CHAPTER 3

DIMENSIONLESS GROUPS AND UNCERTAINTY ANALYSIS

3.1 Dimensionless Groups

The non-dimensional parameters associated with the flow considered here are the jet Reynolds number Re_j, based on the mean gas speed V_j at the injection pipe exit and the exit diameter of the injection pipe D_j, and the Rayleigh number Ra, based on the temperature difference between the heated disk and inlet gas ΔT and the jet-to-disk separation distance H. They are defined as

$$Re_j = \frac{V_j D_j}{\nu} = \frac{4}{\pi}\frac{Q_j}{\nu D_j}, \tag{3.1}$$

and

$$Ra = \frac{g\beta(T_f - T_j)H^3}{\alpha\nu} = \frac{g\beta\Delta T H^3}{\alpha\nu} \tag{3.2}$$

where α is the thermal diffusivity, g is the gravitational acceleration, β is the thermal expansion coefficient, and ν is the kinematic viscosity.

3.2 Uncertainty Analysis

An uncertainty analysis is carried out here to estimate the uncertainty levels in the experiment. Kline and McClintock [34] proposed a formula for evaluating the uncertainty in the result F as a function of independent variables, $X_1, X_2, \Lambda\,\Lambda\,\Lambda\,\Lambda, X_n$,

$$F = F(X_1, X_2, X_3, \Lambda\,\Lambda, X_n) \tag{3.3}$$

The absolute uncertainty of F is expressed as

$$\delta F = \left\{ \left[\left(\frac{\partial F}{\partial X_1}\right)\delta X_1\right]^2 + \left[\left(\frac{\partial F}{\partial X_2}\right)\delta X_2\right]^2 + \left[\left(\frac{\partial F}{\partial X_3}\right)\delta X_3\right]^2 + \Lambda + \left[\left(\frac{\partial F}{\partial X_n}\right)\delta X_n\right]^2 \right\}^{1/2} \tag{3.4}$$

and the relative uncertainty of F is

$$\frac{\delta F}{F}=\left\{\left[\left(\frac{\partial \ln F}{\partial \ln X_1}\right)\left(\frac{\delta X_1}{X_1}\right)\right]^2+\left[\left(\frac{\partial \ln F}{\partial \ln X_2}\right)\left(\frac{\delta X_2}{X_2}\right)\right]^2+\Lambda+\left[\left(\frac{\partial \ln F}{\partial \ln X_n}\right)\left(\frac{\delta X_n}{X_n}\right)\right]^2\right\}^{1/2} \quad (3.5)$$

If $F = X_1^a X_2^b X_3^c$ .... , then the relative uncertainty is

$$\frac{\delta F}{F}=\left[\left(a\frac{\delta X_1}{X_1}\right)^2+\left(b\frac{\delta X_2}{X_2}\right)^2+\left(c\frac{\delta X_3}{X_3}\right)^2+\Lambda\,\Lambda\right]^{1/2} \quad (3.6)$$

where $(\partial F/\partial X_i)$ and δX_i are, respectively, the sensitivity coefficient and uncertainty level associated with the variable δX_i. The values of the uncertainty intervals δX_i are obtained by a root-mean-square combination of the precision uncertainty of the instruments and the unsteadiness uncertainty, as recommended by Moffat [35]. The choice of the variable X_i to be included in the calculation of the total uncertainty level of the result F depends on the purpose of the analysis. The uncertainties for the chosen parameters are calculated as follows:

(1) Uncertainty of the measured temperature difference, $\Delta T = T_f - T_j$

$$\delta(T_f - T_j) = [(\delta T_f)^2 + (\delta T_j)^2]^{1/2} \quad (3.7)$$

(2) The dependence of the air properties k, μ, and ν on the temperature (T in K) [36] is

$$k = 1.195\times10^{-6}T^{1.6}/(T+118)$$
$$\mu = 1.448\times10^{-6}T^{1.5}/(T+118) \quad (3.8)$$
$$\nu = \mu/\rho$$

The uncertainties of the properties are

$$\frac{\delta k}{k}=\frac{T}{k}\frac{\partial k}{\partial T}\frac{\delta T}{T}$$
$$\frac{\delta \rho}{\rho}=\frac{T}{\rho}\frac{\partial \rho}{\partial T}\frac{\delta T}{T} \quad (3.9)$$
$$\frac{\delta \mu}{\mu}=\frac{T}{\mu}\frac{\partial \mu}{\partial T}\frac{\delta T}{T}$$

(3) Uncertainty of Rayleigh number, Ra,

$$Ra = \frac{g\beta(T_f - T_j)H^3}{\alpha\nu} = \frac{g\beta\Delta T H^3}{\alpha\nu} \tag{3.10}$$

$$\frac{\delta Ra}{Ra} = \left[\left(\frac{\delta g\beta}{g\beta}\right)^2 + \left(3\frac{\delta H}{H}\right)^2 + \left(\frac{\delta \Delta T}{\Delta T}\right)^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2 + \left(\frac{\delta\nu}{\nu}\right)^2\right]^{1/2} \tag{3.11}$$

(4) Uncertainty of jet Reynolds number, Re_j,

$$Re_j = \frac{V_j D_j}{\nu} = \frac{4}{\pi}\frac{Q_j}{\nu D_j} \tag{3.12}$$

$$\frac{\delta Re_j}{Re_j} = \left[\left(\frac{\delta\nu}{\nu}\right)^2 + \left(\frac{\delta Q_j}{Q_j}\right)^2 + \left(\frac{\delta D_j}{D_j}\right)^2\right]^{1/2} \tag{3.13}$$

The results from this uncertainty analysis are summarized in Table 3.1.

Table 3.1 Summary of uncertainty analysis

Parameter and Estimate Uncertainty	
Parameters	Uncertainty
D_j, D_w, H (m)	± 0.00005 m
T (℃)	± 0.2℃
ΔT (℃)	0.4℃
Q_j (slpm)	± 2%
μ (Nm/s^2) ρ (kg/m^3) υ (m^2/s)	± 0.05% ± 0.05% ± 0.07%
Ra	± 8.6%
Re_j	± 2.3%