利用社群特性於社區網路影響力最大化之研究

Efficient Influence Maximization in Social Network Via Community Characteristics

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摘要

近幾年來，因為很多大型社群網站的興起，在社群網路中影響力最大化問題已經引起了很多關注。 影響力最大化問題是在社群網路中找尋一群節點，使得影響力的散播最大化。雖然近幾年已有很多研究在解決影響力最大化的問題，但是用以模擬社群網路的模型不能真實反映現實、網路情境，且效率不佳。然而因為大規模社群網路不斷的增加，效率和實際可行性已經是重要的課題。在此篇論文中，我們使用熱流模模擬切實際的網路，並在此模型下提出兩種解決影響力最大化的演算法。我們利用社群結構來避免影響力重疊，再從所找出來的社群結構中找出最具有影響力的關鍵性節點。藉由社群結構的特性可以大量的減少需要考慮的節點數目。我們使用合成和真實的資料實驗的結果顯示我們所提出的演算法在效能上有很大的改善。

Efficient Influence Maximization in Social Network via Community Characteristics

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Abstract

In recent years, considerable concern has arisen over the *influence maximization* in social network, due to the surge of social network web sites. *Influence maximization* is the problem of finding a small subset of nodes in a social network that could maximize the spread of influence. Although many recent studies are focused on influence maximization, these works in general are not realistic nor efficient. Nevertheless, with the increasing number of large-scale social networks, efficiency and practicability requirement for influence maximization have become more critical. In this thesis, we propose two novel algorithms, CDH-Kcut and CDH-Shrink, to solve the influence maximization problem in the realistic model, i.e., heat diffusion model. Our algorithms use the community structure, which could significantly decrease the number of candidates of influential nodes, to avoid information overlapping and to find the influential nodes according to the community structure. The experimental results on synthetic and real datasets show our algorithm significantly outperforms in efficiency.

Acknowledge

I greatly appreciate the kind guidance of my advisor, Prof. Suh-Yin Lee. Without her graceful encouragement, I cannot complete my thesis.

Besides, thanks are extended to all my friends and all the members in the information system laboratory for suggestion and instruction, especially Mr. Yi –Cheng Chen.

Finally, I would like to express my appreciation to my parents for their supports and consideration. This thesis is dedicated to them.

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Chapter 1

Introduction

In the last decade, many studies have been made in the area of social networks, in which users are linked to each other by a binary relationship such as friendship, co-working relation, business contact, to name a few. Fig 1.1 is an example of social network in which nodes tend to cluster together. Due to millions of users in social networks such as blogs, there are a lot of applications on social networks, such as *viral marketing*, *community detection*, etc.

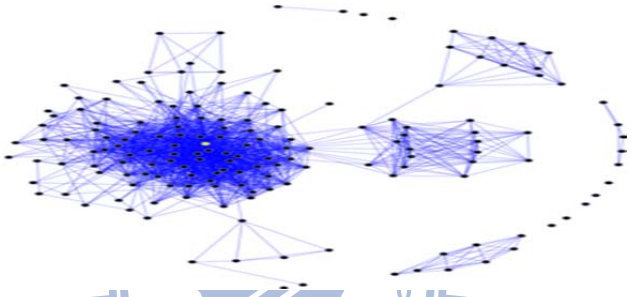


Fig.1.1. An example of a social network diagram. A common property of social network is that nodes tend to cluster together.

Nowadays, many large-scale web sites, such as Facebook and Twitter, become very popular since users can easily share everything with their friends and also bring small and disconnected social networks together. In 2011, Facebook already has more than 600 million active users and Twitter has about 90 million active users. Due to the flourishing of social network websites, marketing on online social networks shows great potential to be much more successful than traditional marketing techniques. According to eMarketer, advertisement spending on worldwide social networking sites in 2008 reached $23.4 billion and will expect to achieve about $23.6 billion in 2010 and $25.5 billion in 2011.

Consider the following motivating example. A company develops a software “cooler” and wants to market it to a social network. The company has limited budget so that it can only

give the free “cooler” to a small number of initial users. The company wishes the initial users could influence their friends to use the product, and their friends could influence their friends’ friends. Through the word-of-mouth effect, the company makes a large number of users adopt the “cooler”. Influence maximization problem is how to select initial users (refered to as *seeds*) so that the number of users that adopt the product or innovation is maximized. That is, the problem is how to find the influential individuals in a social network.

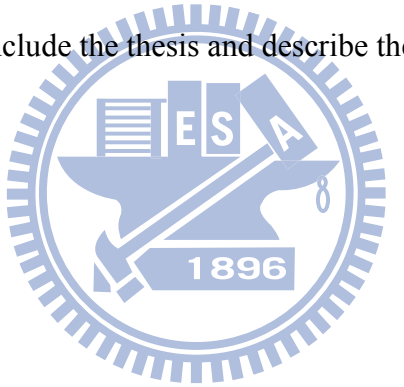
In reality, more social networks become large-scale, so the issue of efficiency becomes more and more crucial. Kempe et al. [6] also proved the influence maximization problem is *NP-hard*, and [6] proposed a climbing-up greedy algorithm. However, the climbing-up greedy algorithm is too time-consuming. If it takes long time for companies to decide which set of individuals should be given free samples to promote their products, they may lose the advantage due to non-timeliness. Moreover, the selected set of individuals will not be useful since the input network may change a lot during this week. Thus, some efficient approximate algorithms were proposed [2, 15]. Although those algorithms are efficient, they are only appropriate for diffusion models, which are not realistic enough.

Realistic modeling is also a very important issue for influence maximization problem. For an example, different social networks have different kinds of information flows. Hot social networks may transfer information faster than other social networks. The effect of time also has to be considered. Sometimes we want to know which set of individuals could trigger more adoptions of products after 3 days, 7 days, a week, etc. Therefore, realistic diffusion models are necessary for making actual predictions of the future behavior of the network.

In this thesis, we tackle the issues of time efficiency and realistic modeling of the influence maximization problem. We propose the novel *community and degree heuristic* (CDH) under heat diffusion model. Our CDH strategy is the unique combination of utilizing the community characteristics and modified *degree centrality*. Firstly, as shown in Fig 1.1, we can see that node clustering is an important characteristic of social networks. Therefore, we

utilize the community detection algorithm to avoid overlapping of influence spread. Secondly, we use the modified *degree centrality* to select influential individuals taking into account the information of community. We develop two approximate algorithms, CDH-Kcut and CDH-Shrink. Compared with the approximate algorithm in [5], CDH-Kcut *and* CDH-Shirnk are more efficient. Besides, both their influence spread are significantly better than classic degree centrality [22].

The rest of this thesis is organized as follows. In chapter 2, we introduce background knowledge required for influence maximization problem. We survey previous works on selection of seeds under different diffusion models in chapter 3. In chapter 4, we will present the details of CDH framework and two algorithms, CDH-Kcut and CDH-Shrink. We analyze the time complexity of each algorithm in chapter 5. In chapter 6, the experimental results will be presented. Finally, we conclude the thesis and describe the future works in chapter 7.



Chapter 2

Background

2.1 Diffusion of Innovation

Rogers [1] theorizes that diffusion is the process by which an innovation is communicated through certain channels over time among the members of a social system. Diffusion is a type of communication concerned with the spread of messages that are perceived as new ideas. Besides, innovations spread through society as the early adopters select the technology first, followed by the majority, until a technology or innovation is common. We use diffusion models, in generally, to simulate the diffusion of innovation.

2.2 Diffusion Model

In this section, we will introduce two basic and one realistic diffusion models. We model a social network as an undirected graph $G(V,E)$, where V is the vertex set and $V=\{v_1,v_2,...v_n\}$.$E=\{(v_i,v_j)|$ there is an edge from v_i to $v_j\}$ is the set of edges. Each node represents an individual and an edge between two nodes represents some kind of relationship (friends, co-authorships etc.). Each node is marked active (an adoption of an idea or innovation) or inactive. V and E also mean vertex set and the set of edges in rest of this thesis.

2.2.1 Linear Threshold Model (LTM)

For an undirected graph $G(V,E)$, we define $N(v) = \{u|(u,v) \in E\}$ as the neighbor set of node v and b_{uv} as influence of active node u on its inactive neighbor v. We define $A(v)$ as the set of active nodes in $N(v)$($A(v) \subseteq N(v)$) .Besides, the activation threshold θ is defined. For a given node, if $\sum_{u \in A(v)} b_{uv} \geq \theta$, node v becomes active. Intuitive meaning is that for an

inactive node v, if total influence exerted on u by all its active neighbors exceeds a pre-defined activation threshold θ, node v becomes active. In turn, it will exert influence on its inactive neighbors and bring some inactive neighbors become active. This process will continue on until no node can be activated.

2.2.2 Independent Cascading Model (ICM)

Another fundamental diffusion model is independent cascading model [24]**.** If a node v is activated at step t and it then tries to activate all its inactive neighbors with success probability p for each inactive neighbor u. If it is successful, then u will be active in step t+1, else v failed and will no longer have chance to activate u. In addition, each active node has only one chance to activate its neighbor u. While some other models [2, 3, 4] are proposed, they all are variations of the two core models, LTM and ICM, we have introduced.

2.2.3 Heat Diffusion Models (HDMs)

Heat diffusion is a physical phenomenon. Heat always flows from a position with high temperature to a position with low temperature. The phenomenon is actually similar to the process of people influencing others. The innovators and early adopters of a product or innovation act as heat sources, and have a very high amount of heat. These people start to influence others, and diffuse their influence to the early majority, then the late majority. Finally, at a certain time, the heat is diffused to the margin of this social network. In reality, different social networks have different information flows. Information on popular websites transfer information faster than other types of social networks. The time aspect needs to be considered when modeling social network marketing since different marketing strategies are required for different duration of information. It is not reasonable that only activated nodes could spread information to. ICM and LTM are built at a very coarse level, typically with only

a few global parameters, and are not useful for marketing actual predictions of the future behavior of the network [23]. Consequently, Ma proposed the realistic model, i.e., heat diffusion model [5]. It provides more parameters to simulate the conditions of real world, such as time and thermal conductivity. Therefore, heat diffusion model can easily simulate time effect in information and different types of information flow. Non-activated nodes can also spread information. HDMs originally has three different models, (1) diffusion on undirected social network, (2) diffusion on directed social network and (3) diffusion on directed social networks with prior knowledge of diffusion probability. In practice, most popular websites, such as Facebook, twitter and plurk are all undirected social networks, so he undirected social network are our focus.

The value $f_i(t)$ describes the heat at node v_i at time t, beginning from an initial distribution of heat given by $\mathbf{f}_i(0)$ at time zero. $\mathbf{f}(t)$ denotes the vector consisting of $\mathbf{f}_i(t)$. Suppose at time t, each node v_i receives an amount $M(i, j, t, \Delta t)$ of heat from its neighbor v_j during a period Δt. The heat $M(i, j, t, \Delta t)$ should be proportional to the time period Δt and the heat difference $\mathbf{f}_j(t) - \mathbf{f}_i(t)$. Moreover, the heat flows from node v_j to node v_i through the edge that connects nodes v_i and v_j. Based on this consideration [5], $M(i, j, t, \Delta t) = \alpha(\mathbf{f}_j(t) - \mathbf{f}_i(t))\Delta t$, where α is the thermal conductivity, i.e., the heat diffusion coefficient. As a result, the heat difference at node v_i between time time t and $t + \Delta t$ will be equal to the sum of the heat that it receives from all its neighbors. This is formulated as Eq (2.1):

$$\frac{\mathbf{f}_i(t+\Delta t) - \mathbf{f}_i(t)}{\Delta t} = \alpha \sum_{j:(v_j, v_i) \in E} (\mathbf{f}_j(t) - \mathbf{f}_i(t)) \tag{2.1}$$

The closed form solution of Eq (1) is :

$$\frac{\mathbf{f}_i(t+\Delta t) - \mathbf{f}_i(t)}{\Delta t} = \alpha \mathbf{H} f(t) \tag{2.2}$$

where

$$\mathbf{H}_{ij} = \begin{cases} 1, & (v_i, v_j) \in E \\ -d(v_i), & i = j, \\ 0 & \text{otherwise.} \end{cases}$$

and $d(v)$ denotes the degree of the node v,

As the limit $\Delta t \to 0$, Eq(2.2) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{f}(t) = \alpha \mathbf{H}\mathrm{f}(t) \tag{2.3}$$

Solving this differential equation in Eq(2.3), we have:

$$\mathrm{f}(t) = \mathrm{e}^{\alpha t \mathrm{H}}\mathrm{f}(0), \tag{2.4}$$

$\mathrm{e}^{\alpha t \mathrm{H}}$ could be extended as:

$$\mathrm{e}^{\alpha t \mathrm{H}} = \mathbf{I} + \alpha t \mathrm{H} + \frac{\alpha^2 t^2}{2!}\mathrm{H}^2 + \frac{\alpha^3 t^3}{3!}\mathrm{H}^3 + \cdots.$$

The matrix $\mathrm{e}^{\alpha t \mathrm{H}}$ is called the diffusion kernel in the sense that heat diffusion process continues infinitely many time from the initial heat diffusion. When the graph of a social network is very large, a direct computation of $\mathrm{e}^{\alpha t \mathrm{H}}$ is very time-consuming. [5] adopts its discrete approximation to compute the heat diffusion equation:

$$\mathrm{f}(t) = (\mathrm{I} + \frac{\alpha t}{p}\mathrm{H})^P \mathrm{f}(0) \tag{2.5}$$

Consider the example network in Fig 2.1 .

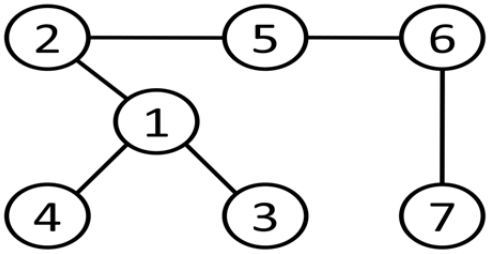


Fig. 2.1 An undirected network

The vector $\mathbf{f}(0)$ equals $[7\ 0\ 0\ 0\ 0\ 0\ 0]^{\mathrm{T}}$ and matrix $\mathbf{H}$ is

$$\mathrm{H} = \begin{bmatrix} -3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Fig 2.2 illustrates the curves of the variation of amount of heat of each node with heat diffusion model in Fig 2.1 (X-axis indicates time and Y-axis indicates amount of heat). We could see that only node 1 has heat at time 0. With time elapsing, the amounts of heat of other nodes are increasing and more close. Besides, assume node 2 is non-activated, but it can still spread information to node 5.

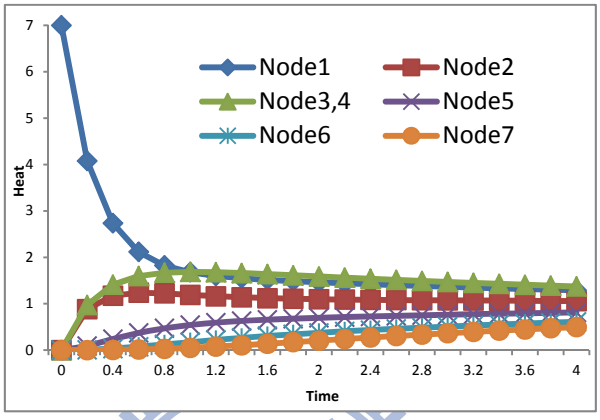


Fig. 2.2 the curve of amount of heat of each node with heat diffusion model in Fig. 2.1

If the amount of heat of node v_i exceeds the activation threshold θ, we think node v_i purchase a product or adopt an innovation.

2.3 Influence maximization problem

The problem of influence maximization [21] posed by Domingos and Richardson is stated below: if we can try to convince a subset of individuals to adopt a new product and the goal is to trigger a large cascade of further adoptions, which set of individuals should we target in order to achieve a maximized influence? In reality, a person's decision to buy the product is often strongly influenced by his friends and acquaintances. That is to say, the influence maximization problem is how we select the most influential early adopters. Better early adopters cause the more people to adopt the product. Online social networks provide good opportunities to address this problem, since we can easily share information with our friends. Influence maximization problems under the LTM, ICM and HDM are all

NP-problems, as already proved in [5, 6].

2.4 Community Detection Algorithm

A community is characterized as a subset of individuals who interact with each other more frequently than other individuals outside the community [22]. Community discovery is similar but not equivalent to the conventional graph partitioning problem. Both community discovery and conventional graph partitioning problem aim to cluster vertices into groups. A key challenge for the former, however, is that the algorithm has to decide what is the "the best", or in other words, the "most natural" partition of a network. In this thesis, we need the "most natural" partitioning without providing any information such as the number of partitions. Furthermore, if there is no good community structure, the network needs not be partitioned. That is why we use the community detection algorithm rather than conventional graph partitioning algorithm.

A quantitative measure, called modularity (Q), was proposed [7] to assess the quality of community structures, and community discovery was formulated as an optimization problem. Because Optimizing Q is an NP-problem, several heuristic methods have been proposed, as surveyed in [8]. Assume M is the number of edges and N is the number of nodes. The time complexity of most community detection algorithms are between O(Mlog$N$) and O(N^3). In this thesis, the efficiency of algorithms are most concerned, so we select KCUT [9] and SHRINK [10], which have low time complexity O(Mlog$N$), as our community detection algorithms. Besides, the two algorithms are not only efficient but also have good modularity. We will briefly introduce the Kcut and SHRINK algorithms in the section 2.4.1 and 2.4.2.

2.4.1 Kcut Algorithm

Kcut algorithm [9] is spectral graph partitioning. There is a family of methods on spectral graph partitioning. These methods depend on the eigenvectors of the Laplacian

matrix of a graph. Depending on the way a graph is partitioned, spectral methods can be classified into two classes. The first class uses the leading eigenvector of a graph Laplacian to bi-partition the graph. The second class of approaches computes a *k*-way partitioning of graph using multiple eigenvectors. We briefly review some representative algorithms of these two classes below.

SM algorithm [11], the representative of first class, works as follows. SM computes μ_2, the second smallest generalized eigenvector of Laplacian matrix. Then a linear search is conducted on μ_2 to find a partition of the graph to minimize a normalized cut criterion [11]. To find more than two clusters, the *SM* algorithm can be applied recursively

The representative of second class is NJW algorithm. NJW algorithm [12] finds a *k*-way partition of a network directly, where *k* is given by the user. NJW computes the *k* smallest generalized eigenvectors of Laplacian matrix and stack them in columns to form a matrix $Y = [\mu_1, \mu_2, ..., \mu_k]$. Each row of *Y* is normalized to have unit length. NJW treats each row as a point in R^k, and then applies standard *k*-means algorithm to group these points into clusters.

Kcut is a unique combination of recursive partitioning and direct *k*-way method. Kcut will achieve the efficiency of a recursive approach, while also having the same accuracy as a direct *k*-way method. It has been empirically observed that if there are multiple communities, using multiple eigenvectors to directly compute a k-way partition is better than recursive bi-partitioning method [12]. To optimize the performance measure of modularity *Q*, Kcut algorithm uses a greedy strategy to recursively partition a network. Unlike the most algorithms that always seek a bi-partition, it adopts a direct *k*-way partitioning. In summary, we compute the best *k*-way partition with $k = 2,3,\ldots,l$ using the *NJW* algorithm, and select the *k* that gives the highest *Q* value. Then for each subnetwork, the algorithm is recursively applied.

Given a network *G* and a small integer *l* that is the maximum number of partitions to be considered for each subnetwork and *Q* is the value of modularity , Kcut executes the steps as

shown in Algorithm 1:

Algorithm 1: Kcut

Input : Graph of social network G; l : the maximum number of partitions to be considered for each subnetwork

Output: Γ : set of clusters

1. Initialize Γ to be a single cluster with all vertices, and set Q=0.
2. **For** each cluster P in Γ,
3. Let g be a subnetwork of G containing the vertices in P;
4. **For** each integer k from 2 to l
5. Apply NJW to find a k-way partitioning of g, denoted by Γ_k^g;
6. Compute new Q value of network as Q'_k=Q(Γ ∪ $\Gamma_k^g \setminus p$);
7. Find the k that gives the best Q value, i.e., k^* = argmax$_k$ Q'_k;
8. **If** $Q'_{k^*} > Q$
9. accept the partition by replacing P with $\Gamma_{k^*}^g$, i.e., Γ = Γ ∪ $\Gamma_{k^*}^g \setminus P$,
10. and set Q = Q'_{k^*};
11. Advance to the next cluster in Γ, if there is any;

Fig 2.3 is an example network for Kcut. Assume l is 3. Fig 2.4 is the eigenvectors of Laplacian matrix of Fig 2.3 and stack them in columns to form a matrix $[\mu_1, \mu_2, \mu_3]$. Apply NJW to find a k-way partitioning of Fig 2.3. We find k = 2 that gives the best Q value. Fig 2.5 is the partitioning of Fig. 2.3. Then no more partitioning could gain the modularity.

