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LETTER TO THE EDITOR

Multiple branches of acoustic plasma oscillations in thin wires

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Abstract. Multiple non-Landau-damped acoustic plasma modes are shown to exist for electrons in a slender wire. These slender acoustic plasmons (SAP) arise as a result of the collective, longitudinal oscillations of the electrons grouped in one of the discrete transverse-motion levels against those grouped in neighbouring levels. We present numerical examples of these (SAP) modes in GaAs, showing that they should be observable with the present techniques of thin semiconductor wire fabrication. In addition, another mode arising from the screened ion oscillations as modified by the slenderness of the wire is analysed.

It is well known that plasma oscillations can arise in metals and semiconductors (Platzman and Wolff 1973). The familiar optical branch of plasma waves consists of electrons and ions (or holes) vibrating out of phase with each other. In three dimensions this branch has a high characteristic frequency due to the strong Coulomb force between the spatially separated positive and negative charges in the wave. This frequency does not reduce to zero in the limit of long wavelengths because the restoring Coulomb force between the surface polarisation charges on the two end faces perpendicular to the wavevector $\boldsymbol{q}$ is independent of the distance between them. These optical plasmons also have phase velocities larger than the Fermi velocity v_F of the system.

Less familiar, perhaps, is the acoustic branch of plasma oscillations. In this case, two different types of charge carriers, differing in effective mass, with the carriers of lesser mass screening the Coulomb interaction between those of greater mass, vibrate in phase with each other. The carriers may be electrons and ions, or s and d electrons with effective mass $m_d \gg m_s$ (Ruvalds 1981). More recently, the possibility of acoustic plasmons in thin films meeting certain special conditions such as spatial separation of the carriers has been suggested (Takada 1977, Das Sarma and Madhukar 1981).

In this Letter we study the multiple branches of acoustic plasma oscillations in a new physical situation, namely, in a slender structure such as a quasi-one-dimensional wire. The physics that gives rise to these collective longitudinal electronic oscillations with phase velocities less than v_F is unique to such a system, although an analogy can be traced to the acoustic modes that were predicted for a bulk metal in the presence of a quantising magnetic field (Ginzburg *et al* 1968, Konstantinov and Perel 1968, Chock and Lee 1970).

These slender acoustic plasmons (SAPs) can first be explained qualitatively as follows. The smallness of the transverse dimension of the slender wire gives rise to widely separated transverse single-particle levels. An electron in a particular transverse energy level n will have a corresponding maximum longitudinal velocity u_n, with the total energy limited by ε_F, the Fermi energy; i.e. u_n is the effective longitudinal Fermi velocity. Let us temporarily ignore the Coulomb interaction among the electrons. A longitudinal disturbance of wavevector q will transfer a momentum $\hbar q$ to each electron. Only those electrons within an $\hbar q$ neighbourhood of the one-dimensional Fermi surfaces characterised by the u_n are allowed by the Pauli principle to have a real transition, with each of them gaining an energy between $\hbar q u_n^-$ and $\hbar q u_n^+$, where $u_n^{\mp} \equiv u_n \pm \hbar q/2m$. In the limit of $\hbar q/m \ll \Delta u_n$ where $\Delta u_n = u_{n+1} - u_n$, if the disturbance has a frequency $\omega \sim q u_n$ (or a phase velocity $s = \omega/q \sim u_n$), all these electrons within the $\hbar q$ neighbourhood of the effective one-dimensional Fermi surface labelled by n will be resonantly excited, resulting in a macroscopically large number of particles participating in such a mode. If we denote the polarisability of the system by $\chi^{(0)}(q, \omega)$ which relates the polarisation to the perturbing fields, the resonance means $\chi^{(0)}(q, \omega) = \pm\infty$ as $\omega = q u_n^{\mp}$. Correspondingly the ratio of the induced charge density $\rho_i(q, \omega)$ to the perturbing external potential $\phi_{ex}(q, \omega)$ is the coefficient $\alpha^{(0)}(q, \omega) = -q^2\chi^{(0)}(q, \omega)$. If we now turn on the Coulomb interactions, the particles will be coupled with each other and the single-particle states will cease to be eigenstates. However, it is well known that, due to the Coulomb screening effect, the effective or total perturbing potential is $\phi_{total}(q, \omega) = \phi_{ex}(q, \omega)/\varepsilon(q, \omega)$, where $\varepsilon(q, \omega) = 1 + 4\pi\chi^{(0)}(q, \omega)$ is the dielectric constant. The induced charged density is then modified to become

