Reduction of Transfer Functions from the Stability-Equation Method and Complex Curve Fitting

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ABSTRACT: *In this paper, the reduction method uses the concepts of stability-equation and important poles to find the denominator of the reduced model. Then the numerator of the reduced model is found by complex curve fitting. This method tends to simultaneously guarantee a stable reduced model from a stable system and obtain a satisfactory result, since it considers the distribution of important poles. Examples are presented to illustrate this advantage.*

I. Introduction

The simplification of a linear dynamic system is often desirable and sometimes necessary in the analysis and design of a complex system. Several simplification methods exist in frequency domain. The approximation method of Chen and Shieh (**1**) is a continued fraction expansion method. Vittal-Rao and Lamba (**2**) present a method based on the least squares approximation of the frequency response data, and this method is simplified by Reddy (**3**). Several methods have also been developed for obtaining a guaranteed stable low-order system if the original high-order system is stable. Hutton and Friedland (**4**) use the stability criterion of Routh for obtaining the reduced model. Appiah (**5**) employs the Hurwitz polynomial approximant as characteristic polynomials and the partial Padé approximation to guarantee a stable reduced-model. These two methods are equivalent (**6**) and called the *Routh–Hurwitz method.* It is also pointed out that if the dominant poles are not closest to the origin, the Routh–Hurwitz approximation fails to produce a good lower order model. Apparently the stability-equation method (**7–9**) also suffers from the same drawback because far-off poles and/or zeros of the stability equation of the characteristic polynomial are discarded. To overcome this defect, a generalization of the Routh method is developed by Shamash

(**10**) to obtain several different reduced models. All methods considering the stability problem mentioned above are developed by first approximating the characteristic polynomial followed by finding the numerator of the simplified transfer function. Different approaches in consideration of the stability problem are Chebyshev polynomial techniques (**11**, **12**) which deal with the stability problem from the whole transfer function, then the defect mentioned by Shamash (**6**) is eliminated. However, these methods may fail due to the existence of pure imaginary poles after the transformation from ζ plane (defined by Langholz and Bistritz) to s plane. Recently, a method similar to the stability-equation method was given by Wan (**13**) using the Mihailov criterion and the Padé approximation technique.

From (**7–9**, **13**), it has been shown that the reduction methods based on stability-equation theory are simple and powerful. In this paper, a method for model reduction is developed from the concept of stability-equation; important poles and complex curve fitting to avoid the defect were discussed by Shamash (**6**). This method guarantees a stable reduced model if the original system is stable.

II. Properties of Reducing a Hurwitz Polynomial from Stability-Equation Method

Consider a Hurwitz polynomial

$$M(s) = a_0 + a_1 s + \cdots + a_n s^n. \tag{1}$$

The stability equations of $M(s)$ can be written as (**7–9**)

$$M_e(s) = a_0 \prod_{i=1}^{l_1} (1 + s^2/z_i^2) \tag{2}$$

and

$$M_0(s) = a_1 s \prod_{i=1}^{l_2} (1 + s^2/p_i^2) \tag{3}$$

where l_1 and l_2 are the integer parts of $n/2$ and $(n-1)/2$, respectively, and

$$z_1^2 < p_1^2 < z_2^2 < p_2^2 < z_3^2 < p_3^2 < \cdots \tag{4}$$

Then the reduced Hurwitz polynomial $M(s)$ can be obtained by discarding the factor with larger magnitudes of p_i^2 or z_i^2 (**7–9**) as

$$\begin{aligned} M'(s) &= M_e'(s) + M_0'(s) \\ &= a_0' + a_1' s + a_2' s^2 + \cdots + a_{n'}' s^{n'}, \quad n' < n \end{aligned} \tag{5}$$

where

$$M_e'(s) = a_0 \prod_{i=1}^{m_1} (1 + s^2/z_i^2) \tag{6}$$

$$M_0'(s) = a_1 s \prod_{i=1}^{m_2} (1 + s^2/p_i^2) \tag{7}$$

and m_1 and m_2 are the integer parts of $n'/2$ and $(n'-1)/2$, respectively.

