國立交通大學

統計學研究所

碩士論文

容許區間建構的 c 管制圖

Improved c Control Charts based on Tolerance Intervals

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摘　　　要

傳統的 c 管制圖通常是根據卜瓦松分佈的信賴區間來建構的管制圖。當監控分佈的參數已知時，現有的 c 管制圖有令人滿意的監控結果，然而，當此參數未知時，現有的 c 管制圖則會遭遇到型一誤差過大的缺點。我們在這研究論文中，提出利用卜瓦松分佈的容許區間來建構 c 管制圖。藉由統計模擬和實際資料的分析顯示，新的 c 管制圖在許多情況下都優於現有的 c 管制圖。

關鍵詞：c 管制圖、容許區間、卜瓦松分佈。

Improved c Control Charts based on Tolerance Intervals

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Abstract

In the previous studies, c charts are usually constructed by confidence intervals for the mean of a poisson distribution. When the mean is known, the existing c charts may lead to a satisfactory result, However, when the mean is unknown, the existing c charts suffer the drawback of large type I error. In this study, c charts based on tolerance intervals for the poisson distribution are proposed. A numerical study shows the proposed c charts outperform the existing ones.

Key words: c chart, tolerance interval, poisson distribution.

誌　　謝

就讀交大統研所的兩年，時光飛逝，卻令我擁有著充實與美好的記憶。很高興能在此認識許多師長以及同學，沒有你們的教導和陪伴，我不會如此順利的邁向人生的下一段旅程。

首先，感謝我的指導教授王秀瑛老師。研究期間，從確立方向、尋找現有方法、提出新方法，直到本文完成，您總是用心的引導我的思緒，使我培養出做學問的態度，非常榮幸能夠成為您的指導學生。同時感謝口試委員，盧鴻興老師、黃榮臣老師、吳謂勝老師，提供許多建議和修正，使我的論文更加完善。

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羅玉芬　謹誌于

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1 Introduction

Attribute Charts are a set of control charts specifically designed for attribute data, which is also known as "count" data. The c-chart is a type of attribute control chart used to monitor the number of nonconformities per unit.

In most cases, the inspection unit is a single unit of product. However, sometimes the inspection unit could consist of more than a single unit. A nonconforming item contains at least one nonconformity. The c control chart is used to detect if the number of nonconformities in an inspection unit is in control and it is signal when a deviation from stability occurs.

The random Y of the number of nonconformity in an item can be assumed to follow a Poisson distribution with the probability function

$$p(Y = y) = \frac{e^{-\lambda}\lambda^y}{y!}$$

where $\lambda > 0$ is the parameter of the Poisson distribution. Assume that an inspection unit has n units, that is, subgroup size being n. Let X denote the total number of nonconformities in this inspection unit. Suppose that the nonconformities in this inspection unit is done independently. Then X follows the poisson distribution $Poi(n\lambda)$. Let x_i denote the observation of the ith inspection unit. A monitor process is to monitor $x_i/n, i = 1, \ldots$. The evaluation of c control chart can be based on the type I error, which is the probability that x_i/n does not fall between the upper and the lower limits of the chart. When λ is known, the widely used c control chart with type I error 0.0027 is the control chart with

3-sigma control limits defined as follows

$$UCL = \lambda + 3\sqrt{\lambda}$$

$$CL = \lambda \tag{1}$$

$$LCL = \lambda - 3\sqrt{\lambda}$$

If $LCL < 0$, then assume $LCL = 0$.

When λ is unknown, the widely used c control chart with type I error 0.0027 is the control chart with 3-sigma control limits defined as follows

$$UCL = \overline{\lambda} + 3\sqrt{\overline{\lambda}}$$

$$CL = \overline{\lambda} \tag{2}$$

$$LCL = \overline{\lambda} - 3\sqrt{\overline{\lambda}}$$

where $\overline{\lambda}$ is the observed average number of nonconformities in a preliminary sample of inspection units. (Shewhart 1926)

For the case when λ is known, the type I error of the control chart (1) is not far away from the nominal level when λ is large. However, when λ is unknown, the type I error of the conventional control chart (2) does not only depend on inspection unit n, but also depends on the value of true λ. Since the control chart (1) is constructed based on the normal approximation, the performance of the control chart (2) may perform poorly when the sample size n is not large enough.

There are other existing c charts in the literature Bartlett (1936) used a transformation of Y to approximate a normal distribution to construct a c chart as well as Anscombe (1948). Bartlett (1947) has further discussion based on this chart. Ryan and Schwertman (1997) used regression equations to construct a c chart and Winterbottom (1993) applyed

Cornish and Fisher asymptotic expansion to construct a c chart.

In this study, we use tolerance interval approach to construct a c chart. Tolerance intervals are useful tools to capture characteristics of the underlying distribution of collected data in industrial, clinical trials, pharmaceutical and life insurance applications (see Hahn and Chandra, 1981; Hahn and Meeker, 1991; Zaslavsky, 2007; Wang, 2007; Gebizlioglu and Yagci, 2008; Cummings, Zhou and Dive, 2011). A tolerance interval is a statistical interval within which, with some probability, a specified proportion of a population falls.

Let W be a random variable with cumulative distribution function F. An interval $(L(W), U(W))$ is said to be a β-content, $(1-\alpha)$-confidence tolerance interval for F (called a $(\beta, 1-\alpha)$ tolerance interval for short) if

$$P\{[F(U(W)) - F(L(W))] \geq \beta\} = 1 - \alpha. \tag{3}$$

In this paper, we use an adjusted tolerance interval by the Edgeworth expansions to construct an improved c control chart. Through the simulation study, this new chart can successfully improved the monitor accuracy when true λ is small and is better than several existing charts. Edgeworth expansions have also been used very successfully for the construction of confidence intervals in discrete distributions (see Brown, Cai and DasGupta, 2002, 2003; Cai, 2005).

The thesis is organized as follows. The existing control charts of the number of nonconformities in an inspection unit when the true λ is known or unknown are briefly discussed in Section 2. The performances of these charts are evaluated in terms of the type I error. In Section 3, a new chart based on an adjusted tolerance interval which can improve the existing chats by reducing the type I error when the true λ is small. The proposed chart is compared with the existing methods by evaluating their expected widths in Section 4. The performances of the new charts are illustrated by a real data example in Section 5. Finally, a conclusion remark in given Section 6.

2 Existing methods

Besides the conventional control chart (2), the four existing charts with nominal type I error 0.0027 discussed in Bartlett (1936), Anscombe (1948), Ryan and Schwertman (1997), and Winterbottom (1993) are introduced as follows.

Bartlett c chart for λ known

Bartlett (1936) based on the fact that the transformation $V = 2\sqrt{X}$ approximates a normal distribution with $\mu = 2\sqrt{n\lambda}$ and $\sigma^2 = 1$. Define $v_i = 2\sqrt{x_i}$. We can plot $v_2, v_3, \ldots$ on a chart with control limits defined as follows

$$\begin{aligned} UCL &= 2\sqrt{n\lambda} + 3 \\ CL &= 2\sqrt{n\lambda} \\ LCL &= 2\sqrt{n\lambda} - 3 \end{aligned} \tag{4}$$

Bartlett c chart for λ unknown

Define $\hat{\lambda}_i = x_{i1} + \ldots + x_{in}$, and for $i = 2, 3, \ldots$ define $v_i = 2\sqrt{x_i}$, where $x_{i1}, \ldots, x_{in}$ denote the n units in the ith inspection unit. We can plot $v_2, v_3, \ldots$ on a chart with control limits defined as follows

$$\begin{aligned} UCL &= 2\sqrt{n\hat{\lambda}_{i-1}} + 3 \\ CL &= 2\sqrt{n\hat{\lambda}_{i-1}} \\ LCL &= 2\sqrt{n\hat{\lambda}_{i-1}} - 3 \end{aligned} \tag{5}$$

Anscombe c chart for λ known

Anscombe (1948) showed that the transformation $U = 2\sqrt{X + \frac{3}{8}}$ is approximately a normal distribution with $\mu = 2\sqrt{n\lambda + \frac{3}{8}}$ and $\sigma^2 = 1$.

Define $u_i = 2\sqrt{x_i + \frac{3}{8}}$. We can plot $u_2, u_3, \ldots$ on a chart with control limits defined as follows

$$\begin{aligned} UCL &= 2\sqrt{n\lambda + \frac{3}{8}} + 3 \\ CL &= 2\sqrt{n\lambda + \frac{3}{8}} \\ LCL &= 2\sqrt{n\lambda + \frac{3}{8}} - 3 \end{aligned} \tag{6}$$

Anscombe c chart for λ unknown

Define $\hat{\lambda}_i = x_{i1} + \ldots + x_{in}$, and for $i = 2, 3, \ldots$ define $u_i = 2\sqrt{x_i + \frac{3}{8}}$, where $x_{i1}, \ldots, x_{in}$ denote the n units in the ith inspection unit. We can plot $u_2, u_3, \ldots$ on a chart with control limits defined as follows

$$\begin{aligned} UCL &= 2\sqrt{n\hat{\lambda}_{i-1} + \frac{3}{8}} + 3 \\ CL &= 2\sqrt{n\hat{\lambda}_{i-1} + \frac{3}{8}} \\ LCL &= 2\sqrt{n\hat{\lambda}_{i-1} + \frac{3}{8}} - 3 \end{aligned} \tag{7}$$

The Ryan and Schwertman c chart for λ known

Ryan and Schwertman (1997) use regression forms to propose the c chart control limits. Define $y_i = x_i$. Plot y_i for i=2,3,. . . on a chart with control limits defined as follows

$$\begin{aligned} UCL &= 0.6195 + 1.0052n\lambda + 2.983\sqrt{n\lambda} \\ LCL &= 2.9529 + 1.01956n\lambda - 3.2729\sqrt{n\lambda} \end{aligned} \tag{8}$$

The Ryan and Schwertman c chart for λ unknown

Define $y_i = x_i$ and $\hat{\lambda}_i = x_{i1} + \ldots + x_{in}$. Plot y_i for $i = 2, 3, \ldots$ on a chart with control limits defined as follows

$$\begin{aligned} UCL &= 0.6195 + 1.0052n\hat{\lambda}_{i-1} + 2.983\sqrt{n\hat{\lambda}_{i-1}} \\ LCL &= 2.9529 + 1.01956n\hat{\lambda}_{i-1} - 3.2729\sqrt{n\hat{\lambda}_{i-1}} \end{aligned} \tag{9}$$

The modified c Chart by Cornish-Fisher expansion for λ known

Winterbottom (1993), apply a Cornish and Fisher asymptotic expansion to construct a c chart. Let Y^* be an asymptotically normal random variable which is unbiased for a parameter θ, has variance $\mu_2 = \sigma^2$ and μ_3 is the third moment about θ. Let z_α be the

$100\alpha th$ percent point of a standard normal distribution and Y^*_α be the $100\alpha th$ percent point of Y^*. Then

$$Y^*_\alpha \approx \theta + z_\alpha \sigma + \mu_3(z^2_\alpha - 1)/(6\sigma^2). \tag{10}$$

Now suppose X is a poisson random variable with mean $n\lambda$. Then $Y^* = X/n$ is the number of nonconformities per inspection and has mean λ and variance λ/n. From (10),

$$Y^*_\alpha \approx \lambda + z_\alpha \sqrt{\lambda/n} + (z^2_\alpha - 1)/(6n)$$

Define $y^*_i = x_i/n$. Plot y^*_i for $i = 2, 3, ...$ on a chart with modified control limits defined as follows

$$\begin{aligned} UCL &= \lambda + 3\sqrt{\frac{\lambda}{n}} + \frac{4}{3n} \\ CL &= \lambda \\ LCL &= \lambda - 3\sqrt{\frac{\lambda}{n}} + \frac{4}{3n} \end{aligned} \tag{11}$$

The modified c Chart by Cornish-Fisher expansion for λ unknown

Define $y^*_i = x_i/n$ and

$$\hat{\lambda}_i = \frac{x_1 + ... + x_i}{n}.$$

Plot y^*_i for $i = 2, 3, ...$ on a chart with control limits defined as follows

$$\begin{aligned} UCL &= \hat{\lambda}_{i-1} + 3\sqrt{\frac{\hat{\lambda}_{i-1}}{n}} + \frac{4}{3n} \\ CL &= \hat{\lambda}_{i-1} \\ LCL &= \hat{\lambda}_{i-1} - 3\sqrt{\frac{\hat{\lambda}_{i-1}}{n}} + \frac{4}{3n} \end{aligned} \tag{12}$$