$$\rho_i(q, \omega) = -q^2\chi^{(0)}(q, \omega)\phi_{ex}(q, \omega)/[1 + 4\pi\chi^{(0)}(q, \omega)],$$

which is no longer divergent at $\omega = q u_n^{\mp}$, reflecting a shift of the resonances from the transition frequencies associated with the free, single-particle levels. The positions of the resonances will now be determined by the zeros of $\varepsilon(q, \omega) = 1 + 4\pi\chi^{(0)}(q, \omega)$. Yet, since $\chi^{(0)}(q, \omega)$ changes from $-\infty$ to $+\infty$ as ω varies from $q u_n^+$ to $q u_{n+1}^-$, $\varepsilon(q, \omega)$ must go through zero in this n-interval of ω. Thus, every such interval contains a resonance mode. The mode in the nth interval is obviously dominated by the interplay of the transverse levels n and $n + 1$. With $q u_n^+ < \omega < q u_{n+1}^-$ the effective perturbing field will cause a parallel polarisation in the longitudinally oscillating particles with a higher characteristic frequency $q u_{n+1}^-$ at the $(n + 1)$-Fermi surface but antiparallel polarisation in those with a lower characteristic frequency $q u_n^+$ at the n-Fermi surface†. Thus, the oscillating particles in levels n and $n + 1$ will vibrate against each other in the nth resonance mode. Since the numbers of particles associated with the characteristic frequencies $q u_n^+$, $q u_{n+1}^-$ are macroscopic, the resonant ω oscillation is also collective in nature, which is closely related to its vanishing damping rate. Furthermore, the dispersion relation is of the acoustic type since the phase velocity $s \equiv \omega/q$ of the nth collective mode is trapped between u_n^+ and u_{n+1}^-, whose values are independent of q.

The same physical argument cannot be used to deduce acoustic modes in a thin film. A longitudinal disturbance of wavevector $\boldsymbol{q}$ will cause free electron transitions in the $\hbar\boldsymbol{q}$ neighbourhood of the effective two-dimensional Fermi surfaces. However, since the effective Fermi velocity is now a vector $\boldsymbol{u}_n$, the corresponding single-particle transition

† This is like a forced harmonic oscillator. The displacement will be in phase with the driving force if the driving frequency is below the characteristic frequency of the oscillator; they will be opposite in phase if the reverse is true.

frequencies are given by $\omega - \boldsymbol{q}\cdot\boldsymbol{u}_n$, which are all different for particles with u_n of different directions on the nth Fermi surface. Thus, a perturbation of frequency $\omega \sim \boldsymbol{q}\cdot\boldsymbol{u}_n$ can only excite selectively few particles rather than all of them in the neighbourhood of the nth Fermi surface. Although Coulomb interaction will again cause particles at one Fermi surface to vibrate against those at the other, only a few particles will be involved in each vibration mode—a fact that spells the demise of these modes. Except under special conditions (Takada 1977 and Das Sarma and Madhukar 1981), generally there are no multiple branches of acoustic plasmons in a thin film geometry.

