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Computer Integrated Manufacturing Systems Vol. 10, No. 3, p. 205–215, 1997

0951-5240/97 $17.00+0.00

PII: S0951-5240(97)00009-8

Assembly-part automatic positioning using high-level entities of mating features

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We present a new and convenient method for automatically positioning parts in an assembly. High-level entities of mating features and mating relations are used to describe the composed state of parts in an assembly. The characteristics of volume, reference origin, and boundary face of mating features and parts are used to determine automatic part-positioning operations. Using the proposed approach, a hierarchical model of the assembly can be constructed more easily with fewer input data. This approach thus makes computer-aided assembly design systems more user-friendly and greatly reduces the amount of input data required. An example is presented to demonstrate the simplicity and effectiveness of the proposed approach.

Keywords: assembly part-positioning, mating relation, mating feature

Introduction

One important issue in developing a feature-based computer-aided assembly design system is provision of a convenient and powerful part-positioning mechanism. In many older studies, however, designers are usually asked to input a 4×4 transformation matrix for each part to be assembled[1]. Because the physical meaning of such a transformation matrix is difficult to understand, and the input data are quite large, the assembly design process is tedious. So, this study is aimed at developing an approach that makes assembly modelling less difficult such that the design intent can more easily be expressed. We use the high-level entities of mating features and mating relations for assembly design.

Over the last decade, using features for part modelling has come to be thought of as a key to integrating design and manufacturing[2–4]. However, the applications of the feature concept are limited to the design of simple parts, not to the general assemblies. This paper uses high-level features to simplify part-positioning for assembly modelling. Instead of directly specifying the transformation operations for each part, we use mating features of the parts to automatically derive their positions.

In this paper, each feature represents a physical volume to be subtracted from or added to a block-type raw material of a part[5–8]. Mating features are defined as those features that comprise mating relations between parts to be assembled. Part feature information may include feature names and feature parameters (e.g. reference points, dimensions, orientations, and so on), which can be obtained from a feature-based part design system[5–7]. Feature orientation is assumed to be parallel/perpendicular to the coordinate axes of the global coordinate system. For example, to model assembly of part 1 and part 2 in *Figure 1*, using the proposed approach, the designer only has to specify that the *cylinder* feature of part 1 is to be *fitted* into the *hole* feature of part 2. Our method is based on the definition of volume feature, when to consider the mating relations of two features it will be easy for users to specify the candidate features to be mated.

The process of designing a feature-based assembly is given in *Figure 2*. The flow chart for automatic

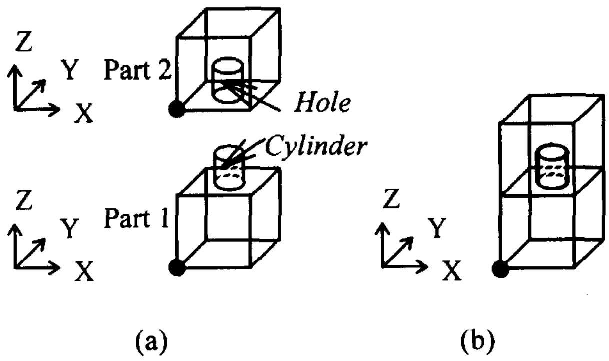


Figure 1 An assembly modelling process: (a) Part 1 and Part 2; (b) after assembling

part-positioning is given in *Figure 3*. Details of each step are described in the sections that follow.

In the remainder of this paper, we first review previous work on part-positioning for assembly modelling. Second, we introduce the issues concerning the relations among features, parts, and assemblies. Third, an approach to automatically positioning parts in an assembly is proposed. Fourth, we give a simulation example to illustrate the proposed approach in detail. Finally, we state conclusions of the paper and suggestions for future work.

Related work

In many older assembly modelling systems, the positions (locations and orientations) of the parts in an assembly are usually specified using a 4×4 homogeneous transformation matrix[9,10]. The designer has to specify the transformation matrix for positioning each part. It is quite awkward and prone to error. Lee *et al.*[1,11–13] tried to avoid the matrix-specification problem by solving a number of equations generated from mating conditions. They computed the transformation matrix for each part after all mating conditions had been interactively assigned. The mating conditions considered were of low-level graphic entities such as faces and centre-lines. Though a large number of redundant equations could be eliminated, the performance was still unsatisfactory[14]. According to the data structure developed by Rocheleau and Lee[1], Ko and Lee[15] proposed a rule-based approach to generate assembly plans.

