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碩士論文

利用設限比例方法建構設限資料之容忍區間

Tolerance Intervals for Twice-censored Data Based on the Censored Rate Approach

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摘要

在工業、臨床試驗及藥理學的應用上，容忍區間是個能分析資料特徵之有用工具之一。在實際應用上，尤其是可靠度分析及臨床試驗，在蒐集資料時常難免會碰到資料遺失或不完整之情形。在現行的研究中，有不少關於容忍區間之建構方法在特定分配上。但較少有一般性的方法來建構設限資料之容忍區間。故在此篇研究，我們會探討如何建構設限資料之容忍區間。並利用設限比例方法去估計資料未知之參數。且將常態分配及一般性分配分別提供演算法步驟。最後在將設限方法應用在實際案例上。

關鍵詞:容忍區間、設限資料、覆蓋率、設限比例方法

Tolerance Intervals for Twice-censored Data Based on the Censored Rate Approach

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Abstract

Tolerance intervals are useful tools to capture characteristics of the underlying distribution of collected data in industrial, clinical trials and pharmaceutical applications. In real applications, especially in reliability testing and clinical trial, it is common that the collected data with censored outcomes. Although there are existing methods for constructing tolerance interval for specific distributions or models, there lacks a unified approach for constructing tolerance intervals with censored data for any distribution. In this study, we consider the problem of constructing tolerance intervals for parametric distributions with censored data. A censored rate approach is proposed to estimate the parameters. Algorithms based on the estimation to construct tolerance intervals for the normal and other distributions are provided in this study. A simulation study and a real data example study show the superiority of the proposed methods.

Key words: Tolerance Interval, censored data,coverage probability, censored rate approach

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Contents

List of Tables

List of Figures

1 Introduction

Tolerance intervals are useful tools to capture characteristics of the underlying distribution of collected data in industrial, clinical trials, pharmaceutical and life insurance applications (Hahn and Chandra 1981; Hahn and Meeker 1991;Zaslavsky 2007; Wang 2007; Gebizlioglu and Yagci 2008; Cummings, Zhou and Dive 2011). A tolerance interval is a statistical interval within which, with some probability, a specified proportion of a population falls.

There are two kinds of tolerance intervals proposed in the literature, the $\beta-$content and $\beta-$expectation tolerance intervals. More specifically, let X be a random variable with cumulative distribution function F. An interval $(L(X), U(X))$ is said to be a β-content, $(1-\alpha)$-confidence tolerance interval for F (called a $(\beta, 1-\alpha)$ tolerance interval for short) if

$$P\{[F(U(X)) - F(L(X))] \geq \beta\} = 1 - \alpha. \tag{1}$$

On the other hand, an interval $(L(X), U(X))$ is said to be a β-expection tolerance interval if

$$E\{[F(U(X)) - F(L(X))]\} = \beta. \tag{2}$$

The approaches of constructing the tolerance interval for the normal distribution and exponential family are widely discussed in Wang and Tsung (2009), Cai and Wang (2009) etc. In real applications such as the reliability and clinical trial problems, the tolerance interval approach is widely used. It is common that the collected data are censored in these applications. There has been discussions on

TI for the censored data (Krishnamoorthy,Mallic and Mathew 2011, Emura and Wang 2010, Hahn and Meeker 1991). It is worth noting that the results are mainly for some specific models such as the weibull, exponential and lognormal distributions. Compared with these models, the approaches for TI with censored data for the normal distribution and exponential distribution has not been studied as depth as them. In this study, we propose a general method to construct TIs with censored data for parametric distribution, which mainly based on a censored rate estimation approach.

In this study, we consider parametric distribution and twice-censored data. Let $X_i, i = 1, ..., n$ be a random sample following a parametric distribution with a distribution function $F_\theta(x)$ and a probability or density function $p_\theta(x)$, where θ is vector of unknown parameters. Let L and U be the left censoring and right censoring times. We consider independent identically distributed random vectors $X_i = (Z_i, \delta_i)\ \ i = 1, ..., n$, where Z_i are the variables of interest and δ_i is an indicator variable with $\delta_i = 1$ if the data X_i is not censored and $X_i = Z_i$, $\delta_i = 2$ if the data X_i is right censored and $Z_i = U$, $\delta_i = 3$ if the data X_i is left censored and $Z_i = L$. Then the likelihood function can be expressed as

$$L(\theta, \underset{\sim}{x}) = \prod_{i=1}^{n} F_\theta(x_i)^{I_{\{\delta_i=3\}}} f_\theta(x_i)^{I_{\{\delta_i=1\}}} S_\theta(x_i)^{I_{\{\delta_i=2\}}} \tag{3}$$

where $S_\theta(x) = 1 - F_\theta(x)$ is the survival function.

Nonparametric approaches are widely-adopted for the twice-censored data (Patilea and Rolin 2006; Shen 2009). But when the data is known to be drawn from a parameter distribution, a more suitable approach is to adopt the parameter distribution to construct TIs. However, for parametric distributions, only the models with a simple survival function form can be dealt with accurately such as the exponential distribution and Weibull distribution etc (Miller 1981, Emura and Wang 2010). For the parameter without a simple survival function form, it is difficult to obtain an accurate estimators for the parameters. A widely-used method is to derive the maximum likelihood estimators of the parameters based on the likelihood function (3). However, it is not easy to find the maximum likelihood estimators based on the likelihood function (3), even for the normal distribution. For example, the normal likelihood function can be express as :

$$L(\mu,\sigma^2|\underset{\sim}{x}) = \prod_{i=1}^{n}\left(\int_{-\infty}^{L}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(x_i-\mu)^2}{2\sigma^2}}dx\right)^{I_{\{\delta_i=3\}}}\left(\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(x_i-\mu)^2}{2\sigma^2}}\right)^{I_{\{\delta_i=1\}}}\left(\int_{U}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(x_i-\mu)^2}{2\sigma^2}}dx\right)^{I_{\{\delta_i=2\}}} \tag{4}$$

Since (4) involves integration, it is difficult to solve the μ and σ^2 theoretically or numerically. Even adopting the EM algorithm to find the MLE, we cannot not guarantee to obtain an accurate result and it is time consumption.

The thesis is organized as follows: Several existing Tolerance intervals are reviewed in Section 2. Section 3 gives the proposed censored rate method. A simulation study for the normal distribution and gamma distribution is give in Section 4. A real data example is give in Section 5.

2 Preliminaries

In this study, we focus on constructing TI for the normal distribution and other parametric distribution with censored data. In this section, we give a review of the widely-used TIs for the normal distribution and gamma distribution as well as the distribution free TIs.

Wald and Wolfowitz (1946) proposed a two-sided β-content, $(1-\alpha)$-confidence tolerance interval for a normal distribution with the form

$$(L(X), U(X)) = (\bar{x} - c_{(1-\alpha;n)}s, \bar{x} + c_{(1-\alpha;n)}s) \tag{5}$$

where $\bar{x}$ and s are sample mean and standard deviation of a sample $x_1, ...x_n$ of size n, and $c_{(1-\alpha;n)}$ depends on α and sample size n. The $c_{(1-\alpha;n)}$ values are tabulated in Odeh and Owen (1980) (see Table 1).