Property 1: $M'(s)$ is approximated to $M(s)$ as $s \to 0$. It is easily seen that the factor $(1 + s^2/z_i^2)$ or $(1 + s^2/p_i^2)$ is less significant as $s \to 0$ if z_i^2 or p_i^2 is of larger magnitude. Then $M'(s)$ obtained by discarding these factors with larger magnitudes of p_i^2 or z_i^2 is to approximate $M(s)$ as $s \to 0$.

Property 2: The roots of $M'(s)$ tend to be approximated to the n' least magnitude roots of $M(s)$. From experience, this phenomenon becomes more apparent if the magnitudes of the $n - n'$ largest magnitude roots of $M(s)$ are much larger than the magnitudes of the n' least magnitude roots of $M(s)$. This property can be stated as follows. Since $M(s)$ can be expressed as

$$M(s) = a_0 \prod_{i=1}^{n} (1 + s/x_i) \tag{8}$$

where $-x_i$ are roots of $M(s)$ which may be complex numbers. Another way of approximating the $M(s)$ as $s \to 0$ can also be obtained by deleting the terms $(1 + s/x_i)$ with the $n - n'$ largest magnitudes of x_i. Thus, the reduced polynomial is

$$M''(s) = a_0 \prod_{i=1}^{n'} (1 + s/x_i). \tag{9}$$

Hence

$$M' \approx M''(s) \tag{10}$$

as $s \to 0$.

Property 3: If the roots of $M'(s)$ tend to be approximated to the n' largest magnitude roots of $M(s)$, $M'(s)$ can still be obtained by the stability-equation method. It can be done by first finding the reciprocal polynomial $M_r(s)$ of $M(s)$ defined by

$$M_r(S) = a_0 S^n + a_1 S^{n-1} + \cdots + a_{n-1} S + a_n \tag{11}$$

and then reducing $M_r(S)$ from the stability-equation method to obtain

$$M_r'(S) = c_{n'} S^{n'} + c_{n'-1} S^{n'-1} + \cdots + c_0. \tag{12}$$

Finally, the reduced polynomial is found by finding the reciprocal polynomial of $M_r'(S)$ as

$$M'(s) = c_{n'} + c_{n'-1} s + \cdots + c_0 s^{n'}. \tag{13}$$

It is because the reciprocal transformation transforms a small value of s to a large value of S and *vice versa*.

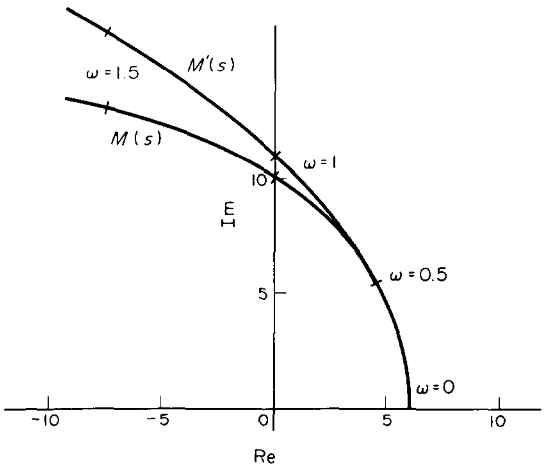

FIG. 1. The loci of $M(s)$ and $M'(s)$ in Example 1.

Example 1

Consider the following polynomial

$$M(s) = 6 + 11s + 6s^2 + s^3. \quad (14)$$

The roots of $M(s)$ are -1, -2 and -3. If $n' = 2$, $M'(s)$ in (5) can be obtained as

$$M'(s) = 6 + 11s + 6s^2. \quad (15)$$

The loci of $M(s)$ and $M'(s)$ are shown in Fig. 1. It is easily seen that $M'(s) \approx M(s)$ if $\omega < 0.5$. This is as stated in Property 1. The roots of $M'(s)$ are $= 0.9167 \pm 0.3997j$. Although these two roots are not close to -1 and -2, there is a tendency to depart from the largest magnitude root -3.

