國立交通大學

電信工程研究所

碩士論文

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A Phasor Domain Acquisition Method for Code Synchronization

研究生：林群明 Student: Chiun-Ming Lin

指導教授：高銘盛 博士 Advisor: Dr. Ming-Seng Kao

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摘要

「碼同步」在展頻通訊系統中是一項重要且值得探討的議題，而傳統的作法是在時域上做序列比對。 藉著快速傅立葉轉換(FFT)，我們可以將碼序列轉變為相量，接著利用逆傅立葉轉換(IFFT)，即可找到輸入序列和本地序列的相位差。當碼序列長度增加時，所耗費的運算量也隨之增加。

本論文中，我們提出一個新的「碼同步」方法，它不需作逆傅立葉轉換以減少運算量。這個方法主要利用複數相量之間的相位關係，藉此可以擷取所需的相位差。在分析複數相量的統計特性之後，我們提出一個簡單的方法求相位差。接著， 我們進一步提出一個進階的方法以提升系統的準確度。最後，我們利用電腦模擬驗證所提的理論，其中分析的重點在於相位變異數的變化以及估測的準確度。模擬的結果顯示，所提方法可減少許多運算量並在高 SNR 的環境下可以獲致良好的結果。

A Phasor Domain Acquisition Method for Code Synchronization

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Abstract

Code synchronization is a critical issue in spread spectrum communication systems. The traditional method to achieve code synchronization is via serial search in the time domain. By performing Fast-Fourier-Transform (FFT), the code sequence can be projected to the phasor domain. Then, code phase between local sequence and input sequence can be found via Inverse-Fast-Fourier Transform (IFFT). However, as the code length increases, the computation increases considerably.

In this thesis, we propose a new method for code synchronization in the phasor domain without IFFT. The method is based on the phase relationship of complex phasors, whereby we can extract code phase via phasor phases. We analyze the statistical property of complex phasors and design a simple method to extract the code phase. Moreover, we further design an improved method to enhance the accuracy under noisy condition. Computer simulation is performed to verify the proposed scheme, which focuses on variance reduction and estimation accuracy. It can be demonstrated that the proposed method works well without much computation while acceptable accuracy is achievable in high SNR environment.

誌　　謝

在本論文中，首先我最感謝的是我的指導教授，高銘盛 博士。不論在課業或是研究，甚至是人生規劃方面，他總是在旁指導著我。並且老師的指導相當細心且有耐心，不論是多小的問題他都能很有條理的教導我們。因此，老師就像父親也像朋友。從老師身上可以學習到許多做人處事的態度及方法，包括老師的好教養以及做事謹慎的態度。相信這是能從老師身上學到且受用一生的好習慣。

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謹將此論文獻給所有幫助過我的人們，共勉之。

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Chapter 1

Introduction

I. Overview

In the traditional digital communication system as shown in Fig-1.1, the information source generates particular symbols at a specific rate. The source encoder translates these symbols into sequences of 0's and 1's. Next, the channel encoder further translates the obtained sequences into another sequences of 0's and 1's, so as to realize high transmission reliability and efficiency. Following the channel encoder, the modulator accepts the encoded bit stream and converts them to signal waveforms suitable for transmission. After passing through the channel, the waveform suffers from amplitude/phase distortion due to the noise, transmission delay and multipath effect. Therefore, the waveform we get at the receiving end might not be the same as the transmitting one. So the primary objective of a communication system is to suppress the bad effects coming from the channel as much as possible.

The inverse process takes place at the destination side. The demodulator converts the signal waveforms to sequences of 0's and 1's, and then the channel decoder translates this sequence to the original one. It also performs error correction and clock recovery. The source decoder finally translates the sequence of into original signals. Recovering the information sequence from the distorted waveform is the main purpose of a communication receiver.

Synchronization in telecommunications networks is the process of aligning the time scales of transmission and switching equipment so equipment operations occur at the correct time and in the correct order. Synchronization requires the receiver clock to acquire and track the periodic timing information in a transmitted signal. The transmitted signal consists of data that is clocked out at a rate determined by the transmitter clock. Signal transitions between 0's and 1's contain the clocking information and detecting these transitions allows the clock to be recovered at the receiver.

II. Our work

Our work will not focus on how to recover the information sequence from the distorted waveform, instead the main objective we are interested in is the "code synchronization", i.e. to find the code phase shift between the incoming sequence and the local sequence at the receiver. This issue is critical in spread spectrum systems where code synchronization must be achieved before signal demodulation.

In order to simplify the problem, we consider the noiseless condition first. When there is no noise accompany with the input sequence, some phase delay exists between the input sequence and the local one. We denote q as the particular shift between them. If the accurate value of q is got, the synchronization is completed. Assume $S_I = (x_0, x_1, ..., x_{N-1})$ is the input sequence and $S_L = (y_0, y_1, ..., y_{N-1})$ is the local sequence, where $y_k = x_{k+q}$ and N is the code length. Our goal is to find the code-phase shift between S_I and S_L, i.e. the value of q. We'll introduce the classic way on dealing with this problem, i.e. the Fast Fourier Transform (FFT) method. By transforming the sequences to the frequency domain via FFT, we find that "q" is imbedded in the phases of complex phasors. Thus we can get it accurately by performing Inverse Fast Fourier Transform (IFFT). In general, N is a large number so that the computation of FFT and IFFT grows rapidly when N increases. To reduce the computation load, we propose a new scheme which can accurately estimate “q” without IFFT.

The main idea is a little bit tricky. Because the information of q is embedded in the phase of each phasor in frequency domain, we can estimate the code phase via inner product of different phasors. However, even though in high SNR condition, we can’t be sure if the observed phase is close enough to the correct one because of the disturbance coming from noise. The intuitive solution is to reduce the noise variance to some extent such that the estimated code phase will have little probability to be erroneous. Therefore, we design a simple but effective method to decrease the noise variance. It can be proved that the proposed scheme can reduce the computation without losing much accuracy.

The thesis is organized as follows. In Chapter 2, we will introduce the phasor concept of FFT and analyze its properties. Next, the details of the proposed system will be described in Chapter 3 and simulation results will be demonstrated in Chapter 4. Finally, we present the conclusions of this study in Chapter 5.

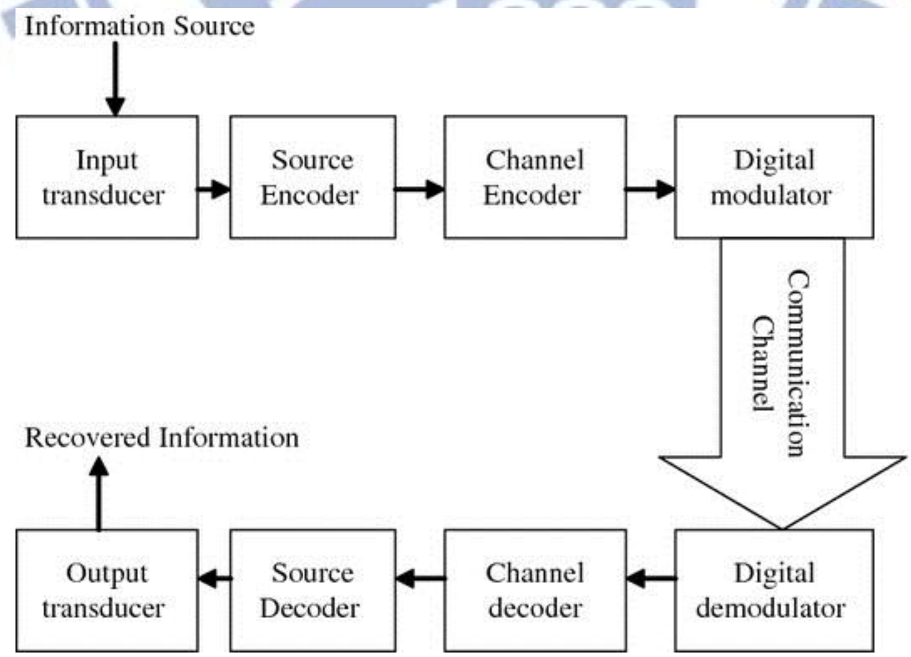

Fig-1.1: The traditional communication system

Chapter 2
Properties of Phasors

Suppose that S_I is the input sequence and S_L is the local sequence, given as $S_I = (x_0, x_1, ..., x_{N-1})$ and $S_L = (y_0, y_1, ..., y_{N-1})$, where $x_i, y_i \in (1,-1)$. We assume q phase shift between these two sequences, i.e. $y_k = x_{k+q}$.

I. Noiseless condition

In the beginning, we consider the noiseless condition which is easy to be analyzed. For the two sequences of interest, when FFT is performed we get

$$X_i = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}ik}, i = 0,1,2,..., N-1 \tag{2.1}$$

$$Y_i = \sum_{k=0}^{N-1} y_k e^{-j\frac{2\pi}{N}ik}, i = 0,1,2,..., N-1 \tag{2.2}$$

Let

$$X_i = |X_i| e^{j\theta_i} \tag{2.3}$$

$$Y_i = |Y_i| e^{j\phi_i} \tag{2.4}$$

where θ_i and ϕ_i denote the phases of X_i and Y_i, respectively.