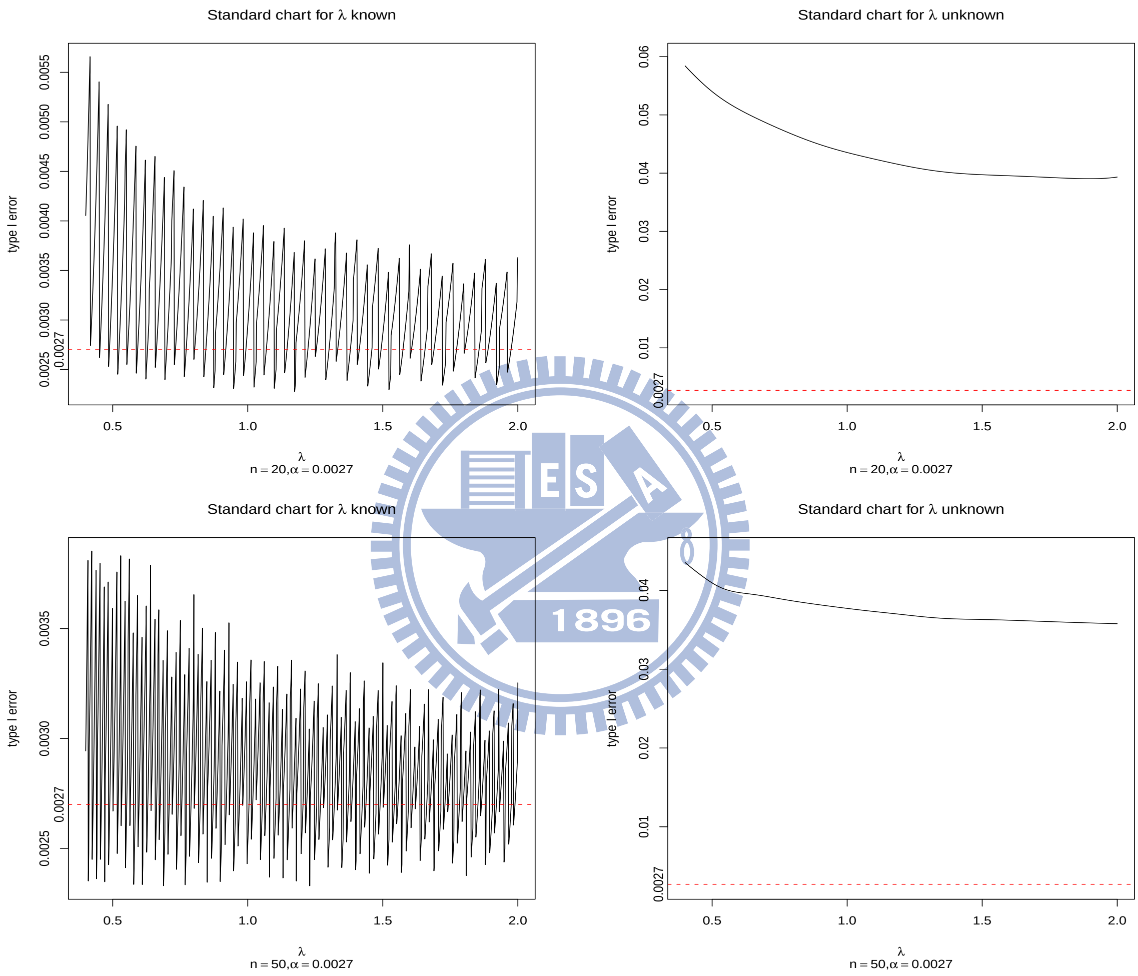


Figure 1: Type I errors of the standard chart (2) with the nominal level 0.0027 when λ is known and unknown for n=20 and n=50.

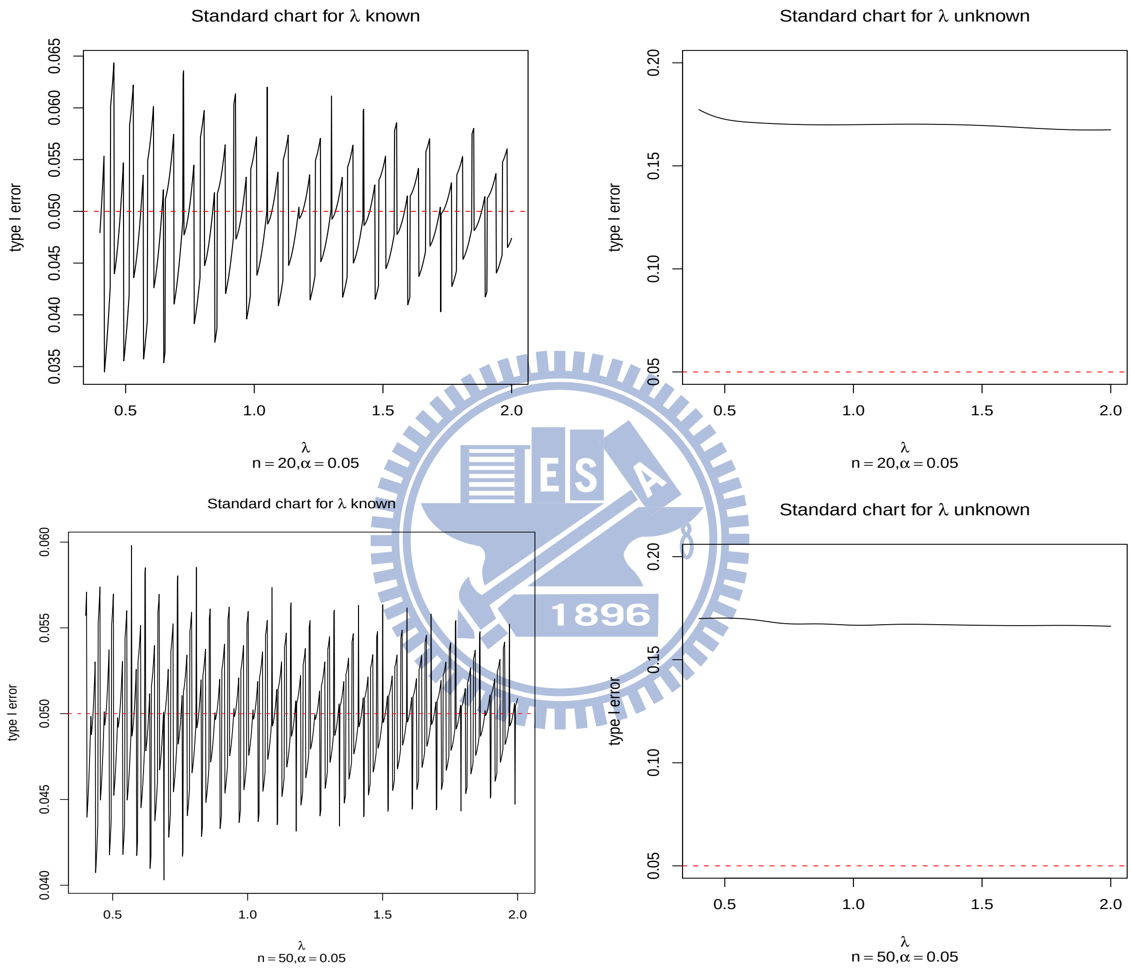


Figure 2: Type I errors of the standard chart (2) with the nominal level 0.05 when λ is known and unknown for n=20 and n=50.

We evaluate the existing methods in terms of their performances of the type I error. In order to simplify the calculation of type I error, we approximate it by the type I error calculated by assuming that $\hat{\lambda}_{i-1}$ and $\hat{\lambda}_i$ follow the same distribution $Poi(\lambda)$. Hence, for the standard chart and the modified chart by Cornish-Fisher expansion, the type I error of control chart with limits LCL and UCL at $\lambda = \lambda_0$ is computed by calculating the probability

$$1 - (Pr_{\lambda_0}(X \leq nUCL) - Pr_{\lambda_0}(X < nLCL)). \tag{13}$$

Because of $nUCL$ and $nLCL$ are not always integers, we use $1 - (P_{\lambda_0}(X < [nUCL] + 1) - P_{\lambda_0}(X \leq [nLCL]))$ to approximate (13), where $[x]$ denotes the largest number equal to or less than x. The type I errors of Bartlett chart and Anscombe chart, and the Ryan and Schwertman chart can be calculated by a similar way. The type I errors of the charts based on the three existing methods with respect to λ known and λ unknown for $n = 20$ and $n = 50$ cases are shown in Figures 1-10. For the standard control chart with λ known case, Figure 1 shows that the type I error oscillates in the nominal level 0.0027 and it is getting closer to the nominal level when λ is larger. Although when the true λ is small, the type I error is not very close to the nominal level, the bias is less than 0.02, which is not very large. Nevertheless, for the λ unknown case, the type I error has a smooth curve and has the same trend as λ known case, but it is much larger than λ known case. In real application, the situation of small λ is important because true λ may be very small. Figure 1 shows that the standard chart may be acceptable if λ is known, but the chart (2) is not satisfactory when λ is unknown.

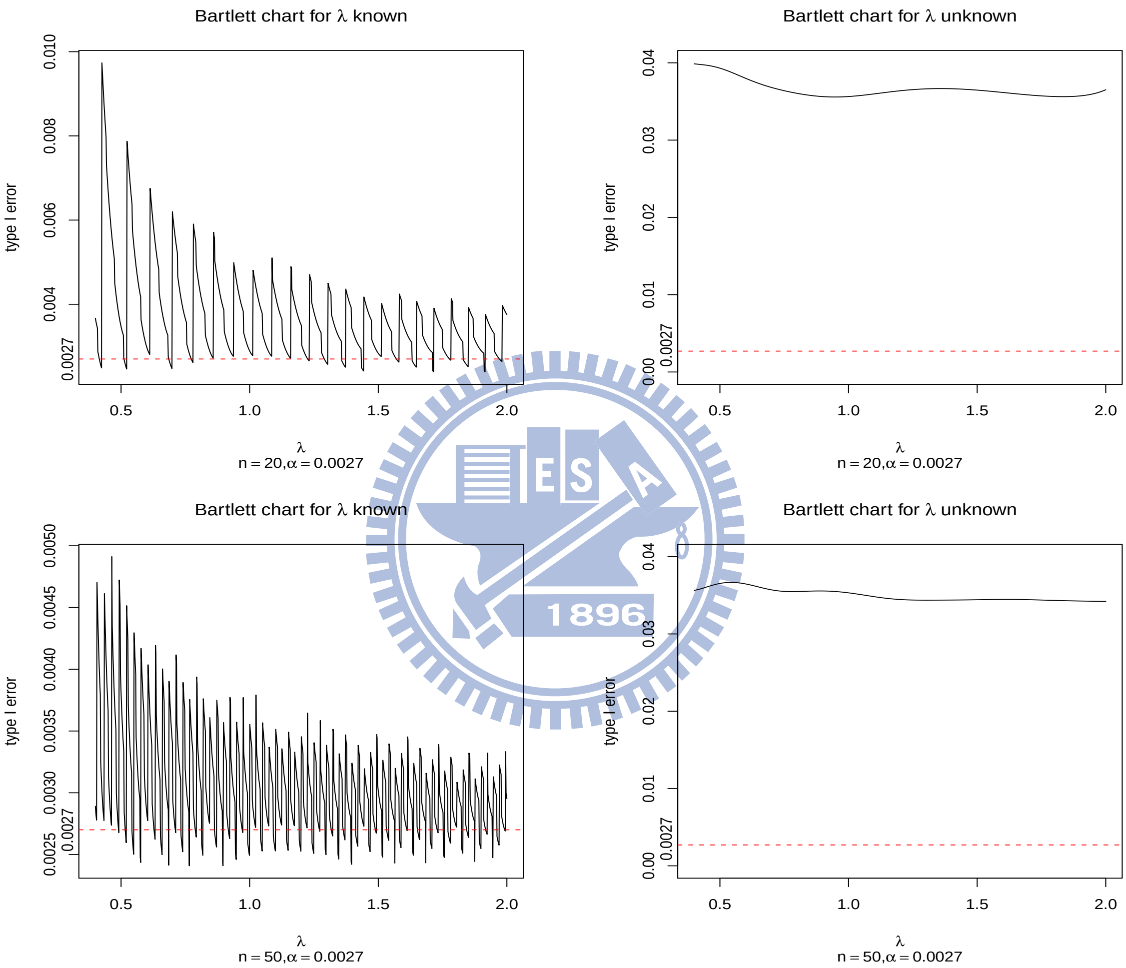


Figure 3: Type I errors of Bartlett chart (5) with the nominal level 0.0027 when λ is known and unknown for n=20 and n=50.

