國立交通大學

財務金融研究所

碩 士 論 文

在交易雜訊下估計具違約邊界之結構化信用風險模型

Estimating the Structural Credit Risk Model with Default Boundaries in the Presence of Equity Trading Noise

研 究 生：袁倫賜

指導教授：李漢星 博士

中 華 民 國 一 百 零 一 年 六 月

在交易雜訊下估計具違約邊界之結構化信用風險模型

Estimating the Structural Credit Risk Model with Default Boundaries in the Presence of Equity Trading Noise

研 究 生：袁倫賜　　Student：Luen-Tsz Yuan

指導教授：李漢星　博士　　Advisor：Dr. Han-Hsing Lee

國 立 交 通 大 學

財務金融研究所

碩 士 論 文

A Thesis

Submitted to Graduate Institute of Finance

College of Management

National Chiao Tung University

in partial Fulfillment of the Requirements

for the Degree of

Master of Science

in

Finance

June 2012

Hsinchu, Taiwan, Republic of China

中華民國一百零一年六月

在交易雜訊下估計具違約邊界之結構化信用風險模型

研 究 生：袁倫賜　　　　　　　　　　指導教授：李漢星　博士

國立交通大學財務金融研究所碩士班

摘要

在 2007 年的全球金融風暴過後，不僅學術界對企業的違約風險非常的重視，實務界亦然，因此，如何能夠更準確的預測企業違約風險成為一個重要的研究課題。本篇研究根據 Duan and Fulop (2009) 所提出的平滑局部化取樣/重要性重新取樣粒子濾波器(smoothed localized sampling/importance resampling particle filter)架構去處理在有交易雜訊(trading noise)下之結構式模型估計。我們的模型在障礙選擇權的架構下以結構式方法進行公司有價證券訂價，本研究結果指出交易雜訊在流動性差的股票上會有顯著的影響，而且可能對於波動度與破產機率的估計產生影響。

關鍵字：障礙選擇權模型、粒子濾波器、結構式信用風險模型、交易雜訊

Estimating the Structural Credit Risk Model with Default Boundaries in the Presence of Equity Trading Noise

Student：Luen-Tsz Yuan Advisor：Dr. Han-Hsing Lee

Graduate Institute of Finance
National Chiao Tung University

Abstract

After the worldwide financial crisis in 2007, credit risk of a company is getting vast attention not only from academic but also from practitioners. It is of interest for researchers to more accurately model and estimate the default risk of a firm. In this paper, we apply the method proposed by Duan and Fulop (2009), the smoothed localized sampling/importance resampling (SL-SIR) particle filter, to deal with the structural model estimation in the presence of trading noise. Our model employs the structural approach for valuing corporate securities under the barrier option framework. Our results suggest that trading noise can be substantial for the less liquid stocks and may potentially affect volatility and default probability estimation.

Keywords: Barrier model, Particle filter, Structural credit risk model, Trading noise.

Contents

List of Figure

List of Table

致謝

終於等到這一天的到來，學校的結束代表著我的人生要進入下一個階段，有點捨不得，有點緊張，但還是很高興，可以開始賺錢了。好不容易適應了新的環境，適應了之後，又要馬上離開了。

回想這兩年在學校的學習，最重要的就是我的指導教授-李漢星教授。不論在生活上或是在課業上都給了我很多的支持與幫助，讓我感到非常的驕傲，有一個這麼棒的老師一直在幫助我。在老師的指導下，才能循序漸進的走向畢業之門。也要感謝其他老師，在課業上都讓我學習到許多知識。更要感謝我的口試委員們給我的建議與指點，讓我的論文能夠更加完整。

退伍後再回到學校，一直覺得非常的幸運。來到交通大學後，也遇到了很多好同學，大家一起在課堂上學習，一起討論著作業，一起熬夜準備考試，還有一起出遊的美好回憶，這些人生經驗讓我每每在回想的時候，都會在心中默默地感到開心。因為有你們，讓我在研究所這個最後的學生生涯更完美。

最後還要感謝我的家人，在我最無助與徬徨的時候，家總是最能讓我放鬆的地方。父母對我的照顧與生活上的幫助，讓我能無憂無慮地讀完碩士，父母總是會不斷地給我鼓勵，但又不會造成我的壓力。姊姊辛苦工作貼補家用，讓我的生活也更加沒有負擔。畢業之後我也會努力工作，換我來照顧這個家。

袁倫賜 謹誌

交通大學財務金融所

民國一百零一年七月

1. Introduction

After the worldwide financial crisis in 2007, credit risk of the company is getting vast attention not only from academic but also from practitioners. Specifically, many firms had good performance but suddenly default during the financial crisis. It raises the importance to more precisely model and estimate default risk of a firm.

One of the fundamental approaches to model the default risk of a corporation is the structural approach. Structural credit risk modeling or defaultable claim modeling is pioneered by the seminal paper by Black and Scholes (1973) and Merton (1974) in which corporate liabilities can be viewed as a covered call — own the asset but short a call option. Later on, the idea is further advanced by Black and Cox (1976), Leland (1994), and others. This approach to model default claims is named structural approach since the model explicitly ties default risk to the firm value process and its capital structure. Currently, structural models are mostly built upon default barrier assumptions proposed by Black and Cox (1976).

Brockman and Turtle (2002) employed a simple proxy approach to calculate asset values by approximating the market value of corporate assets as the sum of the market value of equity and the book value of liabilities. The proxy forces the default barrier to be greater than the future promised payment of liabilities under the down-and-out call option framework.

To overcome this problem, Duan (1994) devised a transformed-data maximum likelihood estimation (MLE) method for structural models. The MLE method hinges on a recognition that the equity value should result from one-to-one differentiable transformation of the firm's unobserved asset value under the given structural credit risk model even though the transformation depends on some unknown model parameters.

However, it has been well documented in the market microstructure literature that observed equity prices can diverge from their equilibrium value due to microstructure noises like as illiquidity, asymmetric information, price discreteness. Microstructure noises have also been shown to have material effect on volatility estimation in the realized volatility literature. The transformed-data MLE method can no longer be applied when trading noises are present in equity prices.

The smoothed localized sampling/importance resampling (SL-SIR) particle filter methods proposed by Duan and Fulop (2009), can effectively estimate structural model in the presence of trading noise in equity value. They devised a nonlinear filtering scheme using the auxiliary particle filtering idea of Pitt and Shephard (1999). They used the Merton model to estimate firm's value and show that their method has good estimation performance.

In our study, we attempt to investigate the performance of SL-SIR particle filter method under barrier option framework. We first conduct Monte Carlo simulation analysis for testing the performance of SL-SIR particle filter method on the Brockman and Turtle (BT) models. Next, we perform an empirical investigation to compare the default forecasting ability of the Merton and the Brockman and Turtle models. Our empirical results show the trading noise indeed impact the volatilities and default probabilities of firms when we estimate both of Merton model and BT models.

2. Literature Review

2.1 Structural Form Model

Credit risk modeling has gained increasing prominence over the years. Structural model and reduced form models are the two most commonly used approaches to model credit risk. The first structure model for default risky bonds was proposed by Blake and Scholes (1973).

They explained how equity owners hold a call option on the firm.

Merton (1974) extended the framework and analyzed risk debt behavior with the model. The basic idea is that a company defaults on its debt if the value of the assets of the company falls below a certain default point. The firm's asset value is assumed to follow a geometric Brownian motion and the firm's capital structure to consist of a zero-coupon debt and common equity. This structural approach then yields formulas for the risky corporate bond and the default probability of the firm. There are two disadvantages of using Black-Sholes model when pricing equity value. One is bond default happen only at maturity, and other is ignoring that low liquidity makes corporate bond default.

