Model Reduction for Keeping Gain Margin and Phase Margin Unchanged

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Abstract—**The effects of model reduction on system characteristics, such as gain margin, phase margin, and bandwidth are investigated. A practical method is proposed so that the reduced models can approximate the frequency response of the original transfer function not only at $S = 0$ and $S = \infty$ but also at some specific points on the frequency response curve of the original system; thus the gain margin and/or phase margin can be kept unchanged. Examples are given, and comparisons with the methods given in current literature are made.**

I. Introduction

In the current literature, most of the methods for model reduction, such as continued-fraction expansion [1]–[3], moment matching [4], [5], Padé approximations [6], [7], and modified Padé approximation [8] are taken either at $S = 0$ and/or $S = \infty$, or at $S = a$ ($a > 0$). None of these methods has considered the problem of matching the frequency responses at the practically important frequencies such as phase crossover, gain crossover, and cutoff; therefore, the system characteristics such as gain margin (GM), phase-margin (PM), and bandwidth cannot be preserved by a system with such kinds of reduced models.

Recently, Lin and Han [9] have proposed a method of model reduction which can be used to make the reduced models approximate the frequency response of the original transfer function at $S = 0$, $S = \infty$, and at a fixed point on the frequency response curve. This paper extends the aforementioned method to obtain reduced models which can match not only at $S = 0$ and $S = \infty$ but also match some desirable points on the frequency response curve of the original transfer function [10]; thus the system characteristics mentioned previously can be preserved.

II. The Proposed Method

Assume that the original transfer function and the reduced model are

$$G(S)=\frac{A_{21}+A_{22}S+A_{23}S^2+\cdots+A_{2n}S^{n-1}}{A_{11}+A_{12}S+A_{13}S^2+\cdots+A_{1,n+1}S^n} \tag{1}$$

and

$$(\omega_1,\ \omega_2,\ \cdots,\ \omega_m)R[r-1,\ r]_j^i(S) = \frac{d_0+d_1S+d_2S^2+\cdots+d_{r-1}S^{r-1}}{c_0+c_1S+c_2S^2+\cdots+c_rS^r}, \tag{2}$$

respectively. In (2), r and $r - 1$ represent the numbers of poles and zeros of $R(S)$, respectively; i and j are the numbers of times of the continued-fraction expansion of $G(S)$ about $S = 0$ and $S = \infty$, respectively; and $\omega_1, \omega_2, \cdots, \omega_m$ are the frequencies at which the frequency response of $G(S)$ are matched by $R(S)$. The procedure for finding the reduced models is as follows.

Step 1: Expand $G(S)$ about $S = 0$ for i (even number) times, i.e.,

$$G(S)=\cfrac{1}{\cfrac{A_{11}+A_{12}S+A_{13}S^2+\cdots+A_{1,n+1}S^n}{A_{21}+A_{22}S+A_{23}S^2+\cdots+A_{2n}S^{n-1}}}$$

$$=\cfrac{1}{h_1+S\cfrac{A_{31}+A_{32}S+A_{33}S^2+\cdots+A_{3n}S^{n-1}}{A_{21}+A_{22}S+A_{23}S^2+\cdots+A_{2n}S^{n-1}}}$$

$$=\cfrac{1}{h_1+\cfrac{S}{\cfrac{A_{21}+A_{22}S+A_{23}S^2+\cdots+A_{2n}S^{n-1}}{A_{31}+A_{32}S+A_{33}S^2+\cdots+A_{3n}S^{n-1}}}}$$

$$=\cfrac{1}{h_1+\cfrac{S}{h_2+S\cfrac{A_{41}+A_{42}S+\cdots+A_{4,n-2}S^{n-2}}{A_{31}+A_{32}S+A_{33}S^2+\cdots+A_{3n}S^{n-1}}}}$$

$$=\left[h_1+\left[\frac{h_2}{S}+\left[\cdots+\left[\frac{h_i}{S}+\frac{H_N(S)}{H_D(S)}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1} \tag{3}$$

where $h_i = A_{i,1}/A_{i+1,1}$ and

$$H_N(S)=A_{i+2,1}+A_{i+2,2}S+\cdots+A_{i+2,n-i/2}S^{n-1-i/2} \tag{4}$$

and

$$H_D(S)=A_{i+1,1}+A_{i+1,2}S+\cdots+A_{i+1,n+1-i/2}S^{n-i/2}. \tag{5}$$

Step 2: Reverse the polynomial sequences in (4) and (5) and continue to expand (3) about $S = \infty$ for j (even number)