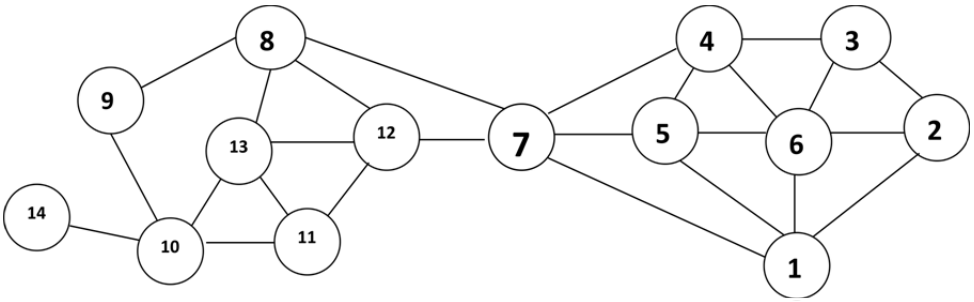


Fig. 2.3 An example network for Kcut

	μ_1	μ_2	μ_3
Node 1	-0.1361	-0.1302	0.0087
Node 2	-0.1361	-0.1737	-0.1142
	-0.1361	-0.1737	-0.1142
...	-0.1361	-0.1302	0.0087
	-0.1361	-0.1207	0.0498
	-0.1361	-0.1627	-0.0648
	-0.1361	-0.0093	0.1465
	-0.1361	0.1127	0.1440
	-0.1361	0.1616	-0.0610
	-0.1361	0.1728	-0.2561
	-0.1361	0.1158	0.0459
	-0.1361	0.1027	0.1797
	-0.1361	0.1486	0.0211
	-0.1361	0.1930	-0.5151

Fig. 2.4 Eigenvectors of Laplacian matrix of Fig 2.3 and stack them in columns to form a matrix $[\mu_1, \mu_2, \mu_3]$

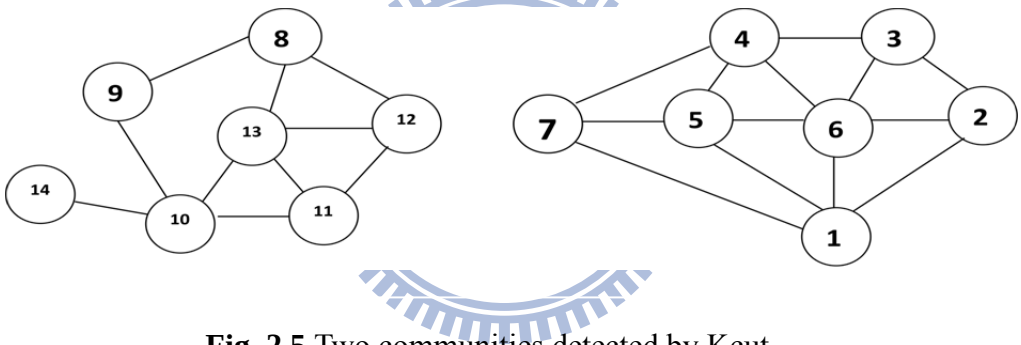


Fig. 2.5 Two communities detected by Kcut

2.4.2 SHRINK Algorithm

SHRINK [10] is a parameter-free hierarchical network clustering algorithm by combining the advantages of density-based clustering and modularity optimization methods. It uses density-based method to quickly know which set of nodes may be the same cluster. Then it uses modularity optimization to decide whether results of clustering are good or not. It not only detects hierarchical communities, but also identifies hubs and outliers. Therefore, local connectivity structure of the network is used in SHRINK. We briefly review the details

of SHRINK as follows.

Given a weighted undirected network $G = (V, E, w.)$. $w(e)$ is the weight of edge e. We formalize some notions and properties of the hierarchical structure-connected clusters. Firstly, we define the *structure similarity*. The structural similarity effectively denotes the local connectivity density of any two adjacent nodes in a weighted network. For a node $u \in V$, we define $w(\{u, u\}) = 1$. The structure neighborhood of a node u is the set $\Gamma(u)$ containing u and its adjacent nodes : $\Gamma(u) = \{v \in V \mid \{u, v\} \in E\} \cup \{u\}$. The structural similarity between two adjacent nodes u and v is then

$$\sigma(u,v) = \frac{\sum_{x \in \Gamma(u) \cap \Gamma(v)} w(\{u,x\}) \cdot w(\{v,x\})}{\sqrt{\sum_{x \in \Gamma(u)} w^2(\{v,x\})} \cdot \sqrt{\sum_{x \in \Gamma(v)} w^2(\{v,x\})}} . \quad (2.6)$$

Therefore, if node u and node v have more mutual and familiar friends, structure similarity of $\{u, v\}$ will be higher. The above structural similarity is extended from a cosine similarity used in [13]. It can be replaced by other similarity definitions such as Jaccard similarity. However, [10] shows that the cosine similarity is better. We define the *dense pair*. $\sigma(u, v)$ is the structure similarity of nodes u and v. If $\sigma(u, v)$ is the largest similarity between nodes u, v and their adjacent neighbor nodes: $\sigma(u, v) = \max\{\sigma(x, y) \mid (x = u, y \in \Gamma(u) - \{u\}) \vee (x = v, y \in \Gamma(v) - \{v\})\}$, then $\{u, v\}$ is called a dense pair in G.

That is to say, a dense pair $\{u, v\}$ is the largest similarity edge from all edges of u and v. As shown in Fig 2.6, {9,13} is a dense pair with structure similarity 0.8165 in the example network since {9, 13} is the largest similarity edges from all edge of node 9 and 13.

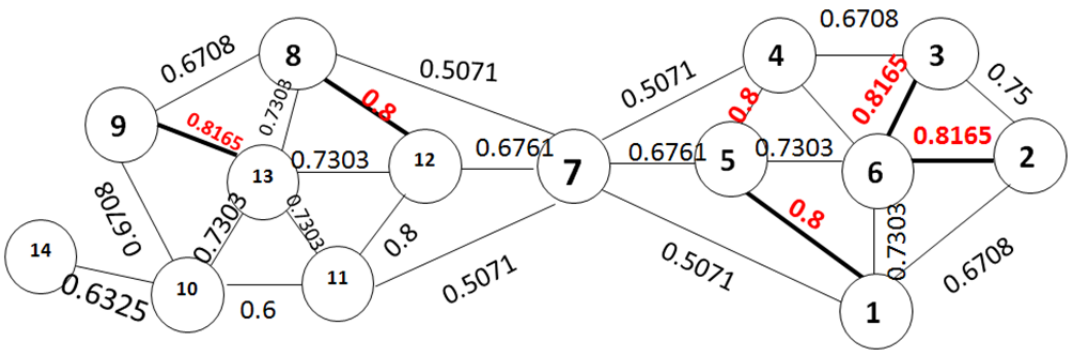


Fig. 2.6 An example network for SHRINK algorithm

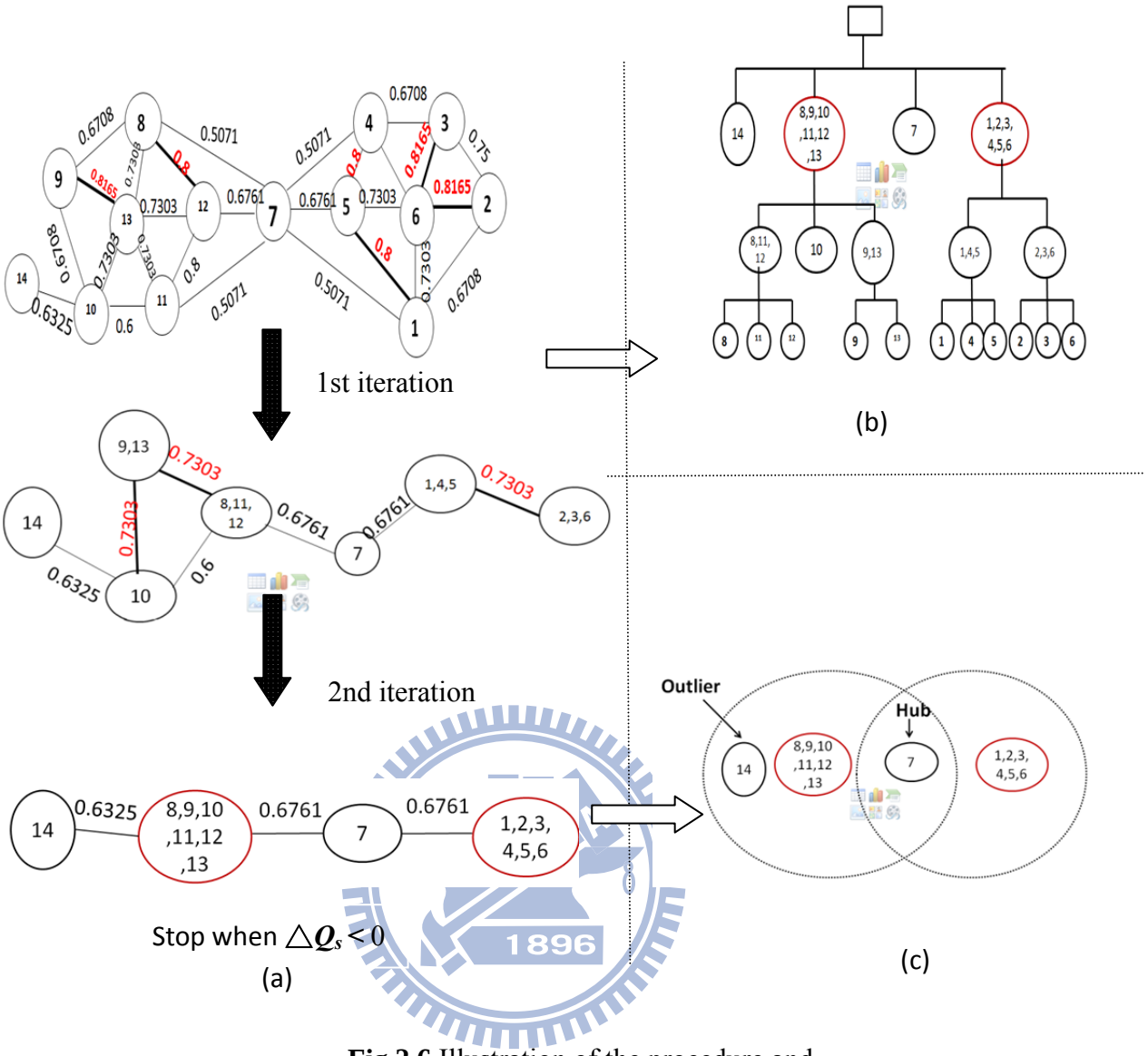


Fig.2.6 Illustration of the procedure and result of the hierarchical network clustering algorithm SHRINK.

The main process can be divided into two phases that are repeated iteratively. Given a network with N nodes, first we initialize each node with a different community label. In this initial partition, the number of communities is the same as the number of nodes. Then, for each node u we combine the corresponding nodes in the dense pairs of u to form a super-node. This process is applied sequentially for all nodes. We record all different communities which represent a partition of the network. The second phase of the algorithm is to build a super-network. We evaluate the modularity gain of Q_s for the shrinkage of the communities found during the first phase. If the modularity gain is positive, the corresponding local

community is replaced by a super-node. The above two phase are executed in turns until there is no community with positive modularity gain. For an example in Fig 2.7(a), at 1st iteration, we separately combine node set {9, 13}, {8, 11, 12}, {1, 4, 5} and {2, 3, 6} as four super nodes with structure similarity 0.8165, 0.8, 0.8, and 0.8165. Then since the modularity gains after shrinkage of communities are positive, the above four node sets are replaced by four super nodes. At 2nd iteration, node set {{8, 11, 12}, {9, 13}, 10} and {{1, 4, 5}, {2, 3, 6}} are separately combined as two super-nodes with 0.7303 and 0.7303. The modularity gains after shrinkage of communities in 2nd are positive, so we replace two node set {{8, 11, 12}, {9, 13}, 10} and {{1, 4, 5}, {2, 3, 6}} by their super-nodes. Since no more shrinkage of communities could gain the modularity gain of Q_s, SHRINK stops. Then the hierarchy of communities naturally occurs, as shown in Fig 2.7(b). Fig 2.7(c) represents final two-layers overlapping communities. Since node 7 connects two communities, it is a hub. In addition, node 14 is identified as an outlier which is loosely connected with the community {8, 9, 10, 11, 12, 13}.

SHRINK is not only efficient but also accurate. Besides, it can detect hubs which are very useful information in maximal influence problem. We could see that in the same graph in Fig 2.6 and Fig 2.3, Kcut only detect two communities, but SHRINK detect not only two communities but also a hub, node 7. Hub is very useful for influence maximization problem.

2.5 Seeds Selection Algorithm

We discuss previous works for seeds selection in this section. Influence maximization problem is an NP-problem. Hence, many works have been proposed to achieve approximate solutions. In social network, we often consider the person who has the most friends as the most influential person, since he can possibly influence most people. Therefore, the intuitive strategy, in general, is selecting seeds based on their degree, called *degree centrality*. Nevertheless, the members of large communities often have larger degree than other members

of smaller communities. Consequently, degree centrality easily selects seeds in the same large community. Influence spreads of each seed in the same community tend to be overlapped. As a result, degree centrality does not have good performance on influence spread. *Distance centrality* is another common used method for influence maximization problem. It selects seeds in the order of increasing average distance to other nodes. However, nodes in the larger communities usually have smaller average distance. As a result, most seeds may also be clustered. Simply stated, degree centrality and distance centrality result in the phenomenon of clustering of seeds, which deteriorates sharply in influence spread.

Pable A. Estevez et al. proposed *set cover greedy algorithm* [2] under independent cascading model (ICM). It kept selecting node with highest "uncover degrees". Once a node is selected, all its neighbors as well as itself are labeled as "covered". This procedure continues until k seeds are selected. This algorithm is computationally fast under simpler models, i.e., ICM. However, it has good influence spread only in high successful probability.

The *Climbing-up greedy algorithm* [6] under ICM and LTM was proposed by David kempe et al.. They also provided the first provable $\left(1-\frac{1}{e}\right)$ approximation guarantees for influence spread. The number e is Euler's number. Recently, since social network websites are getting more popular, we have to pay more attention to efficiency of algorithms. In reality, at the beginning of the innovation diffusion process, several seeds in the network spread the information at the same time, not just one single seed. The information from his (her) social network may come from several seeds. At each iteration of *climbing-up greedy algorithm*, we select most "influential" node on the condition of considering all seeds selected before. This procedure continues until k seeds are selected. If a node could make more nodes to be activated, it seems to be more "influential". For selecting the most influential node, we have to compute each node's influence. Due to the heavy computing load of climbing-up greedy algorithm, it is not appropriate for large social networks. Besides, [5] proposed *enhance greedy algorithm* under heat diffusion model, i.e. the climbing-up greedy algorithm specially

under heat diffusion model. Nevertheless, enhance greedy algorithm is also a climbing–up greedy algorithms. Consequently, we cannot solve influence maximization problem under heat diffusion model in acceptable time.

Yitong Wang et al [14] proposed a *potential-based node selection*. It selects some inactive nodes that might not be optimal at starting phase but could trigger more nodes in later stage of diffusion. It can save half time of totally using *Climbing-up greedy algorithm* and cause more adoptions than that in [6]. However, in practice it is still not efficient enough. Therefore, the extremely efficient algorithm, *degree discount heuristic,* was presented by Wei Chen et al. [15]. It obtains the approximate solutions in large datasets for only a few seconds. Besides, its performance is close to [6]. However, both of [14, 15] are only under LTM or ICM, which are not very realistic diffusion models. In addition, degree discount heuristic is only for very low successful probability, i.e., people are extremely hard to be influenced in very low successful probability.

Chapter 3

Community Degree Heuristic (CDH)

In this chapter, we will describe our community degree heuristic (CDH) that quickly detect seeds under the heat diffusion model. In section 4.1, we present the overview of the system architecture and explain principles of CDH. In section 3.2 and 3.3, we will go into details about our CDH-Kcut and CDH-Shrink.

3.1 Overview of System Architecture

CDH is the unique combination of the community detection algorithm and modified *degree centrality*. Suppose we have data on a social network which has N individuals. The problem we need to solve is: given the quota number k, how to select the initial k "influential" individuals who will be delivered a free sample product, in order to maximize the number of cascade adoptions by which these individuals will influence their friends or individuals on their direct contact list.

In this thesis, we model social network marketing process by heat diffusion process. Initially, we select k individuals as seeds for heat diffusion, denoted by the set S and the k seeds are given a certain amount of heat h_0. At time zero of the heat diffusion process, we set $f_i(0) = h_0$, where $i \in S$. As time elapses, the heat will diffuse through the whole social network. If the amount of heat of individual i at time t is greater than or equal to an activation threshold θ, this individual i will be considered as having been successfully influenced on activated by others, and will adopt the product. We define the influence set of the set of k individuals S, denoted as $I_s(t)$, to be the expected number of individuals who will adopt the product at time t. Now the above problem could be interpreted as: finding the most influential k-size set S to maximize the size of set $I_s(t)$ at time t, where $I_s(t) = \{ i \mid f_i(t) \geq \theta, i \leq N \}$.

This problem is NP-hard, as already proven in [5]. We select the heat diffusion model to be our diffusion model since it can realistically simulate the real world.