Since $y_k = x_{k+q}$, we get the relationship between X_i and Y_i as follows:

$$
\begin{aligned}
Y_i &= \sum_{k=0}^{N-1} y_k e^{-j\frac{2\pi}{N}ik} \\
&= \sum_{k=0}^{N-1} x_{k+q} e^{-j\frac{2\pi}{N}ik} \\
&= \sum_{k=0}^{N-1} x_{k+q} e^{-j\frac{2\pi}{N}i(k+q)} \cdot e^{j\frac{2\pi}{N}iq} \\
&= X_i \cdot e^{j\frac{2\pi}{N}iq}
\end{aligned} \tag{2.5}
$$

Note that X_i and Y_i share the same amplitude but differ by $\frac{2\pi}{N}iq$ in their phases, where $i = 0,1,2,\ldots,N$. Thus we have

$$
|X_i| = |Y_i| \tag{2.6}
$$

$$
\phi_i = \theta_i + \frac{2\pi}{N}iq \tag{2.7}
$$

Next, we define

$$
\begin{aligned}
V_i &= X_i Y_i^* \\
&= |X_i|^2 \cdot e^{-j\frac{2\pi}{N}iq}
\end{aligned} \tag{2.8}
$$

Assume S_I and S_L are maximum-length sequences (m-sequences), it can be proved that

$$
\begin{aligned}
|X_i| = |Y_i| &= 1 \qquad if \quad i = 0 \\
&= \sqrt{N+1} \qquad if \quad i \neq 0
\end{aligned} \tag{2.9}
$$

Using (2.9) we get

$$V_i = 1 \qquad\qquad if \quad i = 0$$
$$= (N+1)e^{-j\frac{2\pi}{N}iq}, \quad if \quad i \neq 0 \tag{2.10}$$

From (2.10) we know that the code phase "q" is embedded in the phase of V_i. Note that V_0 contains no information about q, which is ignored in the following analysis.

Using (2.10), we can get the value of q via IFFT. Let v_k denote the IFFT of V_i, given as follows:

$$v_k = \sum_{i=0}^{N-1} V_i e^{j\frac{2\pi}{N}ik} \quad , \quad k\text{=0,1,2.....,N-1} \tag{2.11}$$

When $k = q$, we get

$$\begin{aligned} v_q &= \sum_{i=0}^{N-1} V_i e^{j\frac{2\pi}{N}iq} \\ &= \sum_{i=0}^{N-1} |X_i|^2 e^{-j\frac{2\pi}{N}iq} \cdot e^{j\frac{2\pi}{N}iq} \\ &= \sum_{i=0}^{N-1} |X_i|^2 \\ &= 1 + (N-1)(N+1) \\ &= N^2 \end{aligned} \tag{2.12}$$

When $k \neq q$, we get

$$
\begin{aligned}
v_k &= \sum_{i=0}^{N-1} V_i e^{j\frac{2\pi}{N}ik} \\
&= \sum_{i=0}^{N-1} |X_i|^2 e^{-j\frac{2\pi}{N}iq} \cdot e^{j\frac{2\pi}{N}ik} \\
&= \sum_{i=0}^{N-1} |X_i|^2 e^{j\frac{2\pi}{N}i(q-k)} \\
&= |X_0|^2 + \sum_{i=1}^{N-1} |X_i|^2 e^{j\frac{2\pi}{N}i(q-k)} \qquad (2.13) \\
&= 1 + (N+1)\cdot \sum_{i=1}^{N-1} e^{j\frac{2\pi}{N}i(q-k)} \\
&= 1 + (N+1)(-1) \\
&= -N
\end{aligned}
$$

Therefore, by looking for the maximum among $\{v_k\}$, the correct code phase can be got.

II. Noisy condition

A. Signal model

Under noisy condition, a gaussion noise denoted as ξ_k is added to the transmitted sequence. In this case, the input sequence becomes $S_I = (w_0, ..., w_{N-1})$ and w_k is given by

$$w_k = \mathrm{sgn}(x_k + \xi_k) \tag{2.14}$$

where "$\mathrm{sgn}(\cdot)$" denotes the sign function, i.e. $\mathrm{sgn}(z) = 1$ if $z \geq 0$, $\mathrm{sgn}(z) = -1$, if $z < 0$. To ease the analysis, we model w_k as

$$w_k = x_k + \beta_k \bar{x}_k \tag{2.15}$$

where $\beta_k \in (0,2)$ and $\bar{x}_k$ denotes the inverse of x_k. When $\beta_k = 2$, we have

$$w_k = \bar{x}_k \tag{2.16}$$

which means error occurs due to ξ_k. When $\beta_k = 0$, we have

$$w_k = x_k \tag{2.17}$$

which means no error occurs.

Let P_e denote the chip error probability. From (2.15), it is easy to have the following probabilities:

$$P(\beta_k = 2) = P_e \tag{2.18}$$

$$P(\beta_k = 0) = 1 - P_e \tag{2.19}$$

Thus, if P_e is known, the statistical property of w_k can be derived.

For the input sequence $(w_0, w_1, ..., w_{N-1})$, we perform FFT to get

$$W_i = \sum_{k=0}^{N-1} w_k e^{-j\frac{2\pi}{N}ik} \quad , \qquad i = 0,1,...., N-1 \tag{2.20}$$

Note that W_i is a complex random variable whose mean is given as

$$
\begin{aligned}
E[W_i] &= E[\sum_{k=0}^{N-1} w_k e^{-j\frac{2\pi}{N}ik}] \\
&= E[\sum_{k=0}^{N-1} (x_k + \beta_k \bar{x}_k) e^{-j\frac{2\pi}{N}ik}] \\
&= E[\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}ik} + \sum_{k=0}^{N-1} \beta_k \bar{x}_k e^{-j\frac{2\pi}{N}ik}] \\
&= E[X_i + \sum_{k=0}^{N-1} \beta_k \bar{x}_k e^{-j\frac{2\pi}{N}ik}] \\
&= X_i + \sum_{k=0}^{N-1} E[\beta_k] \bar{x}_k e^{-j\frac{2\pi}{N}ik}
\end{aligned}
\tag{2.21}
$$

Since

$$E[\beta_k] = 2 \cdot P_e + 0 \cdot (1 - P_e) = 2P_e \tag{2.22}$$

Thus

$$
\begin{aligned}
E[W_i] &= X_i + (2P_e) \cdot \sum_{k=0}^{N-1} \bar{x}_k e^{-j\frac{2\pi}{N}ik} \\
&= X_i + (2P_e)(-X_i) \\
&= (1 - 2P_e) X_i
\end{aligned}
\tag{2.23}
$$

Let A_i denote the mean of W_i, then

$$A_i = (1 - 2P_e) X_i \tag{2.24}$$

Using A_i, we can model W_i as

$$W_i = A_i + n_i \tag{2.25}$$

where n_i is a zero-mean complex random variable.

The variance of n_i, denoted as σ_n^2, can be derived as follows:

$$\begin{aligned}\sigma_n^2 &= E[n_i n_i^*] \\ &= E[(W_i - A_i)(W_i - A_i)^*] \\ &= E[W_i W_i^* - A_i W_i^* - A_i^* W_i + A_i A_i^*] \\ &= E[W_i W_i^*] - |A_i|^2\end{aligned} \tag{2.26}$$

It can be proved that

$$E[W_i W_i^*] = |X_i|^2 - 4P_e |X_i|^2 + 4P_e N \tag{2.27}$$

Thus

$$\begin{aligned}\sigma_n^2 &= |X_i|^2 - 4P_e |X_i|^2 + 4P_e N - (1-2P_e)^2 |X_i|^2 \\ &= 4P_e N - 4P_e^2 |X_i|^2\end{aligned} \tag{2.28}$$

From (2.9) , we get

$$\sigma_n^2 = 4P_e N - 4P_e^2 (N+1) \approx 4P_e (1-P_e) N \tag{2.29}$$

When N is large, we can model n_i as a zero-mean gaussian random variable whose variance is given as above. This signal model will be used in the following analysis.

B.. Vector analysis

Now, we would like to apply the concept of vector to help analysis. Let

$$W_i = | W_i | e^{j\Omega_i} \tag{2.30}$$

where Ω_i is the phase of W_i. As shown in Fig. 2.1, if we look W_i, A_i and n_i as vectors in the complex plane, from (2.25) the following relationship holds:

$$\vec{W}_i = \vec{A}_i + \vec{n}_i \tag{2.31}$$

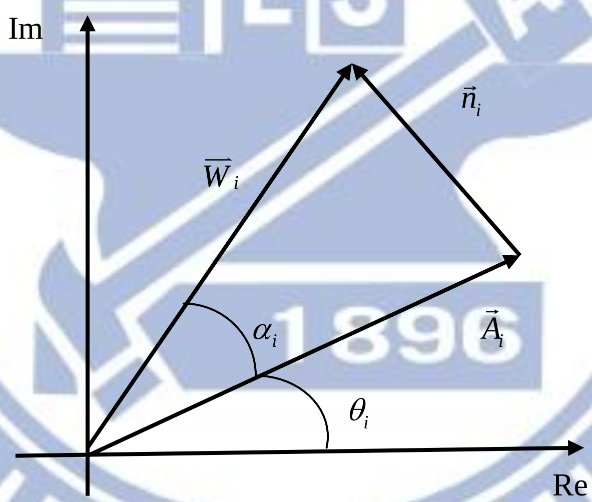

Fig. 2.1: The vector concept of $\overrightarrow{W}_i$

In Fig. 2.1, we have

$$\Omega_i = \theta_i + \alpha_i \tag{2.32}$$

where θ_i is the phase angle of A_i, which is the same as that of X_i, while α_i is the phase angle between $\vec{W}_i$ and $\vec{A}_i$. Note that α_i is a random phase caused by noise. Let

$$\begin{aligned} V_i &= W_i Y_i^* \\ &= |W_i||Y_i| \cdot e^{j(\Omega_i - \phi_i)} \\ &= |W_i||Y_i| \cdot e^{-j\frac{2\pi}{N}iq + \alpha_i} \end{aligned} \tag{2.33}$$

