國立交通大學

電信工程研究所

碩士論文

針對無線感測器網路的訊息中繼考慮通道參數不匹配的低冗餘波束形成設計

Low-Overhead Cooperative Beamforming for Information Relaying in Wireless Sensor Networks Under Mismatched Inter-Node Link CSI

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摘要

於低訊號冗餘合作式通訊系統中的高效能訊號處理演算法在實現節能的第四代中繼網路中扮演著重要的角色。在這篇論文中，我們研究在無線感測器網路中針對訊息中繼來考慮在通道參數不匹配的情況下的低冗餘合作式波束形成設計。在我們所考慮的系統中，為了達到降低傳送通道狀態資訊的訊號冗餘，每一個中繼端會將其測得之訊源與中繼端的鏈結訊雜比量化成一位元的訊息。為了反映無線感測器網路嚴格的傳送功率限制，每一個訊雜比的量化訊息在傳輸過程中因此假設為不理想的，於數學上視為經由一個帶有非零交叉機率的二元對稱通道來傳送。當接收端接收這些可能有位元翻轉的一位元訊雜比訊息，且假設中繼端與接收端間鏈結的通道估測是完美的，則我們首先可以考慮藉由最大化接收訊號的訊雜比期望值所得的波束形成系數設計，其中此訊雜比公式是針對二元對稱通道中位元翻轉的統計特性所求得的期望值。接著，我們進一步延伸考慮中繼端與接收端間鏈結的通道估測可能會產生誤差的情況，在數學上我們以獨立同分佈的高斯隨機變數才表示。在這兩種情況下，我們可以透過針對訊雜比不確定性或是通道狀態資訊不確定性作平均，以分別推導出具封閉形式的條件平均接收訊雜比。因為如此推導出來的訊雜比公式對波束形成系數來說是一高度非線性函數，所以針對這兩種訊雜比公式，我們分別進一步地推導出各自的可分析下界以利於分析。藉由最大化各自的訊雜比下界，相應的亞最佳波束形成系數解即可透過解廣義特徵值問題來求得。最後，電腦模擬結果用以檢驗我們提出方法的成效。

Abstract

High-performance signal processing algorithms for cooperative communication systems with reduced signaling overhead play a key role toward realizing energy-efficient relay networks for 4G and beyond. In this thesis, we study the problem of low-overhead cooperative beamforming design for information relaying in wireless sensor networks (WSNs) by taking account of the effect of mismatched inter-node channel state information (CSI). In the considered system, each relay node quantizes the signal-to-noise ratio (SNR) of the source-to-relay (S-R) link into one bit in order to reduce the signaling overhead dedicated to CSI transmission. To reflect the severe transmit power limitation of WSNs, the transmission link of the quantized SNR message is assumed to be non-ideal, and is modeled by a binary symmetric channel (BSC) with a non-zero crossover probability. With the flipped one-bit SNR messages received at the destination and assuming that the relay-to-destination (R-D) link channel estimation is perfect, we first study the beamforming design based on maximization of the expected receive SNR, averaged with respect to the bit-flipping distributions of BSC's. Next, the proposed approach is extended to the scenario wherein the R-D link channel estimation errors occur, and are modeled as i.i.d. Gaussian random variables. In both cases, we derive closed-form expressions for the conditional receive SNR averaged over the distributions of the SNR/CSI uncertainty. Since the SNR measures thus obtained are highly nonlinear functions of the beamforming coefficients, we further derive for each case a tractable SNR lower bound to facilitate analyses. By conducting maximization with respect to the derived SNR lower bounds, suboptimal beamformers can be obtained via solving generalized eigenvalue problems. Computer simulations are used to illustrate the performances of the proposed schemes.

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Contents

List of Figures

Chapter 1

Introduction

1.1 Overview

Due to the myriad real-world applications, the study of wireless sensor networks, in particular, the development of high-performance distributed signal processing algorithms, has received considerable attention in the recent years [1-6]. Typically, sensor nodes are of a small size, powered by battery, and therefore subject to limited communication and message processing capability. Under such constraints, *cooperation* among sensors for information relaying or data forwarding becomes necessary in order to enable various global target tasks, e.g., data aggregation, decision fusion, and signal retrieval. Among the various cooperative transmission schemes, cooperative beamforming is a promising technique capable of realizing distributed spatial diversity toward link reliability enhancement [7-10]. Since the design of beamforming coefficients requires the knowledge of the channel state information (CSI) of inter-node channel links, communication/signaling overheads dedicated to CSI transmission, or feedback, in cooperative beamforming systems are thus unavoidable. To meet the high energy-efficiency demand for wireless sensor networks, the reduction of physical-layer signaling overhead is crucial [11-16]. There have been many studies of low-overhead cooperative beamforming techniques aimed at realizing energy-efficient information relaying, e.g., see [17-20]. In all of these works, it is commonly assumed that CSI transmission and feedback are errorless. Such an assumption, however, is impractical in the sensor network scenario. Indeed, since sensor nodes are subject to stringent power and decoding complexity constraints, implementation of forward error correction codes for improving the error resilience of quantized CSI (or information bits) may be prohibitive due to unacceptable system complexity and decoding latency [1, Chap. 6]. As a result, the transmission of the local CSI from the far-end relay nodes could be deteriorated by large path loss and severe fading, resulting in distorted CSI received at the destination. In point-to-point multi-antenna communications with limited feedback, system designs in the presence of CSI transmission/feedback errors have been well documented, see, e.g., [21-25]. Related study in the context of cooperative communications, however, remains much to be investigated.

1.2 Contribution

In this thesis, we study the cooperative beamforming design for information relaying in wireless sensor networks in the presence of mismatched inter-node CSI. The cooperative beamforming scheme employing the amplify-and-forward (AF) relaying protocol as in [10] is considered. Also following [10], the information symbols are assumed to be BPSK modulated so as to reflect the rate and decoding complexity/latency constraints in wireless sensor networks [1], [6]. The design of the beamforming weights is aimed at maximizing a certain signal-to-noise ratio (SNR) metric at the destination. To reduce the signaling overhead, each relay node quantizes the SNR of the source-to-relay (S-R) link into one bit[1], which is then sent to the destination for beamforming design. Rather than assuming that the quantized SNR messages are received at the destination without errors, we consider the realistic case that the transmission link of the one-bit message is imperfect, and is mathematically modeled as a binary symmetric channel (BSC) with a known crossover probability[2]. Specific technical contributions of this thesis can be summarized as follows.

(I) Beamforming Design in the Presence of Imperfect Quantized S-R Link SNR

We first consider the beamforming design by taking account of the fact that each one-bit message of S-R link SNR could be flipped by the BSC, while the R-D link channel estimation is assumed to be perfect. Given the one-bit messages received from all relays, the beamforming coefficients are designed at the destination via

1. While multiple-bit quantization at relays is considered in the problem formulation in [10], the simulation study therein shows that the beamformer designed based on even one-bit quantization can perform quite close to that designed in accordance with the full S-R link CSI. Hence, to minimize the signaling overhead, this paper focuses on the specific one-bit case. The generalization of our current solution to the multiple-bit case is under investigation.

2. When referring to "BSC" we specifically focus on the transmission link of the one-bit quantized S-R link SNR.

maximization of the receive SNR averaged with respect to the conditional bit flipping distributions of BSC's. A closed-form formula for the proposed SNR metric is first derived. The formula is seen to be a highly nonlinear function of the beamforming factors, and direct maximization of this objective function is quite difficult. For analytic tractability, we then derive a lower bound of the conditional average SNR that admits the form of a generalized Rayleigh quotient [26]. By conducting maximization with respect to this lower bound, a closed-form suboptimal beamformer can be obtained as the solution to a generalized eigenvalue problem. Computer simulation shows that the proposed scheme does outperform the solution [10] under imperfect quantized S-R link SNR.

(II) Beamforming Design Under Imperfect Quantized S-R Link SNR and R-D Link Channel Estimation Errors

Next, we generalize the results in part (I) by further taking into account the effect of R-D link channel estimation errors. It is assumed that the destination only knows a set of R-D link channel estimates, which are modeled as the true channel gains corrupted with additive white Gaussian errors. Conditioned on the received one-bit quantized S-R link SNR from all relays and estimated R-D link channel parameters, the design criterion of the beamformers is to maximize the receive SNR (at the destination) averaged over the distributions of both the bit-flipping effect of BSC's and R-D link channel estimation errors. An exact formula of the adopted conditional average SNR is first derived. Since the formula is quite complicated, to ease analysis we further resort to certain approximation techniques to derive an associated SNR lower bound which also admits the form of a generalized Rayleigh quotient [26]. As in part (I), we propose to instead conduct maximization of this lower bound. A suboptimal beamformer can be obtained as the solution to a generalized eigenvector problem. Computer simulations show that, compared to the beamformer [10] and the solution derived in part (I), the proposed design is more robust against the R-D link channel estimation errors.

1.3 Organization

The rest of thesis is organized as follows. Chapter 2 is the preliminary, which introduces the system model and the problem statement. Chapter 3 presents the beamforming design under imperfect S-R link SNR, while R-D link channel estimation is assumed to be perfect. Chapter 4 discusses the design of the beamforming weights under mismatched S-R link SNR and R-D link CSI. Finally, Chapter 5 concludes this thesis. Detailed proofs of key mathematical results are relegated to Appendix.

Chapter 2

System Model and Overview of Previous Work

We consider the dual-hop cooperative beamforming system in [10] that is depicted in Figure 1, in which L relays employ the AF protocol to collaboratively transmit the common source signal $x[n] \in \{-1,1\}$ to the destination. During the signal broadcasting phase, the received signal at the ith relay is

$$y_{s_i}[n] = \sqrt{P_s} h_{s,i} x[n] + v_i[n], \tag{2.1}$$

where P_s is the source transmit power, $h_{s,i} \sim \mathcal{CN}(0, \sigma_s^2)$ is the channel gain of the ith S-R link[3], and $v_i[n] \sim \mathcal{CN}(0, \sigma_v^2)$ is the receive noise at the ith relay. Based on (2.1), the instantaneous SNR of the ith S-R link is thus

$$\gamma_{s_i} \triangleq \frac{P_s \left| h_{s,i} \right|^2}{\sigma_v^2}. \tag{2.2}$$

At the information relaying phase, the received signal at the destination reads

$$y_d[n] = \sum_{i=1}^{L} h_{r,i} G_i g_i y_{s_i}[n] + w[n], \tag{2.3}$$

where $h_{r,i} \sim \mathcal{CN}(0, \sigma_r^2)$ denotes the ith R-D channel gain, $G_i = \dfrac{1}{h_{s,i}\sqrt{P_s(1+\gamma_{s_i}^{-1})}}$ is the power normalization factor, g_i is the ith beamforming weight, and $w[n] \sim \mathcal{CN}(0, \sigma_w^2)$ represents the receive noise at the destination. With (2.1), $y_d[n]$ in (2.3) can be expressed as [10]

3. The notation $\mathcal{CN}(0, \sigma^2)$ denotes the circularly complex Gaussian random variable with zero mean and variance σ^2.

$$y_d[n] = \sum_{i=1}^{L} \frac{h_{r,i} g_i}{\sqrt{1+\xi_{s_i}}} x[n] + \sum_{i=1}^{L} \frac{h_{r,i} g_i \sqrt{\xi_{s_i}}}{\sqrt{1+\xi_{s_i}}} \bar{v}_i[n] + w[n], \tag{2.4}$$

where $\xi_{s_i} \triangleq 1/\gamma_{s_i}$ is the reciprocal of the SNR of the ith S-R link, and $\bar{v}_i[n] \sim \mathcal{CN}(0,1)$. To design the beamforming weights g_i's, one commonly used approach is to conduct SNR maximization based on the knowledge of the CSI of the S-R and R-D communication links (e.g., [27-28]). This paper focuses on the low-overhead cooperative beamforming scheme, wherein the ith relay quantizes the SNR of the ith S-R link (see (2.2)) into one bit $q_i \in \{0,1\}$. Assuming that (i) $\{q_1, \cdots, q_L\}$ are received at the destination without errors, and (ii) the CSI of all the R-D links is perfectly known at the destination, the SNR conditioned on either $x[n]=1$ or $x[n]=-1$, is shown to be [10][4]

$$\gamma_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \mathbf{q}) = \frac{\left| \sum_{i=1}^{L} g_i h_{r,i} \phi(q_i) \right|^2}{\sum_{i=1}^{L} |g_i|^2 |h_{r,i}|^2 (1 - \phi^2(q_i)) + \sigma_w^2}, \tag{2.5}$$

where $\mathbf{q} \triangleq [q_1, \cdots, q_L]^T$, $\mathbf{g} \triangleq [g_1, \cdots, g_L]^T$, $\mathbf{h}_\mathbf{r} \triangleq [h_{r,1}, \cdots, h_{r,L}]^T$,

$$\phi(q_i) \triangleq \begin{cases} \dfrac{1}{1 - e^{-\tau_i/(\bar{\gamma}_s)}} \displaystyle\int_0^{\tau_i} \frac{1}{\sqrt{1+(1/\mu)}} \exp(-\frac{\mu}{\bar{\gamma}_s}) d\mu, & \text{when } q_i = 0; \\ e^{\tau_i/(\bar{\gamma}_s)} \displaystyle\int_{\tau_i}^{\infty} \frac{1}{\sqrt{1+(1/\mu)}} \exp(-\frac{\mu}{\bar{\gamma}_s}) d\mu, & \text{when } q_i = 1. \end{cases} \tag{2.6}$$

is the mean of $1/\sqrt{1+\xi_{s_i}}$ given that $\gamma_{s_i} = 1/\xi_{s_i}$ belongs to the quantization interval associated with q_i, $\bar{\gamma}_s \triangleq \frac{P_s \sigma_s^2}{\sigma_v^2}$ is the average S-R link SNR, and $\tau_i > 0$ is the quantization threshold determined according to equation (66) in [10, p-4780][5]. The

4. For a finite L, the number of relays, an analytic SNR formula is difficult to find [10]. The expression (2.5) was obtained in [10] based on asymptotic analyses in the regime $L \to \infty$.