Blake and Cox (1976) have evolved to extend the BSM model to multiple periods. Barrier structural models, these models view default as a down-and-out barrier option. It allows for corporate bond default anytime that before maturity only if the bond value hits a pre-specific level. Whenever the bond value reaches the pre-specific level, the corporation goes into default or is liquidated immediately.

Leland (1994) had extended the model to include taxes, bankruptcy costs, and protective covenants. He considers two possible bankruptcy determinants. One is when bankruptcy is endogenously by the inability of the firm to raise sufficient equity capital to meet its current debt obligations and one is the Brennan and Schwartz case with a positive net-worth covenant. He used to derive closed-form solutions for optimal capital structure when firm asset value follows a diffusion process with constant volatility.

2.2 Transformed-Data Maximum Likelihood Estimation

The structural approach is conceptually elegant but is laden with implementation problems. That is the firm's asset values are unobservable and the model parameters are unknown. It is meaning that the assets' expected return and volatility in the case of Merton's model are unknown.

In the academic literature, there exist at least three other ways of dealing with the unobservability issue. First is a proxy asset value may be computed as the sum of the market value of the firm's equity and the book value of liabilities. The second way is based on solving a system of equation. And the MLE method is third way of dealing with the unobservability issue. The approach put forward in Duan (1994) is based on maximum likelihood estimation (MLE) which views the observed equity time series as a transformed data set with the theoretical equity pricing formula serving as the transformation. The benefits of using the MLE method are well understood in statistics and econometrics.

The estimation problem associated with unobserved asset values can be naturally cast as a transformed-data MLE problem. Such an approach was first developed in Duan (1994). The obvious advantages are that (1) the resulting estimators are known to be statistically efficient in large samples, and (2) sampling distributions are readily available for computing confidence intervals or for testing hypotheses.

2.3 Particle Filter

Since 1993 years particle filter have become a very popular class of numerical methods for the solution of optimal estimation problems in non-Gaussian scenarios. In comparison with standard approximation methods, the principal advantage of particle methods is that they do not rely on any local linearisation technique or any crude functional approximation.

Now we describe the problem of principle filter. Assume that the state of a financial model at time k is described by a random vector x_k whose dynamics follows the transition equation

$$x_{k+1} = Q(x_k, \varepsilon_{k+1}) \tag{1}$$

where $Q(\ )$ is an arbitrary function and ε_k is a sequence of independent random vectors.

When x_k is continuous, this defined the conditional probability density $q(x_{k+1} | x_k)$. x_k is not directly observable, instead at time k a noisy observation y_k is available, linked to x_k through the measurement equations

$$y_k = G(x_k, v_k) \qquad (2)$$

where $G(\ )$ us an arbitrary function and v_k the observation noise is a sequence of random vectors, independent across time and form ε_k. When y_k is continuous, this defines the conditional probability density $g(y_k | x_k)$.

Further, assume some prior distribution, $q_0(x_0)$, for the initial state variable. Then, the objective of filtering is to come up with the distribution of the hidden variable, x_k, given the observed data up to k. This quantity is the filtering distribution of x_k and is denoted by $f(x_k | y_{0:k})$. In the algorithms that follow these distributions are obtained sequentially, as new observations arrive.

Kalman Filtering is applicable to linear normal systems. Here the filtering distributions are normally distributed with a mean and variance that can be recursively updated using the Kalman recursion.

Extended Kalman Filter (EKF) is useful when the transition and measurement equation are not linear but differentiable. This method using a first-order Taylor expansion around $E_{k-1}(x_{k-1})$ and applies Kalman Filtering on the approximating linear system. This approach is often applied in term structure modeling and in commodities modeling. The methods have a big problem that is the system only up to a first order and provide poor results when the nonlinearity of the measurement or transition equation is serious.

Unscentes Kalman Filter (UKF) avoids the linearization altogether and provides a higher order approximation to the nonlinear system than the EKF. This method approximates the normal filtering distribution using a discrete distribution that matches the mean and covariance matrix of the target Gaussian random variable. Then, these points are passed through directly the nonlinear functions to obtain the quantities necessary for the Kalman recursion.

When the system is non-linear or non-gaussian, the filtering distribution may not be normal and the Kalman Filter is not valid any more. To appreciate the difficulty of the task, in the following we describe the sequential filtering problem in the general model.

The joint filtering distribution of $x_{0:k}$ given $y_{1:k}$ is

$$f(x_{0:k} \mid y_{1:k}) = \frac{f(x_{0:k}, y_{1:k})}{f(y_{1:k})} = \frac{f(x_{0:k}, y_{1:k})}{L(y_{1:k})}$$

where $L(y_{1:k})$ is the likelihood of the data observed up to k

$$L(y_{1:k}) = \int f(x_{0:k}, y_{1:k}) dx_{0:k}$$

Now derive the recursive formula connecting the filtering distributions at k and $k+1$

$$\begin{aligned} f(x_{0:k+1} \mid y_{1:k+1}) &= \frac{f(x_{0:k+1}, y_{1:k+1})}{L(y_{1:k+1})} \\ &= \frac{g(y_{k+1} \mid x_{k+1}) q(x_{k+1} \mid x_k) f(x_{0:k}, y_{1:k})}{L(y_{1:k})} \frac{L(y_{1:k})}{L(y_{1:k+1})} \\ &= \frac{g(y_{k+1} \mid x_{k+1}) q(x_{k+1} \mid x_k)}{f(y_{k+1} \mid y_{1:k})} f(x_{0:k} \mid y_{1:k}) \end{aligned}$$

This equation gives the recursion of filtered distributions over the whole path space. Integrating over $x_{0:k-1}$ one gets the following relationship

$$\begin{aligned} f(x_{k:k+1} \mid y_{1:k+1}) &= \frac{g(y_{k+1} \mid x_{k+1}) q(x_{k+1} \mid x_k)}{f(y_{k+1} \mid y_{1:k})} f(x_k \mid y_{1:k}) \\ &\propto g(y_{k+1} \mid x_{k+1}) q(x_{k+1} \mid x_k) f(x_k \mid y_{1:k}) \end{aligned}$$

shows that $f(x_{0:k} \mid y_{1:k})$ is a sufficient statistic. Integrating out x_k, one arrives at the filtering distribution of x_{k+1}

$$f(x_k \mid y_{1:k+1}) \propto \int g(y_{k+1} \mid x_{k+1}) q(x_{k+1} \mid x_k) f(dx_k \mid y_{1:k})$$

The Kalman Filter is a special case where this recursion can be executed in closed-form due to the joint normality of the system. In general, the filtering distributions do not belong to a known parametric family and the integration has to be done using numerical methods.

Our target is the joint filtering distribution of the hidden states

$$f(x_{0:k} \mid y_{1:k}) \propto \prod_{t=1}^{k} g(y_t \mid x_t) q(x_t \mid x_{t-1}) q_0(x_0) \tag{3}$$

Ideally, we would like to sample directly from the densities $g(y_t \mid x_t) q(x_t \mid x_{t-1})$, providing a straightforward recursive Monte Carlo scheme. Unfortunately this is usually impossible because the complexity of these densities.