For the gamma distribution, let $X_1, ..., X_n$ be a random sample from $gamma(\alpha, \lambda)$. Krishnamoorthy, Mathew and Mukherjee (2008) pointed out the transformed sample $Y_1 = X_1^{1/3}, ..., Y_n = X_n^{1/3}$ can be approximated a normal distribution with an arbitrary mean μ and arbitrary variance σ^2. Then by TI (5), let $(L(Y), U(Y))$ be a two-sided β-content, $(1-\alpha)$-confidence tolerance interval for this normal distribution. Then the $gamma(\alpha, \lambda)$ distribution approximate tolerance interval with the form

$$(L(X), U(X)) = (L^3(Y), U^3(Y)) \tag{6}$$

For the data not following the normal distribution but a parametric distribution, we can consider the TI with respect to the distribution (Patel 1986). An alternative approach is to consider the distribution free TI (Gibbons 1971,1975, Hahn and Meeker 1991). A two-sided β-content, $(1-\alpha)$-confidence distribution free tolerance interval has the form

$$(L(X), U(X)) = (X_{(l)}, X_{(u)}) \tag{7}$$

where $X_{(l)}$ and $X_{(u)}$ denote the lth and uth order statistics of the sampled data $x_1, ..x_n$ and l and u are chosen symmetrically and as close together as possible around the integer less than or equal to $(n+1)/2$ such that

$$\sum_{i=0}^{u-l-1} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1-\alpha.$$

In the situation without censored data, the above mentioned TIs can be directly used. When the data is twice-censored, directly applying the above TI can not lead to a satisfactory result. Therefore, in this study, we propose approaches based on the above mentioned TIs to construct TI with censored data and show that the proposed algorithm can obtain a desirable result.

3 Methods

3.1 Censored rate method for the normal distribution

For a twice-censored data, a feasible way to construct TI of the sample population is to estimate the unknown parameter and then generate a new sample from the distribution based on the estimated parameter value to reconstruct TIs. As mentioned in the introduction section, a widely-used method to estimate the parameter is to adopt the maximum likelihood method. However, it is hard to derive the maximum likelihood approach from the likelihood function when $F(x)$ and $S(x)$ cannot be simplified to forms without involving integrations. In this study, we propose a censored rate approach to estimate the unknown parameter. First, we consider the sampled population is the normal distribution.

Suppose we have a sample $X_i = (Z_i, \delta_i), i = 1, ..., n$ from the normal distribution $N(\mu, \sigma^2)$ with the left censored time L and the right censored time U. Suppose that there are n_1, n_2 and n_3 data corresponding to $\delta_i = 1, 2$ and 3 respectively. We call n_2/n and n_3/n as the right and left censored rates. In the proposed approach, we use the left and right censored rates to approximate the probabilities $P(X > U)$ and $P(X < L)$. Therefore, we intend to obtain estimators for μ and σ by solving

$$P(X < U) = \Phi(\frac{U - \mu}{\sigma}) = 1 - n_2/n$$

$$P(X < L) = \Phi(\frac{L - \mu}{\sigma}) = n_3/n \tag{8}$$

where $\Phi(\cdot)$ denotes the cumulative normal distribution function.

Let q_1 and q_2 denote the $(1-n_2/n)$ and n_3/n quantiles of the standard normal distribution. Then (8) can be rewritten as

$$U = \mu + q_1\sigma$$
$$L = \mu + q_2\sigma \tag{9}$$

By solving (9), we have:

$$\hat{\sigma} = \frac{U-L}{q_1 - q_2} \text{ and } \hat{\mu} = (U - q_1\hat{\sigma}). \tag{10}$$

Based on the estimators (10), we can generate n_3 data greater than U and n_2 data less than L from the normal distribution $N(\hat{\mu}, \hat{\sigma^2})$. Then based on the new $n_2 + n_3$ data and the n_1 uncensored data, we can derive the TIs based on (5) and (7). If the sample size $n = n_1 + n_2 + n_3$ is large, we can directly use the n data. If the sample size $n = n_1 + n_2 + n_3$ is not large, we can generate more data based from the normal distribution $N(\hat{\mu}, \hat{\sigma^2})$ to derive TIs. The approach is summarized as the following procedure.

Procedure 1: constructing TIs for a normal distribution with censored data

For a data following a normal distribution $N(\mu, \sigma^2)$ with a right censored time U and a left censored time L, the steps of deriving TIs are listed below.

Step 1. Calculate the estimators (10) of (μ, σ).

Step 2. If the sample size n of the data is large, generate n_2 data greater than the upper censored time U and n_3 data less than lower censored time L respectively from the normal distribution $N(\hat{\mu}, \hat{\sigma})$ to replace the $n_2 + n_3$ censored data. If the sample size n of the data is small, generate a sample with a larger sample size from the normal distribution $N(\hat{\mu}, \hat{\sigma})$.

Step 3. Base on the n_2+n_3 data obtained from Step 2 and the n_1 uncensored data to derive tolerance interval (5) and distribution-free tolerance interval(7) when the sample size n is large. Or base on the generated data with a larger sample size to derive tolerance intervals.

3.2 Censored rate approach for general distributions

For a distribution with a density function or probability function $f(x|\theta)$, we propose a general procedure based on the censored rate estimation to estimate the unknown parameters, where θ is a vector of unknown parameters. Similarly as for the normal distribution, we use n_2/n, and n_3/n to estimate $P_\theta(X \geq U)$ and $P_\theta(X \leq L)$. Since we expect that the probabilities $P(X \geq U|\hat{\theta})$ and $P(X \leq L|\hat{\theta})$ are close to n_2/n, and n_3/n asymptotically. Thus, for an estimator $\hat{\theta}$, we propose an error function

$$\varepsilon(\hat{\theta}) = (P(X \leq L|\hat{\theta}) - \frac{n_3}{n})^2 + (P(X \geq U|\hat{\theta}) - \frac{n_2}{n})^2 \tag{11}$$

to evaluate the estimator $\hat{\theta}$. An estimator with a smaller error function value is better than an estimator with a larger error function value.

The method we propose is first to use the n_1 uncensored data to derive a maximum likelihood estimator of θ, say $\hat{\theta}^{(1)}$. Then the second step is to generate data based on the $\hat{\theta}^{(1)}$ estimator. Then we calculate the maximum likelihood estimator based on the new generate data, say $\hat{\theta}^{(2)}$. Then we repeat the process to derive $\hat{\theta}^{(i)}, i = 3, ..., m$, where m can be chosen as 100. Then we calculate $\varepsilon(\hat{\theta}^{(i)}), i = 1, ..., m$. Under the error function (11), the $\hat{\theta}^{(i)}$ with the smallest $\varepsilon(\hat{\theta}^{(i)})$ is regarded as the desired estimator. The steps of deriving TIs with censored data for a distribution is give in the following procedure.

Procedure 2: constructing tolerance interval for general distributions with censored data

For data following a distribution with a density function or a probability function $f(x|\theta)$ with a right censored time U and a left censored time L, the steps of deriving TIs are listed below.

Step 1. Use n_1 uncensored data to derive the maximum likelihood estimators of θ, say $\hat{\theta}^{(1)}$ and then calculate $\varepsilon(\hat{\theta}^{(1)})$ for the maximum likelihood estimator.

Step 2. Generate n_2 and n_3 data which are greater than U and less than L respectively based on the density or probability function $f(x, \hat{\theta}^{(i)})$.