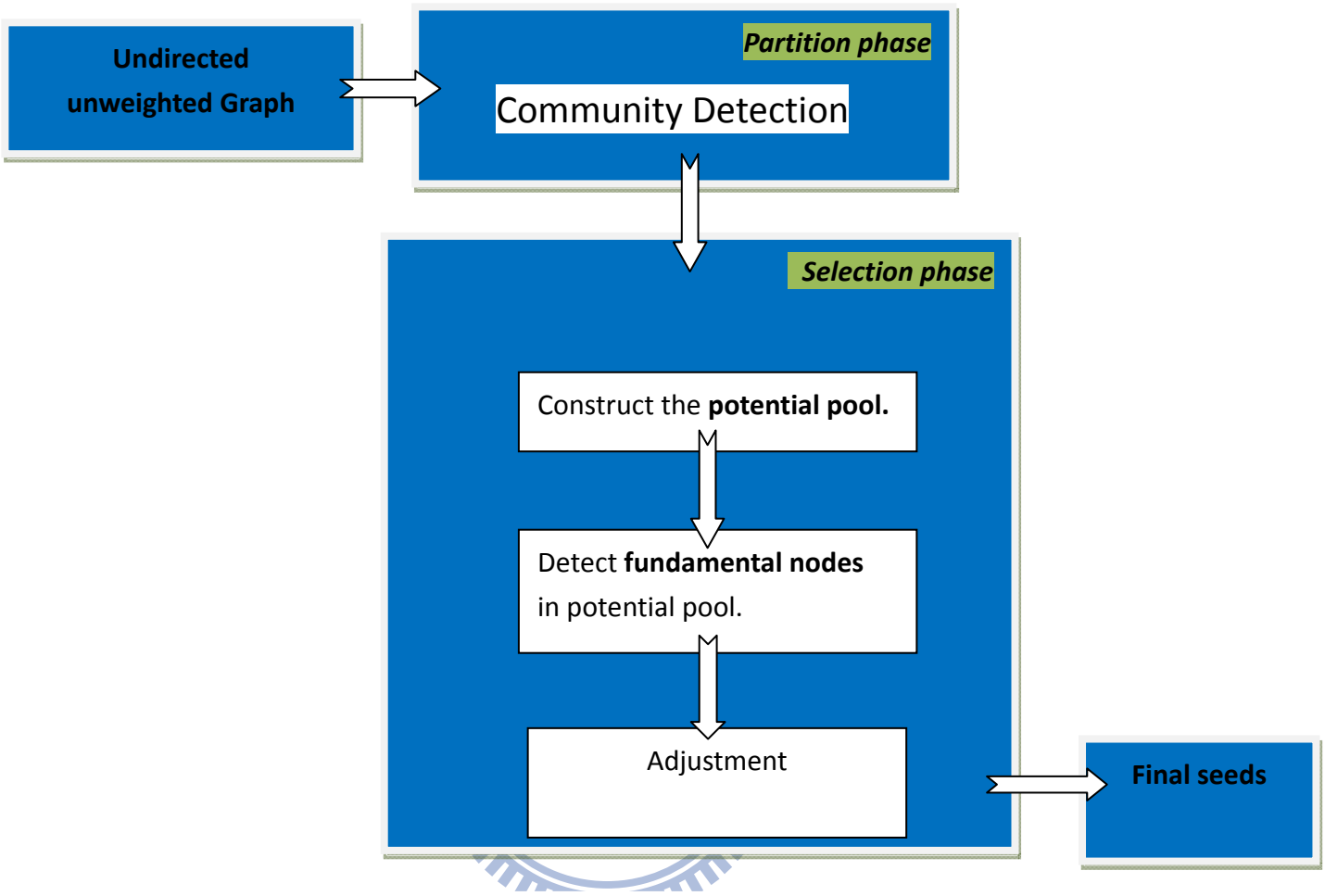


Fig. 3.1 Framework of CDH

The proposed CDH is composed by two phases, *partition phase* and *selection phase*. Fig. 3.1 illustrates the framework of CDH. **The first phase**, *partition phase*, detects the communities of the network. Community is a subset of individuals who interact with each other more frequently than other individuals outside the community. In real life, one's information often spread in his or her circle of friends. That is, most of someone's influence clearly spread in his or her community. We find the same phenomenon in heat diffusion model. In Fig. 3.2, node 1 is a seed. The color of each node means its amount of heat. More dark blue means larger amount of heat. Nodes circled by dotted circle are in the same community. We

can see that most gains of heat are in the community of node 1. In Fig 3.3, node1 and node 2 are seeds. Most gains of heat are also in the community of node 1. Nodes in the other community gains very little amount of heat. We can conclude that if we choose nodes in the same community as seeds, most gains of heat are in their own community. Other communities gain little amount of heat. Therefore, information of community is a very useful tool to avoid influence overlapping in heat diffusion model.

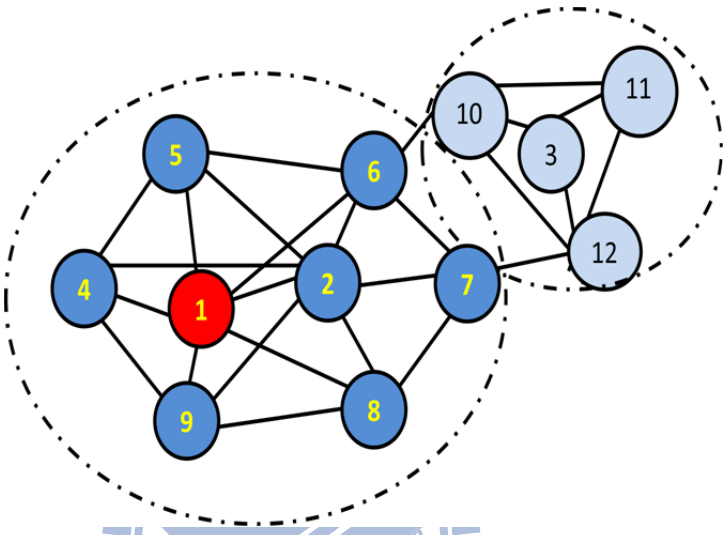


Fig. 3.2. The distribution of heat as node 1 is seed. The color of each node means its amount of heat. More dark blue means larger amount of heat. Nodes circled by dotted line are in the same community.

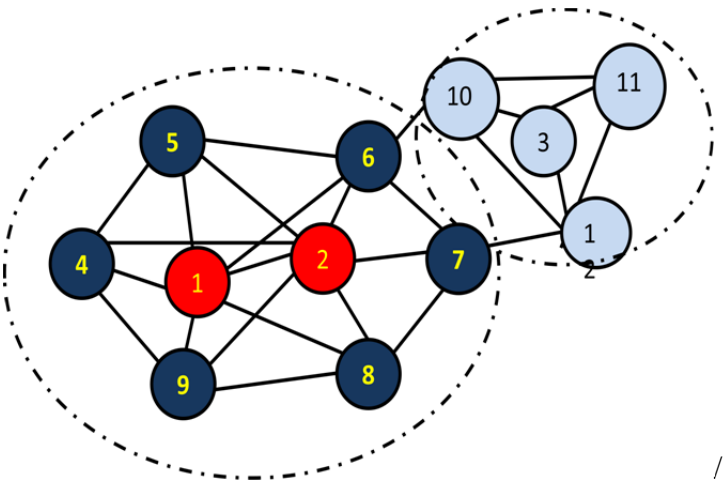


Fig. 3.3. The distribution of heat as node 1 and node 2 are seeds. The color of each node means its amount of heat. More dark blue means larger amount of heat. Nodes circled by dotted line are in the same community.

The reason for using community detection algorithms rather than conventional graph partitioning algorithm is that we want to detect "the best", or in other words, the "most natural" partitioning of a network without providing any information such as the number of partitions. For example, if the network is natural to be partitioned to 3 communities, we should not force the network to be partitioned to 4 communities.

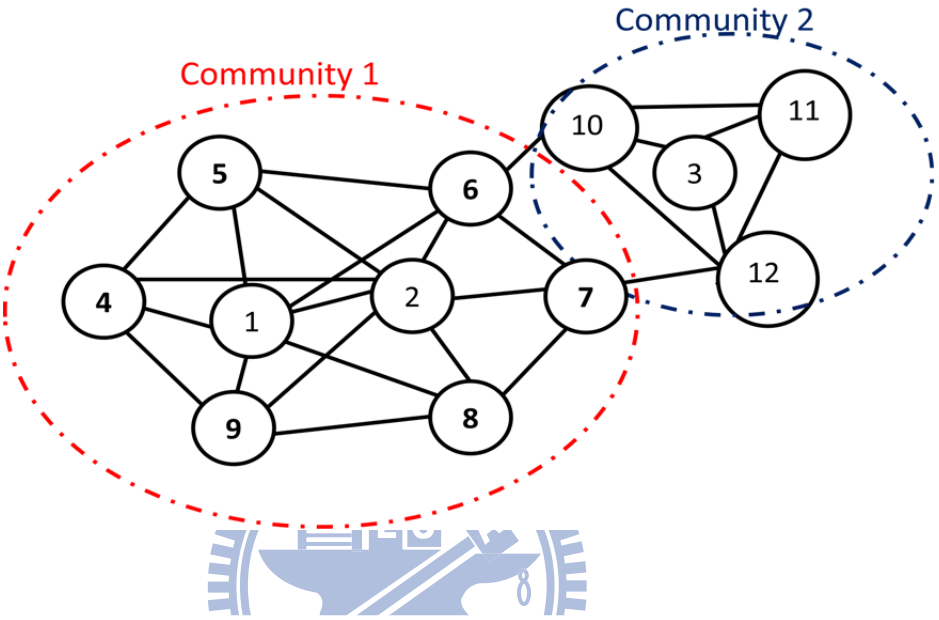


Fig. 3.4 Comparison with three seeds in different community size. Community 1 is a larger community than community 2. Select the same number of seeds from community 1 could trigger more individuals to be activated than from community 2.

The second phase, *selection phase*, finds the most influential nodes based on the result of *partition phase* and parameters of heat diffusion model, such as flow duration, thermal conductivity and activation threshold. In Fig. 3.4, community 1 is a larger community than community 2. It shows that select nodes from community 1 as seeds instead of nodes from community 2 could trigger more individual to be activated. The degrees of each node in social network also fit with power-law distribution [17, 18], i.e., a very large number of nodes have very small numbers of neighbors. Hence, most large-degree nodes are in large communities. Due to the above reasons, we only consider nodes in the large communities as seed candidates.

The candidates are put in the potential pool. Therefore, we intend to select seeds from potential pool. Next, we detect the "**fundamental node**" from the potential pool. Fundamental nodes have more potential to be seeds since it has larger degree than that of other nodes in the same community, or it is located on the important position in the network. The important position means connecting many communities. Fig. 3.4 and 3.5 show two kinds of fundamental node. In Fig.3.4, node 3 is the fundamental node. It has the largest degree among all nodes. In Fig. 3.5 node 12 is the fundamental node. It has better position which can easily influence two node sets {1,2 , 3, 4, 5 } and {6, 7, 8, 9, 11}

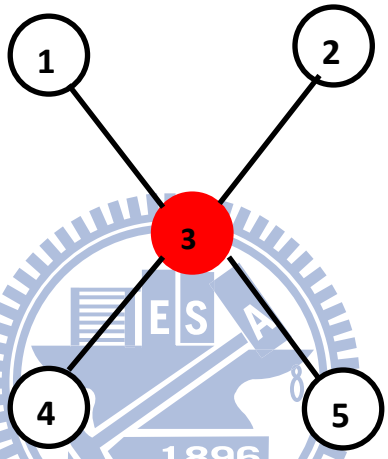


Fig 3.4. An example of Fundamental node. Node 3 is the fundamental node. It is has the largest degree among all nodes.

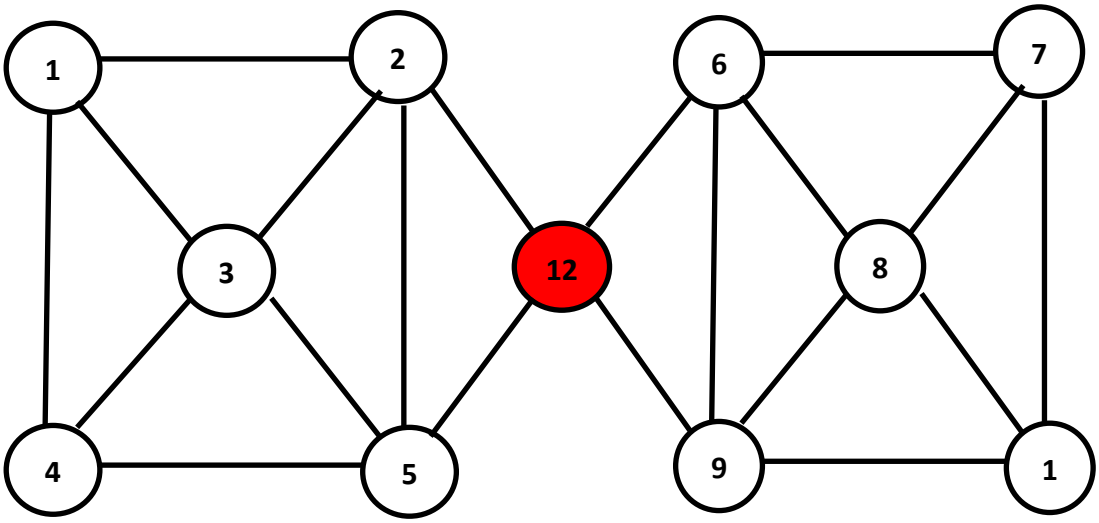
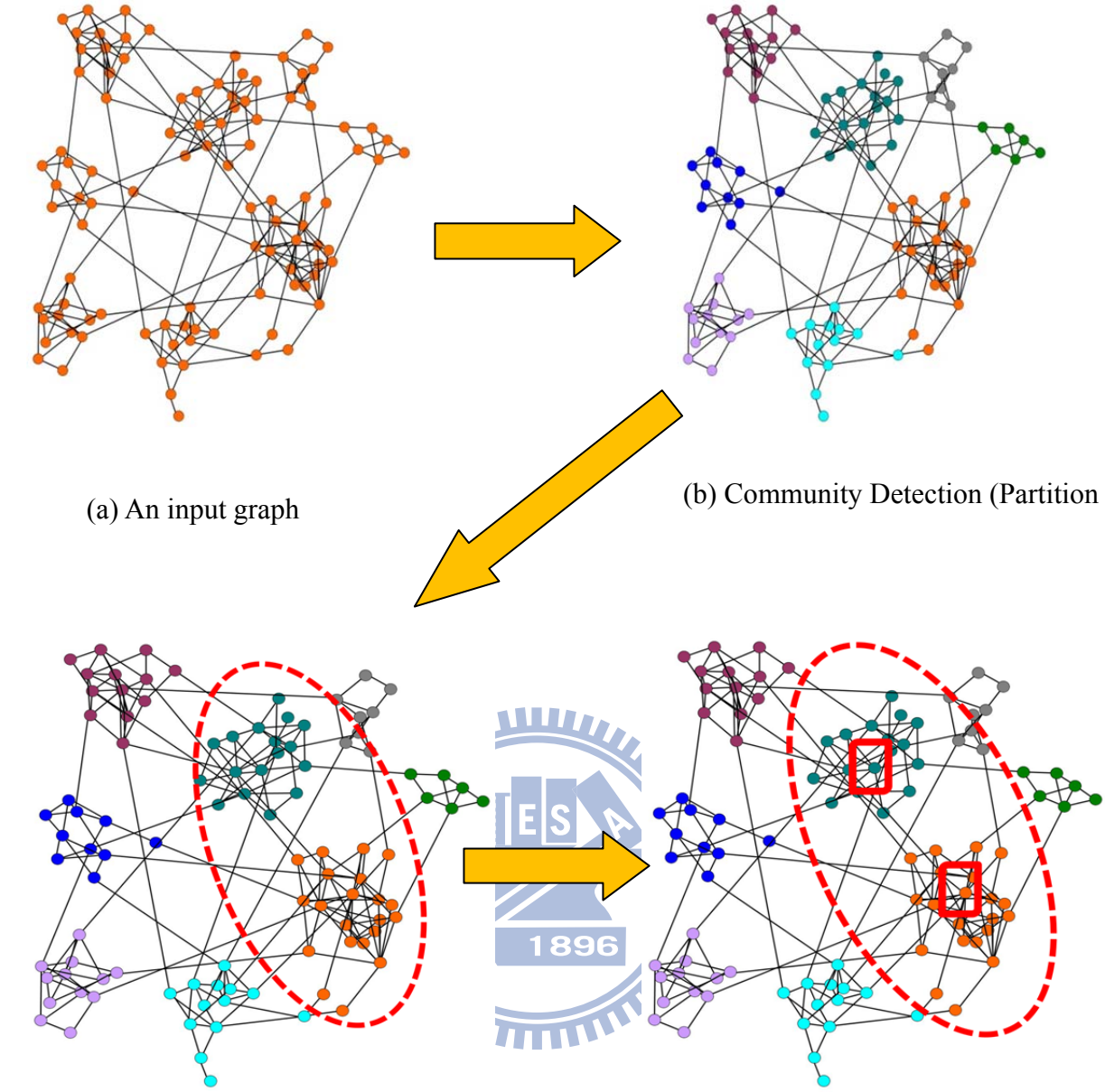


Fig. 3.5. An example of fundamental node. Node 12 is the fundamental node. It has better position which can easily influence two node sets {1, 2, 3, 4, 5} and {6, 7, 8, 9, 11}.



(a) An input graph

(b) Community Detection (Partition Phase)

(c) Construct potential pool (selection phase)

(d) Find fundamental nodes in potential pool (assume two fundamental nodes)

Fig. 3.6 An example of the concept of CDH

How to detect the fundamental nodes is one of differences between CDH-Kcut and CDH-Shrink algorithms. Although these nodes have good chance to become the final seeds, they are not the best seeds in different situations (parameters) of heat diffusion model. For example, seeds which perform well in short flow duration may not be good in long flow duration. Therefore, adjusting the fundamental nodes to become more ideal seeds is essential.

Fig. 3.6(a) is an example of input graph. Fig. 3.6(b) is the result of community detection of Fig. 3.6(a). Different color means different community. After partition phase, the first step of selection phase is constructing the potential pool. Two communities circled by red dotted circle are potential pool since the two communities are the two largest communities among all communities in Fig. 3.6(c). Assume two fundamental nodes are to be selected. Fig. 3.6(d) shows two fundamental nodes in the potential pool since they have large degree. After the step of constructing potential pool and finding fundamental nodes, we effectively narrow down the scope of seed candidates. CDH-Kcut and CDH-Shrink are two algorithms using different community detection algorithms and different strategies of potential pool, fundamental nodes and adjustment. We explain details of two algorithms in next two sections.

3.2 CDH-Kcut

CDH-Kcut is composed of *partition phase* and *selection phase*. The purpose of partition phase is to detect communities. The selection phase finds the influential nodes in communities. The strategies used by the CDH-Kcut are presented below:

(Partition phase):

We partition the network into communities by Kcut algorithm [9]. Every node will belong to only one community, and overlapping community is not allowed in Kcut. We assume the graph G is partitioned to the l communities. In most cases, l is larger than k, so in this paper we don't discuss the case $l < k$.

(**Selection phase**)

After detecting communities, we have l communities. If we want to find k seeds, firstly we construct the potential pool, $PP(G)$. We define potential pool as :

$$PP(G) = \{SC_1^p, SC_2^p, \ldots, SC_k^p\}, \quad (3.1)$$

where SC_i^p is the set of top-p degree nodes in i-th largest community SC_i, $i = 1,2,..,k$. Therefore, $PP(G)$ keeps the top-p degree nodes in each community of top-k largest communities. In most cases, p = 10% of community size is enough for selecting good seeds. Therefore, we significantly narrow down the range of possible seeds. Then, we select the fundamental nodes from the potential pool. Since Kcut cannot identify importance of location of nodes in each SC_i^p, degree has been considered as the only attribute that distinguishes good fundamental nodes from poor fundamental nodes. Thus, we select the largest degree node in each SC_i^p as the fundamental nodes. $S = \{s_1, s_2, \ldots, s_k\}$ is the set of seed candidates s_i. Fundamental nodes are seed candidates. Fig. 3.7 is an example of finding fundamental nodes. Node 1 and node 7 are the largest degree nodes in respective community. Finally, we adjust the fundamental nodes to be the final seeds. Our basic idea of adjustment is a heuristics that tries to use an add-node *a_node* to replace a delete-node *d_node*. If the influence spread after node replacing is larger than before replacing, we do the replacement,

Algorithm 1: CDH-Kcut

Input : Graph of social network G ; number of total seeds k, Parameters p

Output: k seeds

1. Execute the Kcut(G) ;
2. Select top-k biggest communities from the communities in Kcut(G) ;
3. **foreach** selected community SC_i **do**
4. Add top-p degree nodes into set SC_i^p ;
5. **end**
6. **foreach** SC_i^p **do**
7. select the most degree node S_i from SC_i^p ;
8. **End**
9. $IM = I_s(t)$;//$I_s(t)$: influence spread of current seed set, IM : record max influence spread
10. **foreach** community SC_i **do**
11. **if** size(SC_i) > avg($\sum_{i=1}^{k}$ size(SC_i)) **then**
12. Add SC_i in LC ; //LC : the set of large communities
13. **end**
14. **end**
15. Sort LC based on community size ;
16. $ci = 0$; // ci : community index
17. $di = 0$; // di : index of d_node
18. **foreach** C_i in LC
19. $ai = 2$; //ai : index of a_node
20. **while** *ture* **do**
21. Select the ai-*th* large degree node from SC_{ci+1}^p as a_node;
22. Select the seed candidates s_{k-di} from S as d_node;
23. **if** $I_s(t) < IM$ **then**
24. Cancel the replacement in line 21 and line 22.
25. **break ;**
26. **end**
27. $IT = I_s(t)$;
28. $ai = ai + 1$;
29. $di = di + 1$;
30. **End**
31. $ci = ci + 1$;
32. **end**
33. Output individuals in S

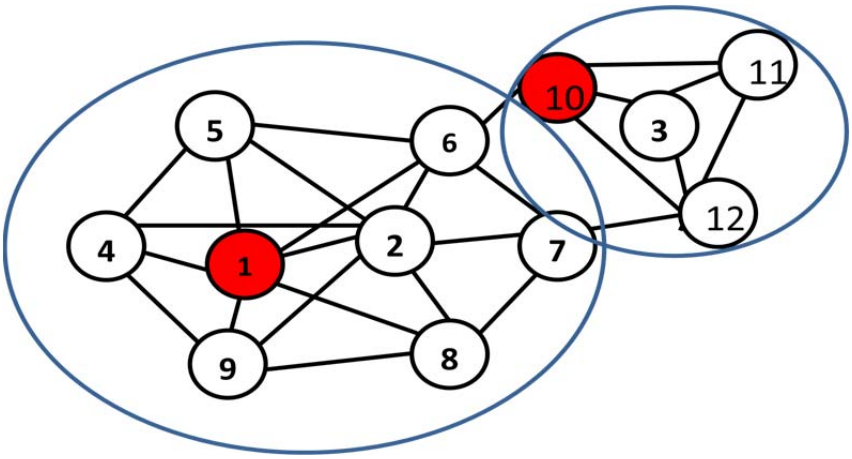


Fig. 3.7 An example of finding fundamental nodes in CDH-Kcut. Red nodes are fundamental nodes.

else we cancel the replacement.

