Chapter 2

LITERATURE REVIEW

The theories to estimate lateral earth pressure due to a strip surcharge loading will be introduced in this chapter. Commonly geotechnical engineers apply the equations suggested in the U.S. Navy Design Manual DM-7.2 in estimating the horizontal pressure on the retaining walls caused by surcharge loading. The charts and formulas in DM-7.2 manual were based on the theory of elasticity and method of images. Experimental results obtained regarding the lateral pressure increment due to surcharge loading will also be discussed in this chapter.

2.1 Earth Pressure At-Rest Theory

2.1.1 Coefficient of Earth Pressure At-Rest

As shown in Fig.2.1(a), the soil element A formed in a horizontal sedimentary deposit is compressed by the overburden pressure $v = z$. During the formation of the deposit, the element is consolidated under the pressure θ . The vertical stress tends to produce a lateral deformation against surrounding soils due to the Poisson's ratio effect. However, the surrounding soil resists the lateral deformation with a developed lateral stress σ_h . Over the geological period, the horizontal strain is kept to be zero. A stable stress state will develop in which the principal stresses $_1$ and $_3$ acts on the vertical and horizontal planes, as shown Fig.2.1(b). The equilibrium condition produced at this stress in commonly termed as the K_o condition. The ratio of the horizontal stress σ_h to

vertical stress $v_{\rm y}$ is defined as the coefficient of earth pressure at-rest, $K_{\rm o}$, or

$$
K_o = \frac{\sigma_h}{\sigma_v} \tag{2.1}
$$

since $\bar{y} = z$, then $\sigma_h = K_0 \chi$, where γ is the unit weight of soil.

2.1.2 Jaky's Formula

Attempts have been made to establish a theoretical relationship between the strength properties of a soil and the coefficient K_0 . The empirical relationship to estimate Ko of coarse-grained soils is discussed in this section. Mesri and Hayat (1993) reported that Jaky (1944) arrived at the relationship between K_0 and the internal friction angle ϕ by analyzing a talus of granular soil freestanding at the angle of repose. Jaky assumed that the angle of repose is equal to the internal friction angle ϕ . This is a reasonable assumption for sediment, normally consolidated materials. Jaky reasoned that the sand cone OAD illustrated in Fig.2.2 is in a state of equilibrium and its surface and inner points are motionless. The horizontal pressure acting on OC is the earth pressure at rest. Slide planes exist in the inclined sand mass. However, as OC is a line of symmetry, shear stresses cannot develop on it. Hence OC is a principal plane. Based on the equations of equilibrium, Jaky was able to expressed the coefficient of earth pressure at rest K_0 with the angle of internal friction ϕ :

$$
K_o = (1 - \sin \phi) \frac{1 + \frac{2}{3} \sin \phi}{1 + \sin \phi}
$$

Jaky 1948 , without any further explanation, adopted

$$
K_0 = 1 - \sin \phi \tag{2.3}
$$

Mayne and Kulhawy (1982) reported that, the approximate theoretical relationship for K_0 for normally consolidated soils introduced by Jaky appears valid for cohesionless soils. Using Jaky's equation to estimate the in situ lateral earth pressure is reliable enough for most engineering purposes.

2.2 Theorectical Study of Earth Pressure Due to Surcharge

2.2.1 Boussinesq Equation

Boussinesq (1883) advanced theoretical expressions for determining stresses at a point within an "ideal" mass due to a surface point load. The theory was based on the assumptions that the mass is an elastic, isotropic, homogeneous, and semi-infinite medium that extends infinitely in all directions from a level surface. Boussinesq's equations provide a widely used basis for estimating the stresses within a soil mass caused by a concentrated load applied perpendicularly to the soil surface.