Example 2

Consider the following polynomial

$$M(s) = 1000000 + 1010100s + 10101s^2 + s^3. \quad (16)$$

The roots of $M(s)$ are -1, -100 and -10000. If $n' = 2$, the reduced polynomial obtained from (5) is

$$M'(s) = 1000000 + 1010100s + 10101s^2. \quad (17)$$

The roots of the above polynomial are -1 and -99 which are close to the two least magnitude roots of $M(s)$. In this case, the magnitude of -10000 is much larger than the magnitude of -1 or -100. If the roots of $M'(s)$ are to approximate the larger magnitude roots of $M(s)$, then $M'(s)$ in (13) is obtained as

$$M'(s) = 1010100 + 10101s + s^2. \tag{18}$$

The roots of the above polynomial are -101.01 and -9999.99 which are indeed close to the two largest magnitude roots of $M(s)$.

III. Model Reduction Based on Complex Curve Fitting and Stability Equation Methods

Consider a linear time-invariant system having the transfer function

$$H(s) = \frac{b_0 + b_1 s + \cdots + b_{n-1} s^{n-1}}{a_0 + a_1 s + \cdots + a_n s^n} = \frac{N(s)}{M(s)}. \tag{19}$$

The problem of model reduction is to approximate the transfer function expressed in (19) by a lower order transfer function

$$H'(s) = \frac{b'_0 + b'_1 s + \cdots + b'_{n'-1} s^{n'-1}}{a'_0 + a'_1 s + \cdots + a'_{n'} s^{n'}} = \frac{N'(s)}{M'(s)}, \quad n' < n \tag{20}$$

where a'_i and b'_i are to be determined, while keeping the response of (20) as close to that of (19) as possible.

In this section a procedure for reducing a model while preserving its stability property is given. This procedure consists of three steps: First, test for the relative importance of the poles of $H(s)$; Second, find $M'(s)$, denominator of the reduced transfer function $H'(s)$ as defined by (20), based on the stability-equation method; Third, determine $N'(s)$, numerator of $H'(s)$, via minimization of the frequency response errors between the original and the reduced models. The procedure is described in detail as follows.

Step 1. Test for the relative importance of poles

Express a given transfer function $H(s)$ by partial fraction expansion as follows:

$$H(s) = \sum_{i=1}^{n_0} \frac{A_i}{(s+p_i)} + \sum_{i=1}^{r_1} \frac{B_i}{(s+p_{n_0+1})^i} + \sum_{i=1}^{r_2} \frac{C_i}{(s+p_{n_0+2})^i} + \cdots + \sum_{i=1}^{r_l} \frac{D_i}{(s+p_{n_0+l})^i} \tag{21}$$

where $n_0 + r_1 + r_2 + \cdots + r_l = n$. We define the significance measure of each individual term in (21) by its steady-state response at any specified frequency. For the special case where a transfer function exhibits low-pass characteristic, as most control systems do, the dc steady-state response of each term in (21) can be taken as the significance measure. Thus the relative importance of all individual terms in (21) is reflected by the set

$$S=\left\{\left|\frac{A_1}{p_1}\right|,\left|\frac{A_2}{p_2}\right|,\ldots,\left|\frac{A_{n_0}}{p_{n_0}}\right|,\left|\frac{B_1}{p_{n_0+1}}\right|,\ldots,\left|\frac{B_{r_1}}{p^{r_1}_{n_0+1}}\right|,\ldots,\left|\frac{D_1}{p_{n_0+l}}\right|,\ldots,\left|\frac{D_{r_l}}{p^{r_l}_{n_0+l}}\right|\right\}. \tag{22}$$

For example, if $|A_2/p_2|$ has the largest magnitude in S, then the term $A_2/s+p_2$ is the most important term and $-p_2$ is considered the most important pole. If $|B_r/p^r_{n_0+1}|$, $1\leq r\leq r_1$, has the next largest magnitude in S, then the term $B_r/(s+p_{n_0+1})^r$ is the next important term and $-p_{n_0+1}$ with multiplicity r are considered the next important poles, etc. The first n' important poles of $H(s)$ can be found easily in this manner. Based on this test, we know how the n' important poles are distributed. The n' important poles may be the least magnitude roots of $M(s)$, or the largest magnitude roots of $M(s)$, or the poles which are neither the least magnitude nor the largest magnitude roots of $M(s)$, or both the least and largest magnitude roots of $M(s)$.