Obviously, the phase of V_i still contains the information of q, but is disturbed by α_i. When IFFT is performed, we obtain:

$$v_k = \sum_{i=0}^{N-1} V_i e^{j\frac{2\pi}{N}ik} \quad , \quad k = 0,1,\ldots,N-1 \tag{2.34}$$

If $k = q$, then

$$\begin{aligned} v_q &= \sum_{i=0}^{N-1} V_i e^{j\frac{2\pi}{N}iq} \\ &= \sum_{i=0}^{N-1} |W_i||Y_i| e^{j(\Omega_i - \phi_i)} \cdot e^{j\frac{2\pi}{N}iq} \\ &= \sum_{i=0}^{N-1} |W_i||Y_i| e^{j(\alpha_i + \theta_i - \phi_i)} \cdot e^{j\frac{2\pi}{N}iq} \end{aligned} \tag{2.35}$$

From (2.7) we get

$$\theta_i - \phi_i = -\frac{2\pi}{N} iq \tag{2.36}$$

Hence

$$
\begin{aligned}
v_q &= \sum_{i=0}^{N-1} |W_i||Y_i| e^{j\alpha_i} \\
&= \sqrt{N+1} \cdot \sum_{i=0}^{N-1} |W_i| e^{j\alpha_i} \\
&= \sqrt{N+1} \cdot \left(\sum_{i=0}^{N-1} |W_i| \cos\alpha_i + j \sum_{i=0}^{N-1} |W_i| \sin\alpha_i\right)
\end{aligned}
\tag{2.37}
$$

Owing to the symmetric property of FFT, the imaginary part of v_q is zero so that

$$
v_q = \sqrt{N+1} \cdot \sum_{i=0}^{N-1} |W_i| \cos\alpha_i \tag{2.38}
$$

Note that for a given W_i, because α_i is a random variable so that $|W_i|\cos\alpha_i$ is random too.

Chapter 3
The Proposed Method

Since the computation of FFT and IFFT is proportional to $N \cdot \log N$, it results in considerable computation when N is large. Therefore, we intend to reduce the computation by using the phase information of complex phasors. It is found that the proposed method can much reduce the computation in high SNR condition. The details of our method are described below.

I. Noiseless condition

First, we review the formulas got in the previous chapter under noiseless condition. Assume $S_I = (x_0, x_1, ..., x_{N-1})$ is the input sequence and $S_L = (y_0, y_1, ..., y_{N-1})$ is the local sequence, where $y_k = x_{k+q}$. When FFT is performed, we get

$$X_i = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}ik} \quad , i = 0,1,2,..., N-1 \tag{3.1}$$

$$Y_i = \sum_{k=0}^{N-1} y_k e^{-j\frac{2\pi}{N}ik} \quad , i = 0,1,2,..., N-1 \tag{3.2}$$

$$\begin{aligned} V_i &= X_i Y_i^* \\ &= |X_i|^2 \cdot e^{-j\frac{2\pi}{N}iq} \end{aligned} \quad , i = 0,1,2,..., N-1 \tag{3.3}$$

When $i \neq 0$, it is obvious that the phase of V_i contains the information about q. Note that because

$$e^{-j\frac{2\pi}{N}iq} = e^{-j(\frac{2\pi}{N}iq+2n\pi)} \tag{3.4}$$

for any integer n, there is phase ambiguity in the phase of V_i. This fact should be carefully considered in estimating q.

From (3.3) we get

$$V_1 = X_1 Y_1^* = |X_1|^2 \cdot e^{-j\frac{2\pi}{N}q} \tag{3.5}$$

Because $0 \le q \le N-1$ so that $0 \le \frac{2\pi}{N} q < 2\pi$, there is no phase ambiguity presents in the phase of V_1. Thus we can obtain q directly from the phase of V_1 under noiseless condition. However, the problem becomes complicated when noise presents.

II. Noisy condition

In noisy condition, the input sequence becomes $S_I = (w_0, w_1, ..., w_{N-1})$, where $w_k = x_x + \xi_k$ and ξ_k is a gaussian noise. As before, we perform FFT to get

$$W_i = \sum_{k=0}^{N-1} w_k e^{-j\frac{2\pi}{N}ik} \quad , \qquad i = 0,1,...., N-1 \tag{3.6}$$

We can express W_i as

$$W_i = |W_i| e^{j\Omega_i} \tag{3.7}$$

where Ω_i is the phase of W_i. Let

$$V_i = W_i Y_i^* = |V_i| e^{j\Psi_i} \tag{3.8}$$

From (2.34), the phase of V_i is given as

$$\Psi_i = -\frac{2\pi}{N} iq + \alpha_i \tag{3.9}$$

where α_i is a random phase caused by noise. Apparently, Ψ_i contains the information of q, which can be employed to estimate the code phase.

Under noisy condition, from (3.9) we get

$$\Psi_1 = -\frac{2\pi}{N} q + \alpha_1 \tag{3.10}$$

where α_1 is the random phase caused by noise. Define

$$q_1 = round(\frac{N}{2\pi}\Psi_1) \tag{3.11}$$

where "$round(z)$" denotes the closest integer of z. We may take q_1 as the 1st estimation of q and define

$$\Delta q_1 = q - q_1 \tag{3.12}$$

where Δq_1 indicates the error between q_1 and q.

It is noteworthy to mention that $\frac{N}{2\pi}$ is a huge number when N is large, which means the error phase in Ψ_1 could possibly be amplified by a large factor in (3.11). In other words, a tiny phase deviation of α_1 may result in a significant deviation of Δq_1.

III. A simple phase estimation method

Using the phases of V_1 given in (3.10), we can get the 1st estimation and then obtain q_1. However, the random phase α_1 and a large value of N may result in unacceptable estimation error. This problem can be resolved with the help of other phases of V_i .

The intuitive way to improve the estimation accuracy is to reduce the variance caused by α_1 as much as possible. According to the ***Law of Large Numbers,*** the average of random variables obtained from a large number of trials is close to their expected value, and tends to become closer as more trials are performed. It is obvious that the terms V_i (for $i>1$) can be used since they do contain the information of q. However, we can't employ them directly due to the inherent phase ambiguity accompanied with their phases. In order to overcome phase ambiguity, we propose the following approach. Let

$$F_0 = V_0 V_1^* = |V_0| \cdot |V_1| \cdot e^{j(\frac{2\pi}{N}q+\alpha_0-\alpha_1)}$$

$$F_1 = V_2 V_3^* = |V_2| \cdot |V_3| \cdot e^{j(\frac{2\pi}{N}q+\alpha_2-\alpha_3)}$$

$$F_2 = V_4 V_5^* = |V_4| \cdot |V_5| \cdot e^{j(\frac{2\pi}{N}q+\alpha_4-\alpha_5)}$$

$$\vdots$$

$$F_{N-2/2} = V_{N-2} V_{N-1}^* = |V_{N-2}| \cdot |V_{N-1}| \cdot e^{j(\frac{2\pi}{N}q+\alpha_{N-2}-\alpha_{N-1})} \tag{3.13}$$

In general,

$$F_i = V_{2i} V_{2i+1}^* = |V_{2i}| \cdot |V_{2i+1}| \cdot e^{j(\frac{2\pi}{N}q+\alpha_{2i}-\alpha_{2i+1})} \tag{3.14}$$

where each α_i in (3.13) is an independent random variable. Eq. (3.14) could be rewritten as

$$F_i = |V_{2i}| \cdot |V_{2i+1}| \cdot e^{j(\frac{2\pi}{N} q + \beta_i)} \tag{3.15}$$

where $\beta_i = \alpha_{2i} - \alpha_{2i+1}$. Note that the phase of F_i can be divided into two terms, i.e. the co-phase term $\frac{2\pi}{N} iq$ and the random phase term β_i.

When the set of $\{F_i, i = 0,1,..., N-2\}$ is available, we define

$$f = \frac{1}{M} \cdot \sum_{i=0}^{M-1} F_i \tag{3.16}$$

where M is an integer. We may express f as

$$f = |f| \cdot e^{j\Theta} \tag{3.17}$$

where Θ is the phase of f. When N is sufficiently large, according to the **Law of Large Numbers** we get

$$\Theta \rightarrow \frac{2\pi}{N} q \tag{3.18}$$

Therefore, Θ could be expressed as

$$\Theta = \frac{2\pi}{N} q + \beta' \tag{3.19}$$

where β' is a random phase. Note that the standard deviation of β' will be much smaller than of β_i, which will result in more precise estimation of $\frac{2\pi}{N}q$. We will demonstrate it with the simulation to be presented in Chapter 4.