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times. Then one has

$$\frac{H_N(S)}{H_D(S)}=\left[E_1S+\left[E_2+\left[\cdots+\left[E_j+\frac{F_N(S)}{F_D(S)}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1} \quad (6)$$

where

$$F_N(S)=A_{i+j+2,n-(i+j)/2}S^{n-1-(i+j)/2}+\cdots+A_{i+j+2,2}S+A_{i+j+2,1} \quad (7)$$

and

$$F_D(S)=A_{i+j+1,n+1-(i+j)/2}S^{n-(i+j)/2}+\cdots+A_{i+j+1,2}S+A_{i+j+1,1}. \quad (8)$$

Step 3: Let

$$\frac{T_N(S)}{T_D(S)}=\frac{Y_{m-1}S^{m-1}+Y_{m-2}S^{m-2}+\cdots+Y_1S+Y_0}{X_mS^m+X_{m-1}S^{m-1}+\cdots+X_1S+X_0} \quad (9)$$

with

$$Y_{m-1}=1 \quad (10)$$

where m is the number of points on the frequency response curve of $G(S)$ to be matched by $R(S)$, and Y_{m-2}, Y_{m-3}, $\cdots$, Y_1, Y_0, X_m, $\cdots$, X_1, X_0 are to be obtained as follows. Let

$$\left.\frac{T_N(S)}{T_D(S)}\right|_{S=j\omega_k}=\left.\frac{F_N(S)}{F_D(S)}\right|_{S=j\omega_k}=r_k+jm_k, \quad k=1,\ 2,\ \cdots,\ m \quad (11)$$

where r_k and m_k are the real part and the imaginary part of $F_N(S)/F_D(S)$ for $S = j\omega_k$, respectively. Separating the real part and the imaginary part in (11), one can obtain a pair of simultaneously independent equations. Therefore, the $2m$ unknowns Y_{m-2}, Y_{m-3}, $\cdots$, Y_1, Y_0, X_m, $\cdots$, X_1, X_0 can be obtained by solving the $2m$ simultaneously independent equations.

Step 4: Replace $F_n(S)/F_D(S)$ in (6) by $T_N(S)/T_D(S)$ and invert the continued fraction, the reduced model defined in (2) is obtained, i.e.,

$$(\omega_1,\ \omega_2,\ \cdots,\ \omega_m)R[r-1,\ r]_j^i(S)$$
$$=\left[h_1+\left[\frac{h_2}{S}+\left[\cdots+\left[\frac{h_i}{S}+\left[E_1S+\left[E_2+\left[\cdots+\left[E_j+\frac{T_N(S)}{T_D(S)}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}. \quad (12)$$

The order of the denominator of the reduced model is

$$r=m+(i+j)/2. \quad (13)$$

By (3) and (6), one has

$$G(S)=\left[h_1+\left[\frac{h_2}{S}+\left[\cdots+\left[\frac{h_i}{S}+\left[E_1S+\left[E_2+\left[\cdots+\left[E_j+\frac{F_N(S)}{F_D(S)}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}. \quad (14)$$

From (11), (12), and (14) it can be seen that

$$G(S)=(\omega_1,\ \omega_2,\ \cdots,\ \omega_m)R[r-1,\ r]_j^i(S), \quad \text{for } S=j\omega_k,\ k=1,\ \cdots,\ m \quad (15)$$

which indicates that the frequency response of $G(S)$ is matched exactly by that of the reduced model at ω_1, ω_2, $\cdots$, ω_m. If both i and j are odd numbers, foregoing equations may have some minor differences.

