國 立 交 通 大 學

管理科學系碩士班

碩士論文

台灣證券市場權益風險溢酬之估測

_比較 GARCH-M、移動視窗和 MIDAS 模型

Estimation of Equity Risk Premiums in Taiwan Security Market: Comparison in Using GARCH-M, Rolling Window and MIDAS Model

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中文摘要

本篇論文使用 Ghysel (2005)所提出混合數據抽樣模型(Mixed Data Sampling)，探討台灣證券交易市場的權益風險溢酬，在引入市場報酬的條件期望值和條件變異數下，以跨期資本資產定價模型為基礎來進行估計與預測。樣本期間自 2006 年 1 月至 2010 年 12 月，以股票報酬之條件變異數作為風險替代變數，預測對象是以月頻率為單位，觀察資料則為日/週頻率，針對不同的權重函數和波動因子，進行和 GARCH-in-mean 模型與移動視窗模型之比較。

實證結果發現：(1) 此樣本期間之風險和權益風險溢酬有負向關係存在。(2) MIDAS 模型在時間序列資料的迴歸估計能力較顯著，其次是移動視窗法，且樣本外資料的預測誤差偏小，表示預測能力良好。(3) 根據不同的波動因子和抽樣頻率，以日頻率報酬資料的平方多項式有著較顯著估計結果。有別於傳統研究方法，混合數據抽樣模型最大特點為採用不同頻率資料，配適出最佳迴歸模型，以此估計證券市場的條件變異和風險報酬。

關鍵字：權益風險溢酬、混合數據抽樣模型，跨期資本資產定價模型，自我相關條件異質變異模型、風險報酬抵換

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Abstract

This paper investigates risk premiums of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) by using Ghysel's mixed data sampling (MIDAS) model which is a new regression regarding volatility estimation. We study the intertemporal relation between conditional mean and conditional variance of the aggregate stock market return. Compared with various approaches such as GARCH-in-mean, rolling window and MIDAS models, we find that: (i) We support for a negative relation between risk and equity risk premium in TSEC weighted index during the period 2006 - 2010. (ii) MIDAS is more convincing in predicting regression for sampled time-series data. (iii) Empirical results show out-sample forecasting ability of MIDAS model also performs well. Specifically, it has smaller forecasting error. (iv) Under MIDAS model of different volatility predictors and different sampling frequencies, a squared premium polynomial with daily frequency data has better estimation.

Keywords：Equity Risk Premium, GARCH-M, ICAPM, MIDAS, Risk-Return Tradeoff

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2012 年夏 於交通大學 博愛二舍

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1. Introduction

Cornell (1999) suggests that the equity risk premium (ERP) plays an important role in a host of financial decisions such as making asset allocation decisions, corporate investment decisions and etc. As we both realize that the equity risk premium is not just a central component of every risk and return model in finance but also a critical determinant for estimating costs of equity in both corporate finance and valuation. In addition, we must have heard a lot about *cost of equity for the market*, which is also a synonym for *expected return on the market*, that is determined by a forecast of the equity risk premium. Even so, there is no one universally accepted methodology for estimating ERP. A wild variety of premiums are used in practice and recommended by academics and financial advisors.

In general, we are accustomed to apply the GARCH family to estimate the equity risk premium under considering the volatility. The models family of generalized autoregressive hetetoskedasticity (GARCH) that encompasses all the popular existing GARCH models. The nesting clearly shows the connection between the existing models, and permits new standard nested test to determine the relative quality of each of the model's fits. The nested models include Bollerslev's (1986) GARCH model, Nelson's (1991) exponential GARCH (EGARCH) model, Zakoian's (1991) threshold GARCH (TGARCH) model, Glosten et al's (1993) GJR GARCH model, and others. The benefit of this method is that GARCH models family is the most easily derived model from asymmetric absolute value GARCH model. There is one thing important which describes a conditional standard deviation as a linear combination of absolute value of shocks and lagged conditional standard deviation. To conclude, GARCH family models indeed play a suitable and efficient role for estimating volatility of time series data analysis.

However, there are still some restrictions for GARCH model in estimations, which is whether sampling frequency need to be high or low. Because if sampled at low rate, information contained in high rate may be ignored. To solve this, Ghysels, Santa-Clara, and Valkanov (2002), (2004) and (2005) proposed a regression approach that can directly accommodate variables at different frequencies. This approach is called as Mixed Data Sampling (MIDAS) regression, which contains a simple, parsimonious, and flexible class of time series models that allows the left-hand side and right-hand side variables of time series regressions to be sampled at different frequencies.

In this paper, we investigate risk premium of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) compiled by Taiwan Stock Exchange Co., Ltd. (TWSE) by using Ghysel's MIDAS model. The weighted index data sampled daily from January 2006 to December 2010 is used to examine the time-varying risk premium without considering individual variable factors. We also test intertemporal capital asset pricing model (ICAPM, see Merton, 1973) relation on the basis of our data. Besides, some studies discuss with the issue about Taiwan forward exchange contracts or Taiwan futures market by applying MIDAS regression, but it is hardly to find the study that focuses on the MIDAS regression to explore the risk-return relation in Taiwan stock market. The reason therefore urges me to examine the asymptotic properties of MIDAS regression estimation and apply it to explore the risk-return relation. Furthermore, we also compare it with GARCH-M model and rolling window model. All of the research procedures verify a theory which exactly points out that the MIDAS regression indeed plays an important role in Taiwan stock market.

The rest of this paper is structured as follows. Literatures related to risk premiums, risk-return relation, volatility and mixed data sampling are described in Section 2. In Section 3, we explain rolling window estimation, GARCH-in-mean estimation and MIDAS regression including methodologies and details. Section 4 shows empirical results of various estimations and measurement of forecasting errors and we provide our conclusions in Section 5.

2. Literature Review

In this following section, we provide some related review about our thesis. Literature in the first subsection is about the concept and definition of equity risk premiums. The second subsection provides a tradeoff view point of risk-return relation including intertemporal capital asset pricing model of Merton (1973). The third subsection focuses on volatility. When comes to forecasting volatility, we must associate it with the benchmark ARCH/GARCH models, furthermore, GARCH-in-mean model is also involving deeply. Although these reviews are not directly and deeply related to our main study, it indeed provide a well and sufficient knowledge to the study background. Last but not least, there is a brief review for mixed data sampling in the last subsection.

2.1. Equity Risk Premium

ERP (often interpreted as the market risk premium) is defined as extra return (over expected yield on risk-free securities) a investor expects to receive from an investment in a diverse common stocks (see Grabowski, 2010). Cornell (1999) claims the difference between the return on common stock and the return on government securities. The ERP is calculated as: $RP_m = R_m - R_f$, where RP_m denotes equity risk premium, R_m denotes expected return on fully diverse equity securities, and R_f denotes rate of return expected on risk-free securities. In general, ERP is sometimes used as a proxy for the “market return” such as Standard & Poor’s (S&P) 500 index and New York Stock Exchange (NYSE) composite stock index. In the meantime, ERP is a forward-looking concept. By estimating the true expected ERP for future, and in general, ERP could be modeled as a normal or unconditional ERP (i.e., the long-term average) and a conditional ERP based on current levels of the stock market and economy relative to the long-term average.

