梁在軸力及彎矩作用下之挫屈研究

研究生：陳致中　　　　　　　　　　　指導教授：蕭國模 博士

國立交通大學機械工程學系碩士班

摘　要

本文的主要目的在利用二階梁理論(Second order beam theory)共旋轉法(corotational formulation)來探討三維尤拉梁在軸力及不均勻彎矩同時作用下的非線性側向-扭轉挫屈分析。

在分析時將梁結構分成很多小段，每一小段稱為一個梁元素。然後在每個梁元素變形後的最新位置上建立一個元素座標系統，並在其上描述梁元素的變形、建立平衡方程式與構成方程式。用虛功原理與幾何非線性梁理論完全一致二次線性化推導出梁元素的平衡方程式與構成方程式。為了求得軸力及不均勻彎矩作用下梁的主要平衡路徑，在梁元素的靜力平衡關係中加上由不均勻彎矩所造成的垂直反作用力。然後利用梁元素的靜力平衡關係、平衡方程式、構成方程式和節點的相容條件，推導出梁在軸力及不均勻彎矩同時作用下的主要平衡路徑的統御方程式，並以數值解法求得主要平衡路徑之精確解。

本文為了求得梁的次要平衡路徑之統御方程式，先在主要平衡路徑加上擾動位移與擾動旋轉向量，元素座標受擾動後對應到新的位置上，並在受擾動後的元素座標上建立元素節點擾動位移與元素擾動旋轉參數間的關係。然後以一次線性化推導出梁的次要平衡路徑之統御方程式。接著以級數解求出空間梁在軸力及不均勻彎矩同時作用下所造成的挫屈彎矩。本文最後以數值例題驗證本文中提出的方法的正確性及有效性，並探討空間梁在不同軸力及不均勻彎矩作用下，對其挫屈彎矩的影響。

Buckling Analysis of Elastic Beam under Axial Force and Bending Moment

Student：Chih-Chung Chen Advisor：Dr. Kuo-Mo Hsiao

Department of Mechanical Engineering

National Chiao Tung University

ABSTRACT

In this study, a consistent co-rotational formulation of second order beam theory is employed for the nonlinear lateral-torsional buckling analysis of three-dimensional elastic Euler beam under axial force and non-uniform bending moment.

The beam structure is divided into several segments, called beam element for convenience. A set of element coordinate system is constructed at the current configuration of the deformed beam element. The deformations, differential equilibrium equations, and constitutive equations of the beam element are defined in the element coordinates. The principle of virtual work and the consistent second order linearization of the fully geometrically nonlinear beam theory are used to derive the differential equilibrium equations and constitutive equations of the beam element. To obtain the primary equilibrium path for the beam under axial force and non-uniform bending moment, the vertical reaction force included by the non-uniform bending moment is considered in the static equilibrium equations of the beam element. The governing equation for primary equilibrium path for beam under axial force and

non-uniform bending moment is derived using static equilibrium equations, differential equilibrium equations, constitutive equations and the compatibility conditions of the beam element. The exact solution of the primary path is solved using an analytical and numerical combined method.

To derive the governing equations for secondary equilibrium path, disturbing nodal displacement and rotation vectors are applied to the primary path of beam elements. Then element coordinates corresponding to this disturbance can be constructed, and element nodal rotation parameters defined in this element coordinates can be determined in terms of the disturbing nodal displacement and rotation vectors. The governing equations for secondary equilibrium path are derived in this element coordinates by using the first order linearization. A power series solution method is used to solve the buckling moment for spatial beams under axial force and non-uniform bending moment to demonstrate the accuracy and effectiveness of the proposed method. Numerical examples are studied to investigate the effect of compressive force and non-uniform bending moment on the buckling moment of spatial beams.