

國立交通大學

應用數學系

數學建模與科學計算碩士班



碩士論文

一個史蒂芬類型問題的數值研討

Numerical Study of A Stefan-Type Problem

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摘 要

本論文中，我們考慮一個 Stefan 類型的問題。主要利用對稱間斷不連續的數值方法來離散這個問題。其中，我們提出相關半離散和全離散的數值方法並證明這兩個數值方法在 L^2 -norm 都是最佳收斂的。最後，我們執行相關的數值實驗，驗證其理論結果。其數值結果與理論是符合一致的。



Numerical Study of A Stefan-Type Problem

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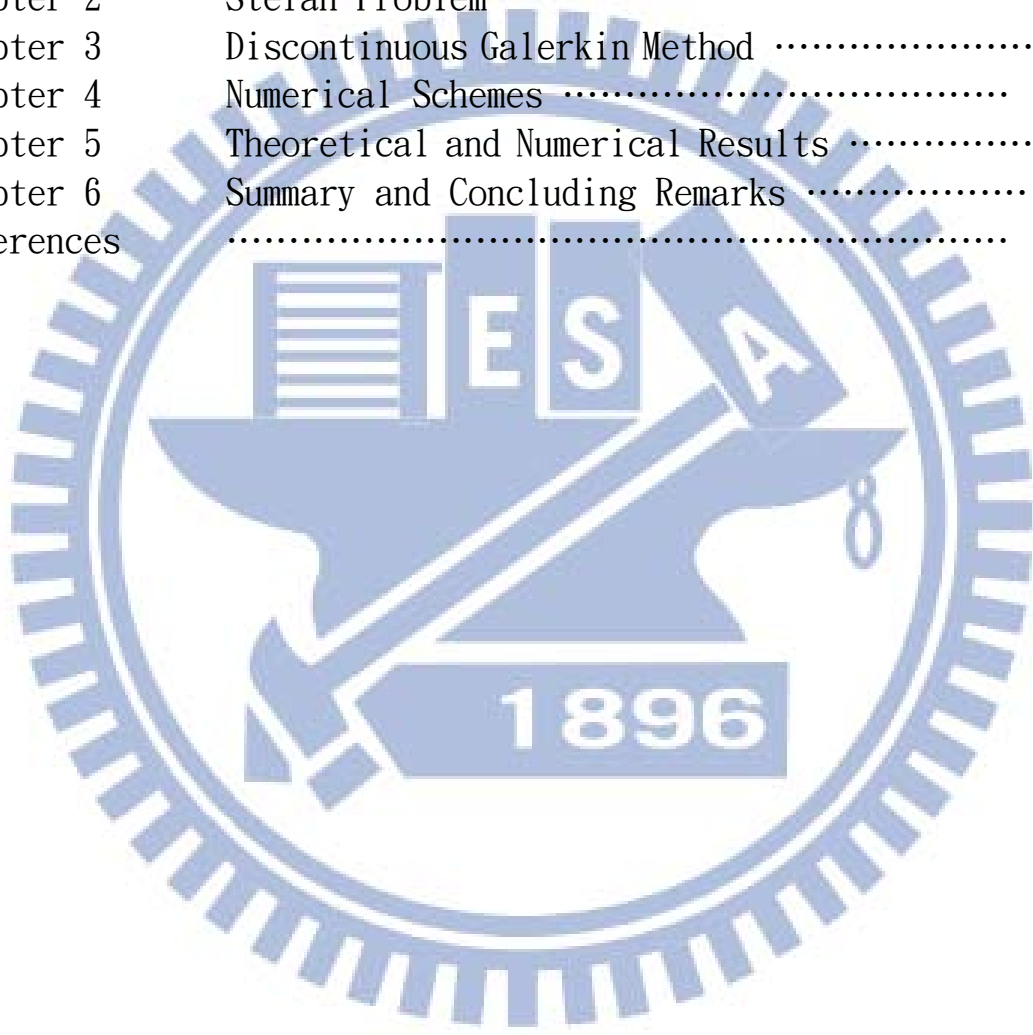
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ABSTRACT

In this thesis, concerning a Stefan-type problem, we study the discontinuous Galerkin approximation of the problem. Based on the symmetric interior penalty Galerkin method, both the semidiscrete and fully discrete schemes are presented and the optimal orders of convergence in L^2 -norm are also proven. Some numerical experiments are also performed to confirm our theoretical results.