Note that Lin and Han's method [9] is a special case of the method proposed here. From (13) it can be seen that, for the reduced models, the lowest order of the denominator is $m + 1$. Therefore, for a reduced model with fixed reduced order, the reduced model may not exist if more points on the original frequency response curve should be matched. Note also that since the steady-state response of the original transfer function must be matched by that of the reduced models, the value of i should not be zero.

The advantage of the proposed method is that by the selection of matching points on the frequency response curve of the original transfer function, and the numbers of times of the continued-fraction expansion about $S = 0$ and $S = \infty$, the frequency responses at low, intermediate, and high frequencies can be matched; thus the system characteristics such as gain margin, phase margin, and bandwidth can be preserved by the system with reduced models.

III. Applications

For illustrative purposes, three examples are presented in this section.

Example 1

Consider the system shown in Fig. 1. Let $k = k_1 = 1.2$, $H_1(S) = 1/S$, and

$$G(S)=G_1(S)=\frac{(S+0.5)(S+10)(S+14)(S^2+8S+32)}{(S+1)(S^2+4S+8)(S^2+4S+20)(S^2+6S+18)}. \quad (16)$$

The gain crossover frequency (ω_{g1}) and the phase-crossover frequency (ω_{p1}) of the loop transfer function $L(S)$ $(=k_1G_1(S)H_1(S))$ are found at $\omega_{g1} = 1.740$ rad/s and $\omega_{p1} = 2.396$ rad/s, i.e.,

$$|k_1G_1(S)H_1(S)|=1, \quad \text{for } S=j\omega_{g1} \quad (17)$$

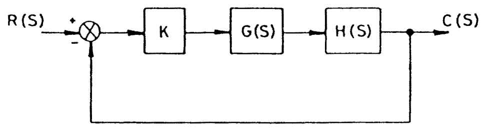

Fig. 1. Block diagram of control system.

and

$$\angle k_1 G_1(S) H_1(S) = -180°, \qquad \text{for } S = j\omega_{p1} \tag{18}$$

Let the reduced model be

$$A_{11}(S) = (\omega_{p1}) R[2, 4]_2^4(S). \tag{19}$$

By steps 1 and 2, after expanding $G_1(S)$ about $S = 0$ and $S = \infty$, respectively, for four and two times, one has

$$G_1(S) = \left[h_1 + \left[\frac{h_2}{S} + \left[h_3 + \left[\frac{h_4}{S} + \left[E_1 S + \left[E_2 + \frac{F_N(S)}{F_D(S)}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1} \tag{20}$$

where

$$h_1 = 1.28571428571 \qquad E_1 = -0.626380774734$$

$$h_2 = -2.00408997956 \qquad E_2 = -4.92255525277$$

$$h_3 = -0.120067213658$$

$$h_4 = 3.60056298821 \tag{21}$$

and

$$\frac{F_N(S)}{F_D(S)} = \frac{A_{84}S^3 + A_{83}S^2 + A_{82}S + A_{81}}{A_{75}S^4 + A_{74}S^3 + A_{73}S^2 + A_{72}S + A_{71}} \tag{22}$$

where

$$A_{75} = 0.324317946 \qquad A_{84} = 53.3510640422$$

$$A_{74} = 15.675709561 \qquad A_{83} = 788.271947667$$

$$A_{73} = 196.058766559 \qquad A_{82} = 4546.62256298$$

$$A_{72} = 1061.65572162 \qquad A_{81} = 11480.2689699.$$

$$A_{71} = 2585.44888817 \tag{23}$$

Since there is only one point at ω_{p1} on the frequency response curve of $G_1(S)$ to be matched by $A_{11}(S)$, by (9) in step 3, let

$$\frac{T_N(S)}{T_D(S)} = \frac{1}{X_1 S + X_0} = \frac{F_N(S)}{F_D(S)}, \qquad \text{for } S = j\omega_{p1}, \tag{24}$$

i.e.,

$$X_1 = I_m[F_D(S)/F_N(S)]/|S|, \qquad \text{for } S = j\omega_{p1}$$
$$= 3.44354873657 \times 10^{-3} \tag{25}$$