Step 2. *Determine* M'(s)

Based on the distribution of the n' important poles, $M'(s)$ can be found as follows:

(i) If n' important poles are the least magnitude roots of $M'(s)$ then $M(s)$ can be found by using the stability-equation method (**7–9**) directly.

(ii) If n' important poles are the largest magnitude roots of $M(s)$, then $M'(s)$ can be found in two steps. First we find the reciprocal polynomial of $M(s)$ by (11). Then the stability-equation method can be applied to approximate $M_r(S)$ by an n'-th degree polynomial $M'_r(S)$. Then $M'(s)$ is taken to be the reciprocal polynomial of $M'_r(S)$.

(iii) If the n' important poles do not belong to the above mentioned distribution types, the methods of (i) and (ii) can still be applied if we separate the system $H(s)$ into two (or more) subsystems as

$$H(s)=H_1(s)+H_2(s),\ H_1(s)\triangleq N_1(s)/M_1(s),\ H_2\triangleq N_2(s)/M_2(s) \tag{23}$$

where $H_1(s)$ has n'_1 important poles which are the least magnitude poles of $H_1(s)$ and $H_2(s)$ has n'_2 important poles which are the largest magnitude poles of $H_2(s)$, where $n'_1+n'_2=n'$. Then the reduced polynomial $M'_1(s)(M'_2(s))$ of order $n'_1(n'_2)$ can be obtained from method (i) (method (ii)). Finally, $M'(s)$ is obtained from the product of $M'_1(s)$ and $M'_2(s)$.

Remark: If either $n'_1=0$ or $n'_2=0$, temporary reduction of the model $H(s)$ is required before obtaining the n'-th order reduced model. It can easily be seen from Example 5.

Step 3. *Determine* N'(s)

Normalize coefficients a'_0 in $H'(s)$ and a_0 in $H(s)$ to $a'_0=a_0$; thus we have

$$\frac{(-b'_2\omega_i^2+b'_4\omega_i^4-\cdots)+j(b'_1\omega_i-b'_3\omega_i^3+\cdots)}{M'(j\omega_i)}=\frac{-b'_0}{M'(j\omega_i)}+H(j\omega_i)+\epsilon_i \tag{24}$$

where ϵ_i is the frequency response error between actual model $H(j\omega_i)$ defined by (19) and the reduced model $H'(j\omega_i)$ defined by (20). By setting $b'_0 = b_0$ for consistent d.c. steady-state response and comparing the imaginary and real parts on both sides of (24), we have

$$\frac{\text{Re}\,[M'(j\omega_i)]}{|M'(j\omega_i)|^2}(-b'_2\omega_i^2 + b'_4\omega_i^4 - \cdots) + \frac{\text{Im}\,[M'(j\omega_i)]}{|M'(j\omega_i)|^2}(b'_1\omega_i - b'_3\omega_i^3 + \cdots)$$
$$= \frac{-\text{Re}\,[M'(j\omega_i)]b_0}{|M'(j\omega_i)|^2} + \text{Re}\,[H(j\omega_i)] + \text{Re}\,[\epsilon_i] \qquad (25)$$

$$\frac{\text{Im}\,[M'(j\omega_i)]}{|M'(j\omega_i)|^2}(b'_2\omega_i^2 - b'_4\omega_i^4 + \cdots) + \frac{\text{Re}\,[M'(j\omega_i)]}{|M'(j\omega_i)|^2}(b'_1\omega_i - b'_3\omega_i^3 + \cdots)$$
$$= \frac{\text{Im}\,[M'(j\omega_i)]b_0}{|M'(j\omega_i)|^2} + \text{Im}\,[H(j\omega_i)] + \text{Im}\,[\epsilon_i]. \qquad (26)$$

