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Effect of non-parabolic band structure on amplification of surface phonons in n-type InSb films†

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Received 2 December 1981

Abstract. The effect of non-parabolic band structure of surface phonons in n-type InSb films has been investigated quantum mechanically in the GHz frequency region. Numerical results show that the amplification coefficient is enhanced for the non-parabolic band structure due to the non-linear nature of the energy band in semiconductors. Moreover, the amplification coefficient depends also on the temperature, the piezoelectric coupling mechanisms, the frequency of sound waves, the electric field, and the thickness of the semiconductor film.

1. Introduction

Elastic surface waves have been the subject of considerable interest in surface physics, since they can be used as probes for understanding the properties of solids near the surfaces, such as surface electronic states (Prange 1969) and surface irregularities (Steg and Klemens 1970, 1974, Sakuma 1972, 1973, De Vries and Miller 1972, Nakayama and Sakuma 1975, 1976, Maradudin and Mills 1976, Nakayama 1976). The interaction of elastic surface waves with conduction electrons in a piezoelectric semiconductor provides a useful tool to study the electronic band structure of matter. Acoustic waves can be propagated along the boundary of an elastic half-space (Ezawa 1971), the amplitude of which falls off rapidly as one goes away from the surface. Such elastic excitations are called Rayleigh waves (Grishin and Kaner 1972). The surface phonons are the quanta of elastic waves that satisfy the proper boundary condition on solid surfaces (Ezawa 1971). They are to be used in place of the bulk phonons when one is dealing with surface phenomena. In piezoelectric semiconductors, the interaction of surface phonons with conduction electrons is dominated by the deformation potential and piezoelectric fields. The deformation of the crystal due to the surface phonons determines directly the deformation potential force acting on the conduction electrons. On the other hand the piezoelectric field, which is due to a polarisation of the medium, can only be found by a self-consistent solution involving Maxwell's equations. The interaction of surface phonons with conduction electrons in piezoelectric semiconductors for the parabolic band structure has been investigated in the intermediate–high-frequency region by Tamura and Sakuma (1977a, b). They calculated and discussed the amplification coef-

† Partially supported by National Science Council of China in Taiwan.

ficients of Rayleigh waves using the Born approximation and the approximation including the effect of the finite relaxation time phenomenologically. This effect of the finite relaxation time reduces the amplification in the lower frequency region. However, their numerical results reflect the qualitatively correct features of the surface phonon amplification. Therefore, if we consider the high-frequency phonons for which the relation $\omega\tau > 1$ holds, where ω is the angular frequency of the surface phonons and τ is the relaxation time of the carriers, we may safely neglect the effect of the relaxation time of the electronic states. In Tamura and Sakuma's calculations, it was assumed that the energy band structure in semiconductors is parabolic. It has been shown that the non-parabolicity of the band structure in piezoelectric semi-conductors can be used to explain the longitudinal magnetoacoustic phenomena (Wu and Spector 1971). Moreover, the discrete part of electronic energy levels is proportional to the square of the level quantum number. Thus, when we take the summation over the higher quantum numbers, the electronic energy becomes comparable with the band gap energy of semiconductors. Consequently, the non-parabolic band model in the electronic energy surface would play an important role for the amplification of the surface phonons. In our present study, we wish to survey the qualitative features of the effect of non-parabolic band structure on the amplification coefficient of surface phonons in n-type InSb films using the quantum treatment. In the calculation of the amplification coefficient of surface phonons, we make the following assumptions:

- (i) the energy band structure of the piezoelectric semiconductor n-type InSb is non-parabolic;
- (ii) the media are elastically isotropic and the quasi-free-electron description of conduction electrons is valid;
- (iii) the interaction of the surface phonons and conduction electrons is via deformation-potential and piezoelectric couplings.

In § 2, we describe the configuration of the layered system of a piezoelectric semiconductor and an insulator which we shall use for determining the amplification characteristics and specify the eigenfunctions of conduction electrons for the non-parabolic band structure. In § 3, the calculation of the interaction Hamiltonian of the surface-phonon-conduction-electron system due to the deformation-potential and piezoelectric couplings is performed quantum mechanically. In § 4, we calculate the amplification coefficient using the Born approximation. In § 5, we present some numerical results of the amplification coefficient for the epitaxial layer of n-type InSb grown on a semi-insulating InSb substrate, and give a brief discussion.

2. Electronic states in a thin layer of semiconductors for non-parabolic band structure

The configuration of the amplifier for Rayleigh-wave amplification can be described as shown in figure 1. A thin layer with the thickness t of a piezoelectric semiconductor is grown epitaxially on an insulating substrate with the same elastic properties as the semiconductor layer (Sze 1981). Therefore, the same kind of semiconductor as the film but of semi-insulating character is a good candidate for the substrate. We fix the cartesian coordinates so that the material occupies the half-space $z \geq 0$ and has the stress-free surface parallel to the xy plane.