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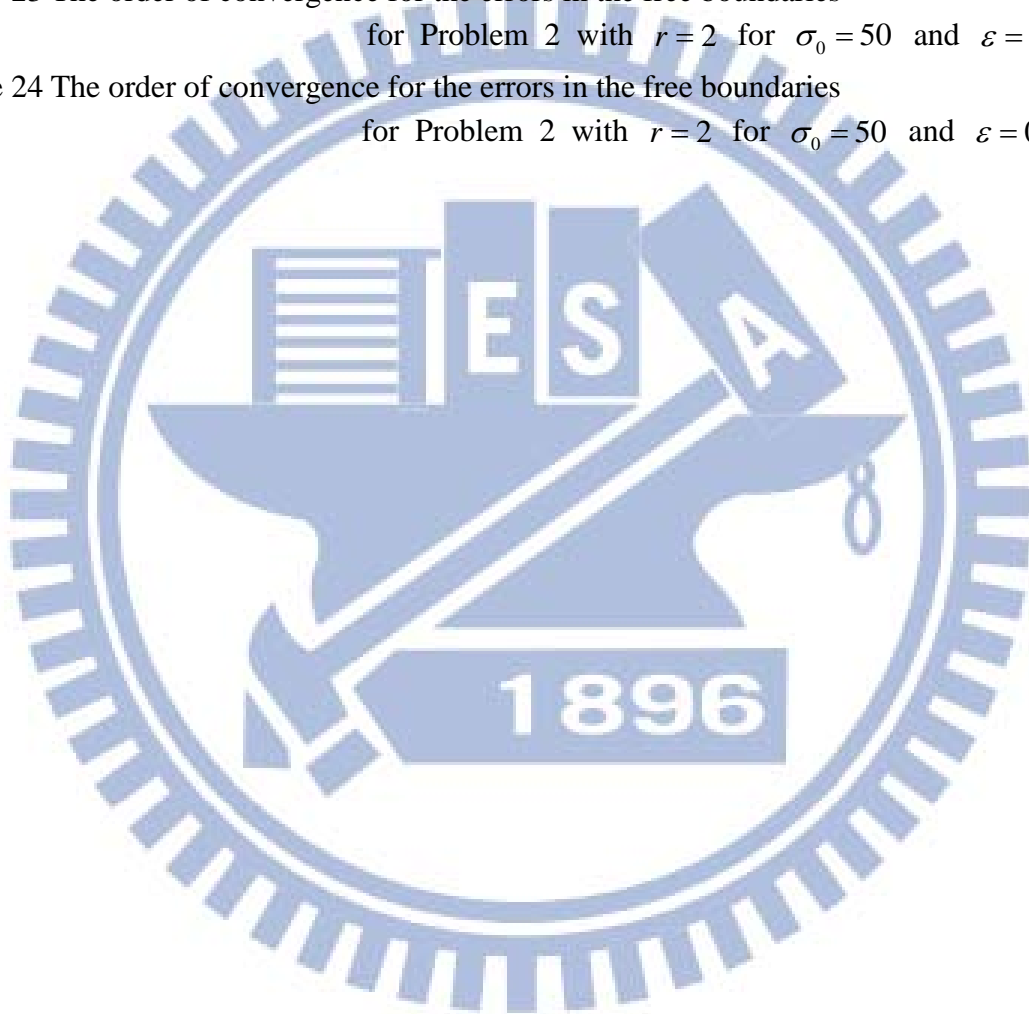
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