Step 3. Calculate the maximum likelihood estimator of θ, say $\hat{\theta}^{(i+1)}$, based on the $n_1 + n_2 + n_3$ data and calculate $\varepsilon(\hat{\theta}^{(i+1)})$.

Step 4. Repeat Steps 2 and 3 k times to obtain $\hat{\theta}^{(i+1)}, i = 1, ..., k$

Step 5. Find the $\hat{\theta}^{(i)}$ with the smallest $\varepsilon(\hat{\theta}^{(i)})$ value among the (k+1) $\varepsilon(\hat{\theta}^{(i)})$ values.

Step 6. Generate data based on the θ value derived in Step 5. And base on these generated data to construct tolerance intervals.

4 Simulation

In this section, we conduct a simulation study to evaluate the TIs derived by the procedures presented in Section 3. We use the $\beta-$expection criterion to evaluate a TI. That is, we evaluate the performance of a tolerance interval $(L(X), U(X))$ by its expected coverage proportion, which is defined to be

$$e_\theta(L(X), U(X)) = E_\theta(F(U(X)) - F(L(X))). \tag{12}$$

It is worth noting that we use the $\beta-$expectation criterion to evaluate TIs instead of using the $\beta-$content criterion, which is to evaluate the performance of TIs by calculating the coverage probability $r_\theta(L(X), U(X)) = P_\theta(F(U(X)) - F(L(X)) > \beta)$. The $\beta-$content criterion is stricter than $\beta-$expectation criterion because $r_\theta(L(X), U(X)) > 1 - \alpha$ implies $e_\theta(L(X), U(X)) > \beta$ if $\alpha < 1/2$.

In this simulation study, we use the normal distribution and the gamma distribution as examples to show the performance of the proposed methods. First, the coverage probabilities of TIs without censoring case are calculated for different sample sizes. Then we generate data and censor the data greater than a upper censored time U and less than a lower censored time L. By applying **Procedures 1 and 2** to the uncensored data to derive TIs, we calculate the coverage probabilities of 0.9-content, 0.95 level TIs for different censored rates.

4.1 Normal Distribution

Table 1 presents the coverage proportions of TIs for the case without censored data and the cases for different censored rates for the normal distribution.

Table 1 shows the expected coverage probability with different censored rates:

Table 1: Coverage proportions of 0.9 content, level $1 - \alpha = 0.95$ TIs (5) and (7) for the standard normal distribution $N(0, 1)$ for different censored rate s

Sample size n	50	100	500	1000
Uncensored TI	0.9426848	0.9341925	0.9179708	0.9115069
$(L, U) = (-1.4, 1), s = 0.239$				
TI (5)	0.947882	0.93905	0.916313	0.911379
Distribution-free TI (7)	0.956301	0.954294	0.919415	0.914434
$(L, U) = (-0.6, 1), s = 0.433$				
TI (5)	0.942749	0.929939	0.915095	0.910275
Distribution-free TI (7)	0.955512	0.945512	0.91995	0.913309
$(L, U) = (-0.6, 0.6), s = 0.548$				
TI (5)	0.939381	0.93231	0.914093	0.914511
Distribution-free TI (7)	0.953178	0.949264	0.920145	0.917852
$(L, U) = (-0.2, 0.6), s = 0.695$				
TI (5)	0.937867	0.938966	0.907528	0.911114
Distribution-free TI (7)	0.95215	0.952404	0.913157	0.913473
$(L, U) = (0.2, 1), s = 0.737$				
TI (5)	0.946476	0.929956	0.917268	0.910407
Distribution-free TI (7)	0.956862	0.942909	0.921643	0.913668

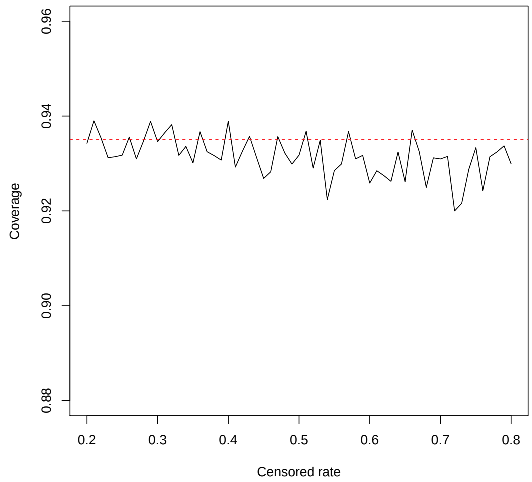

Figure 1: N(0,1) coverage proportions of TI (5) for sample size n=100 with different censored rate (solid line) and coverage proportions with uncensored case (dashed line)

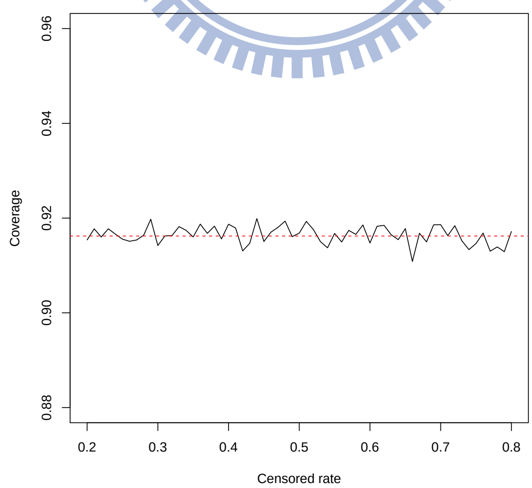

Figure 2: N(0,1) coverage proportions of TI (5) for sample size n=500 with different censored rate (solid line) and coverage proportions with uncensored case (dashed line)

Table 1 shows the coverage proportions of TIs are always greater than the setted β value 0.9. The coverage proportions tend to the setted $\beta - value$ 0.9 when the sample size increases. To achieve a better result for the small sample size case, we generate more data from the distribution based on the estimators $(\hat{\mu}, \hat{s})$ and then based on the data to calculate TIs.

Table 2 shows the coverage proportion with different censored rates using the generated 1000 data:

Table 2: Coverage proportions by regenerated 1000 data of 0.9 content, level $1-\alpha = 0.95$ TIs (5) and (7) for the standard normal distribution $N(0,1)$ for different censored rate s

Sample size n	50	100	500	1000
Uncensored TI	0.9426848	0.9341925	0.9179708	0.9115069
$(L,U) = (-1.4, 1), s = 0.239$				
TI (5)	0.901595	0.901327	0.91058	0.911992
Distribution-free TI (7)	0.905144	0.904295	0.912937	0.914608
$(L,U) = (-0.6, 1), s = 0.433$				
TI (5)	0.889236	0.906238	0.912777	0.910127
Distribution-free TI (7)	0.892327	0.90933	0.916351	0.913837
$(L,U) = (-0.6, 0.6), s = 0.548$				
TI (5)	0.898635	0.903914	0.91148	0.911942
Distribution-free TI (7)	0.901709	0.906403	0.914477	0.915684
$(L,U) = (-0.2, 0.6), s = 0.695$				
TI (5)	0.895989	0.9002	0.907408	0.911784
Distribution-free TI (7)	0.899081	0.903822	0.911118	0.914475
$(L,U) = (0.2, 1), s = 0.737$				
TI (5)	0.880731	0.894976	0.911529	0.910671
Distribution-free TI (7)	0.883636	0.89777	0.914798	0.91385