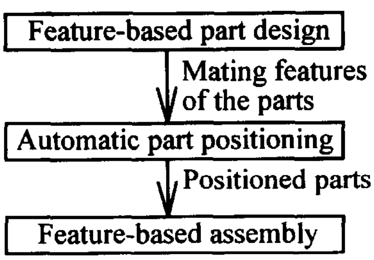


Figure 2 The process of designing a feature-based assembly

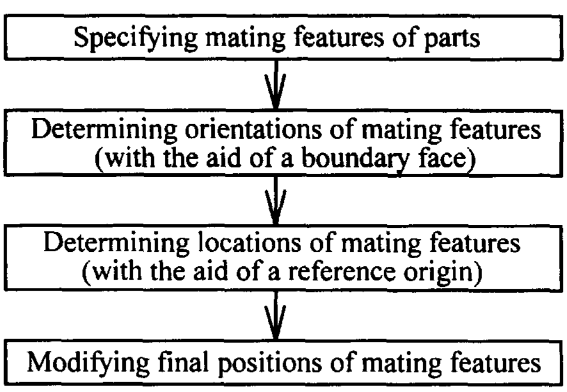


Figure 3 The flow chart for automatic assembly part-positioning

Srikanth *et al.*[14] adopted a sequential strategy for positioning the parts. They assumed that parts can be positioned sequentially in most assemblies. For each pair of parts to be positioned, a relative positioning operator was identified. A complex process for positioning the parts was proposed.

Researchers as Tokyo University have conducted a series of research projects that address the issue of 'relational product modelling'[16–21]. They considered the development of the relational model from the following viewpoints: functionality, integrity, complexity, interface, and realization[18]. For the application of assembly representation, they used a *frame* to represent a part, a subassembly, or an assembly which can be recognized by a human. Suzuki *et al.*[21] developed a method by which two- and three-dimensional geometric models are parametrically modified by changing dimensions. This approach was also applied to the prototype variational design system developed by Kimura and Suzuki[19].

Mullineux[22] thought the task of assembly could be automated if the various constraints imposed as rules were to be formulated. He looked at a way of formulating both equality and inequality constraints in a single form. A goal function was defined that takes a minimum value when all constraints hold and algorithms are used to numerically seek the minimum. The concept of design-with-features was not used in this approach.

Libardi *et al.*[23] in their review suggested that the concept of design-with-features will provide a basis for developing the required representation of spatial relationships in assemblies, while Lim *et al.*[24] and Rosen[25] in their reviews described similar prospects. For single parts, Silva *et al.*[26] described a strategy for the representation and manipulation of spatial relationships between features, especially for interacting and interfeature relationships of mechanical components. Shah and Tadepalli[27] applied the concept of assembly features to their assembly modeller by employing knowledge-based techniques. De Fazio *et al.*[28] used assembly directions and locations of features to align parts. However, the positioning method was not clearly described in the last two references[27,28]. Turner[29] proposed a mathematical programming approach to the problem of relative positioning of parts in assemblies. The position of each part was specified based on the geometric relationships between various features of the part and the mating features of its neighbouring parts. These feature relationships were treated as inequalities. Mathematical programming was used to find the optimal configuration of the parts. Only 2D examples were discussed in his approach.

Anantha *et al.*[30] proposed a framework for using a symbolic geometry approach with 'intermediate geometry' elements[31]. 'Ghost objects' were used to determine feasibility of satisfying mating conditions, i.e. assemblability, interferences, or motion limits on degrees of freedom were not taken into account. Moreover, Shah and Rogers[31] developed concepts for modelling all levels of product relations with a uniform set of structures and relationships. They did not check interference between part volumes.

Several researchers have applied AI techniques to assembly modelling[32-35]. The Edinburgh Design System (EDS), developed by Popplestone, is based on two types of entities, namely concrete modules and interface modules[34,35]. Each module might in turn have subassemblies or parts. Concrete modules contain parameters, variables, etc. whose values need to be explicitly specified at the time of design (e.g. the diameter of a shaft, and shaft speed). Interface modules encode the constraints between these parameters and variables which form the edges of the graph structure. The representation of assemblies was carried out in terms of a high-level language. RAPT[36] was used for input language and a complex AI technique[32] was applied to derive the positions of bodies relative to others given these spatial relationships.

To simplify the assembly design task, a new approach that provides a convenient method for automatically positioning the parts in an assembly is developed in this paper.

Relations among features, parts, and assemblies

Before describing the detailed part-positioning method, we first discuss some issues concerning relations among features, parts and assemblies in this section. The first issue concerns the mating relations of mating features for easy description of assembling processes. The second issue concerns the local reference origin of parts and mating features. The third issue concerns boundary faces used to describe the orientations of parts and mating features.

Mating relations of mating features

The mating relation is usually used to simplify description of the assembled state of two parts. In the past, low-level graphic entities such as faces, centrelines, and so forth were used to represent the mating relation. Instead of low-level graphic entities, we use high-level mating features of the parts to be assembled to represent mating relations. The concept of male and female mating features of the parts in mating relations can be used to clearly describe the assembly of two parts. The spatial relationships between pairs of mating features described by the mating relations are the focus of our discussions.

In this paper, the commonly used mating relations 'fit', 'contact', and 'align' are considered. Discussion of other kinds of mating relations can be found in Perng and Chang[8]. A set of eight sample mating relations is shown in *Table 1*. In a mating relation, a male mating feature usually corresponds to a protrusion feature, while a female mating feature usually corresponds to a depression feature. When a mating relation consists of two depression mating features, such as types 7 and 8, designers must specify which one acts as the male mating feature and which one acts as the female mating feature[8]. When two parts are to be assembled, the part containing the male mating feature is called a male mating part; the other part containing the female mating feature is called a female mating part.

