ($\beta_i = \beta$) (for $i = 0, 1, 2, 3$).

The following lemma is useful in order to determine whether the center of gravity $q(t)$ is inside the support pattern.

Lemma 1: The gait is said to have the static stability margin $S, S > 0$, if and only if $\min(\min S_-(t), \min S_+(t)) = S$.

Proof (Necessity): From the definition of $S_-(t), S_+(t)$

and Definition 12, it is clear that the quadruped is stable (i.e., $q(t)$ is inside support pattern) if and only if $S_+ \times S_- > 0$. Furthermore, $\min(\min S_-(t), \min S_+(t)) = S$.

Proof (sufficiency): If $\min(\min S_-(t), \min S_+(t)) = S$, then certainly the shortest distance from $q(t)$ to the nearest boundary during one complete cycle of locomotion is S , hence the static stability margin is S .

Remark 1: The leg which has minimum value of B should be chosen as the transfer leg, other legs are support legs.

Remark 2: In the stable locomotion cycle, the distance of transfer provided by the transfer leg i should be equal to the total distance that the body moves while leg i is in support state within one locomotion cycle, and this amount should be equal to E .

Remark 3: The regular gait is symmetry. Each leg is equally important while it is in the supporting state and each leg transfers the same distance while it is in transfer state. Note that the symmetry means there is a symmetric pair that have the same duty factor and the same characteristics.

Corollary 1: The regular gait of the quadruped walking robot has zero stability margin when each leg has the stride length and duty factor of

$$\begin{aligned}\lambda &= 4A(0) \\ \beta &= 3/4\end{aligned}\quad (2)$$

respectively, where

$$A(0) = \frac{E}{3} \quad (3)$$

Proof: Consider the sequence of the support states as shown in Fig. 4 for the crawl gait moving in the $+x$ direction. Assume that the initial foothold positions of leg 0 is at "0," leg 1 is at "1," leg 2 is at "3," and leg 3 is at "2," as shown in the Fig. 4. Since $B_2 = 0$, it follows that leg 2 is the transfer leg. After lifting leg 2 (phase 1), the body moves a distance $A(0)$ while leg 2 swings from the location "3" to the location "0." At this stage $B_3 = 0$. Therefore leg 3 is lifted and moved from position "3" to position "0" while the body moves a distance $A(0)$. This procedure is repeated similarly for leg 1 and leg 0. Therefore, the stride length $\lambda = 4A(0)$ and duty factor

$$\beta = \frac{3A(0)}{4A(0)} = \frac{3}{4} \quad \text{Q.E.D.}$$

Note that Corollary 1 implies that in order to obtain a zero static stability margin gait, the body will transfer a distance $A(0) = E/3$ while a leg is in motion. Furthermore, at any moment there is only one leg in motion at one time; the other three legs are in support state. Therefore, the duty factor $\beta = 3/4$ is the smallest one that the quadruped can possibly have. In the following, we will show that the regular gait of quadruped with static stability margin $S(S > 0)$ requires that all the legs will be support legs during sometimes in the locomotion cycle.

Corollary 2: The regular gait of the quadruped walking vehicle that provides stability margin S requires that each leg

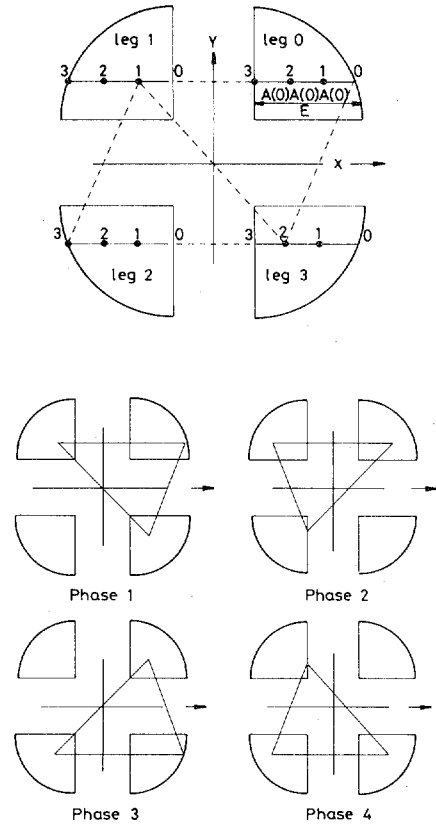


Fig. 4. Foothold position-support states sequence of the quadruped walking robot having zero stability margin. The dashed line denotes the initial support pattern when each leg is at its specified initial foothold position.

has the following values for the stride length, duty factor, and maximum value of S

$$\lambda(S) = 4A(S) + 4S \quad (4)$$

$$\beta(S) = \frac{3E}{4(E-S)} \geq \frac{3}{4} \quad (5)$$

$$S_{\max} = \frac{E}{4} \quad (6)$$

respectively.