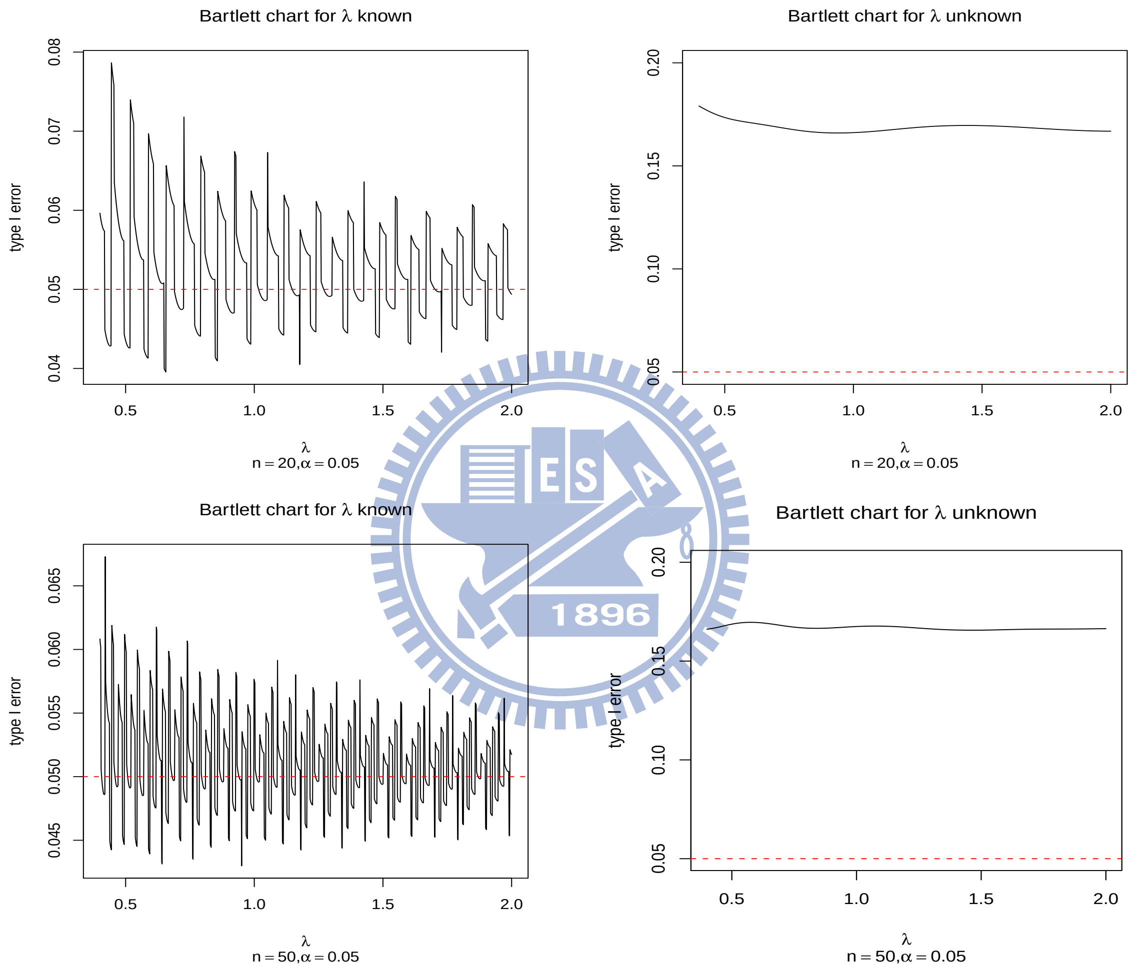


Figure 4: Type I errors of Bartlett chart (5) with the nominal level 0.05 when λ is known and unknown for n=20 and n=50.

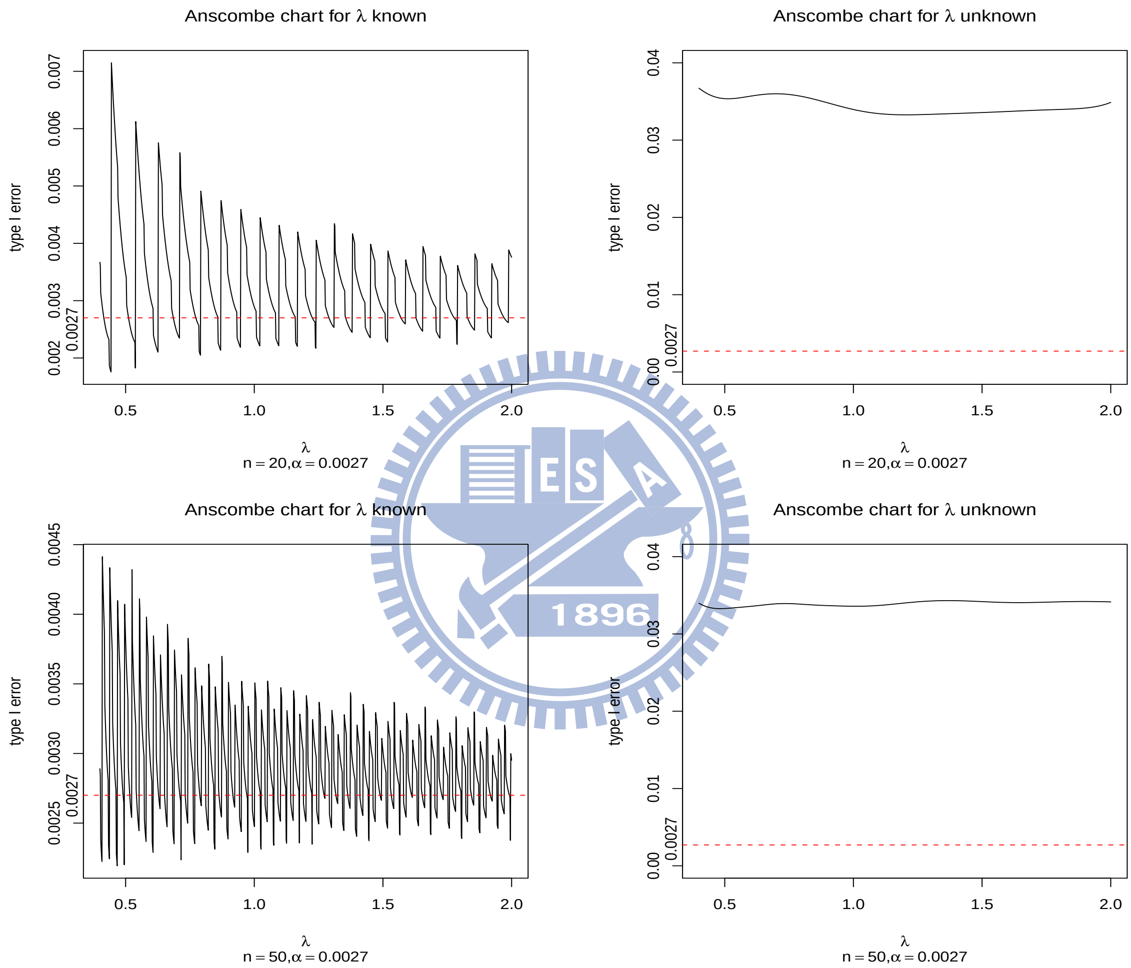


Figure 5: Type I errors of Anscombe chart (7) with the nominal level 0.0027 when λ is known and unknown for n=20 and n=50.

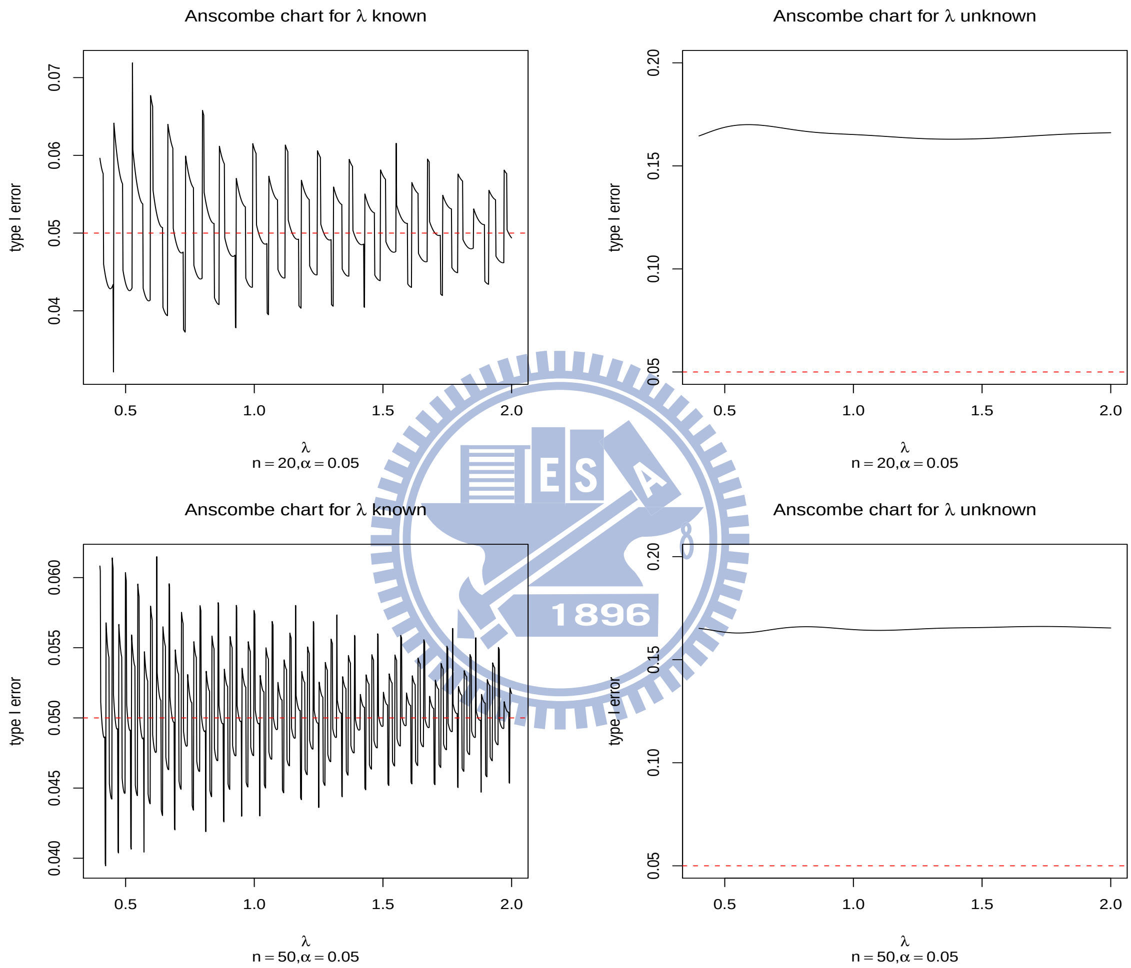


Figure 6: Type I errors of Anscombe chart (7) with the nominal level 0.05 when λ is known and unknown for n=20 and n=50.

For the Bartlett chart and Anscombe chart, Figures 3 and 5 show that the type I error for the λ known, is not far away from the nominal level 0.0027, which is similar to standard chart. However, when λ is unknown, the type I error is improved a little, but it is too large when λ is small.

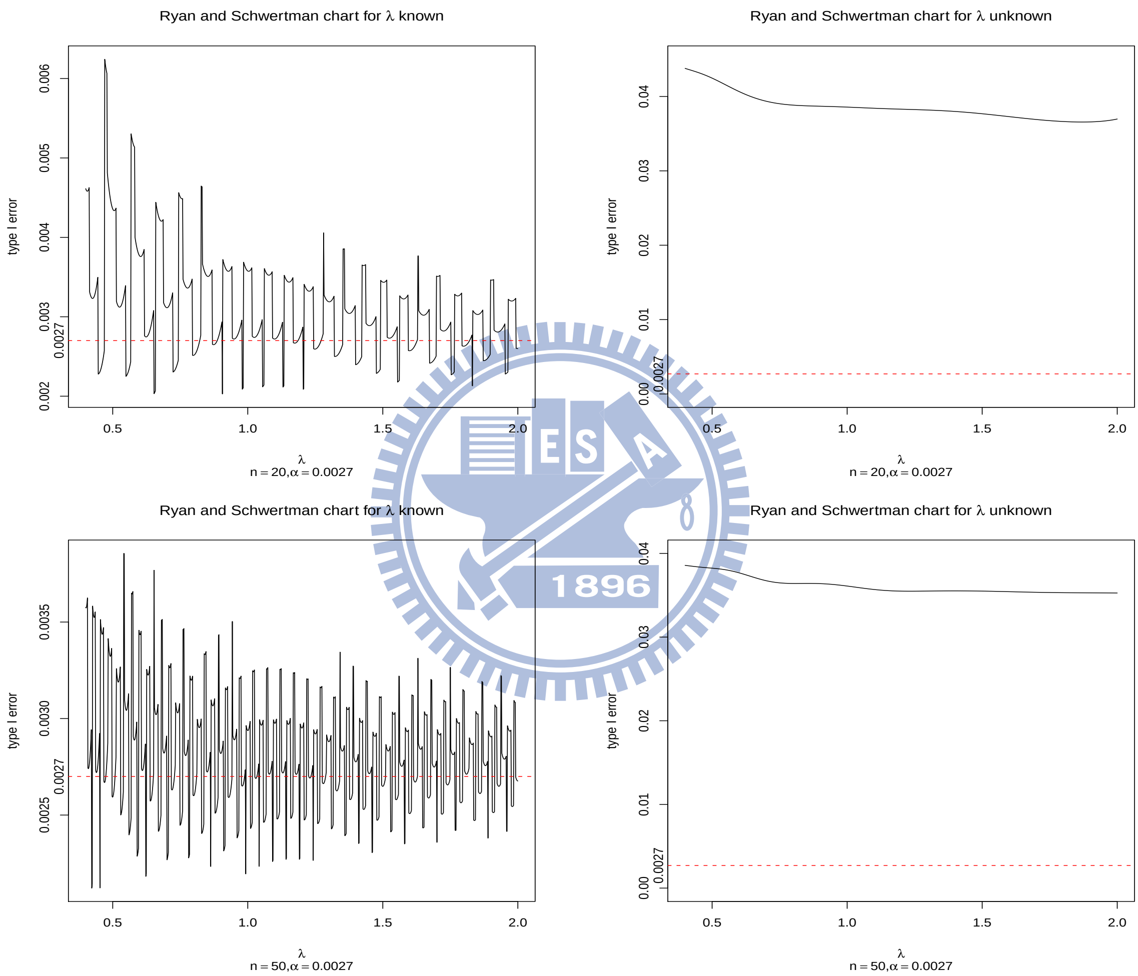


Figure 7: Type I errors of the Ryan and Schwertman chart (9) with the nominal level 0.0027 when λ is known and unknown for n=20 and n=50.