Plenty of studies on the risk premium in securities market have been also demonstrated. Aswath Damodaran (2010) suggests a standard approach for estimating equity risk premiums called – the **“History Returns”**. In fact, the most widely used approach to estimating equity risk premiums is the historical premium approach, where the actual returns earned on stocks over a long time period is estimated, and compared to the actual returns earned on a default-free (usually government security). There are still two other approaches for estimating equity risk premium – **“Survey Approach”** and **“Implied Approach”**. If the equity risk premium is what investors demand for investing in risky assets today, the most logical way to estimate it is to ask these investors what they require as expected returns. This approach is called as Survey Approach, and it is also likely that these survey premiums will be more reflections of the recent past rather than good forecasts of the future. On the other hand, Implied Approach is a forward-looking estimation of the premiums.

There are, however, three reasons for the divergence in risk premiums: different time periods for estimation, differences in risk-free rates and market indices and differences in the way in which returns are averaged over time. As above, risk premiums even can vary dramatically. This paper discusses the risk-return tradeoff relations by extending the field of equity risk premium. Numerous studies have investigated the risk-return tradeoff relations between the market's risk premium and conditional volatility.

2.2. Risk-Return Tradeoff

According to some scholars' researching findings, Christian Lundblad (2007) finds a statistically significant positive relation between risk and returns by using American stock market index about lasting 200 years. Before that, Engle (1987) also finds a typically positive relation about American T-bonds. Similarly, French, Schwert, and Stambaugh (1987); Baillie and DeGennaro (1990); Campbell and Hentschel (1992); Bansal and Lundblad (2002); Ludvigson and Ng (2005) also have the similar conclusions pointing out there is a positive albeit mostly insignificant relation between the conditional variance and the conditional expected return. It means that a tradeoff relation does exist, and the more the conditional variance the greater the expected return.

In contrast, Abel (1988), Nelson (1991), Backus and Gregory (1993) have the opposite conclusions. They find a significantly negative relation between the conditional variance and the conditional expected return. Among them, Campbell (1987) test in monthly U.S. data for 1959–1979 and 1979–1983. He has a finding that there is a perverse negative relationship between stock returns and their conditional variance. Glosten, Jagannathan, and Runkle (1993) provided a classical study showing there actually is a slightly negative relation by using the weighted monthly stock index price of CRSP(Center for Research in Security Prices). Besides, Scruggs (1998) has a study using the CRSP value-weighted return index of NYSE-AMEX stock. He also finds the partial relation between the market risk premium and conditional market covariance is negative and significant. Campbell (1987) and Scruggs (1998) provide a view point that future studies of the intertemporal risk-return relation may wish to consider a more broadly defined proxy for the market portfolio. In addition, Glosten et al. (1993) and Harvey (2001) respectively suggest the third situation. No matter the relation is, the conclusion actually depends on the methods which are applied as the researching frameworks.

These studies as above are based on a fundamental theory which is called as CAPM(Capital Asset Pricing Model). CAPM was independently introduced by Treynor (1961,1962), Sharpe (1964), Lintner (1965) and Mossin (1966), building on the earlier work of Harry Markowitz on diversification and modern portfolio theory. In finance, CAPM is used to determine a theoretically appropriate required rate of return of an asset as the asset is added to an already well-diversified portfolio, given the asset's non-diversifiable risk.

Non-diversifiable risk is also known as "systematic risk" or "market risk", and it is often represented by the quantitative beta (β) in the financial industry as well as the expected return of the market and the expected return of a theoretical risk-free asset. In addition to find the excess return of the stock, it also examines whether the liner relationship exists between the stock expected return and the market risk (β). After the passing forty years, this model is widely used to assess the performance of the investing portfolio. However, in 1980s some scholars pointed out in succession that the market risk (β) is not the only reason to explain the stock expected return, but there are also other factors such as the firm size (Banz, 1981), the company net book-to-market ratio (Rosenberg, Reid and Lanstein, 1985), the price-to-earning ratio (Basu, 1983), the leverage effect (Bhandari, 1988) and etc. It is fundamental for Fama-French (1992) to propose the three-factor model for expected returns.

Extending the CAPM, Robert Merton (1973) provides the ICAPM (Intertemporal Capital Asset Pricing Model). ICAPM suggests that the conditional expected excess return on the stock market should vary positively with the market's conditional variance:

$$\mathrm{E}_t(\mathrm{R}_{t+1}) = \mu + \gamma \mathrm{Var}_t(\mathrm{R}_{t+1}) \ , \tag{1}$$

where γ is the coefficient of relative risk aversion of the representative agent and, according to the model, μ should be equal to zero (see French, Schwert and Stambaugh, 1987). The expectation and the variance of the market excess return are conditional on the information available at the beginning of the return period, time t. As we said before, the risk-return tradeoff is so fundamental in financial economics that it could be described as the "first fundamental law of finance".

Besides, there are some related literatures about discussing the trade-off relation with various risk proxy variables of Taiwanese scholars' studies. Lee (2007) apply ICAPM model with TSEC weighted index monthly data of returns from Jan 1998 to Dec 2006, and then find the significant negative relation between expected return and risk. Cho (2008) examines U.S. S&P500 and NASDAQ-100 stock's mean-variance relationship. His study provides strong evidence of a positive relation between risk and return for the S&P 500 futures. However, there is no such a significant relation between risk and return for the NASDAQ-100 futures. Hsu (2008) investigates the risk premiums of Taiwan's U.S. dollar forward rates and the results also show that there is a positive relationship between premium and risk.

However, the tradeoff is not usually easy to be found in the data. And there is one point we still can't neglect: the main difficulty in testing the ICAPM relation is that the conditional variance of the market is not observable and must be filtered from past returns. On the other hand, the risk-return relation of ICAPM is also used to test the variations for GARCH-in-mean model. It is sometimes leading the empirical evidence and the related literature to a mutual contradiction.

2.3 Volatility

In conventional econometric models, variance of the disturbance term is assumed to be a constant, just like:

$$var(y_t|y_{t-1}) = \sigma^2 \tag{2}$$

however, many empirical economic time series data exhibit periodicity of unusually large volatility, not always followed by periods of relative tranquility. Autoregressive Conditional Heteroscedasticity (ARCH) by Engle (1982) measures time-varying conditional variance as a motivation of development for the ARCH model:

$$y_t = a\, x_t + \varepsilon_t \ , \ \varepsilon_t \,|\, \Omega_{t-1} \sim N(0\,, \sigma_t^2) \tag{3}$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 \ , \tag{4}$$

where y_t denotes dependent variable of interest, x_t is independent variable observed at period t, ε_t is a white-noise disturbance term with variance σ^2, and q denotes the orders of lagged terms. Besides on this, Engle's student Bollerslev (1986) develops a generalized Autoregressive Conditional Heteroscedasticity (GARCH) model which exploit U.S. deflator index data from Q2 in 1948 to Q4 in 1983 as samples and consider variance under ARCH models as an Autoregressive moving average (ARMA) which comprises AR (Average Regressive) components and MA (Moving Average) components for estimating conditional variance. Simply speaking, GARCH model could be regarded as an improvement of ARCH model.