As in Fig 3.3, seeds selected from large community could trigger more adoptions than that selected from small community. As a result, we incline to select add-node in large communities and delete-node in the small communities since we want to know whether selecting nodes from large communities can gain more influence spread or not. If the size of a community is larger than $AvgSC = \text{avg}(\sum_{i=1}^{k} \text{size}(SC_i))$, then this community is deemed as large community. Notice that delete-nodes must be fundamental nodes. Due to the adjustment, we can avoid influence spread from being spoiled for the effect of different value of parameters, such as flow duration, activation threshold and thermal conductivity. We discuss the effect of **flow duration**, **activation threshold** and **thermal conductivity** individually. Comparing the difference caused by long time and short time, information will diffuse farther in long flow duration. That is, in long flow duration the seeds would influence more individuals than that in short low duration. Therefore, we should not select too many seeds in one community in long flow duration. In contrast, it is appropriate selecting more seeds in one community in short flow duration.

It is more difficult to make individuals adopt products in high **activation threshold**. Individuals need more heat to be activated in higher activation threshold, so we tend to select more seeds in one community with high activation threshold.

High thermal conductivity makes information diffuse more quickly. Compare with low thermal conductivity, information of high thermal conductivity makes information diffuse longer distance. Hence, we do not select many seeds in one community. We conclude that differences caused by different parameters are the level of seeds clustering. In the simulation of short flow duration, high threshold and low thermal conductivity it is better to select more seeds in one community, i.e., higher level of seed clustering. On the other hand, long flow duration, low threshold and high thermal conductivity social network, not many seeds in one community is needed, i.e., lower level of seed clustering. Therefore, the adjustment in CDH-Kcut is to test and verify whether large communities should need more seeds. Lines between 9 and 29 in algorithm 1 show the steps in seeds adjustment.

3.3 CDH-Shrink

Our proposed CDH-Shrink is also composed of two phase, partition phase and selection phase. Besides, *Shrink* algorithm [10] could detect *hub*, i.e., a node connecting different communities. It provides us more information about the community structure property. The communities detected by Shrink are more precise than by Kcut. Due to above reasons, we can select more productive fundamental nodes than in CDH-Kcut. We describe it as follows:

(Partition phase):

We get information of community structure and hubs by Shrink algorithm.

(Selection phase):

In selection phase, we construct the potential pool and select fundamental nodes. Then adjust fundamental nodes to find final seeds. As shown in Fig 3.8, if we totally find *k* seeds, we have to select *k* fundamental nodes. Therefore, we have *k* iterations of selecting fundamental nodes. At *i*-th iteration, we only select largest community SC_i among all remaining communities, and then select a fundamental node from SC_i.

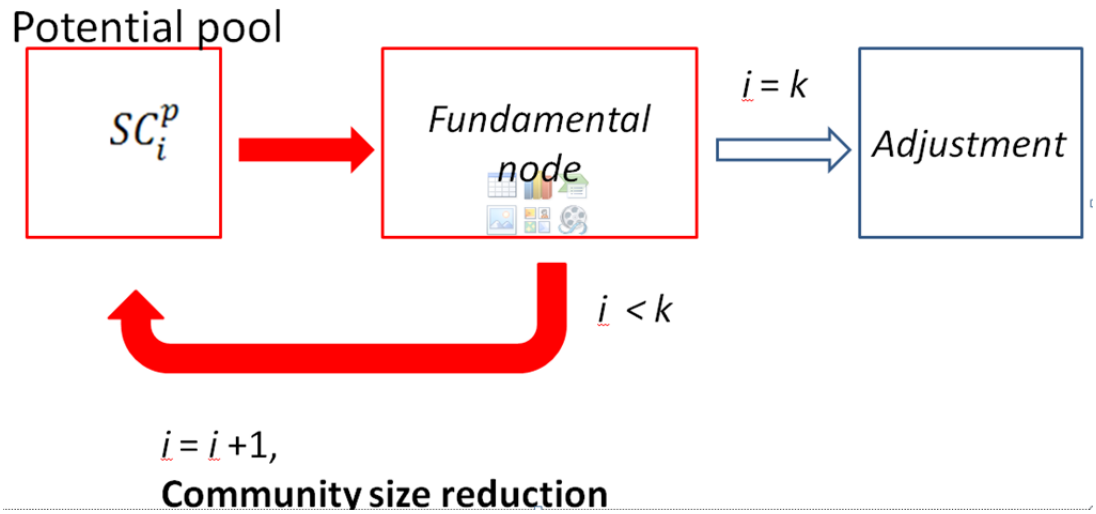


Fig 3.8. Flowchart of selection phase. Assume select *k* seeds. We have *k* iterations of selecting fundamental node. Before selecting the fundamental node of $\boldsymbol{SC_{i+1}^p}$, the size of each community covered by the fundamental node of $\boldsymbol{SC_i^p}$ has to reduce the degree of fundamental node of $\boldsymbol{SC_i^p}$. $PP(G) = \{\boldsymbol{SC_1^p}, \boldsymbol{SC_2^p}, \ldots, \boldsymbol{SC_k^p}\}$. $\boldsymbol{SC_i^p}$: top-p degree nodes in i-th largest community. *p*: 10% of community size. *i*= 1~*k*

We first select top-p degree nodes from SC_i, denoted as SC_i^p. p = 10% of SC_i's size is enough to select good fundamental nodes. Before selecting the fundamental node of SC_{i+1}^p, the size of each community covered by the fundamental node of SC_i^p has to reduce the degree of fundamental node of SC_i^p. As shown in Fig. 3.9, assume that node 7 is the fundamental node in C_1. The size of C_2 will be reduced to 3. Community size reduction is for reducing the influence overlapping. Fundamental nodes should "cover" communities as much as possible while having much influence on their communities. In Fig.3.10, node u belongs to community C_1, C_2 and C_3. That is, u covers C_1, C_2 and C_3.

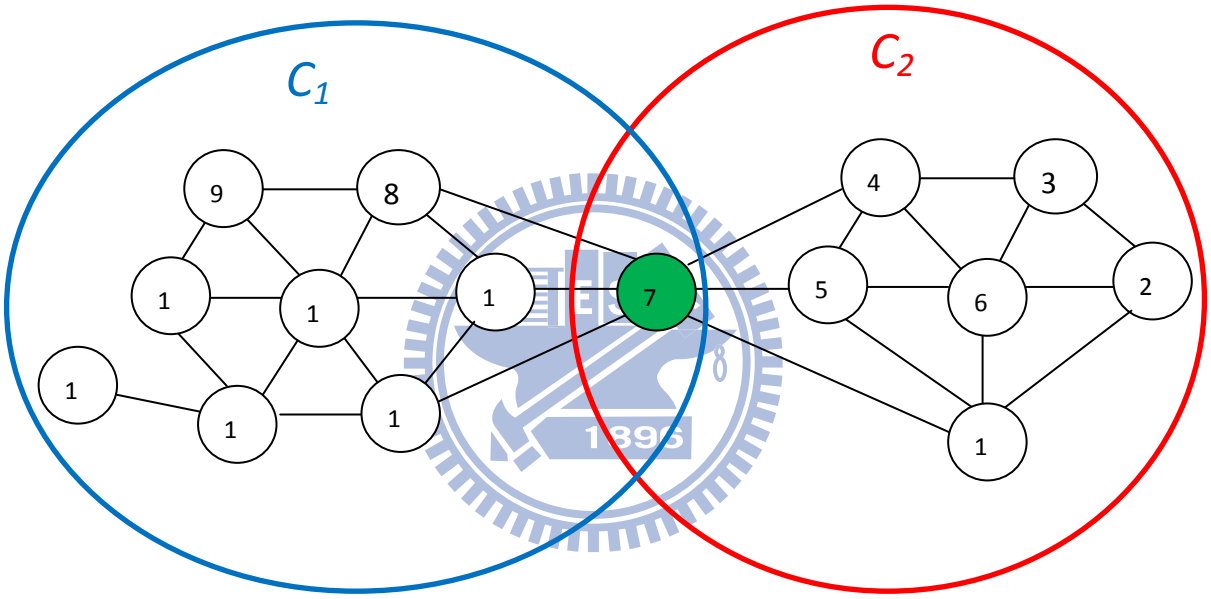


Fig 3.9 Illustration of community size reduction. We select node 7 as the fundamental node in C_1. The size of C_2 will be reduced to 3.

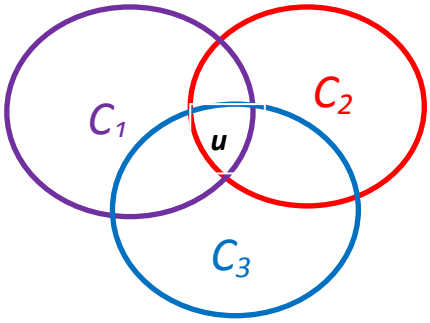


Fig. 3.10 An example of "cover". C_1, C_2 and C_3 are communities. Node u belongs to C_1, C_2 and C_3. That is, u covers C_1, C_2 and C_3.

To select good fundamental nodes, we define "**position_score**" as:

$$\text{position\_score}(u) = |\{C_i \mid u \in C_i, u \in G(V) and\ C_i \in \text{set of communities}\}|, \quad (3.2)$$

to evaluate the importance of node's position in network. If the node u is a nonhub, the position_score(u) is 1. Otherwise, the position_score(u) is the number of communities which u belongs to. We also define "**hub_purity**" as:

$$\text{hub\_purity}(h) = \frac{|\{C_i|\ h \in C_i,\ h \in hubs \text{ and } C_i \notin FC\}|}{\text{position\_score}(h)}, \quad (3.3)$$

where FC is the set of communities which contain fundamental nodes and h is a hub. In Fig. 3.11, C_1 and C_2 are communities containing fundamental node u. C_1, C_2 and C_3 are communities containing node z. C_2 and C_4 are communities containing node v. Therefore, purity(z) = 1/3 and purity(v) = 1/2.

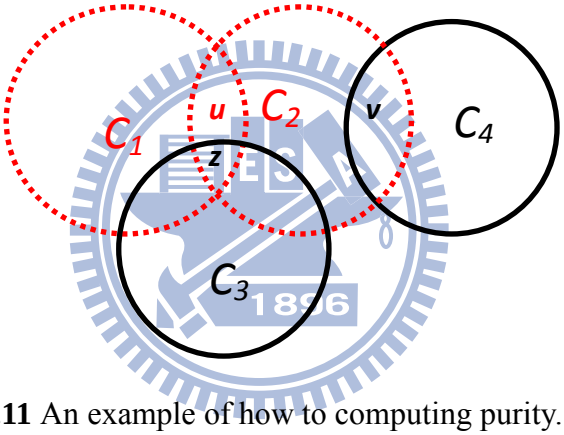


Fig 3.11 An example of how to computing purity. C_1, C_2, C_3 and C_4 are communities. Node u is the fundamental nodes. Purity(z) = 1/3 and purity(v) = 1/2

We choose the *"MAX priority"* nodes from SC_i^p as fundamental nodes, i = 1,2..,k. Selecting fundamental nodes in CDH-Shrink is different from that in CDH-Kcut. ***Function compare_priority*** shows how to compare *priority* of nodes. If both nodes are hub, we compare their position_score other than their degree. We compare hub with hubsize since we want to cover more communities. That is, we want to choose the hub which has important positions in the network. Besides, if the node is a hub, its purity must exceed the threshold of purity. We do not want to select low-*purity* fundamental nodes to reduce information overlapping. The hub with low purity easily covers too many covered communities, and this

hub, consequently, has the lowest priority while comparing purity. When comparing nonhub with hub or nonhub, we compare their influence on their neighbors, i.e., degree, due to no information about importance of location of non_hubs. After ***comparing*** top-p degree nodes in SC_i , we can find the fundamental node . The fundamental node may have a very good position which connecting SC_i with many other communities or have much influence on their neighbors or have both. $S = \{s_1, s_2, \ldots, s_k\}$ is the set of seed candidates s_i. Fundamental nodes are seed candidates

Finally, we adjust fundamental nodes to be final seeds. Adjustment in CDH-Shrink is also a heuristics. Try to choose an add-node to replace a delete-node. Then test whether the influence spread after replacing is larger than that before replacing. **left** and **seedLoad** play important roles in adjustment. *left* and *seedLoad* help us to determine add-nodes and delete-nodes . We define "left" and " seedLoad" as:

$$\text{left}(C_i) = \text{the number of non-activated nodes in community } C_i, \qquad (3.4)$$

$$\text{seedLoad}(C_i) = \frac{\text{size}(C_i)}{|\{v \mid v \in C_i, v \in S\}|}, \qquad (3.5)$$

Left(C_i) might be thought of as "the need of adding more seeds in C_i ". As left(C_i) is increasing, the need of selecting more seeds in C_i is increasing. Implied in the seedLoad(C_i) is whether too many seeds in C_i. When seedLoad(C_i) is small, that, perhaps, means too many seeds in C_i..

In each iteration, we select the add-node *a_node* , where

$$a\_node = u \mid \max\{\textstyle\sum_{i=1}^{t} \text{left}(C_i | u \in C_i),\ u \in G(V), t = \text{position\_score}(u)\}\ .$$

In the meanwhile, we select a delete-community *d_comm*, which has the smallest seedLoad among *SC*, SC $= \{SC_1, SC_2, \ldots SC_k\}$. Then select delete-node *d_node* which has minimun size(A(*d_node*)) among all seeds in *d_comm*. $A(u)$ is a set of active nodes adjacent to node u. We test if we should substitute add-node *a_node* for delete-node *d_node*. If the influence spread after substitution is more than that before, we make a substitution. To quickly find a

productive *a_node*, we do not consider very low degree node. We could assume selecting low degree nodes as seeds is not productive. Lines between 15 and 34 in CDH-Shrink show the details of adjustment. The adjustment has *r* iterations to test the substitution. In most cases, r = 2*k*~3*k* is efficient to get satisfactory influence spread.

Function compare_priority (node *a*, node *b*)

1. **if** *a* is hub and hub_purity(*a*) < *purity_threshold*
2. return *b*
3. **if** *a* is hub and *b* is hub **then**
4. return max(position_score(*a*), position_score (*b*))**;**
5. **else if** *a* is nonhub **then**
6. return max(degree(*a*), degree(*b*));
7. **end**

Algorithm 2: CDH-Shrink

Input : Graph of social network *G* ; number of total seeds *k*, Parameters *p*, *purity_threhold* , adjustment time *r*

Output: *k* seeds

1. Execute the Shrink(*G*);
2. **while** |*SC*| < *k* **do**
3. Add the biggest community SC_i into *SC*;
4. // select the fundamental node
5. **foreach** top-*p* degree nodes in SC_i **do**
6. //n_i : the *i*-th largest degree node in SC_i
7. *maxnode* = compare(n_i, *maxnode*);
8. **end**
9. S_i = *maxnode***;**
10. **foreach** community *Ci* which has *max* **do**
11. Size(C_i) = Size(C_i) – degree(*max*);
12. **end**
13. **end**
14. //adjustment
15. *IM*= $I_s(t)$; // $I_s(t)$: influence spread of current seed set, *IM* : record max influence spread
16. **for** 1 to *r* **do**
17. select *a_node* = $u \mid \max\{(\sum_{i=1}^{t} \text{left}(C_i | u \in C_i)), u \in G(V),$
18. $t = \text{position\_score}(u)\}$;
19. select $d\_comm = \text{argmin}_{C_i \in SC}\, seedLoad(C_i)$
20. select $d\_node = \text{argmin}_{u \in s, u \in d_{comm}} |\{v \mid v \in A(u), A(u) \subseteq G(V)\}|$

```
21.          delete d_node in S and add a_node into S ;
22.          if I_s(t) < IM then
23.               Cancel the replacement between line18 to 22.
24.          end
25.          IM = I_s(t) ;
26.     end
27.     Output individuals in S
```

3.4 Time Complexity of Approximation Algorithms

We now consider the time complexity of CDH-Shrink, CDH-Kcut and enhance greedy algorithm [5]. Suppose that a social network is composed of N individuals and M edges. The time complexity of heat diffusion process is $O(RM)$[5], which means the number of iterations R multiplied by the number of edges M in a social network. In most cases, $R = 30$ is enough for approximating the heat diffusion process. We select k seeds. The complexity of each algorithm is as follows:

For CDH-Kcut, l is the number of communities. Each top-k community selects top-p degree nodes in potential pool. The partition phase in CDH-Kcut is O(Mlog$N$). Assume average number of nodes in a community is $\frac{N}{l}$. Constructing the potential pool is O($kp\frac{N}{l}$). Finding fundamental nodes in potential pool is O(kp). Finding large communities is O(k). Assume we have b large communities. Sorting large communities is O($b$log$b$). Adjustment in CDH-Kcut is O(kRM). Therefore, the time complexity is $O(M\log N + kp\frac{N}{l} + kp + k + b\log b + kRM)$.

For CDH-Shrink, assume the number of adjustment iterations is r, and the average community number of a node is d. The partition phase is O(Mlong$N$). The time complexity of community size reduction is O(d), so finding fundamental nodes in potential pool is O($k(l + p\frac{N}{l}+d)$). In the adjustment, selecting add_node is O(Nd+N), selecting delete_community is O(k), and selecting delete_node is O(k). Therefore, the time complexity of adjustment in

CDH-Shrink is $r(Nd + N + k + k + RM)$. The total time complexity is $O(M\log N + k(l + p\frac{N}{l} + d) + r(Nd + N + k + k + RM))$.

The time complexity of greedy algorithm in enhance greedy algorithm is $O(kNCM)$ since selecting a seed is $O(NRM)$ and we have to select k seeds. In most cases, $r = 2k \sim 3k$ is enough for adjustment. We could see that in terms of time complexity, the ranking is CDH-Shrink ＝ CDH-Kcut ＞ greedy algorithm in [5].

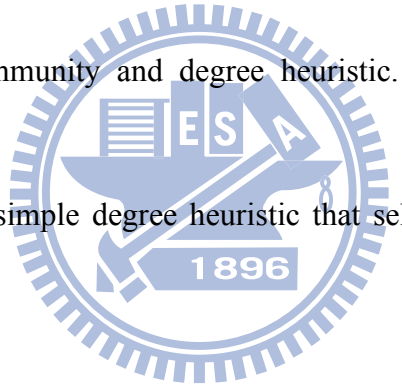
Chapter 4

Experiments

To measure the performance of our proposed algorithms, we conduct experiments on a co-authorship network [26], the zachary's karate network from Newman [25], the network of facebook, and two synthetic networks. The goal of the experiments is to show that our algorithms are very efficient and with satisfying influence spread.

We run the following set of algorithms under heat diffusion model.

- **EGA**: the original enhanced greedy algorithm [5].
- **CDH-Kcut:** the community and degree heuristic. Community detection algorithm used is Kcut. .
- **CDH-Shrink:** the community and degree heuristic. Community detection algorithm used is SHRINK.
- **DH:** As a baseline, a simple degree heuristic that selects the k nodes with the largest degrees.



The performance metrics of the algorithms compared and the parameter setting are listed below.