The motion of electrons parallel to the surface may be described by plane waves and those perpendicular to the surface will be described by some kind of standing waves

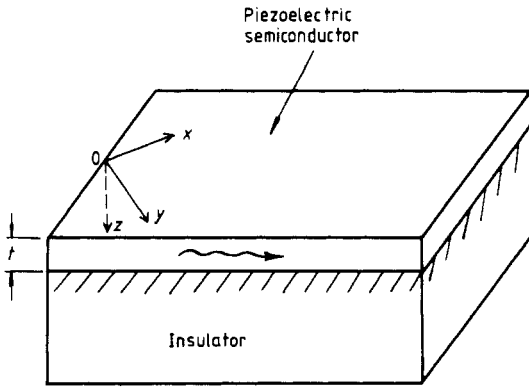


Figure 1. A thin layer with the thickness t of a piezoelectric semiconductor and an insulating material. Surface phonons propagate parallel to the surface of the film (xy plane).

depending on the structure of the potential. It is assumed that the potential along the z axis is a square well which has infinitely high potential barriers at $z = 0$ and $z = t$ neglecting possible depletion layers on both sides near $z = 0$ and $z = t$. Under this assumption, the field operator $\Psi(\mathbf{r})$ of conduction electrons in the second quantised form takes the form (Tamura and Sakuma 1977a, b)

$$\begin{aligned} \Psi(\mathbf{r}) &= S^{-1/2} \sum_{n=1}^{\infty} \sum_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{x}) \Phi_n(z) \\ &= (2/V)^{1/2} \sum_{n=1}^{\infty} \sum_{\mathbf{k}} b_{\mathbf{k},n} \exp(i\mathbf{k} \cdot \mathbf{x}) \sin(n\pi z/t), \end{aligned} \tag{1}$$

where $\mathbf{r} = (\mathbf{x}, z) = (x, y, z)$, $\mathbf{k} = (k_x, k_y)$, $V = tS$ is the volume of the film with a surface area S , $b_{\mathbf{k},n}$ and its Hermitian conjugate $b_{\mathbf{k},n}^\dagger$ are annihilation and creation operators of conduction electrons, respectively, satisfying the commutation relations of the Fermi type. The energies of the conduction electrons $E_{\mathbf{k},n}$ for the non-parabolic band structure are given by the relation

$$E_{\mathbf{k},n} \left(1 + \frac{E_{\mathbf{k},n}}{E_g} \right) = \frac{\hbar^2 k^2}{2m^*} + \frac{\pi^2 \hbar^2 n^2}{2m^* t^2} \quad n = 1, 2, 3, \dots \tag{2}$$

where E_g is the energy gap between the conduction and valence bands, m^* is the effective mass of conduction electrons. Since $(\hbar^2 k_{\max}^2)/2m^* + (\pi^2 \hbar^2 n^2)/2m^* t^2 \approx k_B T \ll E_g$ for semiconductors at low temperatures in which we are interested ($T < 100$ K), then equation (2) can be expanded as

$$E_{\mathbf{k},n} \approx -\frac{1}{2} E_g + \frac{1}{2} E_g a_n + \frac{\hbar^2 k^2}{2m^* a_n} \tag{3}$$

with

$$a_n = \left(1 + \frac{2\pi^2 \hbar^2 n^2}{m^* t^2 E_g} \right)^{1/2}. \tag{4}$$

If the thickness of the layer t is assumed to be $1 \mu\text{m}$, the term $(2\pi^2 \hbar^2 n^2)/(m^* t^2 E_g)^{-1}$ in

equation (4) is $5.8 \times 10^{-4} n^2$. Then the factor a_n coming from the non-parabolicity in energy bands becomes important when the quantum number of energy levels is larger than 20. However, if the electronic energy due to the quantised motion along the z direction is much smaller than the energy gap E_g , then equation (3) can be reduced to the energy of the electronic states for the parabolic band structure used by Tamura and Sakuma (1977a, b), i.e.,

$$E_{k,n} = \frac{\pi^2 \hbar^2 n^2}{2m^* t^2} + \frac{\hbar^2 k^2}{2m^*} \tag{5}$$

The quantum number of energy levels which contributes to the summation in the expression for the amplification coefficient in Tamura and Sakuma’s work is about 50. Thus the factor a_n will play an important role in calculating the amplification coefficient of surface phonons in semiconductor films. Therefore, the energy band structure of n-type InSb in our present work is assumed to be non-parabolic.