Look at Eq. (3) which is mean equation of GARCH model, and then variance equation is defined:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2 \tag{5}$$

if $p = 0$ and $q = 1$, it is clearly shown that the first-order ARCH model is simply a GARCH$(0,1)$ model. Hence, if all β_j equal to zero, the GARCH (p, q)model is equivalent to an ARCH(q) model. For example, ARCH or GARCH model is not trivial but meaningful estimation. There are several interpretations for this formula: (1) Take σ_t^2 for an example, in spite of being a non-observable variable, still can be estimated over time via GARCH model. (2) Furthermore, estimated σ_t^2 has more flexibility in setting parameter, which is also regarded as volatility. As we know, French, Schwert and Stambaugh (1987) use the statistical approaches including ARIMA model and GARCH model to estimate volatility and find that the expected market risk premium is positively related to the predictable volatility of stock returns. Chou (1988) studied the issue of volatility persistence using GARCH-M model and estimates the risk aversion. He shows conclusions that the decline in stock prices is directly related to the increase in volatility. They conclude that mean-variance tradeoff relation is positive but insignificant. To sum up, these empirical results indicate the need of research about the measures of risk.

In recent years, several studies related ERP estimations are presented by using MIDAS regression. Based on ICAPM, Ghysels, Snata-Clara and Valkanov(2005) initially apply monthly returns as proxies of expected returns and daily squared returns over the last years from 1928 to 2000 for estimating the conditional variance by using CRSP value-weighted return data. They find a significantly positive relation between market volatility and return in the U.S. stock market. This is a beginning of all the studies of MIDAS. Furthermore, Ghysels, Sinko and Valkanov (2007) extensively study different lag polynomial specifications and various predictors at one-, two-, three-, four-week frequencies to parameterize the regressions. They find that there is a robustly positive and statistically significant risk-return trade-off across horizons and across predictors.

In addition to U.S. empirical results, Leon, Nave and Rubio (2006) find that the relation between risk and return in most European stock indices is a significant and positive relationship by using MIDAS. On the other hand, Li and Wu (2007) show no significantly positive relation between risk and expected return in Asia Pacific region.

2.4 Mixed Data Sampling

Mixed Data Sampling (MIDAS) regressions are introduced by Ghysels et al. (2005) and it allows us to run parsimoniously parameterized regressions of data observed at different frequencies. There are several advantages of using MIDAS regressions which involve: (1)

data sampled at different frequencies; (2) various past data window lengths; and (3) different regressors. The specification of the regressions combines recent developments regarding estimation of volatility and distributed lag models. MIDAS regressions are used to examine whether future volatility is well predicted by past daily squared returns, absolute daily returns, realized daily volatility, realized daily power, and daily range. Since all of the regressors are used within a framework with the same number of parameters and the same maximum number of lags, the results from MIDAS regressions are directly comparable.

Hence, the MIDAS setup allows us to determine if one of the regressors dominates others. Ghysels, Santa-Clara, Valkanov(2006) found that, for the Dow Jones Index and six individual stock return series, the realized power clearly dominates all other daily predictors of volatility at all horizons. Importantly, the predictive content of the realized power is evident not only from in-sample goodness of fitting measures, but also from out-of-sample forecasts. The daily range is also a good predictor in the sense that it dominates squared and absolute daily returns. The method is a significant departure from the usual autoregressive model building approach embedded in the ARCH literature and its recent extensions such as high-frequency data-based approaches. A comparison of the MIDAS regressions with purely autoregressive volatility models reveals that the MIDAS forecasts are better at forecasting future realized volatility in-sample and out-sample sample.

3. Methodology

Beginning with the explanations of MIDAS estimation, GARCH-in-mean estimation and rolling window estimation as follows, and then followed by the basic assumptions and algorithms. In this paper, we take a new look at risk-return relation and try to estimate conditional variance with various approaches.

3.1. MIDAS Estimation

In this subsection, we introduce the specification of MIDAS regression including various *lag polynomials* and *volatility predictors* (will be both mentioned latter). MIDAS regressions have wide applications in macroeconomics and finance. A typical time series regression model involves data sampled with the same frequency, however, MIDAS regression involves regressors with different sampling frequencies. Actually, this situation also matches the real macroeconomic financial time series data, which might be sampled with almost relatively higher frequencies such as daily frequency, even 5-minute frequency data. From empirical perspectives, this approach does not have to specify the functional form of the high frequency process and is not confined to a window of lags defined over a specific temporal aggregation horizon. Instead, we consider regression models where the variables have different sampling frequencies such that the high frequency process is projected into the low frequency process with a parsimonious weighting scheme.

Back to Eq. (1), returns on the left-hand side are measured monthly because high frequency returns could be too noisy to estimate conditional means. On the right-hand side of Eq. (1), we use daily (or weekly) data in second moments to exploit the advantages of high-frequency returns in variance estimators explained by well-known continuous-record argument of Merton(1980). MIDAS regression is written as:

$$ERP_{t+1} = \mu + \gamma \, Var_t^{MIDAS}(ERP_{t+1}) + \varepsilon_{t+1} \qquad (6)$$

The MIDAS estimator of conditional variance of monthly risk premium, $Var_t^{MIDAS}(ERP_{t+1})$, is also based on the function of prior risk premium data:

$$Var_t^{MIDAS}(ERP_{t+1}) = \sum_{d=0}^{D} \mathcal{W}(d; k1, k2) \, \mathrm{F}(ERP_{t-d}) \qquad (7)$$

where $\mathrm{F}(ERP_{t-d})$ is the function of historical lagged risk premiums. It plays a role similar to ε_t^2 in the GARCH-M model. The corresponding subscript $t-d$ stands for the date t minus d days, $\mathrm{F}(ERP_{t-d})$ denotes specification function including the daily return d days before date t. D is the length of lag, in the meanwhile, $D=22$ (corresponding to one month because a month typically has 22 days) is chosen as our lagged terms. The weight polynomials $\mathcal{W}(d; k1, k2)$ of the MIDAS estimator implicitly capture the dynamics of conditional variance. As follows, there are introductions related the basic properties of weight (lag) polynomials and volatility predictors.