For the Ryan and Schwertman control chart, Figure 7 shows that when λ is known case, the bias of the type I error is less than 0.02 and it is improved a lot. Besides, the

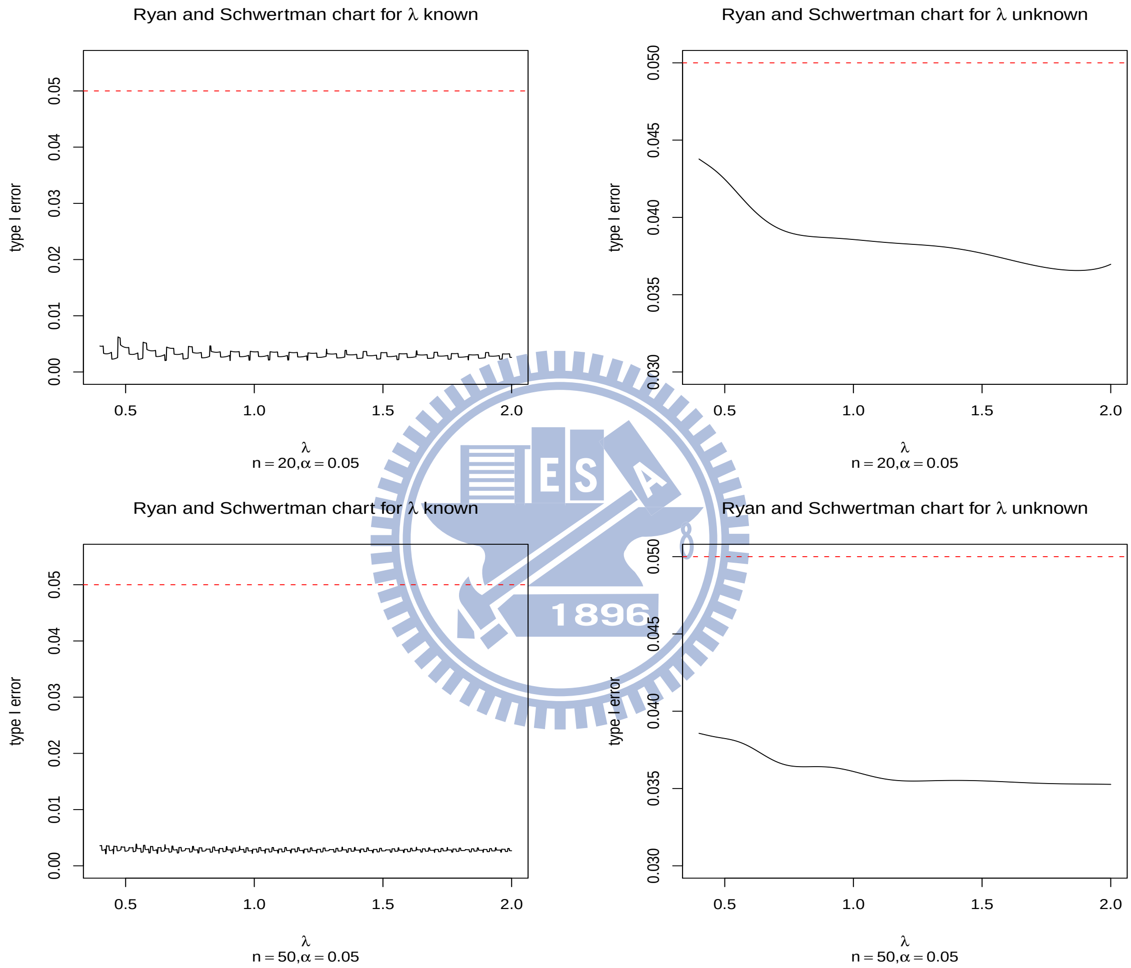


Figure 8: Type I errors of the Ryan and Schwertman c charts (9) with the nominal level 0.05 when λ is known and unknown for n=20 and n=50.

type I error for the λ unknown case is much larger than the nominal level 0.0027 when the true λ is small.

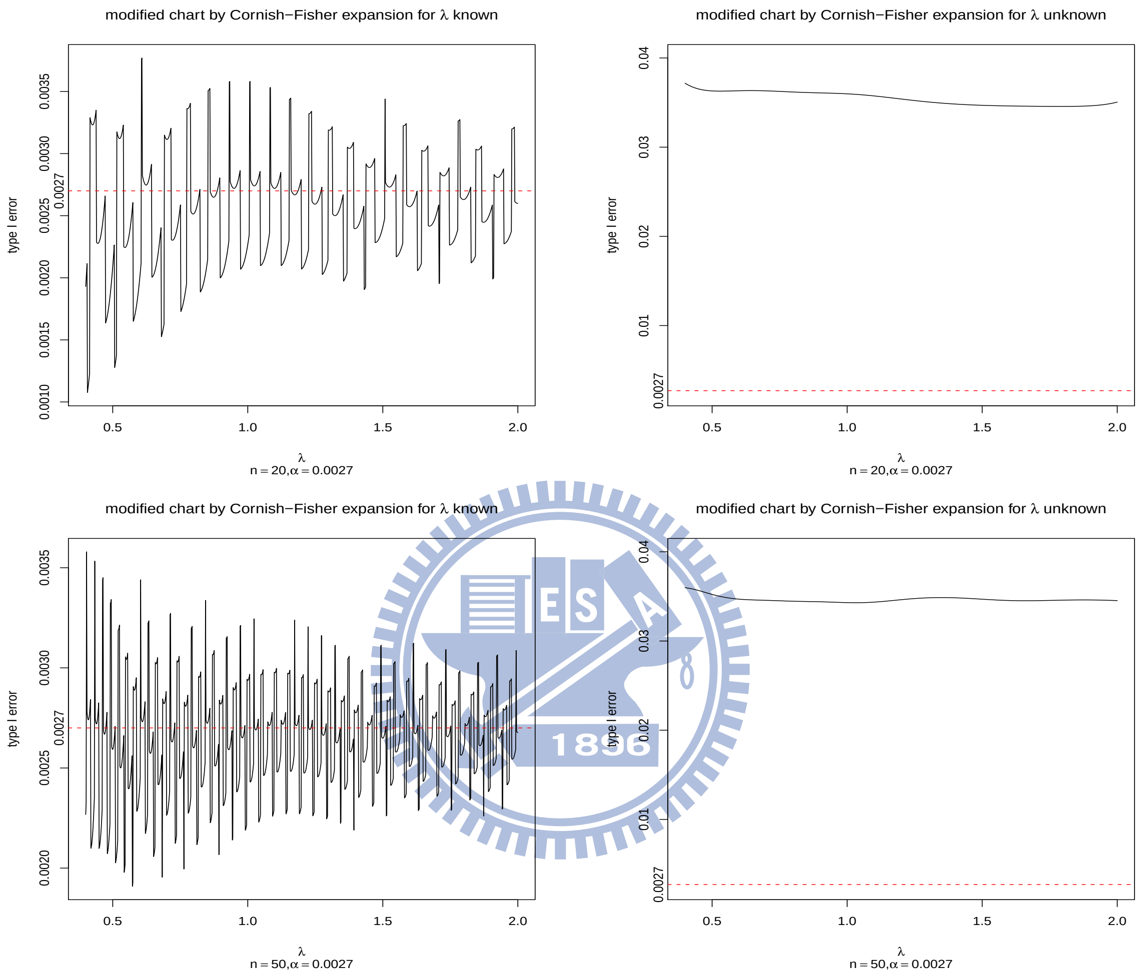


Figure 9: Type I errors of the c charts (12) with the nominal level 0.0027 modified by Cornish-Fisher expansion when λ is known and unknown.

For the modified chart by Cornish-Fisher expansion, the type I error for the λ known case is close to the nominal level 0.0027. However, for the λ unknown case, the type I error is not close to the nominal level, resulting in a unsatisfactory performance.

Combined the above results, when λ is known, the standard chart, the Bartlett chart,

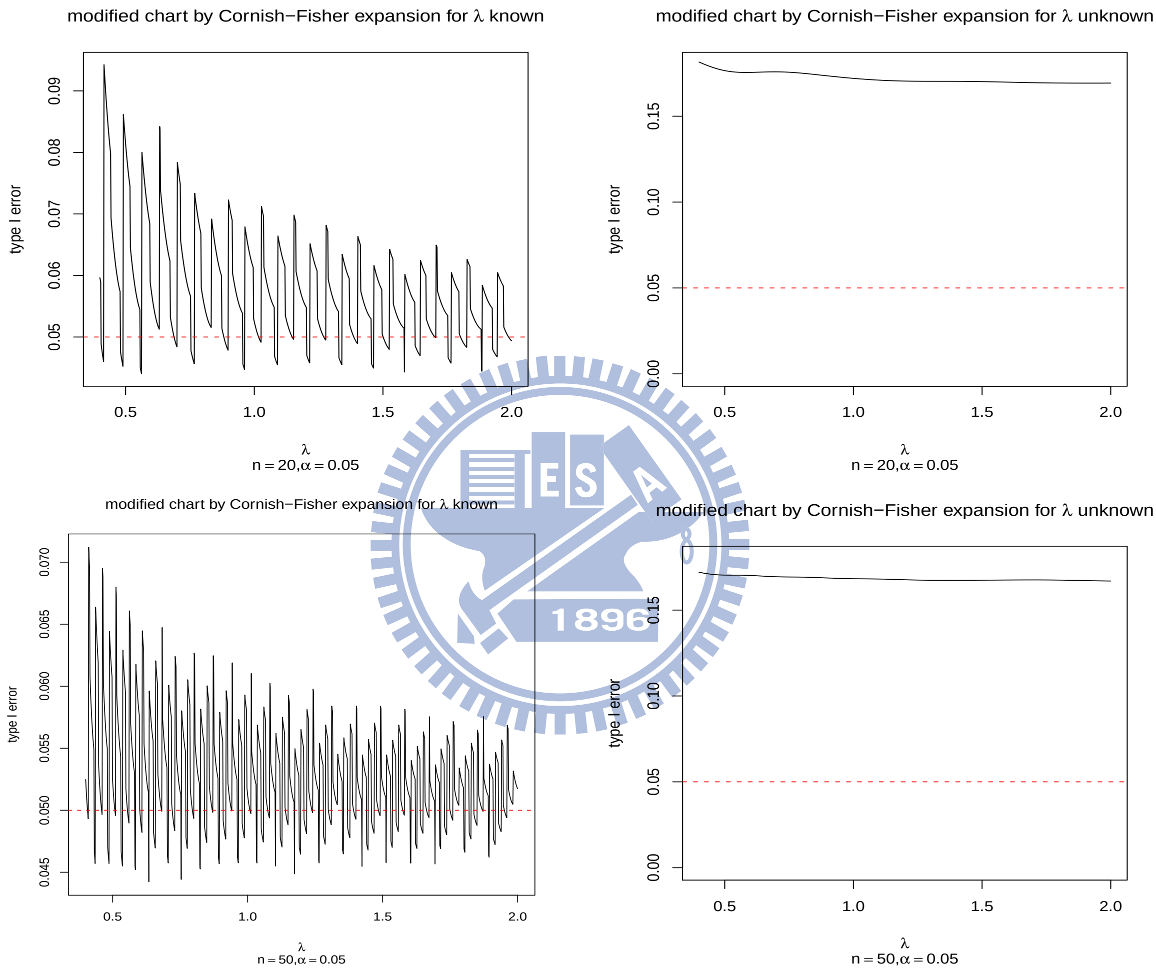


Figure 10: Type I errors of the c charts (12) with the nominal level 0.05 modified by Cornish-Fisher expansion when λ is known and unknown.

the Anscombe chart, and the modified chart by Cornish-Fisher expansion can monitor $\hat{\lambda}_i$ well. The Ryan and Schwertman chart performs well, but this control limits do not depend on confidence level α. When λ is unknown, the four charts are not satisfactory. Therefore, a better chart which can reduce the type I error is proposed in the next section.