In order to study quantitatively the response of the electron gas in a thin wire to longitudinal electric fields of wavevector $\boldsymbol{q} = q\hat{\boldsymbol{z}}$ and frequency ω, we use the self-consistent field method (Ehrenreich and Cohen 1959). Consider a thin wire along the z-axis and, for simplicity, with a square cross section of side a or area $A = a^2$. A Fourier component of the perturbed charge per unit length inside the wire is $\lambda_{q,\omega} = A\rho_{q,\omega}$, where $\rho_{q,\omega}$ is the perturbed (oscillating) charge density. Correspondingly the potential is given as

$$V(\boldsymbol{r}, z, t) = V_{q,\hat{z},\omega}\exp(\mathrm{i}qz)\exp(-\mathrm{i}\omega t) \tag{1}$$

where

$$V_{q,\hat{z},\omega} = -e\lambda_{q\hat{z},\omega}C_1(q) \tag{1a}$$

$$C_1(q) = \frac{1}{A}\int \mathrm{d}^2\boldsymbol{r}\int_{-\infty}^{\infty}\mathrm{d}z''\frac{\exp(\mathrm{i}qz'')}{|(\boldsymbol{r}-\boldsymbol{r}') + \hat{z}z''|}. \tag{2}$$

Here $\boldsymbol{r}$ is a two-dimensional vector in the xy plane. In the limit of a thin wire or $qa \ll 1$, we formally let $|\boldsymbol{r}-\boldsymbol{r}'|$ approach zero in the denominator of equation (2) but simultaneously introduce a cut-off at small $|z''| \sim a$ to restore the main effect of $|\boldsymbol{r}-\boldsymbol{r}'|$ on the integral with respect to z''. Thus

$$C_1(q) \simeq 2\int_a^{\infty}\frac{\cos(qz)}{z}\mathrm{d}z = -2\mathrm{Ci}(qa) \tag{2a}$$

where $\mathrm{Ci}(x)$ is the cosine integral.

Since the contribution from the region of small z to the integral of equation (2a) varies as $\sim\ln(qa)$, we expect any error introduced by the approximate cut-off value $\sim a$ to be insignificant as long as $aq \ll 1$. In the extreme limit of a wire of vanishing cross section we should still have a cut-off imposed by quantum mechanics that two charges of opposite signs cannot get much closer than the Bohr radius a_0 (Lee *et al* 1968). Then in this case we expect $C_1(q) \simeq -2\mathrm{Ci}(qa_0)$.

We should mention that the small-q limit

$$\lim_{qa\to 0} C_1(q) \to -\ln(qa) \tag{3}$$

is a physically reasonable behaviour also obtained by other workers using a different model (Friesen and Bergersen 1980).

The self-consistent-field method applied to this restricted three-dimensional system gives the following expression for the dielectric function $\varepsilon(q, \omega)$:

$$\varepsilon(q\hat{z}, \omega) = 1 - \frac{e^2C_1(q)}{L}\sum_{k,\sigma}\frac{f(\mathscr{E}_{k+\hat{z}q}) - f(\mathscr{E}_k)}{\mathscr{E}_{k+\hat{z}q} - \mathscr{E}_k + \hbar\omega + \mathrm{i}0^+} \tag{4}$$

where L is the length of the wire, $\mathscr{E}_k = \hbar^2k^2/2m$ is the unperturbed electron energy, $\boldsymbol{k}$ is a three-dimensional wavevector and $f(\mathscr{E}_k)$ is the equilibrium Fermi–Dirac distribution

function for the electron system. Notice that due to the finite thickness of the wire, the two-dimensional momentum $\boldsymbol{k}_2 \equiv \boldsymbol{k} - \hat{z}k_z$ of the electron has a discrete spectrum with well separated values while the k_z component assumes quasi-continuous values due to the large L.