3.1.1. Weight Polynomials

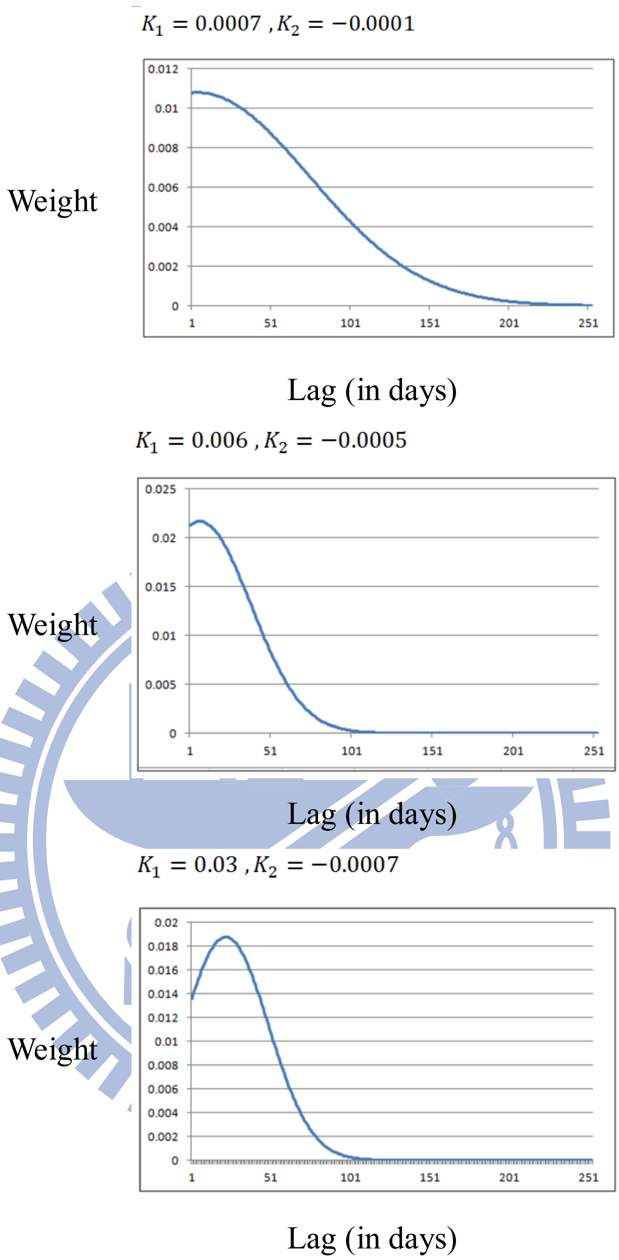
The parameterization of lagged coefficient $\mathcal{W}(d; k1, k2)$ is one of the key MIDAS features. Here we introduce two specifications of MIDAS regression polynomials. The first is:

$$\mathcal{W}(d; k1, k2) = \frac{exp\,(k1d + k2d^2)}{\sum_{d=0}^{D} exp\,(k1d + k2d^2)} \tag{8}$$

We call it as "Exponential Almon Lag", since it is related to "Almon Lags" that is popular in the distributed lag literature (see Almon, 1965). The function $\mathcal{W}(d; k_1, k_2)$ is known to be quite flexible and can take various shapes with only a few parameters. In order to analyze potential shapes, we introduce a quadratic function $f(d) = k_1 d + k_2 d^2$ with derivatives given by $f' = k_1 + 2k_2 d$ and $f'' = 2k_2$. If $k_2 > 0$, there will be a maximum value and the weight has a ascending form. From an economic point of view, this case doesn't make much sense.

Therefore, a descending weight with $k_2 \leq 0$ is reasonable and guaranteed. A slowly declining weight is obtained as we move far away from the beginning of forecasting date. Leon, Nave, and Rubio (2007) provide further analysis and we all know that the parameter k_2 plays a key role in weighting scheme. Besides, k_1 has two possibilities as follows: the first case is $k_1 > 0$ and $k_2 < 0$, which implies that the exponential weight function has a hump-shaped pattern, this case seems to be plausible from an economic point of view in Figure 1. The second case is $k_1 < 0$ and $k_2 < 0$, and we conclude that this form is the most likely reasonable. In addition, it is easy to realize under assumption of $k_1 = k_2 = 0$, we have equal weight which corresponds to a rolling estimator of volatility. As follows, Figure 1 illustrates the various shapes of Exponential.

Figure 1: MIDAS Weight with Exponential Polynomial



The figure plots weight shapes of the mixed data sampling estimator. The weights are calculated by substituting the estimated values of k_1 and k_2 into the weight equation (8). In the top panel, slowly declining weights are displayed. The middle panel shows rapidly declining weights, where the bottom panel contains a weight that has a hump-shape.

We clearly notice that the declining rate determines how many lags are included in MIDAS regression Eq. (7). Once the weight form of $\mathcal{W}(d;k_1,k_2)$ is specified, the lag length selection is totally data driven. When the function decays slowly, a large number of observations need to be taken into consideration for the forecast of variances with small measurement error. Conversely, a fast decay corresponds to using a small number of observations with potentially large measurement error.

The second parameterization is also shown as follows:

$$\mathcal{W}(d;k1,k2)=\frac{f(\frac{d}{D};k1,k2)}{\sum_{d=0}^{D}f(\frac{d}{D};k1,k2)} \tag{9}$$

where $(x\,,k1\,,k2)=\frac{x^{k1-1}(1-x)^{k2-1}\Gamma\,(k1+k2)}{\Gamma(k1)\,\Gamma(k2)}$, and $\Gamma(k1)=\int_0^\infty e^{-x}x^{k1-1}dx$. Eq. (9) is based on Beta function so that we called it as "Beta Lag". For example, we know that under an assumption of $k_1=k_2=1$ we have equal weights. As "Exponential Lag" case, the weight declining rate determines how many lags are included in the MIDAS regression. The two specifications both have two important characteristics. First, they provide positive coefficients, which is necessary for positive definiteness of estimated volatility. Second, they sum up to one. In this paper, we use Exponential Lag as the specification, which is theoretically more parsimonious. We choose the lagged period D as 22 days which is corresponding to one month while comparing various predictors of conditional variance in MIDAS regression.

3.1.2. Volatility Predictors

Volatility predictors with various specifications also affect the risk premiums (see Ghysels, Sinko, and Valkanov, 2007). In particular, some different ways are considered such as: squared returns, absolute returns, return ranges, realized volatility, and realized power (the sum of high frequency absolute returns). In general, we apply daily lagged squared risk premiums and absolute range risk premiums as our volatility predictors. Here are MIDAS general formulations:

$$\mathrm{ERP}_{t+1} = \mu + \gamma \sum_{\mathrm{d}=0}^{\mathrm{D}} \mathcal{W}(d; k1, k2)\, \mathrm{ERP}_{t-d}^{2} + \varepsilon_{t+1} \tag{10}$$

$$\mathrm{ERP}_{t+1} = \mu + \gamma \sum_{\mathrm{d}=0}^{\mathrm{D}} \mathcal{W}(d; k1, k2)\, |\, \mathrm{ERP}_{t-d} \,| + \varepsilon_{t+1} \tag{11}$$

where ERP_{t-d}^{2} is the lagged squared risk premium and $|\,\mathrm{ERP}_{t-d}\,|$ is the absolute risk premium in the MIDAS polynomial volatility predictor.

To estimate the parameters in MIDAS estimation, we use the variance estimator Eq. (7) with the weight function Eq. (8) into the ICAPM relation Eq. (1) and estimate the parameters μ and γ by maximizing the likelihood function. Assuming that the conditional distribution of return is normal:

$$\mathrm{ERP}_{t+1} \sim \mathrm{N}(\, \mu + \gamma\, Var_t^{MIDAS}, Var_t^{MIDAS}\,). \tag{12}$$

Because the true conditional distribution of returns could depart from normality, our estimator applies only quasi-maximum likelihood (see Bollerslev and Wooldridge, 1992).

Using higher frequency returns at daily or weekly interval could improve the estimate of γ because of the availability of additional data points. On the other hand, quarterly returns could increase the efficiency of the estimator of γ because they are less volatile.

