

Theory of total-reflection-mode acoustic-surface-wave amplification in piezoelectric semiconductor films

Chhi-Chong Wu

Department of Applied Mathematics and Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

(Received 24 January 1983; revised manuscript received 10 August 1983)

The effect of nonparabolicity on amplification of total-reflection-mode acoustic surface waves in piezoelectric semiconductor films has been investigated quantum mechanically in the gigahertz-frequency region. Numerical results show that the amplification coefficient oscillates with the frequency in the low- and high-frequency regions due to the intersubband transitions and these oscillations can be affected by the nonparabolicity of the energy band in semiconductors. Moreover, the amplification coefficient depends also on the electronic screening effect, the temperature, the applied electric field, and the thickness of semiconductor films.

I. INTRODUCTION

The interaction of elastic surface waves with conduction electrons in a piezoelectric semiconductor provides a useful tool to study the electronic band structure of solids. In an elastic medium with a stress-free plane boundary, acoustic waves can be propagated along the boundary of an elastic half-space and reflected on the boundary.¹ Thus the electrons localized in the surface region can be affected by the reflection of waves. When the sound velocity of the propagation of waves c lies between the transverse sound velocity c_t and the longitudinal sound velocity c_l , the longitudinal pressure wave will be totally reflected by the solid surface.^{1,2} This is called the total-reflection mode (TR mode). It has been shown that the effect of nonparabolicity on amplification of surface waves becomes considerably important for the Rayleigh waves due to the nonlinear nature of the energy band in semiconductors.³ In piezoelectric semiconductors the interaction of elastic acoustic surface waves with conduction electrons is dominated by the deformation potential and piezoelectric fields. The deformation of the crystal due to the surface waves determines directly the deformation-potential force acting on the conduction electrons. On the other hand, the piezoelectric field, which is due to a polarization of the medium, can be found only by a self-consistent solution involving Maxwell's equations. In this paper, we wish to investigate the amplification characteristics of TR-mode acoustic surface waves in a piezoelectric semiconductor layer such as n -type GaAs using the quantum-mechanical treatment in the gigahertz-frequency region such that $ql > 1$, where q is the wave number of acoustic waves and l is the mean free path of electrons. In the calculation of the amplification coefficient of surface-mode waves, we make the following assumptions.

- (1) The energy-band structure of the piezoelectric semiconductor is nonparabolic.
- (2) The media are elastically isotropic, and the quasi-free-electron description of conduction electrons is valid.
- (3) The interaction of the acoustic surface waves and conduction electrons is via deformation-potential and

piezoelectric couplings.

(4) We disregard all defects and consider a thin film of an ideal semiconductor with perfectly parallel boundary planes, hence the scattering of surface irregularities^{2,4,5} is neglected.

In Sec. II we describe the configuration of the layered system of a piezoelectric semiconductor and an insulator which we shall use for determining the amplification characteristics and specify the eigenfunctions of conduction electrons for the nonparabolic band structure. In Sec. III, we calculate the amplification coefficient of TR-mode acoustic surface waves in n -type GaAs films for the nonparabolic band structure using the Born approximation. In Sec. IV, some numerical results of the amplification coefficient are presented for the epitaxial layer of an n -type GaAs on a semi-insulating GaAs substrate. Finally, a brief discussion is given.

II. ELECTRONIC STATES IN A THIN LAYER OF SEMICONDUCTORS FOR NONPARABOLIC BAND STRUCTURE

The configuration of the amplifier that we consider for amplification of the TR-mode acoustic surface waves is shown in Fig. 1(a). A thin layer with the thickness d of a piezoelectric semiconductor is grown epitaxially on an insulating substrate with the same elastic properties as the semiconductor layer.⁶ The Cartesian coordinates are fixed so that the material occupies the half-space $z \geq 0$ and has the stress-free surface parallel to the x - y plane. If a shear wave polarized in the vertical plane (SV wave) of a medium is incident upon the stress-free boundary surface, the SV wave and a pressure wave (P wave) come out as reflected waves from the surface. The longitudinal P wave is totally reflected by the surface as shown in Fig. 1(b). For this TR mode, the amplitude of the P wave decays exponentially away from the surface. In the work given by Tamura and Sakuma,⁷ the TR mode was considered as the special case of the SV- P mode in which the SV wave is incident upon the surface with the angle of incidence being larger than the critical angle θ , and the P wave is thus to-

