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Storm resampling for uncertainty analysis of a multiple-storm unit hydrograph

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Abstract

Due to various types of uncertainties involved in the estimation of a unit hydrograph (UH), the UH derived by any method is subject to uncertainties. Based on the concept of the bootstrap resampling technique, a practical methodology called storm resampling is proposed to quantify the uncertainties of multiple-storm UH ordinates and any parameters involved in the estimation of the multiple-storm UH. The important UH ordinates and parameters may include UH peak discharge, UH time-to-peak, UH volume, condition number related to the effective rainfall matrix, mean square error of the UH, and ridge parameter. The proposed bootstrap-based storm resampling technique, along with the least squares and ridge least squares solution techniques, is applied to typhoon storm events over a watershed in Taiwan. The methodology can be applied to other UH solution techniques and other hydrological and hydraulic simulation/optimization models. © 1997 Elsevier Science B.V.

1. Introduction

Hydraulic structures are subject to uncertainties. Uncertainties are the primary contributors to hydraulic structures failure. It is important to recognize the uncertainties involved in hydrologic and hydraulic analyses and designs. The uncertainties in hydrology and hydraulics may be attributed to the following sources: (1) natural uncertainties associated with the inherent randomness of natural processes; (2) model uncertainty reflecting the inability of the simulation model or design technique to represent precisely

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the system's true physical behavior; (3) model parameter uncertainties resulting from inability to quantify accurately the model inputs and parameters; (4) data uncertainties including measurement errors, inconsistency and non-homogeneity of data, and data handling and transcription errors; and (5) operational uncertainties including those associated with construction, manufacture, deterioration, maintenance, and other human factors that are not accounted for in the modeling or design procedure (Yen et al., 1986). It is important to not only recognize but also to quantify the uncertainties involved in hydrologic and hydraulic analyses and designs. Quantification of uncertainties provides the information needed for reliability analysis and risk-based design. Detailed discussions on uncertainty analysis in hydrological and hydraulic applications can be found in Tung and Yen (1993).

Like other hydrological and hydraulic models, the unit hydrograph (UH) model, which is one of the most widely used rainfall–runoff tools, is also subject to uncertainties. The UH theory is based on the assumptions that the surface watershed is a linear, lumped, and time-invariant system. However, most watershed systems are non-linear, time-variant, and spatially distributed. Failure to satisfy the assumptions may result in model uncertainty. Furthermore, measurement and data-processing errors in the effective rainfall hyetograph and direct runoff hydrograph lead to data uncertainties. In the process of deriving a UH, these uncertainties will be transmitted to the resulting UH. The uncertainties are indicated by the fact that UHs derived individually from different storm events vary from one storm to another.

Once a UH is derived, it is often used in conjunction with a design storm to calculate the design runoff hydrograph which is then routed through the hydraulic structure located downstream of the surface watershed. The routed hydrograph provides important information for the evaluation and modification of the hydraulic structure. Due to the presence of uncertainties in the UH, the resulting design runoff hydrograph is also subject to uncertainties, which could have important implications on the safety performance and the design of hydraulic structures. Therefore, quantification of uncertainties associated with the derived UH is essential. Uncertainty analysis for runoff prediction based upon a stochastic integral equation method has been presented by Hromadka et al. (1992). Also, the stochastic differential equation technique has been used (Sarino and Serrano, 1990; Hjelmfelt and Wang, 1994) to quantify uncertainties of an instantaneous UH based on Nash's model. Yang et al. (1992) have shown that the estimates of N and K in Nash's instantaneous UH model through using a non-linear programming method to fit storm events exhibit mutual correlation between N and K. It was also found that the estimates of N and K are non-normal random variables.

The conventional way to quantify uncertainties associated with the discrete-time UH is to apply the single-storm analysis by which the UH corresponding to each individual storm event is derived. Then, based on the available UHs from a number of storm events, statistical features for the UH, such as the mean and standard deviation, can be computed. However, many studies have indicated that the UH resulting from a multiple-storm analysis is more representative than the single-storm UH because of noise fluctuations (Bree, 1978a). Two theorems were proposed by Zhao et al. (1994) to support this claim.

However, the conventional statistical analysis cannot be directly applied to multiple-storm analysis because multiple-storm analysis uses all storms to derive only one UH.

Suppose there exist data of the effective rainfall hyetograph and direct runoff hydrograph for several, say R, storms. If the multiple-storm analysis is used along with a solution technique, such as the least squares method, then one can only obtain one UH. It is necessary to find a general and practical method in order to obtain the uncertainties such as mean, standard deviation, or higher moments of multiple-storm UH ordinates. This method should be also suitable to any solution technique when solving for the UH. In this paper, a storm resampling technique is proposed to compute the uncertainties for a multiple-storm UH. This technique is based on a relatively new but very powerful statistical resampling technique called bootstrap.

The bootstrap resampling technique is revolutionary for the following reasons given by Efron and Tibshirani (1993): (1) it allows the data analyst to assess the statistical accuracy of complicated procedures by exploiting the power of the computer (no matter how mathematically complicated the procedures are); (2) it relieves the analyst from having to do complex mathematical derivations; (3) it relieves the analyst from having to make parametric assumptions about the form of the underlying population; (4) it provides more accurate answers than textbook formulas; and (5) it can provide answers to problems for which no analytical answers can be obtained. Since the bootstrap resampling technique was first proposed by Efron (1977) as a computer-based simulation method for estimating the standard error of an estimate, it has attracted the attention of many researchers in the field of statistics. The only assumption for the bootstrap resampling technique is that the data points to be used are a random sample, which is the case for most observed data. Excellent descriptions about the bootstrap resampling technique and its applications can be found in Efron (1982) and Efron and Tibshirani (1993).

The basic idea of the bootstrap resampling technique is to extract more information from one observed random sample to better understand the population (or the statistics associated with the population) by resampling this observed random sample a very large number of times. Suppose there is an observed random sample with n data points. The conventional statistical analysis relies on the observed random sample in such a way that the statistics are directly computed from the sample and then an assumed probability distribution may be fitted to the data. However, the bootstrap resampling technique goes much beyond this because it extracts more information from this single observed random sample. This is achieved by randomly selecting n data points, with replacement, from the original observed random sample a large number of times, say 1000 times. (It must be pointed out ‘‘with replacement’’ mentioned in the previous sentence is crucial to the bootstrap resampling technique.) Then, 1000 new samples are obtained, each of which is called a bootstrap sample consisting of n data points. For each of the 1000 bootstrap samples, one can compute the value of any function of the observed random sample. Then, the statistics can be obtained. For example, mean is one of the commonly sought statistics. Thus, the mean value for each of the 1000 bootstrap samples is computed, resulting in 1000 values for the mean. Then, the standard deviation for the mean can be found by using these computed 1000 values.

Suppose there exist data of effective rainfall and direct runoff for several, say R, storms. Consider these R storms as an observed random sample in which each data point corresponds to the data of a storm. The proposed storm resampling technique is to randomly draw, with replacement, R storms from the original R storms to form a bootstrap storm sample. This drawing process is repeated a large number of times, say 1000 times. Then, one will obtain 1000 bootstrap storm samples. From each of the 1000 bootstrap storm samples, a multiple-storm UH is derived by any solution technique, resulting in 1000 multiple-storm UHs. Therefore, the uncertainties (mean, standard deviation, covariance matrix, confidence interval or higher moments) of the multiple-storm UH ordinates can be computed. More interestingly, the uncertainties of any parameter such as UH time-to-peak, condition number of the design matrix, and others in the process of computing a multiple-storm UH can also be found as well as those of the UH ordinates. The methodology can be applied to uncertainty analysis for other UH solution techniques.

