CHAPTER 3

UNCERTAINTY ANALYSIS

All of the data from experimental results may not be equally good to adopt. Their accuracy should be confirmed before the analyses of experimental results are carried out. Uncertainty analysis (or error analysis) is a procedure used to quantify data validity and accuracy [19.] Errors always are presented in experimental measuring. Experimental errors can be categorized into the fixed (systematic) error and random (non-repeatability) error, respectively [19]. Fixed error is the same for each reading and can be removed by proper calibration and correction. Random error is different for every reading and hence cannot be removed. The objective of uncertainty analysis is to estimate the probable random error in experimental results.

From the viewpoint of reliable estimation, it can be categorized into single-sample and multi-sample experiments. If experiments could be repeated enough times by enough observers and diverse instruments, then the reliability of the results could be assured by the use of statistics [20]. Like such, repetitive experiments would be called multi-sample ones. Experiments of the type, in which uncertainties are not found by repetition because of time and costs, would be called single-sample experiments.

3.1 Analyses of the Propagation of Uncertainty in Calculations

Uncertainty analysis is carried out here to estimate the uncertainty levels in the experiment. Formulas for evaluating the uncertainty levels in the experiment can be found in many papers [20, 21] and textbooks [19, 22, 23]. They are presented as follows:

Suppose that there are n independent variables, $x_1, x_2, \ldots, x_n$, of experimental measurements, and the relative uncertainty of each independently measured quantity is estimated as u_i. The measurements are used to calculate some experimental result, R, which is a function of independent variables, $x_1, x_2, \ldots, x_n$; $R = R(x_1,\ x_2\ ,\ldots,\ x_n)$.

An individual x_i, which affects error of R, can be estimated by the deviation of a function. A variation, δx_i, in x_i would cause R to vary according to

$$\delta R_i = \frac{\partial R}{\partial x_i}\delta x_i\,. \tag{3.1}$$

Normalize above equation by dividing R to obtain

$$\frac{\delta R_i}{R} = \frac{1}{R}\frac{\partial R}{\partial x_i}\delta x_i = \frac{x_i}{R}\frac{\partial R}{\partial x_i}\frac{\delta x_i}{x_i} \tag{3.2}$$

Eq. (3.2) can be used to estimate the uncertainty interval in the result due to the variation in x_i. Substitute the uncertainty interval for x_i,

$$u_{R_i} = \frac{x_i}{R}\frac{\partial R}{\partial x_i}u_{x_i} \tag{3.3}$$

To estimate the uncertainty in R due to the combined effects of uncertainty intervals in all the x_i's, it can be shown that the best representation for the uncertainty interval of the result is [21]

$$u_R = \pm\left[\left(\frac{x_1}{R}\frac{\partial R}{\partial x_1}u_1\right)^2 + \left(\frac{x_2}{R}\frac{\partial R}{\partial x_2}u_2\right)^2 + \ldots + \left(\frac{x_n}{R}\frac{\partial R}{\partial x_n}u_n\right)^2\right]^{1/2} \tag{3.4}$$

3.2 Uncertainty Level Analysis in the Experiment

Several parameters are selected to demonstrate the process of uncertainty level analyses as follows.

The cross-section area of exhaust fan, A_h, is

$A = a \times b$, $a = 800 \pm 0.5\,mm$, $b = 600 \pm 0.5mm$

$A = A(a,b)$

$$u_A = \pm\left[\left(\frac{a}{A}\frac{\partial A}{\partial a}u_a\right)^2 + \left(\frac{b}{A}\frac{\partial A}{\partial b}u_b\right)^2\right]^{1/2} = \pm\left[(u_a)^2 + (u_b)^2\right]^{1/2} = \pm 0.00832$$

$$(u_a = \frac{0.5}{800} = 0.000625,\ u_b = \frac{0.5}{600} = 0.0083)$$

The surface area of pool, A_{Pool}, is

$A = a \times b$, $a = 1000 \pm 0.5mm$, $b = 1000 \pm 0.5mm$

$A = A(a,b)$

$$u_A = \pm\left[\left(\frac{a}{A}\frac{\partial A}{\partial a}u_a\right)^2 + \left(\frac{b}{A}\frac{\partial A}{\partial b}u_b\right)^2\right]^{1/2} = \pm\left[(u_a)^2 + (u_b)^2\right]^{1/2} = \pm 0.000707$$

$$(u_a = \frac{0.5}{1000} = 0.0005,\ u_b = \frac{0.5}{1000} = 0.0005)$$

3.3 The asymmetric uncertainties of thermocouple

Room temperatures are measured by a 1mm diameter K-typed thermocouple, whose signals are sent to a PC-record (Ethernet). The accuracy of the thermocouple itself without coating is ±0.2%. Due to the effects of conduction, convection, and radiation, it is worthwhile to check the correctness of gas temperature measured by such K-typed thermocouple. Via an application of energy balance, i.e.,

Energy in = Energy out, or

Convection to the junction of thermocouple = Radiation from the junction of thermocouple + Conduction loss from the probe

Because of the fine thermocouple (1mm), the conduction term can be neglected. Then, the steady-state energy equation can be rewritten as follows [30].

$$\frac{4h}{D_w}(T_g - T_t) - \frac{4\sigma}{D_w}(\varepsilon T_t^4 - \varphi T_w^4) = 0 \tag{3.5}$$

In practice, the flame temperature is usually above $1000^\circ C$, so the absorption term, φT_w^4, from the relatively low wall temperature can be removed from Eq. (3.5). According to Eq. (3.5), the expression of correlation is given as:

$$T_g = T_t + \frac{\varepsilon\sigma T_t^4}{h} \tag{3.6}$$

where $T_g =$ the true gas temperature

$\varepsilon =$ emissivity of the thermocouple

$\sigma =$ Stefan Boltzmann constant

$h =$ convection heat transfer coefficient at thermocouple wire surface

The expressions of h and ε are mentioned in the paper [24]. Now, the analysis method of uncertainty can be utilized to obtain the uncertainty in the flame temperature from the correlation associated with h, T_t, and ε. The relationship between temperature and error is shown in Fig. 3.1.

3.4 Correction in response time of gas concentration

Combustion products would be moved into exhaust duct after the burner is ignited. Then, the gas is sucked into oxygen analyzer through sampling tube, but oxygen analyzer has a response time in itself. The response time results in an error of oxygen concentration. So acquired data can only be used after correction. The main reasons resulted in the time delay are as follows.

(1) Transmission time from sampling tube to oxygen analyzer.

(2) Response time of oxygen analyzer itself.

In order to solve this problem, the present study adopts the corrective formula as follows.

$$\frac{y(t)}{y(\infty)} = 0 \qquad \text{for } t < t_0$$

$$\frac{y(t)}{y(\infty)} = 1 - \exp\left[\frac{-(t - t_0)}{T}\right] \qquad \text{for } t \geq t_0$$

Then

$$X(t) = y(t) + Ty^{'}(t)$$

$X(t)$: true value

$y(t)$: measured value

$y^{'}(t)$: differential value of $y(t)$

t : time constant