A New Analytical Three-Dimensional Model for Substrate Resistance in CMOS Latchup Structures

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***Abstract*—A new analytical model based on solving the three-dimensional Laplace equation has been developed to calculate the substrate-spreading resistance of a latchup-sensitive path in internal CMOS structures. This model also provides an analytical closed-form expression for the substrate potential as functions of the structural parameters in the substrate, the dimensions of majority-carrier injector, and the majority-carrier current density across the injector. The calculated results based on the developed model have been compared with existing experimental results, and good agreement has been obtained.**

I. INTRODUCTION

IN AN n-well CMOS circuit, the substrate majority-carrier current caused by the forward-biased p$^+$ emitter inside the well can produce the resistive potential drop in the latchup-sensitive path. This substrate potential drop may trigger the CMOS circuit into the latchup state and results in the malfunction of the circuit if the induced potential drop is large enough. Therefore, it is valuable to develop an analytical model to calculate the substrate current required to induce a given potential drop. In addition, it is desirable to take the three-dimensional effects into account since the spreading of majority carriers in the substrate is three-dimensional in nature. Terrill and Hu [1] first published such a model and its accuracy has been verified experimentally [1], [2]. However, this model has been derived by using assumptions that are not applicable for the case of an epitaxial layer or a buried layer in the substrate. Moreover, for some other geometries of majority-carrier injectors, numerical analyses are required. In this paper, a new analytical three-dimensional model is presented to calculate the substrate resistance or equivalently the potential induced by the majority-carrier current in the substrate. The calculated results based on the developed model are compared with existing experimental results measured from various substrate structures and majority-carrier injectors.

II. MODEL

A rectangular parallelepiped structure proposed to simulate the current spreading of majority carriers in the p-type substrate is shown in Fig. 1 in which the majority-carrier injector at the surface $z = z_3$ is appropriately used to simulate the current injected into the substrate through the forward-biased p$^+$ emitter inside the n-well. For the case of a heavily doped layer buried in the homogeneous substrate, as shown in Fig. 1, the low-doped regions ($0 \leq z \leq z_1$ and $z_2 \leq z \leq z_3$) and the heavily doped region ($z_1 \leq z \leq z_2$) are assumed to have uniform doping in order to simplify the analysis. The cases of epitaxial and bulk substrates can be obtained by letting $z_1 \rightarrow 0$ and $z_2 \rightarrow z_1$, respectively.

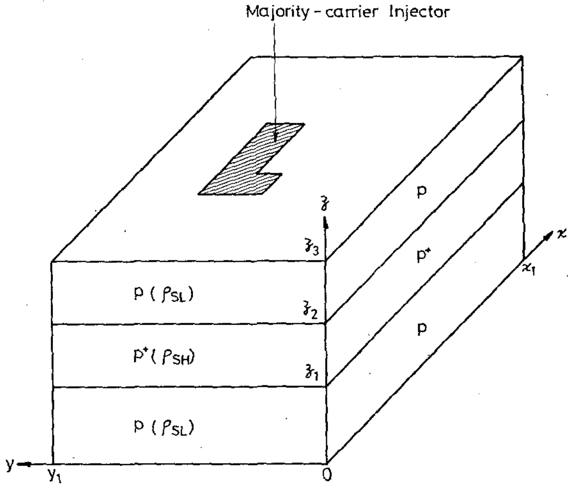

Fig. 1. The rectangular parallelepiped structure used to simulate the current spreading of majority carriers in the p-type substrate.

To find the substrate potential induced by the majority-carrier current, the following Laplace equation is used:

$$\nabla^2 V(x, y, z) = 0 \tag{1}$$

where $V(x, y, z)$ is the substrate potential distribution.