This paper consists of five major parts: Section 1, introduction; Section 2, review of the least squares method and ridge least squares method for deriving a multiple-storm UH with/without storm-scaling technique; Section 3, step-by-step procedures for the proposed bootstrap-based storm resampling technique for quantifying the uncertainties associated with a multiple-storm UH and any other parameters; Section 4, application of the proposed uncertainty analysis methodology to typhoon storm events over a watershed in Taiwan; and Section 5, summary and conclusions. Pertinent mathematical equations for the general bootstrap algorithm and bootstrap-based confidence intervals can be found in Appendix A and Appendix B.

2. Estimating a multiple-storm UH by least squares and ridge least squares methods

This section is a review of the least squares and ridge least squares methods. The UH, proposed by Sherman (1932), is defined as the direct runoff hydrograph resulting from one unit of the effective rainfall distributed uniformly over a watershed for a specified duration. Based upon the UH theory, the surface watershed is considered as a linear, time-invariant, and lumped system with an effective rainfall hyetograph being the input, a direct runoff hydrograph being the output, and a UH being the transformation function. The input, output, and the transformation function can be related by the convolution relationship. In matrix form, the convolution relationship can be expressed as (Chow et al., 1988)

$$\boldsymbol{p}\boldsymbol{u}=\boldsymbol{q} \tag{1}$$

in which $\boldsymbol{u}$ is an $(N-M+1)\times 1$ column vector of UH ordinates and $\boldsymbol{q}$ is an $N\times 1$ column vector of direct runoff hydrograph ordinates and $\boldsymbol{p}$ is an $N \times (N - M + 1)$ matrix

$$\boldsymbol{p} = \begin{pmatrix} p_1 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ p_2 & p_1 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \cdot & p_2 & p_1 & \dots & 0 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & p_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{M-1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot \\ p_M & p_{M-1} & \cdot & \cdot & \cdot & \cdot & \cdot & P_1 & 0 \\ 0 & p_M & p_{M-1} & \cdot & \cdot & \cdot & \cdot & p_2 & p_1 \\ \cdot & 0 & p_M & \cdot & \cdot & \cdot & \cdot & \cdot & p_2 \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & p_{M-1} & \cdot \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & p_M & p_{M-1} \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & p_M \end{pmatrix}_{N \times (N-M+1)} \tag{2}$$

in which p_1, p_2,…, and p_M are the effective rainfall hyetograph ordinates.

There are different types of algorithms for solving Eq. (1) for the UH. They are linear programming method (Mays and Coles, 1980; Zhao and Tung, 1994) and non-linear programming techniques including the least squares methods (Diskin and Boneh, 1975; Bree, 1978a,b; Kitanidis and Bras, 1979; Mawdsley and Tagg, 1981; Morel-Seytoux, 1982; Mays and Taur, 1982; Singh et al., 1982; Bruen and Dooge, 1984; Singh, 1988; Dooge and Bruen, 1989; Nalbantis et al., 1995). Recently, Rao and Tirtotjondro (1995) applied a Bayesian method to UH derivation, and it was shown that the Bayesian method is a general method which can give UHs with few oscillations among the ordinates. In this paper, the discussions will be based on the widely used least squares and ridge least squares methods.

By the ordinary least squares (OLS) method, the UH can be obtained as

$$\boldsymbol{u}_{\mathrm{OLS}} = (\boldsymbol{p}^{\mathrm{t}}\boldsymbol{p})^{-1}\boldsymbol{p}^{\mathrm{t}}\boldsymbol{q} \tag{3}$$

in which the superscripts ‘t’ and ‘ − 1’ represent the transpose and inverse of a matrix, respectively. Eq. (3) is derived by looking for an optimal $\boldsymbol{u}$ such that the sum of the squared errors $(\boldsymbol{pu} - \boldsymbol{q})^{\mathrm{t}}(\boldsymbol{pu} - \boldsymbol{q})$ is minimized. This can be done by taking the first derivative of the sum of the squared errors with respect to $\boldsymbol{u}$, setting the result to zero, and solving for $\boldsymbol{u}$.

Eqs. (1)–(3) are associated with single-storm analysis in which the data of the effective rainfall hyetograph and direct runoff hydrograph of only one storm are used. If the data of more than one storm, say R storms, are available, then the matrix convolution equations representing R storms are

$$\begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{p}_R \end{pmatrix} \boldsymbol{u} = \begin{pmatrix} \boldsymbol{q}_1 \\ \boldsymbol{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{q}_R \end{pmatrix} \tag{4}$$

where $\boldsymbol{p}_{\mathbf{r}}$ and $\boldsymbol{q}_{\mathbf{r}}$ correspond to the rth storm and are defined as those in Eq. (1), and $\boldsymbol{u}$ is the multiple-storm UH.

In a more compact form, Eq. (4) becomes

$$\boldsymbol{Pu} = \boldsymbol{q} \tag{5}$$

where

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{p}_R \end{pmatrix}, \quad \boldsymbol{q} = \begin{pmatrix} \boldsymbol{q}_1 \\ \boldsymbol{q}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{q}_R \end{pmatrix} \tag{6}$$

Recall that the number of UH ordinates for a single storm, say the rth storm, is $J_r = N_r - M_r + 1$ which may vary with storms. Therefore, before the matrices and column vectors are stacked as in Eq. (4), adjustment for the direct runoff hydrographs must be made in order that the individual effective rainfall hyetograph matrices $\boldsymbol{p}_r$, $r = 1, 2, \ldots, R$, have the same number of columns. To make this adjustment, a suggestion has been made to let the number of the multiple-storm UH ordinates, J, be the maximal value of the individual UH ordinate number, that is, $J = \max\{J_1, J_2, \ldots, J_R\}$ (Diskin and Boneh, 1975; Bree, 1978a; Singh, 1988). Then, $J - J_r$ zeros are added to the end of the direct runoff hydrograph ordinates for the rth storm, $r = 1, 2, \ldots, R$.

Because Eq. (5) has the same mathematical format as Eq. (1), except that Eq. (5) has larger dimensions of matrix and vector, the ordinary least squares solution of the multiple-storm UH has the same mathematical format as the single-storm UH

$$\boldsymbol{u}_{\mathrm{OLS}} = (\boldsymbol{P}^{\mathrm{t}}\boldsymbol{P})^{-1}\boldsymbol{P}^{\mathrm{t}}\boldsymbol{q} \tag{7}$$

$$= \left(\sum_{r=1}^{R} \boldsymbol{p}_r^{\mathrm{t}} \boldsymbol{p}_r \right)^{-1} \left(\sum_{r=1}^{R} \boldsymbol{p}_r^{\mathrm{t}} \boldsymbol{q}_r \right)$$

in which the second step in Eq. (7) is derived by using Eq. (6).