To mate two parts is to match their corresponding mating features in shapes and volumes. The mating relation of every pair of parts in an assembly can be considered as the mapping of a pair of mating features. The volumes of the pairs of mating features corresponding to the mating relations in *Table 1* have the following spatial relationships.

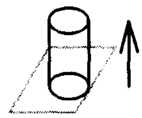
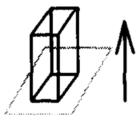
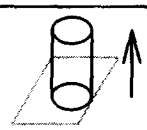
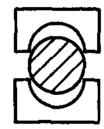
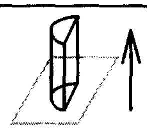
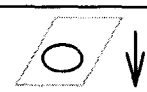
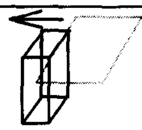
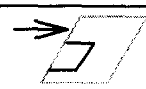
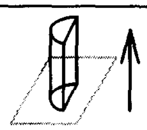
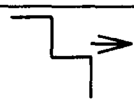
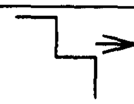
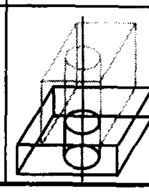
(a) In the fit mating relation, the pair of mating features share the same volume space.
(b) In the contact mating relation, the pair of mating features does not share the same volume space but contact with each other.
(c) In the align mating relation, the pair of mating features does not share the same volume space but align with each other.

Moreover, we assume that an object in an assembly can be positioned according to its location and orientation in the assembly. The object may be a part, or a mating feature. A subassembly is treated as a part in this paper. The location is described by the reference origin, while the orientation is described by the boundary face.

The reference origin (RO) and the location of a mating feature/part

The location of an object can be described in terms of its reference origin. The reference origin (RO) of an object is the point of origin of a right-handed coordinate system that defines an object's location in space, see *Figure 4*. To simplify the automatic part-positioning task for assembly design, the initial RO of each part is set at the lower left corner in a feature-based design system[5]; the RO of the male mating part

Table 1 Eight sample mating relations and corresponding mating features. The symbol '→' indicates the orientation of the feature respect to a right-handed coordinate system that defines the feature[5]

Type	Mating relation	Male mating feature	Symbol of male mating feature	Female mating feature	Symbol of female mating feature	Examples
1	fit_a	*Cylinder*		*Hole*		
2	fit_b	*Boss*		*Pocket*		
3	fit_c	*Cylinder*		*Half-hole*		
4	fit_d	*Half-cylinder*		*Hole*		
5	fit_e	*Plug*		*Uslot*		
6	fit_f	*Half-cylinder*		*Half-hole*		
7	contact	*Step*		*Step*		
8	align	*Hole*		*Hole*		

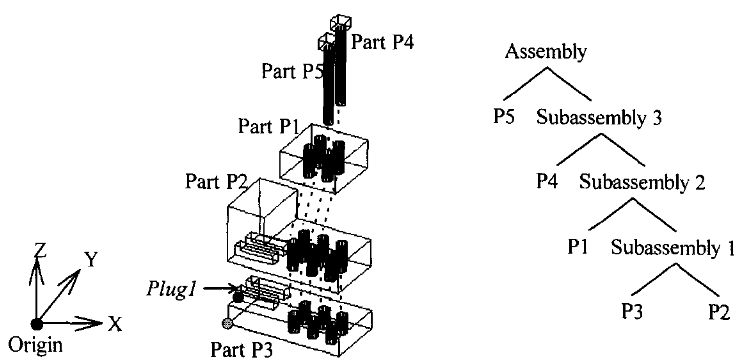


Figure 4 The ROs of sample parts and corresponding mating features. The symbol '◎' indicates the RO of a part, indicates the RO of a mating feature

is used as the RO of the newly-assembled subassembly.

As to a mating feature, the RO is a predefined location for each type of mating feature as listed in *Table 2*. The RO of a mating feature represents the location by which the mating feature can be located in a part as shown in *Figure 4*. Because the relative distance between the ROs of a feature and its associated mating part is fixed, in any mating relation, when the RO of a mating feature is transformed from the initial location to the final assembled location, the associated mating part is placed at the proper location accordingly. When the transformation operations performed on the ROs of the mating features are propagated to the associated mating part, the RO of the mating part can then be located. In each mating step, the automatic part-positioning task must compute the new position of the female mating part to derive the corresponding transformation operations.

Therefore, our strategy for determining the RO of a female mating part is: (a) retrieve the locations of the initial ROs of associated mating features; (b) generate transformation operations such that the female mating feature can be set at the final assembled location. For different mating relations, the ROs of the mating features of the mating parts have the following spatial relationships.