3.2 GARCH-in-Mean Estimation

In finance, the returns of a security may depend on its volatility. Engle et al. (1987) have s study for three-month U.S. Treasury bills and six-month U.S. Treasury bills from 1960Q1 to 1984Q2. They claim that the expected return varies while the risk changes, therefore they take varying conditional variance into GARCH consideration. That approach is known as GARCH-in-mean model, where conditional mean is linearly related to the conditional variance. (see Engle, Lilien and Robins, 1987). General GARCH-M models can be written as:

$$y_t = \mu + \gamma\sigma_t^2 + \varepsilon_t \tag{13}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{14}$$

where μ and γ are constants. The parameter μ is called risk aversion parameter. The formulation implies that there are serial correlations in the return series $\{y_t\}$, the mean model. These serial correlations are introduced by those in the volatility process $\{\sigma_t^2\}$. As we can see, the GARCH-M model incorporates heteroskedasticity into the estimation procedure and allows for direct estimate of time-varying risk premiums. Related to Merton's ICAPM, some scholars claim that if the changes in the investment opportunity set are captured by some steady variables except for the conditional variance, then these variables must be included in the expected return equation (mean equation). The general formulation is as:

$$Var_t^{GARCH} = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta\, Var_{t-1}^{GARCH} \tag{15}$$

where $\varepsilon_t = ERP_t - \mu - \gamma\, Var_t^{GARCH}$. The squared error ε_t^2 (regression error term) in the variance estimator plays a role similar to the squared risk premium functions in MIDAS approach.

3.3 Rolling Window Estimation

A moving window is commonly used with time series data to smooth out short-term fluctuations and highlight longer-term trends or cycles. As an example of rolling window approach, French, Schwert and Stambaugh (1987) use within-month daily squared returns to forecast next month's variance:

$$Var_t^{RW} = \sum_{d=0}^{D} \frac{1}{D} ERP_{t-d}^2 \tag{16}$$

where D is the number of days used in the variance estimator. They apply the autoregressive moving average (ARMA) process for one-month rolling window estimator to model the conditional variance. In the meanwhile, daily squared returns are multiplied by 22 to measure the variance in monthly unit. Here we still choose the window size to be one month, or D = 22. Besides its simplicity, the use of daily data has a number of advantages. First, as with MIDAS approach, the application of using daily data increase the precision of the variance estimator.

Second, the stock market variance is very persistent (see Officer, 1973; Schwert, 1989), so the realized variance on a given month ought to be a good forecast of next month's variance. Then we estimate the parameters μ and γ of risk-return tradeoff in Eq. (1) with maximum likelihood using the rolling window estimator Eq. (16) for the conditional variance.

Based on the literature of Ghysels, Santa-Clara and Valkanov (2005), they suggest the window size should not be limited. A larger window size corresponding to a more than one month, even up to six months, is used because the choice of lagged period has a greater impact on the estimate of γ. In this paper, we choose fixed window size D = 22 to estimate the risk premium in Taiwan stock market.

4 Empirical Results

4.1 Data

Here we use the daily risk premium of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) compiled by Taiwan Stock Exchange Co., Ltd. (TWSE) in our empirical test. The period is from January 2006 to December 2010, including 1246 daily observations. Entire samples are all collected from TEJ (Taiwan Economic Journal). TEJ was founded in April 1990 to provide quality, in-depth and extensive historical financial data and information in the major financial markets in Asia. There is a definition about equity risk premium in this paper: we use the difference, return rates of TSEC weighted index minus two-year Taiwan treasury-bill rates, as a proxy to be explored, including various frequencies such as daily, weekly and monthly data form. In the meantime, statistical software E-Views is applied to analyze and compute some relevant data.

Table 1 shows the descriptive statistics about the sampled equity risk premium. We find the mean for ERP is negative. That means on average there is no premium investors acquire in the stock market during this period. Conversely, they even get some losses. Variances are used in this table because of the relation between risk and return. Specifically, we focus on connections of average return and conditional variance, not standard deviations

Table 1: Descriptive Statistics

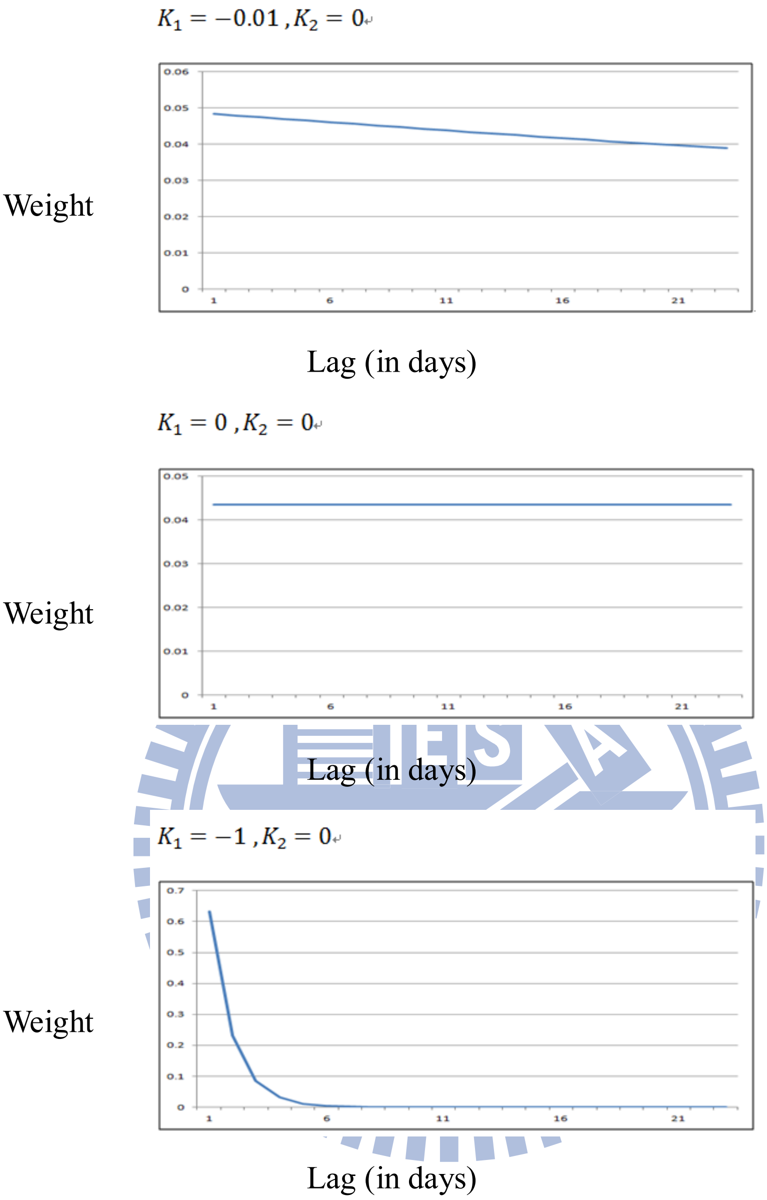
Descriptive statistics of ERP with different sampling frequencies from 2006 to 2010, included are mean, variance, skewness, kurtosis. The number of samples for each frequencies is also reported in the table.