Sometimes, the computed UH by the ordinary least squares method may have significant noise fluctuation among UH ordinates. In this case, the ridge least squares method can be applied. Ridge least squares essentially adds a positive number called the ridge parameter k to the diagonal of the design matrix $\boldsymbol{P}^{\mathrm{t}}\boldsymbol{P}$. The multiple-storm UH by ridge least squares is (Zhao et al., 1994)

$$\boldsymbol{u}_{\mathrm{RLS}} = (\boldsymbol{P}^{\mathrm{t}}\boldsymbol{P} + k\boldsymbol{I})^{-1}\boldsymbol{P}^{\mathrm{t}}\boldsymbol{q} \tag{8}$$

$$= \left(\sum_{r=1}^{R} \boldsymbol{p}_r^{\mathrm{t}}\boldsymbol{p}_r + k\boldsymbol{I}\right)^{-1} \left(\sum_{r=1}^{R} \boldsymbol{p}_r^{\mathrm{t}}\boldsymbol{q}_r\right)$$

in which $\boldsymbol{I}$ is an identity matrix with the same dimension as $\boldsymbol{P}^{\mathrm{t}}\boldsymbol{P}$.

The ridge parameter k in Eq. (8) can be determined based on one of two criteria: (1) minimization of the mean square error of the UH and (2) minimization of the mean square error of the direct runoff hydrograph. The mean square error measures the expected Euclidean distance between the estimated UH (or direct runoff hydrograph) and the true but unknown UH (or direct runoff hydrograph). After the optimal ridge parameter is found by a minimization technique, it is substituted into Eq. (8) to compute the corresponding UH. For the purpose of demonstration, ridge analysis by minimizing the mean square error of the UH is used herein. Detailed equations for the mean square error of the UH can be found in Zhao et al. (1994).

In the framework of multiple-storm analysis, storms with larger effective rainfalls and direct runoffs will dominate the derived UH, leaving smaller storms with little contribution to the determination of the UH. This could lead to a potentially biased estimation of the UH. Zhao et al. (1994) proposed a storm-scaling technique to reduce such a bias. The storm-scaling technique simply divides the effective rainfall hyetograph and direct runoff hydrograph ordinates by the effective rainfall amount of the corresponding storm as

$$\tilde{\boldsymbol{p}}_r = \frac{\boldsymbol{p}_r}{\boldsymbol{1}^{\mathrm{t}}\boldsymbol{p}_r}, \quad r = 1, 2, \ldots, R \tag{9}$$

$$\tilde{\boldsymbol{q}}_r = \frac{\boldsymbol{q}_r}{\boldsymbol{1}^{\mathrm{t}}\boldsymbol{p}_r}, \quad r = 1, 2, \ldots, R \tag{10}$$

in which $\boldsymbol{1}$ is the column vector with all elements being ones. After the effective rainfall hyetograph ordinates and direct runoff hydrograph ordinates are scaled, they will replace the original hyetograph and hydrograph ordinates, and will be used in the setup of the convolution matrix form. It should be pointed out that very small storms may not be included in the derivation of a multiple-storm UH when the UH is intended for use in hydraulic structures design, since very small storms may only cover limited portions of the watershed and have quite different hydrologic processes from large storms.

3. Bootstrap-based storm resampling for multiple-storm UH uncertainty analysis

Consider R storms as an observed random sample $\boldsymbol{S} = (S_1, S_2, \ldots, S_R)$ in which S_r corresponds to the data of the effective rainfall hyetograph and direct runoff hydrograph for the rth storm, $r = 1, 2, \ldots, R$. The step-by-step procedures for the proposed bootstrap-based storm resampling are as follows:

1. Randomly draw R storms from $\boldsymbol{S}$, with replacement, to yield $\boldsymbol{S}^* = (S_1^*, S_2^*, ..., S_R^*)$, which is called the bootstrap storm sample. For example, suppose there are ten storms which form $\boldsymbol{S}$, the original storm sample. A storm is randomly drawn from $\boldsymbol{S}$ (each storm has an equal chance to be drawn), and is denoted by S_1^*. Then, the drawn storm is returned to $\boldsymbol{S}$ so that $\boldsymbol{S}$ still consists of the original observed ten storms. Then, another storm is randomly drawn from $\boldsymbol{S}$ and is denoted by S_2^*. The drawn storm is then returned to $\boldsymbol{S}$. This process of 'drawing and returning' is repeated ten times to yield the bootstrap storm sample $\boldsymbol{S}^* = (S_1^*, S_2^*, \ldots, S_{10}^*)$. It may be noted that a storm in the original storm sample $\boldsymbol{S}$ may be drawn more than once (because it is returned after it is drawn), and thus may appear more than once in the bootstrap storm sample $\boldsymbol{S}^*$. It is also possible that a storm is never drawn, and thus does not appear in the bootstrap storm sample. 'Returning' the randomly drawn storm to the original storm sample is what statisticians usually call 'with replacement.'
2. Use a UH solution technique to calculate a multiple-storm UH* based on the R storms in the bootstrap storm sample $\boldsymbol{S}^*$. Other parameters corresponding to the multiple-storm UH* can also be computed. These parameters may include UH time-to-peak, UH volume, condition number of the design matrix $\boldsymbol{P}^t\boldsymbol{P}$, MSE of the UH*, and the ridge parameter. These parameters form a parameter vector $\boldsymbol{PARA}^*$. Consider the example in Step 1. Calculate a multiple-storm UH* and $\boldsymbol{PARA}^*$ based on the ten storms in the bootstrap storm sample $\boldsymbol{S}^* = (S_1^*, S_2^*, \ldots, S_{10}^*)$ by using a solution technique such as least squares or ridge least squares with/without the storm-scaling technique. It should be noted that any solution technique can be used, and any other parameters can be appended to $\boldsymbol{PARA}^*$.
3. Repeat Steps 1 and 2 a large number of times, say B, to obtain UH^{*1}, UH^{*2}, …, UH^{*B} and $\boldsymbol{PARA}^{*1}, \boldsymbol{PARA}^{*2}, \ldots, \boldsymbol{PARA}^{*B}$. For example, suppose $B = 1000$. Then, 1000 UHs are computed along with 1000 ***PARA***s.
4. Calculate the statistics such as the mean, standard deviation, skew coefficient, and confidence intervals for each ordinate of the UH and ***PARA*** over 1000 values from Step 3. Covariance matrices for the UH and ***PARA*** can also be computed. The formulas for computing the mean, standard deviation, covariance matrix, and skew coefficient can be found in any statistics textbooks. The confidence intervals can be computed by the simple percentile method. Suppose a 90% confidence interval for a UH ordinate is of interest. Increasingly rank the 1000 values of the ordinate. Solve $1 - 2\alpha = 90\%$ for α, which is found to be $\alpha = 0.05$. Then, the lower and upper limits of the 90% confidence interval is found from the ranked 1000 values. The lower limit is chosen such that $100\alpha\%$ or 5% of the 1000 values are smaller than it. The upper limit of the 90% confidence interval is chosen such that $100(1 - \alpha)\%$ or 95% of the 1000 values are smaller than it. Detailed mathematical formulas for the percentile method and other confidence interval methods can be found in Appendix B.

It should be pointed out that the bootstrap technique requires a considerable amount of computation time because a large number of bootstrap samples are involved. Efron (1982) suggested that $B = 200$ is generally large enough for estimating the standard deviation of the statistics. $B = 1000$ or more would be required for estimating the confidence intervals with reasonable accuracy. However, the concern about the computation time is diminishing as the power of computers is greatly enhanced.