$$X_0 = R_e[F_D(S)/F_N(S)], \qquad \text{for } S = j\omega_{p1}$$
$$= 0.223499481125 \tag{26}$$

where $R_e[\cdot]$ and $I_m[\cdot]$ denote the real part and the imaginary part, respectively. Replacing $F_N(S)/F_D(S)$ in (20) by (24) and inverting the continued fraction, one has the reduced model

$$A_{11}(S) = (\omega_{p1}) R[2, 4]_2^4(S)$$
$$= \frac{(S + 0.5033)(S + 36.2328)}{(S + 1.1141)(S + 2.2331)(S^2 + 2.6470S + 9.4251)}. \tag{27}$$

Since

$$A_{11}(S) = G_1(S), \qquad \text{for } S = j\omega_{p1}, \tag{28}$$

therefore,

$$|k_1 A_{11}(S) H_1(S)| = |k_1 G_1(S) H_1(S)| = 1, \qquad \text{for } S = j\omega_{p1} \tag{29}$$

which means that the original and the reduced system have the same gain-crossover frequency ω_{g1} and phase margin.

By using a similar procedure, the results of the model $A_{12}(S)$ which matches two points $(\omega_{g1}, \omega_{p1})$ on the frequency response curve of $G_1(S)$ are listed in Table I. For comparison, the continued-fraction method [1] and the Routh stability array method [11] are applied to reduce $G_1(S)$; the results are also listed in Table I. The Nyquist plots of $G_1(S)$ and its reduced models are shown in Fig. 2. The unit step responses of the original system and the system with reduced models are shown in Fig. 3. The maximum deviations (E_M), integral of absolute errors (IAE) and integral of squared errors (ISE) between the unit step responses of the original system and the system with reduced models are shown in Table I. It can be seen that the reduced model $C_1(S)$ obtained by the continued-fraction method is unstable, and that the model $A_{12}(S)$ of the proposed method is the best one.

Example 2

Consider the system shown in Fig. 1. Assume $k = k_2 = 7.5$, $H(S) = 1$, and

$$G(S) = G_2(S)$$
$$= [1441.53(S + 1.4706)(S + 6.1350)(S + 46.7248)]$$
$$/[(S + 1.8972)(S + 49.3777)(S + 52.5174)$$
$$(S^2 + 0.5456S + 1.1621)(S^2 + 7.7022S + 108.0056)]. \tag{30}$$

TABLE I

Items \ Models	Original $G_1(S)$	Lin and Han's Method $A_{11}(S)$ $(\omega_{p1})R[2, 4]_2^4$	$A_{12}(S)$ $(\frac{\omega_{p1}}{\omega_{g1}})R[2, 4]_2^2$	Continued-Fraction Method $C_1(S)$	Routh Stability Array Method $T_1(S)$
Poles	-1 $-2 \pm j2$ $-2 \pm j4$ $-3 \pm j3$	-1.1141 -2.2331 $-1.3235 \pm j2.7701$	$-1.3734 \pm j2.9043$ $-1.4425 \pm j0.5390$	-1.0009 $+3.7235$ $-1.6831 \pm j2.0901$	$-0.5912 \pm j2.9497$ $-0.8429 \pm j0.6102$
Zeros	-0.5 -10 -14 $-4 \pm j4$	-0.5033 -36.2328	-0.5428 -35.0704	-0.5 $+4.4856$ $+5.2353$	-0.4763 $-2.1642 \pm j3.1913$
ω_{g1}(rad/s)	1.74	1.706	1.74	*	1.476
PM (°)	34.8300	37.2960	34.8300	*	60.8217
ω_{p1}(rad/s)	2.396	2.396	2.396	*	3.001
GM (dB)	3.0802	3.0802	3.0802	*	5.0574
E_M	0	2.2304 E − 2	1.4126 E − 2	*	3.4067 E − 1
IAE	0	8.6356 E − 2	3.8642 E − 2	*	1.3724 E + 0
ISE	0	7.8869 E − 4	2.0085 E − 4	*	2.6418 E − 1

* Unstable.