After Θ is got, we define

$$q_2 = round(\frac{N}{2\pi}\Theta) \tag{3.20}$$

where q_2 denotes the 2nd estimation of q. Obviously, q_2 will be more accurate than q_1. Moreover, the error of the 2nd estimation is given as

$$\Delta q_2 = q - q_2 \tag{3.21}$$

IV. The improved method

As shown above, we may apply the ***Law of Large Numbers*** to lower the variance in phase estimation. However, the estimation error may not be small enough since it should be less than $2\pi / N$ in order to get a correct estimate of q. When N is large, as is usually the case, Δq_2 may not be small enough. Therefore, we are obliged to further reduce the variance by using the special relationship given in (3.8).

In probability theory, if X is a random variable and a is a constant, we have the following equality:

$$Var(\frac{X}{a}) = \frac{1}{a^2} \cdot Var(X) \tag{3.22}$$

where "$Var(z)$" denotes the variance of a ransom variable z. When $a > 1$, the variance of X / a is reduced by a factor of a^2. This fact will be employed to reduce phase variance.

After the 2nd estimation, we take Θ as the estimate of $\frac{2\pi}{N}q$, which may deviate from $\frac{2\pi}{N}q$ by a random phase β'. Assume k is an integer and $k \geq 2$. If Θ is close to $\frac{2\pi}{N}q$, it is expected that $k \cdot \Theta$ is close to $\frac{2\pi k}{N}q$. The idea is: if we can get a good estimate of $\frac{2\pi k}{N}q$, then a better estimate of $\frac{2\pi}{N}q$ is got due to the multiplication factor of k. However, the phase ambiguity problem possibly occurring in the term $\frac{2\pi k}{N}q$ should be carefully considered.

Now, we begin to describe the proposed method. Assume $k \geq 2$ is a given integer. Let

$$
\begin{aligned}
G_0 &= V_0 V_k^* = |V_0| \cdot |V_k| \cdot e^{j(\frac{2\pi}{N}kq + \alpha_0 - \alpha_k)} \\
G_1 &= V_1 V_{k+1}^* = |V_1| \cdot |V_{k+1}| \cdot e^{j(\frac{2\pi}{N}kq + \alpha_1 - \alpha_{k+1})} \\
G_2 &= V_2 V_{k+2}^* = |V_2| \cdot |V_{k+2}| \cdot e^{j(\frac{2\pi}{N}kq + \alpha_2 - \alpha_{k+2})} \\
&\vdots \\
G_{(N-1)/2} &= V_{(N-1)/2} V_{k+(N-1)/2}^* = |V_{(N-1)/2}| \cdot |V_{k+(N-1)/2}| \cdot e^{j(\frac{2\pi}{N}kq + \alpha_{(N-1)/2} - \alpha_{k+(N-1)/2})}
\end{aligned} \tag{3.23}
$$

In general,

$$
G_i = V_i V_{k+i}^* = |V_i| \cdot |V_{k+i}| \cdot e^{j(\frac{2\pi}{N}q + \alpha_i - \alpha_{k+i})} \tag{3.24}
$$

Note that due to the symmetric property of FFT, we only take i=1~$\frac{N-1}{2}$ in the above equation.

As α_i, $i=0,1,2,\ldots, N-1$, are independent random variables, the term $(\alpha_i - \alpha_{i+k})$ could be expressed as a random variable γ_i whose variance is the same as β_i. Then we obtain

$$G_i = V_i V_{k+i}^* = |V_i| \cdot |V_{i+k}| \cdot e^{j(\frac{2\pi}{N}kq+\gamma_i)}, \text{ i=1,2,3,.....,} \frac{N-1}{2} \tag{3.25}$$

where $\gamma_i = \alpha_i - \alpha_{i+k}$ and

$$Var(\gamma_i) = Var(\beta_i) \tag{3.26}$$

Next, by summing G_i we get

$$g = \frac{1}{M}\sum_{i=0}^{M-1} G_i \tag{3.27}$$

Assume

$$g = |g| \cdot e^{j\varepsilon} \tag{3.28}$$

where

$$\varepsilon = \left(\frac{2\pi k}{N} q + \gamma'\right) \quad \text{mod} \quad 2\pi \tag{3.29}$$

In (3.29), the variance of γ' will be much smaller than of γ_i. Note that because of the mod-2π operation, in general ε is not equal to $\frac{2\pi}{N}kq + \gamma'$. This fact should be taken into account in estimating q.

For a given ε, from (3.29) we have the following equality:

$$\frac{2\pi k}{N}q+\gamma' = \varepsilon + 2n\pi \tag{3.30}$$

where n is an unknown integer since q is still unknown. Suppose that n is known, then

$$\frac{\varepsilon+2n\pi}{k} = \frac{2\pi}{N}q+\frac{\gamma'}{k} \tag{3.31}$$

Let

$$h = \frac{\varepsilon+2n\pi}{k} = \frac{2\pi}{N}q+\frac{\gamma}{k} \tag{3.32}$$

We take h as the estimation of $\frac{2\pi}{N}q$, whose mean is given by

$$\mu_h = \frac{2\pi}{N}q+\frac{E[\gamma']}{k} = \frac{2\pi}{N}q \tag{3.33}$$

where $E[\gamma']=0$ is assumed. On the other hand, the variance of h is given by

$$\begin{aligned}\sigma_h^2 &= E[(h-\mu_h)^2] = E[(\frac{\gamma'}{k})^2] \\ &= \frac{1}{k^2}\sigma_{\gamma'}^2\end{aligned} \tag{3.34}$$

where $\sigma_{\gamma'}^2$ denotes the variance of γ'. It is easy to see the variance of h can be much reduced if k is large.

It's noteworthy to mention that in estimating the phase of $\frac{2\pi k}{N}q$, we approximate it as

$k \cdot \Theta$, where $\Theta \approx \frac{2\pi}{N} q$. When we multiply Θ by *k*, the error phase accompanied with Θ is also amplified. As a result, although a large *k* is desired to reduce the variance of h, it also increases the risk of erroneous mod-2π operation. Therefore, choosing an appropriate value of *k* is a critical issue.

When $k \cdot \Theta$ is close to $2n\pi$, it is probably for erroneous mod-2π operation to occur. Accordingly, we choose $k \cdot \Theta$ to be around $(2n+1)\pi$ to avoid it. Because the larger of *k*, the more risk of erroneous mod-2π operation, for safety reason in the beginning we take

$$k \cdot \Theta \cong 3\pi \tag{3.35}$$

In turn k is calculated as

$$k = round(\frac{3\pi}{\Theta}) . \tag{3.36}$$

When $k \cdot \Theta \cong 3\pi$ is taken, we get $n = 1$ in (3.30). After a more accurate value of $\frac{2\pi}{N} q$ is got, we can increase k by taking a larger n so as to further reduce the error phase. Thus, we may design a recursive algorithm to gradually increase k. The algorithm is shown in Fig. 3.1, whose operation is described below.

Assume $\Theta \approx \frac{2\pi}{N} q$ has been obtained. First, we take $n = 1$ and get $k = k_1$ with (3.36). After a more accurate estimate of $h \approx \frac{2\pi}{N} q$ is got, we set $n = n + 1$ and get $k = k_2$ to obtain a more accurate estimate of $\frac{2\pi}{N} q$. The process can be repeated until the required accuracy is met.