Proof: From Fig. 5, we have

$$E = 3A(S) + 4S \quad (7)$$

or

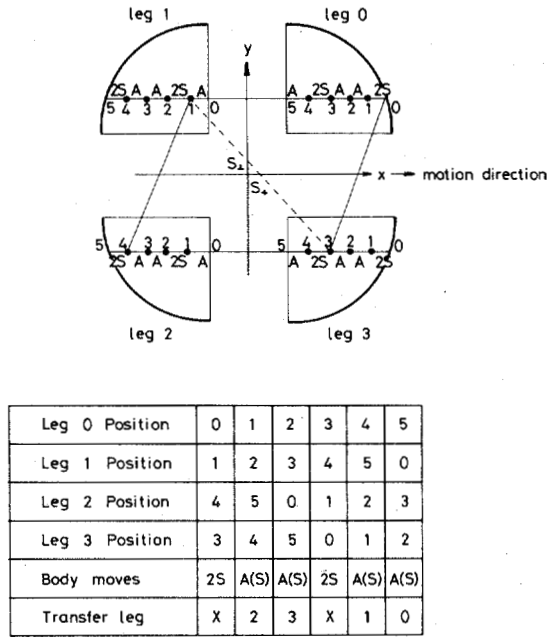
$$A(S) = \frac{E - 4S}{3} \quad (8)$$

Substituting (3) into (8) yields

$$A(S) = A(0) - \frac{4}{3}S \quad (9)$$

Therefore, the stride length

$$\lambda(S) = 4A(S) + 4S = 4A(0) - \frac{4}{3}S \quad (10)$$



Note. "X" denotes that all are support legs

Fig. 5. Foothold positions and state table of the quadruped walking robot with a S_+ longitudinal stability margin and S_+ lateral stability margin.

and duty factor

$$\beta(s) = \frac{3A(s) + 4S}{\lambda(s)} = \frac{E}{4A(0) - \frac{4}{3}S} = \frac{3}{4} \frac{E}{E - S} \geq \frac{3}{4} \quad (11)$$

Moreover, the S_{max} occurs at $A(S) = 0$. Hence, from (8)

$$S_{max} = \frac{E}{4} \quad (12)$$

Q.E.D

Remark 4: Corollary 2 points out a very interesting property that $\lambda(s)\beta(s) = E$, i.e., the product of duty factor and stride length is a constant. While stride length is maximum, the duty factor is minimum, and vice versa. Furthermore, the maximum stride length is $\lambda_{max} = \lambda(0) = 4A(0)$. Hence $\beta_{min} = \beta(0) = 3/4$. The maximum static stability margin occurs at $\beta = 1$. Hence $\lambda_{min} = \lambda(S_{max}) = E = 3A(0)$.

Remark 5: McGhee and Sun [4] have shown that if a vehicle has K legs ($K \geq 4$), then static stability margin implies $\beta \geq 3/K$, where β is the duty factor. Similarly, Corollary 2 shows that $\beta(s) \geq 3/4$, which agrees with McGhee's result for $K = 4$.

Remark 6: McGhee and Frank [2] have proved mathematically that there is a unique optimal gait that maximizes the static stability of a four-legged animal. The resultant gait stability margin S is

$$S = \lambda \left(\beta - \frac{3}{4} \right), \quad \text{for } \frac{3}{4} < \beta < 1. \quad (13)$$

Again, this equation can be derived from Corollary 2 as

follows. From (7), $\lambda\beta = E = 3A(s) + 4S$, it follows that

$$S = \lambda\beta - 3(A(s) + S) = \lambda \left(\beta - \frac{3}{4} \right)$$

using the relation $\lambda(s) = 4A(s) + 4S$.