In general, we should also include the effect of retardation due to the finiteness of the light velocity c. This effect can be shown to change the expression for $C_1(q)$ to

$$C_1(q, \omega) \simeq -\mathrm{Ci}(aq + \omega a/c) - \mathrm{Ci}(aq - \omega a/c) + \mathrm{i}\pi/2[\mathrm{sgn}(q + \omega/c) - \mathrm{sgn}(q - \omega/c)] \tag{5}$$

and the corresponding dielectric function to

$$\varepsilon(q\hat{z}, \omega) = 1 - \frac{e^2 C_1(q, \omega)}{1 + (n_1 e^2/mc^2) C_1(q, \omega)} \frac{1}{L} \sum_{k,\sigma} \frac{f(\mathscr{E}_{k+\hat{z}q}) - f(\mathscr{E}_k)}{\mathscr{E}_{k+\hat{z}q} - \mathscr{E}_k + \hbar\omega + \mathrm{i}0^+} \tag{6}$$

where n_1 is the number of electrons per unit length. From equation (5) we see that as long as $cq > |\omega|$, $C_1(q, \omega)$ is real and positive while if $cq < |\omega|$, $C_1(q, \omega)$ has an imaginary part that implies damping of the oscillation due to electromagnetic radiation into the wire's surroundings. We recover the previous result, equations (2*a*) and (4), if we let $q \gg |\omega|/c$. We will consider this limit in what follows since we are dealing with small phase velocity modes.

If we now evaluate equation (4) at zero temperature, we obtain

$$\varepsilon(q\hat{z}, \omega) = \varepsilon_1 + \mathrm{i}\varepsilon_2 \tag{7}$$

with

$$\varepsilon_1(q\hat{z}, \omega) = 1 + \frac{C_1(q)}{\pi q a_0} \sum_{\mathscr{E}_p < \mathscr{E}_F} \ln \left| \frac{s^2 - u_p^{+2}}{s^2 - u_p^{-2}} \right| \tag{8a}$$

and

$$\varepsilon_2(q\hat{z}, \omega) = \frac{C_1(q)}{\pi q a_0} A_s \, \mathrm{sgn}(s) \tag{8b}$$

where $s = \omega/q$ is the phase velocity of the wave, $a_0 = \hbar^2/me^2$ is the Bohr radius, $\boldsymbol{p}$ is a two-dimensional wavevector

$$u_p^{\pm} = u_p \pm \hbar q/2m \tag{9a}$$

$$u_p = [2(\mathscr{E}_F - \mathscr{E}_p)/m]^{1/2} \tag{9b}$$

$\mathscr{E}_F = \hbar^2 k_F^2/2m$ is the Fermi energy and $A_s = 1$ if $|s|$ lies between u_p^+ and $|u_p^-|$ or $A_s = 0$ otherwise.

We notice that $\varepsilon(q\hat{z}, \omega)$ of equation (7) has divergences at the $|u_p^{\pm}| = |s|$ values, corresponding to the divergences one expects in the polarisability $\chi^{(0)}(q, \omega)$ whenever we approach the unperturbed transition frequencies.

The imaginary part of ε, ε_2, accounts for the Landau damping of the wave. In fact, in order to find the collective excitations of the system, we must solve the dispersion relation (Platzman and Wolff 1973):

$$\varepsilon_1(q\hat{z}, \omega) = 1 + \frac{C_1(q)}{\pi q a_0} \sum_{\mathscr{E}_p < \mathscr{E}_F} \ln \left| \frac{s^2 - u_p^{+2}}{s^2 - u_p^{-2}} \right| = 0 \tag{10a}$$

together with

$$\varepsilon_2(q\hat{z}, \omega) = 0 \tag{10b}$$

if no damping is to occur.

A schematic plot of ε_1 is given in figure 1, where we notice the existence of several zeros within the damping regions, indicating that they are damped modes. We also notice the presence of zeros of ε_1 *outside* the damping regions and these solutions to (10) are precisely the longitudinal SAP modes. Notice also the outermost zero in ε_1 corresponding to the usual high-frequency optical plasma mode, with phase velocity $s > v_F$.