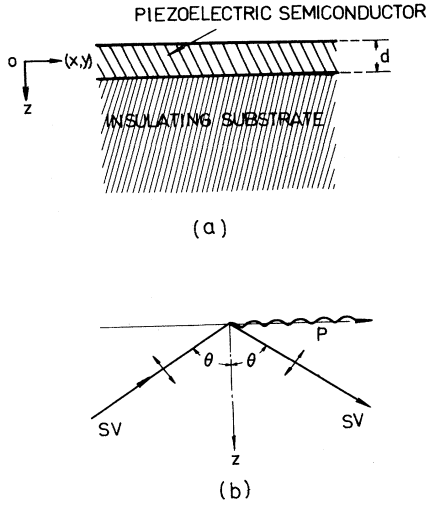


FIG. 1. (a) Thin layer with thickness d of a piezoelectric semiconductor and an insulating material. (b) Longitudinal P wave is reflected and travels along the surface of a medium and penetrates into the medium only above a wavelength from the surface. Transverse SV wave is polarized in the vertical plane.

tally reflected by the surface. However, it is not easy to understand and establish this relationship between the TR mode and the SV - P mode, because $c > c_l$ in the SV - P mode while $c_l < c < c_l$ in the TR mode.¹ Consequently, it seems that we should treat the TR mode and the SV - P mode separately.

The motion of conduction electrons parallel to the surface may be described by plane waves, and those perpendicular to the surface will be described by some type of standing wave depending on the structure of the potential. It is assumed that the potential along the z axis is a square well which has infinitely high potential barriers at $z=0$ and $z=d$. Under this approximation, the field operator $\Psi(\vec{r})$ of conduction electrons in the second quantized form takes the form⁷

$$\begin{aligned} \Psi(\vec{r}) &= \left[\frac{1}{S} \right]^{1/2} \sum_{n=1}^{\infty} \sum_{\vec{k}} b_{\vec{k},n} \exp(i\vec{k} \cdot \vec{x}) \phi_n(z) \\ &= \left[\frac{2}{V} \right]^{1/2} \sum_{n=1}^{\infty} \sum_{\vec{k}} b_{\vec{k},n} \exp(i\vec{k} \cdot \vec{x}) \sin \left[\frac{n\pi z}{d} \right], \end{aligned} \quad (1)$$

where $\vec{r}=(\vec{x},z)=(x,y,z)$, $\vec{k}=(k_x,k_y)$, $V=dS$ is the volume of the film with a surface area S , and $b_{\vec{k},n}$ and its

$$\begin{pmatrix} u_x(\vec{r}) \\ u_y(\vec{r}) \\ u_z(\vec{r}) \end{pmatrix} = \left[\frac{q}{2\pi\beta} \right]^{1/2} \left[H \exp(-\alpha qz) \begin{pmatrix} iq_x/q \\ iq_y/q \\ -\alpha \end{pmatrix} + \exp(-i\beta qz) \begin{pmatrix} i\beta q_x/q \\ i\beta q_y/q \\ i \end{pmatrix} + G \exp(i\beta qz) \begin{pmatrix} i\beta q_x/q \\ i\beta q_y/q \\ -i \end{pmatrix} \right] \exp(i\vec{q} \cdot \vec{x}), \quad (6)$$

where

$$\alpha = [1 - (c/c_l)^2]^{1/2}, \quad \beta = [(c/c_l)^2 - 1]^{1/2}, \quad (7)$$

Hermitian conjugate $b_{\vec{k},n}^\dagger$ are annihilation and creation operators of conduction electrons, respectively, satisfying the commutative relations of the Fermi type. The energy of the conduction electrons $E_{\vec{k},n}$ for the nonparabolic band structure is given by the relation³

$$E_{\vec{k},n} \left[1 + \frac{E_{\vec{k},n}}{E_g} \right] = \frac{\hbar^2 \vec{k}^2}{2m^*} + \frac{\pi^2 \hbar^2 n^2}{2m^* d^2}, \quad n=1,2,3,\dots \quad (2)$$

where E_g is the energy gap between the conduction and valence bands, and m^* is the effective mass of the conduction electron. Since

$$(\hbar^2 \vec{k}_{\max}^2)/2m^* + (\pi^2 \hbar^2 n^2)/2m^* d^2 \simeq k_B T \ll E_g$$

for piezoelectric semiconductors at low temperatures in which we are interested ($T < 100$ K), then Eq. (2) can be expanded as

$$E_{\vec{k},n} \simeq -\frac{1}{2} E_g + \frac{1}{2} E_g a_n + \frac{\hbar^2 \vec{k}^2}{2m^* a_n}, \quad (3)$$

with

$$a_n = [1 + (2\pi^2 \hbar^2 n^2)/(m^* d^2 E_g)]^{1/2}. \quad (4)$$

III. THEORETICAL FORMULATION

The quantization of the elastic wave field $\vec{u}(\vec{r},t)$ can be expanded in terms of the expansion coefficients, a_J and a_J^\dagger , as¹