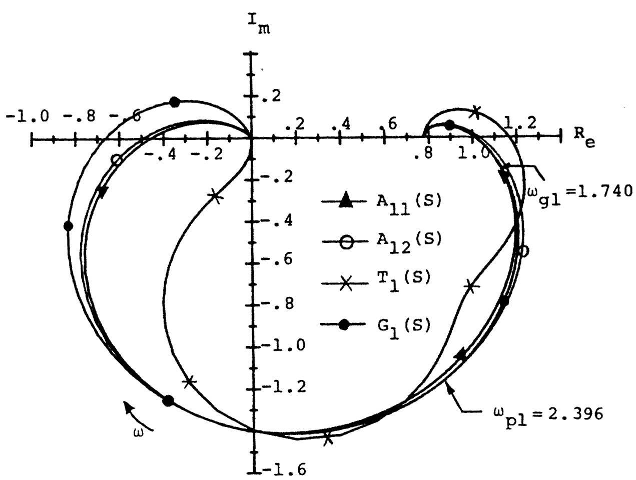

Fig. 2. Nyquist plots of $G_1(S)$ and its reduced models.

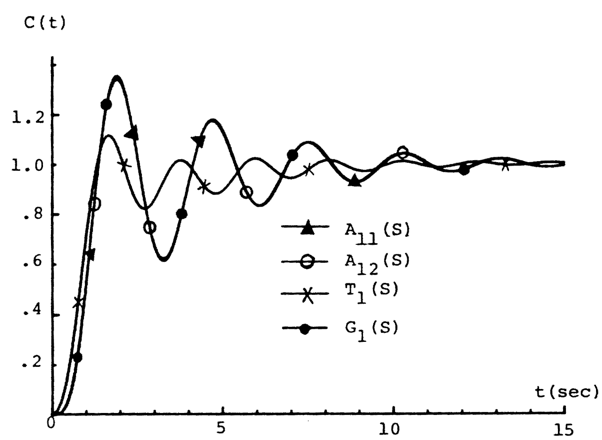

Fig. 3. Closed-loop unit step responses for system with $G_1(S)$ and its reduced models.

The cutoff, gain-crossover, and phase-crossover frequencies of the loop transmission function $L(S)$ are obtained as $\omega_{c2} = 0.613$ rad/s, $\omega_{g2} = 3.85$ rad/s, and $\omega_{p2} = 7.7155$ rad/s, respectively. As in Example 1, the results are shown in Figs. 4 and 5 and listed in Table II. It can be seen that $C_2(S)$ obtained by the continued-fraction method is the one with minimum error, but it is nonminimum phase. In general. the reduced models of the proposed method are better.

Example 3

Consider the flexible rocket control system shown in Fig. 6. The transfer functions are [12]

$$G_R(S) = \frac{7.21}{(S+1.6)(S-1.48)} \tag{31}$$

$$G_S(S) = \frac{2750}{S^2+42.2S+2750} \tag{32}$$

$$G_{SF}(S) = \frac{(S^2+70S+4000)(S^2+22S+12800)}{(S^2+30S+5810)(S^2+30S+12800)} \tag{33}$$

and the structure is defined as

$$\begin{aligned} T(S) = [&0.686(S+53)(S-53)(S^2-152.2S+14500) \\ &\cdot (S^2+153.8S+14500)]/[(S^2+S+605) \\ &\cdot (S^2+45.5S+2660)(S^2+2.51S+3900) \\ &\cdot (S^2+3.99S+22980)]. \end{aligned} \tag{34}$$

The gain-crossover frequency (ω_{g3}) and the phase-crossover frequencies (ω_{p3} and ω'_{p3}) of the open-loop transfer function $F(S)(=\theta_t(S)/E(S))$ are found at $\omega_{g3} = 2.815$ rad/s, $\omega_{p3} = 21.211$ rad/s, and $\omega'_{p3} = 55.25$ rad/s, respectively. As in Example 1, the results are shown in Table III. It can be seen that the reduced models of the proposed method are better and