From equations (9*a*) and (8*b*) we can see that each damping region has a width of $\hbar q/m$. As q becomes sufficiently large, the damping regions overlap each other and all the acoustic modes with such a q will be damped. Thus, the SAP modes owe their existence mathematically to the condition $\hbar q/m \ll u_p$. Physically, this condition means that the transverse levels must be sufficiently widely separated such that each level can house a macroscopic number of electrons with different k_z, leading to the collective nature of the SAP modes.

One can try to obtain analytic expressions for the SAP modes by assuming that the zero of ε_1 is close to one of its divergences. Then the most dominant terms for the determination of that particular zero arise from the two closest asymptotes, say u_1 and u_2. Considering only these two terms and expanding the log for small $q \ll 2m|s - u_p|/\hbar$ we obtain

$$\varepsilon_1(q\hat{z}, \omega) \simeq 1 - \frac{2e^2C_1(q)q^2}{\pi\hbar}\left\{\frac{u_1 f_1}{\omega^2 - u_1^2q^2} + \frac{u_2 f_2}{\omega^2 - u_2^2q^2}\right\} = 0.$$

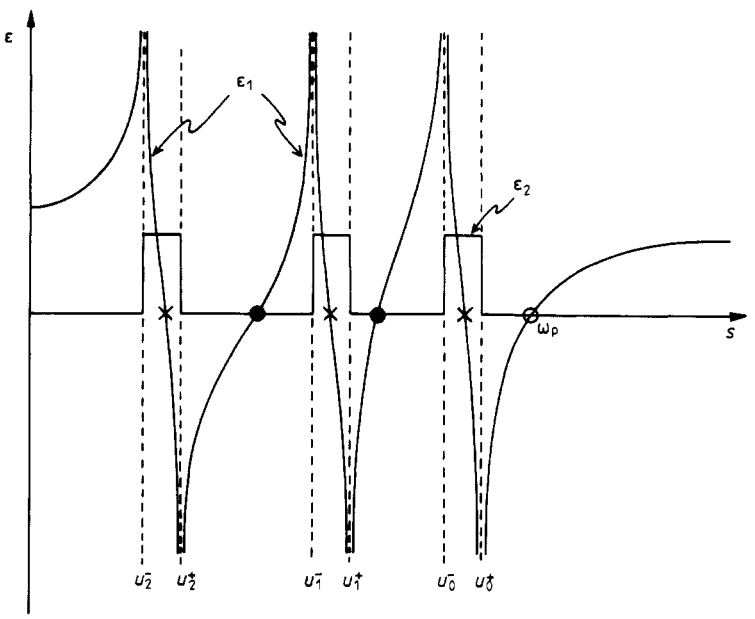

Figure 1. Schematic plot of ε_1 and ε_2 of equation (8), showing the damped modes (×), the SAP modes (●) and the usual plasma mode ω_p.

After some algebra we can solve for ω^2 as

$$s^2 = \frac{\omega^2}{q^2} \simeq \frac{u_1^3 f_1 + u_2^3 f_2}{u_1 f_1 + u_2 f_2} + \frac{2e^2 C_1(q)}{\pi\hbar}(u_1 f_1 + u_2 f_2) + \frac{\pi\hbar(u_1^2 - u_2^2)^2}{8e^2 C_1(q)(u_1 f_1 + u_2 f_2)} \tag{11}$$

where f_i is the degeneracy for the ith transverse energy level.

Expression (11) shows that the dispersion relation is linear or sound-like only in first approximation. However, we cannot completely believe this treatment since, in the case when the zero is not so close to either one of the asymptotes, the other terms in the summation are of similar importance. It will turn out numerically that the dispersion relation is indeed of the acoustic type for a large range of q.