Remark 7: It can be shown that the maximum stability margin of four-legged animals with a gait stability margin given by (13) is $S_{max} = E/4$. In fact, when $\beta = 1$ the maximum stability margin is $S_{max} = 1/4\lambda$. However, $\beta = 1$ implies $E = \lambda$. Therefore $S_{max} = E/4$, which agrees with Corollary 2.

In the above discussions, we all assume that the quadruped is walking on an even terrain. Now let us consider the case that the quadruped is walking on a constant slope terrain as shown in Fig. 6. It is easy to see that the center of gravity projected on the constant slope plane will be located $\Delta(\theta) = h \tan \theta$ away from its location on the horizontal plane, where θ is the slope angle and h is the height between $q(t)$ and the slope plane. Therefore, the regular gait of a quadruped walking vehicle that provides a static stability margin S on a constant slope plane can be derived by substituting $S' = S + \Delta(\theta)$ into the gait of that on even terrain. Thus Corollary 3 is an extension of previous results.

Corollary 3: The regular gait of the quadruped walking vehicle that provides a stability margin S on a constant slope plane with slope angle θ requires that each leg has the stride length

$$\lambda(\theta, S) = 4A(\theta, S) + 4S + \Delta(\theta), \quad 0 \leq \theta < \frac{\pi}{2} \quad (14)$$

and the duty factor

$$\beta(\theta, S) = \frac{3E}{4(E - S - \Delta(\theta))} \quad (15)$$

where $\Delta(\theta) = h \tan \theta$ and

$$A(\theta, S) = \frac{E}{3} - \frac{4}{3}S - \frac{4}{3}\Delta(\theta), \quad 0 \leq \theta < \frac{\pi}{2} \quad (16)$$

Proof: The proof, which follows directly from Fig. 7, is similar to Corollary 2, and therefore, is omitted.

It is interesting to note that (16) reveals that $A(\theta, S)$ decreases as θ increases. Hence there is a plane with maximum slope angle θ , called θ_{max} , that the quadruped walking vehicle can possibly walk with regular gait without causing instability. The following corollary answers this problem.

Corollary 4: The quadruped walking vehicle is climbing up (walking down) a hill on a plane of constant slope angle θ . The maximum θ that the quadruped can walk without causing instability is

$$\theta_{max} = \tan^{-1} \left(\frac{3}{4h} \right). \quad (17)$$

Proof: The maximum θ occurs, when $A(\theta, S)|_{S=0} = 0$. Therefore, from (16), it follows that $\theta_{max} = \tan^{-1}(E/4h)$.

Q. E. D.

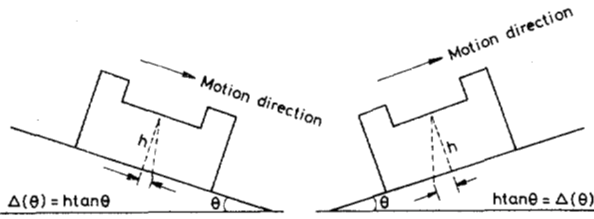
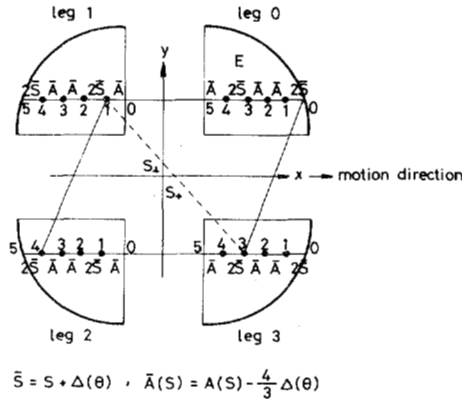


Fig. 6. Quadruped vehicle walks down or climbs up a constant slope plane.



Leg 0 Position	0	1	2	3	4	5
Leg 1 Position	1	2	3	4	5	0
Leg 2 Position	4	5	0	1	2	3
Leg 3 Position	3	4	5	0	1	2
Body moves	2S-bar	A-bar(S)	A(S)	2S-bar	A(S)	A(S)
Transfer leg	X	2	3	X	1	0

Note. "X" denotes that all are support legs

Fig. 7. The foothold positions and state table of the regular gait for a quadruped walking robot that provides a stability margin S on a plane constant of slope with slope angle θ .