1. Influence spread (number of activated nodes)
2. Efficiency (running time)
3. Effect of different values of parameters
   - t : flow duration
   - θ : activation threshold
   - α :thermal conductivity

4.1 Synthetic Networks

For synthetic datasets, we use the Lancichinetti-Fortunato-Radicchi (LFR) benchmark graphs [19, 20] to evaluate the performance of our algorithms. Some important parameters of the synthetic networks are:

- *N*: number of nodes
- *M*: number of edges
- *maxd*: maximum degree
- *mp*: mixing parameter, each nodes shares a fraction mp of its edges with nodes in other communities.

As shown in Table 4.1, we generate five different undirected graphs : (1) 1000*Smp* : the graph with 1000 nodes and small mixing parameter; (2) 1000*Lmp* : the graph with 1000 nodes and large mixing parameter; (3) 1000*Lmaxd* : the graph with 1000 nodes and large maximum degree; (4) 1000*LM* : the graph with 1000 nodes and large number of degree; (5) 5000*Smp* : the graph with 5000 nodes and small mixing parameter. Generally, the higher the mixing parameter of a network is, the more difficult to reveal the community structure.

Table 4.1: The parameters of the omputer-generated datasets for performance evaluation

Dataset	*N*	*M*	*maxd*	*mp*
1000*Smp*	1000	9097	100	0.1
1000*Lmp*	1000	9097	100	0.5
1000*Lmaxd*	1000	9097	200	0.1
1000*LM*	1000	22484	100	0.1
5000*Smp*	5000	47094	100	0.1

Table 4.2 provides the result of different algorithms with different activation threshold, flow duration and thermal conductivity in 1000*Smp*. The influence spreads of CDH-Shrink and CDH-Kcut from θ = 0.2 to θ = 1.4 are almost the same or slightly different. Thus, we only report θ = 0.2, θ=1.5 and θ = 2.0. We can see that CDH-Shrink and CDH-Kcut have same influence spread in most cases and even are better than EGA with θ = 2.0. Fig. 4.1 shows 5 seed selected by CDH-Kcut with t=0.1, θ=0.1, α=0.1 in *1000Smp*. Fig. 4.2 shows 5 seed selected by CDH-Kcut with t=0.1, θ=0.1, α=0.1 in *1000Smp*. We could see that higher activation threshold leads to the phenomenon of seed clustering.

Table 4.2: Influence spread with 4 different algorithms in 1000*Smp*. t is flow duration. θ is activation threshold. α is thermal conductivity

	network	**EGA**	**CDH-Shrink**	**CDH-Kcut**	**DH**
***t*=0.1, *θ*=0.1, *α*=0.1**	*1000Smp*	341	339	339	215
***t*=0.1, *θ*=0.2, *α*=0.1**	*1000Smp*	336	332	332	215
***t*=0.1, *θ*=1.5, *α*=0.1**	*1000Smp*	223	201	197	95
***t*=0.1, *θ*=2.0, *α*=0.1**	*1000Smp*	133	141	166	95
***t*=0.2, *θ*=0.1, *α*=0.1**	*1000Smp*	386	367	345	336
***t*=0.3, *θ*=0.1, *α*=0.1**	*1000Smp*	503	482	499	472
***t*=0.4, *θ*=0.1, *α*=0.1**	*1000Smp*	635	594	649	567
***t*=0.1, *θ*=0.1, *α*=0.2**	*1000Smp*	386	367	345	336
***t*=0.1, *θ*=0.1, *α*=0.3**	*1000Smp*	503	482	499	472
***t*=0.1, *θ*=0.1, *α*=0.4**	*1000Smp*	635	594	649	567

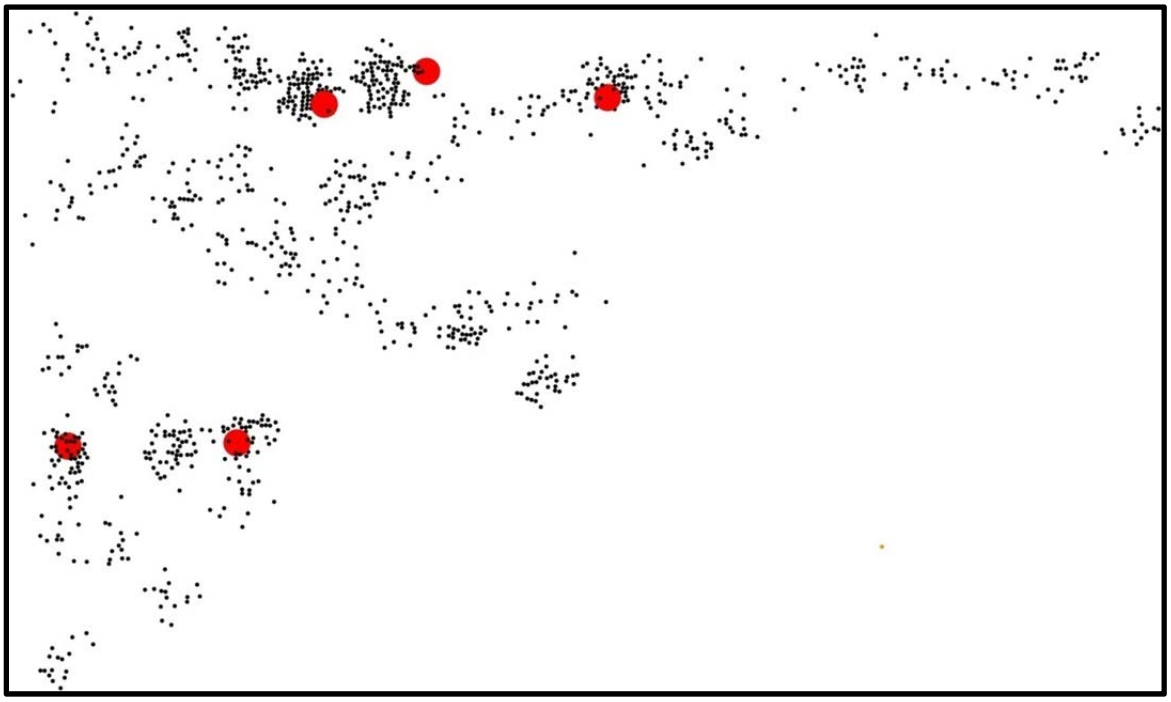


Fig 4.1 5 Seeds (red nodes) selected by CDH-Kcut with t=0.1, θ=0.1, α=0.1 in *1000Smp*.

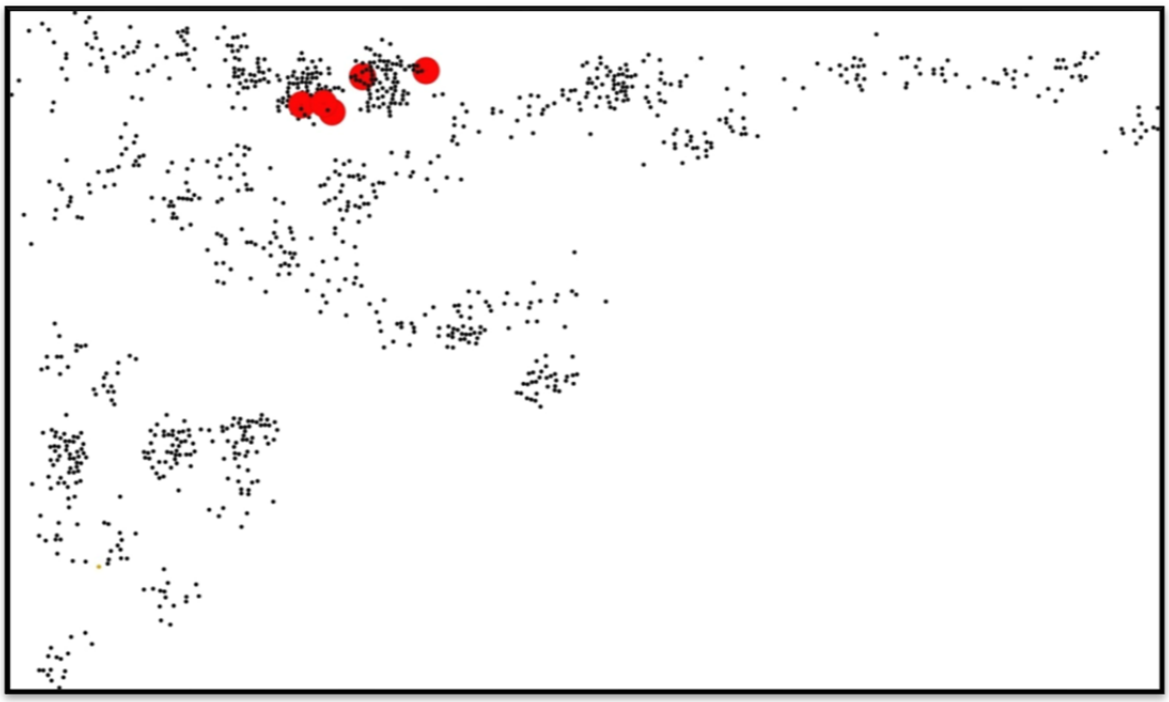


Fig 4.2 5 Seeds (red nodes) selected by CDH-Kcut with t=0.1, θ=2.0, α=0.1 in *1000Smp*.

Table 4.3: Influence spread with 4 different algorithms in **1000*Lmaxd*.**
t is flow duration. θ is activation threshold. α is thermal conductivity

	network	**EGA**	**CDH-Shrink**	**CDH-Kcut**	**DH**
t=0.1, θ=0.1, α=0.1	*1000Lmp*	251	223	213	206
t=0.1, θ=0.2, α=0.1	*1000Lmp*	225	182	175	187
t=0.1, θ=1.5, α=0.1	*1000Lmp*	176	157	149	105
t=0.1, θ=1.6, α=0.1	*1000Lmp*	137	121	93	56
t=0.2, θ=0.1, α=0.1	*1000Lmp*	562	491	467	484
t=0.3, θ=0.1, α=0.1	*1000Lmp*	790	734	712	722
t=0.4, θ=0.1, α=0.1	*1000Lmp*	892	841	813	829
t=0.1, θ=0.1, α=0.2	*1000Lmp*	562	491	467	484
t=0.1, θ=0.1, α=0.3	*1000Lmp*	790	734	712	722
t=0.1, θ=0.1, α=0.4	*1000Lmp*	892	841	813	829

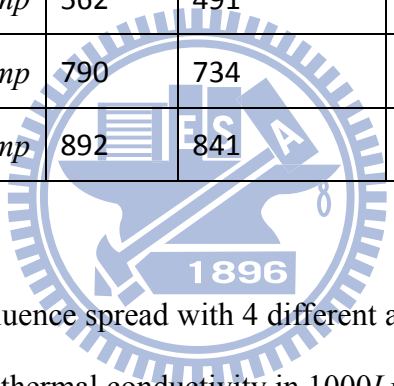
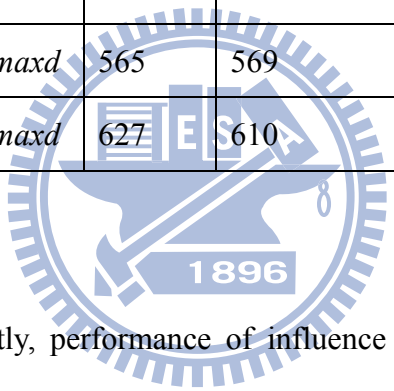


Table 4.3 shows the influence spread with 4 different algorithms with different activation threshold, flow duration and thermal conductivity in 1000*Lmp*. CDH-Kcut performs worse than CDH-Shrink in 1000*Lmp* since SHRINK could detect more accurate community structure than Kcut. Hence, if it is hard to get correct community structure of the graph, CDH-Shrink will probably perform better than CDH-Kcut. Besides, accuracy of detected community structure reflects the performance of influence spread of CDH-Shrink and CDH-Kcut. Therefore, with increasing of mixing parameter, the performance of influence spread of CDH-Shrink and CDH-Kcut have deteriorated.

Table 4.4 indicates the influence spread with 4 different algorithms with different activation threshold, flow duration and thermal conductivity in 1000*Lmaxd*. Nodes in *1000Lmaxd* could have larger degree. That is, some nodes' degree will be extremely larger

Table 4.4: Influence spread with 4 different algorithms in **1000*Lmaxd*.**
t is flow duration. θ is activation threshold. α is thermal conductivity

	network	**EGA**	**CDH-Shrink**	**CDH-Kcut**	**DH**
t=0.1, θ=0.1, α=0.1	*1000Lmaxd*	332	315	298	202
t=0.1, θ=0.2, α=0.1	*1000Lmaxd*	290	278	259	196
t=0.1, θ=1.5, α=0.1	*1000Lmaxd*	170	151	143	146
t=0.1, θ=1.6, α=0.1	*1000Lmaxd*	138	110	113	136
t=0.2, θ=0.1, α=0.1	*1000Lmaxd*	494	493	473	299
t=0.3, θ=0.1, α=0.1	*1000Lmaxd*	565	569	561	404
t=0.4, θ=0.1, α=0.1	*1000Lmaxd*	627	610	599	482
t=0.1, θ=0.1, α=0.2	*1000Lmaxd*	494	493	473	299
t=0.1, θ=0.1, α=0.3	*1000Lmaxd*	565	569	561	404
t=0.1, θ=0.1, α=0.4	*1000Lmaxd*	627	610	599	482



than the others. Consequently, performance of influence spread of DH will be improved, especially in high activation threshold.

Table 4.5 shows the influence spread with 4 different algorithms with different activation threshold, flow duration and thermal conductivity in *1000LM*. In *1000LM*, each node has more neighbors, so information will spread quickly. Hence, we could see that influence spread in 1000*LM* is higher than that in *1000Smp*, *1000Lmp* and *1000Lmaxd*. In most cases, the influence spreads of CDH-Shrink and CDH-Kcut are still better than DH.

Table 4.6 indicates that influence spread with 4 different algorithms with different activation threshold, flow duration and thermal conductivity in *5000Smp*. *5000Smp* has 5000 nodes. In most cases, the influence spread of CDH-Shrink and CDH-Kcut are still better than DH.

Table 4.5: Influence spread with 4 different algorithms in **1000*LM*.**
t is flow duration. θ is activation threshold. α is thermal conductivity

	network	**EGA**	**CDH-Shrink**	**CDH-Kcut**	**DH**
***t*=0.1, θ=0.1, α=0.1**	*1000LM*	663	599	599	521
***t*=0.1, θ=0.2, α=0.1**	*1000LM*	506	439	437	316
***t*=0.1, θ=0.3, α=0.1**	*1000LM*	430	415	402	246
***t*=0.1, θ=1.5, α=0.1**	*1000LM*	226	224	218	184
***t*=0.2, θ=0.1, α=0.1**	*1000LM*	923	848	838	816
***t*=0.3, θ=0.1, α=0.1**	*1000LM*	991	929	936	939
***t*=0.4, θ=0.1, α=0.1**	*1000LM*	1000	993	997	996
***t*=0.1, θ=0.1, α=0.2**	*1000LM*	923	848	838	816
***t*=0.1, θ=0.1, α=0.3**	*1000LM*	991	929	936	939
***t*=0.1, θ=0.1, α=0.4**	*1000LM*	1000	993	997	996

Table 4.6: Influence spread with 4 different algorithms in *5000Smp*.
t is flow duration. θ is activation threshold. α is thermal conductivity

	network	**EGA**	**CDH-Shrink**	**CDH-Kcut**	**DH**
***t*=0.1, θ=0.1, α=0.1**	*5000Smp*	540	467	456	391
***t*=0.1, θ=0.2, α=0.1**	*5000Smp*	438	401	376	252
***t*=0.1, θ=0.3, α=0.1**	*5000Smp*	397	373	326	210
***t*=0.1, θ=1.5, α=0.1**	*5000Smp*	261	235	212	169
***t*=0.2, θ=0.1, α=0.1**	*5000Smp*	1258	1138	1102	843
***t*=0.3, θ=0.1, α=0.1**	*5000Smp*	1284	1175	1113	877
***t*=0.4, θ=0.1, α=0.1**	*5000Smp*	1451	1323	1278	952
***t*=0.1, θ=0.1, α=0.2**	*5000Smp*	1258	1138	1102	843
***t*=0.1, θ=0.1, α=0.3**	*5000Smp*	1284	1175	1113	877
***t*=0.1, θ=0.1, α=0.4**	*5000Smp*	1451	1323	1278	952

In summary, CDH-Shrink, in general, is more productive than CDH-Kcut. With the increasing of activation threshold, seeds will cluster together to trigger larger influence spread. Figure 6.1 and Fig 6.2 indicate the seed clustering phenomenon in high activation threshold, where red nodes are seeds.

4.2 Zachary's Karate Network

Zachary's karate network [25] consists of 34 nodes and 78 edges. Nodes present the members of a karate club in the United States, who were observed during a period of three years. Edges connect individuals who were observed to interact outside the activities of the club. Since Zachary's karate network is a very small network, we only demonstrate the influence spread of different algorithms.

As shown in Fig 4.3, node 0 and 33 are two fundamental nodes if we select two seeds in total. In most cases, the two selected seeds are 0 and 33, so Table 4.7 only lists 5 settings of parameters and the seeds selected by different algorithms in Zachary's karate network. The number in the parentheses is the selected seeds. CDH-Shrink and CDH-Kcut could select seeds according to different value of parameters, DH could not. Consequently, CDH-Shrink and CDH-Kcut get the same influence spread as **EGA** in most cases. Furthermore, in some cases like t = 0.4, θ =0 .6, α = 0.1, CDH strategy gets better influence spread. We could see that the two seeds with t = 0.1, θ = 0.2 and α = 0.1 are 32 and 33. That is, high activation threshold easily cause the phenomenon of seed clustering. High thermal conductivity does not cause the phenomenon of seed clustering. Therefore, two seed with t = 0.1, θ = 0.2 and α = 0.2 are the same seed with t = 0.1, θ = 0.1 and α = 0.1.

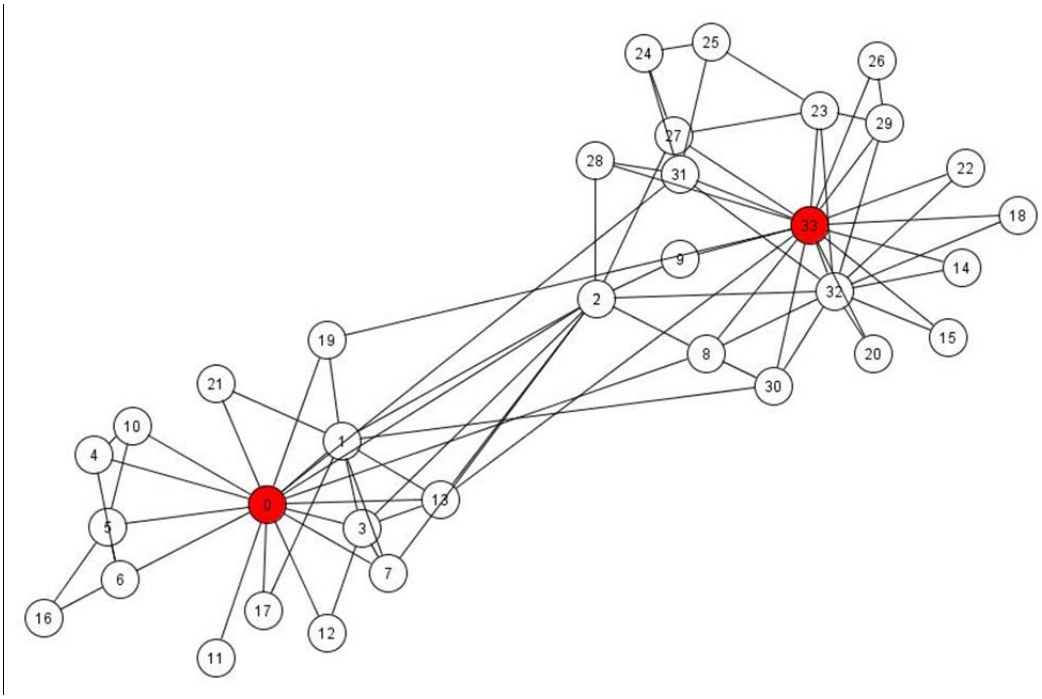


Fig 4.3 Zachary's karate network. If we select 2 seeds in this network. Node 0 and 33 are fundamental nodes in CDH-Kcut and CDH-Shrink.