Using the frequency response data of $H(s)$ at k different frequencies ω_i, $i = 1, 2, \ldots, k$, we obtain from (25) and (26) $2k$ linear equations which can be arranged in the following form:

$$X\boldsymbol{\theta} = \boldsymbol{\beta} + \mathbf{e} \qquad (27)$$

where

$$X = \begin{bmatrix} \frac{\text{Im}\,[M'(j\omega_1)]}{M'(j\omega_1)|^2}\omega_1 & \frac{-\text{Re}\,[M'(j\omega_1)]}{|M'(j\omega_1)|^2}\omega_1^2 & \frac{-\text{Im}\,[M'(j\omega_1)]}{|M'(j\omega_1)|^2}\omega_1^3 & \frac{\text{Re}\,[M'(j\omega_1)]}{|M'(j\omega_1)|^2}\omega_1^4 \cdots \\ \frac{\text{Re}\,[M'(j\omega_1)]}{|M'(j\omega_1)|^2}\omega_1 & \frac{\text{Im}\,[M'(j\omega_1)]}{|M'(j\omega_1)|^2}\omega_1^2 & \frac{-\text{Re}\,[M'(j\omega_1)]}{|M'(j\omega_1)|^2}\omega_1^3 & \frac{-\text{Im}\,[M'(j\omega_1)]}{|M'(j\omega_1)|^2}\omega_1^4 \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\text{Im}\,[M'(j\omega_k)]}{|M'(j\omega_k)|^2}\omega_k & \frac{-\text{Re}\,[M'(j\omega_k)]}{|M'(j\omega_k)|^2}\omega_k^2 & \frac{-\text{Im}\,[M'(j\omega_k)]}{|M'(j\omega_k)|^2}\omega_k^3 & \frac{\text{Re}\,[M'(j\omega_k)]}{|M'(j\omega_k)|^2}\omega_k^4 \cdots \\ \frac{\text{Re}\,[M'(j\omega_k)]}{M'(j\omega_k)|^2}\omega_k & \frac{\text{Im}\,[M'(j\omega_k)]}{|M'(j\omega_k)|^2}\omega_k^2 & \frac{-\text{Re}\,[M'(j\omega_k)]}{|M'(j\omega_k)|^2}\omega_k^3 & \frac{-\text{Im}\,[M'(j\omega_k)]}{|M'(j\omega_k)|^2}\omega_k^4 \cdots \end{bmatrix}$$

$$\boldsymbol{\theta} = [b'_1\ b'_2\ b'_3 \ldots b'_{n'-1}]^T$$

$$\boldsymbol{\beta} = \Bigg[\frac{-\text{Re}\,[M'(j\omega_1)]b_0}{|M'(j\omega_1)|^2} + \text{Re}[H(j\omega_1)]\ \frac{\text{Im}\,[M'(j\omega_1)]b_0}{|M'(j\omega_1)|^2} + \text{Im}\,[H(j\omega_1)] \ldots$$
$$\frac{-\text{Re}[M'(j\omega_k)]b_0}{|M'(j\omega_k)|^2} + \text{Re}[H(j\omega_k)]\ \frac{\text{Im}\,[M'(j\omega_k)]b_0}{|M'(j\omega_k)|^2} + \text{Im}\,[H(j\omega_k)]\Bigg]^T$$

and

$$\mathbf{e} = [\text{Re}\,[\epsilon_1]\ \text{Im}\,[\epsilon_1] \ldots \text{Re}\,[\epsilon_k]\ \text{Im}\,[\epsilon_k]]^T.$$

With the obtained values of $M'(j\omega_i)$, the coefficient vector $\boldsymbol{\theta}$ can be found by ordinary least-squares method as

$$\boldsymbol{\theta} = (X^T X)^{-1} X^T \boldsymbol{\beta}. \tag{28}$$

Thus the transfer function $H'(s)$ of the reduced model is obtained.

Example 3

Consider the system (**2, 3, 7, 8**)

$$H(s) = \frac{1}{s^3 + 6s^2 + 11s + 6} \tag{29}$$

and choose the order of the reduced model as $n' = 2$. Express (29) as

$$H(s) = \frac{0.5}{s+1} - \frac{1}{s+2} + \frac{0.5}{s+3}. \tag{30}$$

It is easily seen that the two important poles of $H(s)$ are -1 and -2 which are the least magnitude roots of $M(s)$. As discussed in (i) of step 2 in this section, we obtain $M'(s) = 6 + 11s + 6s^2$ by applying the stability-equation method. Then based on the 21 frequency response data generated from $H(s)$ at $\omega_1 = 0, 0.25, 0.5, \ldots, 5.0$, we obtain $N'(s)$ from (28) so that the reduced model is