3 Improved c chart

Besides providing control limits with a nominal level 0.0027. We also present the figures for control chart with a nominal level 0.05.

We first introduce a poisson tolerance interval in the literature, and then construct an approximated control chart based on this interval. Let $X = \sum_{i=1}^{n} X_i$ be a random variable following a poisson distribution $Poi(n\lambda)$. It is well known that the coverage probability of the two-sided tolerance interval is too conservative with the nominal level for the poisson distribution. The tolerance interval proposed by Cai and Wang (2009) can be used to construct a useful c chart.

The Two-sided Tolerance interval. Let X be the random variable from the poisson distribution with mean $n\lambda$. Let $a = \frac{1}{6}(z_\alpha + z_{1-\beta'})(2z_\alpha + z_{1-\beta'})$, $b = z_\alpha + z_{1-\beta'}$ and $d = \frac{1}{36}(7 - z_{1-\beta'}^2 + z_\alpha z_{1-\beta'} + 2z_\alpha^2)$. Here β' is the $(1+\beta)/2$ that at least contain β proportion of the population. The first order and second order β-content, $(1-\alpha)$-confidence two-sided tolerance intervals have proposed by Cai and Wang (2009)

$$TI_1(X) = (X + a - b\sqrt{X}, X + a + b\sqrt{X})$$

$$TI_2(X) = (X + a - b\sqrt{X + d}, X + a + b\sqrt{X + d})$$

Based on these intervals, we propose new control charts. The control chart based on the first order TI are

$$\begin{aligned} UCL_{TI1} &= X + a + b\sqrt{X} \\ LCL_{TI1} &= X + a - b\sqrt{X}. \end{aligned} \tag{14}$$

We call it is first order TI control chart.

The control chart based on the second order TI are

$$\begin{aligned} UCL_{TI2} &= X + a + b\sqrt{X + d} \\ LCL_{TI2} &= X + a - b\sqrt{X + d}. \end{aligned} \tag{15}$$

We call it is second order TI control chart.

This new chart can successfully reduce the type I error when λ is unknown, see Figure 11.

There are other improved confidence intervals except the Tolerance interval proposed in the literature to improved the coverage probability of the Wald interval, like the Jeffreys interval and the likelihood ratio interval (see Brown, Cai and DasGupta, 2002). However, chart limits based on the Jeffreys interval and the likelihood ratio test are more difficult to present and compute in an informal environment. The chart based on the Edgeworth expansions tolerance interval does not have the above disadvantage and possess a simple closed form.

4 Width comparison

In this section, we compare the expected width of the new charts with the existing charts. The width does not depend on the observation when λ is known so that we can be directly derived by taking the difference of the upper limit and the lower limit. For the

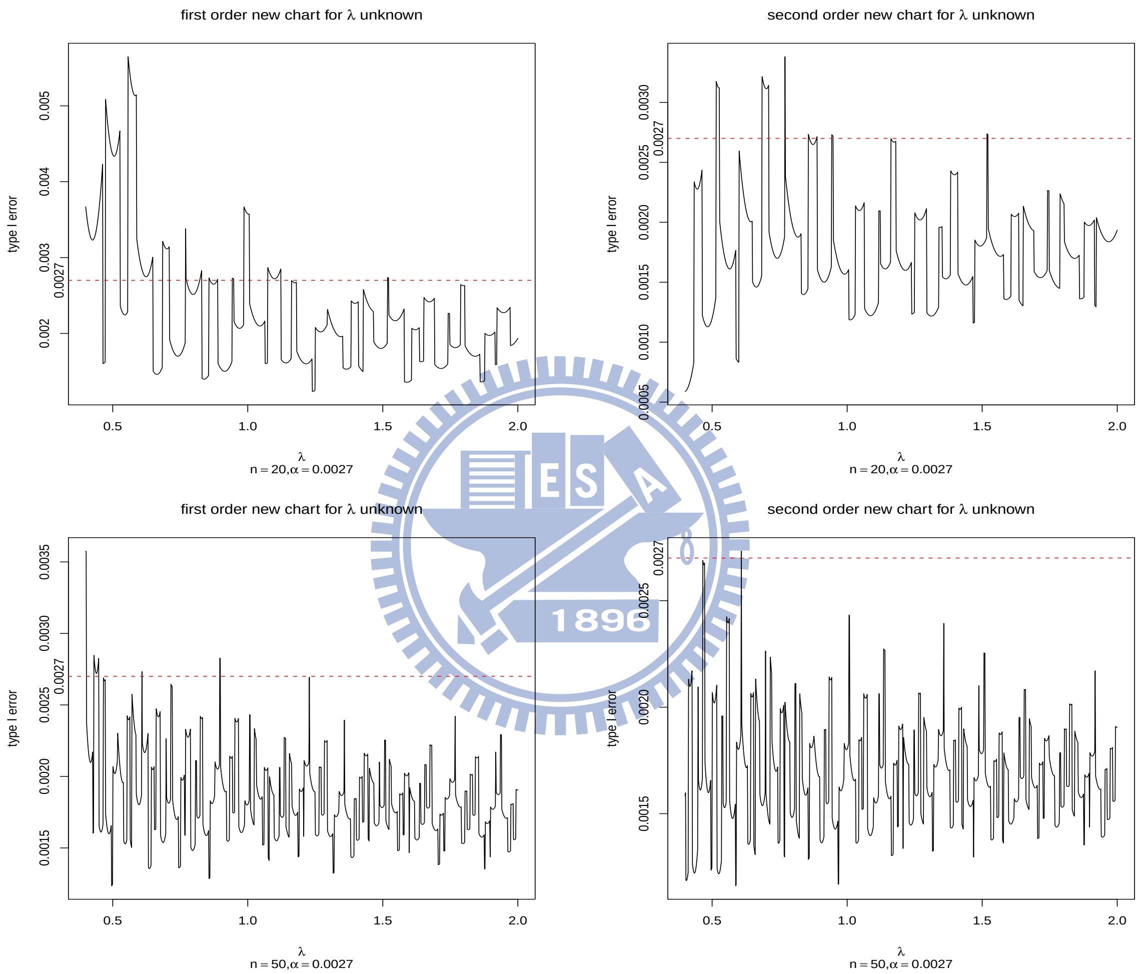


Figure 11: Type I errors of the new c chart with the nominal level 0.0027 when λ is unknown for first order and second order.

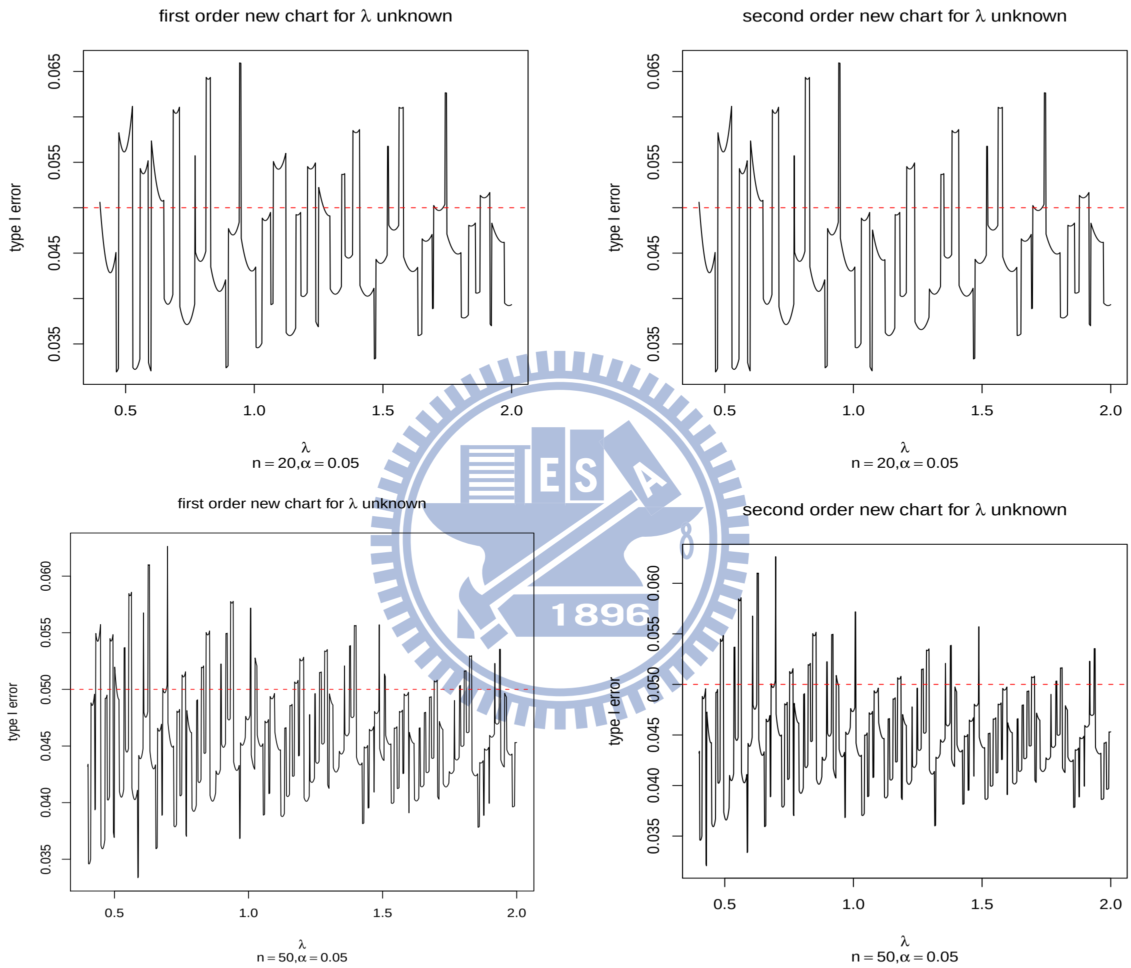


Figure 12: Type I errors of the new c chart with the nominal level 0.05 when λ is unknown for first order and second order.

case of λ unknown, the width depends on the observation. Hence we make a comparison on their expected widths for the λ unknown case. The expected width of a chart is defined as the expect value of the upper limit minusing the expect value of the lower limit, which can be calculated using the formula

$$\sum_{i=0}^{n}(UCL(i) - ICL(i))\frac{e^{-\lambda}\lambda^i}{i!}.$$

The expected widths for the conventional chart, Bartlett chart and Anscombe chart, Ryan and Schwertman chart and the new charts with the nominal level 0.0027 are shown in Figure 13.

Note that the expected width of the modified chart by Cornish-Fisher expansion is the same as that of the standard chart because its upper and lower limits are the upper and lower limits of the standard interval adding the same value $4/(3n)$. Thus, we do not present it in Figure 13.

For the Bartlett chart and Anscombe chart, since the chart is used to detect V and U, which approximates the normal distribution with unit variance. Unlike the other three charts whose limits are used to detect $\hat{\lambda}$, the transformed chart detects another random variable V and U. Thus, we do not directly compare the expected width of the transformed chart with those of the other three charts.

The Ryan and Schwertman control chart and the new charts are detect the number of nonconformities in an inspection unit n so that we can not directly compare the expected width with the other three charts. Thus we ,divided by n, can compared with the other three charts. Figure 13 shows that the expected width of the new charts is relatively lager