Two basic types of bootstrap resampling technique have been developed: unbalanced bootstrap and balanced bootstrap (Davison et al., 1986; Gleason, 1988). The above step-by-step procedures for the storm resampling technique are based on the unbalanced bootstrap. The balanced bootstrap resampling reuses each of the original observed storms exactly equally often. An efficient algorithm was given by Davison et al. (1986). The advantage of balanced bootstrap over unbalanced bootstrap is found in bias and variance reduction.

Table 1
Summary statistics of derived UH and its properties using unbalanced and balanced bootstrap techniques by ordinary least squares without storm-scaling for Tong-Tou watershed

	Unbalanced bootstrap			Balanced bootstrap		
	Mean	SD	Skew	Mean	SD	Skew
UH(1)	7.805e + 00	1.834e + 00	2.694e − 01	7.839e + 00	1.844e + 00	2.547e − 01
UH(2)	6.876e + 00	1.038e + 00	1.114e − 01	6.864e + 00	1.042e + 00	1.797e − 01
UH(3)	2.076e + 00	5.695e − 01	− 3.293e − 01	2.065e + 00	5.783e − 01	− 2.898e − 01
UH(4)	9.513e − 01	4.253e − 01	8.277e − 01	9.463e − 01	4.282e − 01	9.849e − 01
UH(5)	1.519e + 00	3.377e − 01	6.550e − 01	1.525e + 00	3.422e − 01	7.415e − 01
UH(6)	1.310e + 00	3.211e − 01	5.122e − 01	1.314e + 00	3.217e − 01	5.568e − 01
UH(7)	7.252e − 01	1.536e − 01	2.355e − 01	7.198e − 01	1.543e − 01	6.519e − 02
UH(8)	9.343e − 01	1.965e − 01	− 1.962e − 01	9.253e − 01	2.042e − 01	− 1.906e − 01
UH(9)	5.043e − 01	1.359e − 01	− 6.269e − 01	5.049e − 01	1.348e − 01	− 4.504e − 01
UH(10)	5.942e − 01	9.470e − 02	− 2.737e − 01	5.916e − 01	9.575e − 02	− 5.120e − 01
UH(11)	2.525e − 01	1.602e − 01	− 5.062e − 01	2.543e − 01	1.631e − 01	− 5.467e − 01
UH(12)	3.394e − 01	7.979e − 02	1.243e + 00	3.401e − 01	8.094e − 02	1.378e + 00
UH(13)	1.588e − 01	7.442e − 02	6.686e − 02	1.566e − 01	7.814e − 02	3.980e − 01
UH(14)	1.922e − 01	5.148e − 02	1.717e + 00	1.930e − 01	5.420e − 02	2.141e + 00
UH(15)	1.334e − 01	4.070e − 02	− 1.482e + 00	1.328e − 01	4.241e − 02	− 1.527e + 00
UH(16)	1.260e − 01	2.893e − 02	1.759e + 00	1.262e − 01	3.092e − 02	1.932e + 00
UH(17)	6.940e − 02	2.242e − 02	− 4.151e − 01	6.925e − 02	2.487e − 02	− 1.491e + 00
UH(18)	6.229e − 02	1.723e − 02	9.939e − 02	6.199e − 02	1.904e − 02	− 3.859e − 02
UH(19)	2.314e − 02	1.494e − 02	− 6.092e − 01	2.289e − 02	1.519e − 02	− 5.416e − 01
UH(20)	− 6.450e − 03	8.618e − 03	− 2.947e − 01	− 6.512e − 03	8.530e − 03	− 4.454e − 01
Peak	8.506e + 00	1.262e + 00	7.771e − 01	8.526e + 00	1.287e + 00	6.834e − 01
TP	4.164e + 00	1.462e + 00	4.525e − 01	4.123e + 00	1.452e + 00	5.115e − 01
Vol	1.027e + 00	1.498e − 02	− 1.605e + 02	1.027e + 00	1.518e − 02	− 1.543e + 02
Cond	9.288e + 01	6.091e + 01	2.444e + 00	9.435e + 01	6.082e + 01	1.995e + 00
MSE	7.786e + 00	3.603e + 00	1.150e + 00	7.816e + 00	3.503e + 00	1.020e + 00
Ridge	0.00e + 00	0.000e + 00	0.000e + 00	0.000e + 00	0.000e + 00	0.000e + 00

UH(i), the ith ordinate for a multiple-storm UH.
TP, time-to-peak for a multiple-storm UH.
Vol, volume of a multiple-storm UH.
Cond, 2-norm condition number (defined as the ratio of the largest eigenvalue to the smallest eigenvalue of $\boldsymbol{P}^t\boldsymbol{P}$ or $\boldsymbol{P}^t\boldsymbol{P}+k\boldsymbol{I}$).
MSE, mean square error of a multiple-storm UH.
Ridge, ridge parameter (k).
SD, standard deviation.

Much effort has been devoted to the computation of confidence intervals by the bootstrap resampling technique. Generally speaking, there are three bootstrap-based methods for computing the confidence intervals, which are the ‘normality’ method, percentile method, and bias-corrected percentile method (Efron and Tibshirani, 1986; Efron, 1987). The percentile method and bias-corrected percentile method are better than the ‘normality’ method. The percentile method is the most straightforward method. The bias-corrected percentile method is more powerful than the percentile method because it can correct the possible bias.

4. Application and discussion

The proposed storm resampling technique was applied to nine typhoon storm events in Tong-Tou watershed (259.2 km^2) in Taiwan, which occurred from 1970 to 1981. The peak discharges of direct runoff hydrographs varied from 955 m^3 s^{-1} to 3149 m^3 s^{-1}. The

Table 2
Summary statistics of derived UH and its properties using unbalanced and balanced bootstrap techniques by ordinary least squares with storm-scaling for Tong-Tou watershed