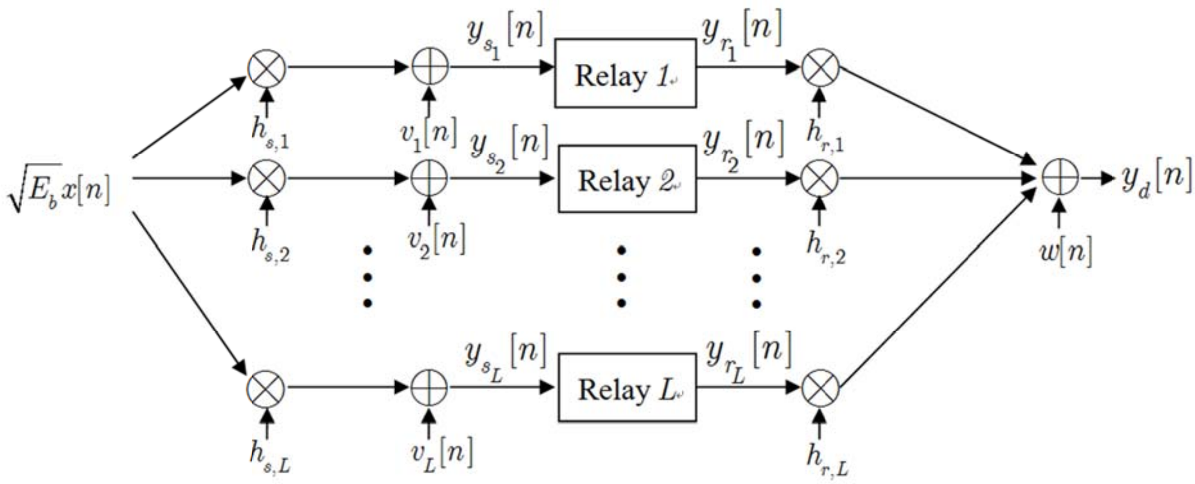
5. The formula (2.6) can be directly obtained based on (33) and (34) in [10] together with the one-bit quantization assumption.

optimal g_i 's, which maximize $\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \mathbf{q})$ in (2.5) subject to the total power constraint

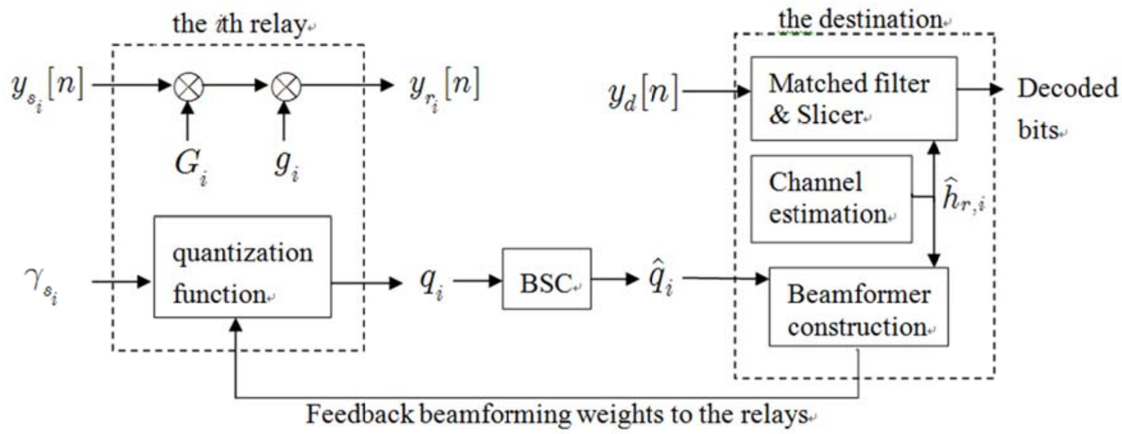
$$\sum_{i=1}^{L} |g_i|^2 = P_d \,, \tag{2.7}$$

are shown to be [10]

$$g_i \propto \frac{h_{r,i}^{-1}\phi(q_i)}{1+\xi_{r_i} - \phi^2(q_i)}, \text{ where } \xi_{r_i} \triangleq 1/\gamma_{r_i} = \frac{\sigma_w^2}{P_d \left|h_{r,i}\right|^2}. \tag{2.8}$$



(a)



(b)

Figure 1. (a) Low-overhead cooperative beamforming system diagram; (b) Depiction of AF relay and S-R link SNR quantizer.

Chapter 3

Beamforming Design in the Presence of Imperfect Quantized S-R Link SNR

We consider the problem of cooperative beamforming design under the assumption that the transmitted one-bit message q_i from each relay is subject to communication channel impairments and R-D channel estimation remains perfect. More specifically, it is assumed that q_i is sent over a BSC with a crossover probability p_i, $1 \le i \le L$. From the perspective of SNR maximization, we propose a new beamforming design method which takes account of the imperfect reception of $\{q_1, \cdots, q_L\}$.

Section 3.1 derives the conditional average SNR, which is the proposed design metric for the beamforming factors. Section 3.2 then derives a lower bound of the considered SNR metric. An analytic suboptimal beamforming scheme is also obtained via the maximization of the lower bound. Computer simulations are given in Section 3.3 to illustrate the performance of the proposed solution.

3.1. Conditional Average SNR

Let $\hat{q}_i \in \{0,1\}$ be the received quantized message associated with q_i, $1 \le i \le L$. Conditioned on the $\hat{\mathbf{q}} = [\hat{q}_1 \cdots \hat{q}_L]^T$, the main purpose is to derive the conditional SNR averaged with respect to all possible transmitted $\tilde{\mathbf{q}} = [\tilde{q}_1 \cdots \tilde{q}_L]^T$'s that are flipped to $\hat{\mathbf{q}}$ by the BSC. Recall that, the SNR conditioned on $\hat{\mathbf{q}} = \mathbf{q} = [q_1 \cdots q_L]^T$ is given by $\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \mathbf{q})$ in (2.5). Hence, the expected $\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}})$ given $\hat{\mathbf{q}}$ is thus

$$\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) \triangleq E_{\tilde{\mathbf{q}}|\hat{\mathbf{q}}}\left[\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}}) \mid \hat{\mathbf{q}}\right] = \sum_{\tilde{\mathbf{q}}} \gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}}) \times \Pr(\tilde{\mathbf{q}} \mid \hat{\mathbf{q}}), \qquad (3.1)$$

where $\Pr(\tilde{\mathbf{q}}|\hat{\mathbf{q}})$ denotes the probability that $\tilde{\mathbf{q}}$ is flipped into $\hat{\mathbf{q}}$. The conditional average SNR (3.1) is obtained by averaging over all possible transmitted $\tilde{\mathbf{q}}$'s given the received $\hat{\mathbf{q}}$. To fix the idea, let us define[6]

$$S_l(\hat{\mathbf{q}}) \triangleq \left\{ \tilde{\mathbf{q}} \middle| \sum_{i=1}^{L} \tilde{q}_i \oplus \hat{q}_i = l \right\}, \quad 0 \le l \le L, \tag{3.2}$$

which denotes the set consisting of all possible $\tilde{\mathbf{q}}$'s that differ from $\hat{\mathbf{q}}$ in exactly *l* bits; there are thus $C_l^L = \frac{L!}{l!(L-l)!}$ possible $\tilde{\mathbf{q}}$'s in $S_l(\hat{\mathbf{q}})$. Associated with each $\tilde{\mathbf{q}} \in S_l(\hat{\mathbf{q}})$, we further collect all indices at which $\tilde{q}_i$ differs from $\hat{q}_i$ to obtain

$$I_l(\tilde{\mathbf{q}}, \hat{\mathbf{q}}) \triangleq \left\{ i \middle| \tilde{q}_i \ne \hat{q}_i \right\}. \tag{3.3}$$

With (3.2) and (3.3), the conditional average SNR is given as

$$\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) = \sum_{l=0}^{L} \sum_{\tilde{\mathbf{q}} \in S_l(\hat{\mathbf{q}})} \Pr(\tilde{\mathbf{q}} | \hat{\mathbf{q}}) \gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}}), \tag{3.4}$$

where

$$\Pr(\tilde{\mathbf{q}} | \hat{\mathbf{q}}) = \left(\prod_{k \in I_l(\tilde{\mathbf{q}}, \hat{\mathbf{q}})} p_k \right) \left(\prod_{m \in I_l^c(\tilde{\mathbf{q}}, \hat{\mathbf{q}})} (1 - p_m) \right), \tag{3.5}$$

and $I_l^c(\tilde{\mathbf{q}}, \hat{\mathbf{q}})$ denotes the complement of $I_l(\tilde{\mathbf{q}}, \hat{\mathbf{q}})$. Through further manipulations an explicit formula of $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})$ is shown in the following theorem.

Theorem 3.1: The conditional average SNR (3.4) admits the following form

$$\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) \triangleq \sum_{l=0}^{L} \left\{ \sum_{k_1=1}^{L} \sum_{k_2=k_1+1}^{L} \cdots \sum_{k_{l-1}=k_{l-2}+1}^{L} \sum_{k_l=k_{l-1}+1}^{L} \frac{\left| \sum_{i=1}^{L} c_i(l, k_1, \cdots, k_l) g_i \right|^2}{\sum_{i=1}^{L} |g_i|^2 d_i(l, k_1, \cdots, k_l)} \right\}, \tag{3.6}$$

where

$$c_i(l, k_1, \cdots, k_l) \triangleq \sqrt{\eta \prod_{j=1}^{l} p_{k_j} \left(\prod_{j=1}^{l} (1 - p_{k_j}) \right)^{-1}} h_{r,i} \phi(\rho_i), \tag{3.7}$$

in which $\eta \triangleq \prod_{l=1}^{L} (1 - p_l)$ and $\phi(\cdot)$ is defined in (2.6),

$$d_i(l, k_1, \cdots, k_l) \triangleq \left| h_{r,i} \right|^2 \left[1 - \phi^2(\rho_i) \right] + \frac{\sigma_w^2}{P_d}, \tag{3.8}$$

6. The notation $\oplus$ denotes the binary addition operation.

and

$$\rho_i \triangleq \begin{cases} \hat{q}_i^t = \hat{q}_i \oplus 1 & , i = k_1, k_2, \cdots, k_l \\ \hat{q}_i & , \text{otherwise.} \end{cases} \tag{3.9}$$

[Proof]: See Appendix. □

To maximize the conditional average SNR $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \hat{\mathbf{q}})$ given in (3.6) with respect to the beamforming weights g_i's, we shall first rewrite $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \hat{\mathbf{q}})$ in a more tractable form. Through further rearranging the indices in the multiple summations in (3.6), $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \hat{\mathbf{q}})$ can be expressed as a single sum of Rayleigh quotients. This is established in the next theorem.

Theorem 3.2: Let $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \hat{\mathbf{q}})$ be defined in (3.6). Then we have

$$\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \hat{\mathbf{q}}) = \sum_{m=1}^{M} \frac{\left|\mathbf{c}_m^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_m \mathbf{g}}, \tag{3.10}$$

in which $M = \sum_{l=0}^{L} C_l^L$, and, for each[7] $1 \le m \le M$,

$$\mathbf{c}_m^H \triangleq [c_1(l, k_1, \cdots, k_l), \cdots, c_L(l, k_1, \cdots, k_l)], \tag{3.11}$$

$$\mathbf{D}_m \triangleq diag\left\{d_1(l, k_1, \cdots, k_l), \cdots, d_L(l, k_1, \cdots, k_l)\right\}, \tag{3.12}$$

for certain $l, k_1, \cdots, k_l$. Given a particular set of indices $l, k_1, \cdots, k_l$ in the multiple summations in (3.6), the corresponding index m in (3.10) is determined according to

$$m = \delta(l) + \sum_{s_0=0}^{l-1} C_{s_0}^L + \sum_{\lambda=1}^{l-1} \sum_{s_\lambda = k_{\lambda-1}+1}^{k_\lambda - 1} C_{l-\lambda}^{L-s_\lambda} + (k_l - k_{l-1}), \tag{3.13}$$

where $k_0 = 0$ and $\delta(\cdot)$ denotes the Kronecker delta function.

[Proof]: See Appendix. □

Based on (3.10), the beamforming weights can be obtained by solving the following optimization problem

7. The dependence of the index *m* in (3.10) on $l, k_1, \cdots, k_l$ is omitted to simplify notation.

$$\text{Maximize} \quad \sum_{m=1}^{M} \frac{\left|\mathbf{c}_m^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_m \mathbf{g}} \quad \text{s.t.} \;\; \|\mathbf{g}\|_2^2 = P_d \,. \tag{3.14}$$

where P_d denotes the total transmit power. However, since the cost function in (3.14) is a highly nonlinear function of **g**, a closed-form solution to (3.14) is hard to find. In the next subsection we propose an alternate approach to finding suboptimal beamforming weights.

3.2 Closed-Form Suboptimal Solution

To facilitate analysis, we go on to derive in the following theorem a tractable lower bound for $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \hat{\mathbf{q}})$. By conducting maximization with respect to this lower bound, we can then obtain a closed-form suboptimal solution.

Theorem 3.3: Let $\mathbf{c}_m$ and $\mathbf{D}_m$ be defined in (3.11) and (3.12). The following inequality holds:

$$\sum_{m=1}^{M} \frac{\left|\mathbf{c}_m^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_m \mathbf{g}} \ge \frac{\mathbf{g}^H \mathbf{c}\mathbf{c}^H \mathbf{g}}{\mathbf{g}^H \mathbf{D} \mathbf{g}} \,, \tag{3.15}$$

where $\mathbf{c} \triangleq \sum_{m=1}^{M} \mathbf{c}_m$ and $\mathbf{D} \triangleq \sum_{m=1}^{M} \mathbf{D}_m$.

[Proof]: See Appendix. □

With the aid of (3.15), a suboptimal beamformer can be obtained based on maximization of the lower bound derived in (3.15):

$$\text{Maximize} \quad \frac{\mathbf{g}^H \mathbf{c}\mathbf{c}^H \mathbf{g}}{\mathbf{g}^H \mathbf{D} \mathbf{g}} \quad \text{s.t.} \;\; \|\mathbf{g}\|_2^2 = P_d \,. \tag{3.16}$$

The solution to (3.16), denoted by $\tilde{\mathbf{g}}$, is precisely the dominant eigenvector of $\mathbf{D}^{-1}\mathbf{c}\mathbf{c}^H$. Since the matrix $\mathbf{D}^{-1}\mathbf{c}\mathbf{c}^H$ is of rank-one, we have

$$\tilde{\mathbf{g}} = c_1 \mathbf{D}^{-1} \mathbf{c} \,, \tag{3.17}$$

where c_1 is chosen so that $\|\tilde{\mathbf{g}}\|_2^2 = P_d$.

Remark:

Though we study the low-overhead scheme in which each R-D link SNR is quantized into a one-bit message, we have tried to extend our first study to the case when multiple bits are used for quantization. Explicit analysis and simulation results are given in Appendix J.

3.3 Simulation Results

In this section computer simulations are used to illustrate the performance of the proposed method. We consider a cooperative beamforming system with four relays ($L = 4$). The channel gains of both the S-R and R-D links are i.i.d. random variables drawn from $\mathcal{CN}(0,1)$; the crossover probability p_i of the BSC obeys the uniform distribution over the interval $[0.05, 0.1]$. The quantization threshold is designed according to the rule in [10, p-4779]. The total power of transmit beamforming is set to be $P_d = 1$. In each Monte-Carlo run, a sequence of $T = 5000$ BPSK source symbols is generated. For fixed average S-R SNR $\overline{\gamma}_s = 20$ dB, Figure 2 compares the BER curves of the proposed beamformer (3.17) with the solution in [10] at various average R-D link SNR, defined to be $\overline{\gamma}_d \triangleq P_d\sigma_r^2 / \sigma_w^2 = \sigma_w^{-2}$ [8]. As can be seen from the figure, the proposed scheme outperforms the method in [10], especially when SNR is high; this is not unexpected since the solution in [10] is designed under the idealized assumption that the one-bit message is received at the destination without errors. Also, it is seen from the figure that the performance improvement is slight when SNR is below 10 dB. This phenomenon is caused by the fact that, for the considered cooperative beamforming scheme, S-R-link CSI mismatch is not a dominant factor for the BER performance in the low-to-medium SNR regime; this fact has been confirmed by the simulation results provided in [10, p-4780]. With fixed $\overline{\gamma}_s = 20$ dB and assuming that the crossover probabilities p_i 's of the BSC are identical for all i (thus $p_i = p,\ 1 \le i \le 4$), Figure 3 further shows the BER of the two methods for $0 \le p \le 0.2$ with respect to three different average R-D link SNR $\overline{\gamma}_d = 5,\ 10,$ and 15 dB. It can be seen that, when $p = 0$ (i.e., the one-bit message is perfectly received), the proposed scheme and the method in [10] yields an identical performance. The result is not unexpected since, with $p = 0$, the considered objective function (3.10) reduces to the single term (2.5) ($M = 1$), and, hence, equality holds in (3.15): this then implies that the proposed solution (3.17) is exactly the beamformer given by (2.8). For $p > 0$, our solution is seen to be quite robust against the increase of p, thereby confirming the advantage of the proposed design.