Boussinesq's equation may be expresses in terms of rectangular coordinates. Referring to the element shown in Fig. 2.3, the equation to determine the horizontal normal stress σ _x is as follow:

$$
\sigma_x = \frac{3Q}{2\pi} \left\{ \frac{x^2}{R^5} + \frac{1 - 2\mu}{3} \left[\frac{1}{R(R+z)} - \frac{(2R+z)x^2}{R^3(R+z)^2} - \frac{z}{R^3} \right] \right\}
$$
(2.5)

where

$$
Q = point \quad load
$$

$$
r = \sqrt{x^2 + y^2}
$$

$$
R = \sqrt{z^2 + r^2}
$$

$$
\mu = Poisson's \quad ratio
$$

2.2.2 Method of Image

Mindlin (1936) pointed out that, as the horizontal displacements at the wall will be zero, the "method of image" may be invoked to predict the horizontal stress. The method of images is based upon the principle of superposition. As indicated in Fig.2.4, the stress in an elastic solid due to a point load P imply horizontal deformations in the x direction at the position of the wall. The horizontal deformation may be brought back to zero by the application of an imaginary point load P', magnitude equal to P. However, by the principle of superposition, the stresses σ_x on the wall will be doubled when P' is applied.

2.2.3 Vertical Strip Loading on Surface of a Semi-Infinite Mass

A simple equation to calculate the lateral pressure increase due to a uniform vertical strip surcharge was mentioned by Jurgenson (1934). Fig. 2.5 shows the case where a uniform vertical load of *q* per unit area is acting on a flexible infinite strip on the surface of a semi-infinite elastic mass. To obtain the stresses at a point $P(x, z)$, consider an elementary strip of width *ds* loaded at a distance *s* from the centerline of the load. The load per unit length of this elementary strip is $q \cdot ds$, and the strip loading can be approximated as a line load. The expression for σ_x given can be presented in a

simplified form:

$$
\sigma_x = \frac{q}{\pi} \left[\alpha - \sin \alpha \cos(\alpha + 2\delta) \right]
$$
 (2.6)

where α and δ are the angle indicated in Fig. 2.5.

2.2.4 U.S. Navy Design Manual

Based on the method of image and experimental data of the Gerber (1929), Terzaghi (1954) proposed equations to estimate the horizontal stress due to a line load. The equations suggested by Terzaghi are widely adopted by design manuals, such as the U.S. Navy Design Manual (Fig. 2.6). It shows a line load Q_L acting on the surface of the backfill at a distance of mH from the retaining wall. The horizontal pressure increase σ_h at the depth of nH can be estimated with the method of image as follows:

$$
\sigma_h = \frac{4q}{\pi H} \frac{m^2 n}{\left(m^2 + n^2\right)^2} \tag{2.7}
$$

where m and n are defined in Fig. 2.6 as $X = mH$, $z = nH$.

Terzaghi found that, for values of m greater than about 0.4, the agreement between theory and observation is fair (Fig.2.7). However, for value of m smaller than 0.4, the discrepancy between observed and computed values increasing values of m (in Fig 2.11 for $m = 0.1$). For m values less than 0.4, by trial and error, Terzaghi suggested that the pressure on the wall due to the line load q could be properly determined with the following equation:

$$
\sigma_h = \frac{0.203 Q_L}{H} \frac{n}{\left(0.16 + n^2\right)^2} \tag{2.8}
$$

In Fig. 2.7, the earth pressure obtained by use of Eq. (2.7) and Eq. (2.8) are generally greater than the measured values. Terzaghi described part of the difference is due to the fact that the Gerber test were made with line loads having a length of not more than 0.8 H, whereas the computed values refer to line loads with infinite length. The remainder of the difference results from the fact that the Boussinesq's equation is strictly an application only to perfectly elastic material.

2. 3 Model Tests and Case Histories

2.3.1 Study of Gerber

Terzaghi (1954) reported that Gerber (1929) adopted clean, uniform river sand with a grain size between 0.2 mm and 1.5 mm as backfills. The lateral support of soil bin was practically rigid. It consisted of the concrete sidewall of a rectangular pit, with a depth of 31 in. (0.80 m). The lateral pressures caused by the line load were measured by means of pressure cells arranged in vertical rows on a rigid, nonyielding model wall.