	Unbalanced bootstrap			Balanced bootstrap		
	Mean	SD	Skew	Mean	SD	Skew
UH(1)	7.635e + 00	1.095e + 00	– 4.991e – 01	7.646e + 00	1.116e + 00	– 5.560e – 01
UH(2)	6.296e + 00	6.792e – 01	1.199e + 00	6.302e + 00	7.065e – 01	1.346e + 00
UH(3)	2.390e + 00	3.974e – 01	– 6.233e – 01	2.376e + 00	4.134e – 01	– 5.461e – 01
UH(4)	1.237e + 00	4.999e – 01	– 7.786e – 01	1.224e + 00	5.106e – 01	– 6.470e – 01
UH(5)	1.633e + 00	5.089e – 01	1.028e + 00	1.647e + 00	5.149e – 01	1.052e + 00
UH(6)	1.394e + 00	2.686e – 01	– 1.188e + 00	1.399e + 00	2.686e – 01	– 1.090e + 00
UH(7)	8.284e – 01	1.462e – 01	– 6.313e – 02	8.219e – 01	1.573e – 01	4.991e – 01
UH(8)	9.035e – 01	1.374e – 01	3.614e – 01	8.959e – 01	1.455e – 01	6.897e – 02
UH(9)	5.928e – 01	1.062e – 01	– 1.561e + 00	5.944e – 01	1.034e – 01	– 1.628e + 00
UH(10)	5.507e – 01	9.210e – 02	1.062e + 00	5.461e – 01	9.329e – 02	2.115e – 01
UH(11)	3.101e – 01	9.906e – 02	– 1.844e + 00	3.094e – 01	1.036e – 01	– 1.854e + 00
UH(12)	3.621e – 01	6.062e – 02	1.606e + 00	3.639e – 01	6.316e – 02	2.269e + 00
UH(13)	2.196e – 01	5.585e – 02	– 9.580e – 01	2.178e – 01	5.896e – 02	– 5.218e – 02
UH(14)	2.107e – 01	3.580e – 02	1.831e + 00	2.115e – 01	4.092e – 02	4.030e + 00
UH(15)	1.578e – 01	3.107e – 02	– 3.076e + 00	1.562e – 01	3.299e – 02	– 3.172e + 00
UH(16)	1.373e – 01	2.005e – 02	2.702e + 00	1.375e – 01	2.218e – 02	3.139e + 00
UH(17)	8.293e – 02	1.838e – 02	– 1.070e + 00	8.288e – 02	2.214e – 02	– 4.964e + 00
UH(18)	6.784e – 02	1.582e – 02	– 1.297e – 01	6.743e – 02	1.813e – 02	– 2.793e – 01
UH(19)	3.173e – 02	1.408e – 02	– 8.536e – 01	3.132e – 02	1.473e – 02	– 7.733e – 01
UH(20)	– 3.102e – 04	7.325e – 03	– 5.969e – 01	– 3.088e – 04	7.051e – 03	– 7.077e – 01
Peak	7.870e + 00	8.383e – 01	9.292e – 03	7.897e + 00	8.491e – 01	– 4.660e – 02
TP	3.516e + 00	1.132e + 00	1.728e + 00	3.510e + 00	1.127e + 00	1.747e + 00
Vol	1.043e + 00	1.990e – 02	– 7.169e + 01	1.043e + 00	1.953e – 02	– 7.583e + 01
Cond	5.430e + 01	5.976e + 01	2.719e + 00	5.494e + 01	5.897e + 01	2.605e + 00
MSE	5.394e + 00	3.843e + 00	2.146e + 00	5.370e + 00	3.704e + 00	2.085e + 00
Ridge	0.000e + 00	0.000e + 00	0.000e + 00	0.000e + 00	0.000e + 00	0.000e + 00

See footnotes to Table 1.

Table 3
Summary statistics of a derived UH and its properties using unbalanced and balanced bootstrap techniques by ridge least squares without storm-scaling for the Tong-Tou watershed

	Unbalanced bootstrap			Balanced bootstrap		
	Mean	SD	Skew	Mean	SD	Skew
UH(1)	7.318e + 00	1.474e + 00	4.257e – 01	7.342e + 00	1.481e + 00	3.931e – 01
UH(2)	6.366e + 00	8.040e – 01	9.045e – 01	6.363e + 00	8.121e – 01	9.552e – 01
UH(3)	2.495e + 00	4.854e – 01	– 8.572e – 01	2.482e + 00	4.891e – 01	– 8.277e – 01
UH(4)	1.073e + 00	3.202e – 01	7.932e – 01	1.070e + 00	3.228e – 01	9.912e – 01
UH(5)	1.349e + 00	2.493e – 01	1.223e + 00	1.352e + 00	2.538e – 01	1.381e + 00
UH(6)	1.244e + 00	2.394e – 01	7.345e – 01	1.246e + 00	2.419e – 01	7.291e – 01
UH(7)	8.335e – 01	1.186e – 01	– 5.113e – 02	8.290e – 01	1.227e – 01	– 9.002e – 02
UH(8)	8.252e – 01	1.620e – 01	2.941e – 01	8.184e – 01	1.654e – 01	2.678e – 01
UH(9)	5.608e – 01	9.517e – 02	– 1.045e + 00	5.589e – 01	9.651e – 02	– 9.330e – 01
UH(10)	5.148e – 01	6.377e – 02	– 7.968e – 02	5.140e – 01	6.497e – 02	– 3.233e – 01
UH(11)	3.080e – 01	1.089e – 01	– 1.036e + 00	3.087e – 01	1.109e – 01	– 1.089e + 00
UH(12)	2.973e – 01	5.200e – 02	1.660e + 00	2.979e – 01	5.361e – 02	1.863e + 00
UH(13)	1.839e – 01	5.632e – 02	– 6.915e – 02	1.825e – 01	5.929e – 02	3.534e – 01
UH(14)	1.716e – 01	4.020e – 02	1.276e + 00	1.718e – 01	4.295e – 02	1.817e + 00
UH(15)	1.416e – 01	2.447e – 02	– 1.061e + 00	1.413e – 01	2.601e – 02	– 1.004e + 00
UH(16)	1.140e – 01	1.921e – 02	1.128e + 00	1.140e – 01	2.058e – 02	1.480e + 00
UH(17)	7.716e – 02	1.464e – 02	5.180e – 02	7.717e – 02	1.666e – 02	– 1.432e + 00
UH(18)	5.495e – 02	1.255e – 02	– 1.560e + 00	5.460e – 02	1.453e – 02	– 1.193e + 00
UH(19)	2.417e – 02	1.128e – 02	– 1.002e + 00	2.385e – 02	1.164e – 02	– 9.535e – 01
UH(20)	– 3.837e – 03	5.857e – 03	– 6.023e – 01	– 3.890e – 03	5.792e – 03	– 6.992e – 01
Peak	7.758e + 00	1.152e + 00	7.997e – 01	7.779e + 00	1.167e + 00	6.929e – 01
TP	3.948e + 00	1.395e + 00	7.846e – 01	3.906e + 00	1.377e + 00	8.555e – 01
Vol	9.979e – 01	2.018e – 02	– 6.031e + 01	9.978e – 01	2.037e – 02	– 5.867e + 01
Cond	3.537e + 01	2.281e + 01	4.459e + 00	3.575e + 01	2.301e + 01	4.526e + 00
MSE	3.545e + 00	9.040e – 01	6.511e – 01	3.555e + 00	8.927e – 01	6.610e – 01
Ridge	1.860e + 04	7.466e + 03	9.230e – 02	1.861e + 04	7.427e + 03	9.340e – 02

See footnotes to Table 1.

duration of the UH was 3 h. The number of bootstrap samples was chosen to be $B = 1000$. Both unbalanced and balanced bootstrap techniques were used. For each bootstrap sample, the multiple-storm analysis was implemented to derive a multiple-storm UH. The solution methods for determining the multiple-storm UH were the ordinary least squares and ridge least squares methods with/without storm-scaling.