Note that since the regular gaits on the constant slope plane can be obtained by substituting $S' = S + \Delta(\theta)$ into the gait of that on even terrain. Therefore, without loss of generality, in the following we consider only the regular gait on even terrain. Thus far only the static longitudinal stability is considered. However, in some cases, the quadruped with longitudinal stability margin $S_+ > 0$ cannot guarantee that the quadruped is statically stable. This is shown as follows. Assume the quadruped is moving in the $+x$ direction (Fig. 5), in this case, the longitudinal stability margin S_+ is reduced, and meanwhile, the distance between $q(t)$ to the boundary of supported pattern measured in $+y$ direction (denoted S_\perp) is also reduced. It is possible that $S_\perp < 0$ occurs even when $S_+ \geq 0$, and thus causing static stability problem. Therefore, it is necessary to define and study the static lateral stability.

Definition 15: The lateral stability margin S_\perp is defined as the distance from $q(t)$ to the boundary of support pattern in the vertical motion direction.

Definition 16: The stability factor α is defined as the ratio of the lateral stability margin to the longitudinal stability margin.

Corollary 5: The optimal value of stability factor of a

quadruped walking robot for maximum stability margin is α equal to one.

Proof: The distance from $q(t)$ to the nearest boundary in the direction of motion and to the vertical direction are always proportional, and according to Definition 16 this value equals α . If $\alpha < 1$, then $S_\perp < S_+$, therefore, this result is not desired. If $\alpha > 1$, then $S_+ < S_\perp$. In general, $\alpha > 1$ implies that W becomes larger (proportional to α), and according to

$$E = (R^2 - W^2)^{1/2} \tag{18}$$

it follows that E becomes smaller. From (6), S_{\max} is also smaller. Therefore, $\alpha = 1$ is the optimal value of maximizing static stability margin of the quadruped. Q. E. D.

Remark 8: Note that Corollary 5 implies that in order to obtain a static stability margin S with stability factor $\alpha = 1$, it is necessary to place the foothold positions at some specific points. In deed, Corollary 5 indicates that in order to maximizes stability margin, it is necessary that: 1) while leg 0 is the transfer leg, the foothold positions of leg 1 and leg 3 are on the straight line $y = -x + S$, where S is the desired static stability margin ($S = S_+ = S_\perp$); 2) while leg 2 is the transfer leg, the foothold positions of leg 1 and leg 3 should be on the straight line $y = -x - S$; 3) while leg 1 is the transfer leg, the foothold positions of leg 0 and leg 2 should be on the straight line $y = x + S$; and 4) while leg 3 is the transfer leg, the foothold positions of leg 0 and leg 2 should be on the straight line $y = x - S$.

Although Corollary 5 characterizes the relation between foothold positions that guarantee the stability margin, this relation is not sufficient to uniquely determine the exact foothold positions for the supported legs. Since for different W (i.e., different lines parallel to the x -axis), one obtains different foothold positions. Therefore, it is necessary to formulate the relations between the stability margin S and W , the distance in y direction from foothold to reachable inner boundary of the leg in the y direction. Following corollary gives the result.

Corollary 6: The optimal W that maximizes static stability margin of a quadruped vehicle, which has symmetry structure is $W = (E - S)/3$.

Proof: Consider the case that the quadruped is moving in $+x$ direction. Assume leg 0 is the transfer leg. The stability margin S occurs when footholds of leg 1 and leg 3 are at the positions

$$\begin{cases} X_1 = -P - A(s) \\ Y_1 = Q + W \end{cases} \text{ and } \begin{cases} X_3 = P + A(s) + 2S \\ Y_3 = -Q - W. \end{cases}$$

respectively. The stability margin is

$$S_+ = (X_1 + X_3)/2 = S \tag{19}$$

and

$$S_\perp = \frac{Y_3 X_1 - Y_1 X_3}{X_1 - X_3} = \frac{(Q + W)S}{P + A(S) + S} \tag{20}$$

For $\alpha = 1$, i.e., $S_\perp = S_+ = S$, it follows that

$$Q + W = P + A(S) + S. \tag{21}$$

If $P = Q$, i.e., symmetry structure, then

$$W = A(S) + S = \frac{E - S}{3}. \quad (22)$$

Similarly, the same result can also be derived when leg i ($i = 1, 2, 3$) is the transfer leg.