$$\begin{aligned} \vec{u}(\vec{r},t) &= \sum_J \left[\frac{\hbar}{2\rho\omega_J S} \right]^{1/2} [a_J \vec{u}_J(\vec{r}) \exp(-i\omega_J t) \\ &\quad + a_J^\dagger \vec{u}_J^*(\vec{r}) \exp(i\omega_J t)], \end{aligned} \quad (5)$$

where ρ is the mass density of the medium, $J=(\vec{q},c,m)$ is a suitable set of quantum numbers, $\vec{q}=(q_x,q_y)$ is a wave vector parallel to the surface, c is the phase velocity defined by $\omega_J = c |\vec{q}| = cq$, and m specifies the propagation mode of the acoustic waves. In the present case m indicates TR mode. a_J and its Hermitian conjugate a_J^\dagger are annihilation and creation operators of surface-mode acoustic waves, respectively, obeying the commutative relations of the Bose type. The explicit expression of the wave function $\vec{u}_J(\vec{r})$ for the TR mode is given by^{1,8}

$$G = \frac{(\beta^2 - 1)^2 - 4i\alpha\beta}{(\beta^2 - 1)^2 + 4i\alpha\beta}, \quad H = \frac{4\beta(\beta^2 - 1)}{(\beta^2 - 1)^2 + 4i\alpha\beta}. \quad (8)$$

Here, the phase velocity c takes a value between c_l and c_l .

In the piezoelectric semiconductor, the conduction electrons interact with the TR-mode acoustic surface waves through the deformation potential, which is proportional to the dilation caused by the acoustic field. These conduction electrons also interact with waves through piezoelectricity. This induced electric field is proportional to the strain in the piezoelectric coupling. The interaction Hamiltonian between conduction electrons and the TR-mode waves may be written as⁷

$$H_I = H_D + H_P, \quad (9)$$

where H_D is the interaction Hamiltonian due to the deformation-potential coupling and H_P is the interaction Hamiltonian due to the piezoelectric coupling. For the deformation-potential coupling, H_D can be expressed by

$$\begin{aligned} H_D &= C \int \Psi^\dagger(\vec{r}) \vec{\nabla} \cdot \vec{u}(\vec{r}) \Psi(\vec{r}) d\vec{r} \\ &= \frac{C}{\sqrt{S}} \sum_{n,n'} \sum_{\vec{k},\vec{q}} (b_{\vec{k}+\vec{q},n}^\dagger b_{\vec{k},n} a_J \Delta_{n'n}^J + \text{H.c.}), \end{aligned} \quad (10)$$

where C is the deformation potential, and

$$\begin{aligned} \Delta_{n'n}^J &= - \left[\frac{\hbar}{4\pi\rho c\beta} \right]^{1/2} \left[\frac{c}{c_l} \right]^2 qH \\ &\times \int_0^d \phi_n^*(z) \exp(-\alpha qz) \phi_n(z) dz. \end{aligned} \quad (11)$$

In a piezoelectric material, the polarization induced by applying a strain can be expressed by⁹

$$\hat{\Phi}_J(z) = \frac{4(2\pi\beta_P^2)^{1/2}}{\epsilon_0} \left[\frac{q}{\beta} \right]^{1/2} \left[\frac{q_x q_y}{q^2} \right] \{ 3\alpha H (c_l/c)^2 \exp(-\alpha qz) + i(2\beta^2 - 1)(c_l/c)^2 [\exp(-i\beta qz) - G \exp(i\beta qz)] \}. \quad (18)$$

For the piezoelectric coupling, semiconductors with zinc-blende crystal structure have only three nonvanishing components of the piezoelectric tensor β_{ijk} , i.e., $\beta_{14} = \beta_{25} = \beta_{36} = \beta_P$. In the acoustic wave model, the displacement vector of the TR-mode wave can be expanded as Eq. (5), then the electric potential can be quantized as follows:

$$\tilde{\Phi}(\vec{r}) = \sum_J \left[\frac{\hbar}{2\rho\omega_J S} \right]^{1/2} a_J \Phi_J(\vec{r}) + \text{H.c.} \quad (19)$$