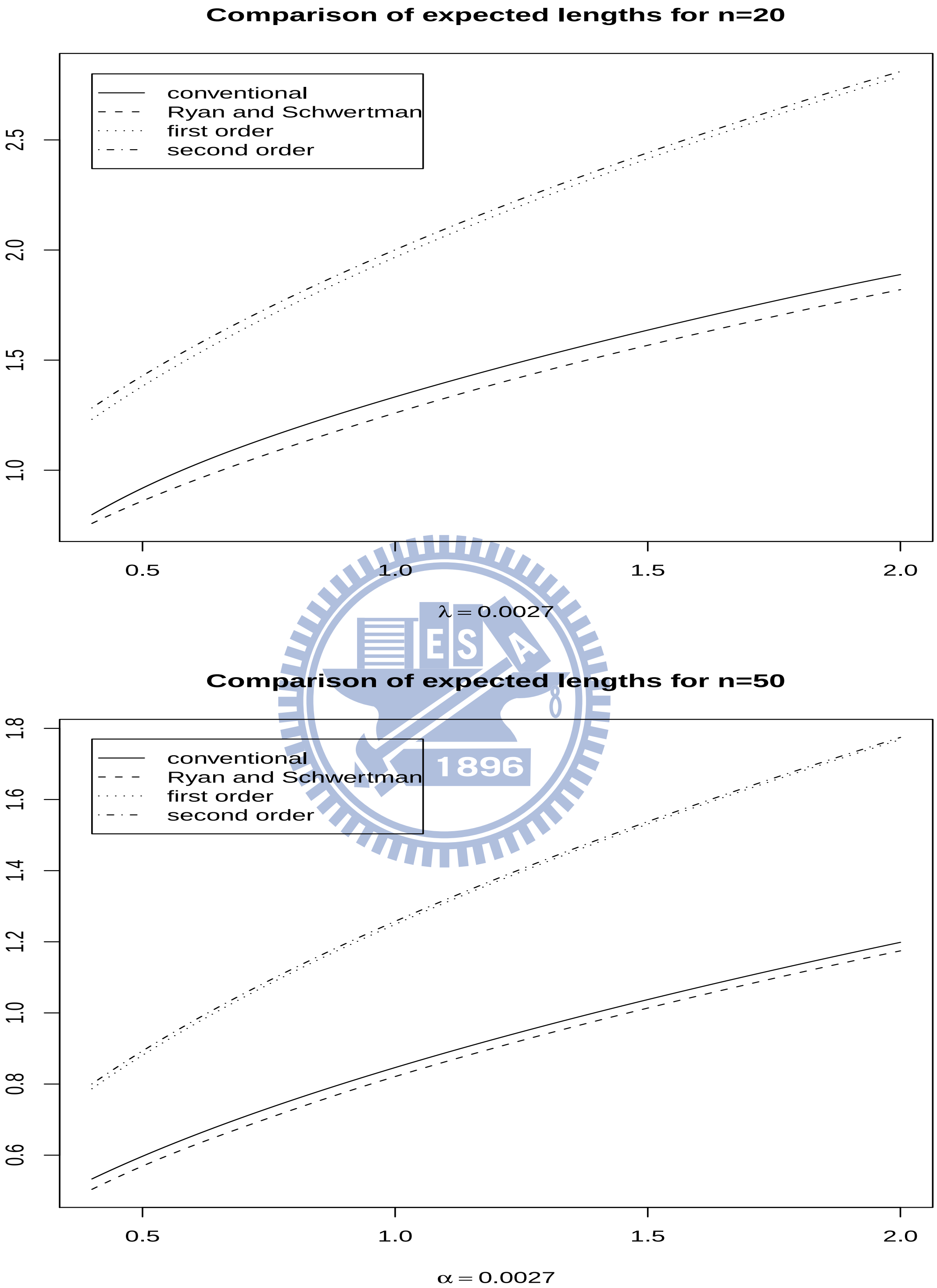
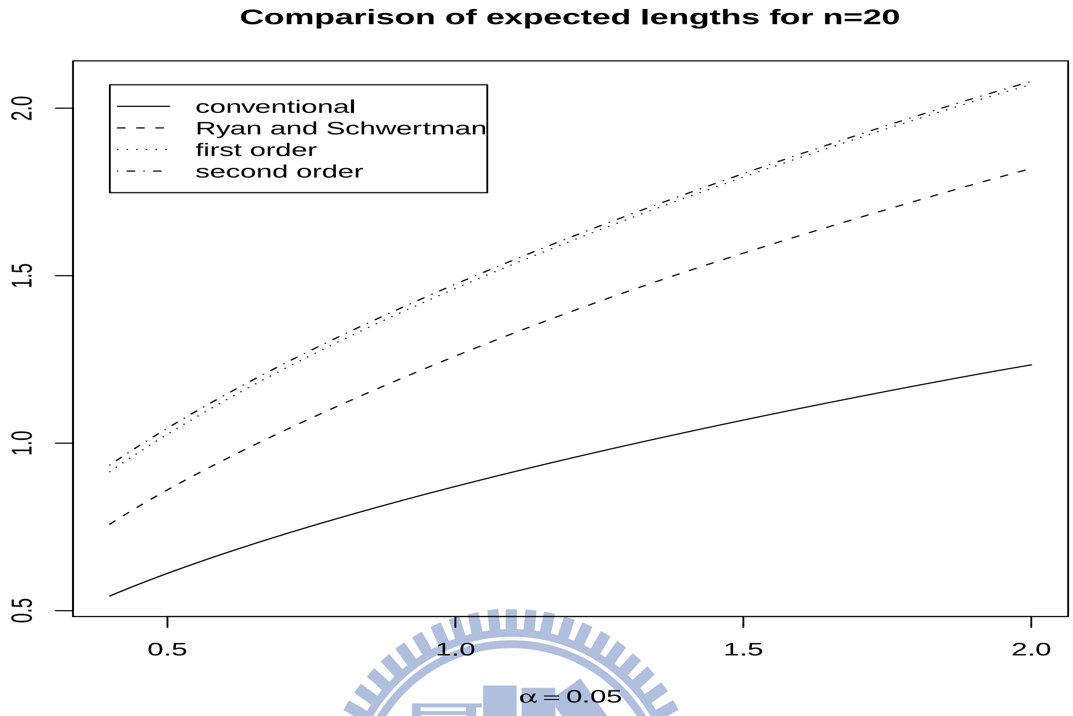


Figure 13: Expected widths of various control charts respectively for the cases of $n = 20$ and $n = 50$ with the nominal level 0.0027. From bottom to top, the expected width of Ryan and Schwertman chart, conventional chart, the first order new chart, the second order new chart.



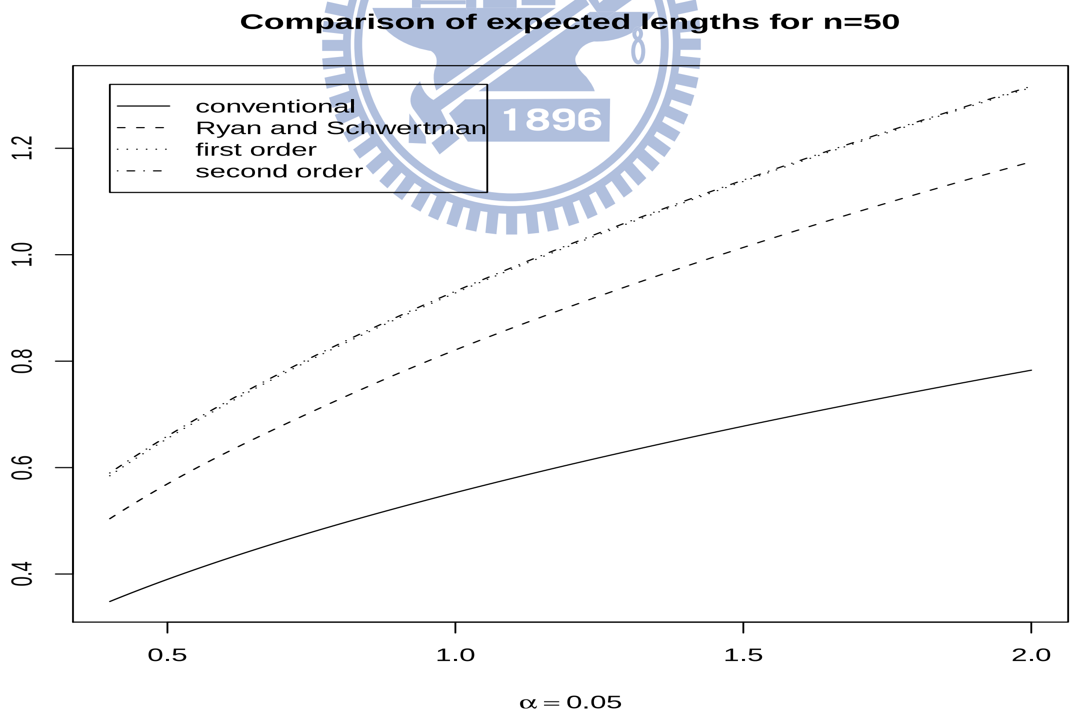


Figure 14: Expected widths of various control charts respectively for the cases of $n = 20$ and $n = 50$ with the nominal level 0.05. From bottom to top, the expected width of Ryan and Schwertman chart, conventional chart, the first order new chart, the second order new chart.

Table 1: The data for the numbers of nonconformities.

Sample number	Number of nonconformities	Sample number	Number of nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

than above existing charts. However, the new charts can be recommended that has better performance for λ unknown case.

5 Example

We illustrate the new charts by a real data example with a small number of defects.

Example 1. The data in this example are about the number of nonconformities printed circuit boards. Table 1 presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. The inspection unit in this example is defined as 100 boards. We found that the the samples 6 and 20 fall outside the control limits because the new inspector had examined the boards in this sample and he did not recognize several of type nonconformities that could have been present and sample 20 resulted from a temperature control problem in the wave soldering machine.

From Figures 15 and 18, there are several points out of the control region for conven-

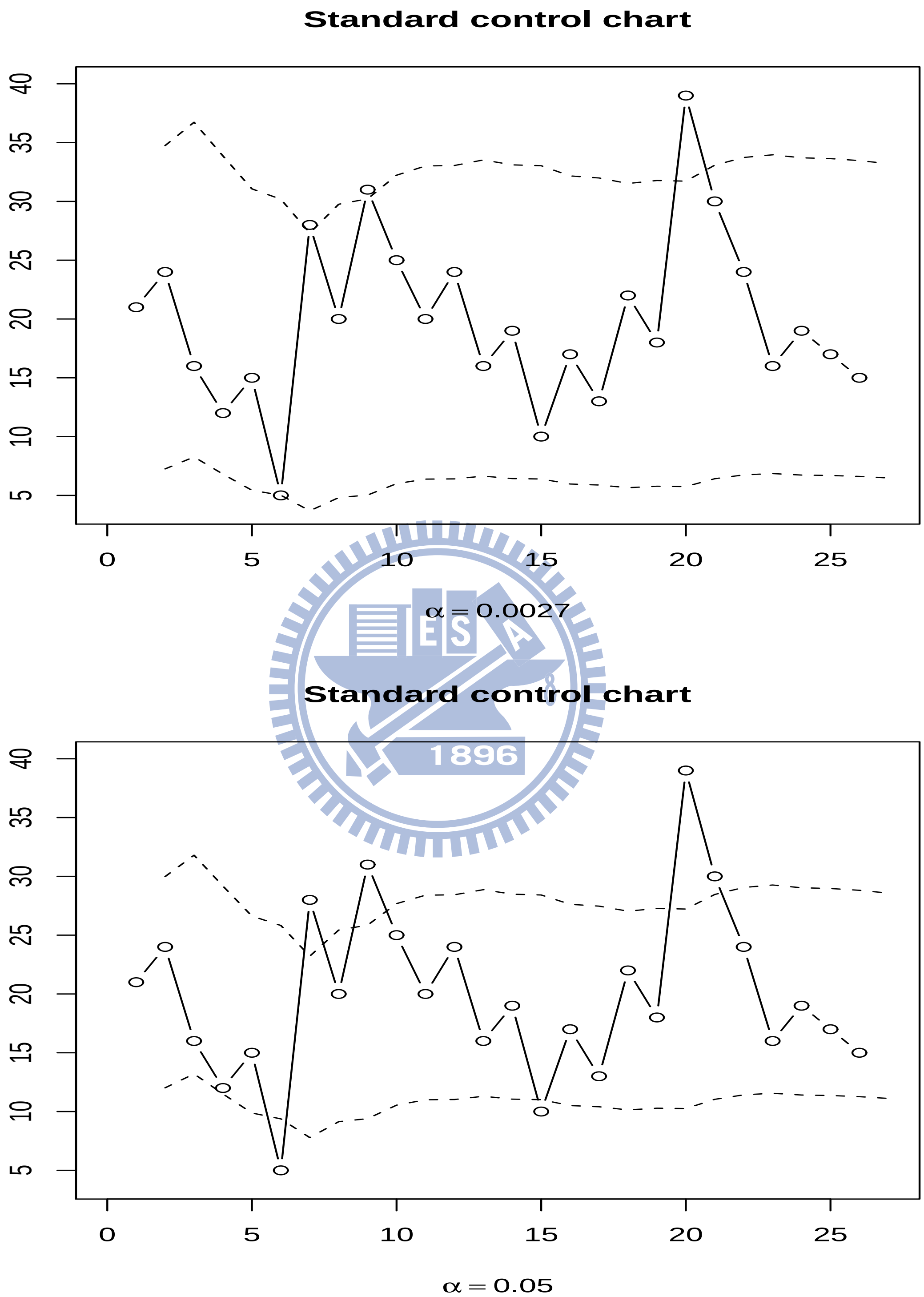
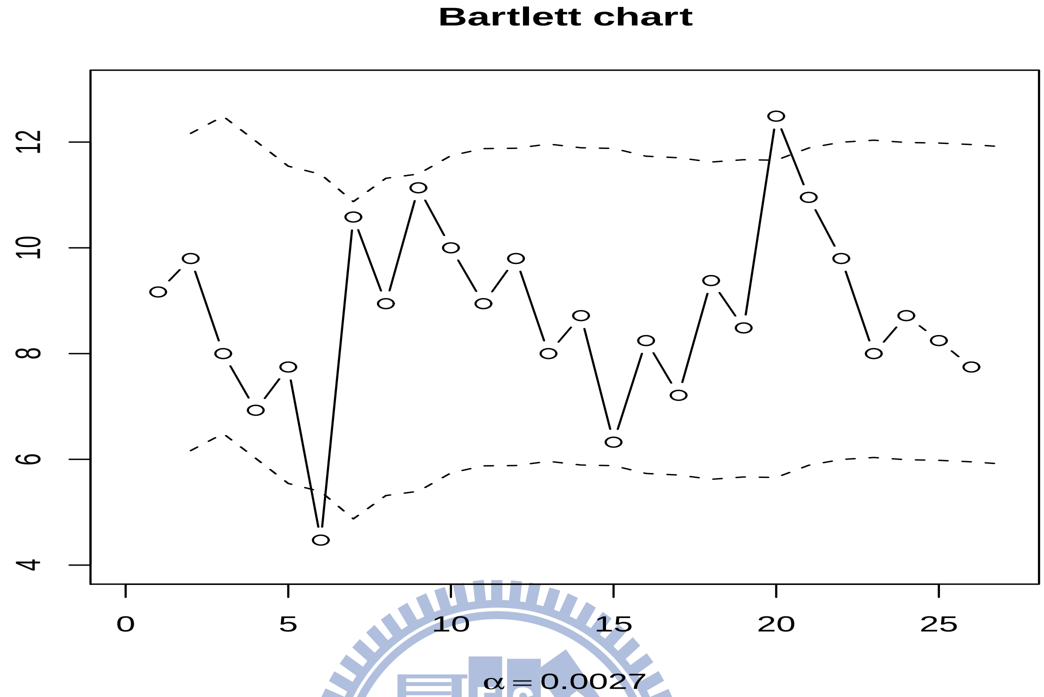


Figure 15: The dashed lines are the UCL and LCL for the conventional chart.