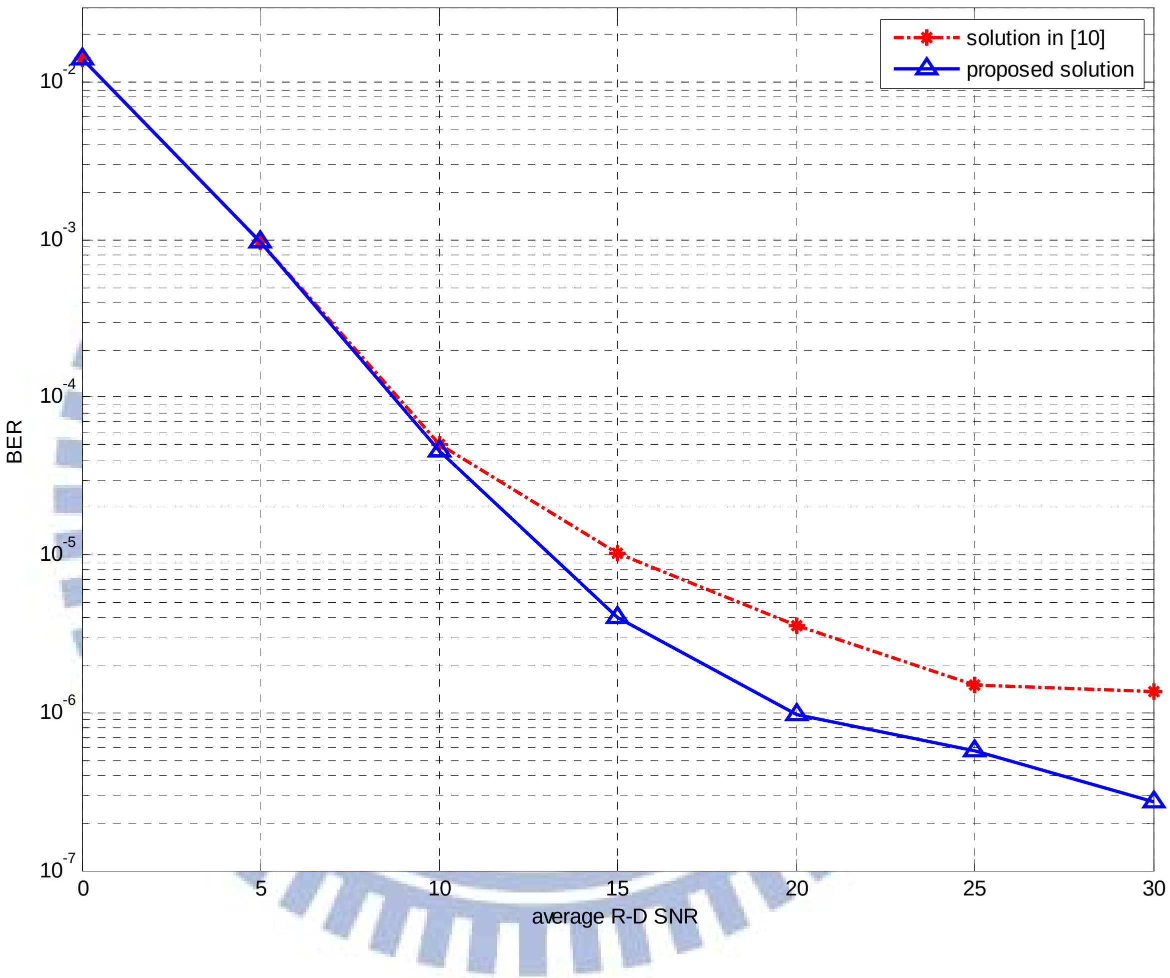


Figure 2. Simulated BER of the proposed beamformer (3.17) and the solution in [10].

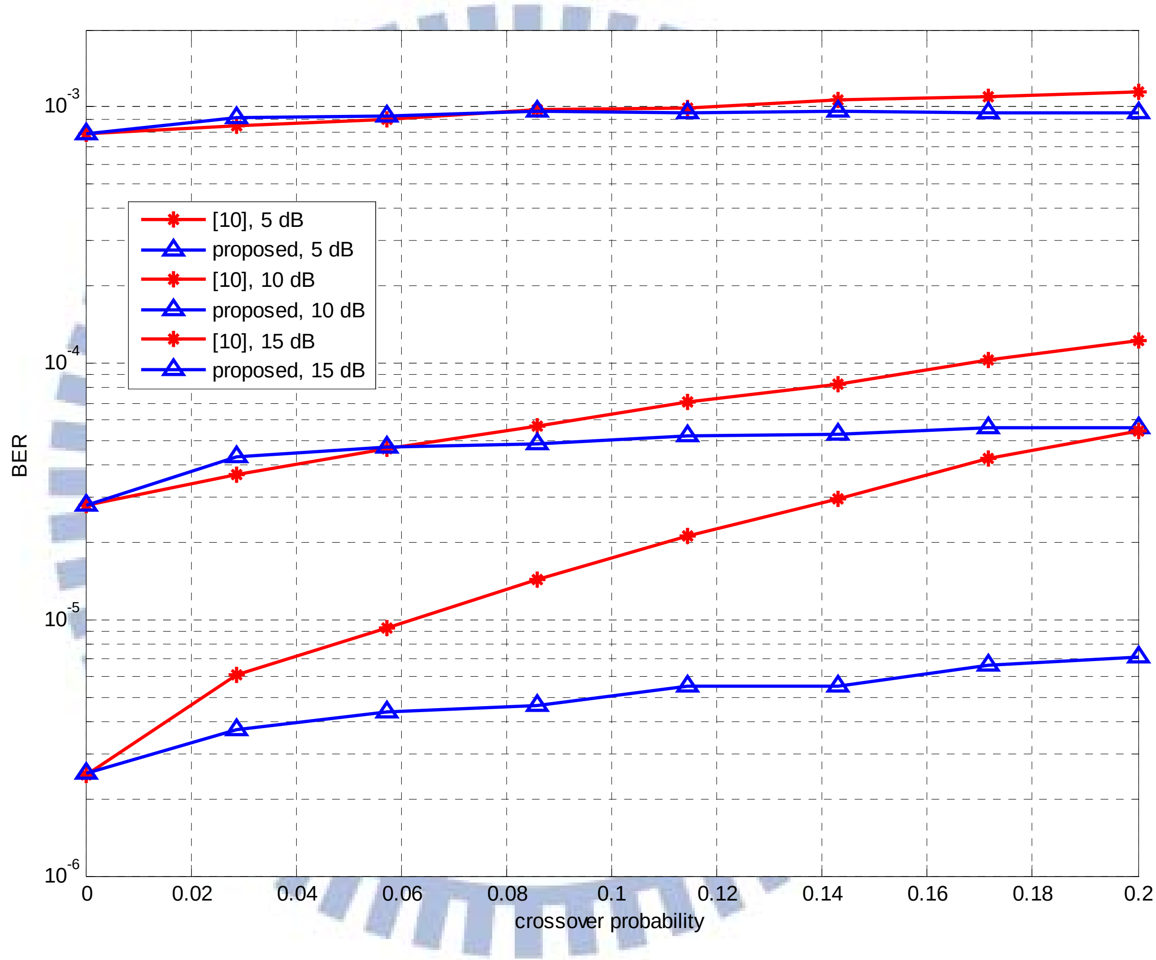


Figure 3. BER results of two methods with respect to different crossover probabilities.

Chapter 4

Beamforming Design Under Imperfect Quantized S-R Link SNR and R-D Link Channel Estimation Errors

Based on the result in Chapter 3, we generalize the bemaforming design by further taking the effect of imperfect R-D channel estimation into consideration. Rather than assuming that knowledge of exact CSI of all R-D links is available, we focus on the practical scenario that the destination only knows the estimated channel coefficients, given by

$$\hat{h}_{r,i} = h_{r,i} - \varepsilon_i,\ 1 \le i \le L, \tag{4.1}$$

where $h_{r,i}$ is the true channel gain of the ith R-D link and $\varepsilon_i \sim \mathcal{CN}(0,\sigma_e^2)$ is the channel estimation error. Given the received one-bit message $\hat{\mathbf{q}} = [\hat{q}_1 \cdots \hat{q}_L]^T$ and the channel estimates $\hat{\mathbf{h}}_{\mathbf{r}} = \left[\hat{h}_{r,1} \cdots \hat{h}_{r,L}\right]^T$, our task is to first derive an analytic expression for the receive SNR averaged over the distributions of the bit-flipping effect and R-D channel estimation error. Then, with the derived conditional SNR as the design criterion, we propose a method for designing the beamforming weights g_i's.

In Section 4.1 the exact expression for the considered conditional average SNR is derived, hereafter denoted by $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$. In Section 4.2, an approximate conditional average SNR formula is derived for intractability of $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ on beamforming design. Based on the approximate SNR formula, Section 4.3 derives a lower bound for $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$. By conducting maximization with respect to this lower bound, a closed-form suboptimal beamforming scheme is then obtained. Section 4.4 shows the simulation results, which illustrates the performance of the proposed method.

4.1. Exact Formula for the Conditional Average SNR

By definition, $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ is precisely the average of $\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_r,\hat{\mathbf{q}})$ defined in (3.10) over the distribution of the estimation errors of the R-D channels, that is,

$$\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}) = E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_\mathbf{r},\hat{\mathbf{q}}}\left[\overline{\gamma}_{dq}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})\right] = E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_\mathbf{r},\hat{\mathbf{q}}}\left[\sum_{m=1}^{M}\frac{\left|\mathbf{c}_m^H\mathbf{g}\right|^2}{\mathbf{g}^H\mathbf{D}_m\mathbf{g}}\right] = \sum_{m=1}^{M} E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_\mathbf{r},\hat{\mathbf{q}}}\left[\frac{\left|\mathbf{c}_m^H\mathbf{g}\right|^2}{\mathbf{g}^H\mathbf{D}_m\mathbf{g}}\right], \tag{4.2}$$

where $\mathbf{e}\triangleq[\varepsilon_1,\cdots,\varepsilon_L]^T$ is the channel estimation error vector. To proceed, we shall first rewrite each summand in (4.2) in the form of the expectation of a ratio of two quadratic forms in the error vector $\mathbf{e}$. Starting from (4.2) together with further manipulations, we have (see Appendix D for the detailed derivations)

$$\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}) = \sum_{m=1}^{M} E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\frac{\mathbf{e}^H\mathbf{b}_m\mathbf{b}_m^H\mathbf{e}+2\,\mathrm{Re}\left[\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\mathbf{e}\right]+\left|\mathbf{b}_m^H\hat{\mathbf{h}}_r\right|^2}{\begin{array}{l}\dfrac{\mathbf{e}^H\mathbf{Z}_m\mathbf{e}}{\eta p(m)}+\dfrac{2}{\eta p(m)}\mathrm{Re}(\mathbf{f}_m^H\mathbf{e})\\ +\dfrac{1}{\eta p(m)}\left[\sigma_w^2+\sum_{i=1}^{L}\left|g_i\right|^2\left[1-\phi^2\left(\rho_i(m)\right)\right]\left|\hat{h}_{r,i}\right|^2\right]\end{array}}\right], \tag{4.3}$$

where

$$p(m)\triangleq\prod_{j=1}^{l}p_{k_j}\left(\prod_{j=1}^{l}(1-p_{k_j})\right)^{-1}, \tag{4.4}$$

$$\mathbf{b}_m^H\triangleq\left[\phi\left(\rho_1(m)\right)g_1\cdots\phi\left(\rho_L(m)\right)g_L\right], \tag{4.5}$$

$$\mathbf{Z}_m = diag\left[\left|g_1\right|^2\left(1-\phi^2\left(\rho_1(m)\right)\right)\cdots\left|g_L\right|^2\left(1-\phi^2\left(\rho_L(m)\right)\right)\right], \tag{4.6}$$

and

$$\mathbf{f}_m = \left[\left|g_1\right|^2 \left(1-\phi^2\left(\rho_1(m)\right)\right)\hat{h}_{r,1}^* \cdots \left|g_L\right|^2 \left(1-\phi^2\left(\rho_L(m)\right)\right)\hat{h}_{r,L}^* \right]^T. \tag{4.7}$$

Now, each summand in (4.3) is the expectation of a Rayleigh ratio of the *complex-valued* Gaussian random vector **e**. To facilitate analysis, let us consider the augmented real-valued random vector associated with **e**, namely,

$$\mathbf{x} = \begin{bmatrix} \mathrm{Re}(\mathbf{e}) \\ \mathrm{Im}(\mathbf{e}) \end{bmatrix}, \tag{4.8}$$

where $\mathrm{Re}(\mathbf{e})$ and $\mathrm{Im}(\mathbf{e})$ denote, respectively, the real and imaginary parts of **e**. With the aid of (4.8), we go on to rewrite each summand in (4.3) in terms of the *real-valued* Gaussian random vector **x**. The result, as shown in the next proposition, will allow us to derive an analytic formula for $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$.

Proposition 4.1: Let $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ be defined in (4.3). Then it follows that

$$\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}) = \sum_{m=1}^{M} E_{\mathbf{x}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}} \left[\frac{\mathbf{x}^T \mathbf{A}_{1,m}\mathbf{x} + \mathbf{a}_{1,m}^T \mathbf{x} + d_{1,m}}{\mathbf{x}^T \mathbf{A}_{2,m}\mathbf{x} + \mathbf{a}_{2,m}^T \mathbf{x} + d_{2,m}} \right], \tag{4.9}$$

where **x** is defined in (4.8),

$$\mathbf{A}_{1,m} \triangleq \begin{bmatrix} \mathrm{Re}(\mathbf{b}_m \mathbf{b}_m^H) & -\mathrm{Im}(\mathbf{b}_m \mathbf{b}_m^H) \\ \mathrm{Im}(\mathbf{b}_m \mathbf{b}_m^H) & \mathrm{Re}(\mathbf{b}_m \mathbf{b}_m^H) \end{bmatrix}, \tag{4.10}$$

$$\mathbf{a}_{1,m} \triangleq \begin{bmatrix} \mathrm{Re}\left[2\mathbf{b}_m^* \mathbf{b}_m^T \hat{\mathbf{h}}_r^*\right] \\ -\mathrm{Im}\left[2\mathbf{b}_m^* \mathbf{b}_m^T \hat{\mathbf{h}}_r^*\right] \end{bmatrix}, \tag{4.11}$$

with **b** defined in (4.5),

$$\mathbf{A}_{2,m} \triangleq \begin{bmatrix} \mathrm{Re}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right) & 0 \\ 0 & \mathrm{Re}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right) \end{bmatrix}, \tag{4.12}$$

$$\mathbf{a}_{2,m} \triangleq \frac{2}{\eta p(m)} \begin{bmatrix} \mathrm{Re}(\mathbf{f}_m^*) \\ -\mathrm{Im}(\mathbf{f}_m^*) \end{bmatrix}, \tag{4.13}$$

$$d_{1,m} = \left|\mathbf{b}_m^H \hat{\mathbf{h}}_r\right|^2, \tag{4.14}$$

and

$$d_{2,m} = \frac{\sigma_w^2 + \sum_{i=1}^{L} |g_i|^2 \left[1 - \phi^2(\rho_i(m))\right] \left|\hat{h}_{r,i}\right|^2}{\eta p(m)}. \tag{4.15}$$

[Proof]: See Appendix. □

Since $\varepsilon_i \sim \mathcal{CN}(0, \sigma_e^2)$, it follows immediately that $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \sigma_e^2 \mathbf{I}_{2L})$, namely, the real $2L$-dimensional Gaussian random vector with zero mean and variance σ_e^2. To find a closed-form expression for $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ based on (4.9), we need the following lemma.