Taking these results into account, the interaction Hamiltonian due to the piezoelectric coupling can be written as

$$H_P = -e \int \Psi^\dagger(\vec{r}) \tilde{\Phi}(\vec{r}) \Psi(\vec{r}) d\vec{r} = \frac{4\pi\beta_P e}{\epsilon_0 \sqrt{S}} \sum_J \sum_{\vec{k}} \sum_{n,n'} (b_{\vec{k}+\vec{q},n}^\dagger b_{\vec{k},n} a_J \phi_{n'n}^J + \text{H.c.}), \quad (20)$$

where

$$\phi_{n'n}^J = \left[\frac{\hbar}{4\pi\rho c\beta} \right]^{1/2} \int_0^d \phi_n^*(z) \{ 3\alpha H (c_l/c)^2 \exp(-\alpha qz) + i(2\beta^2 - 1)(c_l/c)^2 [\exp(-i\beta qz) - G \exp(i\beta qz)] \} \phi_n(z) dz. \quad (21)$$

We have taken the direction of the wave vector \vec{q} along the [110] crystal axis for the TR mode. For the deformation-potential coupling, conduction electrons could interact only with the longitudinal wave; however, for the piezoelectric coupling, conduction electrons interact with both longitudinal and transverse waves. Consequently, we shall consider both longitudinal and transverse coupling mechanisms in our present study.

It is known that conduction electrons never travel freely in a semiconductor but are scattered by a variety of sources before and after they emit or absorb the acoustic surface waves we should observe. To calculate the amplification coefficient α^J we follow the Green's-function method given in Ref. 7. α^J is related to the width of the acoustic surface wave Γ^J as $\Gamma^J = -c\hbar\alpha^J$. By using the Hamiltonians (9), (10), and (20), and neglecting vertex corrections other than the screening effect of electrons, the width Γ^J of the TR-mode waves can be expressed by⁷

$$D_i = \epsilon_{ij} E_j + 4\pi\beta_{ijk} S_{jk}, \quad (12)$$

where D_i is the electric displacement, E_i is the electric field, ϵ_{ij} is the dielectric tensor, β_{ijk} is the piezoelectric tensor, and S_{ij} is the strain tensor which can be written explicitly in terms of the displacement vector \vec{u} of the medium as

$$S_{ij} = \frac{1}{2} \left[\frac{\partial u_i}{\partial r_j} + \frac{\partial u_j}{\partial r_i} \right]. \quad (13)$$

If the electrostatic approximation is valid, the electric field \vec{E} can be derived from a scalar potential Φ_J ,

$$\vec{E} = -\vec{\nabla}\Phi_J. \quad (14)$$

Within insulating crystals, Gauss's law is expressed as

$$\vec{\nabla} \cdot \vec{D} = 0. \quad (15)$$

Since we assume that the medium is isotropic in its elastic properties, that is, $\epsilon_{ij} = \epsilon_0 \delta_{ij}$ (ϵ_0 is the static dielectric constant), then, from Eqs. (12), (14), and (15), we have the equation

$$\nabla^2 \Phi_J = \frac{4\pi}{\epsilon_0} \beta_{ijk} \left[\frac{\partial S_{jk}}{\partial r_i} \right]. \quad (16)$$

By using Eqs. (6), (13), and (16), the electric potential produced by the acoustic vibration of acoustic surface waves traveling with the wave vector \vec{q} can be obtained as

$$\Phi_J(\vec{r}) = \hat{\Phi}_J(z) \exp(i\vec{q} \cdot \vec{x}), \quad (17)$$

with

$$\Gamma^J(\hbar\omega_J) = 2 \sum_{n,n'} \{ [C(q)]^2 |\Delta_{n'n}^J|^2 + [G(q)]^2 |\phi_{n'n}^J|^2 \} \int \frac{d\vec{k}}{(2\pi)^2} \int \frac{d\epsilon}{2\pi} [f(\epsilon - \hbar\omega_J) - f(\epsilon)] A_n(\vec{k}, \epsilon) A_{n'}(\vec{k} - \vec{q}, \epsilon - \hbar\omega_J), \quad (22)$$

where $C(q) = C/\epsilon_i(q)$, $G(q) = 4\pi e\beta_P/\epsilon_0\epsilon_i(q)$, and $\epsilon_i(q) = 1 + (4\pi n_0 e^2 c_i^2/\epsilon_0 c^2 k_B T q^2)$. n_0 is the electron concentration, $i=l$ denotes the electronic screening effect induced by the acoustic vibrations of the longitudinal waves with the longitudinal sound velocity c_l , and $i=t$ denotes the electronic screening effect induced by the acoustic vibrations of the transverse waves with the transverse sound velocity c_t .^{10,11} The Fermi-Dirac distribution function is $f(\epsilon) = \{ \exp[(\epsilon - \mu)/k_B T] + 1 \}^{-1}$, where μ is the chemical potential. $A_n(\vec{k}, \epsilon)$ is the spectral function of the one-electron Green's function with the quantized level n of conduction electrons. In the first approximation, we employ the Born approximation and then simply replace the spectral function by the δ function,