3. Conduction-electron and surface-phonon interaction in piezoelectric semiconductor films

‘Surface phonon’ defined in this paper is a quantum of the Rayleigh wave which is one kind of elastic surface waves in solids (Ezawa 1971, Tamura and Sakuma 1977b). In the configuration shown in figure 1, the isotropic, elastic continuum is filling up the half space ($z > 0$) with a stress-free plane boundary at $z = 0$. The surface-phonon field operator is written, using well known eigenfunctions for the Rayleigh wave, as

$$\mathbf{u}(\mathbf{r}) = \sum_{\mathbf{q}} \left(\frac{\hbar}{2\rho\omega S} \right)^{1/2} [a_{\mathbf{q}} \mathbf{u}_{\mathbf{q}}(z) \exp(i\mathbf{q} \cdot \mathbf{x}) + (\text{Hermitian conjugate})], \tag{6}$$

where ρ is the mass density of the medium, $\mathbf{q} = (q_x, q_y)$ is the wavevector of a surface phonon parallel to the surface, $\omega = c_R |\mathbf{q}| = c_R q$ is the angular frequency of the surface phonon, and c_R is the velocity of the Rayleigh wave. The operator $a_{\mathbf{q}}$ and its Hermitian conjugate $a_{\mathbf{q}}^\dagger$ are the annihilation and creation operators of the surface-phonon field, respectively, obeying the commutation relations of the Bose type. The explicit forms of the wavefunction $\mathbf{u}_{\mathbf{q}}(z)$ are (Ezawa 1971, Tamura and Sakuma 1977b)

$$u_{\mathbf{q}}^j(z) = i \left(\frac{q_j}{q} \right) \left(\frac{q}{J} \right)^{1/2} \left(\exp(-\gamma qz) - \frac{2\gamma\sigma}{1 + \sigma^2} \exp(-\sigma qz) \right) \quad j = (x, y) \tag{7}$$

$$u_{\mathbf{q}}^z(z) = -\gamma \left(\frac{q}{J} \right)^{1/2} \left(\exp(-\gamma qz) - \frac{2}{1 + \sigma^2} \exp(-\sigma qz) \right), \tag{8}$$

γ , σ , and J are the constants defined by the velocity of the longitudinal sound wave c_l and the velocity of the transverse sound wave c_t as

$$\gamma^2 = 1 - (c_R/c_l)^2, \tag{9}$$

$$\sigma^2 = 1 - (c_R/c_t)^2, \tag{10}$$

and

$$J = (\gamma - \sigma) (\gamma - \sigma + 2\gamma\sigma^2)/2\gamma\sigma^2. \tag{11}$$

In the presence of piezoelectric coupling, the sound velocity c is modified to c' as a

result of 'piezoelectric stiffening' (Hutson and White 1962),

$$c'^2 = c^2(1 + K^2), \quad (12)$$

if the conductivity of the medium is small. This is valid in piezoelectric semiconductors. Here K^2 is the electromechanical coupling constant defined by

$$K^2 = 4\pi\beta_p^2/\varepsilon_0c_{44}, \quad (13)$$

where β_p , c_{44} , and ε_0 are respectively the piezoelectric, elastic, and static dielectric constants of the material along an appropriate crystal axis. For n-type InSb, $\beta_p = 1.8 \times 10^4$ esu cm⁻², $c_{44} = 3.02 \times 10^{11}$ dyn cm⁻², and $\varepsilon_0 = 18$, then $K^2 = 7.4 \times 10^{-4}$. Therefore the correction for piezoelectric coupling to the sound velocity is small in our work and can be neglected.

In the piezoelectric semiconductor, the conduction electrons interact with surface phonons through the deformation potential which is proportional to the dilation caused by the acoustic field. These conduction electrons also interact with surface phonons through piezoelectricity. This induced electric field is proportional to the strain in the piezoelectric coupling. The deformation-potential coupling is known to be weaker than the piezoelectric coupling for surface phonons of low frequencies. However, at high phonon frequencies the deformation-potential coupling becomes dominant because of its stronger frequency dependence (Spector 1966). Consequently, we shall take these two coupling mechanisms into account simultaneously in our present study. The interaction Hamiltonian of the conduction-electron-surface-phonon system may be written as

$$H_I = H_D + H_P, \quad (14)$$

where H_D is the interaction Hamiltonian due to the deformation-potential coupling and H_P is the interaction Hamiltonian due to the piezoelectric coupling. For the deformation-potential coupling, H_D can be expressed by

$$\begin{aligned} H_D &= C \int \Psi^\dagger(\mathbf{r}) \nabla \cdot \mathbf{u}(\mathbf{r}) \Psi(\mathbf{r}) \, d\mathbf{r} \\ &= \frac{C}{\sqrt{S}} \sum_{n,n'} \sum_{\mathbf{k},\mathbf{q}} [b_{\mathbf{k}+\mathbf{q},n}^\dagger b_{\mathbf{k},n} a_{\mathbf{q}} \Phi_{nn'}^D(\mathbf{q}) + (\text{Hermitian conjugate})], \end{aligned} \quad (15)$$

where C is the deformation potential, and

$$\Phi_{nn'}^D(\mathbf{q}) = -\left(\frac{\hbar q^2}{2\rho c_R J}\right)^{1/2} \left(\frac{c_R}{c_l}\right)^2 \int_0^l \varphi_n^*(z) \exp(-\gamma q z) \varphi_n(z) \, dz. \quad (16)$$