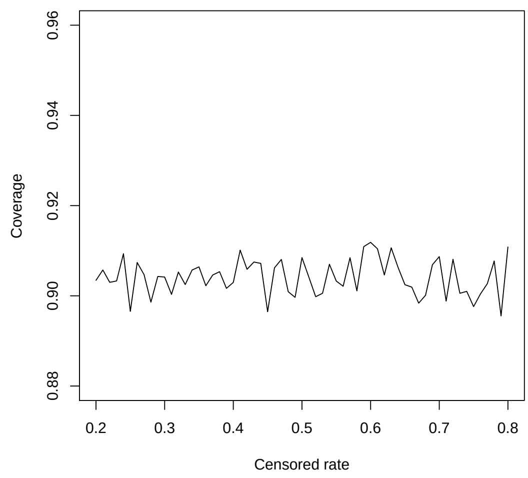

Figure 3: N(0,1) coverage proportions by regenerated 1000 data of TI (5) for sample size n=100 with different censored rate(solid line) and coverage proportions with uncensored case (dashed line)

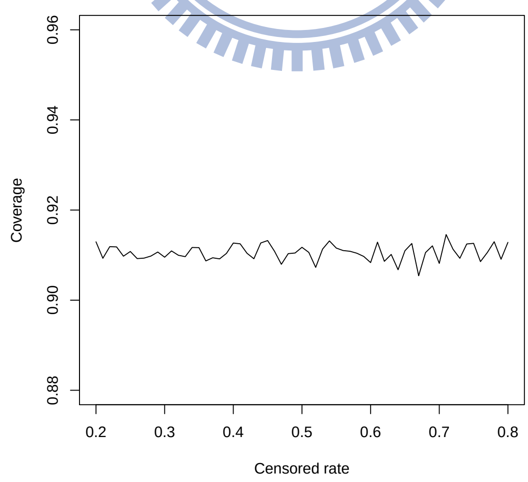

Figure 4: N(0,1) coverage proportions by regenerated 1000 data of TI(5) for sample size n=500 with different censored rate(solid line) and coverage proportions with uncensored case (dashed line)

From Tables 1-2, it reveals that the sample size n increases, the coverage proportions will much close to 0.9. It means that sample size n increases, then the accuracy of tolerance interval will increase too. For small sample size situation. from Table 2 result, we can use regenerate data method to increase accuracy.

Figures 1-2 show that when the censored rate increases, coverage proportion is still close to uncensored situation. And coverage proportion are higher than 0.9 in Figures 1-4. It reveals that the normal method is a good way to construct the tolerance interval for normal distribution.

4.2 Gamma distribution simulation

Then we consider the gamma distribution case. We generate data form the gamma distribution $G(4, 0.05)$ under different censored times L and U, and then adopt **Procedure 2** with $k = 100$ to generate data , $\hat{\alpha}$ and $\hat{\lambda}$. The performance of 0.9-content, 0.95 level TI and distribution free TI (7) are given in Tables 3 and 4.

Table 3: Coverage proportions of 0.9 content, level $1 - \alpha = 0.95$ TIs (6) and (7) for the gamma distribution $G(4, 0.05)$ for different censored rate s

sample size n	50	100	500	1000
Uncensored TI	0.9402119	0.9357238	0.916368	0.9105909
$(L,U) = (10,110), s = 0.203$				
TI (6)	0.947519	0.934563	0.918274	0.911681
Distribution-free TI (7)	0.958208	0.947696	0.923222	0.914777
$(L,U) = (20,100), s = 0.284$				
TI (6)	0.942466	0.937306	0.916717	0.910158
Distribution-free TI (7)	0.948382	0.949671	0.921288	0.912783
$(L,U) = (30,90), s = 0.408$				
TI (6)	0.939538	0.93021	0.917118	0.912648
Distribution-free TI (7)	0.947022	0.946774	0.922243	0.915167
$(L,U) = (35,85), s = 0.487$				
TI (6)	0.949929	0.934746	0.91729	0.911581
Distribution-free TI (7)	0.961445	0.948354	0.922517	0.915089
$(L,U) = (40,80), s = 0.576$				
TI (6)	0.942529	0.935224	0.916078	0.910171
Distribution-free TI (7)	0.953255	0.949475	0.920472	0.912968

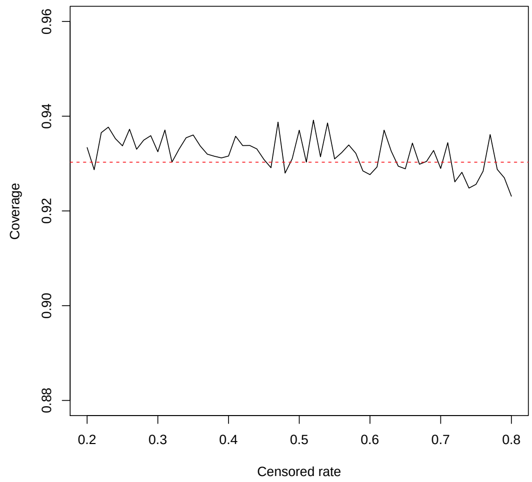

Figure 5: G(4,0.05) coverage proportions of TI (6) for sample size n=100 with different censored rate (solid line) and coverage proportions with uncensored case (dashed line)

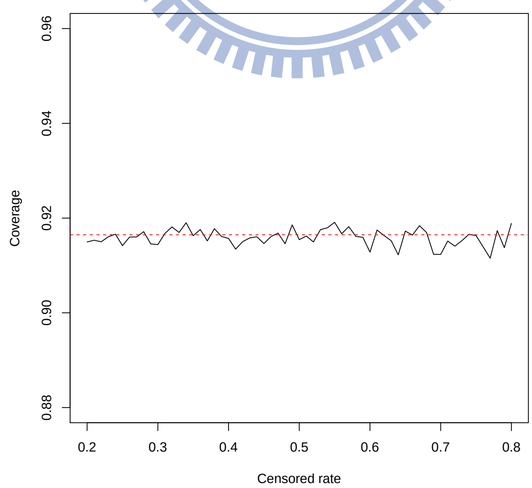

Figure 6: G(4,0.05) coverage proportions of TI (6) for sample size n=500 with different censored rate (solid line) and coverage proportions with uncensored case (dashed line)

Table 4: Coverage proportions by regenerate 1000 data of 0.9 content, level $1-\alpha$ TIs (6) and (7) for the gamma distribution $G(4, 0.05)$ for different censored rate s

sample size n	50	100	500	1000
Uncensored TI	0.9402119	0.9357238	0.916368	0.9105909
$(L,U)=(10,110), s=0.203$				
TI (6)	0.896458	0.907104	0.911554	0.911693
Distribution-free TI (7)	0.899885	0.911698	0.914112	0.914507
$(L,U)=(20,100), s=0.284$				
TI (6)	0.904749	0.914296	0.908947	0.911292
Distribution-free TI (7)	0.907736	0.916863	0.911582	0.915556
$(L,U)=(30,90), s=0.408$				
TI (6)	0.902437	0.904648	0.911802	0.912217
Distribution-free TI (7)	0.905624	0.908264	0.915571	0.916125
$(L,U)=(35,85), s=0.487$				
TI (6)	0.903451	0.902536	0.908789	0.912732
Distribution-free TI (7)	0.906411	0.905598	0.912366	0.91565
$(L,U)=(40,80), s=0.576$				
TI (6)	0.887841	0.909547	0.914453	0.912056
Distribution-free TI (7)	0.892355	0.912906	0.916733	0.915406