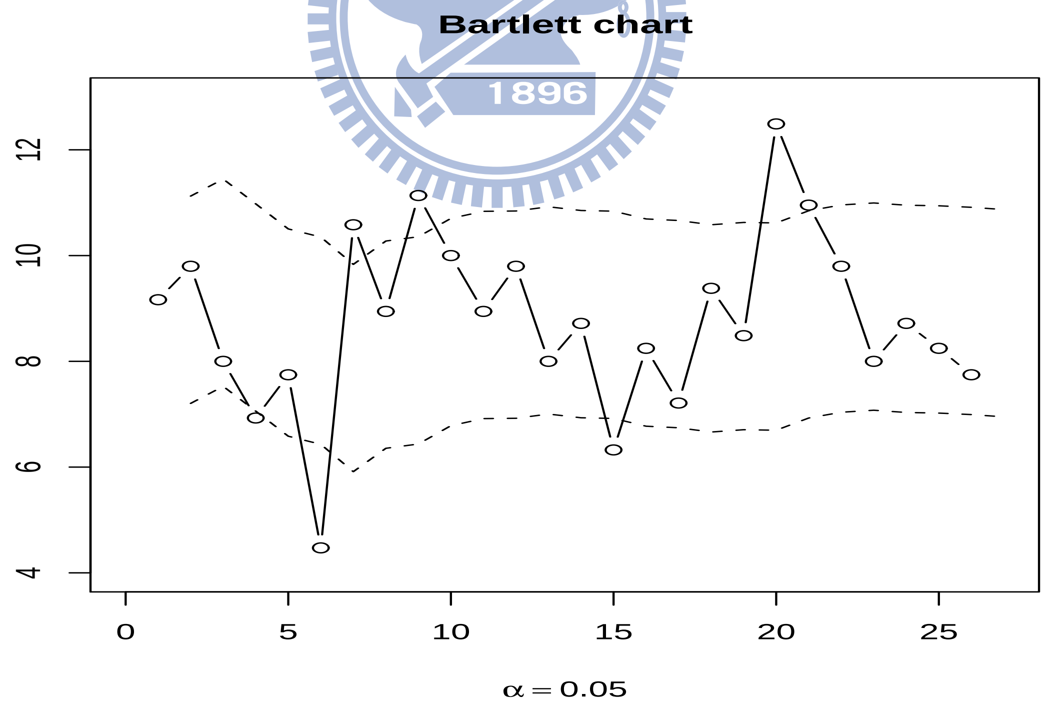


Figure 16: The dashed lines are the UCL and LCL for the Bartlett chart.

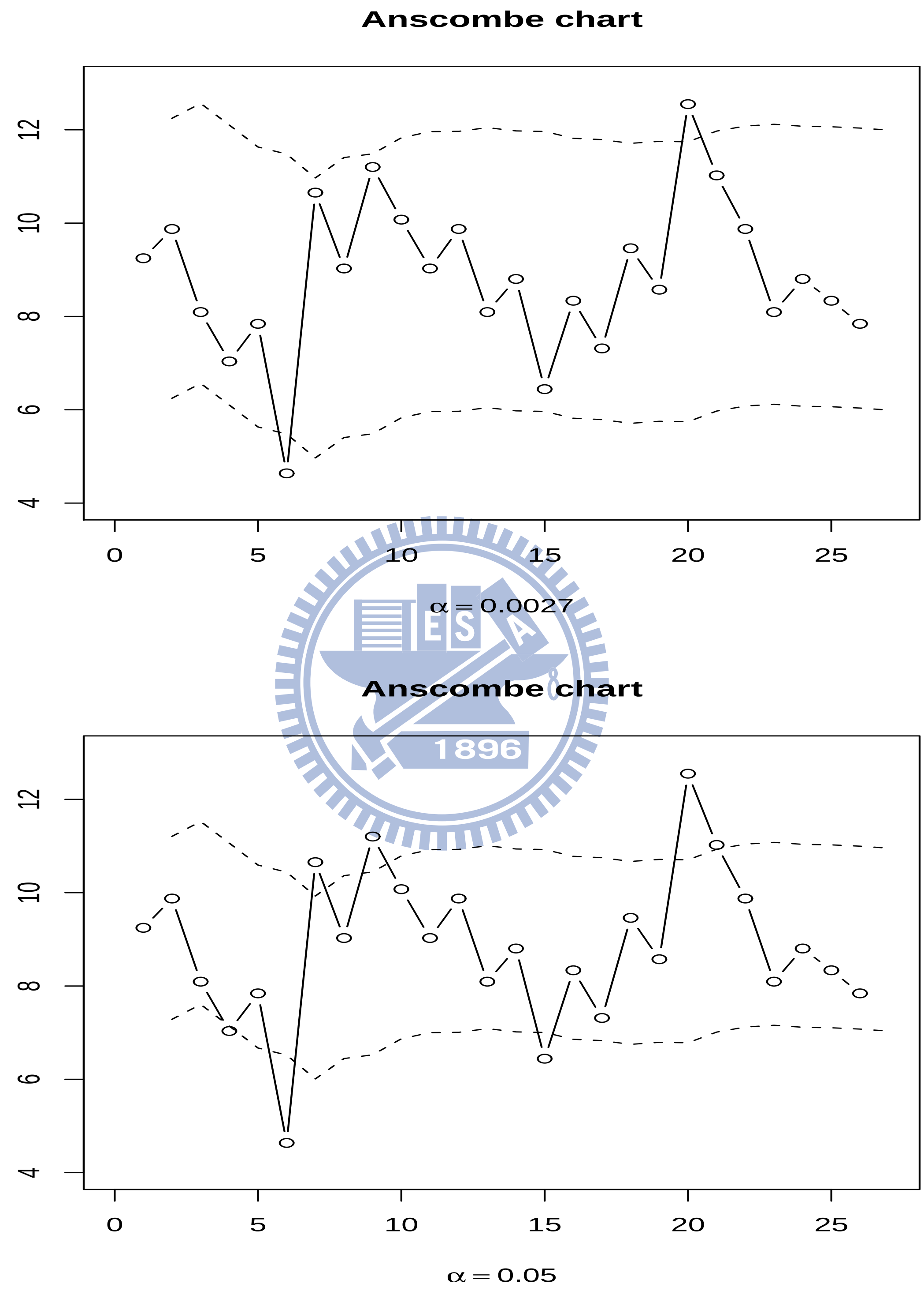


Figure 17: The dashed lines are the UCL and LCL for the Anscombe chart.

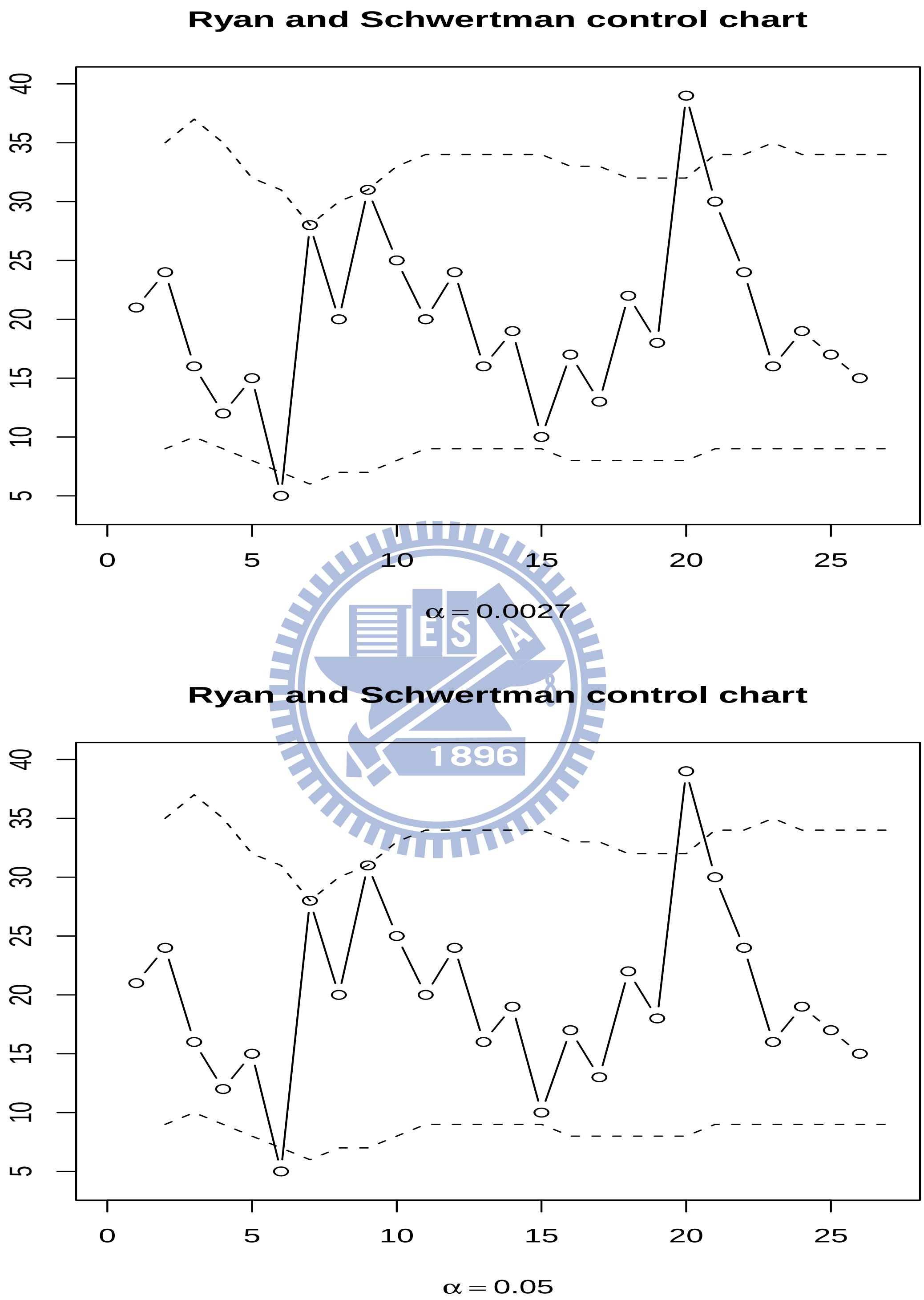
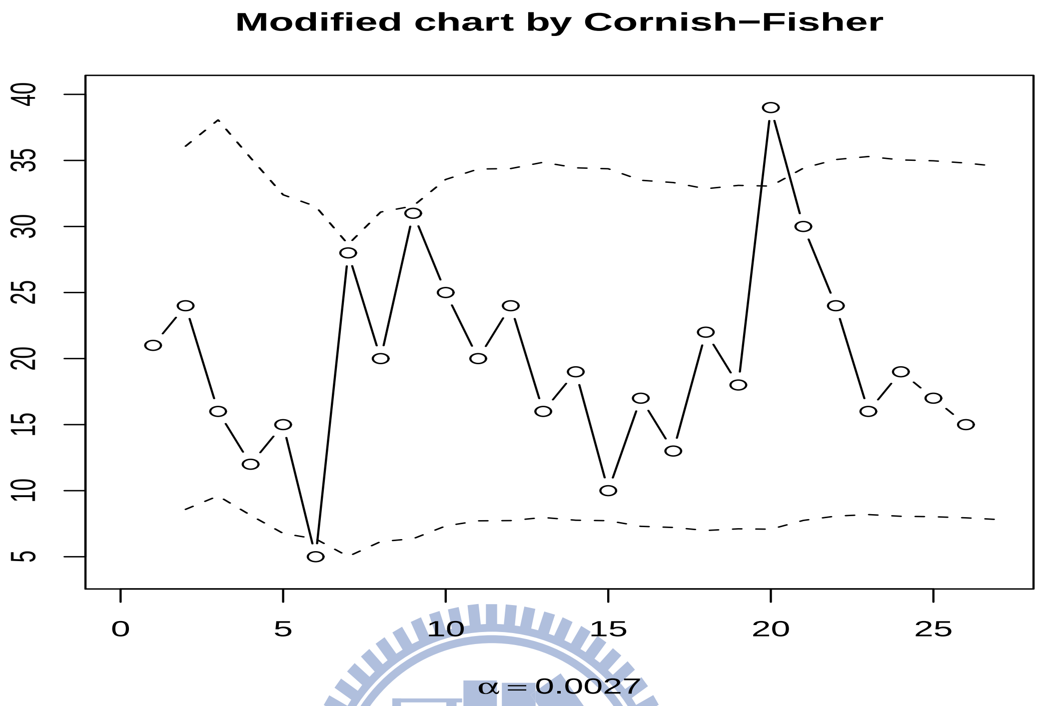


Figure 18: The dashed lines are the UCL and LCL for the Ryan and Shcwertman chart.