$$H'(s) = \frac{0.1667 - 0.0141s}{1 + 1.8333s + s^2}. \tag{31}$$

Several models from other reduction methods are also given below for comparison:

$$H'(s) = \frac{0.1667 - 0.0278s}{1 + 1.6667s + 0.6944s^2} \text{ by Chen and Shieh (1970)} \tag{32}$$

$$H(s) = \frac{1}{6 + 11s + 6s^2} \text{ by Chen } et\ al. \text{ (1980)} \tag{33}$$

$$H'(s) = \frac{11}{66 + 121s + 60s^2} \text{ by Shamash (1980).} \tag{34}$$

The Chebyshev polynomial expansion method (12) can not be applied to this example over the frequency interval of 0 to 5 rad/sec due to the existence of a pair of pure imaginary poles after transformation from ζ to s plane. If Wan's method is used to minimize

$$\int_0^{\omega_f} [\text{amp}\,(H(j\omega)) - \text{amp}\,(H'(j\omega))]^2\, d\omega \tag{35}$$

where $\omega_f = 5$, the reduced model is

$$H'(s) = \frac{1-0.1631s}{6+10.021+6s^2} \text{ by Wan (1981).} \tag{36}$$

If the same frequency interval is considered, both the Vittal Rao and Lambas' and the Reddy's methods result in unstable models as

$$H'(s) = \frac{0.1667(1-0.457s)}{1-7.011s+1.522s^2} \text{ by Vittal Rao and Lamba (1974)} \tag{37}$$

$$H'(s) = \frac{0.1667}{1-0.6667s+s^2} \text{ by Reddy (1976).} \tag{38}$$

The frequency responses of the original system and the above mentioned reduced models are plotted in Fig. 2. It should be noted that the best reduced model of (29) is obtained from the Chen and Shiehs' method. However, this method may result in unstable models if the original system is stable (**9**).

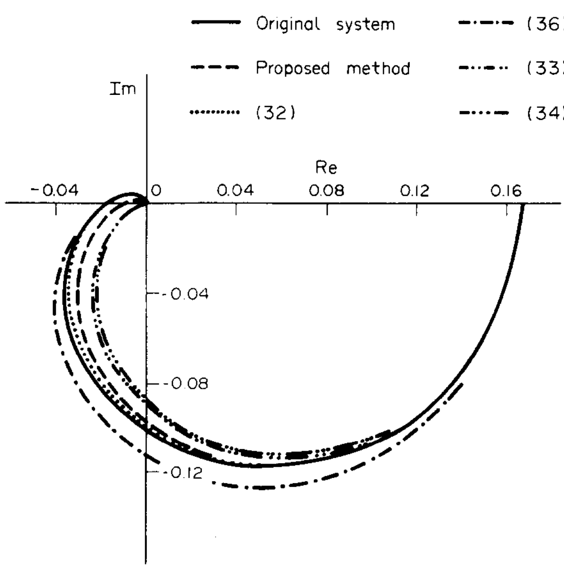

FIG. 2. The frequency responses of the original and the reduced models in Example 3.

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Example 4

Consider the following system (10)

$$H(s)=\frac{8169.13375s^3+50664.96749s^2+9984.32343s+500}{100s^4+10520s^3+52101s^2+10105s+500}. \qquad (39)$$

If n' is selected as 2, then the important poles can be found to be the largest magnitude poles of $H(s)$. As discussed in (ii) of step 2 in this section, $M'(s)$ is obtained as $M'(s)=100s^2+10520s+52100.0403$. Then based on 101 frequency response data generated from (39) at $\omega_i=0, 2, 4, \ldots, 200$, the reduced model is obtained as

$$H'(s)=\frac{8166.0459s+52100.0403}{100s^2+10520s+52100.0403}. \qquad (40)$$

For comparison, the models obtained from other reduction methods are given below:

$$H'(s)=\frac{10105.24135s+500}{52100s^2+10105s+500} \text{ by Chen et al. (1980)} \qquad (41)$$

$$H'(s)=\frac{81.691s+520.048}{s^2+105.2s+520.048} \text{ by Shamash (1980).} \qquad (42)$$

If Wan's method is used to minimize (35) for $\omega_f=200$, the reduced model is