Lemma 4.2: For $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \sigma_e^2 \mathbf{I}_{2L})$ we have

$$E_{\mathbf{x}|\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}} \left[\frac{\mathbf{x}^T \mathbf{A}_{1,m} \mathbf{x} + \mathbf{a}_{1,m}^T \mathbf{x} + d_{1,m}}{\mathbf{x}^T \mathbf{A}_{2,m} \mathbf{x} + \mathbf{a}_{2,m}^T \mathbf{x} + d_{2,m}} \right] = \int_0^\infty \int_0^\infty \frac{\partial^2}{\partial z_1^2} f(-z_1, -z_2) dz_1 dz_2, \tag{4.16}$$

where

$$f(-z_1, -z_2) \triangleq |\mathbf{R}_m|^{-1/2} \exp\left\{-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2\right\}, \tag{4.17}$$

in which

$$\mathbf{R}_m \triangleq \mathbf{I}_{2L} + 2\sigma_e^2 \left(z_1 \mathbf{A}_{1,m} + z_2 \mathbf{A}_{2,m}\right), \tag{4.18}$$

and

$$\mathbf{r}_m \triangleq \sigma_e \left(z_1 \mathbf{a}_{1,m} + z_2 \mathbf{a}_{2,m}\right). \tag{4.19}$$

[Proof]: The result follows directly from Theorem 3.2c.3 in [29]. □

Based on Lemma 4.2, an analytic formula for the conditional average SNR $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ is derived in the following theorem.

Theorem 4.3: Let $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ be defined in (4.9). It follows that

$$\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}) = \sum_{m=1}^{M} \int_0^\infty \int_0^\infty \left\{ \exp(-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2) |\mathbf{R}_m|^{-1/2} f_1(z_1, z_2) \right\} dz_1 dz_2, \tag{4.20}$$

where

$$f_1(z_1,z_2) \triangleq f_2^2(z_1,z_2) + \sigma_e^2 \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \mathbf{a}_{1,m} - 2\sigma_e^3 \begin{bmatrix} -\sigma_e tr(\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}) + 2\mathbf{a}_{1,m}^T\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{r}_m \\ -2\sigma_e \mathbf{r}_m^T \mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{r}_m \end{bmatrix}, \quad (4.21)$$

and

$$f_2(z_1,z_2) \triangleq d_{1,m} - \sigma_e \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1}\mathbf{r}_m + \sigma_e^2 \left[tr(\mathbf{R}_m^{-1}\mathbf{A}_{1,m}) + \mathbf{r}_m^T\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{r}_m\right], \quad (4.22)$$

with $\mathbf{R}_m$ and $\mathbf{r}_m$ defined in (4.18) and (4.19).

[Proof]: See Appendix. □

To the best of our knowledge, the double-integral in (4.20) does not admit further closed-form expressions. In the simulation section, it will be verified that the derived integral formula (4.20) is in close agreement with the corresponding simulation outcome. It can be seen that $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ in the form (4.20) is a very complicated function of the beamforming coefficients g_i's, and direct maximization of $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ based on (4.20) is thus intractable. In the following section, we propose a method that can facilitate an analytic design of g_i's.

4.2 Approximation for $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$

As noted before, the double integral form of $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ in (4.20) makes the beamforming design problem formidable to tackle. To resolve this difficulty, we then turn to the expression of $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ given in (4.9), which is a sum of the expected value of the ratio of quadratic functions in the random vector **x**. To ease subsequent analysis, we propose to adopt the commonly used approximation technique (see, e.g., [30]); more specifically, each summand in (4.9) is approximated as the ratio of the respective means of the numerator and denominator:

$$E_{\mathbf{x}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\frac{\mathbf{x}^T\mathbf{A}_{1,m}\mathbf{x}+\mathbf{a}_{1,m}^T\mathbf{x}+d_{1,m}}{\mathbf{x}^T\mathbf{A}_{2,m}\mathbf{x}+\mathbf{a}_{2,m}^T\mathbf{x}+d_{2,m}}\right]\approx\frac{E_{\mathbf{x}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\mathbf{x}^T\mathbf{A}_{1,m}\mathbf{x}+\mathbf{a}_{1,m}^T\mathbf{x}+d_{1,m}\right]}{E_{\mathbf{x}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\mathbf{x}^T\mathbf{A}_{2,m}\mathbf{x}+\mathbf{a}_{2,m}^T\mathbf{x}+d_{2,m}\right]}. \quad (4.23)$$

Through further rearrangement, the approximation in (4.23) admits a simple form as shown in the next lemma.

Lemma 4.4: Let $\mathbf{x}\sim\mathcal{N}(\mathbf{0},\sigma_e^2\mathbf{I}_{2L})$. Then we have

$$\frac{E_{\mathbf{x}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\mathbf{x}^T\mathbf{A}_{1,m}\mathbf{x}+\mathbf{a}_{1,m}^T\mathbf{x}+d_{1,m}\right]}{E_{\mathbf{x}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\mathbf{x}^T\mathbf{A}_{2,m}\mathbf{x}+\mathbf{a}_{2,m}^T\mathbf{x}+d_{2,m}\right]}=\frac{\sigma_e^2Tr(\mathbf{A}_{1,m})+d_{1,m}}{\sigma_e^2Tr(\mathbf{A}_{2,m})+d_{2,m}}, \quad (4.24)$$

where $Tr(\cdot)$ denotes the trace operator; $\mathbf{A}_{1,m}$, $\mathbf{A}_{2,m}$, $d_{1,m}$, and $d_{2,m}$, are defined in, respectively, (4.10), (4.12), (4.14), and (4.15).

[Proof]: See Appendix. □

Based on (4.23) and (4.24), it directly follows that

$$\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})\approx\sum_{m=1}^{M}\frac{\sigma_e^2Tr(\mathbf{A}_{1,m})+d_{1,m}}{\sigma_e^2Tr(\mathbf{A}_{2,m})+d_{2,m}}. \quad (4.25)$$

By invoking the definitions of $\mathbf{A}_{1,m}$, $\mathbf{A}_{2,m}$, $d_{1,m}$, and $d_{2,m}$, each summand in (4.25) can be further rearranged into a ratio of two standard quadratic forms in terms of the beamforming weights g_i's; hence, the expression for $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ can be further simplified. This is done in the next lemma.

Lemma 4.5: The approximation of $\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})$ in (4.25) can be expressed as

$$\overline{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})\approx\sum_{m=1}^{M}\frac{\mathbf{g}^H\mathbf{W}_m\mathbf{g}}{\mathbf{g}^H\mathbf{Y}_m\mathbf{g}}, \quad (4.26)$$

where

$$\mathbf{W}_m\triangleq\mathbf{u}_m\mathbf{u}_m^H+2\sigma_e^2\eta p(m)diag\left(\phi^2\left(\rho_1(m)\right),\cdots,\phi^2\left(\rho_L(m)\right)\right), \quad (4.27)$$

in which

$$\mathbf{u}^H\triangleq\sqrt{\eta p(m)}\left[\hat{h}_{r,1}\phi\left(\rho_1(m)\right)\cdots\hat{h}_{r,L}\phi\left(\rho_L(m)\right)\right], \quad (4.28)$$

and

$$\mathbf{Y}_m \triangleq diag\left(\begin{array}{l}\frac{\sigma_w^2}{P_d}+\left(1-\phi^2\left(\rho_1(m)\right)\right)\left(2\sigma_e^2+\left|\hat{h}_{r,1}\right|^2\right),\cdots \\ ,\frac{\sigma_w^2}{P_d}+\left(1-\phi^2\left(\rho_L(m)\right)\right)\left(2\sigma_e^2+\left|\hat{h}_{r,L}\right|^2\right)\end{array}\right). \tag{4.29}$$

[Proof]: See Appendix. □

4.3 Design of Beamforming Weights

With the aid of (4.26), the problem of beamforming design for SNR enhancement can be formulated as

$$\text{Maximize} \quad \sum_{m=1}^{M}\frac{\mathbf{g}^H\mathbf{W}_m\mathbf{g}}{\mathbf{g}^H\mathbf{Y}_m\mathbf{g}} \quad \text{s.t. } \left\|\mathbf{g}\right\|_2^2 = P_d. \tag{4.30}$$

where P_d is the total available power budget. The optimization problem (4.30), unfortunately, is not convex and the optimal solution is difficult to find. Toward analytical tractability and complexity reduction, the following theorem further derives a lower bound for the objective function in (4. 30). As will be shown later, by conducting maximization of the derived lower bound, an analytic suboptimal solution can then be obtained.

Theorem 4.6: Let $\mathbf{W}_m$ and $\mathbf{Y}_m$ be defined in (4.27) and (4.29). Then the following inequality holds:

$$\sum_{m=1}^{M}\frac{\mathbf{g}^H\mathbf{W}_m\mathbf{g}}{\mathbf{g}^H\mathbf{Y}_m\mathbf{g}} \geq \frac{\mathbf{g}^H\mathbf{W}\mathbf{g}}{\mathbf{g}^H\mathbf{Y}\mathbf{g}}, \tag{4.31}$$

where $\mathbf{W} = \sum_{m=1}^{M}\mathbf{W}_m$ and $\mathbf{Y} = \sum_{m=1}^{M}\mathbf{Y}_m$.

[Proof]: See Appendix. □

With the aid of Theorem 4.6, we propose to obtain a suboptimal beamformer based on maximization of the lower bound derived in (4.31):

$$\text{Maximize} \quad \frac{\mathbf{g}^H\mathbf{W}\mathbf{g}}{\mathbf{g}^H\mathbf{Y}\mathbf{g}} \quad \text{s.t. } \left\|\mathbf{g}\right\|_2^2 = P_d. \tag{4.32}$$

The solution to (4.32) is precisely the dominant eigenvector of $\mathbf{Y}^{-1}\mathbf{W}$ [26], normalized so that the two-norm equal to P_d.

4.4 Simulation Results

This section uses numerical simulations to illustrate the effectiveness of the proposed scheme. In the cooperative beamforming system, the channel gains of all S-R and R-D links are i.i.d random variables generated from $\mathcal{CN}(0,1)$. The crossover probability p_i of each BSC is drawn from $U(0.05, 0.1)$, the uniform random variable distributed over the interval $[0.05, 0.1]$. The quantization threshold is set according to the rule in [10, p-4779]. The total available power of transmit beamforming is $P_d = 1$. In each Monte-Carlo run, a sequence of $T = 5000$ BPSK source symbols is generated.

A. Validation of the Derived Conditional Average SNR (4.20)

To validate the formula of the derived conditional average SNR $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ in (4.20), we consider a network of $L = 4$ relays, and the proposed suboptimal beamforming scheme (4.32) is employed at each relay. With R-D link channel estimation error variance $\sigma_e^2 = 0.01$, Figure 4 compares the values of $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ obtained based on the integral form (4.20) and corresponding simulated outcome at different average R-D receive SNR, defined to be $\overline{\gamma}_d \triangleq P_d\sigma_r^2 / \sigma_w^2 = \sigma_w^{-2}$ [8]. Also, with R-D receive SNR fixed to be $\overline{\gamma}_d = 15$ dB, Figure 5 compares the theoretical and simulated values of $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ at different channel estimation error variance σ_e^2. In both figures, the simulated results are obtained by averaging over 100 realizations of the R-D link channel estimates. The figures show that the theoretical solutions computed based on (4.20) are indeed very close to the experimental results.

B. BER Performances

For fixed average S-R SNR $\overline{\gamma}_s = 13$ dB, Figure 6 compares the BER curves of the proposed beamforming scheme (4.32) with the solutions in (3.17) and [10] at various average R-D SNR. The number of relays is $L = 6$, and the R-D channel estimation error of each link is drawn according to $\varepsilon_i \sim \mathcal{CN}(0, 0.05)$. As can be seen from the figure, the proposed scheme outperforms the method in [10]; the result is not unexpected since the solution in [10] is designed under the idealized assumption that the one-bit message is received at the destination without errors and that the CSI of the R-D links is perfect. Also, compared with the solution in (3.17), the proposed beamformer (4.32) yields improved performance since the effect of R-D channel estimation error is taken account of in the design. With fixed $\overline{\gamma}_s = 13$ dB, and $\overline{\gamma}_d = 20$ dB, Figure 7 further illustrates the BER of the three methods when the variance σ_e^2 of R-D channel estimation error varies from 0 to 1, i.e. $0 \leq \sigma_e^2 \leq 1$ (the number of relay is $L = 6$). The figure shows that, compared with the solutions in (3.17) and [10], the proposed scheme is indeed more robust against the increase in the channel estimation error variance. It is somewhat unexpected to see that the beamformer in (3.17), which takes into account only the effect of the one-bit S-R-link SNR transmission error, performs worse than [10] when $\sigma_e^2 \geq 0.08$. A plausible rationale behind this phenomenon is that, while the beamformer in [10] admits a simple closed-form expression (see (37) in [10]), the computation of the solution in (3.17) calls for solving a generalized eigenvalue problem. The involved eigen-decomposition increases the algorithmic complexity, and renders the resultant solution more vulnerable to system parameter mismatch. The proposed beamforming scheme, which further takes account of the effect of R-D link channel estimation errors, does improve the robustness against model uncertainty.

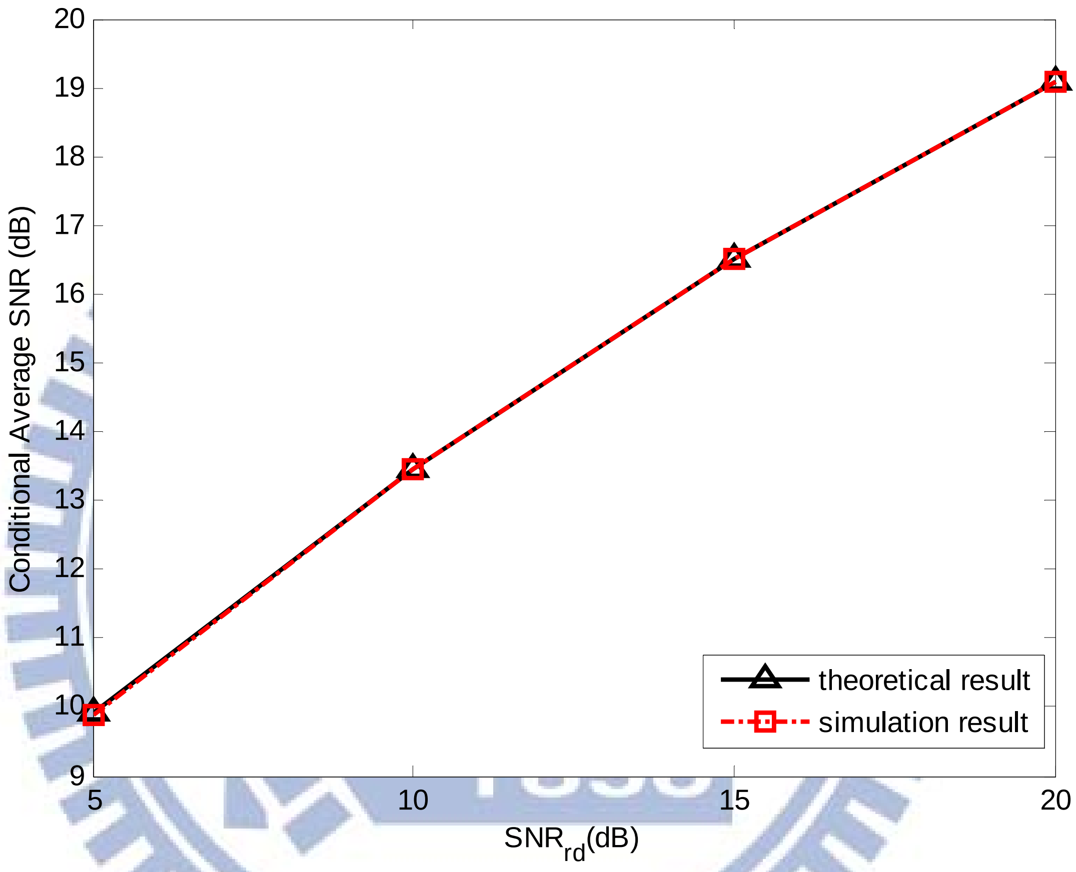


Figure 4. Values of the conditional average SNR $\overline{\gamma}\left(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}\right)$ obtained via the integral solution (4.20) and simulations for different R-D SNR (channel estimation error variance is set to be 0.01).