The SAP modes are very similar to the Ginzburg–Konstantinov–Perel (1968) or Chock–Lee (1970) modes (Konstantinov and Perel 1968). The GKP–CL modes arise when a one-component plasma such as an electron gas is under a magnetic field. The electrons are then grouped into the discrete Landau levels pertaining to the transverse motion. The different groups oscillate against each other, producing a longitudinal acoustic type of oscillation like the SAP modes. Despite the similarities, the dispersion relations for the SAP and the GKP–CL modes are different, mostly because the Coulomb potential is the purely three-dimensional $4\pi e^2/q^2$ in the latter case but the one-dimensional $Ae^2C_1(q)$ in the former case.

We should also mention that other workers on 1D systems such as Friesen and Bergersen (1980) did not find these SAP modes mainly because they considered only a single transverse level and, in that sense, their system was purely one-dimensional.

In quasi-two dimensional systems, it turns out that the Landau damping extends

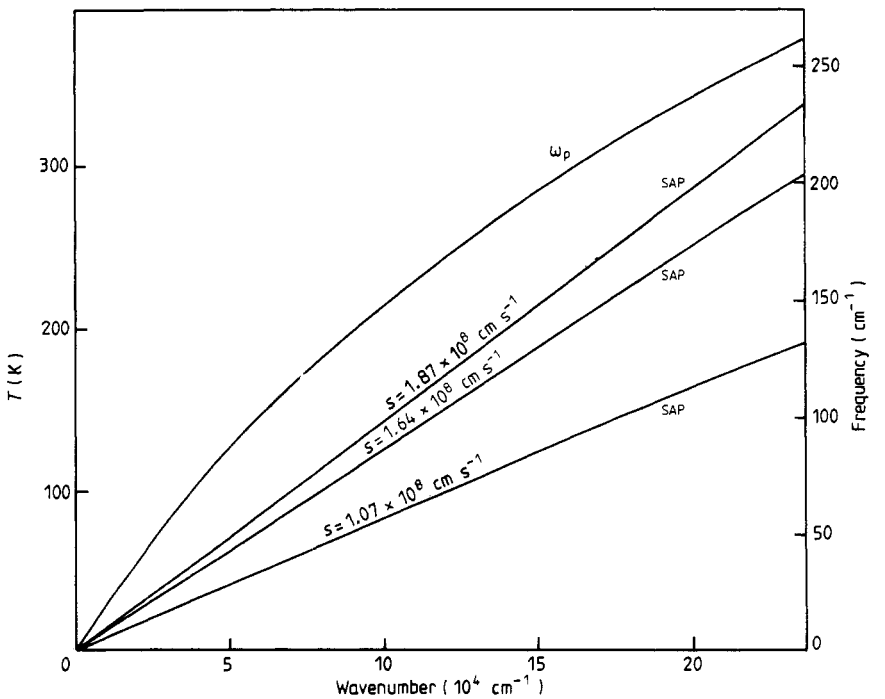

Figure 2. SAP modes for a narrow semiconductor heterojunction. The three modes shown are all the modes present in this case. The usual plasmon ω_p is also shown as reference. Strip width = 200 Å. $n_s = 2.0 \times 10^{13}$ cm^{-2}.

over a larger domain of the variable $s = \omega/q$. It follows that acoustic modes in a thin slab are not possible unless one can achieve spatial separation of the different transverse-motion levels larger than a critical distance. This separation can be accomplished via either a MIS inversion layer structure (Takada 1977) or a superlattice, semiconductor heterojunction (Das Sarma and Madhukar 1981).

Let us consider some specific numerical examples. Our examples will be a metallic wire and a thin strip of semiconductor inversion layer or heterojunction.

The number of SAP modes present in a given system is exactly one less than the number of possible values for the transverse energies. On the other hand, the number of these transverse levels depends on the width a of the wire or the strip as well as the Fermi wavevector of the system k_F. In fact, the number of levels allowed inside the Fermi sphere is characterised by $(k_F a)^2$ which increases with both a and k_F.