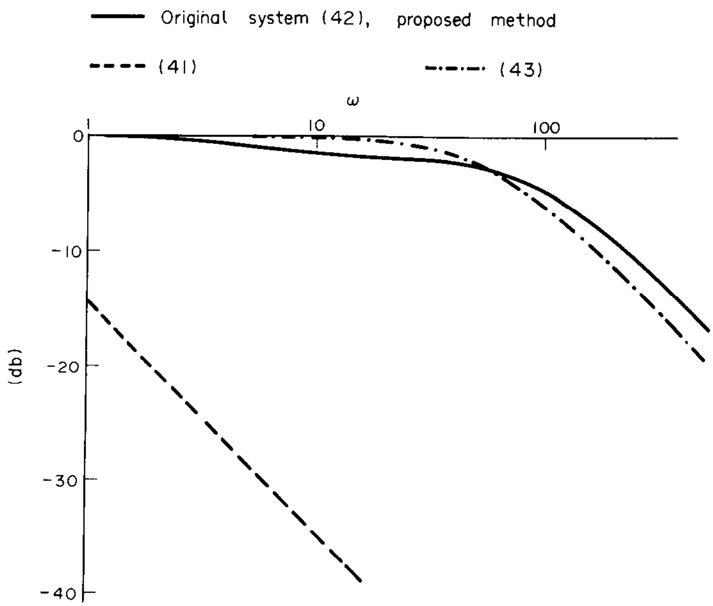

FIG. 3. The frequency responses of the original and the reduced models in Example 4.

obtained as:

$$H'(s) = \frac{3070120.676575s + 500}{52100s^2 + 3070000s + 500} \text{ by Wan (1981).} \tag{43}$$

The frequency response of the original system and the reduced models are shown in Fig. 3.

Example 5

Consider the following system

$$H(s) = \frac{9.01s^2 + 1000.1s + 1000}{s^3 + 111s^2 + 1110s + 1000} \tag{44}$$

$$= \frac{0.01}{s+1} + \frac{10}{s+10} - \frac{1}{s+100}. \tag{45}$$

If the order of the reduced model is selected to be $n' = 1$, the important pole -10 obtained is neither the least nor the largest magnitude pole of $H(s)$. Following the procedure discussed in (iii) of step 2, we express $H(s)$ as

$$H(s) = H_1(s) + H_2(s) = \frac{10.01s + 10.1}{s^2 + 11s + 10} - \frac{1}{s+100}. \tag{46}$$

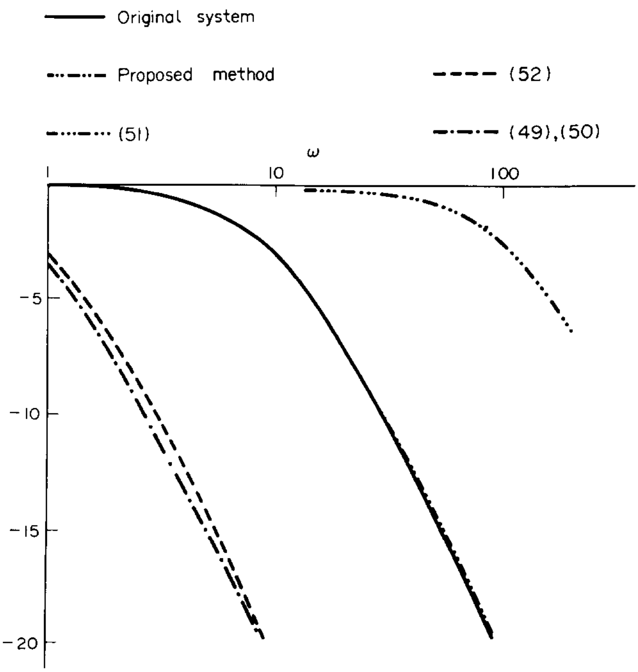

FIG. 4. The frequency responses of the original and the reduced models in Example 5.