Remark 9: Intuitively, it may seem that if the four origins of the reachable sectors are widely separated (Fig. 8(b)), then the quadruped will have a larger static stability margin than that with the origins of the reachable sector closer to the origin (Fig. 8(a)). However, this is not true. In fact, Corollary 6 reveals a very interesting property, that both P and Q have no effect on the stability margin and the stride length. That is to say both Fig. 8(a) and Fig. 8(b) have the same static stability margin and same stride length. This property gives the designer greater freedom to choose the origins of the reachable sectors of the leg that guarantee the stability margin and the stride length. Indeed, the only constraints imposed on the origins of the reachable sectors are the following: a) leg 0 (leg 1) and leg 2 (leg 3) are symmetrical about the point $(x = 0, y = 0)$ and b) leg 0 (leg 1) and leg 3 (leg 2) are symmetrical about the x -axis.

B. Relationship Between Stability Margin, Height and Width

Given a quadruped walking vehicle as shown in Fig. 2, what is the relationship that the width W should satisfy in order to maximize the stability margin of the gait subject to the constraint that each leg is moved within its reachable region as shown in Fig. 3.

Let R_{\max} denote the maximum radius of reachable sector for each leg. Then

$$R_{\max} = \max (l_2^2 + l_3^2 + 2l_2l_3 \sin \theta_3 - H^2)^{1/2} = (L - H^2)^{1/2} \quad (23)$$

where

$$L = l_2^2 + l_3^2 + 2l_2l_3 \sin \theta_{3 \max} \quad (24)$$

and

$$E = (R_{\max}^2 - W^2)^{1/2} = (L - H^2 - W^2)^{1/2}. \quad (25)$$

According to Corollary 2, it shows that $S_{\max} = E/4$. Substituting (26) into (22) yields

$$W = \frac{E}{4} = S_{\max}. \quad (27)$$

Substituting (26) and (27) into (25), we have

$$4S_{\max} = (L - H^2 - S_{\max}^2)^{1/2}. \quad (28)$$

Simple manipulations yields

$$S_{\max}^2 = \frac{(L - H^2)}{17}. \quad (29)$$

From (25), it is easy to see that $R_{\min} = W$. Therefore

$$R_{\min} = W = S_{\max}. \quad (30)$$

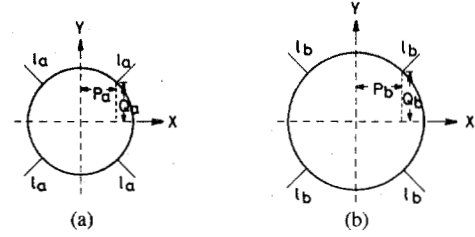


Fig. 8. Origins of reachable Sectors. (a) Close to the origin. (b) Further from the origin. These figures have the same static stability margins and stride length as long as $l_a = l_b$.

Equations (29) and (30) reveal how the maximum stability margin is related to the height H and width W . Intuitively, it follows from (29) that S_{\max} occurs at $H = 0$. However, this is impossible. In fact, the correct S_{\max} can only be determined by taking the configuration of the quadruped shown in Fig. 2 into account. Thus one must seek R_{\min} and H_{\min} obtainable from the quadruped to satisfy equations (29) and (30), respectively, the S_{\max} can then be determined.

From Fig. 2, it is easy to show that

$$H = l_3 \cos (\theta_2 + \theta_3) - l_2 \sin \theta_2 \quad (31)$$

and

$$R = l_3 \sin (\theta_2 + \theta_3) - l_2 \cos \theta_2. \quad (32)$$

For the NCTU Quadruped-1, the dimensions are $l_3 = 60.0$ cm, $l_2 = 40.0$ cm, $l_1 = 23.5$ cm, and $\theta_{3 \max} = 30^\circ$. The reachable values of H and R , given by (31) and (32), respectively, is shown in Fig. 9. The minimum values of R and H that satisfy (29) and (30) can be obtained from Fig. 9. Observed that in general, to minimize H requires maximizing θ_2 , and to minimize R requires maximizing θ_2 and minimizing θ_3 at the same time. Therefore, one can start from the lower right corner point ($\theta_3 = -0.5$ rad and $\theta_2 = 0.5$ rad) and moving along the bottom line of the Fig. 9 (the R_{\min} for $H = \text{const}$) to search the suitable values of H and R that satisfy (29) and (30). In this example, it can be found that when $R_{\min} = S_{\max} = W = 16.25$ cm and $H = 52$ cm, (29) and (30) are satisfied. Now, it is easy to find that the maximum θ that the quadruped can climb up or walk down is 29.6° .