	Mean (%)	Variance	Skewness	Kurtosis
Monthly	-0.49	0.0056	-0.3167	3.2388
Weekly	-1.09	0.0011	-0.5692	4.0098
daily	-1.22	0.0003	-0.3370	4.2252

4.2 MIDAS Estimation

This subsection is integrated from two parts. As we mentioned before, we decide to use Exponential weight specification and apply the 30 days lags length. For first part, we apply the suggestion under setting $k1 = -0.01$ and $k2 = 0$ (see Ghysels, Snata-Clara, and Valkanov, 2006b) as a benchmark. Then we compare it with other two cases: $k1 = 0$ and $k2 = 0$ (shown as equal weight), $k1 = -1$ and $k2 = 0$ (considered as reasonable pattern). We plot the weights that the MIDAS estimator places of the first 22 lagged daily squared risk premiums corresponding to one month in Figure 2. The top panel is case1, the middle panel is case2 and the bottom panel displays case3.

Figure 2: MIDAS Weight on Variables Predictors



The figure plots the estimated weights of conditional variance on the lagged daily squared risk premiums corresponding to one month. Three panels are representative of three different declining weight shapes respectively. We then use the weights to estimate related parameters by MIDAS approach.

Now we jointly estimate the parameters μ and γ by nonlinear least squares (NLS). In Table 2, we show three different weight polynomials and two various types of the volatility predictors. We also explore the estimation results between MIDAS approach and rolling window approach.

Table 2: MIDAS Estimation of Equity Risk Premiums

The table shows estimates of ERP with MIDAS estimation using TAIEX form Jan 2006 to Dec 2010. Exponential lag is used and lagged daily (weekly) squared (absolute) risk premiums are respectively used in the construction of conditional variance estimator. The estimated equations are as follows:

$\mathrm{ERP}_{t+1} = \mu + \gamma \sum_{\mathrm{d=0}}^{\mathrm{D}} \mathcal{W}(d; k1, k2) \mathrm{ERP}_{t-d}^{2} + \varepsilon_{t+1}$ / $\mathrm{ERP}_{t+1} = \mu + \gamma \sum_{\mathrm{d=0}}^{\mathrm{D}} \mathcal{W}(d; k1, k2) |\mathrm{ERP}_{t-d}| + \varepsilon_{t+1}$,

where $\mathcal{W}(d; k1, k2) = \frac{exp\,(k1d + k2d^2)}{\sum_{d=0}^{D} exp\,(k1d + k2d^2)}$

The coefficients and corresponding p-value are shown in the middle columns and the right column is shown as corresponding R-squared value.

MIDAS Estimation			
Panel A : Daily ERP^2			
	μ	γ	R^2
weight 1 (K1=-0.01, K2=0)	**5.47** **(<0.0001)***	**-1.47** **(<0.0001)***	**0.4615**
weight 2 (K1=0, K2=0)	5.41 (<0.0001)*	-1.46 (<0.0001)*	0.4608
weight 3 (K1=-1, K2=0)	0.58 (0.6350)	-0.29 (0.1595)	0.0338

Panel B : Weekly ERP^2			
	μ	γ	R^2
weight 1 (K1=-0.01, K2=0)	3.43 (0.0020)*	-0.31 (<0.0001)*	0.3437
weight 2 (K1=0, K2=0)	3.35 (0.0025)*	-0.30 (<0.0001)*	0.3379
weight 3 (K1=-1, K2=0)	2.50 (0.0194)*	-0.03 (<0.0001)*	0.2795
Panel C : Daily $\|ERP\|$			
	μ	γ	R^2
weight 1 (K1=-0.01, K2=0)	9.50 (<0.0001)*	-6.73 (<0.0001)*	0.4141
weight 2 (K1=0, K2=0)	9.38 (<0.0001)*	-6.70 (<0.0001)*	0.4117
weight 3 (K1=-1, K2=0)	2.62 (0.1509)	-2.09 (0.0472)*	0.0662
Panel D : Weekly $\|ERP\|$			
	μ	γ	R^2
weight 1 (K1=-0.01, K2=0)	7.12 (<0.0001)*	-3.04 (<0.0001)*	0.3686
weight 2 (K1=0, K2=0)	6.96 (<0.0001)*	-3.00 (<0.0001)*	0.3598
weight 3 (K1=-1, K2=0)	4.31 (0.0042)*	-2.00 (0.0001)*	0.2266

*indicates the statistics reach 0.05 of the significant level

This subsection presents the result of MIDAS approach based on the Merton's ICAPM model. We find the coefficients μ and γ are almost statistically significant. First, we start from MIDAS estimation. In daily data, the estimated risk aversion coefficient γ is ranging between -0.29 and -6.73. There is not a very small gap between the both sides. The risk aversion absolute seems greater in daily data than in weekly data, and that means the degree of risk aversion which can be tolerated by investors. In addition, we see that there are just little differences between the weight 1 and weight 2 polynomials and R-square values respectively. We also find such t-statistics of the corresponding estimated coefficient are significant by judging from the p-values. We can conclude that volatility predictors of weight 1 and weight 2 are obviously better than weight 3. Actually, these results with polynomial weight 3 are not explainable enough.

In weekly data, the estimated risk aversion coefficient is of -0.03 to -3.04 and the difference is much closer. However, the result of weight 3 case becomes better because its R-squares value is getting obviously higher, even over 20% extra. While mentioning to R-square value, it is reports to quantify the explanatory power of the variance estimators in predictive regressions for sampled premiums. To sum up, the estimation of daily risk premium performs better than weekly risk premium because the significance of coefficients and variance explanations level performs more outstanding, up to 46%. Moreover, the result of squared daily return volatility is also comparable to the result of absolute daily return volatility. The risk aversion coefficients of weight 1 and 2 are -1.47and -1.46, and the model explained variation levels are around 46.15% and 46.08% respectively. Basically both are almost equivalent, but we still prefer to choose the estimation model under $k1 = -0.01$ and $k2 = 0$ with daily frequency. These results point to the importance of having a flexible functional form for the weights on past daily squared returns. Then we use out-sample to measure forecasting errors in following subsection to make certain whether the estimation is appropriate.

However, one thing important needs to be noticed. We all have negative magnitude of risk aversion coefficients in above cases, no matter whether the squared risk premium or the absolute risk premium is. It clearly points out that the tradeoff relation in our empirical study is negative. These "negative" results are obviously corresponding to some previous classical studies. Actually we think the results may depend on what the estimated method for the conditional variance of returns is used. Campbell (1987) use generalized method moments (GMM) to verify the relationship between expected stock returns and the conditional variance of stock returns. The coefficient estimates of GMM for stock suggest that stocks have a higher expected return when their conditional variance is low. Correspondingly, Nelson (1991) uses the GARCH method to estimate a model of the risk premium on the CRSP value-weighted market index form 1962 to 1987. The outcome is shown as a statistically significantly negative relation between both. In recent studies, Glosten et al. (1993) use the CRSP data and find support of a negative relation between conditional expected monthly return and conditional variance of monthly return, using the modified GARCH-M model. More related interpretation we leave in Section 5.

4.3 GARCH-in-Mean Estimation

Before applying GARCH-M estimation, time series data should be processed by a kind of unit tests and we find out the result shows significant rejections of null hypothesis which mean the risk premium data is not autocorrelated. Then we directly use the data under GARCH-M estimation.

Table 3: GARCH-M Estimation of Equity Risk Premiums

The table shows estimates of ERP with GARCH-M estimation using TAIEX form Jan 2006 to Dec 2010. The estimated equations are as follows:

$ERP_{t+1} = \mu + \gamma\, Var_t^{GARCH} + \varepsilon_{t+1}$, where $Var_t^{GARCH} = \omega + \alpha\, \varepsilon_t^2 + \beta\, Var_{t-1}^{GARCH}$.