Since $H_1(s)$ has the important pole at largest magnitude pole of $H_1(s)$, the first order approximation of $H_1(s)$ is obtained following (ii) of step 2 and step 3 as

$$H_1'(s) = \frac{11.11}{s+11}. \tag{47}$$

Now, comparing $H_1'(s)$ and $H_2(s)$, the important pole is the least magnitude pole of $H_1'(s)+H_2(s)$. Thus, according to (i) of step 2 and step 3, the reduced first order model of $H(s)$ is obtained by reducing the temporarily reduced model $H_1'(s)+H_2(s)$ as

$$H'(s) = \frac{1100}{111s+1100}. \tag{48}$$

For comparison, the first order reduced models from other methods which consider the stability property are listed below.

$$H'(s) = \frac{1000}{1110s+1000} \text{ by Chen } et\ al. \text{ (1980)} \tag{49}$$

$$\left.\begin{array}{ll} H'(s) = \dfrac{1000}{1110s+1000} & (50) \\ H'(s) = \dfrac{111}{s+111} & (51) \end{array}\right\} \text{ by Shamash (1980)}$$

$$H'(s) = \frac{1}{s+1} \text{ by Wan (1981).} \tag{52}$$

The frequency responses of the original and the reduced models are plotted in Fig. 4. It is easy to see that the proposed method provides a superior result.

IV. Conclusion

Since the stability-equation method for reducing a Hurwitz polynomial has been proved to be a simple and powerful technique, the reduction method discussed in this paper is based on this method. We first discuss the properties of using the stability-equation method to reduce a Hurwitz polynomial. Then we use these properties and complex curve fitting for model reduction. This approach can be considered as an extension of the stability-equation method by considering the original pole distribution. The advantages of the proposed method are: (1) all the reduced models are stable if the original system is stable; (2) the reduced system can be emphasized over a desired frequency interval via complex curve fitting. It is also clear that the proposed method can be modified to simplify a discrete-time system by the use of bilinear transformation (7).

References

(1) C. F. Chen and L. S. Shieh, "An algebraic method for control system design," *Int. J. Contr.*, Vol. 11, pp. 717–739, 1970.
(2) S. Vittal-Rao and S. S. Lamba, "A new frequency domain technique for the simplification of linear dynamic systems," *Int. J. Contr.*, Vol. 20, pp. 71–79, 1974.
(3) A. S. S. R. Reddy, "A method for frequency domain simplification of transfer functions," *Int. J. Contr.*, Vol. 23, pp. 403–408, 1976.
(4) M. F. Hutton and B. Friedland, "Routh approximation for reducing order of linear, time-invariant systems," *IEEE Trans. Automat. Contr.*, Vol. AC-20, pp. 329–337, 1975.
(5) R. K. Appiah, "Padé methods of Hurwitz polynomial approximation with application to linear system reduction," *Int. J. Contr.*, Vol. 29, pp. 39–48, 1979.
(6) Y. Shamash, "Failure of the Routh–Hurwitz method of reduction," *IEEE Trans. Automat. Contr.*, Vol. AC-25, pp. 313–314, 1980.
(7) T. C. Chen, C. Y. Chang and K. W. Han, "Reduction of transfer functions by the stability-equation method," *J. Franklin Inst.*, Vol. 308, pp. 389–404, 1979.
(8) T. C. Chen, C. Y. Chang and K. W. Han, "Model reduction using the stability-equation method and the Padé approximation method," *J. Franklin Inst.*, Vol. 309, pp. 473–490, 1980.
(9) T. C. Chen, C. Y. Chang and K. W. Han, "Model reduction using the stability-equation method and the continued-fraction method," *Int. J. Contr.*, Vol. 32, pp. 81–94, 1980.
(10) Y. Shamash, "Stable biased reduced order models using the Routh method of reduction," *Int. Syst. Sci.*, Vol. 11, pp. 641–654, 1980.
(11) Y. Bistritz and G. Langholz, "Model reduction by Chebyshev polynomial techniques," *IEEE Trans. Automat. Contr.*, Vol. AC-24, pp. 741–747, 1979.
(12) G. Langholz and Y. Bistritz, "Model reduction of dynamic systems over a frequency interval," *Int. J. Control*, Vol. 31, pp. 51–62, 1980.
(13) B. W. Wan, "Linear model reduction using Mihailov criterion and Padé approximation technique," *Int. J. Contr.*, Vol. 33, pp. 1073–1089, 1981.