In a piezoelectric material, the polarisation induced by applying a strain can be expressed by (Spector 1966)

$$D_i = \varepsilon_{ij} E_j + 4\pi\beta_{ijk} S_{jk}, \quad (17)$$

where D_i is the electric displacement, E_i is the electric field, ε_{ij} is the dielectric tensor, β_{ijk} is the piezoelectric tensor, and S_{ij} is the strain tensor which is written explicitly in terms of the displacement vector \mathbf{u} of the medium as

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial r_j} + \frac{\partial u_j}{\partial r_i} \right). \quad (18)$$

If the electrostatic approximation is valid, the electric field \mathbf{E} is derived from a scalar

potential φ_q ,

$$\mathbf{E} = -\nabla\varphi_q. \tag{19}$$

Within insulating crystals, Gauss’s law is expressed as

$$\nabla \cdot \mathbf{D} = 0. \tag{20}$$

Since we assume that the medium is isotropic in its elastic properties, that is $\epsilon_{ij} = \epsilon_0\delta_{ij}$ (ϵ_0 is the static dielectric constant), then, from equations (17), (19), and (20), we have the equation

$$\nabla^2\varphi_q = \frac{4\pi}{\epsilon_0} \beta_{ijk} \left(\frac{\partial S_{jk}}{\partial r_i} \right). \tag{21}$$

Using equations (7), (8), (18), and (21), the electric potential produced by the acoustic vibration of the surface phonon travelling with the wavevector \mathbf{q} can be obtained as

$$\varphi_q(\mathbf{r}) = \hat{\varphi}_q(z) \exp(i\mathbf{q} \cdot \mathbf{x}) \tag{22}$$

with

$$\hat{\varphi}_q(z) = \frac{4\pi\beta_P}{\epsilon_0} \left(\frac{q}{J} \right)^{1/2} \left(\frac{q_x q_y}{q^2} \right) \left(-\frac{3\gamma \exp(-\gamma qz)}{1 - \gamma^2} + \frac{2\gamma(1 + 2\sigma^2) \exp(-\sigma qz)}{1 - \sigma^4} \right). \tag{23}$$

In the case of piezoelectric coupling, semiconductors with zincblende crystal structure have only three non-vanishing components of the piezoelectric tensor β_{ijk} , i.e., $\beta_{14} = \beta_{25} = \beta_{36} = \beta_P$. In the phonon picture, the displacement vector of the surface phonon can be expanded as equation (6), then the electric potential can be quantised as follows:

$$\hat{\varphi}(\mathbf{r}) = \sum_q \left(\frac{\hbar}{2\rho\omega S} \right)^{1/2} a_q \varphi_q(\mathbf{r}) + (\text{Hermitian conjugate}). \tag{24}$$

Taking these results into account, the interaction Hamiltonian due to the piezoelectric coupling can be written as

$$\begin{aligned} H_P &= -e \int \Psi^\dagger(\mathbf{r}) \hat{\varphi}(\mathbf{r}) \Psi(\mathbf{r}) \, d\mathbf{r} \\ &= \frac{4\pi\beta_P e}{\epsilon_0 \sqrt{S}} \sum_{\mathbf{k}, \mathbf{q}} \sum_{n, n'} [b_{\mathbf{k}+\mathbf{q}, n}^\dagger b_{\mathbf{k}, n} a_q \Phi_{nn'}^P(\mathbf{q}) + (\text{Hermitian conjugate})], \end{aligned} \tag{25}$$

where

$$\begin{aligned} \Phi_{nn'}^P(\mathbf{q}) &= -\left(\frac{\hbar\gamma^2}{2\rho c_R J} \right)^{1/2} \left(\frac{q_x q_y}{q^2} \right) \int_0^l \varphi_n^*(z) \\ &\quad \times \left(-\frac{3 \exp(-\gamma qz)}{1 - \gamma^2} + \frac{2(1 + 2\sigma^2) \exp(-\sigma qz)}{1 - \sigma^4} \right) \varphi_n(z) \, dz. \end{aligned} \tag{26}$$

We have fixed the direction of the wavevector \mathbf{q} along the [110] crystal axis (the easiest direction in which the Rayleigh wave is accompanied by piezoelectric effects). Although conduction electrons couple only with the longitudinal wave in the deformation-potential coupling, in the piezoelectric coupling the conduction electrons couple with both longitudinal and transverse waves. Consequently, we shall consider both longitudinal and transverse coupling mechanisms in the present study.