The coefficients and corresponding p-value are shown in the middle columns and the right column is shown as corresponding R-squared value.

GARCH-M Estimation

	μ	γ	ω	α	β	R^2
Monthly	228	-0.22	1.59	0.74	0.25	0.0264
2006-2010	(<0.0001)*	(<0.0001)*	(<0.0001)*	(<0.0001)*	(<0.0001)*	
Weekly	0.06	-0.03	0.93	0.60	0.39	~0.000
2006-2010	(0.3686)	(<0.0001)*	(0.0727)	(<0.0001)*	(<0.0001)*	
Daily	0. 86	-0.77	0.001	0.0041	1.00	0.2247
2006-2010	(<0.0001)*	(<0.0001)*	(0.0545)	(<0.0001)*	(<0.0001)*	

*indicates the statistics reach 0.05 of the significant level

Table 3 shows the empirical results of GARCH(1,1)-M estimation of risk premium data with different frequencies. The estimated coefficients are obtained by a sort of maximum likelihood estimations, and we assuming error term ε_t is normally distributed. Compared with other three different frequencies in GARCH-M estimation, the R-squared statistics with daily frequency data is much better. In addition, the GARCH-M model with daily frequency shows the statistical significance of mean equation and variance equation, excluding intercept term. The risk aversion coefficient γ is around of -0.77 for the mean equation and the p-value of the corresponding estimated coefficient looks very significantly. Here we notice that the risk aversion is still negative, consistent with the MIDAS estimation as we mentioned above.

Besides, under the GARCH-M approach the R-squared statistic is around of 22.47%, lower than in the MIDAS approach which is shown as 46.15%. Take this for example, it is because the MIDAS approach estimates two parameters rather than three as GARCH-M model does and employs more observations to forecast market volatility under variance equation. In generally speaking, traditional GARCH-M estimation outcome in explainable range is not superior to the MIDAS approach.

4.4 Rolling Window Estimation

We discuss about the rolling window estimation with daily and weekly frequency data. The results of rolling window approach are shown in Table 4. The estimate of γ is still negative (around of -1.4), and the coefficient is very significant because the p-value is far lower than the significant level. It is shown consistently under this situation with the MIDAS estimation. Besides, R-square value is 46.08% of rolling window estimation, and almost as same as the MIDAS estimation, 46.15%. They are so close but obviously we still recognize that the daily frequency specification is better as a result of the higher R-squared value. The rolling window approach can be thought as a robust check of the MIDAS estimation because it is a simple estimator of conditional variance with no parameters to be estimated. Besides its simplicity, the use of daily data has some advantages: first, as with MIDAS approach, it can increase the precision of the variance estimator. Second, the stock market variance is quite persistent (see Officer, 1973; Schwert, 1989), so the realized variance on a given month ought to be a good forecast of next month's variance.

Table 4: Rolling Window Estimation of Equity Risk Premiums

The table shows estimates of ERP with rolling window estimation using TAIEX form Jan 2006 to Dec 2010. The estimated equations are as follows:

$ERP_{t+1} = \mu + \gamma\, Var_t^{RW} + \varepsilon_{t+1}$, where $Var_t^{RW} = \sum_{d=0}^{D} \frac{1}{D} ERP_{t-d}^2$

The coefficients and corresponding p-value are shown in the middle columns and corresponding R-squared values are shown in the right column.

Rolling Window Estimation			
	μ	γ	R^2
Daily ERP_t^2	5.41 (<0.0001)*	-1.40 (<0.0001)*	0.4608
Weekly ERP_t^2	3.35 (0.0025)*	-0.29 (<0.0001)*	0.3379

*indicates the statistics reach 0.05 of the significant level

4.5 Forecasting

Analysts are often interested in comparing the accuracy of competing forecasts, for a variety of reasons. For example, accuracy comparisons can be used to help discriminate competing models. Accordingly, a number for equal forecast accuracy have been developed. After estimating as above, here we use out-sample data to compare the risk premium forecasting errors. The purpose of forecasting is to understand whether these approaches maintain consistent performances under out-sample by observing how much close is between risk premium estimators and the realized data. The smaller the estimator error is, the better the estimation performs. At first, we use in-sample data between Jan 2006 and Dec 2010 to estimate the original parameters (μ , γ) as shown in Table 2, 3, 4. In general, percentage of in-sample observations to out-sample observations ratio is about 10% or 15% (see Ashley, 2003). Therefore, we decide to choose 12 months as our forecasting period.

Some literatures discuss the various approaches to forecast estimation errors, such as mean error (ME), mean square error (MSE), root mean square error (RMSE), mean absolute error (MAE) and mean absolute percent error (MAPE). In this paper, we apply RMSE and MAE approaches to measure prediction level in various volatility estimation models.

4.5.1 Root Mean Square Error

The root mean square error (RMSE) (see Christiano, 1989) is a frequently used measure of differences between an estimator and the values actually observed. The concept of RMSE is close to MSE, and RMSE is the squared root of MSE, is as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{N}(ERP_t - \widehat{ERP}_t)^2}{N}}$$

where ERP_t is the realized volatility on day t, $\widehat{ERP}_t$ is the foracasted volatility on day t, N denotes sampling days.

4.5.2 Mean Absolute Error

In practice, the mean absolute error (MAE) is to measure how close forecasts are to eventual outcomes. The mean absolute error is given by: $\text{MAE} = \frac{\sum_{t=1}^{N}|ERP_t - \widehat{ERP}_t|}{N}$

4.5.3 Forecasting Results

For the reason we pay attention to the importance of out-sample forecasting is that it can avoid the situations of over-fitting models or of abusing data-mining. Forecasting accuracy comparison can help discriminate among competing models. Several recent studies have examined the small-sample properties of some commonly used tests, too. In fact, we focus not only on comparing with different models, but also on understanding the forecasting accuracy within in-sample and out-sample data under various estimations.

Table 5: Results of Forecasting Error

The table shows the forecasting error results by using RMSE / MAE for out-sample.

Comparing Forecasts		
	In-sample (60)	**Out-sample (12)**
MIDAS		
RMSE	5.4498	8.1490
MAE	4.3284	5.9583
GARCH-M		
RMSE	7.3795	5.5631
MAE	5.6129	5.5631
Rolling Window		
RMSE	5.4536	8.4781
MAE	4.3435	6.5689

All forecasting error unit is of percentage (%)

From Table 5, no matter whether RMSE or MAE estimation is used, obviously we find that out-sample performance of GARCH-M estimation is quite good, which means the smaller errors. However, errors of in-sample under GARCH-M estimation are the largest, even up to 7.3795%, almost 1.7 times to the smallest one. On the other hand, we realize that MIDAS and rolling window estimations are basically developed form similar concepts and the forecasting results vary consistently and stably. Overall, forecasting errors in using MIDAS is close to GARCH-M, which is almost of 0.4% in difference under MAE approach. Basically we still can regard forecasting errors of MIDAS as the same as forecasting errors

GARCH-M under MAE approach in out-sample. These approaches of forecasting error sequentially ranked as GARCH-M, MIDAS and RW estimation from the smallest error to the largest error. Figure 3 shows forecasting graph under MIDAS and Table 6 shows error differences under MIDAS as follows.