The boundary and interface conditions used are

$$\left.\frac{dV}{dz}\right|_{z=z_3} = \begin{cases} \rho_{SL} j_p(x, y) & \text{for the injector} \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

$$\left.\frac{dV}{dz}\right|_{z=z_2^-} = \eta \left.\frac{dV}{dz}\right|_{z=z_2^+} \tag{3}$$

$$V(x, y, z_2^-) = V(x, y, z_2^+) \tag{4}$$

Manuscript received September 12, 1985; revised December 2, 1985.

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IEEE Log Number 8607550.

$$\left.\frac{dV}{dz}\right|_{z=z_1^-} = \frac{1}{\eta}\left.\frac{dV}{dz}\right|_{z=z_1^+} \tag{5}$$

$$V(x, y, z_1^-) = V(x, y, z_1^+) \tag{6}$$

$$V(x, y, 0) = 0 \tag{7}$$

$$\left.\frac{dV}{dx}\right|_{x=0 \& x=x_1} = 0 \tag{8}$$

$$\left.\frac{dV}{dy}\right|_{y=0 \& y=y_1} = 0 \tag{9}$$

where j_p (positive) is the injected majority-carrier current density, $\rho_{SL}(\rho_{SH})$ is the resistivity in the low- (heavily) doped region, and $\eta = \rho_{SH}/\rho_{SL}$. Note that the boundary condition in (7) is formulated by considering the back contact used in existing CMOS technology. The boundary conditions in (8) and (9) result from the fact that the injector is placed around the location $(x_1/2, y_1/2, z_3)$ and both x_1 and y_1 are large enough. In the present case, $x_1 = 1000$ μm and $y_1 = 1000$ μm have been used in our work.

Using a separation of variables, the induced potential V at the surface can be shown to be (see the Appendix)

$$V(x, y, z_3) = \sum_{n,m=0}^{\infty} A_{nm} \cos\left(\frac{m\pi y}{y_1}\right) \cos\left(\frac{n\pi x}{x_1}\right) \tag{10}$$

$$A_{nm} = F_{nm} \frac{4}{x_1 y_1} \left[\int_0^{x_1} dx \int_0^{y_1} dy \left.\frac{dV}{dz}\right|_{z=z_3} \cdot \cos\left(\frac{m\pi y}{y_1}\right) \cos\left(\frac{n\pi x}{x_1}\right)\right] \tag{11}$$

where $F_{oo} = [z_1 + \eta(z_2 - z_1) + (z_3 - z_2)]/4$, $F_{om} = F1/2F2$ for $m \geq 1$, $F_{no} = F1/2F2$ for $n \geq 1$, and $F_{nm} = F1/F2$ for $n \geq 1$ and $m \geq 1$. Note that $F1$ and $F2$ in the above equations are expressed by

$$F1 = \tanh(\delta_0 x_s) + \eta \tanh(\delta_0 x_L) + \tanh(\delta_0 x_E) + \tanh(\delta_0 x_s) \tanh(\delta_0 x_L) \cdot \tanh(\delta_0 x_E)/\eta \tag{12}$$

$$F2 = \delta_0[1 + \tanh(\delta_0 x_s) \tanh(\delta_0 x_E) + \eta \tanh(\delta_0 x_L) \cdot \tanh(\delta_0 x_E) + \tanh(\delta_0 x_s) \cdot \tanh(\delta_0 x_L)/\eta] \tag{13}$$

$$\delta_0 = \pi\sqrt{(n/x_1)^2 + (m/y_1)^2} \tag{14}$$

where $x_s = z_1$, $x_L = z_2 - z_1$, and $x_E = z_3 - z_2$.

Furthermore, the substrate majority-carrier current denoted as I_p can be written as

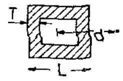
$$I_p = \left[\int_0^{x_1} dx \int_0^{y_1} dy \frac{1}{\rho_{SL}} \left.\frac{dV}{dz}\right|_{z=z_3}\right]. \tag{15}$$

Note that an analytical model for the substrate resistance can be obtained by using (10) divided by (15). If the current density j_p is assumed to be uniform, a simplified expression without the integral form can be easily derived. Finally, for the highly symmetrical spreading of majority carriers in the substrate, a two-dimensional closed-form expression can be easily developed by following the above procedure, which can be used to appropriately represent the actual three-dimensional case in order to greatly improve the computational efficiency.