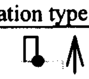
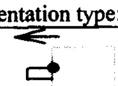
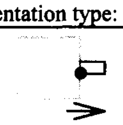
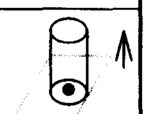
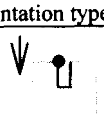
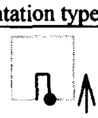
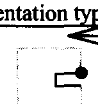
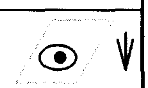
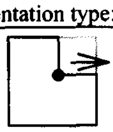
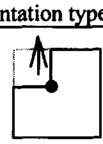
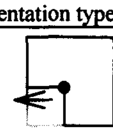
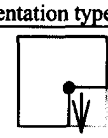
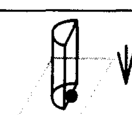
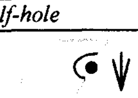
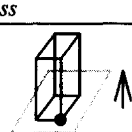
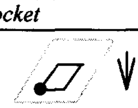
(a) For the fit mating relation, the ROs of the pair of mating features will be at the same location.
(b) For the contact and align mating relations, the ROs of the pair of mating features will not be identical. The final assembled location will be determined by the relative distance between the ROs.

The orientation of a mating feature/part described in terms of the boundary face (BF)

We use the boundary face (BF) concept to describe transformation operations for determining orientations of mating features. A BF is defined as one of the six outermost surfaces, called the 'Front', 'Back', 'Left', 'Right', 'Top', or 'Bottom' face, of a block-type raw material (see *Figure 5*(a)). The normal vector of a BF is defined as pointing outward. A feature's BF can be determined by its feature type and RO. The BF of a depression feature is the face on which the feature's RO is located, see *Figure 5*(b). For a depression feature whose RO is not on any outermost surface of the raw material, one outermost surface with a normal vector opposite to the feature orientation is selected as its BF, see *Figure 5*(c). As for a protrusion feature, the outermost surface whose normal vector is opposite to the feature orientation is set as its BF, see *Figure 5*(d).

Using the BF concept, we can easily express the rotation transformation to determine the orientations of the mating features of different parts. For example, when the part with a *hole* feature, as shown in *Figure 5*(b), is to be combined with the part with a *cylinder* feature, as shown in *Figure 5*(d), this can simply be described as rotating the BF of the *hole* feature from Front to Back. That is, the orientation

Table 2 The RO and orientation of each type of mating feature. (1) The RO is indicated by the symbol '•'; (2) the orientation is indicated by the symbol '→'

Feature	*Plug* (feature-orientation type: 1)	*Plug* (feature-orientation type: 2)	*Plug* (feature-orientation type: 3)	*Plug* (feature-orientation type: 4)	*Cylinder*
RO location					
Feature	*Uslot* (feature-orientation type: 1)	*Uslot* (feature-orientation type: 2)	*Uslot* (feature-orientation type: 3)	*Uslot* (feature-orientation type: 4)	*Hole*
RO location					
Feature	*Step* (feature-orientation type: 1)	*Step* (feature-orientation type: 2)	*Step* (feature-orientation type: 3)	*Step* (feature-orientation type: 4)	*Half-cylinder*
RO location					
Feature	*Half-hole*	*Boss*	*Pocket*		
RO location					

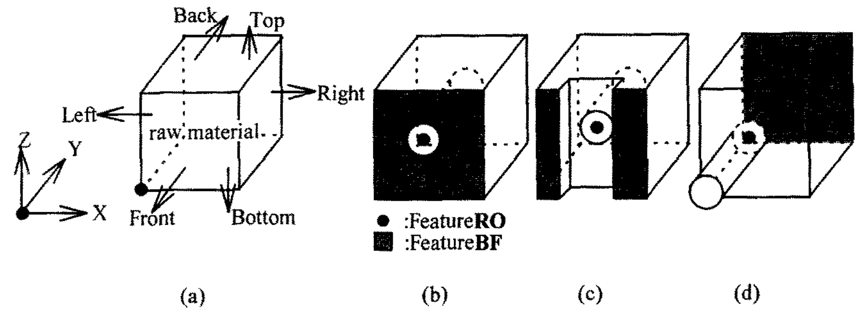


Figure 5 Illustrations of BFs. (a) Each outermost surface of a raw material is defined as a potential BF; (b) a depression feature whose RO is located on an outermost surface; (c) a depression feature whose RO is not located on an outermost surface; (d) a protrusion feature

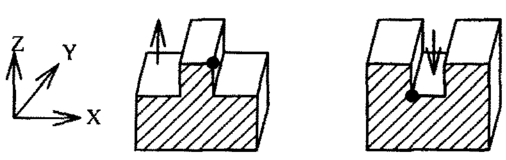
of the female mating part can be determined through the rotation transformation of the corresponding female mating feature's BF. Once the orientation of a female mating feature has been determined, the orientation of the associated female mating part can be determined accordingly.

Following the definition of BF, the BFs of pairs of mating features can be found on the same projection direction if the corresponding mating parts are to be assembled. For example, in *Figure 6*, the BFs of the two mating features are on the same Front view, and the parts to be mated are on the desired orientation accordingly. This general spatial relationship for mating features can be applied to all eight mating relations listed in *Table 1*.

We also define a set of direction numbers on the edges of potential BFs that corresponds to the orientation types of mating features in *Table 2* to facilitate computing the rotation angles of female mating parts when the BFs of the mating features are on the same projection direction. Direction numbers on each BF, as shown in *Figure 7*, are assigned counterclockwise from one to four, and the upper edge of the BF in the normal coordinate system is assigned the number '1' as viewed from the outside of the raw material.