In the case of the metallic wire, assume $a = 50$ Å and $k_F = 1.38 \times 10^8$ cm^{-1} ($\mathscr{E}_F = 7.3$ eV, appropriate for the $Au_{40}Pd_{60}$ alloy). These extremely thin wires are presently attainable experimentally using a very remarkable technique (Chaudhari and Habermeier 1980). The number of SAP modes with these parameters is 50, resulting in mode frequencies too close to each other to be resolved experimentally because of thermal broadening.

A very interesting case is that of a narrow strip of inversion layer or heterojunction. Consider typical parameters for GaAs: effective mass $0.068\,m_e$ and average dielectric constant of the structure 6.95. Since the number of SAP modes is controlled by $(k_F a)^2$ we have chosen two examples of such parameters, one with $k_F = 1.12 \times 10^7$ cm^{-1} ($n_s = 2.0 \times 10^{13}$ cm^{-2}) and $a = 200$ Å as shown in figure 2 and another with $k_F = 5.61 \times 10^6$ cm^{-1} ($n_s = 5.0 \times 10^{12}$ cm^{-2}) and $a = 500$ Å as shown in figure 3. Narrow strips

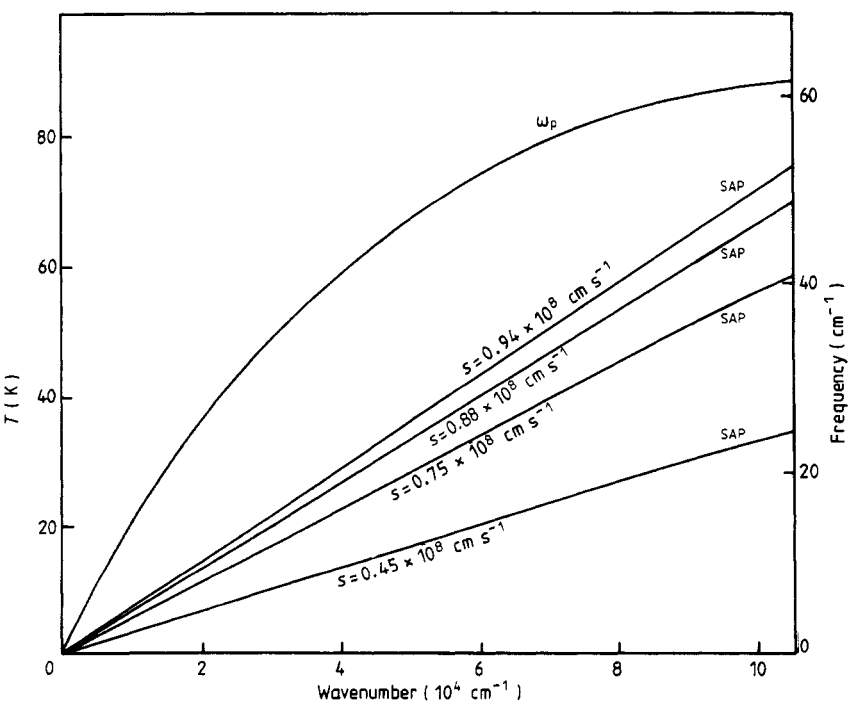

Figure 3. SAP modes for a narrow semiconductor heterojunction. Here the system has four SAP modes. Strip width = 500 Å. $n_s = 5.0 \times 10^{12}$ cm^{-2}.

with widths of this order have been obtained using recent advances in microfabrication technology (Skocpol *et al* 1982 and Wheeler *et al* 1982).

Comparing the numerically obtained results expressed in figures 2 and 3 with those for the metallic wire, we observe that because of the smaller value of k_F, the SAP modes in the inversion layer structures are more widely spaced than those in the wire. Also, since in metals the k_F values are invariably of the order of $10^8\,\mathrm{cm}^{-1}$, a very small width is needed in order to get a few widely spaced SAP modes. However, in inversion layers one can control the electron density or the Fermi momentum over a wide range. This versatility allows for larger widths which are easily attainable experimentally.