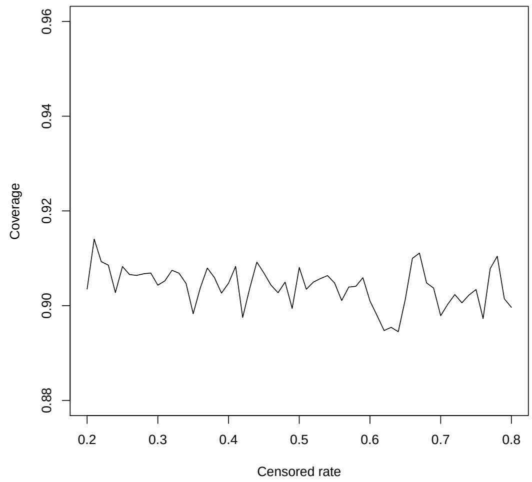

Figure 7: G(4,0.05) coverage proportions by regenerated 1000 data of TI (6) for sample size n=100 with different censored rate(solid line) and coverage proportions with uncensored case (dashed line)

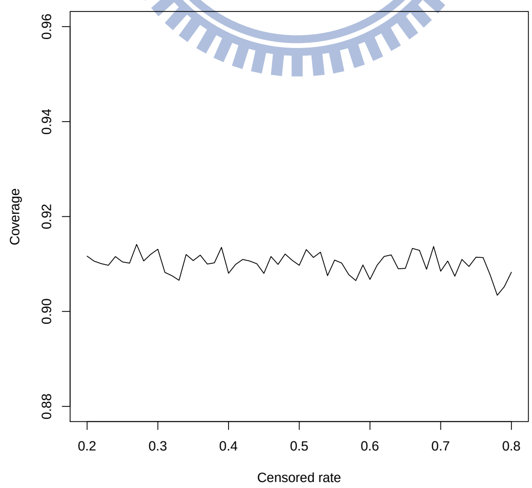

Figure 8: G(4,0.05) coverage proportions by regenerated 1000 data of TI (6) for sample size n=500 with different censored rate(solid line) and coverage proportions with uncensored case (dashed line)

From Tables 3-4, it reveals that the coverage proportions of the tolerance interval and distribution-free tolerance interval for the gamma distribution are higher than 0.9. When the sample size increases, the coverage proportion tend to be close to 0.9. As for small sample size situation, we can adopt **Procedure 2** to generate more data to improve the estimation.

From Figures 5-8, it reveals that when the censored rate increases, coverage proportion is more far away from the coverage proportions with uncensored rate case. But it still higher than 0.9. It reveals that the general approach method is a good way to construct the tolerance interval for continuous distributions.

If we increase sample size n and iteration time k, the general approach method is more accurate. However, if the iteration time k is too large, it is time consumption. So choose the suitable iteration times ($k = 100$) can make the coverage more accuracy with shorter simulation time.

5 A real data example

A data set which record the ages at first stroke of 1205 patients from a hospital in Taiwan is used to illustrate the methods. The data is given in the Appendix. Figure 9 shows an approximately density function fitting the 1205 data. The sample mean and standard deviation of the 1205 data are 68.36 and 13.07 respectively. Since it is not symmetric curve and is skewed to the right, we expect it is not a normal distribution. To obtain a better result, we consider using the generalized extreme value distribution to fit the data (Embrechts, ppelberg, Mikosch 1997, Leadbetter, Lindgren, Rootzen 1983, Resnick 1987 and Stuart 2001), and use Procedure 2 to derive TIs.

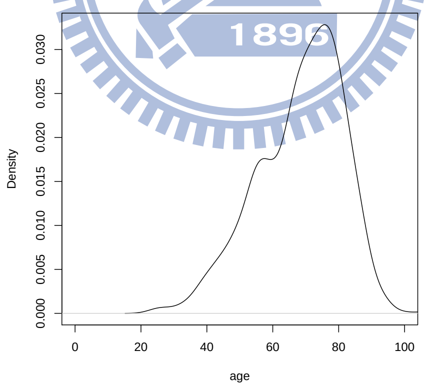

Figure 9: stroke data

The generalized extreme value distribution has the density function :

$$f(x;\mu,\sigma,\xi)=\frac{1}{\sigma}[1+\xi(\frac{x-\mu}{\sigma})]^{(-\frac{1}{\xi})-1}exp\{-[1+\xi(\frac{x-\mu}{\sigma})]^{-\frac{1}{\xi}}\} \qquad (13)$$

for $1+\xi(x-\mu)/\sigma>0$, where μ is a location parameter with $-\infty<\mu<\infty$, σ

is a scale parameter with $\sigma > 0$ and ξ is a shape parameter with $-\infty < \xi < \infty$. Figures 10 and 11 are two density functions with respect to different parameter values.

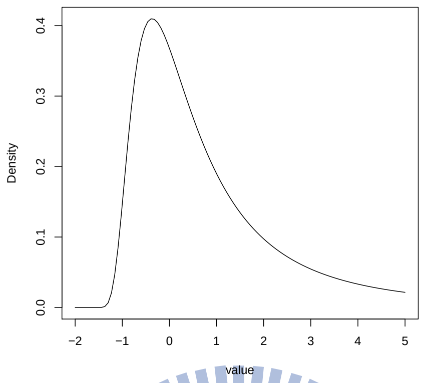

Figure 10: The density function of the generalized extreme value distribution with $\mu = 0, \sigma = 1, \xi = 0.5$

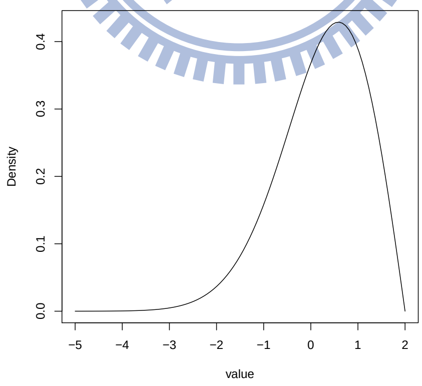

Figure 11: The density function of the generalized extreme value distribution with $\mu = 0, \sigma = 1, \xi = -0.5$

Table 5: Coverage proportions for TI (5), distribution-free TI (7) based on the normal distribution and distribution free TI based on the GEV distribution for the stroke data

True T.I		(43.74,86.38)	
Censored time (L, U)	(55,95)	(60,90)	(65,85)
Censored rate	0.171784	0.273858	0.418257
Normal T.I (5)	(48.09,89.52)	(49.01,89.15)	(51.59,88.53)
Coverage proportions	0.904564	0.892116	0.860580
Distribution-free			
Normal T.I (7)	(47.58,86.87)	(48.16,86.87)	(51.44,88.30)
Coverage proportions	0.882157	0.878838	0.861410
Distribution-free			
GEV T.I (7)	(41.73,86.87)	(40.50,86.87)	(43.12,86.22)
Coverage proportions	0.921991	0.926971	0.903734