Fig. 10 shows how the stride length, the stability margin, and the duty factor are related one another as a function of θ , the slope angle of the terrain. From this figure, the required stride length and the duty factor that corresponds to a regular gait with a specified stability margin and terrain slope angle θ can be obtained. In particular, corresponding to the maximum stability margin with $S_{\max} = 16.25$ cm, the stride length and duty factor when $\theta = 0^\circ$ are $\lambda = 65$ cm and $\beta = 1$, respectively.

Remark 10: These results can also be obtained from Corollary 2. Indeed, from (12), it follows that $E = 4S_{\max} = 65$ cm, and furthermore, from (11) one obtains $\beta(s) = 1$. Therefore, $A(s) = 0$ and $\lambda(s) = 4S_{\max} = 65$ cm. This shows that the graphical method to determine the gait describing above provides the correct results.

Note that Figs. 9 and 10 provide a simple and straightforward method to derive regular gaits graphically. Basically,

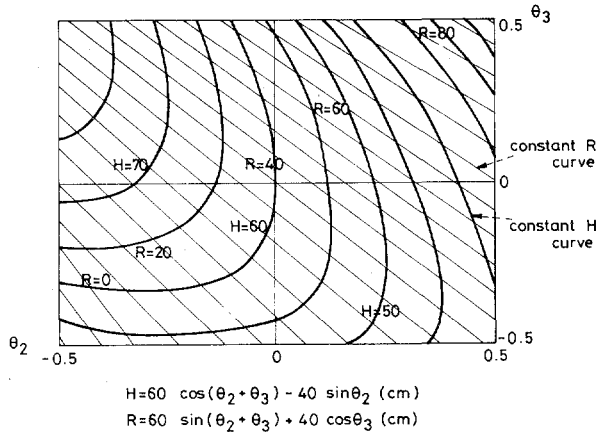


Fig. 9. The reachable value of H and R of a leg.

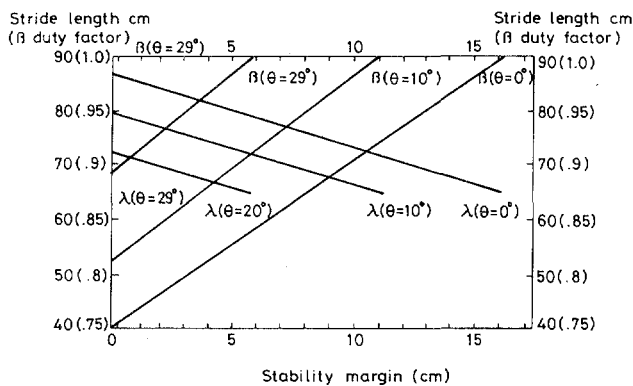


Fig. 10. The relation of stride length and duty factor with stability margin and slope of the travel plane.

this graphical method of deriving gait consists of two steps:

- Step 1) Determine the maximal static stability margins that the quadruped is physically accessible.
- Step 2) Plot the stride length and duty factor, given by (14) and (15), respectively, as a function of slope angle θ for $0 < S < S_{\max}$.

Based on the figure obtained from Step 2, the regular gait with any specified static stability margin (in the range $0 < S \leq S_{\max}$) for various slope angle can easily be observed. This figure is very useful for gait control problems and may turn out to be useful for the leg design of a quadruped walking vehicle. The merits of this graphical method is that it summarizes all the informations relating the stability margin, the stride length, the duty factor, and the terrain slope angle into a series of curves. Once these curves has been set up, various physically realizable regular gaits can be obtained without any further calculations.

IV. CONCLUSION

In this paper some of the fundamental problems of gait control of a quadruped walking vehicle are discussed. In the course of the investigation, the configuration of the constructed NCTU Quadruped-1 is fully taken into consideration. Some mathematical relations between the stability margin and the stride length are formulated. It is found that the product of