We notice in figures 2 and 3 that the energy spacing between modes is of the order of several kelvins, which is larger than the thermal broadening at liquid helium temperatures. At these temperatures, typical collision times (Ando *et al* 1982) τ in semiconductor inversion layers are 10^{-12} s, so that $\omega\tau > 1$ for our SAP modes with larger q values. Therefore, by working at low temperatures and with smaller wavelengths one should be able to observe these modes.

Finally, in a real solid there are also collective oscillations of the ions that give rise to phonons. Since the electrons are coupled to the ions, the phonon frequencies depend on the screening by the electrons. For long-wavelength longitudinal oscillations, the dispersion relation is given by (Chock and Lee 1970)

$$\omega^2 = \Omega_p^2/\varepsilon(q, \omega) \tag{12}$$

where Ω_p is the plasma frequency of the ions when the electrons are treated as a fixed negative background.

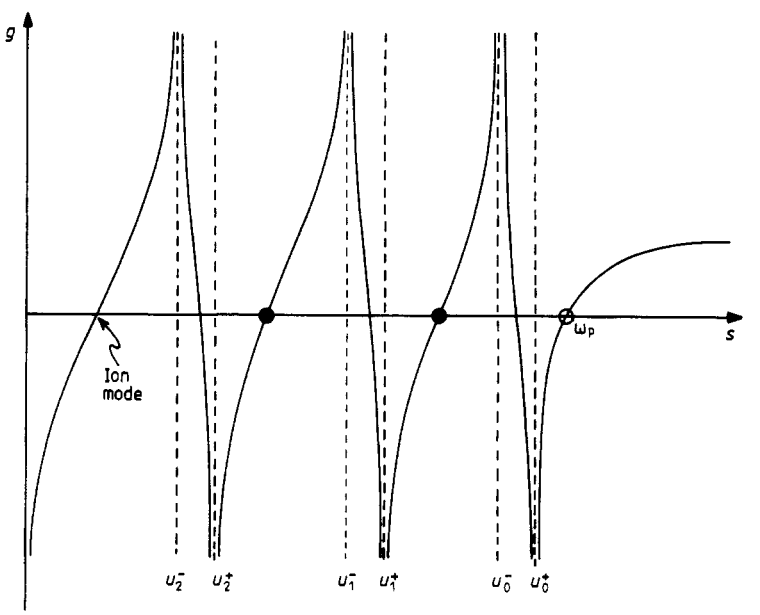

Figure 4. Sketch of function $g(q, \omega)$ of equation (13). Notice the SAP modes, indicated by a heavy dot, as well as the usual plasma mode, and most importantly, the new ion mode at the lowest frequency.

Using equations (8) and (12) we obtain the dispersion relation for the longitudinal modes of the electron–phonon system in the zero temperature limit as

$$g(q,\omega) = 1 - \frac{\Omega_p^2}{\omega^2} + \frac{C_1(q)}{\pi q a_0} \sum_{\mathscr{E}_p < \mathscr{E}_F} \ln \left| \frac{s^2 - u_p^{+2}}{s^2 - u_p^{-2}} \right| = 0. \tag{13}$$

This equation now replaces equation (10*a*). If we schematically plot the function g of equation (13), we obtain figure 4, in which we observe the SAP modes slightly modified by the ion–electron interaction. We also notice the emergence of a new mode which does not exist in figure 1. It has the lowest phase velocity of all modes and is due entirely to ion dynamics, although appropriately screened by the electrons in all the transverse levels (Chock and Lee 1970).

In conclusion, we remark that although the SAP modes are similar to the GKP–CL modes in nature, the SAP modes are much more easily observable. For example, to obtain Landau levels of spacings comparable to those of the transverse levels in the examples discussed here, one needs to apply magnetic fields of the order of 10^3–10^4 T, obviously not easily attainable.

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