Table 4.7 Two seeds selected by different algorithm in Zachary's karate network. t is time. θ is threshold. α is thermal conductivity

	EGA	**CDH-Shrink**	**CDH-Kcut**	**DH**
t=0.1, θ=0.1, α=0.1	31(0, 33)	31(0,33)	31(0,33)	31(0,33)
t=0.1, θ=0.2, α=0.1	12(32, 33)	12(32, 33)	12(32, 33)	6(0,33)
t=0.1, θ=0.3, α=0.1	12(32, 33)	12(32, 33)	12(32, 33)	6(0,33)
t=0.1, θ=0.2, α=0.2	31(0, 33)	31(0, 33)	31(0, 33)	31(0, 33)
t=0.4, θ=0.6, α=0.1	8(4,7)	12(32,33)	12(32, 33)	6(0,33)

4.3 A Collaboration Network

We extract a large real-life academic collaboration network from the e-print. Each node in the network represents an author. If an author *i* co-authored a paper with author *j*, the graph contains an undirected edge from *i* to *j*. If the paper is co-authored by *k* authors this generates a completely connected (sub)graph on *k* nodes. The coauthor network is the "High Energy Physics - Phenomenology collaboration network", denoted as NETHep, with papers in the period from January 1993 to April 2003 (124 months), which contains N=12008 nodes and M=237010 edges. The graph is available for download on the web at http://snap.stanford.edu/data.

On the large real collaboration network, NETHep, we report the efficiency and influence spread of EGA, CDH-Shrink, CDH-Kcut and DH with different numbers of seeds and values of parameters. In CDH-Shrink, we set purity threshold =0.35, which can get satisfying influence spread.

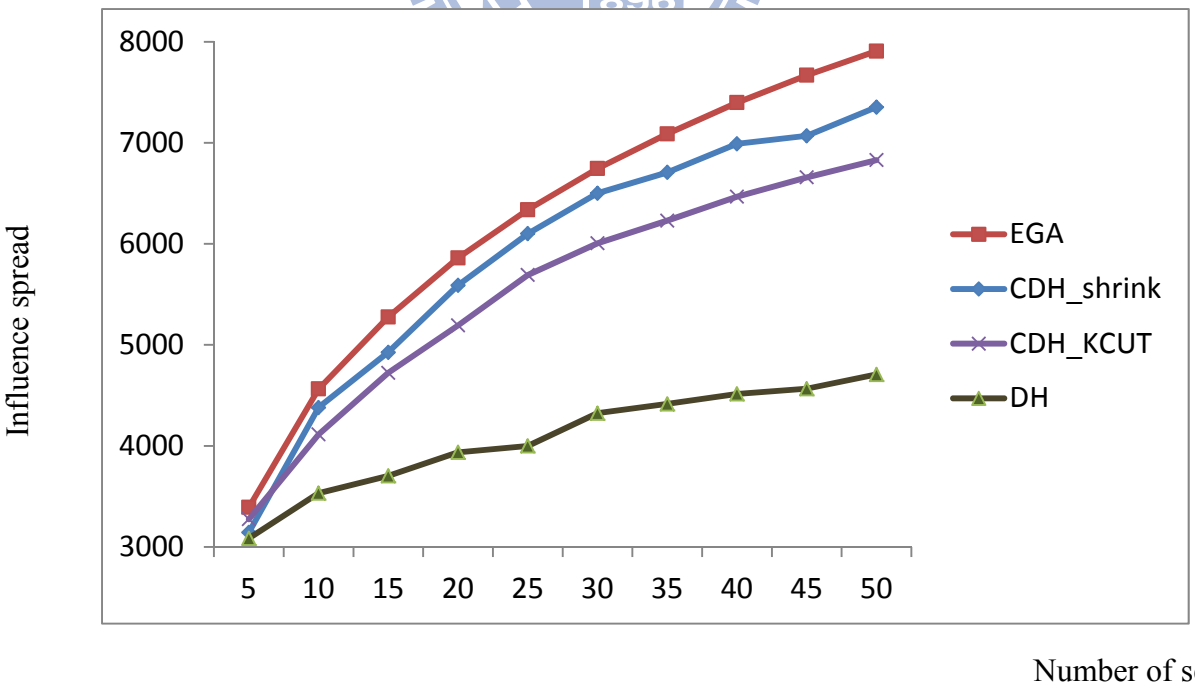


Fig4.4 influence spread of different algorithms on NETHep.
t=0.1, θ=0.1, α=0.1

Fig 4.4 (x-axis indicates the number of seeds and y-axis indicates influence spread) shows the influence spread of different algorithms with different number of seeds on NETHep. In most cases, EGA's influence spread > CDH-Shrink's influence spread > CDH-Kcut's influence spread > DH's influence spread. With the increasing number of seeds, CDH-Shrink and CDH-Kcut is getting better and better than DH since most seeds selected by DH are only in few communities.

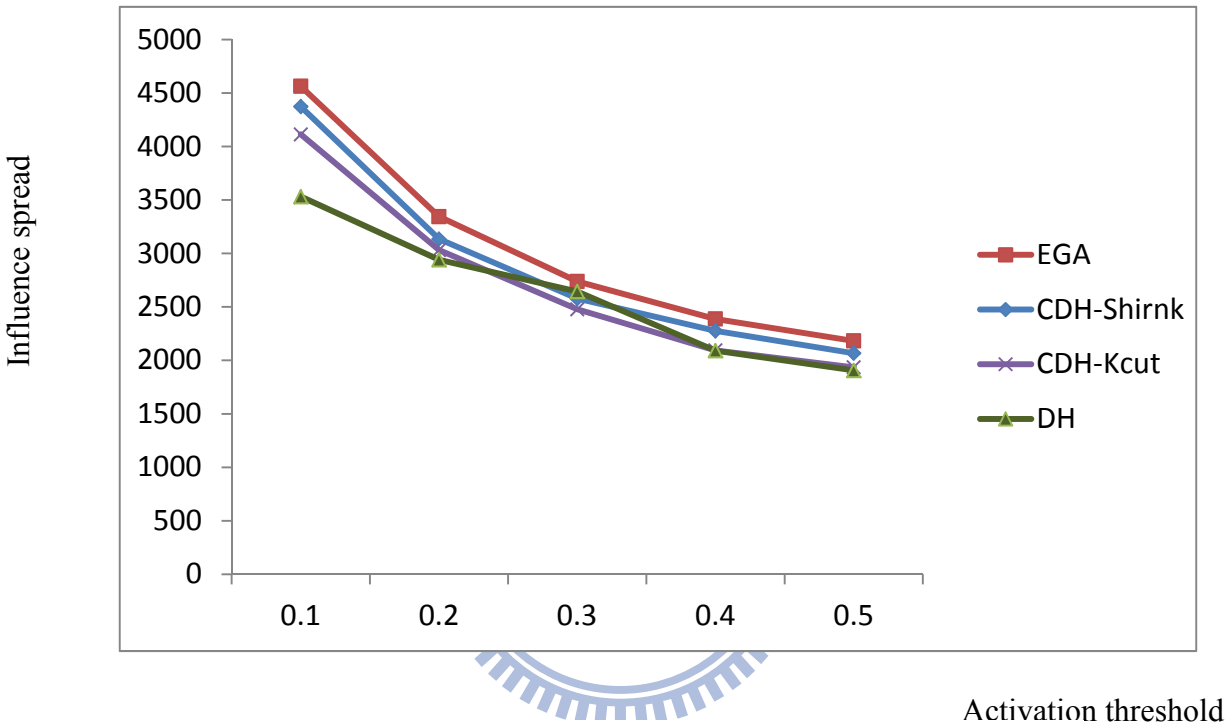


Fig 4.5 influence spread of different algorithms with different activation threshold on NETHep. t=0.1, α=0.1. Select 10 seeds.

Fig 4.5 (x-axis indicates activation threshold and y-axis indicates influence spread) indicates the influence spread of 10 seeds with different θ from 0.1 to 0.5 with a span of 0.1. Fig 4.6 (x-axis indicates activation threshold and y-axis indicates influence spread) indicates the influence spread of 30 seeds with different θ from 0.1 to 0.5 with a span of 0.1. The results reflected in Fig 4.5 and Fig 4.6 indicate that although the total influence in the spread four algorithms will decrease as θ increases, CDH-Shrink and CDH-Kcut still maintain great performance. Notice that DH improves its influence spread with the increasing of θ. That results from the effect of seed clustering in high θ. However, it is still worse than CDH-Shrink

and CDH-Kcut when selecting more seeds.

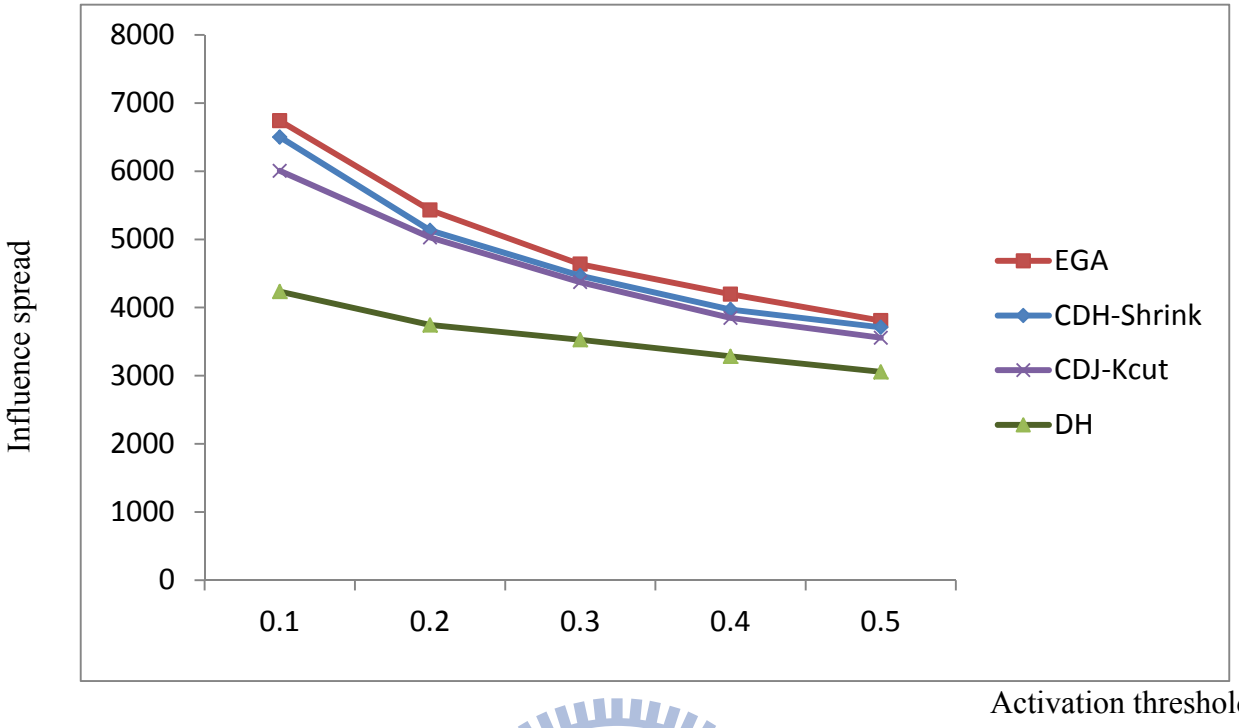


Fig 4.6 influence spread of different algorithms with different activation threshold on NETHep. θ=0.1, α=0.1. Select 30 seeds

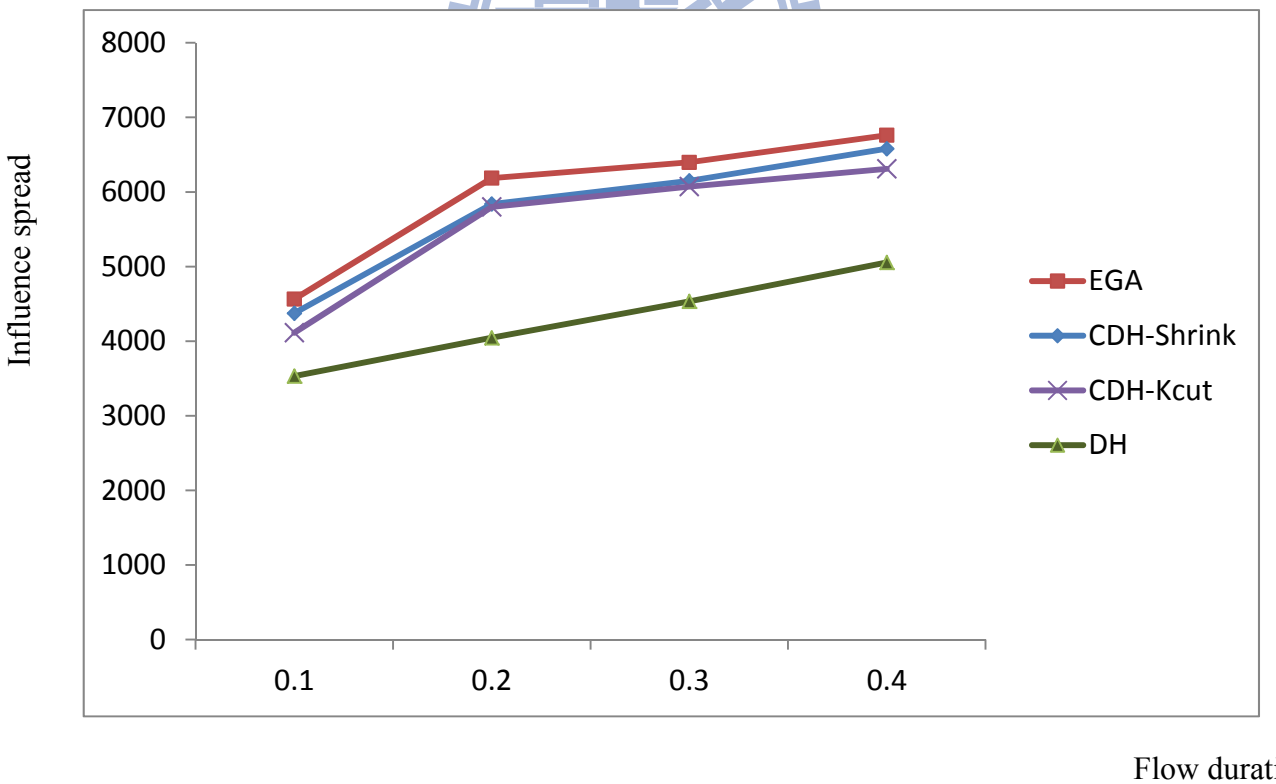


Fig 4.7 influence spread of different algorithms with different flow duration on NETHep. θ=0.1, α=0.1. Select 10 seeds

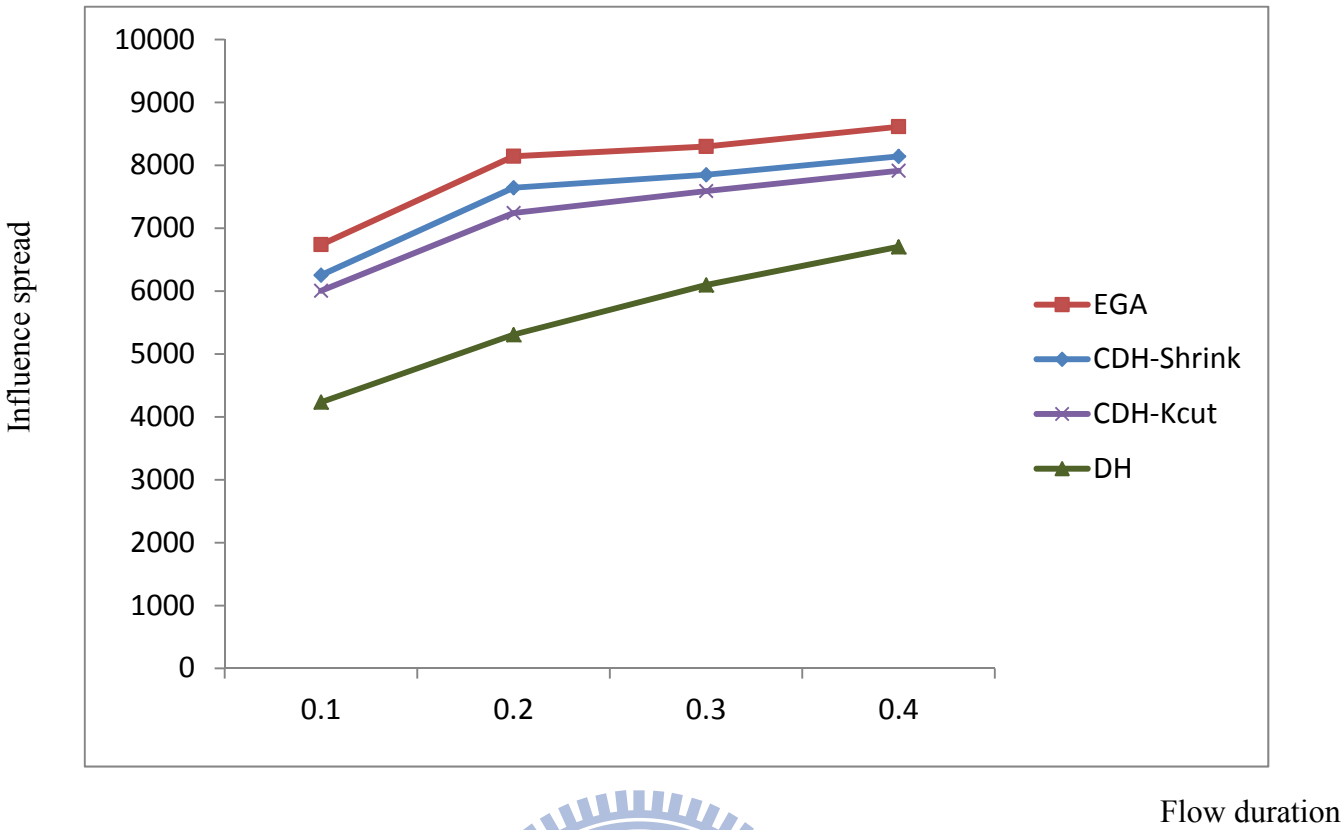


Fig. 4.8 influence spread of different algorithms with different time on NETHep. θ=0.1, α=0.1. Select 30 seeds

Fig 4.7 (x-axis indicates flow duration and y-axis indicates influence spread) indicates the influence spread of 10 seeds with different t from 0.1 to 0.4 with a span of 0.1. Fig 4.8 (x-axis indicates flow duration and y-axis indicates influence spread) indicates the influence spread of 30 seeds with different t from 0.1 to 0.4 with a span of 0.1. Fig 4.7 and Fig 4.8 (x-axis indicates time and y-axis indicates influence spread) show that our proposed algorithms still maintain good influence spread with the increasing of t. We only report results from t = 0.1 to t = 0.4 since too large t will lead to the situation that most nodes are influenced, and thus we cannot easily distinguish the performance of the four algorithms.