4. Calculation of the amplification coefficient

In this section, we shall calculate the amplification coefficient of the surface phonons due to the interaction of surface phonons with conduction electrons through the deformation-potential and piezoelectric couplings. It is known that conduction electrons never travel freely in a semiconductor but are scattered by a variety of sources before and after they emit or absorb the surface phonons we should observe. Using Hamiltonians (14), (15), and (25), and neglecting vertex corrections other than the screening effect of the electrons, the width $\Gamma^{(s)}$ of the surface phonon can be expressed by

$$\Gamma^{(s)}(\mathbf{q}, \hbar\omega) = 2 \sum_{n,n'} \{ [C(q)]^2 |\Phi_{nn'}^D(\mathbf{q})|^2 + [G(q)]^2 |\Phi_{nn'}^P(\mathbf{q})|^2 \} \times \int \frac{d\mathbf{k}}{(2\pi)^2} \int \frac{d\varepsilon}{2\pi} [f(\varepsilon - \hbar\omega) - f(\varepsilon)] A_n(\mathbf{k}, \varepsilon) A_n(\mathbf{k} - \mathbf{q}, -\hbar\omega), \tag{27}$$

where $C(q) = C/\varepsilon_i(q)$, $G(q) = 4\pi e\beta_p/\varepsilon_0\varepsilon_i(q)$, and $\varepsilon_i(q) = 1 + (4\pi N e^2 c_i^2/\varepsilon_0 c_R^2 k_B T q^2)$. N is the electron concentration, $i = l$ denotes the electronic screening effect induced by the acoustic vibrations of the longitudinal waves with the longitudinal sound velocity c_l , and $i = t$ denotes the electronic screening effect induced by the acoustic vibrations of the transverse waves with the transverse sound velocity c_t (Roth and Argyres 1966, García-Moliner and Flores 1979). The Fermi-Dirac distribution function is $f(\varepsilon) = \{\exp[(\varepsilon - \mu)/k_B T] + 1\}^{-1}$, where μ is the chemical potential. In the piezoelectric coupling, $\varepsilon_i(q)$ must be taken as the longitudinal dielectric function $\varepsilon_l(q)$ of the transverse dielectric function $\varepsilon_t(q)$ depending on the coupling induced by the longitudinal or transverse waves, respectively. $A_n(\mathbf{k}, \varepsilon)$ is the spectral function of the one-electron Green function with the quantised level n of conduction electrons. In the first approximation, we employ the Born approximation and then simply replace the spectral function by the δ -function,

$$A_n(\mathbf{k}, \varepsilon) = 2\pi\delta(\varepsilon - E_{kn}). \tag{28}$$

In a situation where the conduction electrons have a drift velocity v in the direction of the surface-phonon wavevector q , we must replace $\hbar\omega$ by $-\hbar\omega x$ with the drift parameter $x = |v|/c_R - 1$. Then

$$f(\varepsilon - \hbar\omega) - f(\varepsilon) = -\frac{\hbar\omega x}{4k_B T} \operatorname{sech}^2\left(\frac{\varepsilon - \mu}{2k_B T}\right). \tag{29}$$

From equations (27), (28), and (29), the surface phonon amplification coefficient α_i for the i -type surface phonon can be obtained as

$$\alpha_i = -\Gamma_{\text{Born}}^{(s)}(\mathbf{q}, -\hbar\omega x)/\hbar c_R$$

$$= \left(\frac{x}{\hbar^3}\right) \left(\frac{2m^*}{\pi k_B T}\right)^{1/2} \sum_{s=1}^{\infty} (-1)^{s+1} s^{1/2} \exp[sP(q)/k_B T]$$

$$\times \left\{ \sum_{n,n'} \exp[-sQ(q, n, n')/k_B T] \left[\frac{8A^2 C^2}{\pi^2 [\varepsilon_i(q)]^2} \left(\frac{\hbar q^2}{\rho c_R J}\right) \right. \right.$$

$$\left. \times \left(\frac{c_R}{c_l}\right)^4 \frac{(nn')^2 [1 - (-1)^{n+n'} \exp(-\pi A)]^2}{[A^2 + (n - n')^2]^2 [A^2 + (n + n')^2]^2} \right\}$$

$$\begin{aligned}
 & + \frac{32(e\beta_p)^2}{\epsilon_0^2[\epsilon_l(q)]^2} \left(\frac{\hbar\gamma^2}{\rho c_R J} \right) \left\{ \frac{3Ann'[1 - (-1)^{n+n'} \exp(-\pi A)]}{(1 - \gamma^2)[A^2 + (n - n')^2][A^2 + (n + n')^2]} \right. \\
 & \left. - \frac{2Bnn'(1 + 2\sigma^2)[1 - (-1)^{n+n'} \exp(-\pi B)]}{(1 - \sigma^4)[B^2 + (n - n')^2][B^2 + (n + n')^2]} \right\}^2 \Bigg\}, \tag{30}
 \end{aligned}$$

where

$$P(q) = \frac{1}{2}E_g + \mu - (m^*/2q^2\hbar^2)[\hbar\omega_x - (q^2\hbar^2/2m^*)]^2,$$

$$Q(q, n, n') = \frac{1}{2}E_g a_n + (m^*/2q^2\hbar^2)\{E_g[\hbar\omega_x - (q^2\hbar^2/2m^*)](a_n - a_{n'}) + \frac{1}{4}E_g^2(a_n - a_{n'})^2\},$$

$$A = \gamma q t / \pi, \quad \text{and} \quad B = \sigma q t / \pi.$$

In equation (30), $i = l$ denotes the electronic screening effect induced by the acoustic vibrations of the longitudinal waves with the longitudinal dielectric function $\epsilon_l(q) = 1 + (4\pi N e^2 c_l^2 / \epsilon_0 c_R^2 k_B T q^2)$ and $i = t$ denotes the electronic screening effect induced by the acoustic vibrations of the transverse waves with the transverse dielectric function $\epsilon_t(q) = 1 + (4\pi N e^2 c_t^2 / \epsilon_0 c_R^2 k_B T q^2)$.