$$A_n(\vec{k}, \epsilon) = 2\pi\delta(\epsilon - E_{\vec{k},n}). \quad (23)$$

In a situation where conduction electrons have a drift velocity \vec{v} in the direction of the acoustic surface wave vector \vec{q} , we must replace $\hbar\omega_J$ by $-\hbar\omega_J X$ with the drift parameter $X = |\vec{v}|/c - 1$. Then

$$f(\epsilon - \hbar\omega_J) - f(\epsilon) \simeq -\frac{\hbar\omega_J X}{4k_B T} \operatorname{sech}^2 \left[\frac{\epsilon - \mu}{2k_B T} \right]. \quad (24)$$

From Eqs. (22)–(24), the amplification coefficient α_i^J of TR-mode waves for the i -type electronic screening effect induced by the acoustic vibrations of longitudinal waves or transverse waves in the piezoelectric coupling can be obtained as

$$\begin{aligned} \alpha_i^J &= -\frac{\Gamma_{\text{Born}}^J(-\hbar\omega_J X)}{\hbar c} \\ &= \left[\frac{X}{\hbar^3} \right] \left[\frac{2m^*}{\pi k_B T} \right]^{1/2} \sum_{s=1}^{\infty} (-1)^{s+1} \sqrt{s} \exp[sP(q)/(k_B T)] \left[\sum'_{n,n'} \exp[-sQ(q, n, n')/(k_B T)] \right. \\ &\quad \left. \times \left[\frac{C^2}{[\epsilon_i(q)]^2} |\Delta_{n'n}^J|^2 + \frac{16\pi^2 e^2 \beta_P^2}{\epsilon_0^2 [\epsilon_i(q)]^2} |\phi_{n'n}^J|^2 \right] \right], \quad (25) \end{aligned}$$

where \sum' indicates a summation over n and n' with $n \neq n'$ due to the Pauli exclusion principle,

$$P(q) = \frac{1}{2} E_g + \mu - (m^*/2q^2\hbar^2) [\hbar\omega_J X - (q^2\hbar^2/2m^*)],$$

and

$$Q(q, n, n') = \frac{1}{2} E_g a_n + (m^*/2q^2\hbar^2) \{ E_g [\hbar\omega_J X - (q^2\hbar^2/2m^*)] (a_n - a_{n'}) + \frac{1}{4} E_g^2 (a_n - a_{n'})^2 \}.$$

Functions $|\Delta_{n'n}^J|^2$ and $|\phi_{n'n}^J|^2$ are given by

$$|\Delta_{n'n}^J|^2 = \left[\frac{64\hbar q^2}{\pi^3 \rho c} \right] \frac{A^2 \beta (1 - \beta^2)^2 (\beta^2 - 1)^2 (nn')^2 [1 - (-1)^{n'+n} \exp(-\pi A)]^2}{[(\beta^2 - 1)^4 + 16\alpha^2 \beta^2] [A^2 + (n' - n)^2] [A^2 + (n' + n)^2]^2}, \quad (26)$$

$$\begin{aligned} |\phi_{n'n}^J|^2 &= \left[\frac{2\hbar}{\pi^3 \rho c} \right] \frac{B^2 (\beta^2 - 1)^2}{\beta [(\beta^2 - 1)^4 + 16\alpha^2 \beta^2]} \left[\frac{288\alpha^4 (nn')^2 [1 - (-1)^{n'+n} \exp(-\pi A)]^2}{(1 - \alpha^2)^2 [A^2 + (n' - n)^2]^2 [A^2 + (n' + n)^2]^2} \right. \\ &\quad - \frac{96\alpha^2 (\beta^2 - 1) (2\beta^2 - 1) (nn')^2 [1 - (-1)^{n'+n} \exp(-\pi A)] [1 - (-1)^{n'+n} \cos(\pi B)]}{(1 - \alpha^2) (1 + \beta^2) [A^2 + (n' - n)^2] [A^2 + (n' + n)^2] [B^2 - (n' - n)^2] [B^2 - (n' + n)^2]} \\ &\quad + \frac{(\beta^2 - 1)^2 (2\beta^2 - 1)^2 [8(nn')^2 + (n^2 + n'^2)^2 + (2nn' + B^2)^2]}{(1 + \beta^2)^2 [B^2 - (n' - n)^2]^2 [B^2 - (n' + n)^2]^2} \\ &\quad \left. - \frac{16(\beta^2 - 1)^2 (2\beta^2 - 1)^2 (-1)^{n'+n} (nn')^2 \cos(\pi B)}{(1 + \beta^2)^2 [B^2 - (n' - n)^2] [B^2 - (n' + n)^2]^2} - \frac{(\beta^2 - 1)^2 (2\beta^2 - 1)^2 \cos(2\pi B)}{(1 + \beta^2)^2 [B^2 - (n' - n)^2] [B^2 - (n' + n)^2]} \right], \quad (27) \end{aligned}$$

where $A = \alpha q d / \pi$ and $B = \beta q d / \pi$.