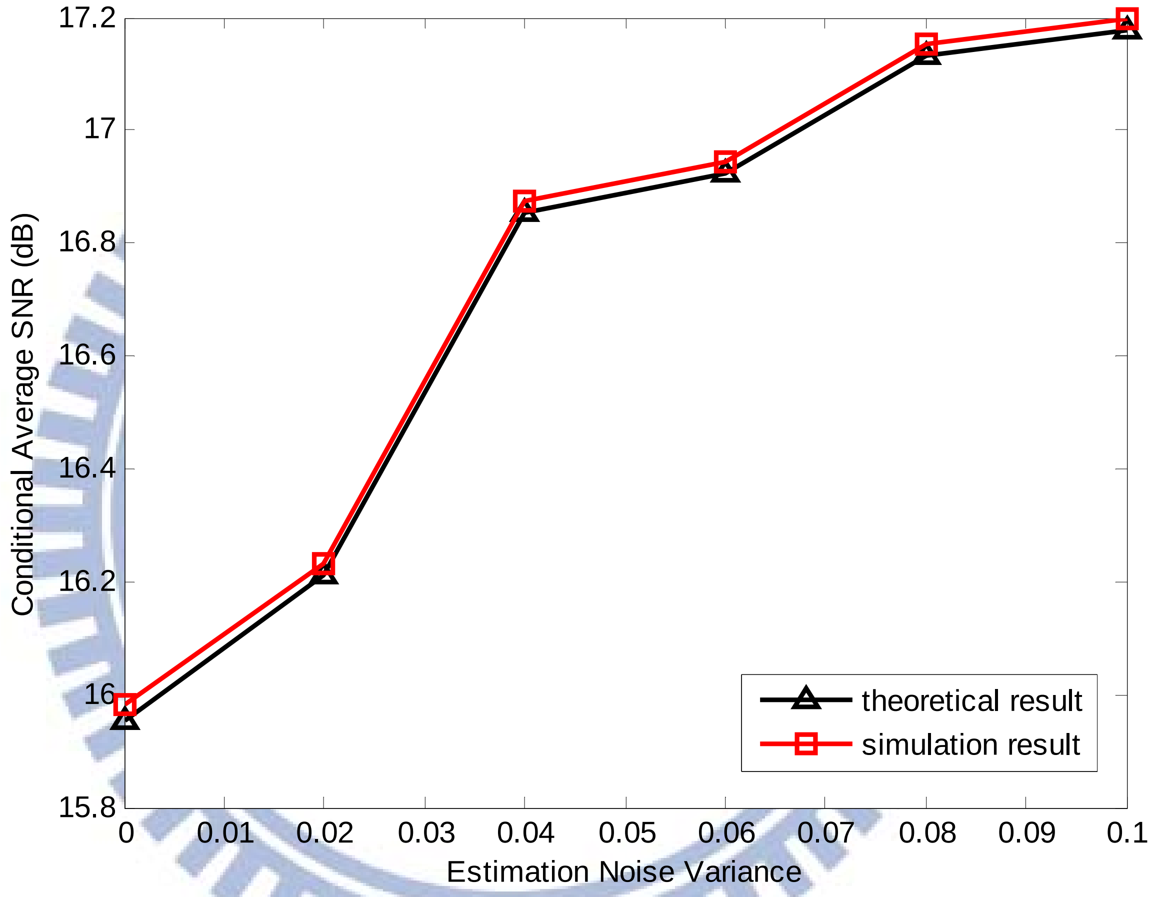


Figure 5. Values of the conditional average SNR $\bar{\gamma}\left(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}\right)$ obtained via the integral solution (4.20) and simulations for different channel estimation error variance (R-D SNR is set to be 15 dB).

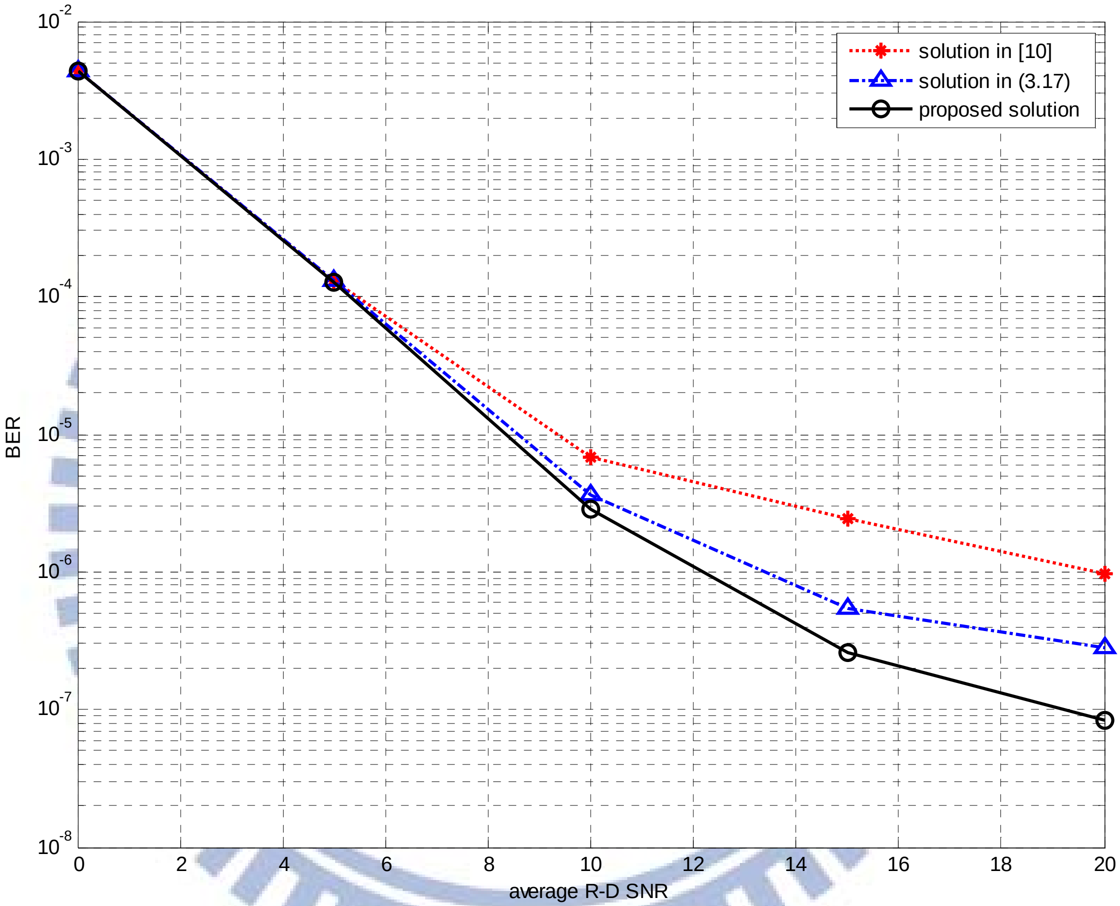


Figure 6. BER performance of the proposed beamformer (4.32) and the two solution in [10] and (3.17) for different R-D SNR.

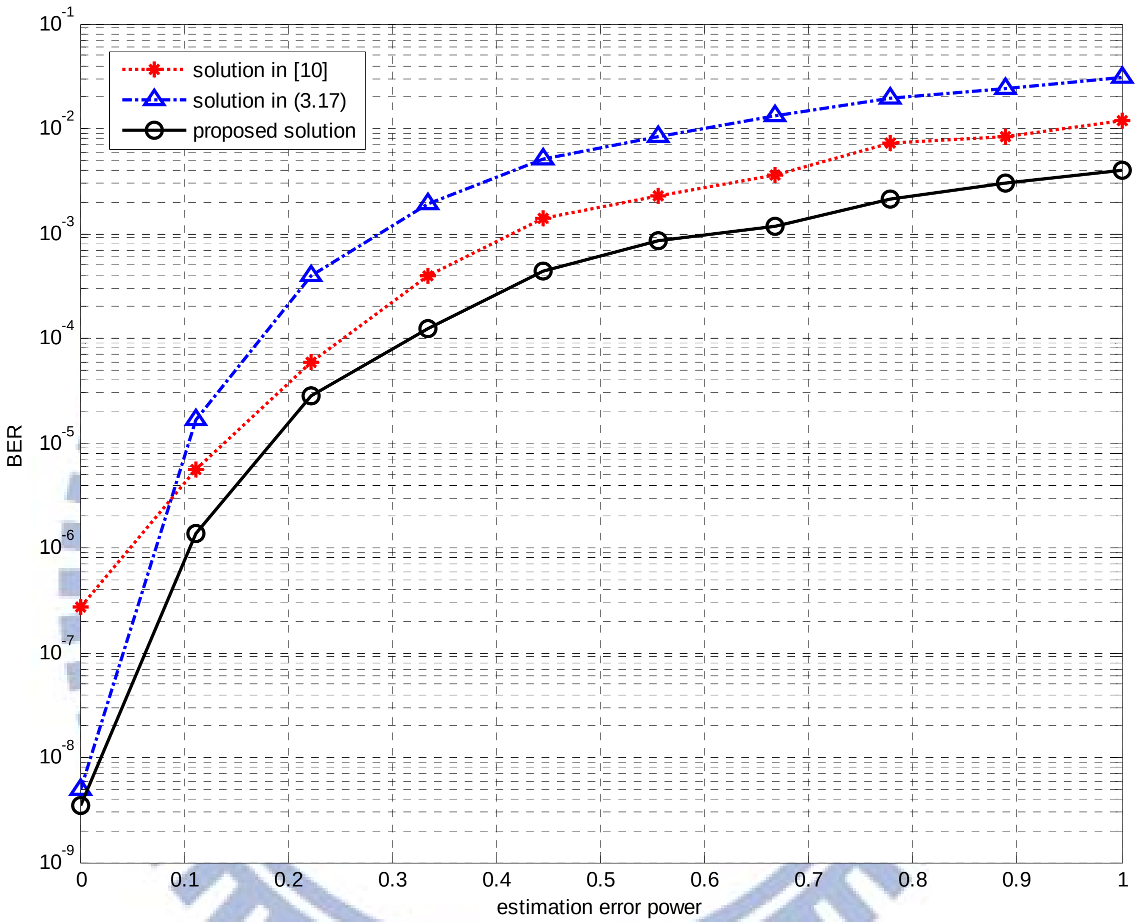


Figure 7. BER performance of the proposed beamformer (4.32) and the two solutions in [10] and (3.17) for different channel estimation error variance.

Conclusion

Low-overhead cooperative beamforming design under mismatched inter-node CSI has been an important research problem in the study of relay-based wireless communications. In this these we investigate this problem by further taking into account the effects of imperfect transmission of quantized S-R link SNR and R-D link channel estimation errors. As in previous study, the transmission link of the one-bit S-R link SNR is modeled as a BSC with a non-zero crossover probability. In the first part of this thesis, we assume R-D link channel estimation is perfect. Given a set of received quantized SNR message, we derive the closed-form formula of the expected conditional SNR, averaged over the bit-flipping distributions of BSCs. While beamforming design via direct maximization of this SNR metric is formidable, we further derive an tractable SNR lower bound. By conducting maximization of this lower bound, a suboptimal beamformer can be obtained as the solution to a generalized eigenvalue problem. In the second part of this thesis, the assumption of perfect R-D channel estimation is relaxed, and the channel estimation errors are modeled as i.i.d. Gaussian random variables. Given the received quantized S-R link SNR and a set of R-D link channel estimates, we further derive the exact formula of the expected conditional SNR averaged over both the distributions of the bit-flipping effect and R-D link channel estimation errors. Since the SNR metric thus obtained is difficult to analyze, we resort to certain approximation techniques to derive a tractable SNR lower bound. Still, through maximization of this lower bound a suboptimal beamformer can be obtained by solving a generalized eigenvalue problem. Computer simulations evidences the performance advantages of the proposed schemes.

Appendix A

Proof of Theorem 3.1

To derive (3.6), we shall find an explicit expression for $\beta_l \triangleq \sum_{\tilde{\mathbf{q}} \in S_l(\hat{\mathbf{q}})} \Pr(\tilde{\mathbf{q}} | \hat{\mathbf{q}}) \gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}})$, $l = 0, \cdots, L$. The term β_0 represents the case with $\mathbf{q} = \hat{\mathbf{q}}$. It then follows immediately that

$$\beta_0 = \Pr(\tilde{\mathbf{q}} = \hat{\mathbf{q}} | \hat{\mathbf{q}}) \gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) \overset{(a)}{=} \underbrace{\prod_{k=1}^{L} (1 - p_k)}_{= \Pr(\tilde{\mathbf{q}} = \hat{\mathbf{q}} | \hat{\mathbf{q}})} \frac{\left| \sum_{i=1}^{L} h_{r,i} g_i \phi(\hat{q}_i) \right|^2}{\sum_{i=1}^{L} \left| h_{r,i} \right|^2 \left| g_i \right|^2 [1 - \phi^2(\hat{q}_i)] + \sigma_w^2}, \quad \text{(A.1)}$$

where (a) follows from (2.5) and (3.5). The term β_1 represents the event that the true $\mathbf{q}$ differs from $\hat{\mathbf{q}}$ in one bit. Given $\hat{\mathbf{q}}$, there are totally $\left| S_1(\hat{\mathbf{q}}) \right| = C_1^L = L$ possible candidate $\tilde{\mathbf{q}}$'s. Therefore, β_1 can be accordingly expressed as

$$\beta_1 = \sum_{\tilde{\mathbf{q}} \in S_1(\hat{\mathbf{q}})} \Pr(\tilde{\mathbf{q}} | \hat{\mathbf{q}}) \gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}}) = \sum_{k_1=1}^{L} \left[p_{k_1} \prod_{j \neq k_1} (1 - p_j) \right] \gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}})$$

$$= \sum_{k_1=1}^{L} \left[p_{k_1} \prod_{j \neq k_1} (1 - p_j) \right] \frac{\left| h_{r,k_1} g_{k_1} \phi(\hat{q}_{k_1}^t) + \sum_{i \neq k_1} h_{r,i} g_i \phi(\hat{q}_i) \right|^2}{\left| h_{r,k_1} \right|^2 \left| g_{k_1} \right|^2 \left[1 - \phi^2(\hat{q}_{k_1}^t) \right] + \sum_{i \neq k_1} \left| h_{r,i} \right|^2 \left| g_i \right|^2 \left[1 - \phi^2(\hat{q}_i) \right] + \sigma_w^2}$$

$$\overset{(b)}{=} \sum_{k_1=1}^{L} \frac{\left| \sum_{i=1}^{L} c_i(1, k_1) g_i \right|^2}{\sum_{i=1}^{L} \left| g_i \right|^2 d_i(1, k_1)},$$

(A.2)

where (b) follows after some straightforward manipulations. The term β_2 stands for the event that $\hat{\mathbf{q}}$ differs from the true $\mathbf{q}$ in two bits. Given $\hat{\mathbf{q}}$, there are totally

$\left|S_2(\hat{\mathbf{q}})\right| = C_2^L = L(L-1)/2$ possible candidate $\tilde{\mathbf{q}}$'s in this case. By repeating the above arguments, it can be directly verified that

$$\beta_2 = \sum_{k_1=1}^{L} \sum_{k_2=k_1+1}^{L} \left\{ \frac{\eta \dfrac{p_{k_1} p_{k_2}}{(1-p_{k_1})(1-p_{k_2})} \times \left| \sum_{i=k_1,k_2} h_{r,i} g_i \phi(\hat{q}_i^t) + \sum_{i \neq k_1,k_2} h_{r,i} g_i \phi(\hat{q}_i) \right|^2}{\sum_{i=k_1,k_2} \left|h_{r,i}\right|^2 \left|g_i\right|^2 \left(1-\phi^2(\hat{q}_i^t)\right) + \sum_{i \neq k_1,k_2} \left|h_{r,i}\right|^2 \left|g_i\right|^2 \left(1-\phi^2(\hat{q}_i)\right) + \sigma_w^2} \right\}$$

$$= \sum_{k_1=1}^{L} \sum_{k_2=k_1+1}^{L} \frac{\left| \sum_{i=1}^{L} c_i(2,k_1,k_2) g_i \right|^2}{\sum_{i=1}^{L} \left|g_i\right|^2 d_i(2,k_1,k_2)}. \tag{A.3}$$