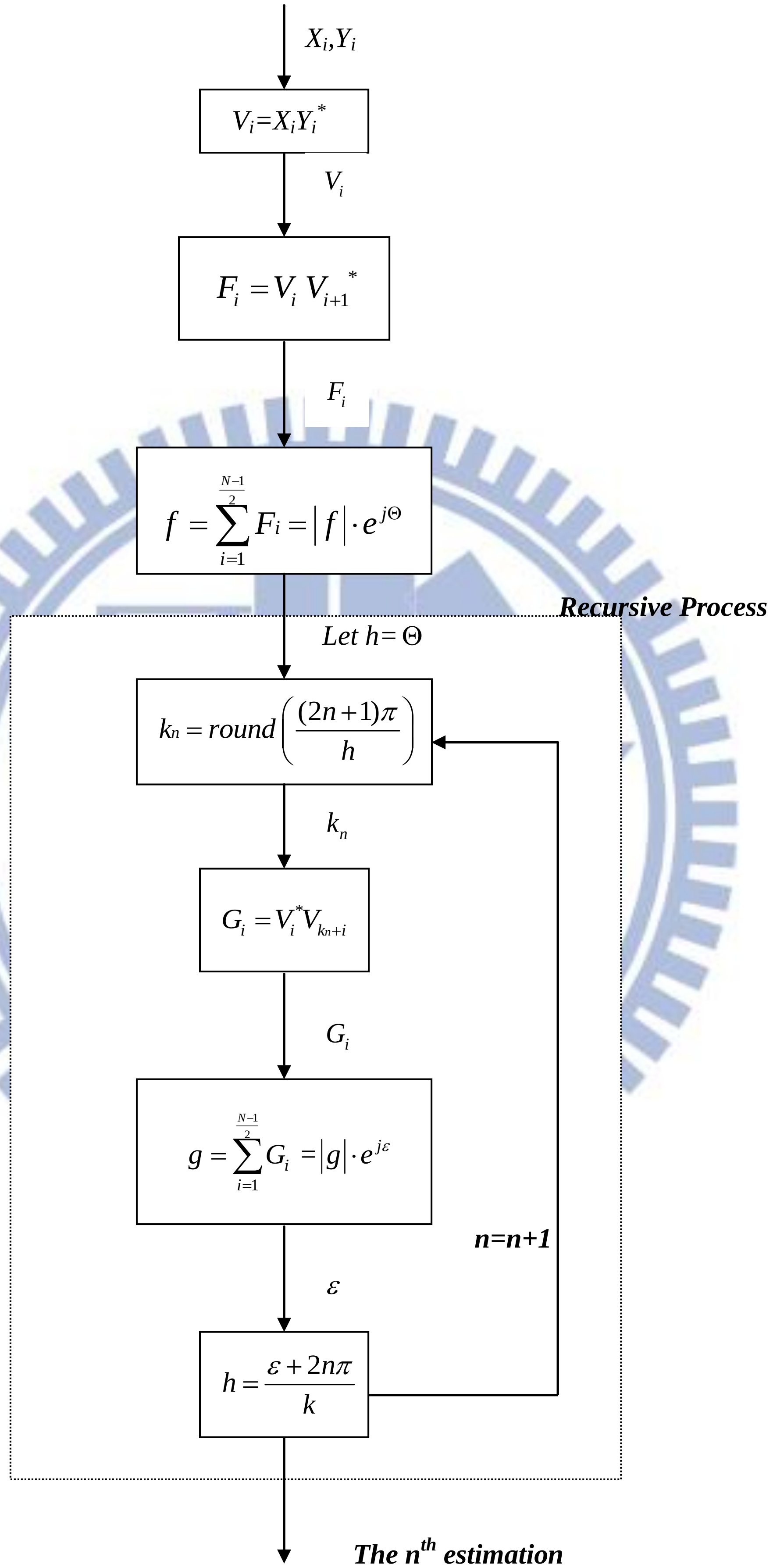

Fig 3.1: The estimation flow chart

V. *New local sequence*

In the above method, when q is small the estimated $\Theta \approx \frac{2\pi}{N} q$ will be close to zero, in this case k will be a rather large number to make $k \cdot \Theta \cong 3\pi$. Under this circumstance, it has high risk for erroneous mod-2π operation. Therefore, we have to design a scheme to resolve this problem.

As shown in Fig. 3.2, we assume the incoming sequence S_I and the local sequence S_L has a small code phase shift in between so that the estimated value of q, denoted as $\hat{q}$, is small. Then we design a local sequence Z_L, where $Z_L = \{z_0, z_1, \ldots, z_{N-1}\}$ and $z_k = x_{k+q'}$. We take q' such that $\frac{2\pi}{N} q'$ is much larger than $\frac{2\pi}{N} \hat{q}$ and $q' = \hat{q} + \Delta q$. Our purpose is to use Z_L as a new local sequence whose code phase shift with S_I is away from zero.

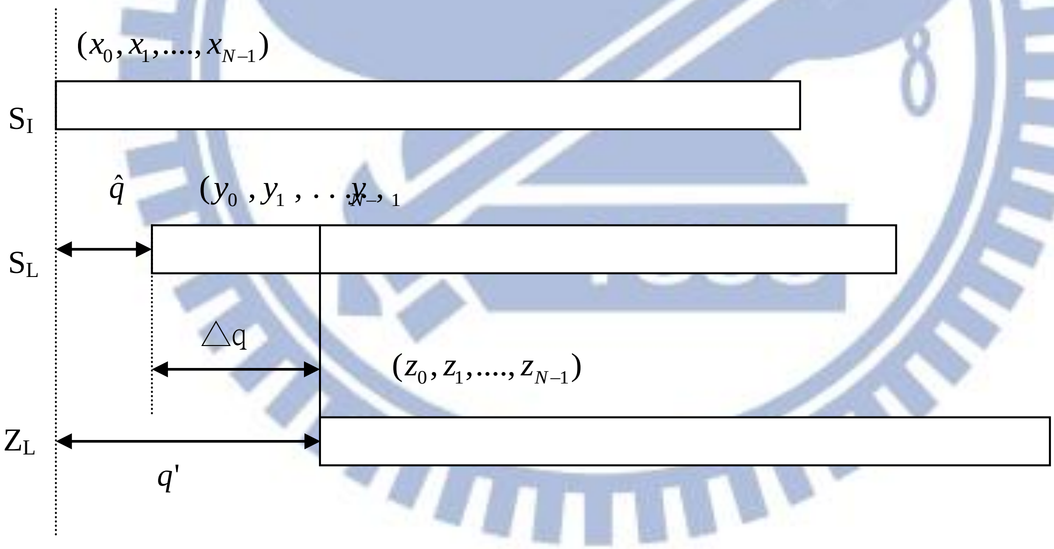

Fig 3.2: A new local sequence Z_L.

Under noiseless condition, we have $\hat{q} = q$. When FFT is performed, we get the following equation:

$$\begin{aligned} Z_i &= \sum_{k=0}^{N-1} z_k e^{-j\frac{2\pi}{N}ik} \\ &= \sum_{k=0}^{N-1} x_{k+q'} e^{-j\frac{2\pi}{N}ik} \\ &= \sum_{k=0}^{N-1} x_{k+q'} e^{-j\frac{2\pi}{N}i(k+q')} \cdot e^{j\frac{2\pi}{N}iq'} \\ &= X_i \cdot e^{j\frac{2\pi}{N}iq'} \end{aligned} \tag{3.37}$$

Let U_i be defined as

$$U_i = X_i \overset{*}{Z} \ , i = 0,1,2,..., N-1 \tag{3.38}$$

The relationship between V_i and U_i is given by

$$\begin{aligned} U_i &= X_i Z_i^* \\ &= |X_i|^2 \cdot e^{-j\frac{2\pi}{N}iq'} \\ &= |X_i|^2 \cdot e^{-j\frac{2\pi}{N}i(q+\Delta q)} \\ &= |X_i|^2 \cdot e^{-j\frac{2\pi}{N}iq} \cdot e^{-j\frac{2\pi}{N}i\Delta q} \\ &= V_i \cdot e^{-j\frac{2\pi}{N}i\Delta q} \end{aligned} \tag{3.39}$$

It's obvious that there is $\frac{2\pi}{N} i\Delta q$ phase difference between V_i and U_i. In (3.25), we have

$$G_i = V_i V_{k+i}{}^* = |V_i| \cdot |V_{i+k}| \cdot e^{j(\frac{2\pi}{N}kq+\gamma_i)}, \text{ i=1,2,3,....., } \frac{N-1}{2} \tag{3.40}$$

Similarly, we define a new factor P_i for U_i as

$$
\begin{aligned}
P_i &= U_i U_{i+k}^* \\
&= V_i\, e^{-j\frac{2\pi}{N}i\Delta q} \cdot V_{i+k}^* e^{j\frac{2\pi}{N}(i+k)\Delta q} \\
&= V_i\, V_{i+k}^* \cdot e^{j\frac{2\pi}{N}k\Delta q} \\
&= G_i \cdot e^{j\frac{2\pi}{N}k\Delta q}
\end{aligned}
\tag{3.41}
$$

As shown in Fig 3.3, the two phasors $\vec{G}_i$ and $\vec{P}_i$ have a phase difference of $\frac{2\pi}{N}\Delta q$ in between. By choosing an appropriate value of Δq, it is easy to make $\frac{2\pi}{N}q'$ away from zero. Thus, when Z_L is used as the new local sequence, the same algorithm can be applied to get the estimate of q'. After q' is got, it is easy to obtain $q = q' - \Delta q$, while Δq is a known parameter which represents the code phase between S_L and Z_L.

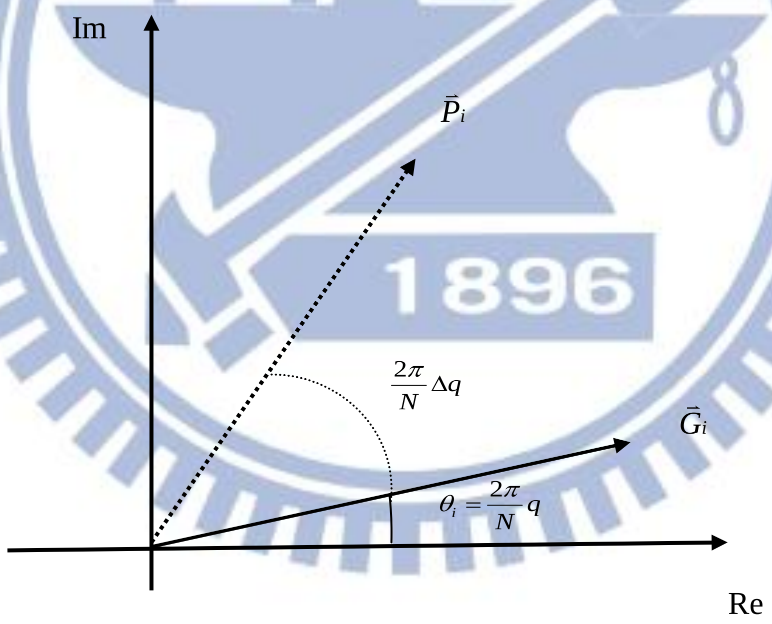

Fig 3.3: The vectors concept of $\vec{P}_i$ and $\vec{G}_i$

Chapter 4
Simulation Results

After introducing the proposed algorithm, we will show simulation results in this chapter. We will perform the algorithm shown in Fig3.1 and find estimation results under different SNR conditions.