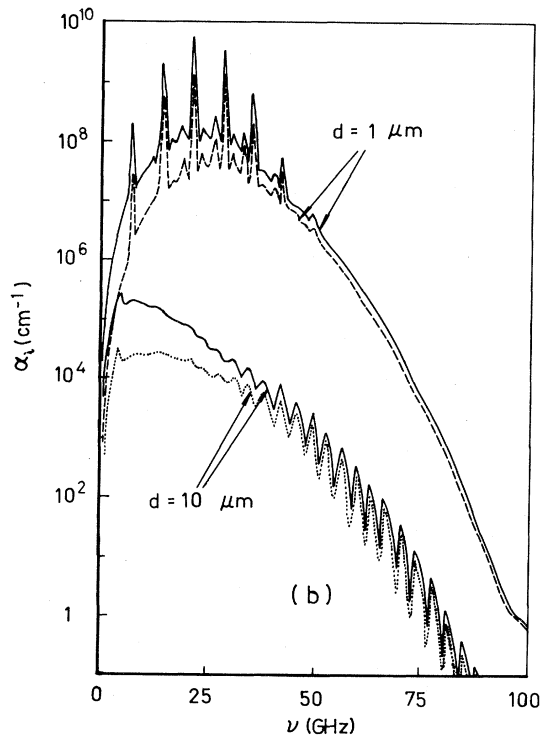
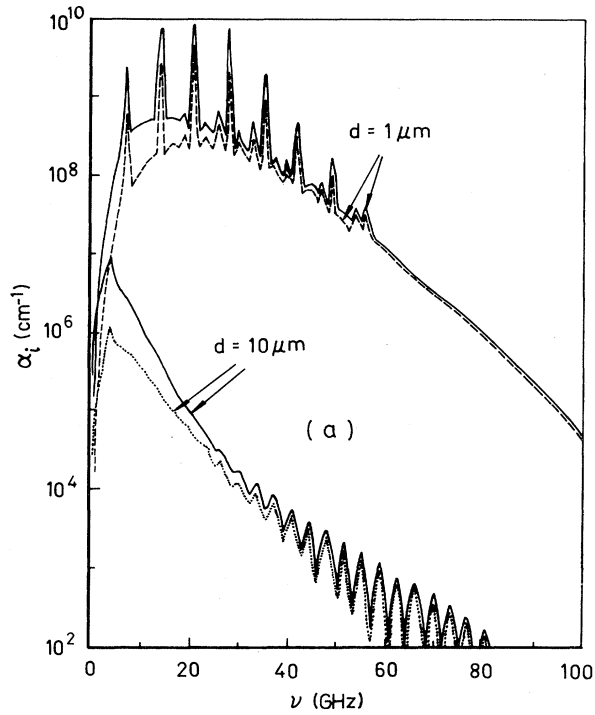


FIG. 2. Amplification coefficient of TR-mode acoustic surface waves vs frequency at $X=10$ ($E=88$ V/cm) with (a) $T=77$ K and (b) $T=19.7$ K. Solid curve: amplification coefficient for the screening effect of the transverse field. Broken curve: amplification coefficient for the screening effect of the longitudinal field.