4.1. Uncertainty of multiple-storm UH ordinates

The uncertainties of UH ordinates derived by different methods are given on the upper parts of Tables 1–4. Comparing the results for UH ordinates in Tables 1–4 between the unbalanced and balanced bootstrap techniques, it can be found that there is no significant difference between the unbalanced and balanced bootstrap techniques in quantifying the uncertainties for UH ordinates. Figs 1–3 show, respectively, the mean, standard deviation, and skew coefficient of multiple-storm UHs derived from the unbalanced bootstrap procedure along with the ordinary least squares method and ridge least squares method

Table 4
Summary statistics of a derived UH and its properties using unbalanced and balanced bootstrap techniques by ridge least squares with storm-scaling for Tong-Tou watershed

	Unbalanced bootstrap			Balanced bootstrap		
	Mean	SD	Skew	Mean	SD	Skew
UH(1)	7.147e + 00	9.502e – 01	– 1.226e – 01	7.154e + 00	9.573e – 01	– 2.165e – 01
UH(2)	5.902e + 00	4.959e – 01	1.311e + 00	5.914e + 00	5.291e – 01	1.588e + 00
UH(3)	2.586e + 00	3.035e – 01	– 5.784e – 01	2.571e + 00	3.114e – 01	– 6.435e – 01
UH(4)	1.378e + 00	3.127e – 01	– 4.695e – 01	1.370e + 00	3.200e – 01	– 2.681e – 01
UH(5)	1.483e + 00	3.116e – 01	8.040e – 01	1.495e + 00	3.206e – 01	1.012e + 00
UH(6)	1.385e + 00	1.832e – 01	– 4.122e – 01	1.389e + 00	1.848e – 01	– 3.314e – 01
UH(7)	8.768e – 01	9.410e – 02	– 7.068e – 01	8.707e – 01	1.033e – 01	3.250e – 02
UH(8)	8.228e – 01	9.713e – 02	3.642e – 01	8.175e – 01	1.019e – 01	9.237e – 02
UH(9)	6.107e – 01	5.581e – 02	– 1.492e + 00	6.092e – 01	5.855e – 02	– 2.687e + 00
UH(10)	5.080e – 01	5.143e – 02	6.844e – 02	5.061e – 01	5.592e – 02	– 2.049e + 00
UH(11)	3.379e – 01	6.209e – 02	– 2.042e + 00	3.366e – 01	6.669e – 02	– 2.149e + 00
UH(12)	3.335e – 01	3.727e – 02	6.814e – 01	3.349e – 01	4.102e – 02	1.711e + 00
UH(13)	2.301e – 01	3.862e – 02	– 7.266e – 01	2.290e – 01	4.184e – 02	1.017e + 00
UH(14)	1.997e – 01	2.599e – 02	3.783e – 01	1.996e – 01	3.029e – 02	3.560e + 00
UH(15)	1.598e – 01	1.837e – 02	– 2.378e + 00	1.589e – 01	1.994e – 02	– 1.995e + 00
UH(16)	1.282e – 01	1.307e – 02	8.832e – 01	1.280e – 01	1.436e – 02	1.050e + 00
UH(17)	8.708e – 02	1.161e – 02	7.938e – 01	8.713e – 02	1.488e – 02	– 6.088e + 00
UH(18)	6.301e – 02	1.229e – 02	– 2.042e + 00	6.263e – 02	1.466e – 02	– 1.802e + 00
UH(19)	3.139e – 02	1.222e – 02	– 1.471e + 00	3.095e – 02	1.287e – 02	– 1.374e + 00
UH(20)	1.961e – 03	5.593e – 03	5.140e – 02	1.920e – 03	5.446e – 03	3.918e – 02
Peak	7.267e + 00	8.334e – 01	1.768e – 01	7.290e + 00	8.325e – 01	8.138e – 02
TP	3.358e + 00	9.733e – 01	2.331e + 00	3.372e + 00	9.889e – 01	2.267e + 00
Vol	1.011e + 00	2.432e – 02	– 3.553e + 01	1.011e + 00	2.377e – 02	– 3.813e + 01
Cond	2.261e + 01	1.743e + 01	5.702e + 00	2.327e + 01	1.901e + 01	5.688e + 00
MSE	2.878e + 00	1.070e + 00	2.754e + 00	2.901e + 00	1.059e + 00	2.551e + 00
Ridge	1.942e – 01	6.163e – 02	5.062e – 02	1.929e – 01	6.264e – 02	1.470e – 01

See footnotes to Table 1.

with/without storm-scaling. Fig. 1 indicates that the ordinary least squares method produces a UH with a slightly higher mean peak than the ridge least squares method. Furthermore, consideration of storm-scaling results in slightly lower mean peak discharge in the derived UH. As clearly indicated by Fig. 2, the ridge least squares method yields smaller values of standard deviation than the ordinary least squares method for either ‘with storm-scaling’ or ‘without storm-scaling’. Using the storm-scaling procedure further lowers the standard deviation associated with UH ordinates. Skew coefficients of UH ordinates could be positive or negative, and there is no clearly discernable pattern indicating the relative magnitude among different methods. From the viewpoint of reducing UH uncertainty, using the ridge least squares method in conjunction with the storm-scaling procedure would be the most desirable of the methods considered herein.

The overall uncertainty of a UH can be represented by the covariance matrix for UH ordinates. Determinant and trace of the covariance matrix can be used to measure the overall uncertainty associated with a random vector. A larger value of the determinant or trace implies larger uncertainty associated with a random vector. Table 5 provides the

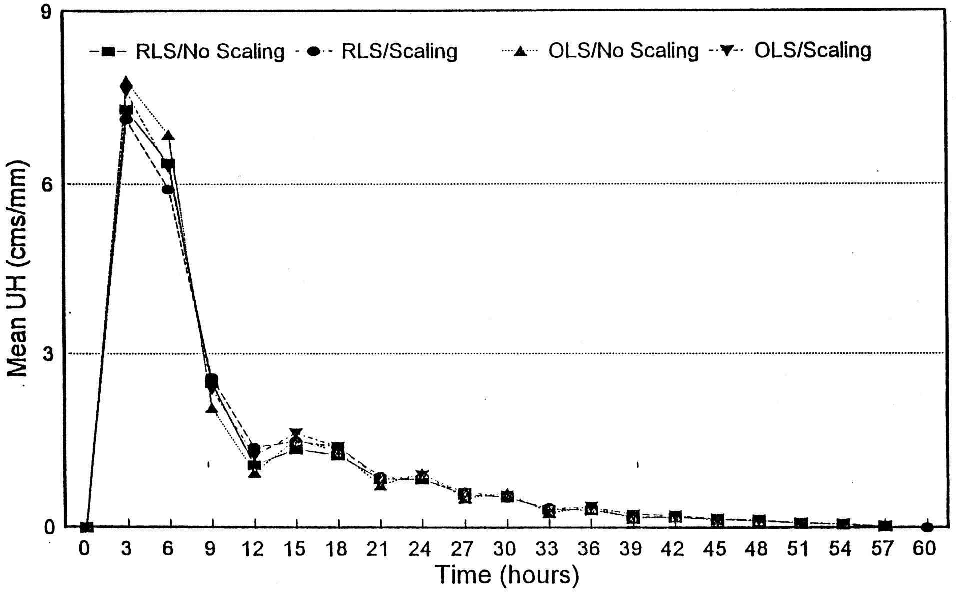

Fig. 1. Comparison of mean UHs by unbalanced bootstrap using ordinary and ridge least squares methods.