5. Numerical results and discussion

In this section, a numerical example is developed for an n-type InSb thin film grown epitaxially on a semi-insulating InSb substrate. The relevant values of physical parameters are taken to be: $\beta_p = 1.8 \times 10^4$ esu cm^{-2} for $q \parallel [110]$, $m^* = 0.013 m_0$ (m_0 is the

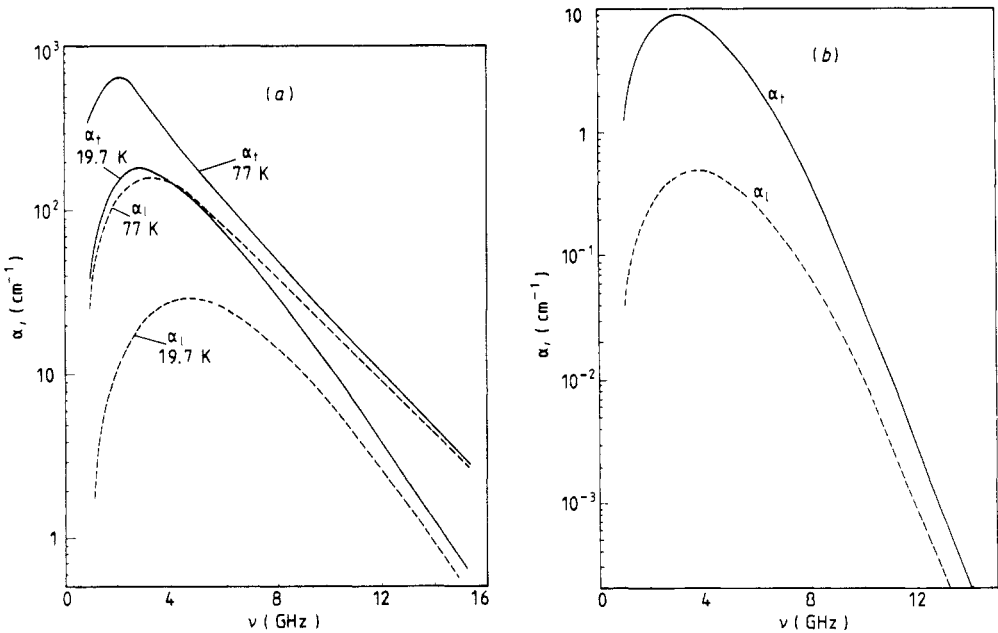


Figure 2. Amplification coefficients of surface phonons versus frequency for the non-parabolic band structure in n-type InSb with $t = 1 \mu\text{m}$ and $x = 10$ at (a) $T = 77$ K and 19.7 K, and (b) $T = 4.2$ K.

mass of free electron), $\rho = 5.8 \text{ g cm}^{-3}$, $N = 1.75 \times 10^{14} \text{ cm}^{-3}$, $\epsilon_0 = 18$, $c_R = 1.35 \times 10^5 \text{ cm s}^{-1}$, $c_l = 3.76 \times 10^5 \text{ cm s}^{-1}$, $c_t = 1.61 \times 10^5 \text{ cm s}^{-1}$, $C = 4.5 \text{ eV}$, $E_g = 0.2 \text{ eV}$. The frequency dependence of the amplification coefficients with $t = 1 \text{ }\mu\text{m}$ and $x = 10$ (corresponding to the applied electric field $E = 2.5 \text{ V cm}^{-1}$) is shown in figure 2. It can be seen that the amplification coefficients increase with the frequency and then decrease monotonically. This is quite different from the results of Tamura and Sakuma (1977a, b) for the parabolic band structure in n-type GaAs. Since the effective mass m^* and the energy gap E_g for n-type GaAs are much larger than those for n-type InSb, from equation (4), the factor a_n for n-type InSb becomes much more important than that for n-type GaAs. We can also see that the amplification coefficients decrease with the