The assembly part automatic positioning

An approach to conveniently positioning parts using the characteristics of volume, RO, and BF of the mating parts/mating features is described in this section. When two mating parts and their associated mating features are specified in designing an assembly, we first consider the specified relations of



(a) *Plug* (b) *Uslot*

Figure 6 Two mating features with the BFs on the same projection direction

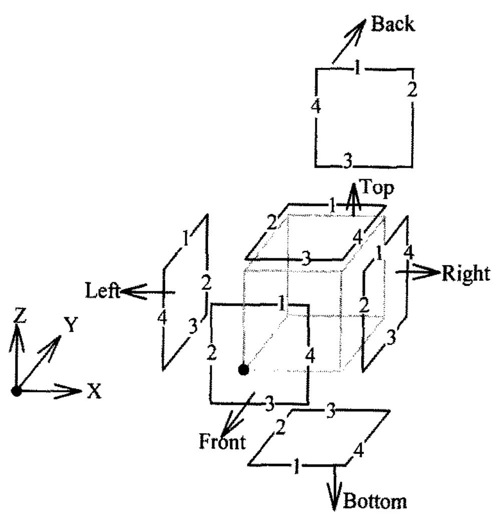


Figure 7 Direction numbers of different feature-orientation types on each BF

Table 3 Ten groups of BF combinations with corresponding rotation axes and angles

Group	The BF combinations (BF of the male feature vs BF of the female mating feature)	Rotation axis	Rotation angle
1	Front–Front, Back–Back, Top–Top, Bottom–Bottom, Left–Left, Right–Right	none	0°
2	Front–Left, Left–Back, Back–Right, Right–Front	z	90°
3	Front–Top, Top–Back, Back–Bottom, Bottom–Front	x	90°
4	Left–Bottom, Bottom–Right, Right–Top, Top–Left	y	90°
5	Front–Right, Right–Back, Back–Left, Left–Front	z	−90°
6	Front–Bottom, Bottom–Back, Back–Top, Top–Front	x	−90°
7	Left–Top, Top–Right, Right–Bottom, Bottom–Left	y	−90°
8	Front–Back, Back–Front	z	180°
9	Left–Right, Right–Left	y	180°
10	Top–Bottom, Bottom–Top	x	180°

Table 4 The contents of the quick-reference matrix Q

	Q [*Bm*, *Bf*]	(*Bf*) Front	Back	Left	Right	Top	Bottom
(*Bm*)	Front	[none, 0]	[z, 180°]	[z, 90°]	[z, −90°]	[x, 90°]	[x, −90°]
	Back	[z, 180°]	[none, 0]	[z, −90°]	[z, 90°]	[x, −90°]	[x, 90°]
	Left	[z, −90°]	[z, 90°]	[none, 0]	[y, 180°]	[y, −90°]	[y, 90°]
	Right	[z, 90°]	[z, −90°]	[y, 180°]	[none, 0]	[y, 90°]	[y, −90°]
	Top	[x, −90°]	[x, 90°]	[y, 90°]	[y, −90°]	[none, 0]	[x, 180°]
	Bottom	[x, 90°]	[x, −90°]	[y, −90°]	[y, 90°]	[x, 180°]	[none, 0]

the mating features of the parts to be assembled, then examine the spatial relationships between the initial state and the final assembled state of the mating features, and finally derive the necessary transformation operations needed to achieve the mated relations. These transformation operations are then performed on the two mating parts as well as mating features such that they finish in the proper locations and orientations.

Thus, the principle of our automatic positioning approach is first to make the BFs of the two mating features have the same projection direction, and then to place the ROs of the two mating features at the same location. Three general steps are proposed to compute the new position of the female mating part to be assembled. The first two steps can be applied to all the mating relations listed in *Table 1*, while the last step can be applied when different combinations of mating features are further considered. Details of these steps are explained below.

Step 1. *Rotate the female mating part to make the BF of the associated female mating feature coincide with the BF of the corresponding male mating feature.*

Before rotation, we first translate the female mating part to make the RO of the female mating feature coincide with the RO of the male mating part. We select the RO of the female mating feature as the rotation centre, and select the x, y, or z axis as the rotation axis. The rotation angle used in this paper is a counterclockwise rotation. After rotation, the female mating feature is moved back to its original location. The rotation axis and the rotation angle are determined by the pairing cases of the BFs of the male and female mating features. There are 36($= 6 \times 6$) possible combinations of BFs for the male and female mating features. Combinations with the same rotation axis and rotation angle are clustered in groups. 10 groups of combinations can thus be obtained, as listed in *Table 3*. The process of deriving the rotation axes and the rotation angles for groups 1 to 4 is explained below.

Each pair of BFs in the first group in *Table 3* of male and female mating features is identical, thus the original mating features or associated mating parts are in the desired orientations and no rotation is needed. For group 2, rotating the female mating part 90° counterclockwise about the z axis will place the BFs of the mating features on the same projection direction. Thus, the rotation angle is 90° for group 2. The rotation angle is also 90° for groups 3 and 4, but the rotation axes are the x axis and the y axis, respectively. Other groups can be explained in a similar fashion.