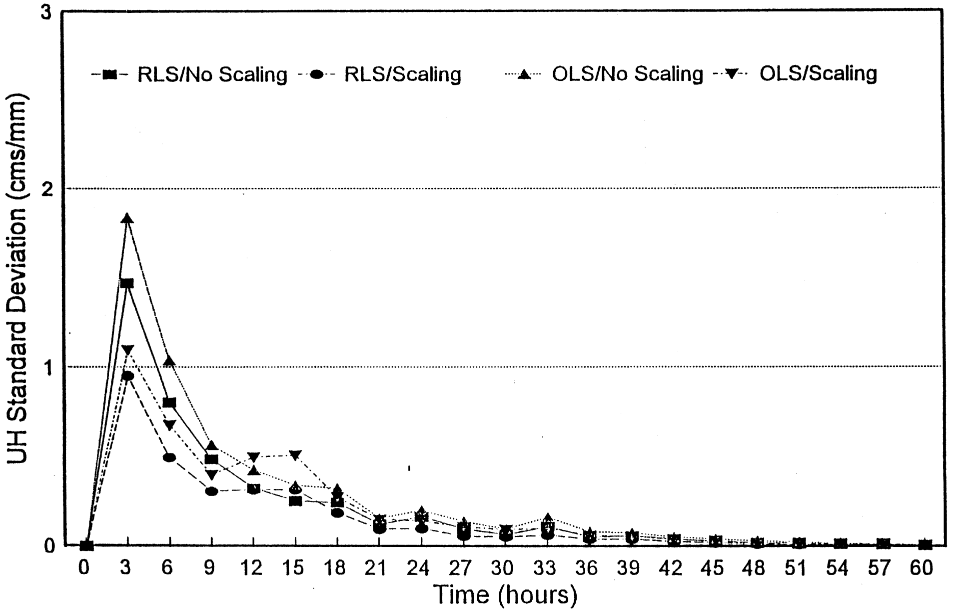

Fig. 2. Comparison of standard deviations of UHs by unbalanced bootstrap using ordinary and ridge least squares methods.

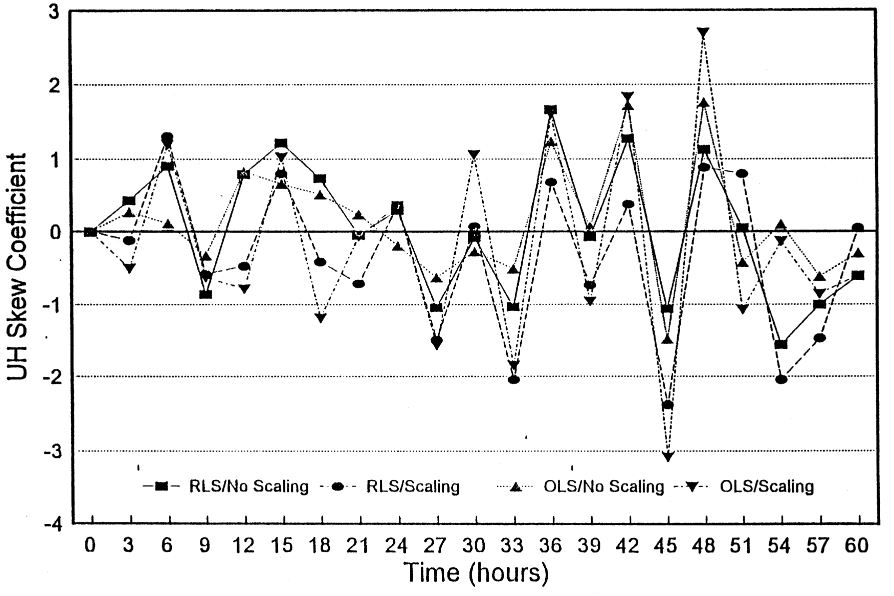

Fig. 3. Comparison of skew coefficients of UHs by unbalanced bootstrap using ordinary and ridge least squares methods.

determinant and trace of the covariance matrix of multiple-storm UHs derived by the various methods. It can be seen that the values of determinant are essentially zero. Therefore, comparison of these values may not be meaningful. Under this condition, the trace of a covariance matrix should be used for comparison. In Table 5, one observes that the trace of the UH derived by the ridge least squares method is smaller than that by the ordinary least squares method for either 'with storm-scaling' or 'without storm-scaling.' Furthermore, the use of storm-scaling reduces the trace compared with 'without storm-scaling.' Although the unbalanced bootstrap yields a slightly smaller trace than the balanced bootstrap, both methods can be considered the same.

Table 5

Properties of covariance matrix (C) of UH derived from multiple-storm analysis by ordinary least squares and ridge least squares for the Tong-Tou watershed

Least squares method	Storm scaling	Bootstrap algorithm	Determinant (C)	Trace (C)
Ordinary least squares	Yes	Unbalanced	1.997e − 53	2.481e + 00
	Yes	Balanced	4.517e − 52	2.602e + 00
	No	Unbalanced	1.163e − 52	5.300e + 00
	No	Balanced	4.464e − 52	5.367e + 00
Ridge least squares	Yes	Unbalanced	4.033e − 60	1.502e + 00
	Yes	Balanced	2.291e − 58	1.570e + 00
	No	Unbalanced	1.312e − 58	3.352e + 00
	No	Balanced	5.746e − 58	3.398e + 00

4.2. Uncertainty of the important UH ordinate and parameters

The lower parts of Tables 1–4 list the uncertainties of the important UH ordinate (i.e. the UH peak discharge) and parameters such as UH time-to-peak, UH volume, 2-norm condition number, mean square error, and ridge parameter. One observes that there is no significant difference between the unbalanced and balanced bootstrap algorithms. Since the unbalanced bootstrap and balanced bootstrap practically yield the same results, the unbalanced algorithm may be recommended because its algorithm is easier than that for the balanced bootstrap.

The storm-scaling results in smaller mean values of UH peak, time-to-peak, 2-norm condition number, mean square error, and ridge parameter (only for the ridge least squares method). Smaller mean values of 2-norm condition number and mean square error of a UH imply that, on average, the corresponding UH has fewer noise oscillations.

4.3. Confidence intervals of the multiple-storm UH

Fig. 4 and Fig. 5 show the 90% confidence intervals for the multiple-storm UH by the various procedures when the ridge least squares method is applied. Each figure shows confidence intervals for three different confidence interval methods. Fig. 4 compares the unbalanced and balanced bootstrap algorithms, indicating that there is no difference between the two. Fig. 5 compares the differences in UH confidence intervals for storm-scaling and no storm-scaling. One sees that the storm-scaling produces a narrower confidence interval around the peak discharge than no storm-scaling does.

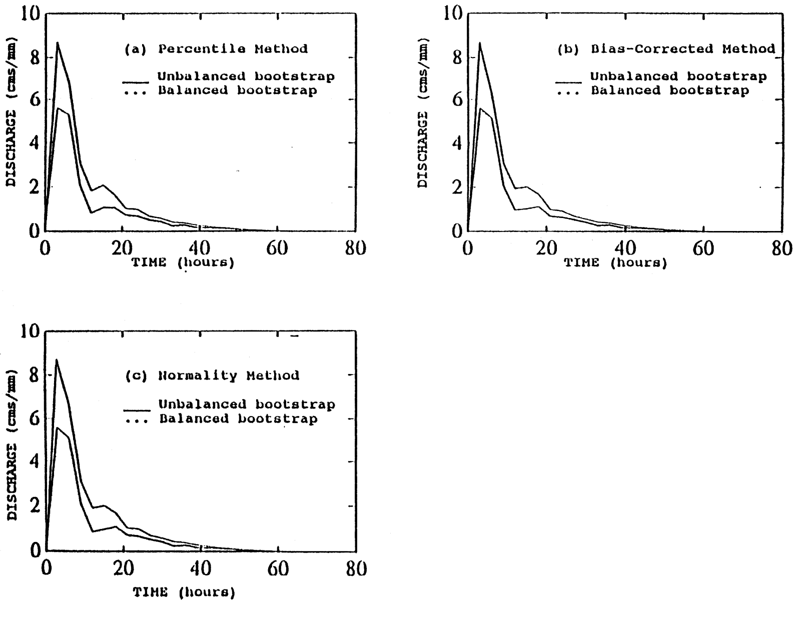

Fig. 4. Comparison of 90% confidence intervals by unbalanced and balanced methods for UH derived from ridge least squares with storm-scaling for Tong-Tou watershed.