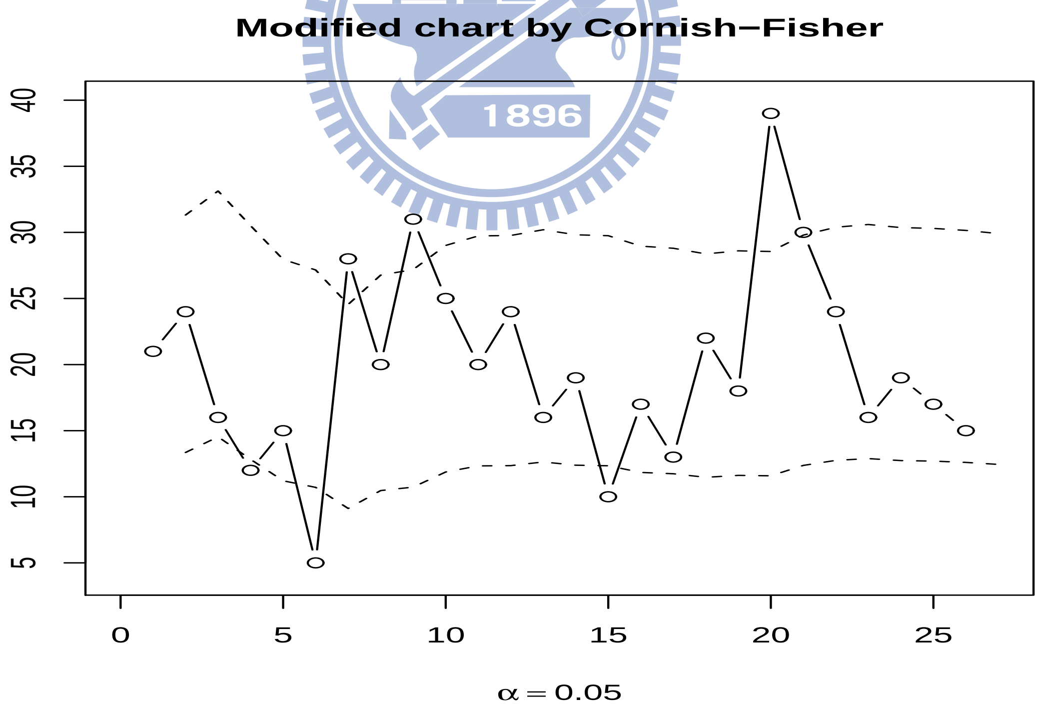


Figure 19: The dashed lines are the UCL and LCL for the chart modified by Cornish-Fisher expansion.

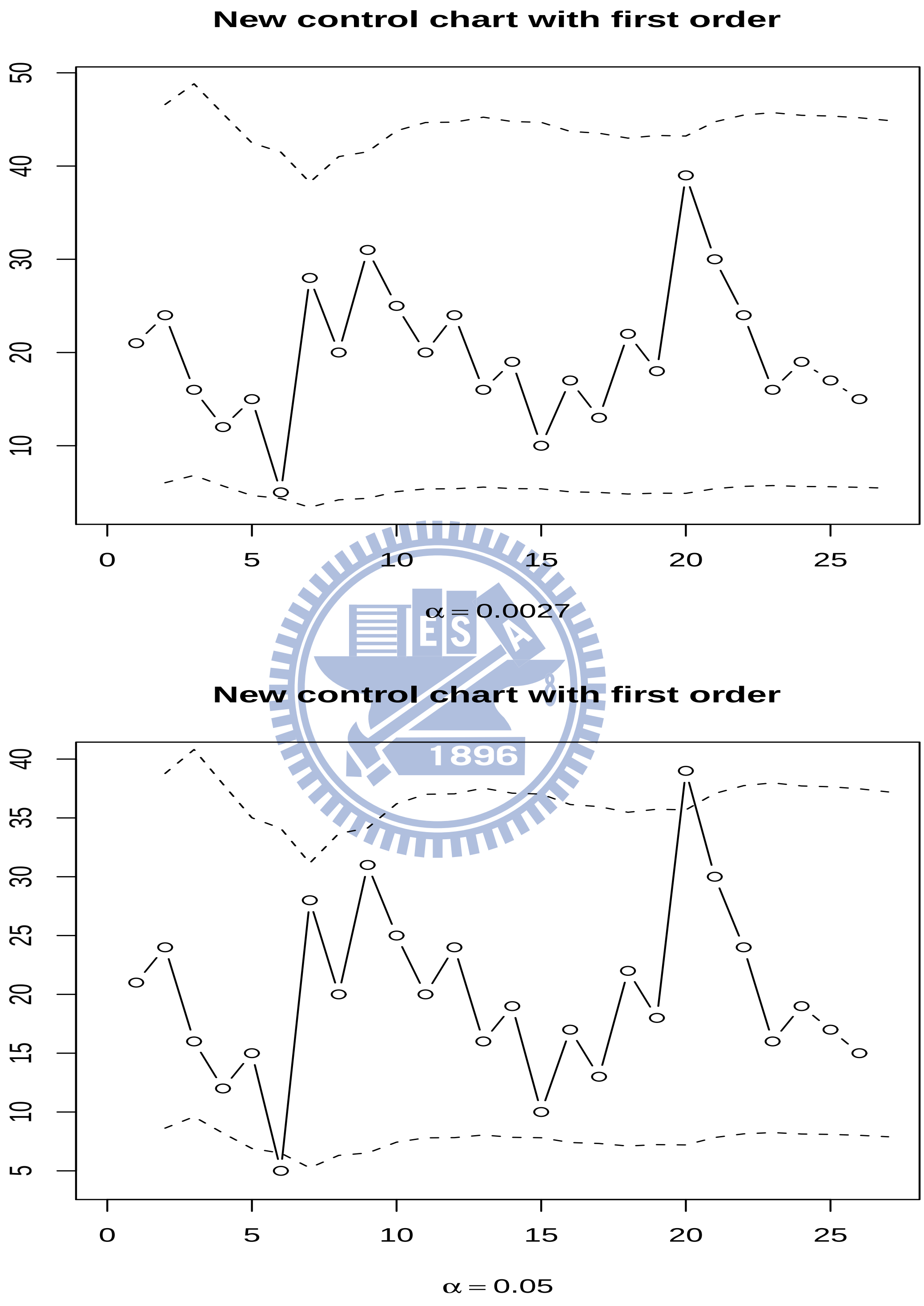


Figure 20: The dashed lines are the UCL and LCL for the new chart with first order.

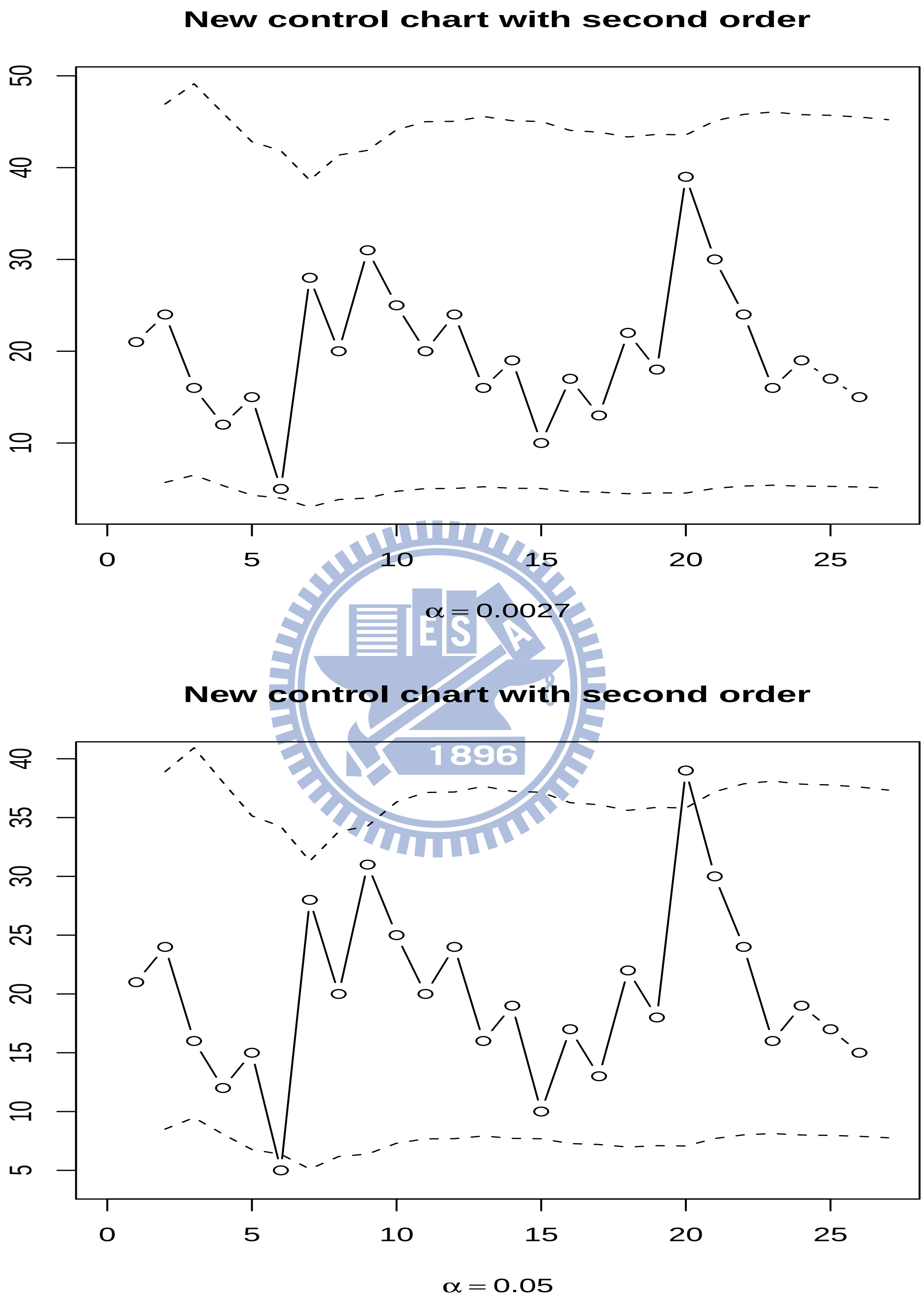


Figure 21: The dashed lines are the UCL and LCL for the new chart with second order.

tional chart and Ryan and Schwertman chart under the nominal level 0.0027. Figures 16, 17, and 19 show that the same two points are plot outside the control limits for the Bartlett chart, the Anscombe charts, and the modified chart by Cornish-Fisher expansion. Figures 20 and 21 show that all of the points are fall between the limits for the first and second order new charts. Therefore, under the nominal level, the new charts are too conservative. When under the level 0.05, the proposed chart can detect the out control points. The new charts can exactly detect the assignable cause under the level 0.05. The other existing charts detect more out control points than the proposed chart with level 0.05. Compared with the existing charts the new charts performs better in this real data example.

6 Conclusion

In this paper, we evaluate the four existing charts, the standard chart, the Bartlett chart, the Anscombe chart, the Ryan and Schwertman chart and the modified chart by Cornish-Fisher expansion, in terms of the type I error and expected width criteria. The existing methods can perform well when the number of nonconformities is known, but they are unsatisfactory when the number of nonconformities is unknown. Therefore, we propose a new chart in this paper for the number of nonconformities unknown case, which is based on the form of the tolerance interval.

Compared with the existing charts, the new charts are too conservative with the nominal level 0.0027 but the new charts can successfully reduce the type I error with the level 0.05. It is certainly true that the expected width is larger than the four existing charts. But it has a simple closed form which can be easily adopted in real applications and it has decisive improvement over the four existing charts for the number of nonconformities unknown case.

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