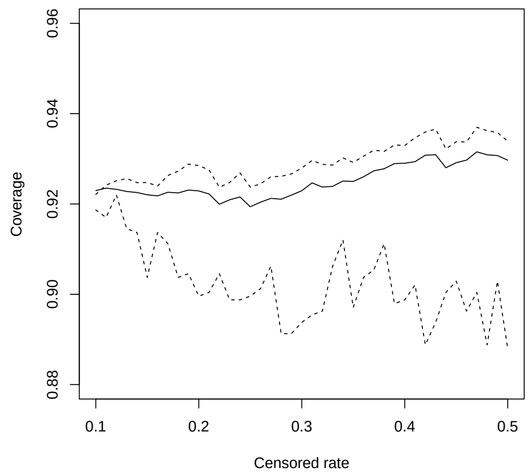

Figure 12: Coverage proportions for stroke of TI (5)(solid line), distribution-free TI (7)(upper dashed line), and GEV distribution-free TI (lower dashed line)

From Table 5 and Figure 12, it reveals that the coverage proportion of GEV distribution-free tolerance interval is much close to 0.9 than tolerance interval and distribution-free tolerance interval based on the normal distribution. And when censored rate increases, the bias of coverage proportion of three tolerance interval increases. The GEV distribution fits the data better than the normal distribution. The reason may be that the data does not follow normal distribution, but we think the uncensored data follow normal distribution.

However, when the censored rate for data increases, the bias of coverage proportion increases. Then using normal method to construct tolerance interval is better than GEV method in this example. Therefore, if the censored rate is too large, using normal distribution method will better than GEV distribution method.

References

[1] Cai, T. and Wang, H. (2009). Tolerance Intervals for Discrete Distributions in Exponential Families. Statistica Sinica, 19, 905-923.

[2] Coles, Stuart (2001). An introduction to Statistical modeling of Extreme Values. Springer Statist.

[3] Cummings, J., Zhou, C and Dive, C. (2011). Application of the β-expectation tolerance interval to method validation of the M30 and M65 ELISA cell death biomarker assays. J Chromatogr B Analyt Technol Biomed Life Sci. Apr 15;879(13-14):887-93.

[4] Embrechts,P,Klppelberg,C , and Mikosch,T. (1997) Modelling extremal events for insurance and finance. Springer,Berlin.

[5] Emura, T. and Wang, H. (2010). Approximate Tolerance Limits Under Log-location-scale Regression Models in the Presence of Censoring, Technometrics, 52, 313-323.

[6] Gebizlioglu, O. L. and Yagci, B. (2008). Tolerance intervals for quantiles of bivariate risks and risk measurement. Insurance: Mathematics and Economics 42, 1022-1027.

[7] Gibbons, J. D. (1971), Nonparametric Statistical Inference, New York: McGraw-Hill Book Co.

[8] Gibbons, J. D. (1975), Nonparametric Methods for Quantitative Analysis, New York: Holt, Rinehart, Winston.

[9] Hahn, G.J.(1970b), Statistical intervals for a normal population. Part I. Tables, examples and applications, Journal of Quality Technology 2, 115-125.

[10] Hahn, G. J. and Chandra, R. (1981). Tolerance intervals for Poisson and Binomial variables. J. Quality Tech. 13, 100-110.

[11] Hahn, G. J. and Meeker, W. Q. (1991). Statistical Intervals: A Guide for Practitioners. Wiley Series.

[12] Krishnamoorthy, K and Mathew, T and Mukherjee, S (2008). Normal-Based Methods for a Gamma Distribution: Prediction and Tolerance Intervals and Stress-Strength Reliability. Technometrics, 50, 69-78.

[13] Krishnamoorthy, K. and Mallick, A. and Thomas Mathew, T. (2011) Inference for the Lognormal Mean and Quantiles Based on Samples With Left and Right Type I Censoring, Technometrics, 53, 72-82.

[14] Leadbetter, M.R., Lindgren, G. and Rootzn, H. (1983). Extremes and related properties of random sequences and processes. Springer: New York.

[15] Miller, R. G., Jr. (1981), Simultaneous Statistical Inference (Second Edition), New York: Springer-Verlag.

[16] Odeh, R. E.,and Owen, D. B.(1980), Tables for Normal Tolerance Limits, Sampling Plans, and Screening, New York: Marcel Dekker, Inc.

[17] Patel, J. K. (1986), Tolerance Intervals, a Review, Communication in Statistics, Theory and Methods, 15, 2719-2762.

[18] Patilea, V. and Rolin, J.-M. (2006), Product-limit estimators of the survival function with twice censored data. Ann Statist., 34(2),Page 925-938.

[19] Resnick, S.I. (1987). Extreme values, regular variation and point processes. Springer-Verlag, New York.

[20] Shen ,P (2009), Nonparametric Estimators of the Survival Function with Twice Censored Data. Ann Inst Stat Math.1-10.

[21] Wald, A. and Wolfowitz, J. (1946). Tolerance limits for normal distribution. Ann. Math. Statist. 17, 208-215.

[22] Wang, H. (2007). Estimation of the probability of passing the USP dissolution test. J Biopharm Stat, 17, 407-413.

[23] Wang, H. and Tsung, F. (2009) . Tolerance Intervals with Improved Coverage Probabilities for Binomial and Poisson Variables. Technometrics, 51, 25-33.

[24] Zaslavsky, BG. (2007). Calculation of tolerance limits and sample size determination for clinical trials with dichotomous outcomes. J Biopharm Stat. ;17(3):481-91.