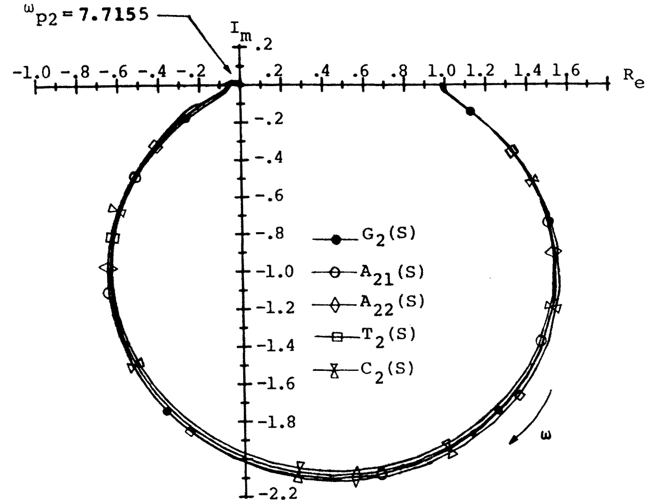

Fig. 4. Nyquist plots of $G_2(S)$ and its reduced models.

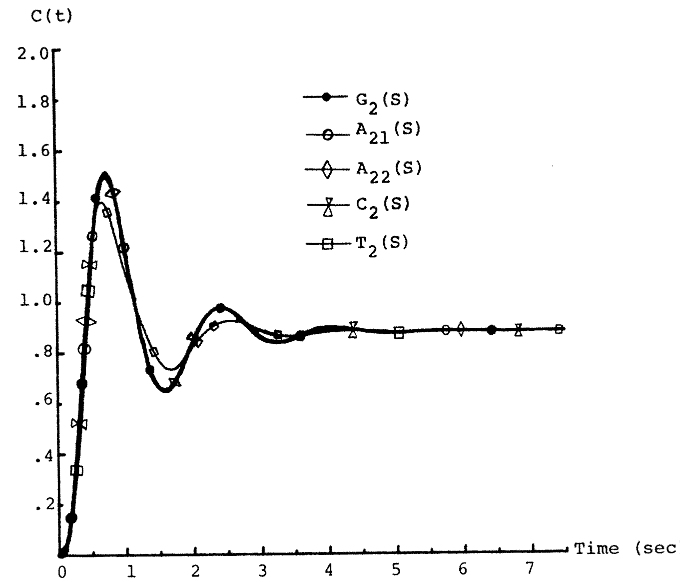

Fig. 5. Closed-loop unit step responses for system with $G_2(S)$ and its reduced models.

TABLE II

Items \ Models	Original $G_2(S)$	Lin and Han's Method $A_{21}(S)$ $(\omega_{c2})R[2, 5]_2^6$	$A_{22}(S)$ $(\frac{\omega_{g2}}{\omega_{p2}})R[2, 5]_2^4$	Continued-Fraction Method $C_2(S)$	Routh Stability Array Method $T_2(S)$
Poles	-1.8972 -49.3777 -52.5174 $-0.2728 \pm j1.0429$ $-3.8511 \pm j9.6527$	-1.9018 $-0.2728 \pm j1.0429$ $-3.3702 \pm j8.8092$	-2.2219 $-0.2743 \pm j1.0399$ $-3.2935 \pm j9.4443$	-1.8972 $-0.2728 \pm j1.0429$ $-3.8811 \pm j9.6608$	-1.7321 $-0.2664 \pm j1.0500$ $-2.9808 \pm j8.9687$
Zeros	-1.4706 -6.1350 -46.7248	-1.4714 -6.7267	-1.5949 -8.3011	-1.4706 -6.1396 $+36.9878 \pm j53.5303$	-1.4706 -6.1350 -46.7248
ω_{c2}(rad/s)	0.613	0.613	0.613	0.613	0.619
ω_{g2}(rad/s)	3.85	3.861	3.85	3.85	3.798
PM (°)	24.585	24.565	24.585	24.585	31.620
ω_{p2}(rad/s)	7.7155	7.371	7.7155	7.721	8.4
GM (dB)	7.554	6.987	7.554	7.563	7.541
E_M	0	2.3061 E − 2	9.7957 E − 3	4.7750 E − 4	1.3213 E − 1
IAE	0	2.2844 E − 2	1.6951 E − 2	3.6338 E − 4	1.9254 E − 1
ISE	0	2.6285 E − 4	8.6641 E − 5	1.0509 E − 7	1.3196 E − 2