The solid curves in Fig. 2.7 represent the test results of one of the Mr. Gerber's series of the tests performed on backfills with a height $H = 0.8$ m. The surcharge, 0.4 ton per square foot (38 kN/m²), covered a strip 6.3 in. (0.16m) wide and 25 in. (0.64m) long. The center line of the loaded strip was placed, respectively, at a distance $x = 3.1$ in., 9.3 in., 15.5 in., and 23.0 in. from the upper edge of the wall, corresponding to values of (m = x / H) of 0.1, 0.3, 0.5, and 0.7.

2.3.2 Study of Spangler

Spangler (1938) measured earth pressures variation on a retaining wall due to a concentrated surface load. The lateral support of backfill consisted of a reinforced concrete cantilever wall, 84 in. (2.13 m) height and 6 in. (0.15 m) thick, which was free to tilt about the outer edge of the base of the base plate.

Based on the experimental data, Spangler thought that, if the wall is of intermediate stiffness, then the pressure due to the wheel load will be somewhere between Boussinesq's solution and the method of images. Field experimental data by Spangler (1938) and Gerber (1929) with these two limits are shown in Fig. 2.8. The elastic solution was adopted as the lower limit and the upper limit was twice of the elastic solution based on the method of images suggested by Mindlin (1936).

Bowles (1988) discussed the test results of the Spangler (1936) and Spangler and Mickle (1956). Bowles argued that the early work of Spangler and Spangler and Mickle introduced an error into general application of the Eq (2.5) by simplifying the elastic solution with the Poisson's ratio $\mu = 0.5$. However, the error can be avoided by the direct use of an appropriate value for μ . Spangler's work measured the lateral pressure against the retaining wall with metal ribbons since earth pressure cells were not readily available in early 1930's. Spangler simply dumped a granular backfill behind the wall with no compaction at all, so the soil behind the wall was in an extremely loose state. Then Spangler backed a truck onto the loose backfill for the rear wheels to simulate two concentrated loads. To simulate a line load, Spangler laid down a 3 m-long railroad cross-tie parallel to the wall and rear wheels of a loaded were backed. From these results, Spangler found that the measured lateral pressure was about twice that predicted by elastic solution with $\mu = 0.5$. Mindlin (1936) discussed Spangler's (1936) work and decided the factor of two could be explained by a rigid wall producing the effect of a mirror load placed symmetrically in front of the wall.

2.3.3 Study of Rehnman and Broms

Rehnman and Broms (1972) investigated the lateral pressure due to surcharge loading on a basement wall by full-scale tests. A 6.0 m-long and 2.5 m-high heavily reinforced-concrete wall with a thickness of 0.23 m was constructed for the tests. The wall was built on a 0.1 m-thick reinforced-concrete slab which extend 6 m from the wall as illustrated in Fig. 2.9. The wall, which was hinged at the bottom, was supported laterally by a hydraulic jack and a rigid steel frame. The steel frame was fixed at the RC slab in front of the wall. For some of the tests, the surface of the wall was covered by 0.5 m-thick rockwool insulation mats. The purpose of these tests was to observed the how a compressible layer affected the magnitude and distribution of the earth pressures on a rigid wall. In the series of experiments, two different backfill materials were investigated, namely a gravelly sand and a silty fine sand, and each was either placed loosely behind the wall without compaction or was compacted in layers. The earth pressures caused by heavy concentrated loads were investigated by driving heavy-wheel loaders close to the wall, and the earth pressure increase on the wall was measured.

The measured pressure increased due to the Michigan 175 wheel loader is shown in Fig. 2.10. The solid lines in the figure correspond to an earth pressure increase which is twice that calculated by the Boussinesq's equation. It can been seen from Figs. 2.10 that the calculated earth pressures (for $\mu = 0.5$) agree relatively well with the measured values for the loosely placed material.