TABLE I

THE SUBSTRATE RESISTANCE R_s AS A FUNCTION OF THE DISTANCE d FROM THE INTERIOR OF THE INJECTOR WITH THE THICKNESS T AS A PARAMETER

(The measured substrate resistances are extracted from [1] and the theoretical results are calculated by the developed model.)

Injector Dimensions	Parameters	d(μm) \ T(μm)	6	12	20
 L = 50 μm	$\frac{R_s(K\Omega)\text{exp.}}{R_s(K\Omega)\text{cal.}}$	0	$\frac{0.95}{1.04}$	$\frac{0.9}{1.16}$	$\frac{0.85}{1.36}$
		25	$\frac{1.1}{1.10}$	$\frac{1.0}{1.09}$	$\frac{0.9}{1.08}$
		70	$\frac{0.45}{0.40}$	$\frac{0.45}{0.40}$	$\frac{0.45}{0.40}$

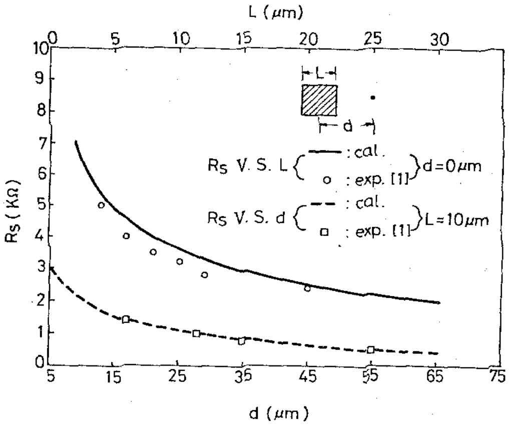


Fig. 2. The calculated and measured dependencies of the substrate resistance (R_s) on the distance d from the interior of the injector and the side length L of the square injector.

III. RESULTS AND DISCUSSION

The accuracy of the developed model can be verified by the experimental results measured from various structures presented by Terrill and Hu [1]. These structures were formed on the same wafer with $z_2 = z_3$, $z_2 - z_1 = 1$ μm, $z_3 = 325$ μm (i.e., wafer thickness), $\rho_{SL} = 21\ \Omega \cdot$ cm, and $\rho_{SH} = 1.2\ \Omega \cdot$ cm. The substrate resistances measured from these structures are shown in Table I and Fig. 2, in which the injector dimensions are marked. Based on the developed model, the calculated substrate resistances for the uniform current density across the injectors are also shown in Table I and Fig. 2. Good agreement between the developed model and the experimental measurements has been obtained. Note that although Terrill and Hu [1] have reported a different method for calculating the substrate resistance, for the injector configuration shown in Table I, however, their theoretical results must be computed by the numerical method.

More recently, Terrill *et al.* [2] have reported the substrate resistances measured from various structures with nonimplanted and implanted regions in the substrate. These experimental results will be used to further verify the accuracy of the developed model. Fig. 3 shows com-

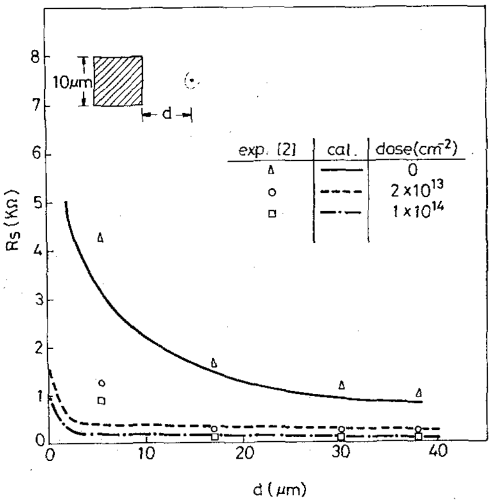


Fig. 3. The calculated and measured substrate resistances (R_s) as a function of the distance d from the edge of the injector with the implantation dose as a parameter.