A quick-reference matrix **Q**[*Bm*, *Bf*] is built up for easy retrieval of information on the rotation axes and rotation angles in *Table 3*. The index *Bm* is the BF of the male mating feature; while *Bf* is the BF of the female mating feature. The corresponding elements of the quick-reference matrix **Q** in the form [rotation axis, rotation angle] are given in *Table 4*.

The transformation operations in Step 1 can be summarized as:

$$\mathbf{T}[-x_f, -y_f, -z_f]\cdot\mathbf{R}[\text{rotation axis, rotation angle}]\cdot\mathbf{T}[x_f, y_f, z_f]$$

where

(1) **T** indicates a translation operation, (x_f, y_f, z_f) are the coordinate values of the RO of the female mating feature, and (0, 0, 0) is the RO of the male mating part;

Table 5 The rotation axis and corresponding BF of different feature-orientation types of a *plug*

BF	Rotation-axis directions
Back/Front	y
Right/Left	x
Top/Down	z

Table 6 Summarized four groups of the feature-orientation types of *plug* and *Uslot*. *Tm* is the direction number of the orientation type of a *plug*, *Tf* is the direction number of the orientation type of a *Uslot*

Group	The combinations of feature-orientation types of a *plug* and a *Uslot* (*Tm* − *Tf*)	Rotation angle: Back, Right, or Top BF of a *plug*	Rotation angle: Front, Left, or Down BF of a *plug*
1	1−1 = 0, 2−2 = 0, 3−3 = 0, 4−4 = 0	180°	180°
2	1−2 = −1, 2−3 = −1, 3−4 = −1, 4−1 = 3	90°	270°
3	1−3 = −2, 2−4 = −2, 3−1 = 2, 4−2 = 2	0°	0°
4	1−4 = −3, 2−1 = 1, 3−2 = 1, 4−3 = 1	270°	90°

Table 7 The rotation axes and associated rotation angles corresponding to *plug* BFs

BF	Rotation axis	Rotation angle
Back	y	eqn (2)
Front	y	eqn (3)
Right	x	eqn (2)
Left	x	eqn (3)
Top	z	eqn (2)
Down	z	eqn (3)

(2) **R** indicates a rotation operation, the rotation axis and the rotation angle are obtained from the quick-reference matrix **Q**.

Step 2. *Translate the female mating part to make the RO of the female mating feature coincide with the RO of the corresponding male mating feature.*

In Step 2, the transformation operation is:

$$\mathbf{T}[\Delta x, \Delta y, \Delta z]$$

where $\Delta x = x_m - x_f$, $\Delta y = y_m - y_f$, $\Delta z = z_m - z_f$, and (x_m, y_m, z_m) are the coordinate values of the RO of the male mating feature.

So, for the mating relations fit_a, fit_b, fit_c, and fit_d, the associated mating parts can be assembled using the following transformation operations:

$$\mathbf{T}[-x_f, -y_f, -z_f]\cdot\mathbf{R}[\text{rotation axis, rotation angle}]\cdot\mathbf{T}[x_f, y_f, z_f]\cdot\mathbf{T}[\Delta x, \Delta y, \Delta z] \quad (1)$$

where [rotation axis, rotation angle] = $\mathbf{Q}[Bm, Bf]$.

Step 3. *According to the properties of the female mating feature, perform additional transformation on the female mating part to reach the final assembled position.*

When some features with different orientation types, as listed in *Table 2*, are to be assembled using the mating relations fit_e, fit_f, contact or align, further transformation operations are required. Below, we describe additional transformation operations for the fit_e mating relation in detail. A description of the transformation operations for other mating relations can be found in Perng[37].

fit_e: *plug* and *Uslot*. After the two part-positioning steps described above, the BFs of the two mating features will be on the same projection direction.

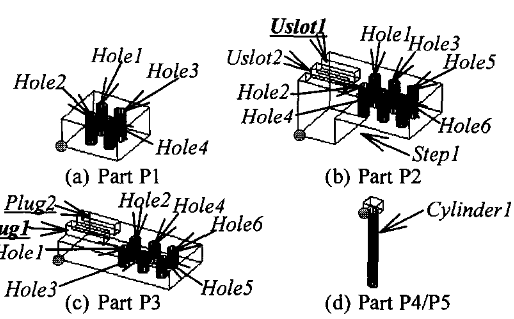


Figure 8 The parts of the sample assembly: the symbol '•' indicates the RO of each part

However, the *plug* and *Uslot* on the same BF will still have four feature orientations each, as shown in *Table 2*. There are 16 possible combinations of feature orientations for a *plug* and a *Uslot* and a rotation transformation must be used to fit the *plug* and the *Uslot* features correctly. The rotation axis is determined by the BF of the *plug* feature as listed in *Table 5*.

Meanwhile, the rotation angle can be determined by the combinations of different feature orientations of a *plug* and a *Uslot*, and the BF of the *plug*. The 16 combinations of feature orientations are grouped into four groups in *Table 6* according to the rotation angles and BF of the *plug*.