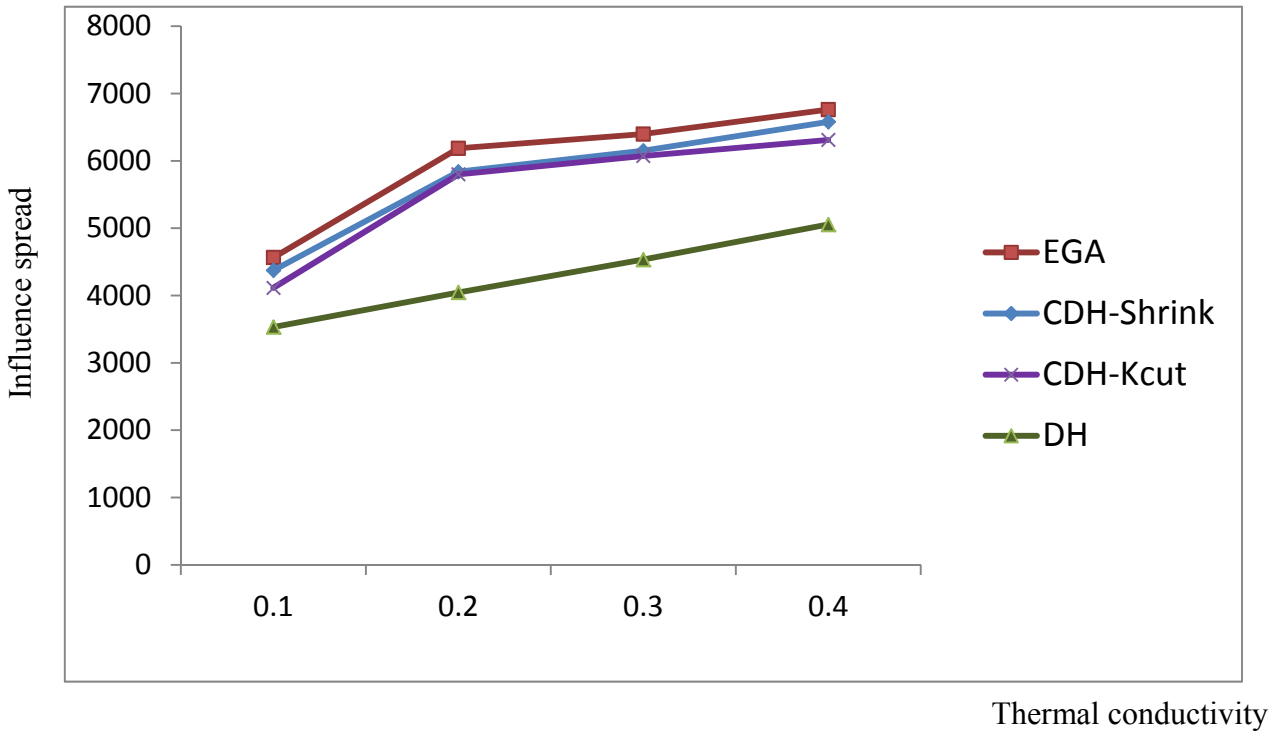


Fig. 4.9 influence spread of different algorithms with different thermal conductivity on NETHep. t=0.1, θ=0.1. Select 10 seeds

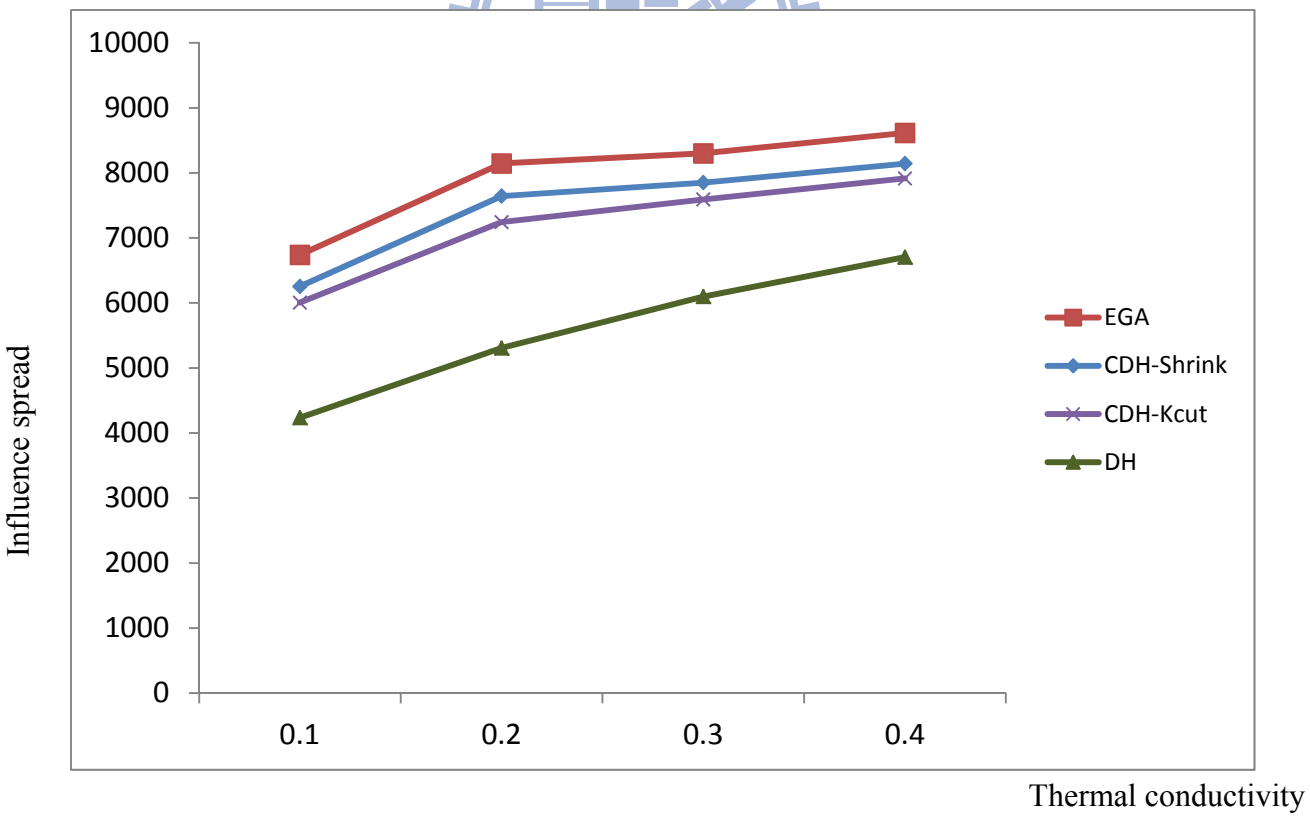


Fig 4.10 influence spread of different algorithms with different thermal conductivity on NETHep. t=0.1, θ=0.1. Select 30 seeds

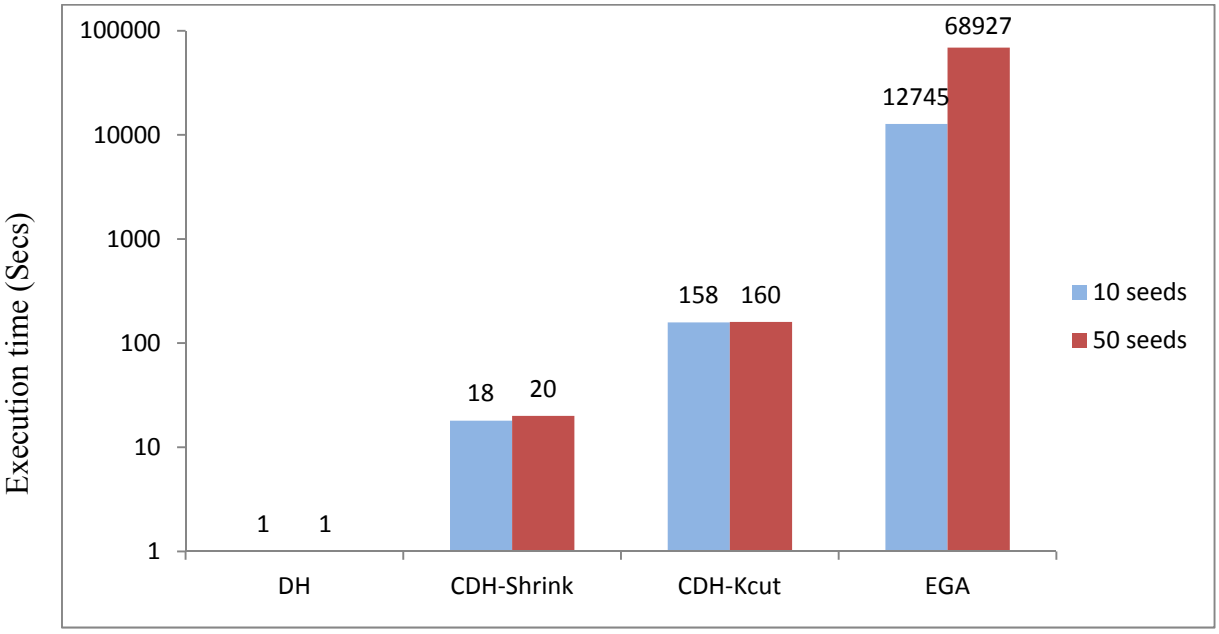


Fig. 4.11 Running time of different algorithms on the collaboration graph NETHep. Selecting 10 seeds and 50 seeds respectively

Fig 4.9 (x-axis indicates thermal conductivity and y-axis indicates influence spread) indicates the influence spread of 10 seeds with different thermal conductivity α from 0.1 to 0.4 with a span of 0.1. Fig 4.10 (x-axis indicates thermal conductivity and y-axis indicates influence spread) indicates the influence spread of 30 seeds with different thermal conductivity α from 0.1 to 0.4 with a span of 0.1. As shown in Fig 4.9 and Fig 4.10(x-axis indicates thermal conductivity and y- axis indicates influence spread), CDH-Shrink and CDH-Kcut remain good influence spread even if in different α. The reason why maximal value of α is 0.4 is the same as that in comparing influence spread with different flow duration. That is, we cannot easily distinguish the performance of four algorithms with too large α.

We present the efficiency of four algorithms in Fig 4.11 (10 and 50 seeds respectively, x-axis indicates different algorithms and y-axis (logarithmic scale) indicates execution time). Since DH only needs to select top-k degree nodes as seeds, DH is extremely efficient. CDH-Shrink has least running time among CDH-Shrink, CDH-Kcut and EGA. We also can see that the running time of EGA is proportional to the number of seeds. Execution time of CDH-Shrink and CDH-Kcut are only slightly different between 10 seeds and 50 seeds. This is because CDH-Shrink and CDH-Kcut only have to spend a little more time on adjustment

when the number of nodes increases.

4.4 Facebook Network

We extract one large real-life networks from Facebook. Each node in the network represents a user. If user *i* is a friend of user *j*, the graph contains an undirected edge from *i* to *j*. The network is denoted as FB, in the period from April 2004 to 2009 January (124 months), which contains N =63731 nodes and M = 817090 edges.

FB is a large real network. Thus, we report not only the efficiency but also influence spread of our algorithms with respect to different numbers of seeds and values of parameters. We set purity threshold =0.2, which get satisfying influence spread in FB.

Fig 4.12 (x-axis indicates the number of seeds and y-axis indicates influence spread) shows the influence spread of different algorithms with different number of seeds on FB. As shown in Fig 4.12, in most cases EGA's influence spread > CDH-Shrink's influence spread > CDH-Kcut's influence spread > DH's influence spread. Since EGA is too time-consuming, we only report the influence spread from 5 seeds to 30 seeds.

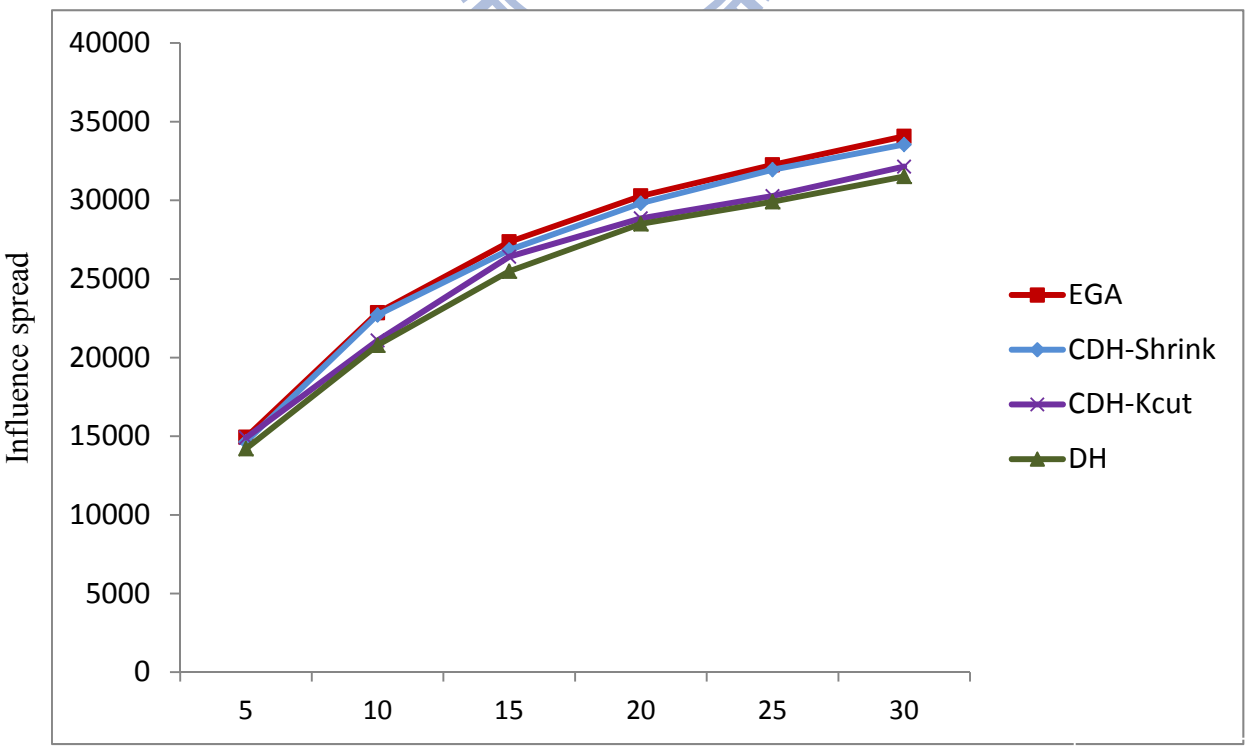


Fig 4.12 influence spread of different algorithms on FB. t=0.1, θ=0.1, α=0.1

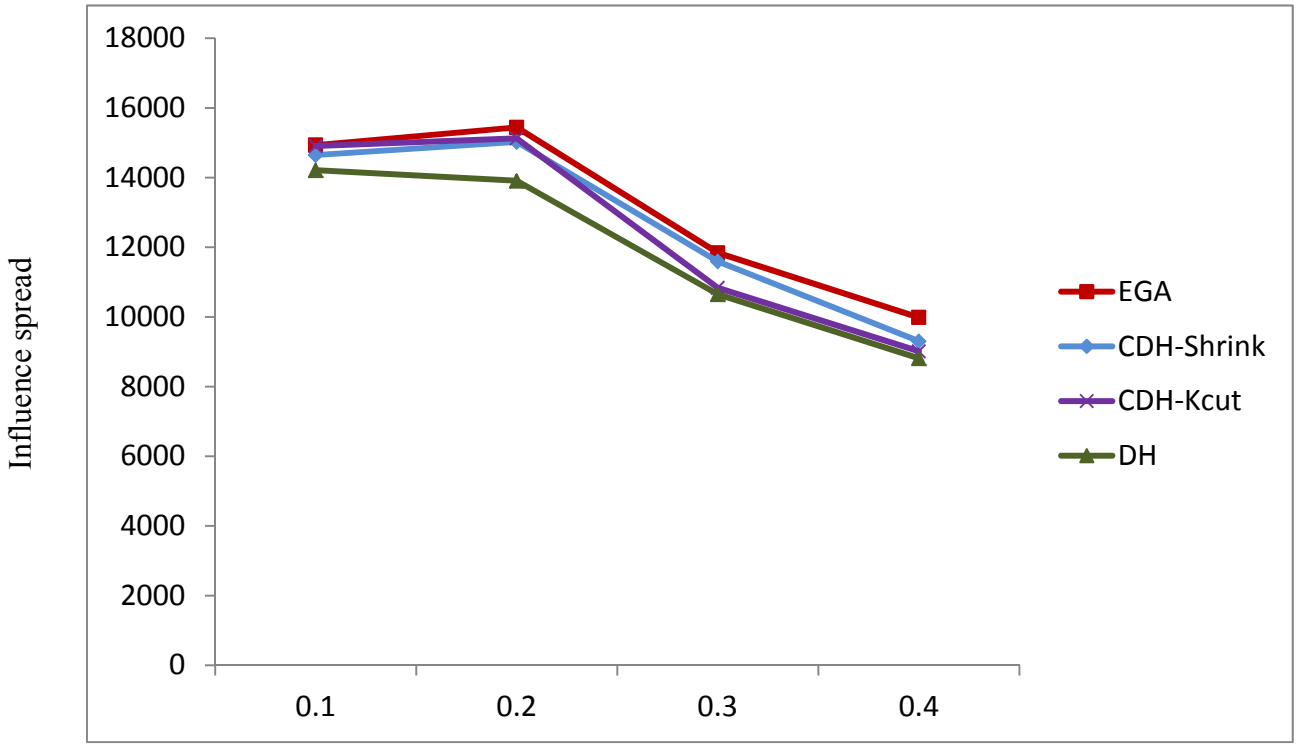


Fig 4.13 influence spread of different algorithms with different activation threshold on FB. θ=0.1, α=0.1. Select 10 seeds

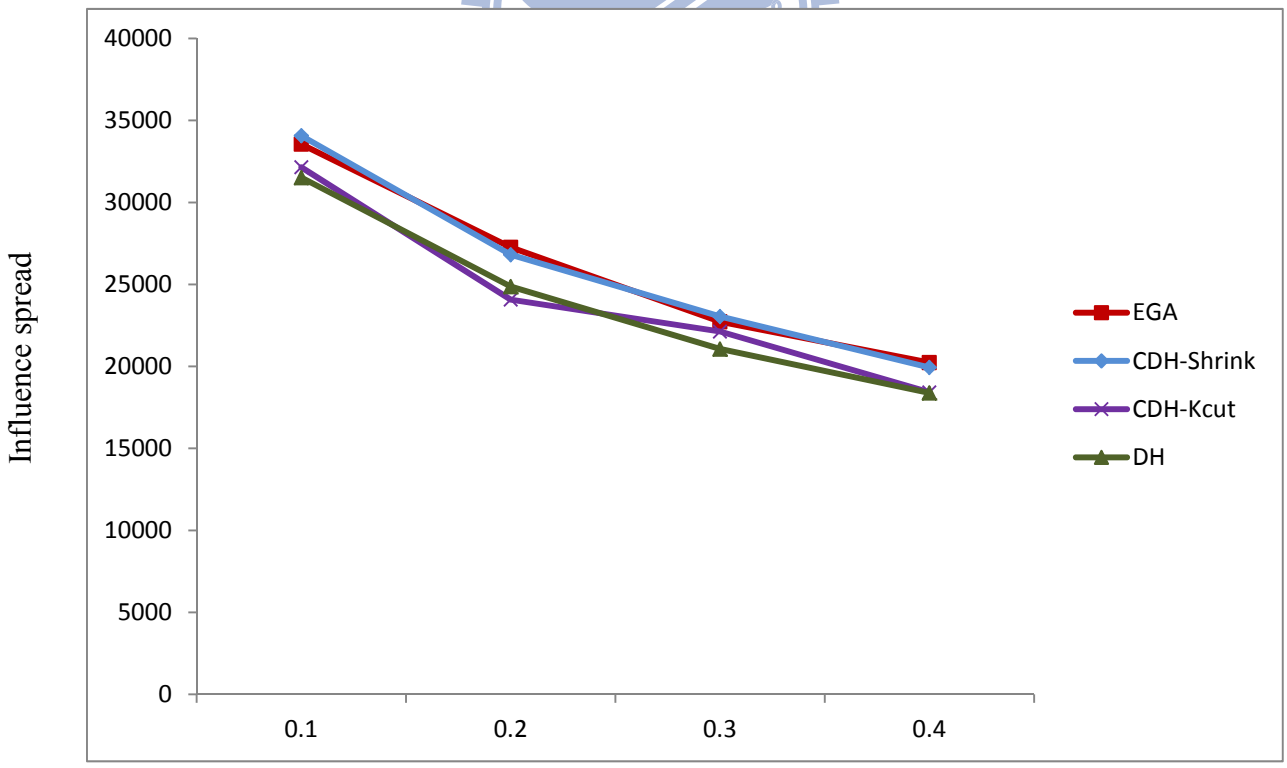


Fig 4.14 influence spread of different algorithms with different activation threshold on FB. θ=0.1, α=0.1. Select 30 seeds

Fig 4.13 (x-axis indicates activation threshold and y-axis indicates influence spread) indicates the influence spread of 10 seeds with different θ from 0.1 to 0.4 with a span of 0.1. Fig 4.14 (x-axis indicates activation threshold and y-axis indicates influence spread) indicates the influence spread of 30 seeds with different θ from 0.1 to 0.4 with a span of 0.1. As shown in Fig 4.13, when only select 10 seeds, EGA's influence spread > CDH-Shrink's influence spread > CDH-Kcut's influence spread > DH's influence spread with α = 0.2, 0.3 and 0.4. In Fig 4.14, we could see that the performance of DH's influence spread is better than that in Fig 4.13. With α = 0.3 in Fig 4.14**,** DH's influence spread is even close to EGA and CDH-Shrink, and better than CDH-Kcut since the phenomenon of seed clustering with high activation threshold. Besides, CDH-Shrink's influence spread is better than EGA with α = 0.1 and 0.3.