the duty factor and stride length is equal to the length of the boundary of reachable area of the leg. Thus, once the configuration of the quadruped is determined, the relation between stride length and duty factor is completely specified. Furthermore, the mathematical expression representing the relationship between the stability margin, the stride length, and duty factor are also formulated. Again, these equations are expressed in terms of quantities that specify the configuration of the quadruped walking vehicle. From these results, the mathematical equation that relates the stability margin, the height of the leg, and the width are derived. This result is the foundation for the gait selection and control, and possibly for the design of any symmetric quadruped walking vehicle. Based on the derived results, a graphical approach has been presented to determine the regular gaits of the quadruped. Indeed, by observing Fig. 10—which shows how the stability margin, the stride length, and the duty factor are related graphically—the regular gaits on even terrain, or the regular gaits on constant slope terrain can be obtained, respectively. Thus we may conclude that Figs. 9 and 10 summarize all the mathematical equations describing the regular gaits of the quadruped walking vehicle, and from the point of view of applications, these two figures provide user a simple and straightforward way for determining the regular gaits. Therefore, this graphical method may reduce the time required for engineers with little knowledge of the walking vehicles to understand the gait control problems and, more important, to determine with confidence a suitable gait of a quadruped walking vehicle.

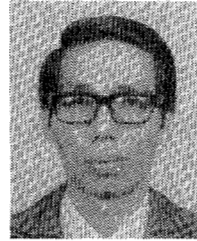
REFERENCES

- [1] R. B. McGhee, "Some finite state aspects of legged locomotion," *Mathematical Biosci.*, vol. 2, no. 1/2, pp. 67-84, Feb. 1968.
- [2] R. B. McGhee and A. A. Frank, "On the stability properties of quadruped creeping gait," *Math. Biosci.*, vol. 3, no. 3, pp. 331-351.
- [3] R. B. McGhee and A. K. Jain, "Some properties of regularly realizable gait matrices," *Mathematical Biosci.*, vol. 13, no. 1, pp. 179-193, Feb. 1972.
- [4] R. B. McGhee and S. S. Sun, "On the problems of selecting a gait for a legged vehicle," in *Proc. Fourth IFAC Symp. Auto. Contr. in Space*, 1974.
- [5] R. B. McGhee et al., "Real-time computer control of a hexapod vehicle," in *Proc. 3rd CISM-IFTOMM Symp. Theory and Practice of Robots and Manipulators*. Amsterdam: Elsevier, 1978, pp. 323-339.
- [6] R. B. McGhee and G. I. Iswandhi, "Adaptive locomotion of a multilegged robot over rough terrain," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-9, no. 4, pp. 176-182, 1979.
- [7] K. Ikeda, et al., "Finite state control of quadruped walking vehicle—control by hydraulic digital actuator," *Biomechanism*, vol. 2, pp. 164-172, (in Japanese), 1973.
- [8] E. I. Kugusheve, and V. S. Jaroshevskij, "Problems of selecting a gait for an integrated locomotion robot," 4th Int. Conf. Artificial Intell., Tbilisi, Georgian SSR, USSR, 1975.
- [9] K. Taguchi, K. Ikeda, and S. Matsumoto, "Four-legged walking machine," in *Proc. 2nd Romansy Symp.*, pp. 173-182, 1976, Warsaw, Poland.
- [10] S. Hirose and Y. Umetani, "Some considerations on a feasible walking mechanism as a terrain vehicle," in *Proc. 3rd CISM-IFTOMM Symp. Theory and Practice of Robots and Manipulators*. Amsterdam: Elsevier, 1978, pp. 357-375.
- [11] ———, "The basic considerations of energetic efficiencies of walking vehicles," *Trans. Soc. Instrument Contr. Engineers*, vol. 15, no. 7, pp. 298-933, 1979 (in Japanese).
- [12] ———, "A Cartesian coordinate manipulator with articulated structure," in *Proc. 11th Int. Symp. Industrial Robots*. London: International Fluidics, 1981.