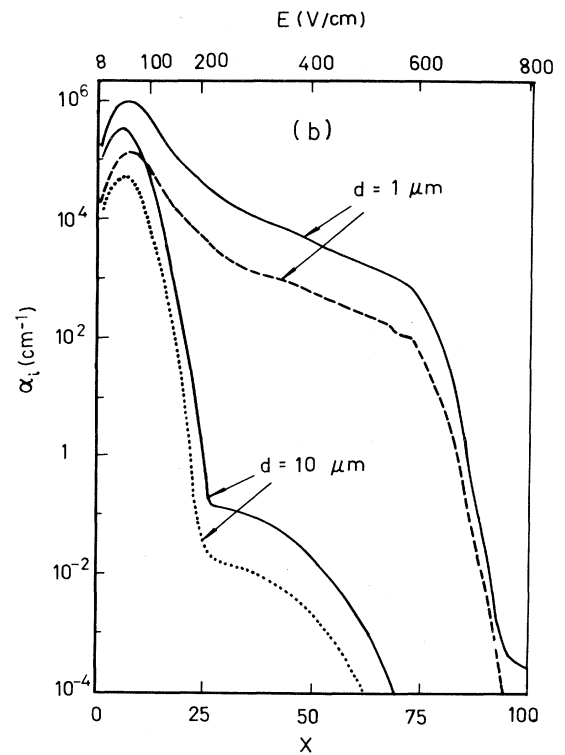
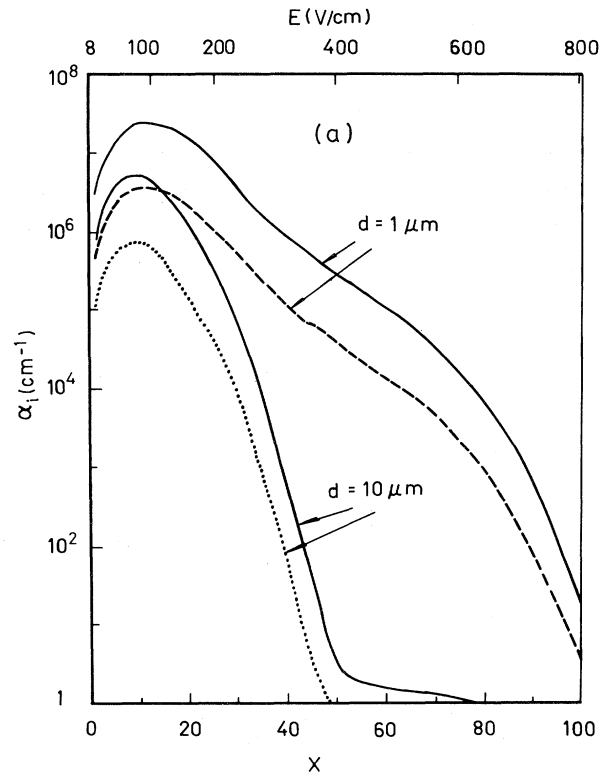


FIG. 3. Amplification coefficient of TR-mode acoustic surface waves vs drift parameter or applied electric field at $\nu=3$ GHz with (a) $T=77$ K and (b) $T=19.7$ K. Solid curve: amplification coefficient for the screening effect of the transverse field. Broken curve: amplification coefficient for the screening effect of the longitudinal field.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, a numerical example is developed for an n -type GaAs thin film grown epitaxially on a semi-insulating GaAs substrate. The relevant values of physical parameters are taken to be $\beta_p = 4.71 \times 10^4$ esu/cm² for $\bar{q} \parallel [110]$, $n_0 = 1.73 \times 10^{15}$ cm⁻³, $m^* = 0.07m_0$ (m_0 is the mass of free electron), $\rho = 5.32$ g/cm³, $\epsilon_0 = 12.9$, $c = 4 \times 10^5$ cm/sec, $c_l = 5.17 \times 10^5$ cm/sec, $c_t = 3.04 \times 10^5$ cm/sec, $C = 7$ eV, and $E_g = 1.51$ eV. The frequency dependence of the amplification coefficient at $X = 10$ ($E = 88$ V/cm) is shown in Fig. 2. It can be seen that the amplification coefficient increases with the frequency up to $\nu = 20$ GHz and then decreases with increasing the frequency for the $1\text{-}\mu\text{m}$ thickness of an epitaxial semiconductor film, while for $d = 10 \mu\text{m}$, the amplification coefficient increases with the frequency up to $\nu = 4$ GHz and then decreases with increasing frequency. Moreover, the amplification coefficient oscillates with the frequency from the frequency around $\nu = 25$ GHz for the case $d = 10 \mu\text{m}$, while in the case $d = 1 \mu\text{m}$ the amplification coefficient oscillates with the frequency in the lower-frequency region and then monotonically decreases with the frequency after $\nu = 60$ GHz. This is quite different from the results of surface-mode elastic waves for the parabolic band structure in n -type GaAs.⁷ It is also shown that the amplification coefficient increases with temperature. Moreover, the amplification coefficient α_t with the transverse dielectric function is larger than that α_l with the longitudinal dielectric function due to the larger electronic screening effect for the longitudinal waves. Figure 3 shows the amplification coefficients versus the drift parameter X (or the applied electric field E) with $\nu = 3$ GHz. It can be seen that there exists a maximum around $X = 9$ (or $E = 80$ V/cm) and the amplification coefficient decreases with the drift parameter (or the applied electric field). We can also see that after passing the maximum point the amplification coefficient decreases monotonically with increasing thickness of the semiconductor film and decreasing of temperature.