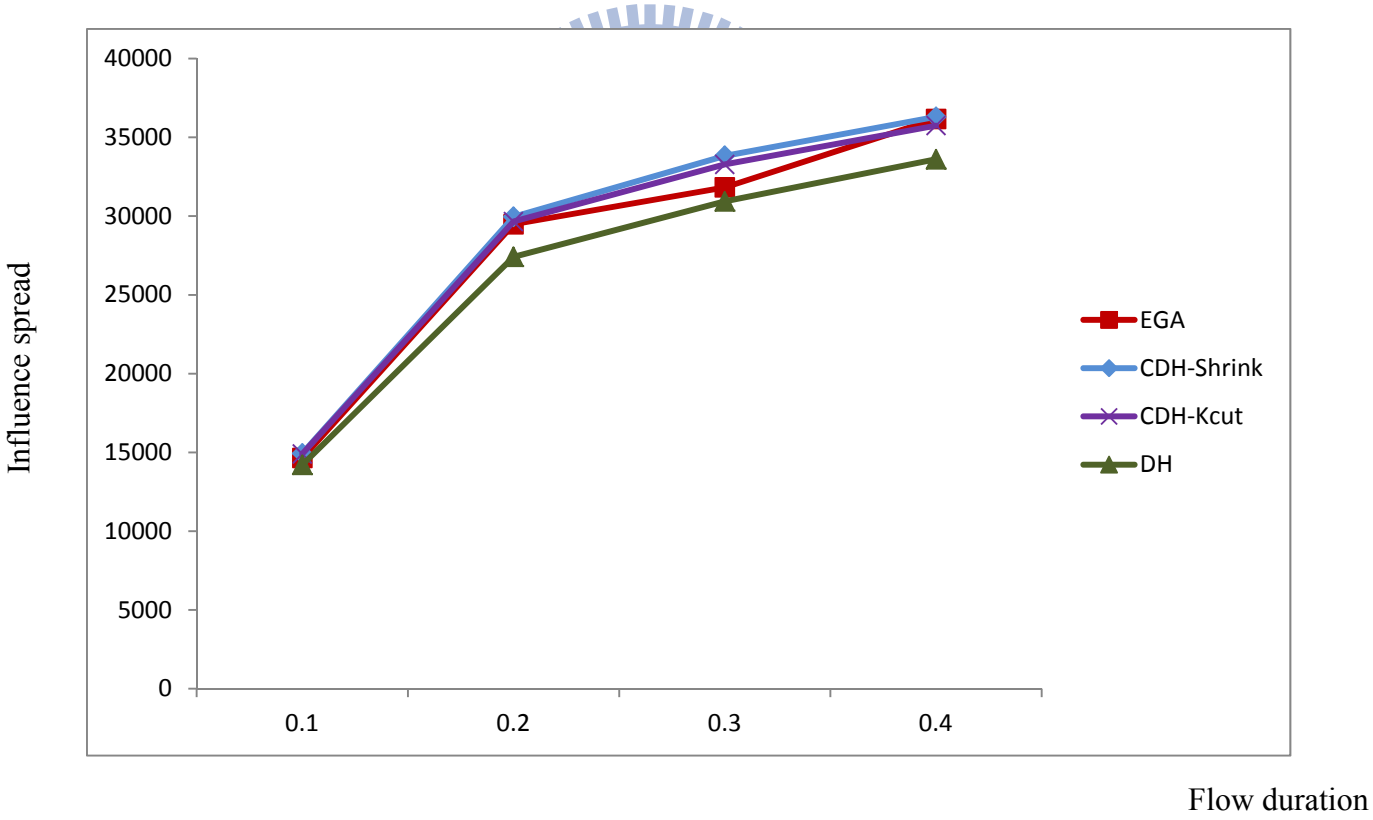


Fig 4.15 influence spread of different algorithms with different time on FB. θ=0.1, α=0.1. Select 10 seeds

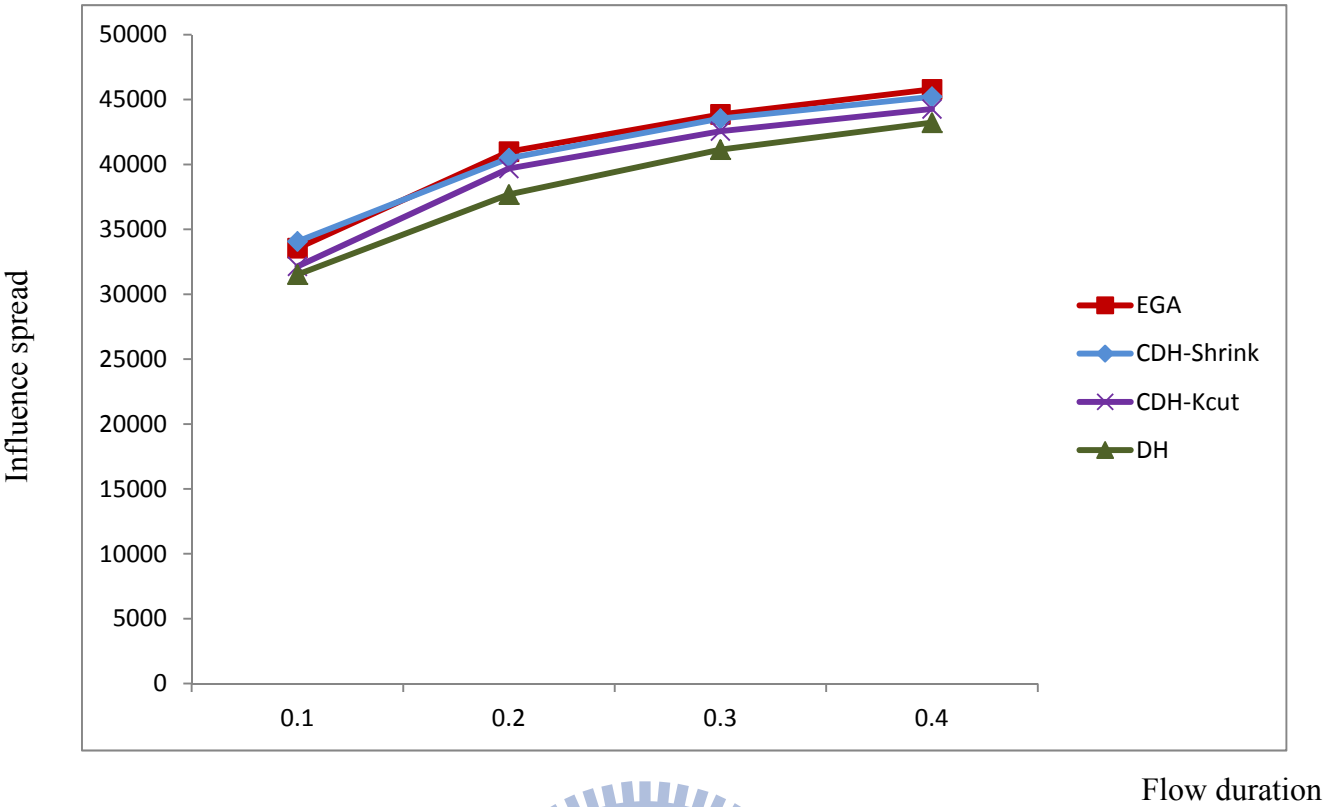


Fig 4.16 influence spread of different algorithms with different flow duration on FB. θ=0.1, α=0.1. Select 30 seeds

Fig 4.15 (x-axis indicates flow duration and y-axis indicates influence spread) indicates the influence spread of 10 seeds with different t from 0.1 to 0.4 with a span of 0.1. Fig 4.16 (x-axis indicates flow duration and y-axis indicates influence spread) indicates the influence spread of 30 seeds with different t from 0.1 to 0.4 with a span of 0.1. Unlike other cases on NETHep or FB, in Fig 4.15 CDH-Shrink's influence spread > CDH-Kcut's influence spread > EGA's influence spread > DH's influence spread. However, when we select 30 seeds as shown in Fig 4.16, the ranking of influence spread becomes EGA > CDH-Shirnk > CDH-Kcut > DH. EGA still has the most influence spread in most cases. To be notices that no matter what values of parameters on FB, CDH-Shrink's influence spreads are very close to EGA.

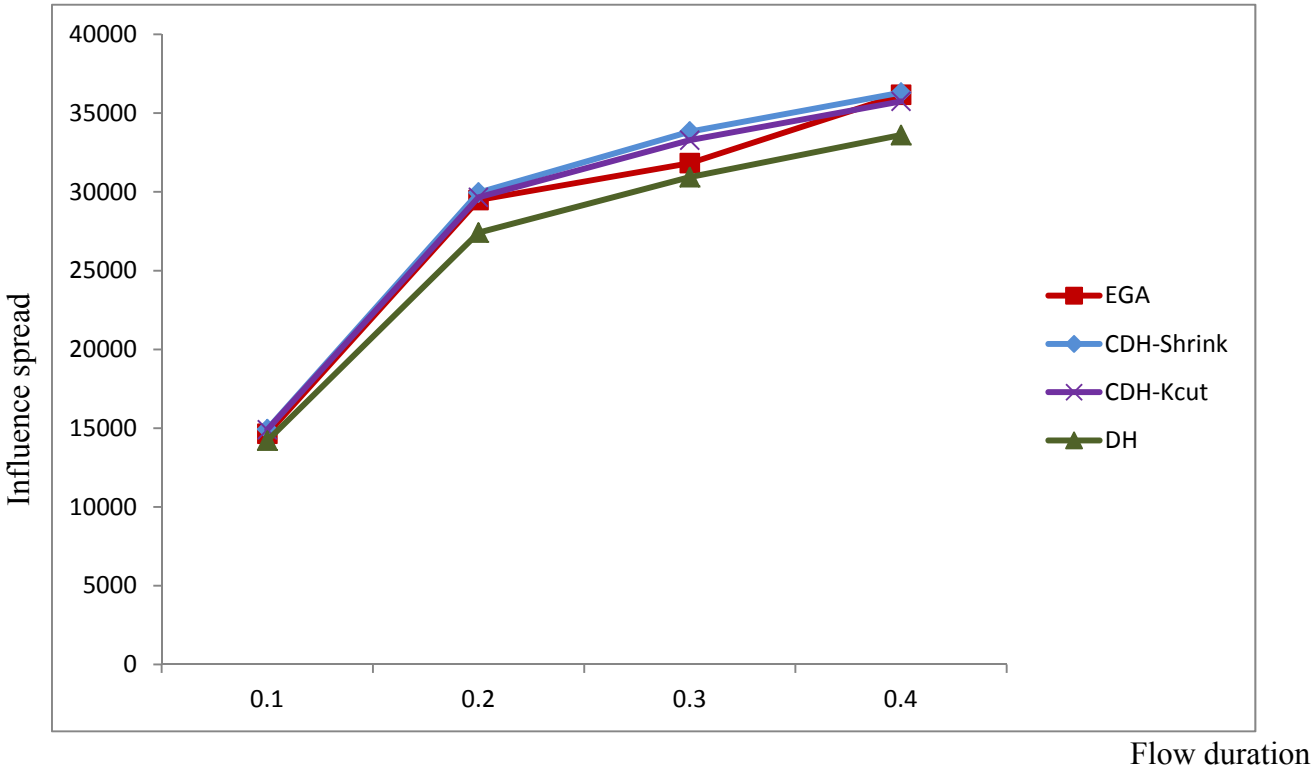


Fig. 4.17 influence spread of different algorithms with different thermal conductivity on FB. t=0.1, θ=0.1. Select 10 seeds

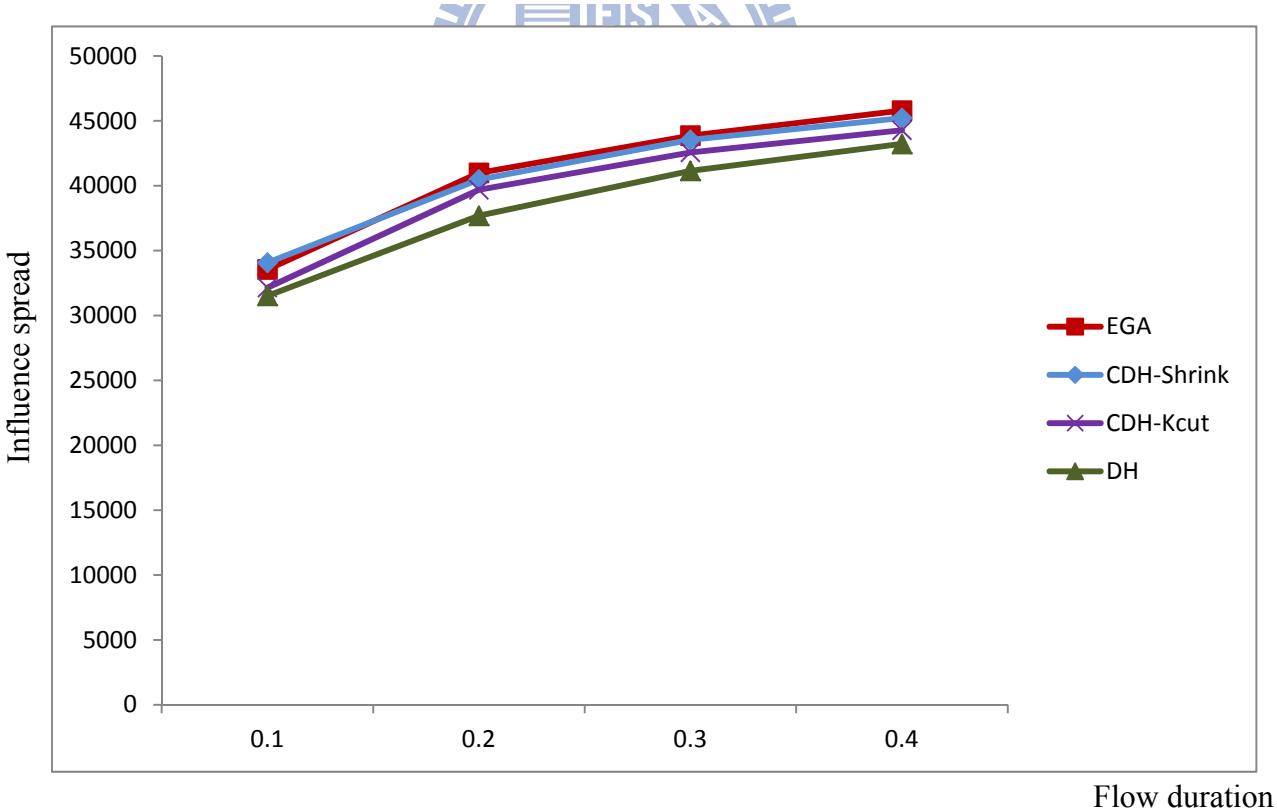


Fig 4.18 influence spread of different algorithms with different thermal conductivity on FB. t=0.1, θ=0.1. Select 30 seeds

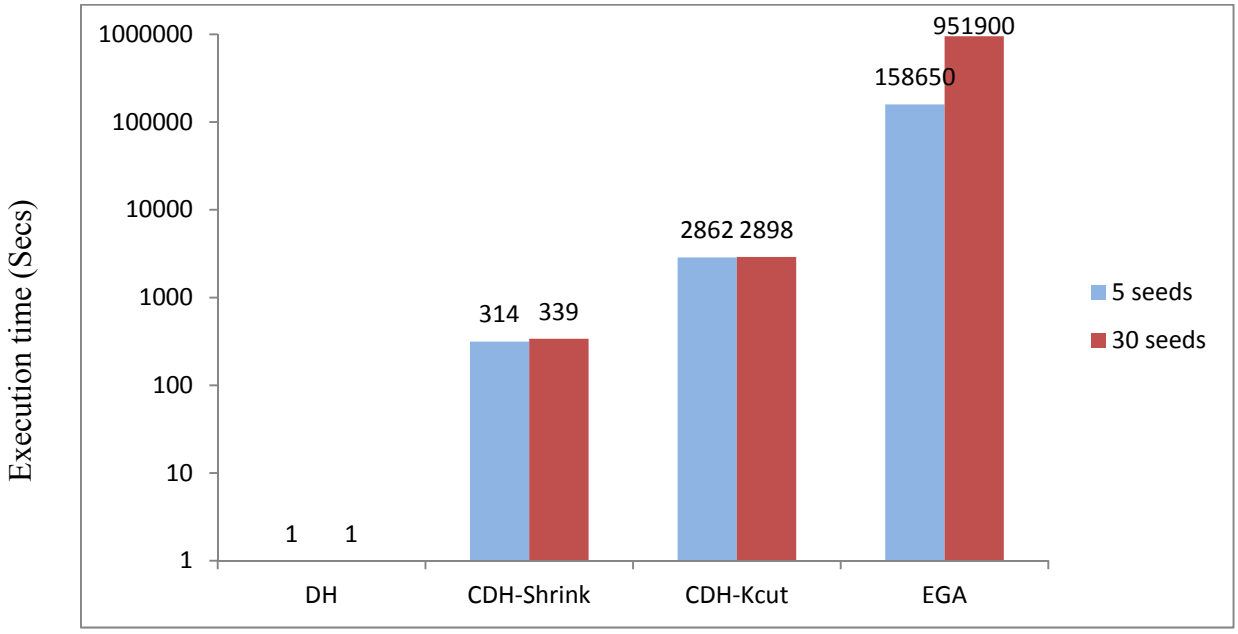


Fig 4.19 Running time of different algorithms on FB. Selecting 5 seeds and 30 seeds respectively

Fig 4.17 (x-axis indicates thermal conductivity and y-axis indicates influence spread) indicates the influence spread of 10 seeds with different t from range 0.1 to 0.4 with a span of 0.1. Fig 4.18 (x-axis indicates thermal conductivity and y-axis indicates influence spread) indicates the influence spread of 30 seeds with different α from range 0.1 to 0.4 with a span of 0.1. We could see that CDH-Shrink still have very good influence spread with α from range 0.1 to 0.4.

We present the efficiency of four algorithms in Fig 4.19 (5 and 30 seeds respectively, x-axis indicates different algorithms and y-axis (logarithmic scale) indicates execution time). CDH-Shrink has the least running time. Just like on NETHep, we can see that the running time of EGA is proportional to the number of seeds.

Overall, in terms of influence spread, EGA > CDH-Shrink > CDH-Kcut > DH. In terms of efficiency, DH > CDH-Shrink > CDH-Kcut > EGA. One thing deserves to be mentioned, due to the phenomenon of seeds clustering, DH will get better performance of influence spread with high activation than that with low activation threshold.

Chapter 5

Conclusions

In this thesis, we present two algorithms CDH-Kcut and CDH-Shrink under the undirected heat diffusion model by combining information of community structure and modified degree-centrality method. The purpose of our work is to efficiently solve the influence maximization problem under the heat diffusion model. By contrast with linear threshold model and independent cascading model, the heat diffusion model is more realistic. The time complexity analysis shows CDH-Shrink and CDH-Kcut is more efficient than enhance greedy algorithm [5]. Experimental results on the real-world and synthetic datasets also validate that our algorithms achieve great performance in efficiency.

SHRINK [10] and Kcut [9] are community detection algorithms used in CDH-Shrink and CDH-Kcut, respectively. Both of SHRINK and Kcut are very efficient and could get community structures satisfactorily. Beside, SHRINK could detect hubs which are very useful information in maximal influence problem. When comparing CDH-Shrink with CDH-Kcut, CDH-Shrink, in general, utilizes hubs and better community structure to achieve better influence spread. Although, both algorithms are efficient with time complexity $O(M\text{long}N)$, CDH-Kcut, in practice, would cost more execution time.

In the future, to interpret the real world more realistically, we will extend our method to weighted graphs. Furthermore, an important property of any social network is evolution. Every social network is evolving all the time. Static community detection algorithms could only detect community structure without considering the evolution of social networks. It is interesting to utilize dynamic community structures to solve the influence maximization problem in the future.

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