Table 6: Real data for the 1205 ages at first stoke

46.74	44.32	75.75	78.73	71.64	44.48	77.97	47.23	69.72	85.08
61.33	61.79	75.71	76.86	88.65	66.26	78.2	86.05	56.19	76.46
49.75	82.76	81.97	78.47	78.52	57.07	69.8	66.81	51.67	73.93
36.47	69.11	43.74	47.93	64.81	67.13	54.32	93.66	73.75	55.37
74.42	51.56	78.07	81.36	76.67	76.13	44.1	23.7	70.5	66.22
59.89	74.88	68.32	79.98	80.25	64.84	76.48	52.87	67.75	64.17
73.04	82.78	84.23	71.59	86.38	49.25	61.91	79.88	77.12	77.33
86	74.58	71.53	68.27	50.95	69.83	61.36	87.5	90.81	84.58
75.36	54.92	68.22	81.97	69.88	57.98	55.9	54.92	67.95	62.19
40.15	80.67	85.76	73.49	51.96	79.96	65.33	67.75	73.59	76
88.12	61.58	79.06	61.09	73.7	80.7	80.89	48.25	55.93	56.88
77.57	73.8	82.82	66.05	51.64	57.11	56.14	75.76	77.82	74.93
64.9	42.5	44.75	74.5	79.67	70.81	81.51	48.78	53.11	70.11
78.51	79.87	64.85	86.68	83.65	52.7	62.69	32.33	70.67	65.32
87.08	57.73	75.49	65.81	88.26	53.64	74.12	80.56	60.17	76.83
86.55	75.3	53.99	65	66.37	46.64	40.87	60.18	68.75	75.4
86.75	67.45	68.08	78.14	42.95	74.21	43.38	75.3	85.92	75.29
85.33	91.42	77.25	68.27	56.86	64.66	83.79	70.7	72.7	81.76
64.62	62.33	76.86	85.94	77.1	55.65	72.83	57.04	69.01	85.63
52.96	51.74	69.95	74.79	70.8	63.09	39.47	73.1	68	66.98
31.16	72.05	61.81	65.96	77.96	75.33	79.06	41.23	67.82	71.09
78.56	66.19	88.93	83.47	76.73	74.07	68.96	66.39	75.32	83.31
86.58	70.34	71.2	83.28	64.47	77.93	58.3	48.38	75	47.46
46.41	68.9	75.59	75.65	63.83	81.38	75.94	67.72	89.52	78.59
79.71	84.57	78.23	81.26	51.74	58.92	81.83	56.78	76.35	71.87
81.22	76.5	72.3	76.2	63.85	79.52	53.34	73.26	68.22	76.61
63.19	56.21	64.56	65.54	80.44	61.28	76.89	61.01	107.07	78.38
53.13	85.43	86.39	63.95	51.11	73.08	56.3	66.01	80.07	84.44
49.48	50.02	77.43	39.18	46.98	52.65	44.05	44.47	46.04	80.54
41.43	61.03	68.57	88.31	39.19	73.52	78.6	63.23	65.05	67.2
67.09	58.33	79.82	35.36	58.22	61.98	84.85	93.98	64.79	82.18
83.72	76.87	83.92	47.04	71.03	86.31	55.65	58.2	81.71	79.22
55.97	66.31	49.24	72.93	65.92	77.7	71.04	69.19	95.14	63.22
42.58	82.74	57.2	72.36	59.72	72.1	78.79	67.99	66.65	66.42
76.34	65.21	61.55	53.9	55.9	56.74	79.59	78.65	83.81	61.22
67.52	60.43	69.4	77.84	63.81	75.2	66.45	67.47	55.12	74.22
66.3	69.19	68.1	64.81	84.03	77.35	81.51	58.71	68.34	59.44
75.43	68.32	68.47	62.36	67.37	78.02	27.62	82.3	78.67	83.52
60.65	64.45	72.66	45.62	57.72	74.31	67.26	88.7	58.31	55.47

80.37	67.28	51.17	80.51	77.19	70.56	67.61	53.45	71.54	84.44
67.07	76.38	75.37	56.81	76.45	68.85	83.32	84.13	79.46	56.22
78.61	76.15	75.81	77.18	71.64	88.07	88.08	89.96	76.82	65.31
56.87	71.98	66.77	77.16	72.92	80.84	65.47	63.87	81.44	82.78
78.39	71.59	61.53	73.28	70.7	42.13	68.52	71.68	73.79	70.93
59.12	69.19	59.99	76.98	68.67	70.94	71.61	79.21	74.99	52.82
57.76	66.56	48.24	79.66	79.1	73.64	56.82	84.03	87.4	76.11
72.22	65.73	48.54	65.87	65.71	64.97	82.54	72.8	54.54	75.91
55.5	78.37	78.44	63.21	72.18	79.16	86.62	76.68	61.53	81.91
70.04	76.98	88.77	41.53	60.34	76.27	53.41	53.78	77.58	49.95
54.03	85.63	70.32	70.72	78.54	75.64	77.44	31.86	72.02	62.12
75.45	79.81	81.87	59.56	68.09	59.77	73.71	56.57	57.44	62.63
60.42	80.46	65.23	69.76	53.41	54.1	60.98	80.87	79.61	65.32
38.15	81.32	82.83	81.02	80.17	55.91	64.85	72.75	78.35	83.15
75.54	71.38	68.15	72.43	68.24	76.86	67.09	86.94	72.17	58.5
68.69	64.86	64.42	39.1	73	46.83	75.45	77.47	81.41	73.11
64.04	72.46	88.58	82.43	58.55	83.79	80.58	79.93	72.03	74.63
60.16	34.64	72.3	54.13	72.63	68.62	80.22	44.65	79.48	77.72
49.16	71.34	67.08	39.68	71.84	72.05	75.7	54.12	65.24	68.62
82.43	37.46	78.41	69.36	82.16	51.56	85.78	73.95	62.58	56.42
74.72	74.19	66.5	55.95	66.32	82.25	48.53	68.5	80.08	72.47
38.76	56.96	66.5	59.44	71.98	74.27	27	91.93	71.31	62.83
63.67	58.34	78.53	68.65	89.59	72.27	68.85	43.72	69.05	48.38
65.28	50.87	76.25	73.4	77.53	74.03	51.41	71.7	93.84	53.64
78.42	67.3	73.08	84.31	56.58	70.04	64.84	81.55	76.22	70.54
76.42	76.67	69.37	71.08	53.15	76.32	57.71	43.07	59.18	76.02
78.04	48.92	58.92	73.78	62.78	69.55	51.48	64.82	56.33	27.13
55.61	76.81	86.95	62.9	46.5	75.71	70.82	76.13	68.82	45.25
70.45	77.14	80.72	66.64	62.41	80.97	73.8	80.12	55.13	58.73
54.14	82.35	72.96	42.79	64.94	39.04	75.83	74.47	62.37	61.98
75.45	67.42	55.11	68.54	64.65	53.08	72.38	74.59	79.4	56.38
65.52	64.15	70.99	95	58.66	57.88	63.62	87.07	71.64	67.42
79.39	46.02	49.67	73.2	66.98	61.05	25.74	55.68	76.19	79.07
82.54	91.76	66.21	82.02	75.2	83	76.73	81.44	79.94	78.42
79.92	73.95	57.03	72.91	61.98	61.5	93.75	73.84	83.32	63.72
79.46	67.54	52.12	75.36	69.78	80.86	85.57	49.92	54.06	80.81
68.08	78.85	76.04	80.51	74.93	58.32	64.04	82.62	74.35	78.89
76.88	57.58	36.59	86.13	87.43	73.97	89	75.79	79.15	89.08
73.8	86.56	80.04	67.15	78.21	39.82	77.14	82.8	79.47	52.61
71.93	55.56	76.42	77.72	80.32	80.24	84.29	71.45	78.54	52.17