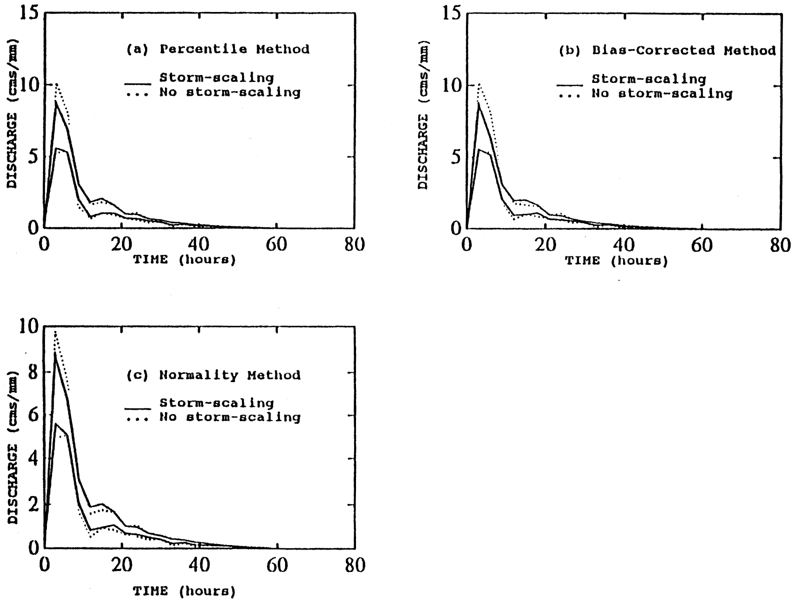

Fig. 5. Comparison of 90% confidence intervals by storm-scaling and no storm-scaling with balanced bootstrap for UH derived from ridge least squares for Tong-Tou watershed.

5. Summary and conclusions

A computed UH can be used along with a design rainfall to develop a design runoff hydrograph. Quantification of uncertainties associated with the derived UH provides essential information for safety evaluation of hydraulic structures and risk-based designs. This paper proposed a practical methodology called storm resampling based on the bootstrap resampling technique to quantify the uncertainties such as mean, covariance matrix, skewness, and confidence intervals for multiple-storm UH ordinates and any parameters involved in the process for estimating the UH. The important UH ordinates and parameters included UH peak discharge, UH time-to-peak, UH volume, 2-norm condition number related to the effective rainfall matrix, mean square error of the UH, and ridge parameter. The observed storms were considered as an observed random sample in which each data point involves the data of the direct runoff hydrograph and effective rainfall hyetograph for each storm.

The proposed methodology was applied to typhoon storm events for a watershed in Taiwan. The uncertainties such as the mean, standard deviation, and skewness associated with the ordinates and parameters for the multiple-storm UH were computed. Confidence intervals for the multiple-storm UH were also computed by three different methods. The computed uncertainties will be useful in risk and reliability analysis for hydraulic structures. Numerical results indicate that both balanced and unbalanced bootstrap resampling algorithms produce practically identical results. Therefore, the unbalanced algorithm may be recommended because its algorithm is easier than that for the balanced bootstrap. The results indicate that the storm-scaling produces tighter confidence intervals around the peak than the no storm-scaling does.

Although the storm resampling technique was used to quantify the uncertainties for a multiple-storm UH derived from the ordinary and ridge least squares methods, it can be applied to other solution techniques for UH determination. The storm resampling technique can also be applied to other hydrological and hydraulic simulation/optimization models.

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Appendix A. General algorithm for bootstrap resampling technique

Suppose $\hat{\theta}(\boldsymbol{X})$ is an estimate of interest which is a function of a random sample consisting of R data points $\boldsymbol{X} = (X_1, X_2, \ldots, X_R)$. For example, θ is the sample mean. Suppose we are interested in estimating the standard deviation of the sample mean. The bootstrap technique can be used to estimate any aspects of $\hat{\theta}$'s distribution as follows:

1. Randomly draw R data points from $\boldsymbol{x}$, with replacement, to yield $\boldsymbol{x}^* = (x_1^*, x_2^*, \ldots, x_R^*)$ which is called the bootstrap sample.
2. Calculate $\theta^* = \theta(\boldsymbol{x}^*)$. For example, compute the mean value for this bootstrap sample.
3. Repeat 1–2 a large number of times, say 1000, to obtain θ^{*1}, θ^{*2}, …, θ^{*1000}. For example, 1000 sample mean values are obtained.
4. Calculate whichever aspect of $\hat{\theta}$'s distribution based on θ^*. For example, compute the standard deviation of the mean over these 1000 values.

To see how the bootstrap resampling technique is linked to the proposed storm-resampling technique, consider the observed R storms as $\boldsymbol{x}$, and each ordinate of a multiple-storm UH as θ. Then, any procedures for computing a multiple-storm UH are the functional $\hat{\theta}(\boldsymbol{X})$ (no matter how mathematically complicated they are). Any parameters in the process of computing a UH can also be dealt with as UH ordinates were.

Appendix B. Computation of bootstrap confidence intervals for UH ordinates

Suppose that the number of bootstrap samples is B. For the jth ordinate of the UH, u_j, the 'normality' method assumes that B u_j^* ordinates are observed random samples from a univariate normal distribution. Since the mean and standard deviation of u_j^* are known from the bootstrap distribution of u_j^*, the confidence interval for u_j at a specified coverage probability $1 - 2\alpha$ can be found based on the normal distribution.

The percentile method is to find the interval between the 100α and $100(1 - \alpha)$ of the bootstrap distribution of u_j^* where $1 - 2\alpha$ is the probability that the true u_j will be contained in the interval. Let the cumulative distribution function (CDF) of the u_j^* based on the bootstrap sample be

$$\hat{G}(s) = Pr_*(u_j^* < s) \tag{B1}$$

where $Pr_*()$ indicates the probability computed according to the bootstrap distribution of u_j^*. The percentile method is to take

$$u_j \epsilon [\hat{G}^{-1}(\alpha),\ \hat{G}^{-1}(1-\alpha)] \tag{B2}$$

as an approximated $1 - 2\alpha$ confidence interval for u_j.

Efron and Tibshirani (1986) showed that if

$$\hat{G}(\hat{u}_j) \neq 0.5 \tag{B3}$$

where u_j is the estimate of u_j based on the original storm events, then the bias-corrected percentile method should be used. Eq. (B3) implies that u_j is not the median of the bootstrap distribution of u_j^*. The bias-corrected percentile method (BC method) takes

$$u_j \epsilon [\hat{G}^{-1}(\boldsymbol{\Phi}(2z_0 + z^{(\alpha)})),\ \hat{G}^{-1}(\boldsymbol{\Phi}(2z_0 + z^{(1-\alpha)}))] \tag{B4}$$

where $z^{(\alpha)}$ and $z^{(1-\alpha)}$ are, respectively, the quantiles at α and $1-\alpha$ for the standard normal distribution, and

$$z_0 = \boldsymbol{\Phi}^{-1}[\hat{G}(\hat{u}_j)] \tag{B5}$$

where Φ^{-1} is the inverse function of the standard normal CDF.

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