Firstly, we consider the case of an m-sequence whose length is $N=2^{10}-1$, and $q=100$ is assumed. The algorithm depicted in Fig. 3.1 could be simplified as that in Fig. 4.1.

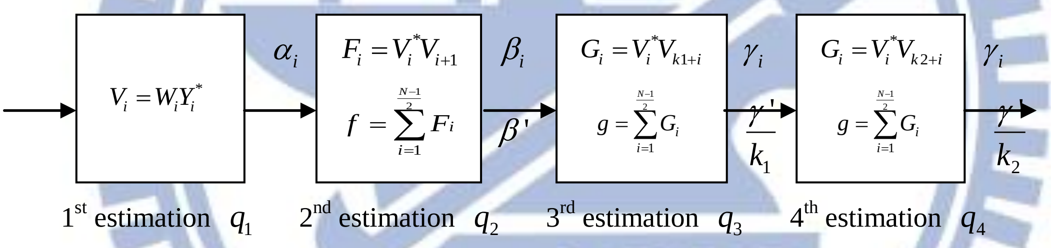

Fig 4.1 The flow chart in simulation.

I. First Estimation

Recall that in the first step, we employ V_1 which contains information of q without phase ambiguity as the initial guess, and then obtain q_1 as the 1st estimation. The equations involved in this estimation are given as follows:

$$V_1 = W_1 Y_1^* = |V_1| e^{j\Psi_1} \tag{4.1}$$

$$\Psi_1 = -\frac{2\pi}{N} q + \alpha_1 \tag{4.2}$$

$$q_1 = round(\frac{N}{2\pi}\Psi_1) \tag{4.3}$$

$$\Delta q_1 = q - q_1 \tag{4.4}$$

Fig 4.2 illustrates the standard deviation (STD) of α_i (in degrees) with respect to SNR. We see that the STD is large when SNR is low. Since our estimation is determined from N legal phases, a wrong estimation occurs if the deviation is over $\frac{1}{2}\cdot\frac{2\pi}{N}(0.176^o)$. Obviously, the accuracy is unacceptable even in high SNR conditions (5dB~10dB).

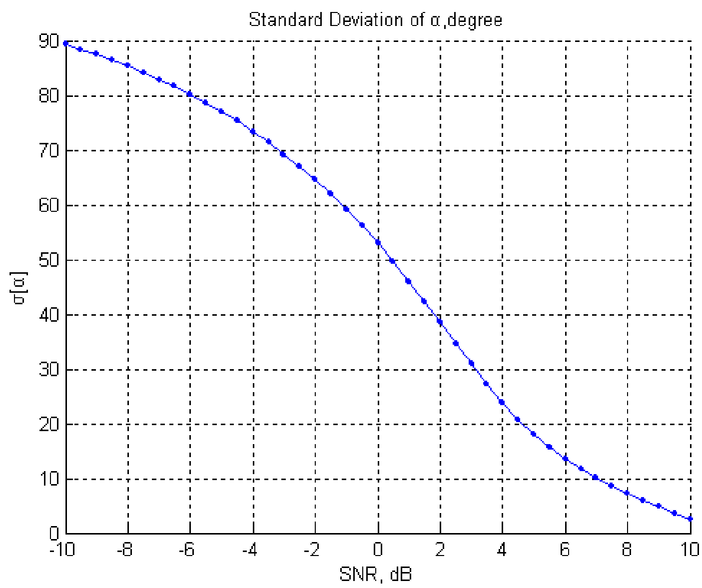

Fig 4.2 The standard deviation of α_i in terms of SNR.

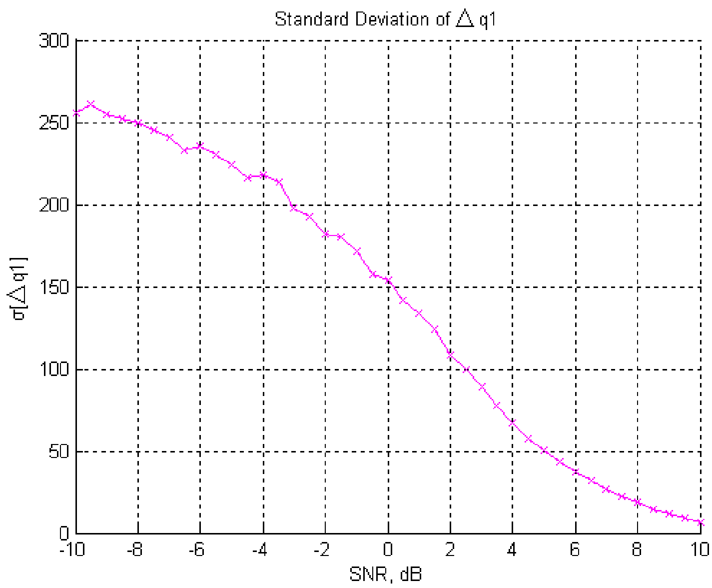

Fig 4.3 The standard deviation of q_1.

Fig 4.3 shows the standard deviation of the error in the 1st estimation, i.e. the STD of Δq_1. As expected, the STD of Δq_1 is proportional to that of α_i and its accuracy is generally unacceptable even in high SNR condition. For example, when SNR=10dB, the STD of Δq_1 is greater than one, which means most of the estimation of q is erroneous. This result is not strange, since the estimation is just based on the phase of V_1, which is not very reliable even in high SNR cases.

II. Second Estimation

In the second estimation, we transfer V_i to be F_i which contains the information of q as well. Since the random phase of F_i, denoted as β_i, is equal to $\alpha_i - \alpha_{i+1}$, the variance of β_i is larger than of α_i. However, we may apply the ***Law of Large Number*** to lower the variance by summing phasors. The equations involved in this procedure are shown as follows:

$$F_i = V_i V_{i+1}^* = |V_i| \cdot |V_{i+1}| \cdot e^{j(\frac{2\pi}{N}q+\beta_i)} \tag{4.5}$$

$$f = \sum_{i=1}^{\frac{N-1}{2}} F_i = |f| \cdot e^{j\Theta} = |f| \cdot e^{j(\frac{2\pi q}{N} + \beta')} \tag{4.6}$$

$$\Theta = \frac{2\pi}{N} q + \beta' \tag{4.7}$$

$$q_2 = round(\frac{N}{2\pi}\Theta) \tag{4.8}$$

$$\Delta q_2 = q - q_2 \tag{4.9}$$

where β' is the error phase in Θ. As shown in Fig. 4.4, the STD of β' is much reduced compared with that of β_i, which indicates the accuracy of Θ is much improved due to the summation process.

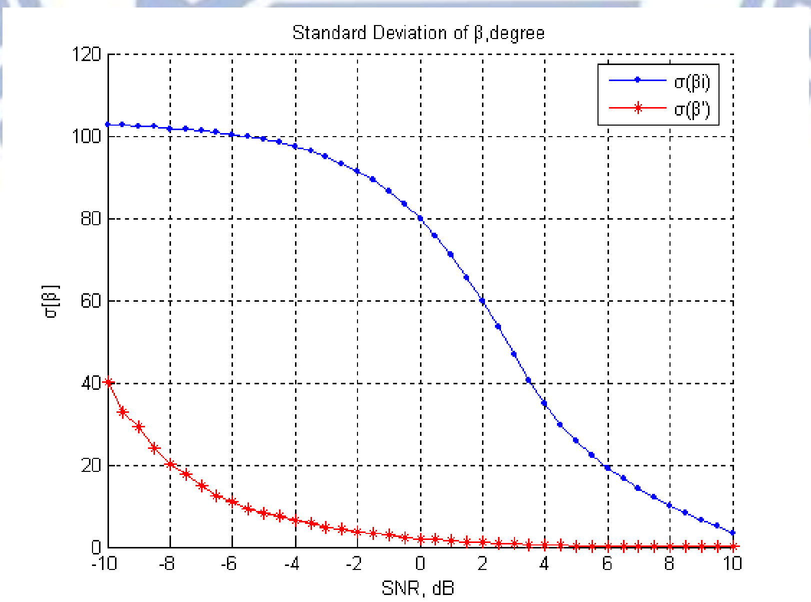

Fig 4.4 The standard deviation of β_i and β' under different SNR conditions.

Now, we would like to analyze the relationship between β_i and β'. In probability theory, if X_i, $i = 1,2,...,n$, are independent and identically distributed random variables, we have

$$Var(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n}Var(X_i) \quad (4.10)$$

Substituting $\frac{N-1}{2}$ for n in (4.1) and take square root in both sides, the relationship between the STDs of β' and β_i are given by

$$\begin{aligned}\sigma(\beta') &= \frac{1}{\sqrt{\frac{N-1}{2}}} \cdot \sigma(\beta_i) \\ &\cong \frac{1}{22.616} \cdot \sigma(\beta_i)\end{aligned} \quad (4.11)$$

Because β_i and β' are random phases in the complex domain, there are two special properties of them:

(1) The range of phase is $[-\pi, \pi]$,

(2) $e^{j\theta} = e^{j(\theta+2k\pi)}$.

These two properties make the relationship between $\sigma(\beta')$ and $\sigma(\beta_i)$ becomes nonlinear, but not the linear one given in (4.11). However, Eq.(4.11) is still an important reference for our simulation, because it is close to the simulated result when SNR is high.