71.36	37.14	47.79	45.17	57.82	84.32	58.83	78.99	72.7	82.86
42.88	83.69	82.56	78.38	86.88	80.57	87.09	86.01	62.89	77.05
72.72	75.51	75.18	84.33	67.75	80.76	56.13	65.25	74.25	81.34
35.33	86.39	44.63	71.32	78.58	65.78	63.27	70.45	61.8	66.35
67.2	53.41	85.28	81.17	85.65	73.71	69.58	72.25	60.02	59.79
91.99	65.28	54.55	54.09	66.56	86.22	73.86	77.22	69.79	58.97
46.02	65.07	75.06	76.8	88.22	88.53	49.16	80.12	84.56	63.24
70.01	90.02	65.36	72.88	51.6	72.13	70.36	82.62	52.43	71.1
46.55	78.6	23.8	67.22	61.06	39.3	73.76	70.91	56.96	73.65
76.21	82.39	57.33	50.82	57.89	73.99	49.68	72.65	72.71	47.13
66.3	79.44	54.04	69.3	32.32	88.15	55.94	56.32	84.49	76.36
33.17	51.99	81.1	50.21	69.1	64.59	108.22	59.31	76.35	82.18
53.5	83.65	68.49	66.45	45.72	73.28	72.02	75.67	69.44	64.75
84.9	70.97	71.38	54.69	58.73	44.9	60.88	62.57	55.35	67.26
70.54	67.38	79.33	63.68	47.79	79.9	72.9	54.21	51.86	82.21
38.38	47.33	82.03	68.52	46.38	43.33	69.25	54.75	53.4	81.62
43.65	69.65	76.45	75.97	65.91	47.85	50.01	48.88	60.2	68.34
51.7	83.68	71.57	50.82	77.84	69.73	100.36	71.9	48.62	55.75
83.23	59.01	67.56	69.48	83.61	70.43	74.18	77.14	49.27	81.27
80.57	70.58	56.95	84.32	51.4	92.25	70.97	81.69	59.42	77.98
66.53	50.46	61.96	39.75	80.69	77.8	72.95	55.44	74.53	54.21
68.07	48.43	78.63	54.73	88.15	49.2	59.47	52.88	65.92	61.48
61.65	64.61	54.14	67.71	37.63	66.47	77.99	78.43	62	67.75
80.98	76.75	40.89	70.09	53.36	81.27	60.48	80.36	69.65	89.13
86.05	54.64	83.77	67.84	84.93	54.19	55.44	74.75	38.22	78
73.45	71	41.98	72.87	84.79	45.16	85.88	71.28	73.16	48.7
68.62	69.76	71.49	79.31	42.15	81.78	60.69	52.67	64.52	65.87
72.89	81.42	58.24	71.5	72.17	76.17	70.76	73.43	73.02	76.59
69.91	71.18	86.67	78.14	77.02	76.56	69.56	59.61	88.16	70.94
65.49	56.06	74.81	71.48	43.68	65.39	76.33	78.97	85.32	74.48
57.15	82.19	71.49	75.78	74.13	72	65.47	93.92	69.16	66
73.26	92.8	59.42	72.91	57.72	43.29	71.65	58.14	65.72	61.74
80.18	56.56	60.17	57.34	74.05	86.89	75.3	81.84	84.97	48.37
75.06	56.85	76.12	80.65	56.03	60.48	61.67	77	77.12	35.61
53.66	76.41	85.86	81.07	66.48	76.94	77.01	69.75	37.67	76.02
62.89	56.01	84.92	54.76	57.99	48.75	79.55	87.57	60.18	73.45
73.55	75.39	54.6	40.85	83.95	69.56	61.19	50.35	55.33	50.26
61.85	85.22	84.49	70.26	77.79	60.89	86.25	42.29	65.98	78.8
68.57	70.85	53.04	67.56	56.89	88.19	69.48	81.38	78.07	78.79
77.79	76.97	55.05	80.62	40.18	67.72	74.48	91.24	64.22	68.63
43.59	81.52	61.19	52.63	65.32	83.37	58.28	54.57	67.82	66.22
57.96	70.39	49.95	80.41	71.61					

Table 7: The $c_{(p;n)}$ for two-sided tolerance interval described by a normal distribution

	$1-\alpha=0.95$			$1-\alpha=0.99$		
		p			p	
n	0.90	0.95	0.99	0.90	0.95	0.99
4	5.37	6.34	8.22	9.42	11.12	14.41
5	4.29	5.08	6.60	6.65	7.87	10.22
6	3.73	4.42	5.76	5.38	6.37	8.29
7	3.39	4.02	5.24	4.66	5.52	7.19
8	3.16	3.75	4.89	4.19	4.97	6.48
9	2.99	3.55	4.63	3.86	4.58	5.98
10	2.86	3.39	4.44	3.62	4.29	5.61
12	2.67	3.17	4.16	3.28	3.90	5.10
15	2.49	2.96	3.89	2.97	3.53	4.62
20	2.32	2.76	3.62	2.68	3.18	4.17
25	2.22	2.64	3.46	2.51	2.98	3.91
30	2.15	2.55	3.35	2.39	2.85	3.74
40	2.06	2.45	3.22	2.25	2.68	3.52
60	1.96	2.34	3.07	2.11	2.51	3.30
∞	1.64	1.96	2.58	1.64	1.96	2.58

Table 8: The number v for two-sided distribution-free tolerance interval contains at least $100p\%$ of the Sampled Population with $100(1-\alpha)\%$ Confidence

	$p = 0.90$			$p = 0.95$			$p = 0.99$		
	$1-\alpha$			$1-\alpha$			$1-\alpha$		
n	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
10	1	1	1	1	1	1	1	1	1
	0.6513	0.6513	0.6513	0.4013	0.4013	0.4013	0.0956	0.0956	0.0956
15	1	1	1	1	1	1	1	1	1
	0.7941	0.7941	0.7941	0.5367	0.5367	0.5367	0.1399	0.1399	0.1399
20	1	1	1	1	1	1	1	1	1
	0.8784	0.8784	0.8784	0.6415	0.6415	0.6415	0.1821	0.1821	0.1821
25	1	1	1	1	1	1	1	1	1
	0.9282	0.9282	0.9282	0.7226	0.7226	0.7226	0.2222	0.2222	0.2222
30	1	1	1	1	1	1	1	1	1
	0.9576	0.9576	0.9576	0.7854	0.7854	0.7854	0.2603	0.2603	0.2603
40	2	1	1	1	1	1	1	1	1
	0.9195	0.9852	0.9852	0.8715	0.8715	0.8715	0.3310	0.3310	0.3310
50	2	2	1	1	1	1	1	1	1
	0.9662	0.9662	0.9948	0.9231	0.9231	0.9231	0.3950	0.3950	0.3950
60	3	2	1	1	1	1	1	1	1
	0.9470	0.9862	0.9982	0.9539	0.9539	0.9539	0.4528	0.4528	0.4528
80	5	4	2	2	1	1	1	1	1
	0.9120	0.9647	0.9978	0.9139	0.9835	0.9836	0.5525	0.5525	0.5525
100	6	5	4	2	2	1	1	1	1
	0.9424	0.9763	0.9922	0.9629	0.9629	0.9941	0.6340	0.6340	0.6340
200	15	13	11	6	5	4	1	1	1
	0.9071	0.9680	0.9919	0.9377	0.9736	0.9910	0.8660	0.8660	0.8660
300	23	22	19	10	9	7	1	1	1
	0.9301	0.9542	0.9903	0.9350	0.9659	0.9934	0.9510	0.9510	0.9510
400	32	30	27	15	13	11	2	1	1
	0.9254	0.9643	0.9908	0.9010	0.9645	0.9906	0.9095	0.9820	0.9820
500	41	39	35	19	17	14	2	2	1
	0.9249	0.9607	0.9921	0.9135	0.9657	0.9945	0.9602	0.9602	0.9934
600	51	48	44	23	21	18	3	2	1
	0.9043	0.9591	0.9901	0.9247	0.9680	0.9938	0.9389	0.9830	0.9976
800	69	66	61	32	30	26	5	4	2
	0.9146	0.9593	0.9912	0.9199	0.9606	0.9935	0.9015	0.9583	0.9971
1000	88	85	79	41	39	35	6	5	3
	0.9801	0.9515	0.9901	0.9194	0.966	0.9907	0.9339	0.9713	0.9973