Importance sampling (IS) is an approach that can be used in such case. Here, one draws from a feasible proposal distribution $r(x_{0:k})$ instead of the target and attaches importance weights to the samples to compensate for the discrepancy between the proposal and the target. If the weighted samples is denoted by $(\xi_{0:k}^{(m)}, w_k^m)$ where m=1,…,M, the samples and weights are obtained as

$$\xi_{0:k}^{(m)} \sim r(x_{0:k})$$

$$w_k^{(m)} = \frac{\prod_{t=1}^{k} g(y_t \mid \xi_t^{(m)}) q(\xi_t^{(m)} \mid \xi_{t-1}^{(m)}) q_0(\xi_0^{(m)})}{r(\xi_{0:k}^{(m)})}$$

The expectation $E(h(x_{0:k} \mid y_{1:k}))$ can be estimated by the estimator

$$h = \frac{\sum_{m=1}^{M} h(\xi_{0:k}^{(m)}) w_k^{(m)}}{\sum_{m=1}^{M} w_k^{(m)}}$$

Using independence of the sample the estimator is asymptotically consistent

$$\hat{h} - E\left(h\left(x_{0:k} \mid y_{1:k}\right)\right) \xrightarrow{P} 0 \quad \text{as} \quad \mathrm{M} \to \infty$$

and asymptotically normal

$$\sqrt{M}\left[\hat{h}-E\left(h\left(x_{0:k}\mid y_{1:k}\right)\right)\right]\xrightarrow{d}N\left[0,\frac{Var_h(h(x_{0:k})w(x_{0:k}))}{[E_r(w(x_{0:k}))]^2}\right] \text{ as } \mathrm{M}\to\infty$$

Note that the asymptotic variance can also be estimated using the simulation output, allowing inference on the reliability of the estimate.

The preceding importance sampling algorithm can be made sequential by choosing a recursive structure for the importance sampling distribution, $r(x_{0:k})$

$$R(x_{0:k})=\prod_{t=1}^{k}r(x_t\mid y_t,x_{t-1})r_0(x_0)$$

Then the importance weight w_k can be written as

$$w_k=\prod_{t=1}^{k}\frac{g(y_t\mid x_t)q(x_t\mid x_{t-1})}{r(x_t\mid y_t,x_{t-1})}\frac{q_0(x_0)}{r(x_t\mid y_t,x_{t-1})}$$

and the importance sampler can be implemented in a sequential manner that is the way which is calling Sequential Importance Sampling (SIS).

This algorithm seems to provide a solution to the recursive filtering problem. Unfortunately after a couple of time steps the normalized weights of most points fall to zero and the weighted sample cease to provide a reliable representation of the target distribution.

Sequential Importance Sampling with Resampling (SIR) is a way that can deal the weight degeneracy problem. The method is resampling the current population of particles using the normalized weights as probabilities of selection. After resampling, all importance weights are reset to one. The intuition behind this procedure is that unlikely trajectories are eliminated and likely ones are multiplied. This approach concentrates on the marginal filtering distribution $f(x_k\mid y_{0:k})$ instead of the joint distribution $f(x_{0:k}\mid y_{0:k})$. Resampling helps to achieve a better characterization of the last state of the system at the expense of representing the past of the full hidden path, $x_{0:k}$.

3. Model and Estimation Method

3.1 Equity value in Brockman and Turtle's (2002) Model

In this section, we describe the model of pricing the equity value using the barrier option model of Brockman and Turtle (BT). They propose a framework for corporate security valuation based on path dependent instead of the commonly used path-independent approach. They argue that path dependency is an intrinsic and fundamental characteristic of corporate direct implication of this framework is that equity will be price as a European down-and-out call option. The equity pricing equation is as follow:

$$S_t = S(V_t;\sigma,K,H,r,T-t)$$

$$= V_t\Phi(a_t) - Ke^{-r(T-t)}\Phi(a_t-\sigma\sqrt{T-t}) - V_t(\frac{H}{V_t})^{2\eta}\Phi(b_t) + Ke^{-r(T-t)}(\frac{H}{V_t})^{2\eta-2}\Phi(b_t-\sigma\sqrt{T-t}) \quad (4)$$

where S_t is the equity value at time t, V_t is the firms value at time t, K is the debt value at time T, H is the barrier, $\Phi(\bullet)$ is the standard normal distribution,

$$a_t = \begin{cases} \dfrac{\ln(V_t/K) + (r+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} & \text{for} \quad K \ge H \\ \dfrac{\ln(V_t/\mathrm{H}) + (r+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} & \text{for} \quad K < H \end{cases},$$

$$b_t = \begin{cases} \dfrac{\ln(\mathrm{H}^2/V_tK) + (r+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} & \text{for} \quad K \ge H \\ \dfrac{\ln(\mathrm{H}/V_t) + (r+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} & \text{for} \quad K < H \end{cases},$$

and $\eta = \dfrac{r}{\sigma^2} + \dfrac{1}{2}$.

3.2 Log Likelihood Function

We can apply the transformed-data MLE method of Duan (1994) to obtain the log-likelihood function of a discretely sampled equity value on a firm that survives in the entire sample period. It turns out to be the log-likelihood function of the firm's asset value conditional on survival evaluated at the implied asset values and then plus a term related to the Jacobian of the transformation. Thus

$$L_{BT}^{S}(\mu,\sigma,H;V_{t_0},V_{t_1},V_{t_2},...,V_{t_n})$$

$$=L_{BT}^{V}(\mu,\sigma,H;V_{t_0},V_{t_1},V_{t_2},...,V_{t_n})-\sum_{j=1}^{n}\ln\left|\frac{\partial S(\hat{V}_{t_j}(\sigma,H);\sigma,K,H,r,T-t)}{\partial V_{t_j}}\right|$$

where

$$L_{BT}^{V}(\mu,\sigma,H;V_{t_0},V_{t_1},V_{t_2},...,V_{t_n})=-\frac{n}{2}\ln(2\pi\sigma^2 h)-\frac{1}{2}\sum_{j=1}^{n}\frac{(R_{t_j}-(\mu-\frac{\sigma^2}{2})h)^2}{\sigma^2 h}-\sum_{j=1}^{n}\ln(V_{t_j})$$

$$+\sum_{j=1}^{n}\ln(1-\exp(-\frac{2}{\sigma^2 h}\ln\frac{V_{t_{j-1}}}{H}\ln\frac{V_{t_j}}{H})) \quad (5)$$