Based on the same idea and procedures, it can be readily shown that, for $l = 1, \cdots, L$,

$$\beta_l = \sum_{k_1=1}^{L} \sum_{k_2=k_1+1}^{L} \cdots \sum_{k_l=k_{l-1}+1}^{L} \frac{\eta \prod_{j=1}^{l} p_{k_j}}{\prod_{n=1}^{l} (1-p_{k_n})} \frac{\left| \sum_{i=k_1,\cdots,k_l} h_{r,i} g_i \phi(\hat{q}_i^t) + \sum_{i \neq k_1,\cdots,k_l}^{L} h_{r,i} g_i \phi(\hat{q}_i) \right|^2}{\left\{ \begin{array}{l} \sum_{i=k_1,\cdots,k_l} \left|h_{r,i}\right|^2 \left|g_i\right|^2 \left(1-\phi^2(\hat{q}_i^t)\right) \\ + \sum_{i \neq k_1,\cdots,k_l}^{L} \left|h_{r,i}\right|^2 \left|g_i\right|^2 \left(1-\phi^2(\hat{q}_i)\right) + \sigma_w^2 \end{array} \right\}}$$

$$= \sum_{k_1=1}^{L} \sum_{k_2=k_1+1}^{L} \cdots \sum_{k_l=k_{l-1}+1}^{L} \frac{\left| \sum_{i=1}^{L} c_i(l,k_1,\cdots,k_l) g_i \right|^2}{\sum_{i=1}^{L} \left|g_i\right|^2 d_i(l,k_1,\cdots,k_l)}. \tag{A.4}$$

Equation (3.6) follows since $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) = \sum_{l=0}^{L} \beta_l$. □

Appendix B

Proof of Theorem 3.2

All we have to do is to rewrite the multiple summations in (3.6) as a single summation in the form of (3.10), and then to provide an explicit relation between *m* in (3.10) and the multiple indices $l, k_1, \cdots, k_l$ involved in (3.6). For ease of discussion, recall that $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_\mathbf{r}, \hat{\mathbf{q}}) = \sum_{l=0}^{L} \beta_l$, where β_0 is defined in (A.1) and β_l for $l \neq 0$ is given by (A.4). Note that, for each $0 \le l \le L$, there are totally C_l^L terms in β_l. The main procedures for deriving (3.10) can be summarized as follows: (1) exhaustively list all the $C_0^L + C_1^L + \cdots + C_L^L$ terms in (3.6) in the increasing order of *l*; (2) particularly, in β_l $(l \ge 1)$, the C_l^L terms in the *l*-fold multiple summations are listed in the following way: starting from $k_1 = 1$, exhaustively list all involved terms indexed by this k_1 in the remaining summations, and then proceed to $k_1 = 2$, and so forth. Based on such procedures, for given $l, k_1, \cdots, k_l$, the corresponding *m* given in (3.13) can then be obtained by induction and some straightforward manipulations.

The first term in (3.10) (indexed by $m = 1$) is simply β_0, thus $\frac{\left|\mathbf{c}_1^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_1 \mathbf{g}} = \frac{\left|\sum_{i=1}^{L} c_i(l=0) g_i\right|^2}{\sum_{i=1}^{L} \left|g_i\right|^2 d_i(l=0)}$. Now consider $l \neq 0$, and our purpose is to determine for the particular indices $(l, k_1, \cdots, k_l)$ the corresponding *m*. For this we first note that the total number of terms contained in $\beta_0, \cdots, \beta_{l-1}$ is $\sum_{i=0}^{l-1} C_i^L$. For the particular *l*, let us likewise exhaustively list all the terms in β_l and collect them into a set $\mathcal{L}_l$. Assume that the considered term indexed by $(l, k_1, \cdots, k_l)$ is exactly the K-th element in $\mathcal{L}_l$.

Then it follows immediately that $m = \sum_{i=0}^{l-1} C_i^L + K$. Hence it suffices to determine *K*. Towards this end, we recall that, for a fixed *l*, thus totally *l* flipped bits, the indices $k_1 < k_2 < \cdots < k_l$, where $k_1, \cdots, k_l \in \{1, \cdots, L\}$, denote the locations at which bit errors occur. Consequently, it is noted that the first C_{l-1}^{L-1} terms[8] in $\mathcal{L}_l$ are those indexed by $k_1 = 1$, the next C_{l-1}^{L-2} terms in $\mathcal{L}_l$ are those indexed by $k_1 = 2$, and so on. Hence, in $\mathcal{L}_l$ there are $\sum_{s_1=1}^{k_1-1} C_{l-1}^{L-s_1}$ terms listed before the terms indexed by a given k_1. Now let us further list the terms indexed by such a k_1 to obtain a set $\mathcal{S}_{k_1}$. By following the same argument, the first $C_{l-2}^{L-(k_1+1)}$ terms in $\mathcal{S}_{k_1}$ are indexed by $k_2 = k_1 + 1$, the next $C_{l-2}^{L-(k_1+2)}$ terms are indexed by $k_2 = k_1 + 2$, and so forth. Hence, it can be readily deduced that there are totally $\sum_{s_2=k_1+1}^{k_2-1} C_{l-2}^{L-s_2}$ terms listed before the terms indexed by a given k_2 in $\mathcal{S}_{k_1}$. As a result, for a given pair of k_1 and k_2, a total number of $\sum_{s_1=1}^{k_1-1} C_{l-1}^{L-s_1} + \sum_{s_2=k_1+1}^{k_2-1} C_{l-2}^{L-s_2}$ terms are listed before the terms indexed by such (k_1, k_2) in $\mathcal{L}_l$. By induction, it can be concluded that there are totally $\sum_{\lambda=1}^{l-1} \sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda}$ terms listed before the terms indexed by a given set of $k_0 = 0, k_1, \cdots, k_{l-1}$ in $\mathcal{L}_l$. Hence, for the considered $l, k_1, \cdots, k_l$, the associated *K* can be computed as

8. Since $k_1 = 1$ and the relation $k_1 < k_2 < \cdots < k_l$ must hold, the plausible values of k_i, $2 \le i \le l$, take only $L-1$ levels, namely, $\{2, 3, \cdots, L\}$. Hence, there are totally C_{l-1}^{L-1} possible error patterns (equal to the total number of combinations of $l-1$ out of $L-1$ levels).

$$K = \sum_{\lambda=1}^{l-1} \sum_{s_\lambda = k_{\lambda-1}+1}^{k_\lambda - 1} C_{l-\lambda}^{L-s_\lambda} + (k_l - (k_{l-1}+1)+1) = \sum_{\lambda=1}^{l-1} \sum_{s_\lambda = k_{\lambda-1}+1}^{k_\lambda - 1} C_{l-\lambda}^{L-s_\lambda} + (k_l - k_{l-1}). \tag{A.5}$$

With (A.5), the desired index *m* is thus

$$m = \begin{cases} \sum_{i=0}^{l-1} C_i^L + \sum_{\lambda=1}^{l-1} \sum_{s_\lambda = k_{\lambda-1}+1}^{k_\lambda - 1} C_{l-\lambda}^{L-s_\lambda} + (k_l - k_{l-1}), l > 0; \\ 1, \qquad l = 0. \end{cases} \tag{A.6}$$

The assertion follows from (A.6). □

Appendix C

Proof of Theorem 3.3

By the Cauchy-Schwartz inequality, we have

$$\underbrace{\left(\sum_{m=1}^{M}\frac{\left|\mathbf{c}_m^H\mathbf{g}\right|^2}{\mathbf{g}^H\mathbf{D}_m\mathbf{g}}\right)}_{\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_\mathbf{r},\hat{\mathbf{q}})}\left(\sum_{m=1}^{M}\mathbf{g}^H\mathbf{D}_m\mathbf{g}\right)\geq\left(\sum_{m=1}^{M}\left|\mathbf{c}_m^H\mathbf{g}\right|\right)^2. \tag{A.7}$$

Then,

$$\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_\mathbf{r},\hat{\mathbf{q}})\geq\frac{\left(\sum_{m=1}^{M}\left|\mathbf{c}_m^H\mathbf{g}\right|\right)^2}{\left(\sum_{m=1}^{M}\mathbf{g}^H\mathbf{D}_m\mathbf{g}\right)}\geq\frac{\left|\sum_{m=1}^{M}\mathbf{c}_m^H\mathbf{g}\right|^2}{\left(\sum_{m=1}^{M}\mathbf{g}^H\mathbf{D}_m\mathbf{g}\right)}=\frac{\left|\mathbf{c}^H\mathbf{g}\right|^2}{\mathbf{g}^H\mathbf{D}\mathbf{g}}=\frac{\mathbf{g}^H\mathbf{c}\mathbf{c}^H\mathbf{g}}{\mathbf{g}^H\mathbf{D}\mathbf{g}}, \tag{A.8}$$

which proves (3.15). $\square$

Appendix D

Derivations of Equation (4.3)

Since $\mathbf{h}_r = \hat{\mathbf{h}}_r + \mathbf{e} = \left[\hat{h}_{r,1} + \varepsilon_1, \cdots, \hat{h}_{r,L} + \varepsilon_L\right]^T$ and $\varepsilon_i \sim \mathcal{CN}(0, \sigma_e^2)$, $\forall i$, based on Lemma 2.1, $\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ in (3.1) can be further expressed as

$$\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}) = \sum_{m=1}^{M} E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\frac{\left|\sum_{i=1}^{L}\sqrt{\eta p(m)}\left(\hat{h}_{r,i} + \varepsilon_i\right)\phi\left(\rho_i(m)\right) g_i\right|^2}{\sum_{i=1}^{L}\left|g_i\right|^2\left|\hat{h}_{r,i} + \varepsilon_i\right|^2\left(1 - \phi^2\left(\rho_i(m)\right)\right) + \sigma_w^2}\right], \tag{A.9}$$

where $p(m)$, $\rho_i(m)$ are defined, respectively, in (4.4) and (3.9). Through straightforward rearrangement (A.9) can be expressed as

$$\overline{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}) = \sum_{m=1}^{M} E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\frac{\begin{array}{c}\eta p(m)\left|\sum_{i=1}^{L}\hat{h}_{r,i}\phi\left(\rho_i(m)\right) g_i\right|^2 + \eta p(m)\left|\sum_{i=1}^{L}\varepsilon_i\phi\left(\rho_i(m)\right) g_i\right|^2 \\ +2\,\mathrm{Re}\left(\eta p(m)\sum_{i=1}^{L}\hat{h}_{r,i}^{*}\phi\left(\rho_i(m)\right) g_i^{*}\sum_{j=1}^{L}\varepsilon_j\phi\left(\rho_j(m)\right) g_j\right)\end{array}}{\begin{array}{c}\sum_{i=1}^{L}\left|g_i\right|^2\left[1 - \phi^2\left(\rho_i(m)\right)\right]\left|\hat{h}_{r,i}\right|^2 + \sigma_w^2 \\ +\sum_{i=1}^{L}\left|g_i\right|^2\left[1 - \phi^2\left(\rho_i(m)\right)\right]\left|\varepsilon_i\right|^2 \\ +2\,\mathrm{Re}\left(\sum_{i=1}^{L}\left|g_i\right|^2\left[1 - \phi^2\left(\rho_i(m)\right)\right]\hat{h}_{r,i}^{*}\varepsilon_i\right)\end{array}}\right]$$

$$\overset{(c)}{=} \sum_{m=1}^{M} E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}}}\left[\frac{\eta p(m)\left|\mathbf{b}_m^H\hat{\mathbf{h}}_r\right|^2 + \eta p(m)\left|\mathbf{b}_m^H\mathbf{e}\right|^2 + 2\eta p(m)\,\mathrm{Re}\left(\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\mathbf{e}\right)}{\sigma_w^2 + \sum_{i=1}^{L}\left|g_i\right|^2\left[1 - \phi^2\left(\rho_i(m)\right)\right]\left|\hat{h}_{r,i}\right|^2 + \mathbf{e}^H\mathbf{Z}_m\mathbf{e} + 2\,\mathrm{Re}(\mathbf{f}_m^H\mathbf{e})}\right], \tag{A.10}$$

where (c) holds by using (4.5), (4.6), and (4.7) together with some manipulations. □

Appendix E

Proof of Proposition 4.1

We will first show that $\mathbf{e}^H\mathbf{b}_m\mathbf{b}_m^H\mathbf{e} + 2\,\mathrm{Re}\left[\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\mathbf{e}\right] + \left|\mathbf{b}_m^H\hat{\mathbf{h}}_r\right|^2 = \mathbf{x}^T\mathbf{A}_{1,m}\mathbf{x} + \mathbf{a}_{1,m}^T\mathbf{x}$ $+d_{1,m}$. To proceed, let us note that

$$
\begin{aligned}
\mathbf{e}^H\mathbf{b}_m\mathbf{b}_m^H\mathbf{e} &= [\mathrm{Re}(\mathbf{e}) + j\,\mathrm{Im}(\mathbf{e})]^H\left[\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right) + j\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right][\mathrm{Re}(\mathbf{e}) + j\,\mathrm{Im}(\mathbf{e})] \\
&= \left\{\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right) + \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) + j\begin{bmatrix}\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) \\ -\,\mathrm{Im}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\end{bmatrix}\right\} \\
&\quad \times[\mathrm{Re}(\mathbf{e}) + j\,\mathrm{Im}(\mathbf{e})] \\
&= \left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right) + \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Re}(\mathbf{e}) \\
&\quad -\left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) - \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Im}(\mathbf{e}) \\
&\quad + j\begin{Bmatrix}\left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) - \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Re}(\mathbf{e}) \\ +\left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right) + \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Im}(\mathbf{e})\end{Bmatrix} \\
&= \left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right) + \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Re}(\mathbf{e}) \\
&\quad -\left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) - \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Im}(\mathbf{e}) \\
&\quad + j\underbrace{\left\{\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\mathrm{Re}(\mathbf{e}) + \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\mathrm{Im}(\mathbf{e})\right\}}_{=0} \\
&\overset{(d)}{=}\left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right) + \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Re}(\mathbf{e}) \\
&\quad -\left[\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) - \mathrm{Im}(\mathbf{e}^T)\,\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\right]\mathrm{Im}(\mathbf{e}) \\
&= \begin{bmatrix}\mathrm{Re}(\mathbf{e}) \\ \mathrm{Im}(\mathbf{e})\end{bmatrix}^T\begin{bmatrix}\mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right) & -\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) \\ \mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right) & \mathrm{Re}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\end{bmatrix}\begin{bmatrix}\mathrm{Re}(\mathbf{e}) \\ \mathrm{Im}(\mathbf{e})\end{bmatrix} = \mathbf{x}^T\mathbf{A}_{1,m}\mathbf{x},
\end{aligned}
\tag{A.11}
$$

where (d) holds since $\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)$ is anti-symmetric, thus $\mathrm{Re}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\mathrm{Re}(\mathbf{e}) =$

$\mathrm{Im}(\mathbf{e}^T)\,\mathrm{Im}\left(\mathbf{b}_m\mathbf{b}_m^H\right)\mathrm{Im}(\mathbf{e}) = 0$. Moreover, with $\mathbf{a}_{1,m}^T$ defined in (4.11), we have

$$\begin{aligned} 2\,\mathrm{Re}\left[\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\mathbf{e}\right] &= 2\,\mathrm{Re}\left[\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\left(\mathrm{Re}(\mathbf{e}) + j\,\mathrm{Im}(\mathbf{e})\right)\right] \\ &= 2\left[\mathrm{Re}\left(\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\right)\mathrm{Re}(\mathbf{e}) - \mathrm{Im}\left(\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\right)\mathrm{Im}(\mathbf{e})\right] \\ &= \left[\mathrm{Re}\left(2\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\right) \quad -\mathrm{Im}\left(2\hat{\mathbf{h}}_r^H\mathbf{b}_m\mathbf{b}_m^H\right)\right]\begin{bmatrix}\mathrm{Re}(\mathbf{e})\\ \mathrm{Im}(\mathbf{e})\end{bmatrix} = \mathbf{a}_{1,m}^T\mathbf{x}. \end{aligned} \tag{A.12}$$

By setting $d_{1,m} = \left|\mathbf{b}_m^H\hat{\mathbf{h}}_r\right|^2$, the assertion then follows by definition of $\mathbf{b}_m$ and $\hat{\mathbf{h}}_r$.