- [13] ———, "The basic motion regulation system for a quadruped walking vehicle," ASME Conf. Mechanisms, Los Angeles, CA, no. ASME 80 DET-34, 1980.
- [14] S. Hirose, "A study of design and control of a quadruped walking vehicle," *Int. J. Robotics Res.*, vol. 3, no. 2, pp. 113-133, 1984.
- [15] M. Hirofumi and S. Isao, "Dynamic walk of the biped," *Int. J. Robotics Res.*, vol. 3, no. 2, pp. 60-74, 1984.
- [16] H. Hemami, "A state space model for interconnected rigid bodies," *IEEE Trans. Auto. Contr.*, vol. AC-27, no. 2, pp. 376-382, 1982.
- [17] ———, "Some aspects of Euler-Newton equations of motion," *Ingenieur-Archiv.*, vol. 52, pp. 167-176, 1982.
- [18] H. Hemami and R. L. Farnsworth, "Postural and gait stability of a planar five link biped by simulation," *IEEE Trans. Auto. Contr.*, vol. AC-22, pp. 452-458, 1977.
- [19] H. Hemami and B. F. Wyman, "Modeling and control of constrained dynamics systems with application to biped locomotion in the frontal plane," *IEEE Trans. Auto. Contr.*, vol. AC-24, pp. 526-535, 1979.
- [20] H. Hemami and A. Katbab, "Constrained inverted pendulum model for evaluating upright postural stability," *J. Dyn. Syst. Measurement Contr.*, vol. 104, no. 4, pp. 343-349, 1982.
- [21] C. L. Golliday, Jr., and H. Hemami, "Postural stability of the two-degree-of-freedom biped by general linear feedback," *IEEE Trans. Auto. Contr.*, vol. AC-21, no. 1, pp. 74-79, 1976.
- [22] M. H. Raibert, "Hopping in legged systems-modeling and simulation for the 2D one-legged case," *IEEE Trans. Syst. Man Cybern.*, vol. 14, no. 3, May 1984.
- [23] M. H. Raibert and H. B. Brown, Jr., "Experiments in balance with a 2D one-legged hopping machine," *ASME J. Dyn. Syst. Measurement Contr.*, vol. 106, no. 2, 1984.
- [24] M. H. Raibert and I. E. Sutherland, "Machines that walk," *Scientific Amer.*, vol. 248, no. 1, pp. 44-53, 1983.
- [25] M. H. Raibert and F. C. Wimberly, "Tabular control of balance in a dynamic legged system," *IEEE Trans. Syst. Man Cybern.*, vol. 14, no. 3, 1984.
- [26] M. H. Raibert, *et al.*, Dynamically stable legged locomotion—Third annual report, tech. rep., CMU-RI-TR-83. Carnegie-Mellon University, Robotics Institute, Pittsburgh, PA.
- [27] M. H. Raibert *et al.*, "Experiments in balance with a 3D one-legged hopping machine," *Int. J. Robotics Res.*, vol. 3, no. 2, pp. 75-92, 1984.
- [28] D. M. Wilson, "Insect walking," *Ann. Rev. Entomol.*, vol. 11, pp. 103-121, 1966.
- [29] I. Kato and H. Tsuiki, "Hydraulically powered biped walking machine with a high carrying capacity," in *Proc. Fourth Int. Symp. External Contr. of Human Extremities*, 1972.
- [30] A. P. Bessenov and N. V. Umnov, "The analysis of gaits in six-legged

vehicles according to their static stability," in *Proc. Symp. Theory and Practice of Robots and Manipulators*, 1973.

- [31] S. S. Sun, "A theoretical study of gaits for legged locomotion systems," Ph.D. dissertation, The Ohio State University, Columbus, OH, Mar. 1974.
- [32] S. J. Tsai, "An experimental study of a binocular vision system for rough terrain locomotion of a hexapod walking robot," Ph.D. dissertation, The Ohio State University, Columbus, OH, 1983.
- [33] S. M. Song, "Kinematic optimal design of a six-legged walking machine," Ph.D. dissertation, The Ohio State University, Columbus, OH, 1984.



Tsu-Tian Lee was born in Taipei, Taiwan, on February 1, 1949. He received the B.S. degree in control engineering from National Chiao Tung University, Hsinchu, Taiwan, in 1970, and the M.S. and Ph.D. degrees in electrical engineering from University of Oklahoma, Norman, in 1973 and 1975, respectively.

In 1975, he was appointed Associate Professor and in 1978 Professor of Department of Control Engineering at the National Chiao Tung University, where he currently holds the rank of Professor of

the Institute of Control Engineering. His current research interests involve the theory and applications of computer control of robots, optimal control theory and applications, and the multilegged walking robots.

Dr. Lee is a member of Phi Tau Phi.



Ching-Long Shih was born in Taiwan, Republic of China. He received the B.S. and M.S. degrees both in control engineering from National Chiao Tung University, Hsinchu, Taiwan, in 1980 and 1984, respectively.

He is currently working toward the Ph.D. degree in electrical engineering at the Ohio State University, Columbus, OH. His main interests are in the areas of robotics, walking machines, and artificial intelligence.