$h=t_j-t_{j-1}$, $R_{t_j}=\ln(\frac{V_{t_j}}{V_{t_{j-1}}})$ and the first derivative of the equity value with respect to the asset value can be derived as follow:

$$\frac{\partial S(\hat{V}_t;\sigma,H)}{\partial V_t}=V_t\frac{\partial\Phi(a_t)}{\partial V_t}+\Phi(a_t)-X\frac{\partial\Phi(a_t-S)}{\partial V_t}$$

$$-\left(V_t\left(\frac{H}{V_t}\right)^{2\eta}\frac{\partial\Phi(b_t)}{\partial V_t}\right)-\left(V_t\frac{\partial\left(\frac{H}{V_t}\right)^{2\eta}}{\partial V_t}\Phi(b_t)\right)-\left(\left(\frac{H}{V_t}\right)^{2\eta}\Phi(b_t)\right)$$

$$+\left(X\left(\frac{H}{V_t}\right)^{2\eta-2}\frac{\partial\Phi(b_t-s)}{\partial V_t}\right)+\left(X\frac{\partial\left(\frac{H}{V_t}\right)^{2\eta-2}}{\partial V_t}\Phi(b_t-s)\right)$$

where

$$X=Ke^{-r(T-t)}$$

$$s=\sigma\sqrt{T-t}$$

$$\frac{\partial\Phi(a_t)}{\partial V_t}=\frac{1}{\sqrt{2\pi}}e^{-\frac{a_t^2}{2}}\frac{1}{sV_t}$$

$$\frac{\partial\Phi(b_t)}{\partial V_t}=-\frac{1}{\sqrt{2\pi}}e^{-\frac{b_t^2}{2}}\frac{1}{sV_t}$$

$$\frac{\partial\left(\frac{H}{V_t}\right)^{2\eta}}{\partial V_t}=-2\eta\left(\frac{H}{V_t}\right)^{2\eta}\left(\frac{1}{V_t}\right)$$

$$\frac{\partial\left(\frac{H}{V_t}\right)^{2\eta-2}}{\partial V_t}=-(2\eta-2)\left(\frac{H}{V_t}\right)^{2\eta-2}\left(\frac{1}{V_t}\right)$$

$$\frac{\partial\Phi(a_t-S)}{\partial V_t}=\frac{1}{\sqrt{2\pi}}e^{-\frac{(a_t-s)^2}{2}}\frac{1}{sV_t}$$

$$\frac{\partial\Phi(b_t-s)}{\partial V_t}=-\frac{1}{\sqrt{2\pi}}e^{-\frac{(b_t-s)^2}{2}}\frac{1}{sV_t}$$

3.3 Contain Trading Noise

3.3.1 Basic Model and Method

It will become much complex estimation if the equity prices contain trading noise. The market microstructure literature indeed strongly suggests that noise should be expected. Then the relationship between the unobserved asset value and the observed equity value predicted by the equity pricing formula is masked by trading noise, which is modeled as an exogenous variable in our pricing model. Following Duan and Fulop (2009), we assume a multiplicative error structure for the trading noise and change the Merton model to become BT's model. We express the logarithmic equity value as follow:

$$\ln S_{t_i} = \ln S(V_{t_i};\sigma, K, H, r, T-\tau_i) + \delta \nu_i \tag{6}$$

where $\{\nu_i, i = 0, ..., N\}$ are i.i.d. standard normal random variables and the nonlinear pricing function $S(V_{t_i};\sigma, K, H, r, T-t_i)$ has been given.

We can derive its discrete-time form of the unobserved asset value which with the process following the geometric Brownian motion as following:

$$\ln V_{t_{i+1}} = \ln V_{t_i} + (\mu - \frac{\sigma^2}{2})h + \sigma\sqrt{h}\varepsilon_{t+1} \tag{7}$$

where $\{\varepsilon_i, i = 1, ..., N\}$ are i.i.d. standard normal random variables.

This two equation constitute a state-space model with the first being the measurement equation and the second the transition equation. To deal with the non-linear filtering problem, we employ Smoothed Localized SIR (SL-SIR) particle filter which is proposed by Duan and Fulop (2009). They construct a localized sampler that takes advantages of the knowledge on $S_{t_{i+1}}$ and localize the sampling of $V_{t_{i+1}}$ around the asset value implied by $S_{t_{i+1}}$ under no trading noise. The complete step of our method is as follow:

- Initial State: Set $V_{t_0}^{(m)} = S^{-1}(S_{t_0})$ where the function $S^{-1}(\bullet)$ is the inverse of the equity pricing function describe above.
- Recursion: For $j = 1, ..., N$

1. Sampling

- Begin with $V_{t_{j-1}}^{(m)}$ in the equal-weight filtering sample. Draw a standard normal $v_j^{(m)}$ and compute $V_{t_j}^{(m)} = V_{t_j}^*(S_{t_j}, v_j^{(m)})$ to obtain the pair $(V_{t_{j-1}}^{(m)}, V_{t_j}^{(m)})$, where $V_{t_j}^*(S_{t_j}, v_j) = S^{-1}(S_{t_j} e^{-\delta v_j}; \sigma, K, H, r, T - t_j)$

- Compute the importance weights

$$w_j^{(m)} = f(V_{t_j}^{(m)} \mid V_{t_{j-1}}^{(m)}, \Theta) \left| \frac{\partial S(V_{t_j} \mid \Theta)}{\partial V_{t_j}} \right|^{-1}$$

- Normalized the importance weights

$$\pi_j^{(m)} = \frac{w_j^{(m)}}{\sum_{m=1}^{M} w_j^{(m)}}$$

2. Resample from the weighted sample $\left\{\left(V_{t_j}^{(m)}, \pi_j^{(m)}\right); m = 1, \ldots, M\right\}$ to obtain a new equal-weight sample of size M.

3.3.2 Smooth resampling

Since the likelihood function based on the typical particle filtering algorithm is not continuous due to a requited resampling step, smoothness must be built into the algorithm to make it suitable for parameters estimation. We used the smooth bootstrapping method that is proposed by Pitt (2002). First, sort the data $\left\{(V_{t_j}^{(m)}, \pi_j^{(m)}); m = 1, \ldots, M\right\}$ and then becomes $\left(x^m, \pi_m\right), m = 1, \ldots, M$. Then we following Pitt to assign $\Pr(1) = \tilde{\pi}_1 = \frac{1}{2}\left(2\pi_1 + \pi_2\right)$, $\Pr(R-1) = \tilde{\pi}_{R-1} = \frac{1}{2}\left(\pi_{R-1} + 2\pi_R\right)$ and $\Pr(i) = \tilde{\pi}_i = \frac{1}{2}\left(\pi_i + \pi_{i+1}\right)$ for $i = 2, \ldots, R-2$. We generate the M sorted uniforms as $u_{(j)} = \frac{\left(j-1\right)}{M} + \frac{u}{M}$ where $j = 1, \ldots, M$ and u is the single random variable with the uniform distributions ($u \sim U\left(0,1\right)$). And we get regions (r_j) and u_j^* for all j, as following:

set $s=0$, $j=1$;

for ($i=1$ *to* R-1)

{

$s=s+\tilde{\pi}_i$;

while ($u_{(j)} \le s$ and $j \le M$)

{

$r_j = i$;

$u_j^* = \frac{\left(u_{(j)} - \left(s - \tilde{\pi}_i\right)\right)}{\tilde{\pi}_i}$;

$j = j+1$;

}

}

Having obtained the region we are in $r_j, j=1,...,M$, we then sample conditional upon the region. If $r_j = 1$, $y_j = x^1 + \max\left(0, u_j^* - \frac{\pi_1}{2\pi_1+\pi_2}\right) \times \frac{2\pi_1+\pi_2}{\pi_1+\pi_2} \times \left(x^2 - x^1\right)$.

If $2 \le r_j \le R-2$ then $y_j = \left(x^{r_j+1} - x^{r_j}\right) \times u_j^* + x^{r_j}$. If $r_j = R-1$ and $u_j^* < \frac{\pi_{R-1}+\pi_R}{\pi_{R-1}+2\pi_R}$ then $y_j = x^{R-1} + \frac{\pi_{R-1}+2\pi_R}{\pi_{R-1}+\pi_R} \times \left(x^R - x^{R-1}\right) \times u_j^*$, else $y_j = x^R$. And $\left\{y_j, j=1,...,M\right\}$ is the new equal-weight sample of size M.