Now, it remains to prove $\dfrac{1}{\eta p(m)}\left[\sum_{i=1}^{L}|g_i|^2\left[1-\phi^2\left(\rho_i(m)\right)\right]\left|\hat{h}_{r,i}\right|^2 + \sigma_w^2\right] + \dfrac{\mathbf{e}^H\mathbf{Z}_m\mathbf{e}}{\eta p(m)}$ $+\dfrac{2}{\eta p(m)}\mathrm{Re}(\mathbf{f}_m^H\mathbf{e}) = \mathbf{x}^T\mathbf{A}_{2,m}\mathbf{x} + \mathbf{a}_{2,m}^T\mathbf{x} + d_{2,m}$. Toward this end, we first rewrite $\dfrac{\mathbf{e}^H\mathbf{Z}_m\mathbf{e}}{\eta p(m)}$ as

$$\begin{aligned} \frac{\mathbf{e}^H\mathbf{Z}_m\mathbf{e}}{\eta p(m)} &= \begin{bmatrix}\mathrm{Re}(\mathbf{e})\\ \mathrm{Im}(\mathbf{e})\end{bmatrix}^T \begin{bmatrix}\mathrm{Re}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right) & -\mathrm{Im}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right)\\ \mathrm{Im}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right) & \mathrm{Re}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right)\end{bmatrix}\begin{bmatrix}\mathrm{Re}(\mathbf{e})\\ \mathrm{Im}(\mathbf{e})\end{bmatrix} \overset{(e)}{=} \mathbf{x}^T\begin{bmatrix}\mathrm{Re}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right) & 0\\ 0 & \mathrm{Re}\left(\dfrac{\mathbf{Z}_m}{\eta p(m)}\right)\end{bmatrix}\mathbf{x} \\ &= \mathbf{x}^T\mathbf{A}_{2,m}\mathbf{x}, \end{aligned} \tag{A.13}$$

where (e) holds since $\dfrac{\mathbf{Z}_m}{\eta p(m)}$ is a real-valued matrix. By following similar procedures as in the derivation of (A.12), the term $\dfrac{2}{\eta p(m)}\mathrm{Re}(\mathbf{f}_m^H\mathbf{e})$ can be rewritten as

$$\frac{2}{\eta p(m)}\mathrm{Re}(\mathbf{f}_m^H\mathbf{e}) = \frac{2}{\eta p(m)}\left[\mathrm{Re}\left(\mathbf{f}_m^H\right) \quad -\mathrm{Im}\left(\mathbf{f}_m^H\right)\right]\begin{bmatrix}\mathrm{Re}(\mathbf{e})\\ \mathrm{Im}(\mathbf{e})\end{bmatrix} = \mathbf{a}_{2,m}^T\mathbf{x}, \tag{A.14}$$

where $\mathbf{a}_{2,m}^T$ is defined in (4.13). By setting $d_{2,m} = \dfrac{\sigma_w^2 + \sum_{i=1}^{L}|g_i|^2\left[1-\phi^2\left(\rho_i(m)\right)\right]\left|\hat{h}_{r,i}\right|^2}{\eta p(m)}$,

the assertion then follows. $\square$

Appendix F

Proof of Theorem 4.3

Recall that each summand of $\bar{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}})$ is expressed in (4.16). To prove the theorem, we first rewrite $\frac{\partial}{\partial z_1} f(-z_1, -z_2)$ as

$$\frac{\partial}{\partial z_1} f(-z_1, -z_2) = e^{-z_2 d_{2,m} - z_1 d_{1,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2} \left\{ \begin{array}{l} \frac{\partial}{\partial z_1}\left[|\mathbf{R}_m|^{-1/2} \right] - d_{1,m} |\mathbf{R}_m|^{-1/2} \\ + \frac{1}{2} |\mathbf{R}_m|^{-1/2} \frac{\partial}{\partial z_1}\left[\mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m \right] \end{array} \right\}. \tag{A.15}$$

Note that in (A.15),

$$\begin{aligned} \frac{\partial}{\partial z_1}\left[|\mathbf{R}_m|^{-1/2} \right] &= -\frac{1}{2} |\mathbf{R}_m|^{-3/2} \frac{\partial}{\partial z_1}\left[|\mathbf{R}_m| \right] \overset{(f)}{=} -\frac{1}{2} |\mathbf{R}_m|^{-3/2} |\mathbf{R}_m| \, tr\left[\mathbf{R}_m^{-1} \frac{\partial}{\partial z_1} \mathbf{R}_m \right] \\ &= -\frac{1}{2} |\mathbf{R}_m|^{-1/2} tr\left[\mathbf{R}_m^{-1} \left(2\sigma_e^2 \mathbf{A}_{1,m} \right) \right] = -\sigma_e^2 |\mathbf{R}_m|^{-1/2} tr\left(\mathbf{R}_m^{-1} \mathbf{A}_{1,m} \right), \end{aligned} \tag{A.16}$$

where (f) follows from [32] and

$$\begin{aligned} \frac{\partial}{\partial z_1}\left[\mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m \right] &= \frac{\partial \mathbf{r}_m^T \mathbf{R}_m^{-1}}{\partial z_1} \mathbf{r}_m + \mathbf{r}_m^T \mathbf{R}_m^{-1} \frac{\partial \mathbf{r}_m}{\partial z_1} \\ &= \left(\mathbf{r}_m^T \left(\frac{\partial \mathbf{R}_m^{-1}}{\partial z_1} \right) + \frac{\partial \mathbf{r}_m^T}{\partial z_1} \mathbf{R}_m^{-1} \right) \mathbf{r}_m + \mathbf{r}_m^T \mathbf{R}_m^{-1} \left(\sigma_e \mathbf{a}_{1,m} \right) \\ &\overset{(g)}{=} \left(-2\sigma_e^2 \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} + \sigma_e \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \right) \mathbf{r}_m + \sigma_e \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{a}_{1,m} \\ &= 2\sigma_e \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \mathbf{r}_m - 2\sigma_e^2 \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{r}_m, \end{aligned} \tag{A.17}$$

where (g) holds for the equality

$$\frac{\partial \mathbf{R}_m^{-1}}{\partial z_1} = -\mathbf{R}_m^{-1} \frac{\partial \mathbf{R}_m}{\partial z_1} \mathbf{R}_m^{-1} = -2\sigma_e^2 \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1}, \tag{A.18}$$

and some straightforward manipulation. With (A.16) and (A.17), (A.15) then admits the form

$$\frac{\partial}{\partial z_1} f(-z_1,-z_2) = e^{-z_2 d_{2,m} - z_1 d_{1,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2} \Big\{ -\sigma_e^2 \left| \mathbf{R}_m \right|^{-1/2} tr\left(\mathbf{R}_m^{-1} \mathbf{A}_{1,m} \right) - d_{1,m} \left| \mathbf{R}_m \right|^{-1/2}$$

$$+ \frac{1}{2} \left| \mathbf{R}_m \right|^{-1/2} \left[2\sigma_e \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \mathbf{r}_m - 2\sigma_e^2 \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{r}_m \right] \Big\}$$

$$= -\left| \mathbf{R}_m \right|^{-1/2} e^{-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2} f_2(z_1, z_2), \qquad \text{(A.19)}$$

where $f_2(z_1, z_2)$ is defined as in (4.22). Then, the second derivative of $f(z_1, z_2)$ with respect to z_1 can be obtained as

$$\frac{\partial^2}{\partial z_1^2} f(-z_1,-z_2) \overset{(h)}{=} \sigma_e^2 e^{-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2} \left| \mathbf{R}_m \right|^{-1/2} tr\left(\mathbf{R}_m^{-1} \mathbf{A}_{1,m} \right) f_2(z_1, z_2)$$

$$- e^{-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2} \left| \mathbf{R}_m \right|^{-1/2} \frac{\partial}{\partial z_1} f_2(z_1, z_2)$$

$$- e^{-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2} \left| \mathbf{R}_m \right|^{-1/2} f_2(z_1, z_2) \begin{Bmatrix} -f_2(z_1, z_2) \\ +\sigma_e^2 tr(\mathbf{R}_m^{-1} \mathbf{A}_{1,m}) \end{Bmatrix}$$

$$= e^{-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{r}_m / 2} \left| \mathbf{R}_m \right|^{-1/2} \left\{ -\frac{\partial}{\partial z_1} f_2(z_1, z_2) + f_2^2(z_1, z_2) \right\}, \qquad \text{(A.20)}$$

where (h) holds based on equalities in (A.16) and (A.17), together with some basic manipulation. Hence, it suffices to derive $\frac{\partial}{\partial z_1} f_2(z_1, z_2)$, which can be obtained via

$$\frac{\partial}{\partial z_1} f_2(z_1, z_2) = \frac{\partial}{\partial z_1} \left\{ \sigma_e^2 tr(\mathbf{R}_m^{-1} \mathbf{A}_{1,m}) + \sigma_e^2 \mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{r}_m - \sigma_e \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \mathbf{r}_m \right\}$$

$$= \sigma_e^2 tr(\mathbf{A}_{1,m} \frac{\partial}{\partial z_1} \mathbf{R}_m^{-1}) + \sigma_e^2 \begin{bmatrix} \frac{\partial \mathbf{r}_m^T}{\partial z_1} \left(\mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{r}_m \right) + \mathbf{r}_m^T \left(\frac{\partial}{\partial z_1} \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \right) \mathbf{r}_m \\ + \left(\mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \right) \frac{\partial \mathbf{r}_m}{\partial z_1} \end{bmatrix}$$

$$- \sigma_e \left[\mathbf{a}_{1,m}^T \left(\frac{\partial \mathbf{R}_m^{-1}}{\partial z_1} \right) \mathbf{r}_m + \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \left(\frac{\partial \mathbf{r}_m}{\partial z_1} \right) \right]$$

$$= -2\sigma_e^4 tr(\mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1}) + \sigma_e^2 \begin{bmatrix} \sigma_e \mathbf{a}_{1,m}^T \left(\mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{r}_m \right) \\ -4\sigma_e^2 \mathbf{r}_m^T \left(\mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \right) \mathbf{r}_m \\ +\sigma_e \left(\mathbf{r}_m^T \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \right) \mathbf{a}_{1,m} \end{bmatrix}$$

$$- \sigma_e \left[-2\sigma_e^2 \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \mathbf{A}_{1,m} \mathbf{R}_m^{-1} \mathbf{r}_m + \mathbf{a}_{1,m}^T \mathbf{R}_m^{-1} \left(\sigma_e \mathbf{a}_{1,m} \right) \right]$$

$$= 2\sigma_e^3 \begin{bmatrix} -\sigma_e tr(\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}) + 2\mathbf{a}_{1,m}^T\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{r}_m \\ -2\sigma_e \mathbf{r}_m^T\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{r}_m \end{bmatrix} - \sigma_e^2\mathbf{a}_{1,m}^T\mathbf{R}_m^{-1}\mathbf{a}_{1,m}. \tag{A.21}$$

Based on (A.20) and (A.21), we have

$$\frac{\partial^2}{\partial z_1^2} f(-z_1, -z_2) = e^{-z_1 d_{1,m} - z_2 d_{2,m} + \mathbf{r}_m^T \mathbf{R}_m^{-1}\mathbf{r}_m / 2} \left|\mathbf{R}_m\right|^{-1/2} \left\{ f_2^2(z_1, z_2) + \sigma_e^2 \mathbf{a}_{1,m}^T\mathbf{R}_m^{-1}\mathbf{a}_{1,m} - 2\sigma_e^3 \begin{bmatrix} -\sigma_e tr(\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}) + 2\mathbf{a}_{1,m}^T\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{r}_m \\ -2\sigma_e \mathbf{r}_m^T\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{A}_{1,m}\mathbf{R}_m^{-1}\mathbf{r}_m \end{bmatrix} \right\}. \tag{A.22}$$

Equation (4.20) follows from (A.22). □

Appendix G

Proof of Lemma 4.4

Since $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \sigma_e^2 \mathbf{I}_{2L})$, it follows that

$$E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_{\mathbf{r}},\hat{\mathbf{q}}}\left[\mathbf{x}^T \mathbf{A}_{1,m}\mathbf{x} + \mathbf{a}_{1,m}^T \mathbf{x} + d_{1,m}\right] = E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_{\mathbf{r}},\hat{\mathbf{q}}}\left[\mathbf{x}^T \mathbf{A}_{1,m}\mathbf{x}\right] + d_{1,m}$$
$$\overset{(i)}{=} tr\left(\sigma_e^2 \mathbf{I}_{2L}\mathbf{A}_{1,m}\right) + d_{1,m} = \sigma_e^2 tr\left(\mathbf{A}_{1,m}\right) + d_{1,m}, \tag{A.23}$$

where (i) holds due to [33, p-414]. Similarly, we have

$$E_{\mathbf{e}|\mathbf{g},\hat{\mathbf{h}}_{\mathbf{r}},\hat{\mathbf{q}}}\left[\mathbf{x}^T \mathbf{A}_{2,m}\mathbf{x} + \mathbf{a}_{2,m}^T \mathbf{x} + d_{2,m}\right] = \sigma_e^2 tr\left(\mathbf{A}_{2,m}\right) + d_{2,m}. \tag{A.24}$$

The result follows from (A. 23) and (A.24). □

Appendix H

Proof of Lemma 4.5

To prove (4.4), first we rewrite (4.3) as

$$\bar{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}) \approx \sum_{m=1}^{M} \frac{2\sigma_e^2 tr\left(\mathrm{Re}\left(\mathbf{b}_m \mathbf{b}_m^H\right)\right) + d_{1,m}}{2\sigma_e^2 tr\left(\mathrm{Re}\left(\frac{\mathbf{Z}_m}{\eta p(m)}\right)\right) + d_{2,m}}$$

$$= \sum_{m=1}^{M} \frac{2\sigma_e^2 \sum_{i=1}^{L} \phi^2\left(\rho_i(m)\right)\left|g_i\right|^2 + \left|\sum_{i=1}^{L} \hat{h}_{r,i}\phi\left(\rho_i(m)\right) g_i\right|^2}{\left\{\begin{array}{l} \frac{2\sigma_e^2}{\eta p(m)} \sum_{i=1}^{L}\left[1-\phi^2\left(\rho_i(m)\right)\right]\left|g_i\right|^2 + \frac{1}{\eta p(m)} \sum_{j=1}^{L}\left|\hat{h}_{r,j}\right|^2\left[1-\phi^2\left(\rho_j(m)\right)\right]\left|g_j\right|^2 \\ + \frac{\sigma_w^2}{\eta p(m)} \end{array}\right\}}. \tag{A.25}$$