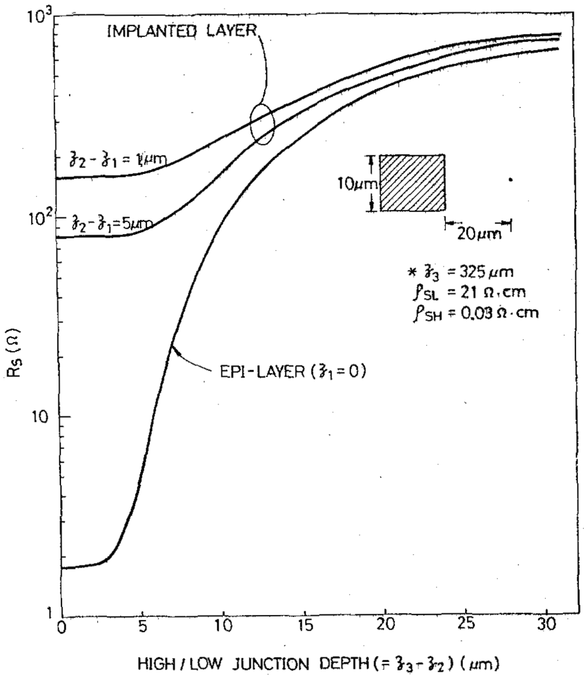


Fig. 4. The calculated dependencies of the substrate resistance (R_s) on the high–low junction depth below the surface for the cases of epitaxial layer and implanted layer in the substrate. The effects of different implanted-layer thicknesses are also shown.

parisons between the calculated substrate resistances based on the developed model and the measured results cited in [2]. The structural parameters used to calculate the substrate resistances for the structures with the implanted layer in the substrate are $z_3 - z_2 = 1$ µm, $z_2 - z_1 = 1$ µm, $z_3 = 325$ µm, $\rho_{SL} = 21\ \Omega \cdot$ cm, and $\rho_{SH} = 0.1\ \Omega \cdot$ cm for a dose of 2×10^{13} cm^{-2} and $0.03\ \Omega \cdot$ cm for a dose of 1×10^{14} cm^{-2}. It is shown in Fig. 3 that satisfactory agreement between the developed model and the experimental measurements is obtained. Furthermore, based on our developed model, the predicted dependencies of the substrate resistance on both epitaxial-layer thickness and buried- (implanted-) layer depth below the surface are shown in Fig. 4. Note that the method used in [1] for calculating the substrate resistance is incapable of simulating these effects.

IV. CONCLUSION

We have presented a new analytical three-dimensional model with a closed-form expression for calculating the substrate resistance of a latchup-sensitive path in internal CMOS structures. This model can also be used to evaluate the dependencies of the substrate potential on the structural parameters in the substrate, the injector dimensions, and the current density across the injector. The calculated results based on the developed model have shown to be in good agreement with existing experimental results. Based on the developed method [3], the substrate resistance model developed in this paper can be used to efficiently calculate the dc triggering currents for the advanced CMOS fabricated on the epitaxial wafer.

APPENDIX
DERIVATION OF EQUATION (10)

The expression

$$\Phi_{nm} = \cos\left(\frac{m\pi y}{y_1}\right) \cos\left(\frac{n\pi x}{x_1}\right) \sinh(\delta_0 z) \tag{A1}$$

can satisfy the Laplace equation (1) and the boundary condition (7), (8), and (9), in which

$$\delta_0 = \pi \sqrt{\left(\frac{n}{x_1}\right)^2 + \left(\frac{m}{y_1}\right)^2}. \tag{A2}$$

Thus the solution of the problem for $0 \le z \le z_1$ can be expanded in terms of these Φ_{nm}, i.e.