3.3.3 Describe the work of finding the MLE

There are four parameters, (σ, δ, μ, H), we need to estimate. First, we use the estimate parameter under traditional model, with no noise, to be the initial parameter. Then use dichotomy to get firm's value under BT's model by equation (4). Second, generate a matrix

$A_{M\times 1}$ with all elements are under standard normal distribution ($N\left(0,1\right)$). Then we take the matrix multiply 0.1 and plus the initial firm's value, V_{t_0}, then we get M point equal-weights sample be the initial firms value. Now we are using localized sampling to generate the proposal distribution:

set $gridlength = 10\delta$, N=25 and $intervallength = gridlength / N$

for $i = 0$ to N

{

$$S(i) = S^t * \exp(-0.5 \times gridlength + (i-1) \times intervallength)$$

}

and use dichotomy to find the correspond $V(i)$.

for $i = 1$ to M

{

$noise(i) = \delta \times v$, (where $v \sim N(0,1)$)

$$bin = \left[\frac{noise(i) + 0.5 \times gridlength}{intervallength} \right]$$

then $upper = -0.5 \times gridlength + bin \times intervallength$

$$w = \left(upper - noise(i)\right) / intervallength$$

(Interpolation Method)

$$\ln V_t^{(i)} = \ln V(bin) + (1-w) \times \ln V(bin+1)$$

}

After that, we calculate the weights of the proposal sample given old particles as we describe above and the log-likelihood value at time t_j can be write as $L_{t_j} = \ln(\sum_{i=1}^{M} w_i^j \Big/ M)$. Then we can calculate that $\pi_j^{(i)} = w_i^j \Big/ \sum_{i=1}^{M} w_i^j$, $i = 1,...,M$. Then we used the smooth resampling method as Section 3.3.2 to find out the new equal weights sample. Finally, the estimate of firm's value is the mean of the new samples.

After the process, we calculate likelihood estimation as $L = \sum_{j=1}^{n} L_{t_j}$, where n is the size of observe equity value. Then repeat the step and find out the maximum likelihood estimation to find the parameter we are interesting.

4. Result

4.1 Simulation Analysis

We conduct Monte Carlo simulation experiment to investigate the finite-sample performance of SL-SIR particle filter on the Brockman and Turtle model. We generate sample paths of noisy equity observations by setting initial firm value as 100, the debt value at maturity as 100, risky free rate as 0.05 and the time to maturity as ten years. In short, we generate 250 daily returns and then compute the firm's asset values backward to a yield a sample of 251 asset values. We keep the simulated data only when asset values are above of the barrier (H) all the time before maturity. Corresponding to the simulated asset values, we compute 251 equity values using the measurement equation (4). For estimating model parameters, we act as if we do not know the asset value as it is in the real-life estimation situation.

The parameters we set in Monte Carlo experiment are taken from Duan and Fulop (2009), expect default boundary H. The base case parameter values are as follow: $\sigma = 0.3$, $\delta = 0.004$ and $\mu = 0.2$. Following Duan et al (2004), we set the barrier to be 80. We also change the three key parameters, σ , δ and H , to investigate the performance of SL-SIR particle filter under these setting. We simulate 500 samples in each case of Tables 1 and 2.

Panel A of Table 1, presents the estimation results for the base case parameter. We next compare the results of different σ (0.15 and 0.7) while keeping other parameters identical as they are in base case. Panel C presents the results for different values of H (50 and 90).

Similar to the result of Duan and Fulop, when σ is increasing the performance is getting worse in estimates of firm value, and δ is overestimated than base case. We observe that δ is closer to true value and the mean of firm's value error is smallest when σ is smaller. In Panel B, we can find that when H is bigger the standard deviation of barrier estimates is smaller. It means that the estimates is more accurate when H is bigger. We also find that when $H = 50$ the trading noise δ is closer to true value. It is reasonable because when the value of H is close to zero, the BT's model will become Merton's model. As a result, the estimates of δ become more accurate and similar to those in Duan and Fulop (2009).

Table 2 reports the results of different size of trading noise from 0.004 to 0.016, 0.02 and 0.025. As we already find in Table 1 that δ is overestimated under the BT's model. We next want to examine in which case the parameter can be better pinned down.

It is apparent that δ s are overestimated when δ is smaller than 0.009. When δ is bigger than 0.01, the estimation performance improves as the percentages of mean's error are smaller than five percent. Although it seems to have underestimate trading noise when δ=0.16, we can find that the SL-SIR particle filter can effectively uncover true parameters when δ=0.2 and δ=0.25. It is shown as δ is getting larger, SL-SIR particle filter can perform better and obtain more accurate results. The percentage of mean's error is 1.4477253% and 0.345951% for $\delta = 0.2$ and $\delta = 0.25$, respectively.

Table 1

Panel A

Estimation performance for Base Case Parameters

	Model parameters				
	σ	$\delta\times100$	μ	H	$\lvert\hat{V}-V\rvert$
True Value	0.3	0.4	0.2	80.0	
Mean	0.289535	0.694017	0.317163	81.354324	6.934573
Median	0.280056	0.487926	0.278804	83.056711	5.832541
Standard deviation	0.073935	0.764775	0.242836	14.128389	5.731099
Mean's Error (%)	3.45	73.50	58.58	1.69	
Median's Error (%)	6.65	21.98	39.40	3.82	

Panel B

Estimation performance for different values of volatility

	Model parameters				
	σ	$\delta\times100$	μ	H	$\lvert\hat{V}-V\rvert$
True Value	0.15	0.4	0.2	80	
Mean	0.166208	0.442701	0.219092	73.356847	3.886265
Median	0.148554	0.460068	0.224714	79.690627	1.380885
Standard deviation	0.075934	0.364792	0.148109	21.769248	8.259415
Mean's Error (%)	10.81	10.68	9.55	8.30	
Median's Error (%)	0.96	15.02	12.36	0.39	
True Value	0.7	0.4	0.2	80	
Mean	0.681307	1.137764	0.871481	80.711376	11.692051
Median	0.665008	0.959083	0.839004	82.489340	11.567329
Standard deviation	0.109159	1.131262	0.453123	16.754879	7.567292
Mean's Error (%)	2.67	184.44	335.74	0.89	
Median's Error (%)	5.00	139.77	319.50	3.11	

Panel C

Estimation performance for different values of *H* (Barrier)

	Model parameters				
	σ	$\delta\times100$	μ	H	$\lvert\hat{V}-V\rvert$
True Value	0.3	0.4	0.2	50	
Mean	0.298248	0.461218	0.194039	44.409513	5.629562
Median	0.295778	0.432980	0.198395	47.273668	4.434005
Standard deviation	0.062164	0.445027	0.287267	22.896787	5.016560
Mean's Error (%)	0.58	15.30	2.98	11.18	
Median's Error (%)	1.41	8.24	0.801	5.45	
True Value	0.3	0.4	0.2	90	
Mean	0.292529	0.732502	0.405107	89.726744	5.528392
Median	0.280429	0.585835	0.384180	92.277832	4.587497
Standard deviation	0.057169	0.776436	0.230903	11.000337	4.968595
Mean's Error (%)	2.49	83.13	102.55	0.30	
Median's Error (%)	6.52	46.46	92.09	2.53	