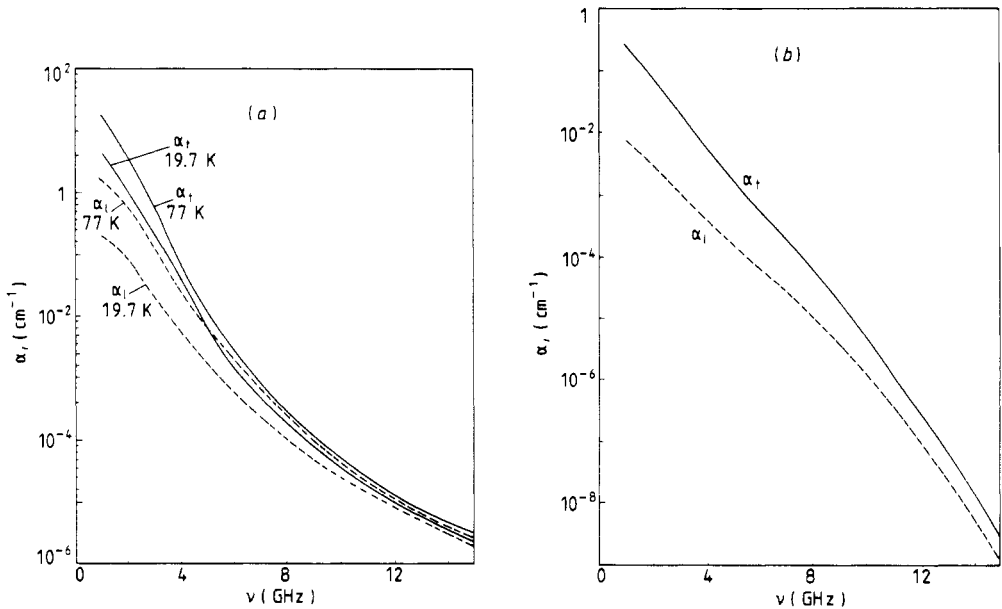


Figure 3. Amplification coefficients of surface phonons versus frequency for the non-parabolic band structure in n-type InSb with $t = 10 \text{ }\mu\text{m}$ and $x = 10$ at (a) $T = 77 \text{ K}$ and 19.7 K , and (b) $T = 4.2 \text{ K}$.

temperature. This means that the effect of amplification of surface phonons will be diminished considerably at very low temperatures. Moreover, the amplification coefficient α_t with the transverse dielectric function is enhanced much more than that α_i with the longitudinal dielectric function owing to the larger electronic screening effect for the longitudinal waves. In figure 3, we show the amplification coefficient as a function of the frequency ν for the case $t = 10 \text{ }\mu\text{m}$. It shows that the amplification coefficients α_t and α_i are reduced considerably in the high-frequency region and the maximum point disappears when the thickness of the thin film increases. It can be seen that the amplification coefficients decrease monotonically with the increasing frequency. Thus the energy of surface phonons is reduced and conduction electrons will not get more energies from surface phonons even if the frequency increases. This is quite different from that for n-type GaAs in which the amplification coefficient increases with the frequency (Tamura and Sakuma 1977a, b). Figure 4 shows the amplification coefficients versus the drift

parameter x or applied electric field E with $\nu = 3$ GHz and $t = 1 \mu\text{m}$. It can be seen that the amplification coefficient increases monotonically with the applied electric field. This is in qualitative agreement with the numerical results of Tamura and Sakuma (1977a, b). However, when the thickness of the semiconductor film increases, the amplification coefficients are reduced considerably for both types of the electronic screening effect. In figure 5 we also find that the local maximum and minimum appear in the lower-temperature region.

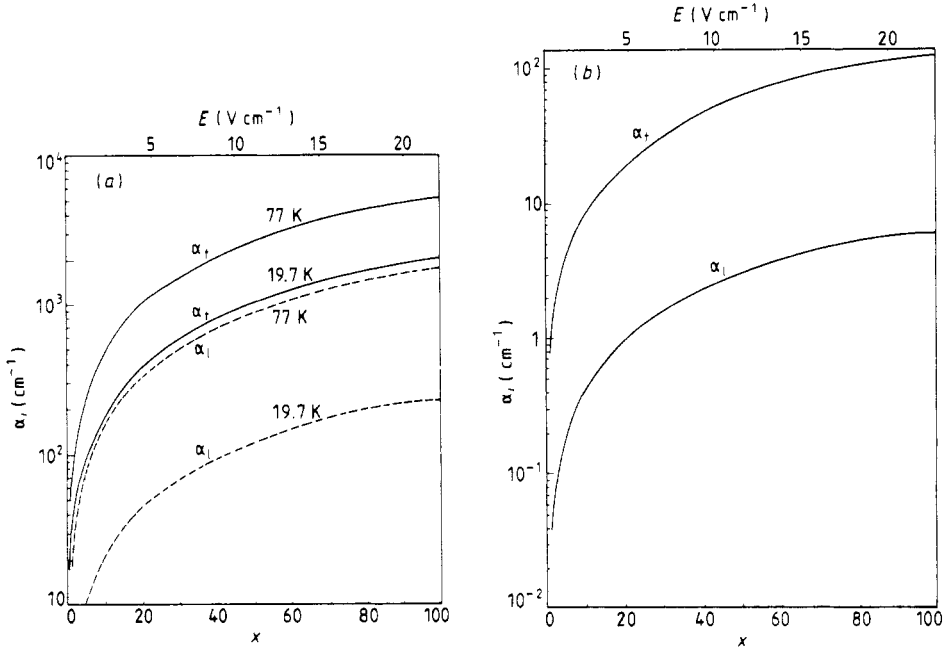


Figure 4. Amplification coefficients of surface phonons versus drift parameter $x = (|v|/c_R - 1)$ or applied electric field E for the non-parabolic band structure in n-type InSb with $t = 1 \mu\text{m}$ and $\nu = 3$ GHz at (a) $T = 77 \text{ K}$ and 19.7 K , and (b) $T = 4.2 \text{ K}$.