We have calculated the amplification coefficient of the TR-mode acoustic surface waves for the nonparabolic band structure in piezoelectric semiconductor films. In the high-frequency region, the contributions of the deformation-potential coupling to the amplification become important,⁹ the function $|\Delta_{n'n}^J|^2$ will thus play an important role. However, the amplification coefficient will decrease very rapidly in the high frequency in our present results and the function $|\phi_{n'n}^J|^2$, due to the piezoelectric coupling, plays a comparable important role. Some oscillations can be observed in the low-frequency region and the high-frequency region depending on the thickness of semiconductor films. These oscillations come from the contributions of the piezoelectric coupling in which the factor $|\phi_{n'n}^J|^2$ contains harmonic functions of the frequency, $\cos(\pi B)$ and $\cos(2\pi B)$, due to the relation $B = \beta q d / \pi = 2\beta d \nu / c$. When the thickness of semiconductor films is small, some large and sharp cusp oscilla-

tions appear in the lower-frequency region and the amplification coefficient decreases monotonically with increasing the frequency in the high-frequency region due to the piezoelectric coupling effect. However, when the thickness of semiconductor films is larger, oscillations can be observed in the high-frequency region. The amplitude of oscillations can be affected by the quantum numbers n and n' ; thus these oscillations arise from the intersubband transitions and are affected by the nonparabolicity of the energy band in semiconductors due to the factor $Q(q, n, n')$. From Eqs. (25)–(27), it can be seen that the amplification coefficient changing with the drift parameter (or the applied electric field) is dominated by the factors $P(q)$ and $Q(q, n, n')$, but not by the functions $|\Delta_{n'n}^J|^2$ and $|\phi_{n'n}^J|^2$. The dominant factor for α_t^J vs X in Eq. (25) is

$$f(X) = X \{ \exp[C_1(X - C_2)^2 - C_3(n, n')(X - C_4)] \}, \quad (28)$$

where C_1 , C_2 , $C_3(n, n')$, and C_4 are independent of X . It can be shown that there exist at most two local extrema in Eq. (28). These extrema depend on the quantum numbers n and n' . Thus the nonparabolicity of the energy-band structure in semiconductors will affect these extrema. In Fig. 3 only one maximum point appears; the second extremum could be degenerated to an inflection point at $d = 10 \mu\text{m}$ and disappears when the thickness of semiconductor films is decreasing. After passing this maximum point, the amplification coefficient decreases with increasing the drift parameter (or the applied electric field), since the energy of conduction electrons in semiconductor films increases for the effect of the nonparabolic band structure in semiconductors.

Figures 2 and 3 show that the amplification coefficient also increases with temperature. This can be explained from the expression for the amplification coefficient given in Eq. (25), which shows that α_t^J is roughly proportional to

$$T^{3/2} \exp[-F(n, n')/T],$$

where $F(n, n')$ is independent of the temperature. The first factor $T^{3/2}$ comes from the dielectric function $\epsilon_t(q)$,¹¹ while the second factor $\exp[-F(n, n')/T]$ is related to the energy band of semiconductors. Consequently, the temperature dependence of the amplification arises from the electronic screening effect and the effect of the nonparabolicity in semiconductors. The numerical analysis presented here, therefore, shows that the amplification coefficient depends on the temperature, the frequency of sound waves, the applied electric field, and the thickness of semiconductor films.

ACKNOWLEDGMENT

The author wishes to acknowledge the partial financial support from National Science Council of China in Taiwan.

- ¹H. Ezawa, *Ann. Phys. (N.Y.)* **67**, 438 (1971).
²T. Sakuma, *Phys. Rev. B* **8**, 1433 (1973).
³C. C. Wu and J. Tsai, *J. Appl. Phys.* **53**, 3344 (1982).
⁴R. G. Steg and P. G. Klemens, *Phys. Rev. Lett.* **24**, 381 (1970).
⁵T. Sakuma, *Phys. Rev. Lett.* **29**, 1394 (1972).
⁶S. M. Sze, *Physics of Semiconductor Devices* (Wiley, New York, 1981).
⁷S. Tamura and T. Sakuma, *Phys. Rev. B* **16**, 3638 (1977).
⁸T. Nakayama, *Phys. Rev. B* **14**, 4670 (1976).
⁹H. N. Spector, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1966), Vol. 19, p. 291.
¹⁰L. M. Roth and P. N. Argyres, in *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic, New York, 1966), Vol. 1, p. 159.
¹¹F. García-Moliner and F. Flores, *Introduction to the Theory of Solid Surfaces* (Cambridge University, Cambridge, England, 1979).