Table 2

Estimation performance for different values of trading noise

	Model parameters				
	σ	$\delta \times 100$	μ	H	$\lvert \hat{V} - V \rvert$
True Value	0.3	0.4	0.2	80	
Mean	0.289535	0.694017	0.317163	81.354324	6.934573
Median	0.280056	0.487926	0.278804	83.056711	5.832541
Standard deviation	0.073935	0.764775	0.242836	14.128389	5.731099
Mean's Error (%)	3.49	73.50	58.58	1.69	
Median's Error (%)	6.65	21.98	39.40	3.82	
True Value	0.3	0.5	0.2	80	
Mean	0.289467	0.695682	0.317105	81.365318	6.936071
Median	0.280759	0.495246	0.279546	82.858325	5.781936
Standard deviation	0.074145	0.766175	0.242746	14.191345	5.780183
Mean's Error (%)	3.51	39.14	58.55	1.71	
Median's Error (%)	6.41	0.95	39.77	3.57	
True Value	0.3	0.6	0.2	80	
Mean	0.294242	0.775547	0.324763	80.180297	6.887912
Median	0.284981	0.596755	0.295044	81.792215	5.790525
Standard deviation	0.079514	0.822876	0.238358	14.387245	6.275649
Mean's Error (%)	1.92	29.26	62.38	0.23	
Median's Error (%)	5.01	0.54	47.52	2.24	
True Value	0.3	0.7	0.2	80	
Mean	0.292950	0.848283	0.325065	80.302363	6.741612
Median	0.285692	0.733541	0.291508	81.816070	5.836279
Standard deviation	0.072632	0.820944	0.235947	13.912915	5.689180
Mean's Error (%)	2.35	21.18	62.53	0.378	
Median's Error (%)	4.77	4.793	45.75	2.27	
True Value	0.3	0.8	0.2	80	
Mean	0.293906	0.903380	0.326171	80.141896	6.757292
Median	0.287266	0.846911	0.291558	81.506543	5.785801
Standard deviation	0.072455	0.830006	0.237592	13.916262	5.629737
Mean's Error (%)	2.03	12.92	63.09	0.18	
Median's Error (%)	4.24	5.86	45.78	1.88	
True Value	0.3	0.9	0.2	80	
Mean	0.296631	0.938760	0.328385	79.776027	6.957787
Median	0.287888	0.897966	0.296226	81.064581	5.832944
Standard deviation	0.076099	0.844382	0.238327	14.362693	5.990341
Mean's Error (%)	1.12	4.31	64.19	0.28	
Median's Error (%)	4.04	0.23	48.11	1.33	

Table 2 (continued)

Estimation performance for different values of trading noise

	Model parameters				
	σ	$\delta\times 100$	μ	H	$\left\lvert\hat{V}-V\right\rvert$
True Value	0.3	1	0.2	80	
Mean	0.301696	1.025207	0.330317	78.723039	7.133954
Median	0.290818	1.071254	0.307786	81.117809	5.830117
Standard deviation	0.077207	0.766523	0.245389	14.922607	6.414935
Mean's Error (%)	0.57	2.52	65.16	1. 60	
Median's Error (%)	3.06	7.13	53. 90	1.40	
True Value	0.3	1.1	0.2	80	
Mean	0.300864	1.099797	0.335889	79.236590	7.155484
Median	0.291033	1.104849	0.300734	80.249527	6.165905
Standard deviation	0.082791	0.968198	0.254974	14.656768	6.110792
Mean's Error (%)	0.29	0.02	67.94	0.95	
Median's Error (%)	2.99	0.44	50.37	0.31	
True Value	0.3	1.2	0.2	80	
Mean	0.301904	1.156085	0.334673	79.061060	7.258010
Median	0.290482	1.190590	0.303950	80.083199	6.108626
Standard deviation	0.082940	0.867500	0.241394	14.947478	6.420176
Mean's Error (%)	0.63	3.66	67.34	1.17	
Median's Error (%)	3.17	0.78	51.97	0.10	
True Value	0.3	1.3	0.2	80	
Mean	0.302884	1.267403	0.332679	78.853835	7.343398
Median	0.290809	1.307893	0.309171	80.286758	6.338478
Standard deviation	0.081462	0.922298	0.242305	14.970130	6.279127
Mean's Error (%)	0.96	2.51	66.34	1.43	
Median's Error (%)	3.06	0.61	54.59	0.36	
True Value	0.3	1.4	0.2	80	
Mean	0.303326	1.366716	0.337996	78.761607	7.374877
Median	0.293122	1.418037	0.309068	80.129119	6.220201
Standard deviation	0.081793	0.867711	0.246041	15.048518	6.297342
Mean's Error (%)	1.11	2.38	69.00	1.55	
Median's Error (%)	2.29	1.29	54.53	0.16	
True Value	0.3	1.5	0.2	80	
Mean	0.304000	1.448024	0.335804	78.676881	7.488319
Median	0.294014	1.514571	0.306532	80.284168	6.193693
Standard deviation	0.083616	0.865700	0.243837	15.233475	6.389743
Mean's Error (%)	1.33	3.47	67.90	1.65	
Median's Error (%)	2.00	0.97	53.27	0.36	

Table 2 (continued)

Estimation performance for different values of trading noise

	Model parameters				
	σ	$\delta \times 100$	μ	H	$\lvert \hat{V} - V \rvert$
True Value	0.3	1.6	0.2	80	
Mean	0.307599	1.528617	0.335275	78.066716	7.836115
Median	0.295355	1.610964	0.307866	79.985345	6.173877
Standard deviation	0.084953	0.783359	0.241210	15.990268	6.893824
Mean's Error (%)	2.53	4.46	67.64	2.42	
Median's Error (%)	1.55	0.69	53.93	0.02	
True Value	0.3	2	0.2	80	
Mean	0.307512	1.971045	0.339919	78.095754	7.947089
Median	0.294395	2.021401	0.312005	80.072815	6.099056
Standard deviation	0.089618	0.817818	0.247985	16.184400	7.016480
Mean's Error (%)	2.50	1.45	69.96	2.38	
Median's Error (%)	1.87	1.07	56.00	0.09	
True Value	0.3	2.5	0.2	80	
Mean	0.310238	2.508649	0.344919	77.577095	8.318177
Median	0.295412	2.541412	0.315167	80.521116	6.373411
Standard deviation	0.096634	0.756537	0.255675	17.381054	7.717444
Mean's Error (%)	3.41	0.35	72.46	3.03	
Median's Error (%)	1.53	1.66	57.58	0.65	

From Table 2, we find that the median of our estimation parameters are more close to the true value than mean's. It may because that the number of our simulation experiment, currently 500, is not large enough to overcome the problem of unstable parameter estimates due to the complexity of the Brockman and Turtle model. We believe that the estimate results can be better with larger samples size.

4.2 Empirical analysis

We conduct empirical analysis on the 35 companies that constitute the index of Dow Jones Industrial Average during 2006 to 2009. Since one expects severe trading noise for the least liquid companies, for the purpose of comparison, we also select the other 35 companies which are least liquid in 2008 to be the comparison group. We use the illiquidity ratio proposed by Amihud (2002) as a proxy for the price impact of a trade, and we select 35 stocks with the largest illiquidity ratio in 2008. The firm-specific illiquidity ratio $AMIHUD_{i,t}$ for stock *i* in year *t* is given by the average daily ratio of the absolute return of a stock to its dollar trading volume over a year.