We have calculated the amplification coefficient for the non-parabolic band structure in piezoelectric semiconductor films using the Born approximation which is valid in the region $\omega\tau > 1$. However, in our present study we investigate the qualitative features of the effect of non-parabolicity on the amplification of surface phonons in the piezoelectric semiconductor films, the effect of finite relaxation time can thus be neglected. In the results of Tamura and Sakuma (1977a, b) it showed that the effect of the finite relaxation time of the electronic state reduces the amplification coefficient considerably at relatively low frequency. Their results for the Born approximation and the approximation including the effect of finite relaxation time phenomenologically are in qualitative agreement. Consequently, we may safely neglect the effect of the relaxation time of the electronic states and can obtain just qualitative agreement using the Born approximation to discuss the effect of non-parabolicity in piezoelectric semiconductors.

Our present calculations show that the amplification coefficient for the transverse dielectric screening effect is enhanced much more than that for the longitudinal effect. It can also be seen that the amplification coefficient increases with increasing tempera-

ture. Since the effective mass for conduction electrons in an energy level of the non-parabolic band structure with the quantum number n is m^*a_n , the effective mass of conduction electrons defined by m^*a_n will thus depend on high quantum numbers in which the effect of the non-parabolicity on the surface-phonon-conduction-electron interaction is important. Therefore, the amplification coefficients depend on the band structure of materials, the temperature, the electronic screening effect, the frequency of sound waves, the applied electric field, and the thickness of the semiconductor films.

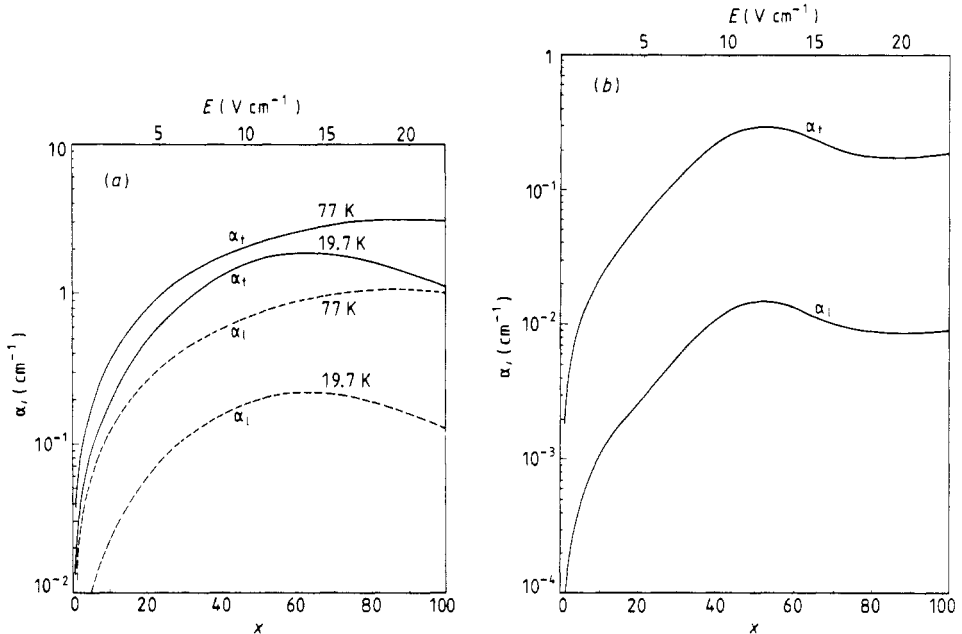


Figure 5. Amplification coefficients of surface phonons versus drift parameter $x = (|v|/c_R - 1)$ or applied electric field E for the non-parabolic band structure in n-type InSb with $t = 10 \mu\text{m}$ and $\nu = 3 \text{ GHz}$ at (a) $T = 77 \text{ K}$ and 19.7 K , and (b) $T = 4.2 \text{ K}$.

However, there exist some relations between these important factors. From the expression for the amplification coefficient given in equation (30), it can be seen that α_i is roughly proportional to $T^{3/2}$ due to the dominant factor $\epsilon_i(q)$. This means that the effect of temperature is dominated by the electronic screening effect. On the other hand, the frequency dependence of the amplification coefficient is not dominated by $\epsilon_i(q)$, but by factors A , B , $P(q)$, and $Q(q, n, n')$. This complicated frequency dependence of the amplification coefficient arises mainly from the effect of the non-parabolic band structure in piezoelectric semiconductors. Similarly, the dependence on the thickness of thin films also comes from the non-parabolic band structure of materials.

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