2.3.4 Study of Sherif and Mackey

Sherif and Mackey (1977) observed the earth pressure on a retaining wall with repeated loading for simulating some structures like bridge abutments and basement

walls, which support loads from moving vehicles. The tests were carried out in a tank (Fig. 2.11) made of steel angles and plates, of inside dimensions 1.2 m \times 1.2 m \times 0.5 m. One side of the tank was the model retaining wall that could be considered very stiff. The backfill was a uniform sand with a specific gravity of 2.65. The relative density of the deposited sand was 90%. Loading the backfill surface with a line load was simulated by loading a steel strip 25 mm \times 25 mm in cross-section and 550 mm in length. The load was applied by a lever arm device at two points on the strip. Repeated loading was achieved by lifting and lever arm to release the load from the steel strip, and lowering the lever arm to reapply the load again.

The test results are shown in Fig. 2.12. It is found that earth pressures due to repeated loading were expected to exceed greatly those due to the first loading. The major increase in the intensity of earth pressures would be near the mid-heights of the walls. The effect of repeating the load would decrease as the load is applied further **EARTH 1896** away from the wall.

2.3.5 Study of Smoltczyk et al.

Smoltczyk et al. (1979) observed the distribution of the earth pressures under strip loads for various soil densities, magnitudes of the loads, and varying distances from the retaining wall. As shown in Fig. 2.13, a model wall 2.5 m-high and 3.5 m-wide was installed in the testing hall of the Institute of Foundation Engineering of Landesgewerbeanstalt Bayren at Nurnberg, Germany. The model wall consisted of IPBsteel profiles with wooden sheeting. The earth pressure was measured with 76 Glotzlcells and 5 compression-shear stress measuring device of Stuttgart University.

Middle to coarse grained sand with a coefficient of uniformity $C_u = 2.0$ was used for tests. The sand was dried for testing in order to avoid any apparent cohesion. The backfill was placed at 10 cm lifts and compacted by a surface vibrator. The sand was placed at two different densities, at the mean relative density $D_r = 50$ % and the dense state $D_r = 80 %$.

The design line load was applied by means of 8 IPB 200-beams of about 35 cm length (Fig. 2.13). Traverse beams were used to achieve a uniform load distribution. The line loads were applied at 0.5 m, 1.0 m and 2.0 m from the wall. The magnitude of the line load applied to dense sand was 100 kN/m.

Fig 2.14 showed the test results for a strip load on dense sand $(D_r = 80\%)$. From the comparison of the test results for the distance $x = 0.5$ m and the data computed after Boussinesq (1883), and Terzaghi (1954), it might be stated that the distribution of the earth pressure is most closely represented by Terzaghi's approach. The point of application of the total lateral force after the Boussinesq's equation is located higher, and the magnitude of the earth pressure is lower than the measured data. As indicated in Fig 2.15 (a) and (b), the test results for the line load located at the distance $x = 1.0$ m and 2.0 m reveal that the Boussinesq's approach seems to be more reasonable.

2.3.6 Study of Van Den Berg

Van Den Berg (1991) investigated the influence of the surface loading on earth retaining wall by the small-scale laboratory tests with homogeneous dry Eastern-Scheldt sand. The tests have been carried out in a rectangular test pit with a basis of $2 \text{ m} \times 1 \text{ m}$ and a height of 1 m as shown in Fig. 2.16. In this soil bin, a rigid, but movable wall was placed. In the first stage of the test, the wall was fixed and the backfill behind the wall was placed. During the preparation of the backfill, the horizontal pressure on the wall was measured. In the second stage of the test, a vertical strip loading F_v was applied, and the pressure increase on the fixed wall was measured. The tests were processed with the variation of the density of the sand $(D_r = 30\%$ and 60%), the distance of the surface loading to the wall ($m = x/L = 0.4$ and 0.8), and the magnitude of the loading $(F_v = 2.5 \text{ kN and } 5.0 \text{ kN}).$

Fig. 2.17 presents the relation between $F_{h,sl}$ (a horizontal force caused by the surface loading) divided by F_v , and the distance between vertical load and the wall. The factor $F_{h,sl}/F_v$ varies from about 0.18 to 0.31 depending on the distance of the distance of the load to the wall and density of the sand.