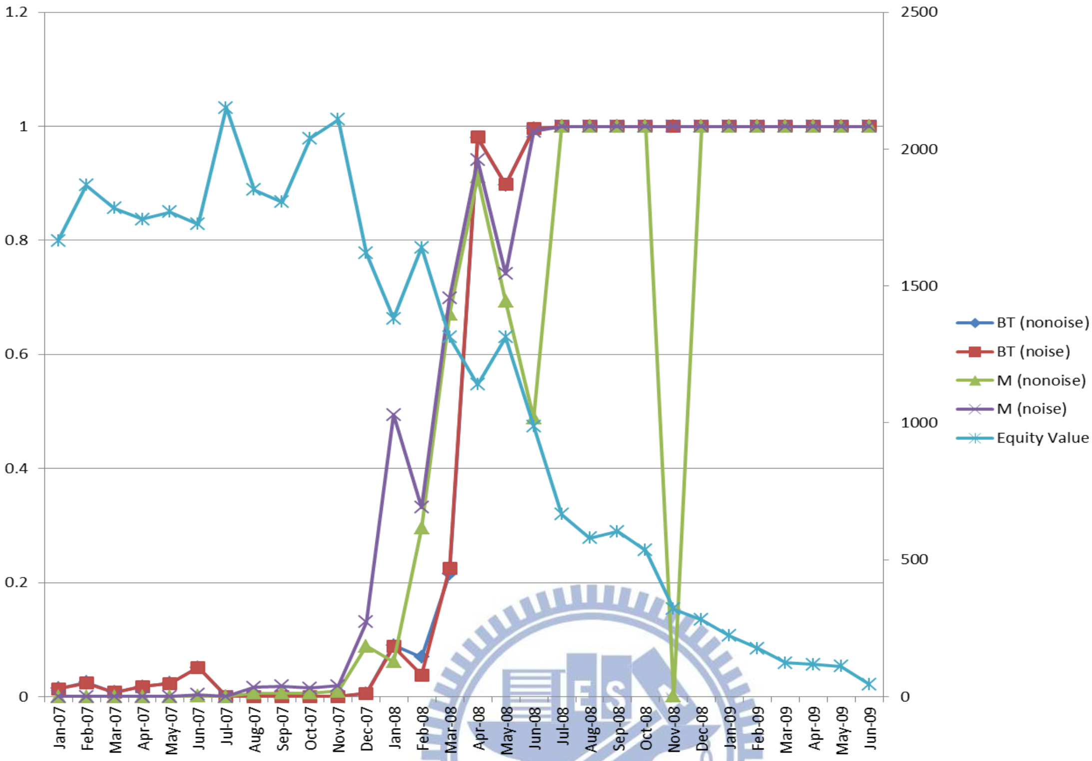
$$AMIHUD_{i,t} = \frac{1}{D_{i,t}} \sum_{d=1}^{D_{i,t}} \frac{|r_{i,d,t}|}{V_{i,d,t}}$$

where $r_{i,d,t}$ and $V_{i,d,t}$ are return and volume (measured in million dollars), respectively, for the stock on day *d* in year *t*, and $D_{i,t}$ is the number of observations for the stock in year *t*. Our data samples consist of daily equity values of these firms over 2006 to 2009. The initial maturity of debt is set to 10 years. To implement the Merton model, *T* is usually taken to be one year and *L* is measured as the firm's book value of short-term debt, plus one half of its long-term debt (see Bharath and Shumway, 2008). Following Duffiee, Saita and Wang (2007), we measure the short-term debt as the maximum of "Debt in current liabilities" and "Total current liabilities". Accordingly, the liability measure *L* is equal to short-term debt plus one half of the long-term debt. The three month T-bill rate from the Federal Reserve website is chosen as the risk-free rate. The resulting value is our proxy for the face value of the debt in our pricing model. We set $h = 1/251$ to reflect the use of daily equity values. And we run the estimation using the 5000-particle SL-SIR filter.

Figure 1 demonstrates the default probabilities of the Merton and the BT models using General Motor (GM) as an example. We use the monthly data to illustrate that considering trading noise or not does influence the implied default probability. We also find that both the Merton and the BT models are feasible in estimating default probability. Although we could not find the root cause of an unreasonable default probability estimate of the Merton model in

2008, this kind of results are rare in our estimation.

Figure 1. Monthly implied default probabilities of GM from 2007 to 2009



We next report the differences of implied default probability and volatility estimates in Tables 3 and 4. In Table 3, BT(noise) represents the BT model in the [presence of trading noise while BT(nonoise) does not consider trading noise. Similarly, M(noise) incorporates trading noise while M(nonoise) does not. Table 3 reports the absolute difference of default probabilities. Because implied default probabilities are low (many of them are close to zero) before financial tsunami, we present the results by percentiles of the absolute difference. It appears that the probabilities are impacted by different pricing models as well as different liquidity conditions in terms of groups of Dow-Jones and illiquidity stocks. The estimates of volatility also have observable difference between these two groups. It means that trading noise indeed influence the estimation of implied default probability and volatility.

Table 3

Panel A.
Absolute Difference of Default Probability

Dow Jones Components	BT(nonoise) -BT(noise)	M(nonoise) -M(noise)	BT(nonoise) -M(nonoise)	BT(noise) -M(noise)
average	0.031864	0.013354	0.264293	0.260407
25 th percentile	0.000002	0.000000	0.000199	0.000091
Median	0.000796	0.000029	0.075084	0.051885
75 th percentile	0.013679	0.010548	0.536559	0.547368
90 th percentile	0.071517	0.040340	0.808958	0.813308
95 th percentile	0.133160	0.065176	0.910398	0.926545
Illiquid Stocks	**BT(nonoise) -BT(noise)**	**M(nonoise) -M(noise)**	**BT(nonoise) -M(nonoise)**	**BT(noise) -M(noise)**
average	0.037535	0.212264	0.329232	0.197663
25 th percentile	0.000000	0.001134	0.009735	0.005404
Median	0.000113	0.039172	0.116780	0.076523
75 th percentile	0.013074	0.292674	0.645902	0.307275
90 th percentile	0.092606	0.821428	0.989598	0.592290
95 th percentile	0.249004	0.926299	0.999351	0.825591

Panel B.
Absolute Difference of Volatility

Dow Jones Components	BT(nonoise) -BT(noise)	M(nonoise) -M(noise)	BT(nonoise) -M(nonoise)	BT(noise) -M(noise)
average	0.039642	0.039686	0.177541	0.151296
25 th percentile	0.001134	0.000229	0.012484	0.009888
Median	0.006527	0.017756	0.116565	0.090000
75 th percentile	0.025453	0.054508	0.280177	0.225390
90 th percentile	0.062772	0.104063	0.388257	0.335491
95 th percentile	0.109684	0.158432	0.544140	0.465794
Illiquid Stocks	**BT(nonoise) -BT(noise)**	**M(nonoise) -M(noise)**	**BT(nonoise) -M(nonoise)**	**BT(noise) -M(noise)**
average	0.262956	0.118848	0.262843	0.174705
25 th percentile	0.026349	0.000000	0.051156	0.013393
Median	0.121739	0.003040	0.197470	0.072725
75 th percentile	0.388565	0.106766	0.437946	0.180761
90 th percentile	0.722370	0.456727	0.588492	0.386295
95 th percentile	0.966690	0.570590	0.680860	0.694316

Table 4

Panel A.