With $\mathbf{u}_m^H$ defined in (4.28), (A.25) can be rewritten as

$$\bar{\gamma}(\mathbf{g}, \hat{\mathbf{h}}_r, \hat{\mathbf{q}}) \approx \sum_{m=1}^{M} \frac{\mathbf{g}^H \left\{2\sigma_e^2 \eta p(m) diag\left[\phi^2\left(\rho_1(m)\right), \cdots, 2\sigma_e^2 \phi^2\left(\rho_L(m)\right)\right] + \mathbf{u}_m \mathbf{u}_m^H\right\} \mathbf{g}}{\mathbf{g}^H diag\left(\begin{array}{l} \frac{\sigma_w^2}{P_d} + \left(1-\phi^2\left(\rho_1(m)\right)\right)\left(2\sigma_e^2 + \left|\hat{h}_{r,1}\right|^2\right), \\ \cdots, \frac{\sigma_w^2}{P_d} + \left(1-\phi^2\left(\rho_L(m)\right)\right)\left(2\sigma_e^2 + \left|\hat{h}_{r,L}\right|^2\right) \end{array}\right) \mathbf{g}}. \tag{A.26}$$

By invoking the definitions of $\mathbf{W}_m$ and $\mathbf{Y}_m$ in (4.27) and (4.29), equation (4.26) follows immediately from (A.26). □

Appendix I

Proof of Theorem 4.6

By Cauchy-Schwartz inequality, we obtain

$$\underbrace{\sum_{m=1}^{M}\frac{\mathbf{g}^H\mathbf{W}_m\mathbf{g}}{\mathbf{g}^H\mathbf{Y}_m\mathbf{g}}}_{\approx\bar{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})}\left(\sum_{m=1}^{M}\mathbf{g}^H\mathbf{Y}_m\mathbf{g}\right)\geq\sum_{m=1}^{M}\mathbf{g}^H\mathbf{W}_m\mathbf{g}\,. \tag{A.27}$$

Then,

$$\bar{\gamma}(\mathbf{g},\hat{\mathbf{h}}_r,\hat{\mathbf{q}})\approx\sum_{m=1}^{M}\frac{\mathbf{g}^H\mathbf{W}_m\mathbf{g}}{\mathbf{g}^H\mathbf{Y}_m\mathbf{g}}\geq\frac{\sum_{m=1}^{M}\mathbf{g}^H\mathbf{W}_m\mathbf{g}}{\left(\sum_{m=1}^{M}\mathbf{g}^H\mathbf{Y}_m\mathbf{g}\right)}=\frac{\mathbf{g}^H\mathbf{W}\mathbf{g}}{\mathbf{g}^H\mathbf{Y}\mathbf{g}}\,. \tag{A.28}$$

□

Appendix J

Extension of the First Study: Multi-bit SNR Quantization

J.1 Analyses

Assume that B bits ($B \geq 1$) are used at each relay for SNR quantization. Assume also that, at the ith relay, the B bits are BPSK modulated, and are then transmitted over a BSC with crossover probability p_i, $1 \leq i \leq L$. Denote by $\hat{q}_{i,j}$ the jth received bit from the ith relay, $1 \leq i \leq L$ and $1 \leq j \leq B$. Let us collect all $\hat{q}_{i,j}$'s into a matrix to form

$$\widehat{\mathbf{Q}} = \begin{bmatrix} \hat{q}_{1,1}, \cdots, \hat{q}_{1,B} \\ \hat{q}_{2,1}, \cdots, \hat{q}_{2,B} \\ \vdots \qquad \vdots \\ \hat{q}_{L,1}, \cdots, \hat{q}_{L,B} \end{bmatrix}, \tag{A.29}$$

in which the ith row consists of the received B-bit message of ith S-R link SNR. For a given $\widehat{\mathbf{Q}}$, let us collect all $\widetilde{\mathbf{Q}} = \begin{bmatrix} \tilde{q}_{1,1}, \cdots, \tilde{q}_{1,B} \\ \tilde{q}_{2,1}, \cdots, \tilde{q}_{2,B} \\ \vdots \qquad \vdots \\ \tilde{q}_{L,1}, \cdots, \tilde{q}_{L,B} \end{bmatrix}$'s that differ from $\widehat{\mathbf{Q}}$ in exactly l rows to obtain

$$S_l(\widehat{\mathbf{Q}}) = \left\{ \widetilde{\mathbf{Q}} \left| \sum_{i=1}^{L} u(a_i) = l, \text{ where } a_i = \sum_{j=1}^{B} \hat{q}_{i,j} \oplus \tilde{q}_{i,j} \text{ and } u(\cdot) \text{ is the unit-step function} \right. \right\}$$

(A.30)

In addition, associated with each $\widetilde{\mathbf{Q}} \in S_l(\widehat{\mathbf{Q}})$ we define

$$I_l(\widehat{\mathbf{Q}}, \widetilde{\mathbf{Q}}) = \left\{ (i,j) \middle| \tilde{q}_{i,j} \neq \hat{q}_{i,j} \right\}, \tag{A.31}$$

which consists of the locations of all different entries between $\widetilde{\mathbf{Q}}$ and $\widehat{\mathbf{Q}}$. Based on (A.30) and (A.31), the conditional average SNR $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_r, \widehat{\mathbf{Q}})$ in the multiple-bit case is obtained as

$$\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_r, \widehat{\mathbf{Q}}) = \sum_{l=0}^{L} \sum_{\widetilde{\mathbf{Q}} \in S_l(\widehat{\mathbf{Q}})} \Pr(\widetilde{\mathbf{Q}} | \widehat{\mathbf{Q}}) \gamma_{dq}(\mathbf{g}, \mathbf{h}_r, \widetilde{\mathbf{Q}}), \tag{A.32}$$

where $\Pr(\widetilde{\mathbf{Q}} | \widehat{\mathbf{Q}}) = \left(\prod_{(i,j) \in I_l(\widetilde{\mathbf{Q}}, \widehat{\mathbf{Q}})} p_i \right) \left(\prod_{(i,j) \in I_l^c(\widetilde{\mathbf{Q}}, \widehat{\mathbf{Q}})} (1 - p_i) \right)$ and $I_l^c(\widetilde{\mathbf{Q}}, \widehat{\mathbf{Q}})$ is the complement of $I_l(\widehat{\mathbf{Q}}, \widetilde{\mathbf{Q}})$. By following the same techniques as in the paper and through further manipulations, $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_r, \widehat{\mathbf{Q}})$ can be further expressed as

$$\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_r, \widehat{\mathbf{Q}}) = \sum_{l=0}^{L} \left(\sum_{k_1=1}^{L} \sum_{k_2=k_1+1}^{L} \cdots \sum_{k_l=k_{l-1}+1}^{L} \sum_{j_1=1}^{\tau} \sum_{j_2=1}^{\tau} \cdots \sum_{j_l=1}^{\tau} \frac{\left| \sum_{i=1}^{L} c_i(l, k_1, \cdots, k_l, j_1, \cdots, j_l) g_i \right|^2}{\sum_{i=1}^{L} |g_i|^2 d_i(l, k_1, \cdots, k_l, j_1, \cdots, j_l)} \right),$$

(A.33)

where

$$\tau = 2^B - 1, \tag{A.34}$$

$$c_i(l, k_1, \cdots, k_l, j_1, \cdots, j_l) \triangleq \sqrt{\eta \prod_{t=1}^{l} p_{k_t}^{v_t} \left(\prod_{t=1}^{l} (1 - p_{k_t})^{v_t} \right)^{-1}} h_{r,i} \phi(\mathbf{r}_i), \tag{A.35}$$

$$d_i(l,k_1,\cdots,k_l) \triangleq \left|h_{r,i}\right|^2\left[1-\phi^2(\mathbf{r}_i)\right]+\frac{\sigma_w^2}{P_d}, \tag{A.36}$$

$$\eta \triangleq \prod_{l=1}^{L}(1-p_l)^B,$$

$$\mathbf{r}_i \triangleq \begin{cases} \left[\hat{q}_{i,1}\cdots\hat{q}_{i,B}\right],\ i\neq k_1,\cdots,k_l \\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2}\cdots\hat{q}_{i,B}\right],\ j_t=1,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \left[\hat{q}_{i,1},\hat{q}_{i,2}^t,\hat{q}_{i,3}\cdots\hat{q}_{i,B}\right],\ j_t=2,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \qquad\vdots \\ \left[\hat{q}_{i,1},\hat{q}_{i,2}\cdots\hat{q}_{i,B}^t\right],\ j_t=C_1^B,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2}^t,\hat{q}_{i,3}\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+1,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2},\hat{q}_{i,3}^t,\hat{q}_{i,4},\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+2,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \qquad\vdots \\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2}\cdots\hat{q}_{i,B-1},\hat{q}_{i,B}^t\right],\ j_t=C_1^B+B-1,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \left[\hat{q}_{i,1},\hat{q}_{i,2}^t,\hat{q}_{i,3}^t,\hat{q}_{i,4},\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+B,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \left[\hat{q}_{i,1},\hat{q}_{i,2}^t,\hat{q}_{i,3},\hat{q}_{i,4}^t,\hat{q}_{i,4}\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+B+1,\ i=k_t,\ t\in\{1,\cdots,l\} \\ \qquad\vdots \end{cases}, \tag{A.37}$$

where $\hat{q}_{i,i'}^t=\hat{q}_{i,i'}\oplus 1$, and $\upsilon_t=n$, where $\sum_{i=0}^{n-1}C_i^B\leq j_t\leq\sum_{i=1}^{n}C_i^B$. Through further rearranging the indices in the multiple summations in (R.12), $\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_\mathbf{r},\widehat{\mathbf{Q}})$ can be expressed as a single sum of Rayleigh quotients, which is shown as

$$\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_r,\widehat{\mathbf{Q}})=\sum_{m=1}^{M}\frac{\left|\mathbf{c}_m^H\mathbf{g}\right|^2}{\mathbf{g}^H\mathbf{D}_m\mathbf{g}}, \tag{A.38}$$

where

$$\mathbf{c}_m^H \triangleq [c_1(l,k_1,\cdots,k_l,j_1,\cdots,j_l),\cdots,c_L(l,k_1,\cdots,k_l,j_1,\cdots,j_l)], \tag{A.39}$$

$$\mathbf{D}_m \triangleq diag\left\{d_1(l,k_1,\cdots,k_l,j_1,\cdots,j_l),\cdots,d_L(l,k_1,\cdots,k_l,j_1,\cdots,j_l)\right\}, \tag{A.40}$$

$$m=\delta(l)+\sum_{s_0=0}^{l-1}C_{s_0}^L+\sum_{\lambda=1}^{l}\sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1}C_{l-\lambda}^{L-s_\lambda}+\sum_{\lambda_2=1}^{l}\tau^{l-\lambda_2}(j_{\lambda_2}-1)+1, \tag{A.41}$$

and $M = \sum_{i=0}^{L} C_i^L \tau^i$, where τ is defined in (A.34).

J.2 Simulation Results

For the two-bit case, i.e., $B = 2$, computer simulation is conducted to compare the BER performances of the proposed method and the solution in [10]. In the simulation setup, the number of relay nodes is $L = 4$, the average SNR of the S-R link is set to be $\overline{\gamma}_s = 20$ dB, and the crossover probability p_i of each BSC follows the uniform distribution over the interval $[0.05, 0.1]$. The simulated BER with respect to different R-D link SNR is then plotted in Figure A.1, shown on the next page. It can be seen from the figure that, as expected, the proposed method outperforms the solution in [18].

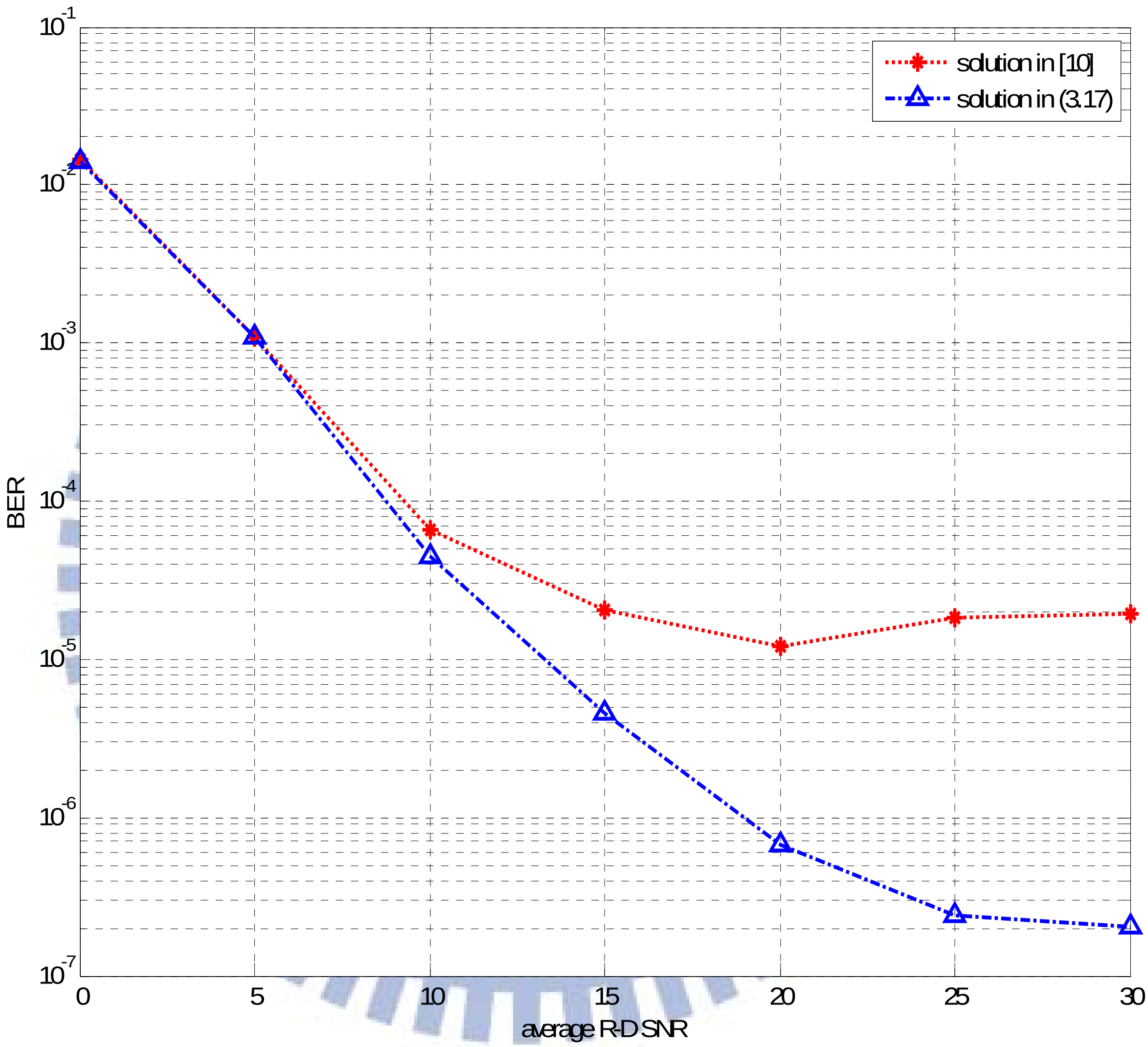


Figure 8. BER results for two methods when 2-bit SNR quantization is adopted at each relay.

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Publication List

A. International Journal Papers

[1] Jwo-Yuh Wu, Chung-Hsuan Hu, Tsang-Yi Wang, and Sheng-Ho Tsai "Design of low-overhead cooperative beamforming under imperfect quantized SNR of source-to-relay links," ***IEEE Trans. Signal Processing***, revised, 2012.

[2] Jwo-Yuh Wu, Chung-Hsuan Hu, Tsang-Yi Wang, Wen-Hsuan Lee, "Low-overhead cooperative beamforming for information relaying in wireless sensor networks under mismatched inter-node link CSI," ***IEEE Trans. Wireless Communications***, submitted, 2012.

B. International Conference Papers

[3] Jwo-Yuh Wu, Chung-Hsuan Hu, and Tsang-Yi Wang, "Low-overhead cooperative beamforming under imperfect quantized SNR of source-to-relay links," ***the Seventh IEEE Sensor Array and Multichannel Processing Workshop (SAM)***, 2012, Hoboken, USA.