Fig 4.5 shows the STD of Δq_2 in terms of SNR. Obviously, the STD of Δq_2 is much improved in high SNR cases due to the reduced variance of β'. Therefore, after the 2nd estimation we would have an accurate estimation of q for $SNR \geq 5dB$.

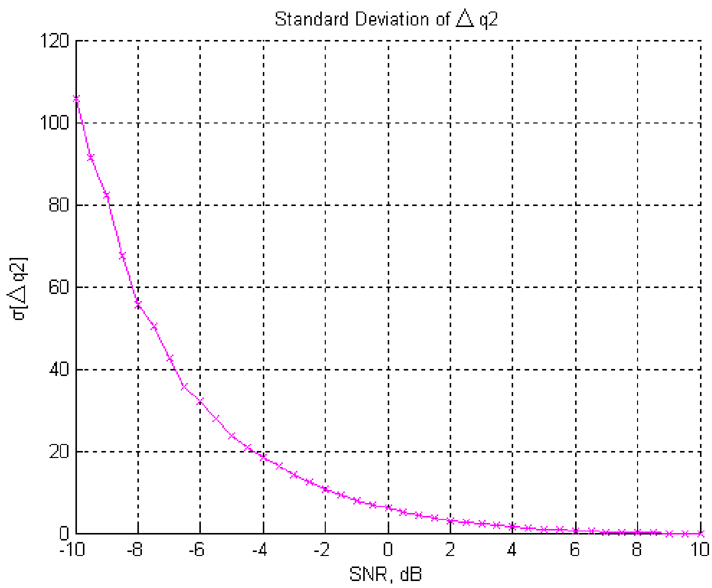

Fig 4.5: The standard deviation of q_2.

III. Third Estimation

In the third estimation, we use the prior estimation to get a more accurate result. The corresponding k_1 is obtained from $round(\frac{3\pi}{\Theta})$, which means the accuracy relies on the prior estimation of Θ. The equations involved in the third estimation are listed as follows:

$$k_1 = round(\frac{3\pi}{\Theta}) \tag{4.12}$$

$$G_i = V_i^* V_{k_1+i} = |V_i| \cdot |V_{i+k_1}| \cdot e^{j(\frac{2\pi}{N}k_1 q + \gamma_i)} \tag{4.13}$$

$$g = \sum_{i=1}^{M-1} G_i = |g| \cdot e^{j\varepsilon} = |g| \cdot e^{j(\frac{2\pi}{N}k_1 q + \gamma'} \tag{4.14}$$

$$h = \frac{\varepsilon + 2\pi}{k_1} = \frac{2\pi}{N} q + \frac{\gamma'}{k_1} \tag{4.15}$$

For the case of $q=100$ assumed in the simulation, the parameter k_1 is calculated as

$$\begin{aligned} k_1 &= round(\frac{3\pi}{\frac{2\pi q}{N}}) \\ &= round(\frac{3N}{2q}) \\ &= round(\frac{3\cdot 1023}{2\cdot 100}) = 15 \end{aligned} \tag{4.16}$$

Since γ_i has the same distribution as β_i, they are supposed to share the same STD as demonstrated in Fig4.6.

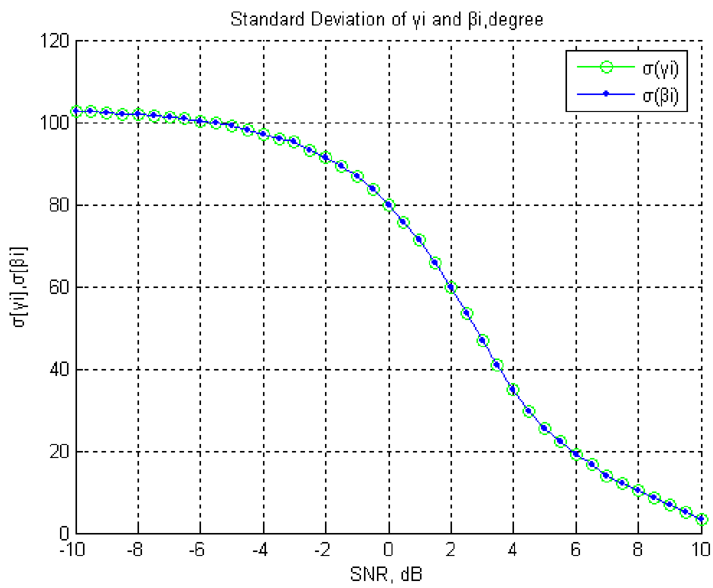

Fig 4.6: The standard deviations of β_i and γ_i.

Next, we focus on the ratio of $\sigma(\gamma')$ to $\sigma(\gamma'/k_1)$. The simulation result is shown in Fig 4.7, where the STD of γ'/k_1 is reduced when SNR > -5dB. But it even increases when SNR < -5dB. It is because of the undesired mod-2π operation that leads to the increase of $\sigma(\gamma'/k_1)$

in low SNR cases. Recall that k_1 is obtained from $round(\frac{3\pi}{\Theta})$, which means if $k_1 \cdot \frac{2\pi q}{N}$ is larger than 4π or smaller than 2π, the undesired mod-2π operation occurs. From (4.15), we could infer that it will result in increased STD since we add or subtract a factor of $\frac{2\pi}{k_1}$ in the calculation.

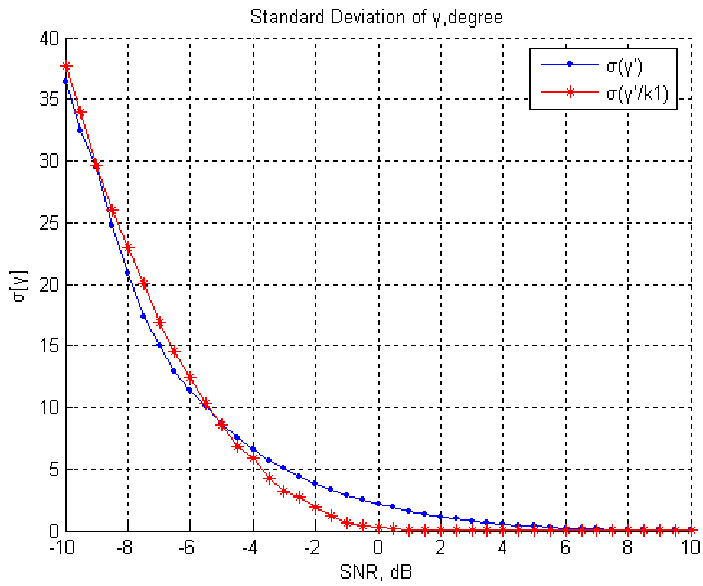

Fig 4.7: The standard deviations of γ' and $\frac{\gamma'}{k_1}$.

The STD of Δq_3 is shown in Fig. 4.8. Apparently, the STD of Δq_3 is much improved for $SNR \geq 0dB$, which indicates the 3rd estimation can significantly improve the performance when SNR is high.

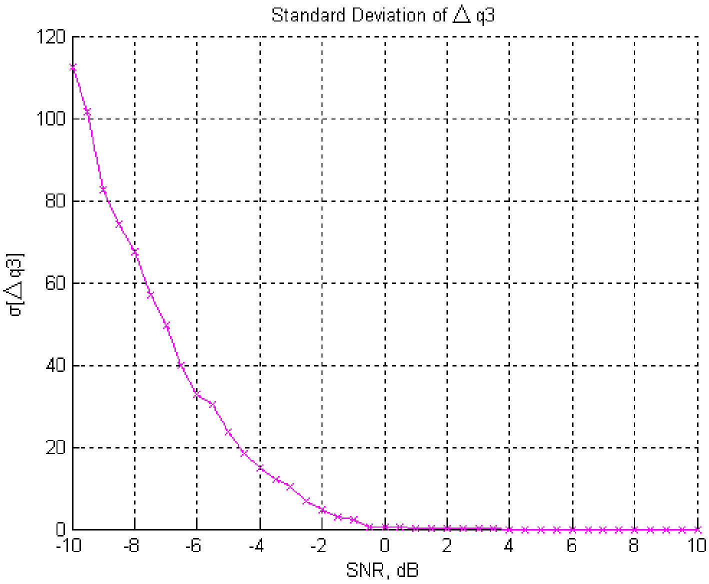

Fig 4.8: The standard deviation of q_3.

IV. Recursive algorithm

In the third estimation, we figure out that the improvement in estimation accuracy is closely relate to k_1. And the reliability of k_1 depends on the accuracy of prior estimation. Now, since we have more accurate result after the third estimation, we can proceed to improve the accuracy with the recursive algorithm described before.