From *Table 6*, we can derive an equation for the desired rotation angle when the BF of a *plug* is Back, Right, or Top:

$$\text{rotation angle} = [(Tm - Tf) \times 90° + 540°] \bmod 360° \quad (2)$$

where Tm is the direction number of the feature orientation type of *plug*, Tf is the direction number of the feature orientation type of *Uslot*, and mod is the modulo division operator.

Thus, a similar equation for the desired rotation angle when the BF of the *plug* is Front, Left, or Down is:

$$\text{rotation angle} = [(Tf - Tm) \times 90° + 540°] \bmod 360° \quad (3)$$

After analysing *Table 5*, *Table 6*, eqns (2) and (3), we summarize the rotation axes and the rotation angles corresponding to each of the BFs of the *plug* feature in *Table 7*.

The final additional transformation operations are:

$$\mathbf{T}[-x_f, -y_f, -z_f]\cdot\mathbf{R}[\text{rotation axis, rotation angle}]\cdot\mathbf{T}[x_f, y_f, z_f] \quad (4)$$

where the rotation axis and the rotation angle are obtained from *Table 7*.

Therefore, when two mating features and the associated mating relation are specified, the proposed approach can automatically generate the transformation matrices accordingly and can assemble the parts properly. To determine the feasibility of assembly, a check for interference between part volumes[38] is done by a solid modelling system, ACIS[39]. Detailed description algorithms and implementations of the automatic part-positioning approach for all eight mating relations listed in *Table 1* can be found in Perng[37].

Example

In this section, we give an example to demonstrate in detail the effects of the proposed automatic part-positioning approach. The sample assembly is composed of parts P1, P2, P3, P4 and P5 as shown in *Figure 8*. The feature information about each part is input by a 3D feature-based design system[8].

The interactive procedure for modelling the sample assembly is given in *Table 8*. The data in columns 2 to 4 are specified by the designer, the final column is the corresponding assembly tree[8]. The detailed steps for automatically computing the female mating part P2 in the first mating step are given in *Table 9*. The designer must specify that parts P3 and P2 are, respectively, the male and female mating parts, and must specify the corresponding mating features of *plug1* and *Uslot1*.

The summarized transformation operations in *Table 9* are: $\mathbf{T}[0, -30, -40]\cdot\mathbf{T}[0, -15, -20]\cdot\mathbf{R}[x, 180°]\cdot\mathbf{T}[0, 15, 20]$. Here, the ACIS solid modelling system is used to compute the matrix transformation operations, **T** and **R**. We need but specify the necessary parameters of the matrix transformation operations and call the corresponding transformation functions of ACIS, then the results of these complex transformation matrices will be computed automatically by our proposed approach. The final positions and transformation matrices of the other mating parts can be computed as positioning female mating part P2 was. Hence, the designer can easily model an assembly by specifying the mating relations using high-level mating features and mating parts.

The transformation matrix and subassembly graph for each step are given in Appendix A.

Table 8 The simulated procedure for modelling the sample assembly

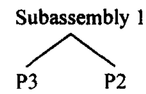
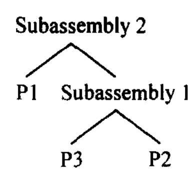
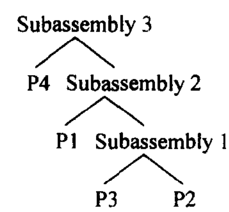
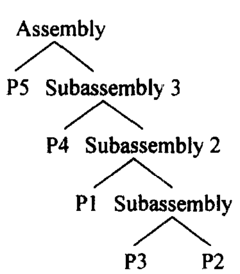
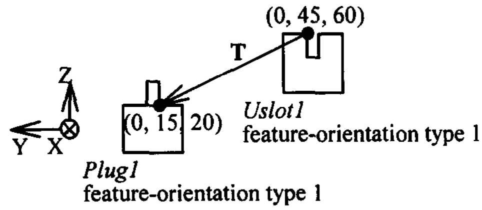
Step	Male mating part	Female mating part	Mating relation	Assembly tree
1	Part P3	Part P2	(1) fit_e1: P3–*plug1*–male mating part P2–*Uslot1*–female mating part (2) fit_e2: P3–*plug2*–male mating part P2–*Uslot2*–female mating part (3) align 1: P3–*hole1*–male mating part P2–*hole1*–female mating part (4) align2: P3–*hole2*–male mating part P2–*hole2*–female mating part (5) align3: P3–*hole3*–male mating part P2–*hole3*–female mating part (6) align4: P3–*hole4*–male mating part P2–*hole4*–female mating part (7) align5: P3–*hole5*–male mating part P2–*hole5*–female mating part (8) align6: P3–*hole6*–male mating part P2–*hole6*–female mating part	
2	Part P1	Subassembly1	(1) align 1: P1–*hole1*–male mating part P2–*hole3*–male mating part (2) align2: P1–*hole2*–male mating part P2–*hole4*–male mating part (3) align3: P1–*hole3*–male mating part P2–*hole5*–female mating part (4) align4: P1–*hole4*–male mating part P2–*hole6*–female mating part	
3	Part P4	Subassembly2	(1) fit_a1: P4–*cylinder1*–male mating part P1–*hole3*–female mating part	
4	Part P5	Subassembly3	(1) fit_a1: P5–*cylinder1*–male mating part P1–*hole4*–female mating part P5	