Difference of Default Probability

Dow Jones Components	BT(nonoise) -BT(noise)	M(nonoise) -M(noise)	BT(nonoise) -M(nonoise)	BT(noise) -M(noise)
average	0.022275	-0.004801	0.260541	0.233465
25 th percentile	-0.000059	-0.000364	0.000066	0.000007
Median	0.000000	0.000000	0.071311	0.039093
75 th percentile	0.002244	0.000001	0.536007	0.518805
90 th percentile	0.054480	0.009397	0.808869	0.802970
95 th percentile	0.109080	0.028934	0.910298	0.916753
Illiquid Stocks	BT(nonoise) -BT(noise)	M(nonoise) -M(noise)	BT(nonoise) -M(nonoise)	BT(noise) -M(noise)
average	-0.006302	-0.153317	0.310956	0.163942
25 th percentile	-0.005831	-0.221848	0.001394	0.000611
Median	-0.000009	-0.008673	0.099461	0.057745
75 th percentile	0.000000	0.000000	0.637916	0.286693
90 th percentile	0.000000	0.062442	0.987768	0.565846
95 th percentile	0.026856	0.184319	0.999316	0.770740

Panel B.

Difference of Volatility

Dow Jones Components	BT(nonoise) -BT(noise)	M(nonoise) -M(noise)	BT(nonoise) -M(nonoise)	BT(noise) -M(noise)
average	0.004440	0.033661	-0.173105	0.350527
25 th percentile	-0.000739	0.000014	-0.278849	0.192099
Median	0.001851	0.014228	-0.116222	0.258953
75 th percentile	0.017170	0.051748	-0.001309	0.390042
90 th percentile	0.038537	0.103409	0.006132	0.541114
95 th percentile	0.065636	0.158292	0.009722	1.162767
Illiquid Stocks	BT(nonoise) -BT(noise)	M(nonoise) -M(noise)	BT(nonoise) -M(nonoise)	BT(noise) -M(noise)
average	0.191549	0.118187	0.141635	0.068273
25 th percentile	0.017597	0.000000	0.007219	0.000000
Median	0.110124	0.002524	0.140785	0.028836
75 th percentile	0.339242	0.106766	0.345676	0.124270
90 th percentile	0.642578	0.456727	0.541707	0.260886
95 th percentile	0.843289	0.570590	0.603962	0.391795

In Table 4, we present the difference among models to explore the relationship among various models. The results show that when we consider trading noise, the default probabilities in general increase. And the implied default probabilities of BT model are higher than those of the Merton model. The estimates of volatility are smaller when we incorporate trading noise, especially for those illiquid stocks with larger trading noise.

Table 5

Estimation results of trading noise ($\delta \times 100$)

Data	Dow Jones	Dow Jones	Illiquid Stocks	Illiquid Stocks
Model	BT	Merton	BT	Merton
2007.Q1~2009.Q4				
average	0.620997	0.512007	6.164850	2.763445
average	0.000034	0.001515	1.140957	0.000286
25 th percentile	0.236182	0.347953	2.226918	1.319713
Median	0.563633	0.872474	3.936719	2.798791
75 th percentile	1.080099	1.309204	5.794606	4.223958
90 th percentile	1.499628	1.701847	8.662014	6.100530
2007.Q1~2008.Q2				
average	0.187281	0.194590	4.440171	0.486229
25 th percentile	0.000001	0.000001	0.706262	0.000001
Median	0.078462	0.023637	1.160575	0.000650
75 th percentile	0.324260	0.349305	1.774399	0.738272
90 th percentile	0.499112	0.555216	2.890811	1.768415
95 th percentile	0.564012	0.612619	4.264387	2.326484
2008.Q3~2009.Q4				
average	1.048487	0.824867	7.807008	4.942600
25 th percentile	0.011285	0.334262	2.509572	2.001414
Median	0.478274	0.824949	3.575181	2.671950
75 th percentile	0.947227	1.183250	5.044821	3.828606
90 th percentile	1.340457	1.699006	7.263062	5.953734
95 th percentile	1.841368	2.010067	9.798447	8.062973

Table 5 shows that no matter what pricing model we use, there exists substantial difference in trading noise estimates between the Dow Jones stocks and the low liquidity stocks. Compare firms, less-liquid stocks, consistently have larger δ than Dow Jones stocks. In the subperiod results in Panels B and C, we can also observe that the trading noise is smaller during the period 2007.Q1 to 2008 Q2. In contrast, trading noise estimates are much larger during the period of global financial tsunami.

5. Conclusion

This paper investigates the performance of the SL-SIR particle filter, proposed by Duan and Fulop (2009), on the popular structural models with default boundaries – the Brockman and Turtle model. Compared with the Merton model, the structural models with default boundaries introduce additional complexities in parameter estimation. In our Monte Carlo simulation analysis, it appears that the parameter estimates of the Brockman and Turtle model are not as stable as the simple Merton model. Fortunately, the problems are less severe and SL-SIR particle filter has good performance when trading noise is getting larger. The empirical analysis shows that trading noises indeed impact the calculation of default probabilities when one applies the structural form models. Our results suggest that one cannot ignore trading noise when one attempts to calibrate credit risk of a firm and to measure its default probability, especially when the impact of trading noise is substantial during the global financial crisis.

In this study, we attempted to investigate if the Brockman and Turtle model can outperform the Merton model when we introduce trading noise into structural credit risk modeling. Due to the time constraint, we have not yet tested the capabilities of bankruptcy prediction accuracy of these two models. In the future, the difference of these two credit risk models in pricing bonds and bankruptcy predictions can be further explored.

References

Amihud, Y., 2002, Illiquidity and stock returns: Cross-section and time-series effects. Journal of Financial Markets 5, 31-56.

Bharath, S.T., Shumway T., 2008. Forecasting default with the Merton distance to default model. Review of Financial Studies, 21, 1339-1369.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637-659.

Black, F., Cox, J. 1976, Valuing corporate securities: Some effects of bond indenture provisions, The Journal of Finance, Vol. 31, No. 2, pp. 351-367.

Brockman, P., Turtle, H., 2003. A barrier option framework for corporate security valuation. Journal of Financial Economics 67, 511-529.

Doucet, A., Johansen, A.M., 2008. A tutorial on particle filtering and smoothing: Fifteen years later.

Duan, J.C., Gauthier, G., Simonato, J.G., 2004. On the equivalence of the KMV and maximum likelihood methods for structural credit risk models. Working paper, University of Toronto.

Duan, J.c., Fulop, A. 2009. Estimating the structural credit risk model when equity prices are contaminated by trading noises. Journal of Econometrics 150, 288-296

Fulop, A., 2011. Filtering methods. Handbook. ESSEC Business School, Paris.

Duffie, D., Saita, L., Wang, K., 2007. Multi-period corporate default prediction with stochastic covariates. Journal of Financial Economics 83, 635-665.

Goodrich, M.A., 2006. A tutorial on simple particle filters.

Leland, H. E., 1994. Corporate Debt Value, Bond Covenants and Optimal Capital Structure, The Journal of Finance, Vol. 49, No. 4, 1213-1252.

Lopes, H.F., Tsay, R.S., 2011. Particle filters and Bayesian inference in financial econometrics. University of Chicago Booth School of Business.

Merton, R.C., 1974. On the pricing of corporate debt: The risk structure of interest rates. Journal of Finance 29, 449-470.

Merton, R. C., 1976, Option pricing when underlying stock return are discontinuous. Journal of Financial Economics 3, pp. 125-144.

Pitt, M., 2002. Smooth particle filters likelihood evaluation and maximization. Working paper. University of Warwick.

Pitt, M., Shephard, N., 1999. Filtering via simulation: Auxiliary particle filter. Journal of the American Statistical Association 94, 590-599.