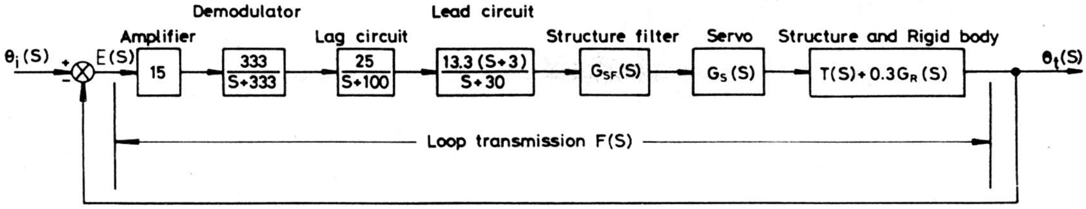

Fig. 6. Block diagram of flexible rocket control system.

TABLE III

Items \ Models	Original $T(S)$	Lin and Han's Method $A_{31}(S)$ $(\omega_{g3})R[4,5]_0^8(S)$	$A_{32}(S)$ $(\frac{\omega'_{p3}}{\omega_{p3}})R[4,5]_0^6(S)$	Continued-Fraction Method $C_3(S)$	Routh Stability Array Method $T_3(S)$
Poles	$-0.5 \pm j24.5917$ $-22.75 \pm j46.2865$ $-1.255 \pm j62.4374$ $-1.995 \pm j151.578$	-41.2593 $-0.487 \pm j24.6094$ $-12.7017 \pm j47.831$	-4.1813 $-0.5644 \pm j24.573$ $-9.1847 \pm j58.551$	-41.3000 $-0.4850 \pm j24.6103$ $-12.700 \pm j47.8175$	-161.1435 $-0.3674 \pm j24.7273$ $-1.0052 \pm j61.6964$
Zeros	$-53, +53$ $+76.1 \pm j93.3209$ $-76.9 \pm j92.6628$	$+61.5494$ $+67.2837$ $-40.7288 \pm j6.5652$	-4.1813 -40.4167 $+45.4054 \pm j19.635$	61.3836 67.5164 $-40.75 \pm j9.3849$	$-53, +53$ -150.2440 $+157.0873$
ω_{g3}(rad/s)	2.815	2.815	2.815	2.815	2.815
PM (°)	36.9431	36.9431	36.9431	36.9430	36.9689
ω_{p3}(rad/s)	21.211	21.219	21.211	21.220	23.315
GM (dB)	6.9073	6.8959	6.9073	6.8835	0.5601
ω'_{p3}(rad/s)	55.25	50.528	55.25	50.522	61.374
GM′ (dB)	7.3394	12.3808	7.3394	12.3732	−7.3560
E_M	0	6.7112 E − 2	4.7497 E − 2	6.6939 E − 2	*
IAE	0	3.9819 E − 2	3.5337 E − 2	3.9843 E − 2	*
ISE	0	1.1764 E − 3	7.4402 E − 4	1.1769 E − 3	*

* Unstable.

that the reduced model $T_3(S)$ produced by the stability array method makes the sytem unstable.

IV. Conclusion

The effects of model reduction on system characteristics such as gain margin, phase margin, and bandwidth have been illustrated. Applications of the proposed method to preserve the aforementioned system characteristics have been presented. In comparison with the results of other methods given in current literature, the proposed method can in general give better results.

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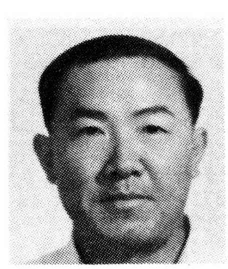

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