$$V(x, y, z) = \sum_{n,m=0}^{\infty} B_{nm} \cos\left(\frac{m\pi y}{y_1}\right) \cdot \cos\left(\frac{n\pi x}{x_1}\right) \sinh(\delta_0 z). \tag{A3}$$

In a similar way, the potential for $z_1 \le z \le z_2$ can be expressed as

$$V(x, y, z) = \sum_{n,m=0}^{\infty} B_{nm} \cos\left(\frac{m\pi y}{y_1}\right) \cos\left(\frac{n\pi x}{x_1}\right) \cdot [\sinh(\delta_0 z_1) \cosh(\delta_0(z - z_1)) + \eta \cosh(\delta_0 z_1) \sinh(\delta_0(z - z_1))]. \tag{A4}$$

Note that both (A4) and (A3) simultaneously satisfy the common boundary conditions (5) and (6).

Furthermore, the potential for $z_2 \le z \le z_3$ can be expressed as

$$V(x, y, z) = \sum_{n,m=0}^{\infty} B_{nm} \cos\left(\frac{m\pi y}{y_1}\right) \cos\left(\frac{n\pi x}{x_1}\right) \cdot F_{ac} \tag{A5}$$

where

$$\begin{aligned}F_{ac} = {} & [\sinh(\delta_0 z_1)\cosh(\delta_0(z_2 - z_1)) + \eta\cosh(\delta_0 z_1) \\ & \cdot \sinh(\delta_0(z_2 - z_1))]\cosh(\delta_0(z - z_2)) \\ & + \left[\frac{1}{\eta}\sinh(\delta_0 z_1)\sinh(\delta_0(z_2 - z_1)) + \cosh(\delta_0 z_1)\right. \\ & \left.\cdot \cosh(\delta_0(z_2 - z_1))\right]\sinh(\delta_0(z - z_2)). \qquad \text{(A6)}\end{aligned}$$

Also, note that both (A5) and (A4) simultaneously satisfy the common boundary conditions (3) and (4).

There remains only the boundary condition at $z = z_3$

$$\left.\frac{dV(x, y, z)}{dz}\right|_{z=z_3} = \sum_{n,m=0}^{\infty} B_{nm}\cos\left(\frac{m\pi y}{y_1}\right) \cdot \cos\left(\frac{n\pi x}{x_1}\right)\left.\frac{dF_{ac}}{dz}\right|_{z=z_3}. \qquad \text{(A7)}$$

This is just a double Fourier series for the function $dV/dz|_{z=z_3}$. Consequently, the coefficients B_{nm} are given by

$$B_{nm} = \frac{\dfrac{4}{x_1 y_1}\left[\displaystyle\int_0^{x_1} dx \int_0^{y_1} dy \left.\frac{dV}{dz}\right|_{z=z_3}\cos\left(\frac{m\pi y}{y_1}\right)\cos\left(\frac{n\pi x}{x_1}\right)\right]}{C\left.\dfrac{dF_{ac}}{dz}\right|_{z=z_3}} \qquad \text{(A8)}$$

where

$$C = \begin{cases} 4, & \text{for } n = 0 \text{ and } m = 0 \\ 2, & \text{for } n = 0 \text{ and } m \neq 0 \\ 2, & \text{for } n \neq 0 \text{ and } m = 0 \\ 1, & \text{for } n \neq 0 \text{ and } m \neq 0 \end{cases} \qquad \text{(A9)}$$

and

$$\begin{aligned}\left.\frac{dF_{ac}}{dz}\right|_{z=z_3} = {} & [\sinh(\delta_0 z_1)\cosh(\delta_0(z_2 - z_1)) \\ & + \eta\cosh(\delta_0 z_1)\sinh(\delta_0(z_2 - z_1))] \\ & \cdot \delta_0\sinh(\delta_0(z_3 - z_2)) \\ & + \left[\frac{1}{\eta}\sinh(\delta_0 z_1)\sinh(\delta_0(z_2 - z_1))\right. \\ & \left. + \cosh(\delta_0 z_1)\cosh(\delta_0(z_2 - z_1))\right] \\ & \cdot \delta_0\cosh(\delta_0(z_3 - z_2)). \qquad \text{(A10)}\end{aligned}$$