Table 9 The process for automatically computing the position of female mating part P2 for assembling

Step	Transformation operation description
1. Get the BF of *plug1* in the male mating part (*Bm*) and the BF of *Uslot1* in the female mating part (*Bf*)	From *Figure 8*, both the *Bm* and *Bf* are Left
2. Rotate the female mating part such that the BF of *Uslot1* coincides with that of *plug1*. Use the *Bm* and *Bf* to find the corresponding rotation axis and rotation angle from **Q**[*Bm*, *Bf*] as given in *Table 4*	Because *Bm* = Left and *Bf* = Left, we have **Q**[Left, Left] = [none, 0]. No rotation is needed
3. Translate the female mating part such that the RO of *Uslot1* is identical to that of *plug1*	 Move (0, 45, 60) to (0, 15, 20). The translation operation is **T**[0, −30, −40]
4. Further rotation transformation is needed	The feature orientation is inconsistent
4a. Translate the female mating part to make the RO of the female mating feature coincide with that of the male mating part	Move (0, 15, 20) to (0, 0, 0). The translation operation is **T**[0, −15, −20]
4b. Determine the rotation axis from the BF of *plug1* in *Table 7*; determine the rotation angle from the combination of feature-orientation types of *plug1* (*Tm*), *Uslot1* (*Tf*), and eqn (3)	*Tm* = 1, *Tf* = 1, and *Bm* = Left, from *Table 7* the rotation axis is the *x* axis; the rotation angle is 180° ($= [(1-1)\times 90° + 540°] \bmod 360°$). The rotation operation is **R**[*x*, 180°]
4c. Translate the female mating part to bring the RO of the female mating feature back to the original location in step 4a	Move (0, 0, 0) to (0, 15, 20). The translation operation is **T**[0, 15, 20]

Conclusions

In this paper, we have proposed a new approach that provides a convenient method for automatically positioning parts in an assembly for a feature-based assembly design system. The proposed approach can transform the part components from the input state to the assembled state using high-level entities of mating features and mating relations. The characteristics of volume, reference origin, and boundary face of mating features and parts are used to determine the automatic positioning approach. When considering the mating relations of two features, our method is based on the definition of volume feature. It is easier for users to specify the candidate features to be mated than those for surface-based features. Using the proposed approach, designers no longer have to input complicated position data for all parts and specify the transformation operations. Assemblies can thus be constructed more easily and efficiently with fewer data inputs. Furthermore, the design intent can be more easily expressed using the proposed approach.

Although the transformation operations derived in this paper can work well for two mating parts with only one pair of mating features, more research is required for cases in which two parts with multiple pairs of features must be mated. More mating relations between mating features have to be considered in further research. When tolerances are considered, validation of loose mating or tight mating during assembling remain to be studied.

Acknowledgements

This work was supported by the National Science Council, Taiwan under contract NSC 85-2213-E-009-038.

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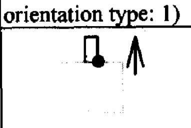
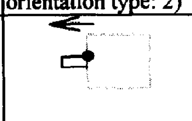
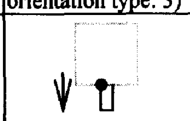
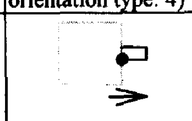
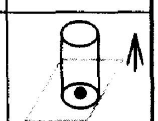
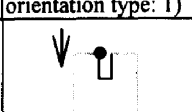
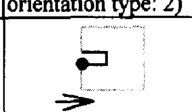
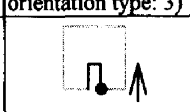
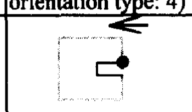
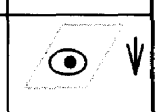
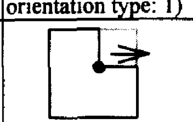
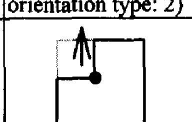
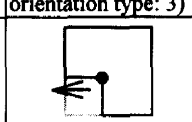
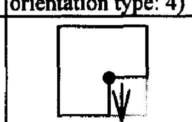
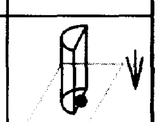
Appendix A

The transformation matrices corresponding to Table 9

Feature	*Plug* (feature-orientation type: 1)	*Plug* (feature-orientation type: 2)	*Plug* (feature-orientation type: 3)	*Plug* (feature-orientation type: 4)	*Cylinder*
RO location					
Feature	*Uslot* (feature-orientation type: 1)	*Uslot* (feature-orientation type: 2)	*Uslot* (feature-orientation type: 3)	*Uslot* (feature-orientation type: 4)	*Hole*
RO location					
Feature	*Step* (feature-orientation type: 1)	*Step* (feature-orientation type: 2)	*Step* (feature-orientation type: 3)	*Step* (feature-orientation type: 4)	*Half-cylinder*
RO location					
Feature	*Half-hole*	*Boss*	*Pocket*		
RO location