Figure 3: Forecasting Graph with MIDAS Estimation

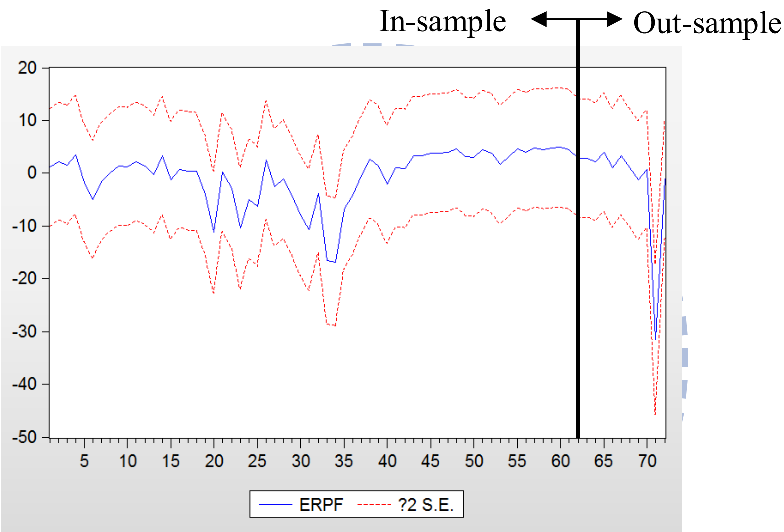


Table 6: Out-Sample Errors (2011.1 – 2011.12)

Period (year/month)	Real ERP	Forecasted ERP	Error (%)
2011.1	1.3564	4.4178	3.0614
2011.2	-6.657	2.8815	9.5385
2011.3	0.1501	2.9229	2.7728
2011.4	2.9379	4.051	1.1131
2011.5	-1.0121	4.004	5.0161
2011.6	-4.5668	2.9358	7.5026
2011.7	-0.9472	3.4425	4.3897
2011.8	-11.2543	-3.1977	8.0566
2011.9	-7.4062	-1.4198	5.9864
2011.10	4.2511	2.7699	1.4812
2011.11	-9.7889	-31.7794	21.9905
2011.12	1.6078	1.0167	0.5911

All forecasting error unit is of percentage (%)

5 Conclusion

This paper take a new look at Merton's ICAPM, focus on the trade-off between conditional variance and conditional mean of the stock market return. We show the existence of a time-varying risk premium in Taiwan stock market by introducing mixed data sampling model estimation. Our results are more conclusive because MIDAS estimation confirms the weighted polynomial with different sampling frequencies performs pretty good. Not the same as with previous studies, added power obtained from the new MIDAS estimator actually makes risk premium estimation more flexible.

According to the previous empirical results, conclusions of this study are as follows:

1) The tradeoff between risk and return has long been an important topic in asset valuation research. Most of this research examine the tradeoff among different securities within a given time period. We find the common evidence of a negative relation between risk and return in Taiwan stock market within these years. In fact, we think that what types of model are used to assume conditional variance of returns as a research framework is highly relevant to the issue of risk-return relation regardless of positive relation or negative relation. However, sometimes the models we used cannot completely capture volatility persistence or reflect positive and negative shocks.

Black (1976) and Christic (1982) propose financial leverage effect for an examination of the risk-return tradeoff with asymmetric variance effect. After that, Campbell and Hentschel (1992) propose volatility feedback effect to explain the same situations. Most empirical studies show that negative relation between risk and return might be attributed to asymmetric effects in the conditional variance. Moreover, the type of relevance is mostly confined by model assumptions which indeed affect these empirical results.

In addition to asymmetric effect, many different approaches for setting risk as a proxy variable could also affect the empirical results, especially for risk-return tradeoff. Moreover, the financial tsunami brings about some potential phenomenon such as increasing difficulty in predicting expected returns and conditional variances. Meanwhile, it also indeed related to the sampling period we selected. Although we acquire negative relation about risk-return tradeoff, which is opposed to some previous research, we still show some advantages in MIDAS estimation as follows.

2) Comparing with the rolling window and GARCH-M estimation, we conclude that MIDAS estimation is better and more suitable. As the model explained variation power, 46.15% of MIDAS is larger than 46.07% of rolling window, also greater than 22.47% of GARCH-M. The rolling window approach can be thought as a robust check of the MIDAS estimation

because it is a simple estimator of conditional variance with no parameters to be estimated. Except for explained variation power, these estimation coefficients are very statistically significant because of p-values are below significance level. That means the MIDAS estimation is indeed a well-performed model.

3) By using MIDAS approach, this estimator is behalf of a weight average of past daily squared returns with flexible functions. MIDAS estimator is not only the superior estimator because it can be appropriately explained by past risk premiums, but also a better forecaster in the stock market than rolling window estimators. Last but not the least, after experiencing investigations of the MIDAS specifications for various volatility predictors, we obtain that higher frequency predictor such as daily squared return provides greater results.

The empirical results are statistically significant, at the same time, the forecasting performance of MIDAS is also reasonable. We still have interests to use MIDAS to process how these different and jointly estimated weights of volatility predictor work. Next, we explore the parameters $k1$ and $k2$ more deeply. Our purpose is to directly and jointly estimate the parameters $k1$, $k2$, μ, γ of Eq. (6) and (7) by nonlinear least error approach. Owing to the smaller the conditional variance is, the smaller the estimated forecasting error is.

Therefore we apply MAE and RMSE forecasting error approaches, and then make certain that the values of MAE and RMSE are both minimum. Our estimated algorithm is as follows, which is based on rules of minimum error. Take MAE for example, our main purpose is to minimize the value of $\frac{\sum_{t=1}^{60}|ERP_t-\widehat{ERP_t}|}{60}$, which is under restrains of $\widehat{ERP}_{t+1} = \hat{\mu} + \hat{\gamma}Var_t(ERP_{t+1})$,

and $Var_t = \sum_{d=0}^{22} \frac{exp\ (k1d+k2d^2)}{\sum_{d=0}^{22} exp\ (k1d+k2d^2)} ERP^2_{t-d}$.

After jointly nonlinear least error calculation with the same period, our results are shown in Table 7.

Table 7: Errors of Jointly Nonlinear Least Calculation

	Forecasting Estimation				
Minimum Error (%)	μ	γ	K_1	K_2	R^2
0.0096 (RMSE)	-0.3163	-0.0503	1.2346	-18.3336	0.0020
0.0564 (MAE)	0.0105	-0.0002	1.2346	-18.3336	~0.000

*indicates the statistics reach 0.05 of the significant level
All forecasting error unit is of percentage (%)

We find that the outcome is not good enough while comparing with previous results under setting the specific parameter K. The both coefficients μ and γ are not statistically significant because the p-values are not below the significance level 0.05. Moreover, the explanatory power is also abnormally low so that we cannot verify this case to be well. Here is a trivial implication that the suggestion of setting K_0 and K_1 as some specific values (see Ghysels et al., 2006) improves the outcomes of MIDAS estimation better. As for the reasons why the effects of estimated weight polynomial parameter such as K are not relatively outstanding, we leave these issues for future research.

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