Substituting (A8) into (A5) we have

$$V(x, y, z) = \sum_{n,m=0}^{\infty} A_{nm}\cos\left(\frac{m\pi y}{y_1}\right)\cos\left(\frac{n\pi x}{x_1}\right), \quad z_2 \leq z \leq z_3 \qquad \text{(A11)}$$

where

$$A_{nm} = \frac{1}{C}\frac{4}{x_1 y_1}\left[\int_0^{x_1} dx \int_0^{y_1} dy \left.\frac{dV}{dz}\right|_{z=z_3} \cdot \cos\left(\frac{m\pi y}{y_1}\right)\cos\left(\frac{n\pi x}{x_1}\right)\right] \cdot \frac{F_{ac}}{\left.\dfrac{dF_{ac}}{dz}\right|_{z=z_3}}. \qquad \text{(A12)}$$

Note that as $\delta_0 \rightarrow 0$, it can be shown that

$$\frac{F_{ac}}{\left.\dfrac{dF_{ac}}{dz}\right|_{z=z_3}} \rightarrow [z_1 + \eta(z_2 - z_1) + (z - z_2)] \qquad \text{(A13)}$$

since the corresponding $\sinh(\delta_0 z) \rightarrow \delta_0 z$ and $\cosh(\delta_0 z) \rightarrow 1$. Also, it can be shown that with $z = z_3$ (A11) is equivalent to (10) in the text.

Special Case

For the case of uniform current density of majority carriers across a square injector ($a_1 \leq x \leq a_2$ and $a_1 \leq y \leq a_2$) with side-length $L (= a_2 - a_1)$, the surface potential drop due to the majority-carrier current (I_p) can be calculated using (A11), in which the coefficient A_{nm} can be written as

$$\begin{aligned} & A_{nm}(n \neq 0, m \neq 0) \\ & = \frac{4 I_p \rho_{sL}}{nm\pi^2 L^2}\left[\sin\left(\frac{m\pi a_2}{y_1}\right) - \sin\left(\frac{m\pi a_1}{y_1}\right)\right] \\ & \cdot \left[\sin\left(\frac{n\pi a_2}{x_1}\right) - \sin\left(\frac{n\pi a_1}{x_1}\right)\right] \cdot \frac{F_{ac}|_{z=z_3}}{\left.\dfrac{dF_{ac}}{dz}\right|_{z=z_3}} \qquad \text{(A14)}\end{aligned}$$

$$A_{om}(m \neq 0) = \frac{2 I_p \rho_{sL}}{m\pi x_1 L}\left[\sin\left(\frac{m\pi a_2}{y_1}\right) - \sin\left(\frac{m\pi a_1}{y_1}\right)\right] \cdot \frac{F_{ac}|_{z=z_3}}{\left.\dfrac{dF_{ac}}{dz}\right|_{z=z_3}} \qquad \text{(A15)}$$

$$A_{no}(n \neq 0) = \frac{2 I_p \rho_{sL}}{n\pi y_1 L}\left[\sin\left(\frac{n\pi a_2}{x_1}\right) - \sin\left(\frac{n\pi a_1}{x_1}\right)\right] \cdot \frac{F_{ac}|_{z=z_3}}{\left.\dfrac{dF_{ac}}{dz}\right|_{z=z_3}} \qquad \text{(A16)}$$

$$A_{oo} = \frac{I_p \rho_{sL}}{x_1 y_1} [z_1 + \eta(z_2 - z_1) + (z_3 - z_2)]. \quad \text{(A17)}$$

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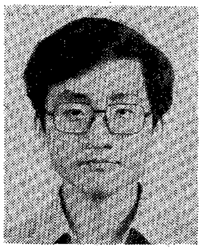

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