In the beginning, we take $k = k_2$ and repeat the process similar to the 3rd estimation as follows:

$$k_2 = round(\frac{11\pi}{h}) \tag{4.17}$$

$$G_i = V_i V_{k_2+i}^{\ *} = |V_i| \cdot |V_{i+k_2}| \cdot e^{j(\frac{2\pi}{N}k_2 q + \gamma_i)} \tag{4.18}$$

$$g = \sum_{i=1}^{M-1} G_i = |g| \cdot e^{j\varepsilon} = |g| \cdot e^{j(\frac{2\pi}{N}k_2 q + \gamma')} \tag{4.19}$$

$$h = \frac{\varepsilon + 2\pi}{k_2} = \frac{2\pi}{N} q + \frac{\gamma'}{k_2} \tag{4.20}$$

The corresponding k_2 should be

$$\begin{aligned} k_2 &= round\left(\frac{11\pi}{\frac{2\pi q}{N}}\right) \\ &= round\left(\frac{11N}{2q}\right) \\ &= round\left(\frac{11\cdot 1023}{2\cdot 100}\right) \\ &= 56 \end{aligned} \tag{4.21}$$

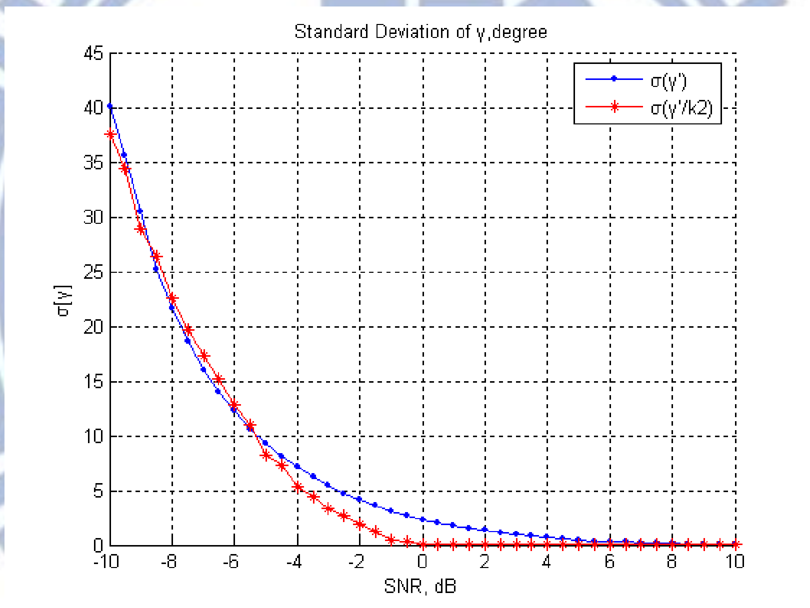

Fig 4.9: The standard deviation of γ' and $\dfrac{\gamma'}{k_2}$

As depicted in Fig 4.9, we find that the performance is still bad when SNR is low, but it is improved for $SNR \geq -5dB$. This result is similar to the 3rd estimation.

The comparison between $\sigma(\gamma'/k_1)$ and $\sigma(\gamma'/k_2)$ is shown in Fig 4.10. Since the proposed method relies on the accuracy of prior estimation, the relationship between $\sigma(\gamma'/k_1)$ and $\sigma(\gamma'/k_2)$ is not meaningful in low SNR condition. Thus, we merely show the simulation result for $SNR = 0 - 10dB$. From Fig 4.10, the ratio of $\sigma(\gamma'/k_1)$ to $\sigma(\gamma'/k_2)$

is pretty close to the expected value of $\frac{15}{56}$. Recall that a wrong estimation occurs once the deviation is over $\frac{1}{2}\cdot\frac{2\pi}{N}(0.176^{o})$, the correct probability of the 4th estimation is much better than that of the 3rd.

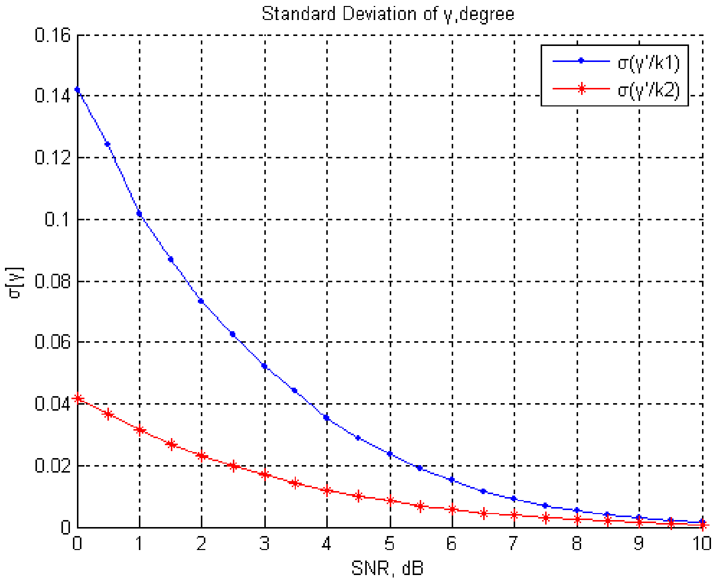

Fig4.10: The standard deviation of $\frac{\gamma'}{k_1}$ and $\frac{\gamma'}{k_2}$

Finally, we present the correct probabilities in detecting the code phase with the recursive algorithm with $n = 3,4,5,6$ in Fig. 4.11. As shown in the figure, the correct probability is enhanced with the recursive algorithm. Since the recursive algorithm is based on the prior estimation, we can’t increase the correct probability even if we increase the number of iterations in low SNR cases. The result reveals that a more accurate estimation method is needed for the 1st estimation when SNR is low.

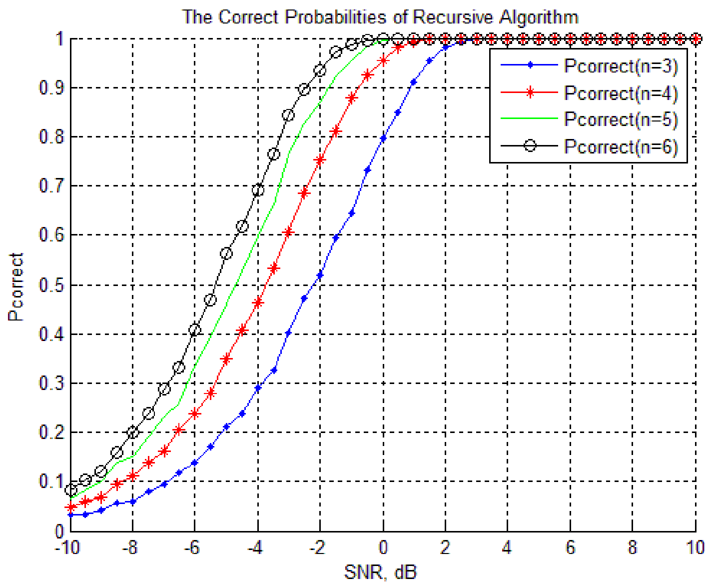

Fig. 4.11: The correct probabilities of recursive algorithm n=3,4,5,6

Chapter 5
Conclusions

In this thesis, we propose an effective method to achieve code synchronization. In Chapter 2, we introduce basic principle of the FFT method to achieve code synchronization. By performing FFT, the input sequence is projected to the phasor domain, where we get X_i. Then we compare the phase of X_i with the local phasor Y_i. It is found that there is a regular phase difference within which the code phase q is embedded. Thus we define a factor V_i as the inner product of X_i and Y_i, and it contains information of q. However, due to the inherent phase ambiguity, we could not interpret the code phase q directly from V_i. Furthermore, we employ the phasor concept and perform analysis to understand the statistic properties of these phasors.

In general, the sequence length N is a large number. It is known that the computation of FFT and IFFT is in the order of $N \cdot \log N$, which means considerable computation is required when the sequence length is long. Therefore, we intend to reduce the computation by using the phase relationship between phasors. By the mathematic analysis, we find every term of V_i contains information of q, but it can't be used to get q directly due to inherent phase ambiguity. If we can solve the phase ambiguity problem, the computation caused by IFFT can be saved. Then, by observing the phase of inner product of V_i and V_{i+1}, we find it contains information of q without phase ambiguity in noiseless condition. However in noisy condition, the estimation will be strongly affected by noise, which leads to erroneous estimation. In order to improve it, we propose a simple solution to reduce error phase by using the **Law of Large Number**.

Because our estimation chooses the most possible phase among N legal phase, if the phase deviation is larger than $\frac{2\pi}{N}$, a wrong estimation occurs. From the simulation result, even a tiny phase deviation may lead to incorrect value of q. Thus, we propose an improved method to further reduce error phase. By referring to the simple estimation before, we can get the approximated phase angel of $V_i^*V_{i+k}$. Likewise, when all the terms $V_i^*V_{i+k}$ are summed, we can obtain a phase whose value is k multiple of $\frac{2\pi}{N}q$, and it has the same noise variance as $V_i^*V_{i+1}$. Then the variance of error phase is divided by k so that a more accurate estimate is got. Note that the idea of the improved method can be recursively operated to get more accurate estimation. However, the more estimation the more computation required, so the number of calculations in the recursive algorithm should depend on the SNR.

In Chapter 3 we also design a method to overcome the problem occurring when q is small. It is equivalent to shifting the estimated phase angle to a large value such that the multiplication factor k will not be too large. This method does not reduce noise variance, but requires additional computation. Thus, it is not a necessary process, which is applied only when the estimated phase is close to zero.

In chapter 4, we demonstrate our theory by simulation under different SNR conditions. By analyzing the performance in each step of estimation, we interpret the simulation results and try to overcome the difficulties when SNR is low. Since our method is based on the prior estimation, which means the error may propagate through the process, the only way to enhance the accuracy in low SNR is doing more estimations and costing more computation.

In the future, we may design a more accurate method for the 1st estimation, which will improve the performance of the recursive algorithm in low SNR conditions. Moreover, when SNR is known, we can determine the optimum k_n so as to have the least computation. This would save much computation in high SNR conditions.

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